Optical Stochastic Cooling: Theory and Design

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U.S. DEPARTMENT OF ENERGY
Fermi Research Alliance LLC

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- Measurements of maximum cooling rates
- Transverse OSC in anticooling mode
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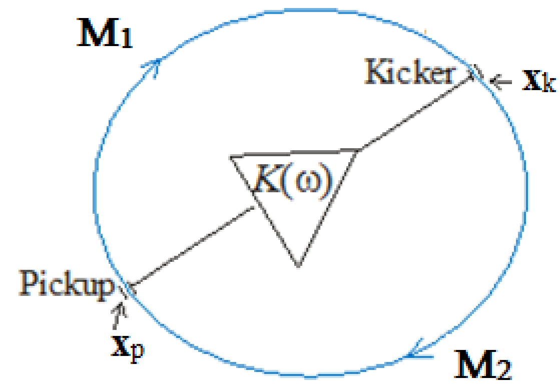
Additional details of OSC theory may be found in
“THE DESIGN OF OPTICAL STOCHASTIC COOLING FOR IOTA”,
JINST 16 T05002 (95 pages)

Small Amplitude Cooling Rates for 3D OSC

One turn map



- OSC maps

$$\mathbf{x}_p = \mathbf{M}_2\mathbf{x}_k \;, \quad \mathbf{x}_k = \mathbf{M}_1\mathbf{x}_p + \mathbf{M}_c\mathbf{x}_p \quad (1)$$

where $\mathbf{M}_c$ is coupling matrix

- For the chicane located purely in horizontal plane

$$\frac{\delta p}{p} = -\xi k_0 \left((\mathbf{M}_1)_{51} x + (\mathbf{M}_1)_{52} \theta_x + (\mathbf{M}_1)_{56} \frac{\Delta p}{p} \right)$$

$$\Rightarrow \quad \mathbf{M}_c = -k_0\xi \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ (\mathbf{M}_1)_{51} & (\mathbf{M}_1)_{52} & 0 & 0 & 0 & (\mathbf{M}_1)_{56} \end{bmatrix}$$

- Combining Eqs. (1) we obtain one turn map related to kicker position

$$\mathbf{x}_k = \left(\mathbf{M}_1 + \mathbf{M}_c\right)\mathbf{M}_2\mathbf{x}_k$$

$$\Rightarrow \quad \boxed{\mathbf{M}_{tot} = \mathbf{M} + \mathbf{M}_c\mathbf{M}_2 \;, \quad \mathbf{M} = \mathbf{M}_1\mathbf{M}_2}$$

where $\mathbf{M}$ is one-turn matrix without OSC

Small Amplitude Cooling Rates for 3D OSC (2)

Sum of cooling rates

- Using the symplectic perturbation theory and the rate-sum theorem (see backup slides) one finds:

$$\sum_{i=1}^{3}\lambda_i = \frac{1}{2}\mathrm{Tr}\left(\mathbf{M}_c\mathbf{U}\mathbf{M}_1{}^T\mathbf{U}\right) = \frac{1}{2}\xi k_0(\mathbf{M}_1)_{56}$$

Cooling rates sum does not depend on eigen-vectors. Only on $\mathbf{M}_1$ & $\mathbf{M}_c(\mathbf{M}_1)$

Longitudinal and transverse cooling rates

- Assume an absence of betatron motion but non-zero dispersion
  - Accounting that $x = D\Delta p/p, \quad \theta_x = D'\Delta p/p$

    one obtains: $$\frac{\delta p}{p} = k_0\xi\,\Delta s = k_0\xi\left((\mathbf{M}_1)_{51}D + (\mathbf{M}_1)_{52}D' + (\mathbf{M}_1)_{56}\right)\frac{\Delta p}{p}$$

⇨ $$\lambda_s = \frac{1}{2}k_0\xi\left((\mathbf{M}_1)_{51}D + (\mathbf{M}_1)_{52}D' + (\mathbf{M}_1)_{56}\right)$$

- Consequently, the sum of transverse cooling rates is:

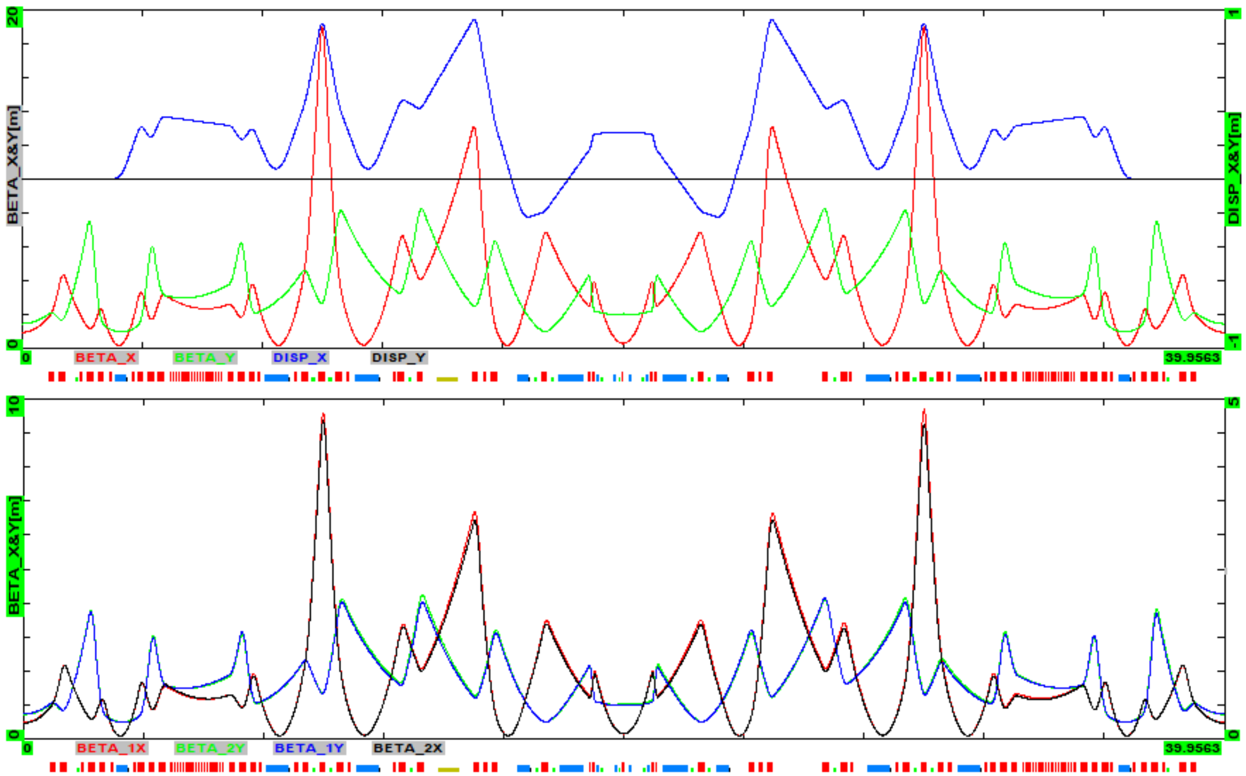
$$\lambda_1 + \lambda_2 = -\frac{k_0\xi}{2}\left((\mathbf{M}_1)_{51}D + (\mathbf{M}_1)_{52}D'\right)$$

Small Amplitude Cooling Rates for 3D OSC (3)

Transverse cooling rates at coupling resonance

- At coupling resonance each plan 4D betas are equal for both eigen vectors

$$\lambda_1 = \lambda_2 = -\frac{k_0 \xi}{4}\left((\mathbf{M}_1)_{51} D + (\mathbf{M}_1)_{52} D' \right)$$



Beta-function for uncoupled and coupled IOTA optics

- Coupling is produced by one skew quad located at zero dispersion after good machine decoupling was achieved => no vertical dispersion

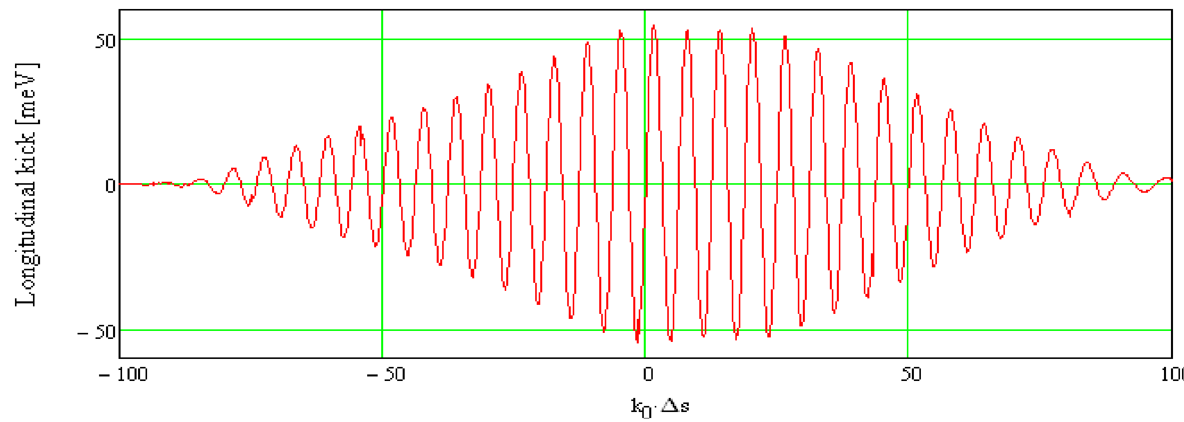
Cooling Rates for Particles with Large Amplitudes

Dimensionless amplitude for longitudinal motion

- The actual cooling force depends on Δs nonlinearly
- In the first approximation the longitudinal kick is

$$\frac{\delta p}{p} = -\xi k_0 \Delta s$$

$$\Rightarrow \quad \frac{\delta p}{p} = -\xi \sin\left(k_0 \Delta s\right)$$



Dependence of long. kick on $k_0\Delta s$ for IOTA passive OSC

- For the longitudinal cooling (no betatron motion)

$$a_s \equiv k_0 \Delta s_{\max} = k_0 \left((\mathbf{M}_1)_{51} D + (\mathbf{M}_1)_{52} D' + (\mathbf{M}_1)_{56} \right) \left. \frac{\Delta p}{p} \right|_{\max}$$

- Summing all degrees of freedom we have

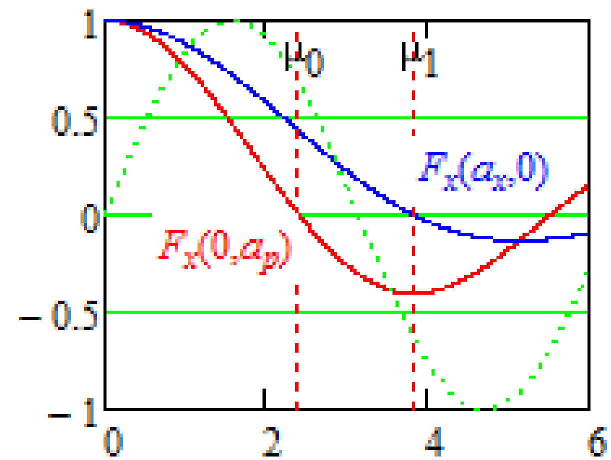
$$k_0 \delta s = a_1 \cos\psi_1 + a_2 \cos\psi_2 + a_s \cos\psi_s$$

- Consequently, the kick dependence on time is

$$\frac{\delta p}{p} = -\xi \sin\left(a_1 \cos(\omega_1 t + \psi_1) + a_2 \cos(\omega_2 t + \psi_2) + a_s \cos(\omega_s t) \right)$$

Cooling Rates for Particles with Large Amplitudes(2)

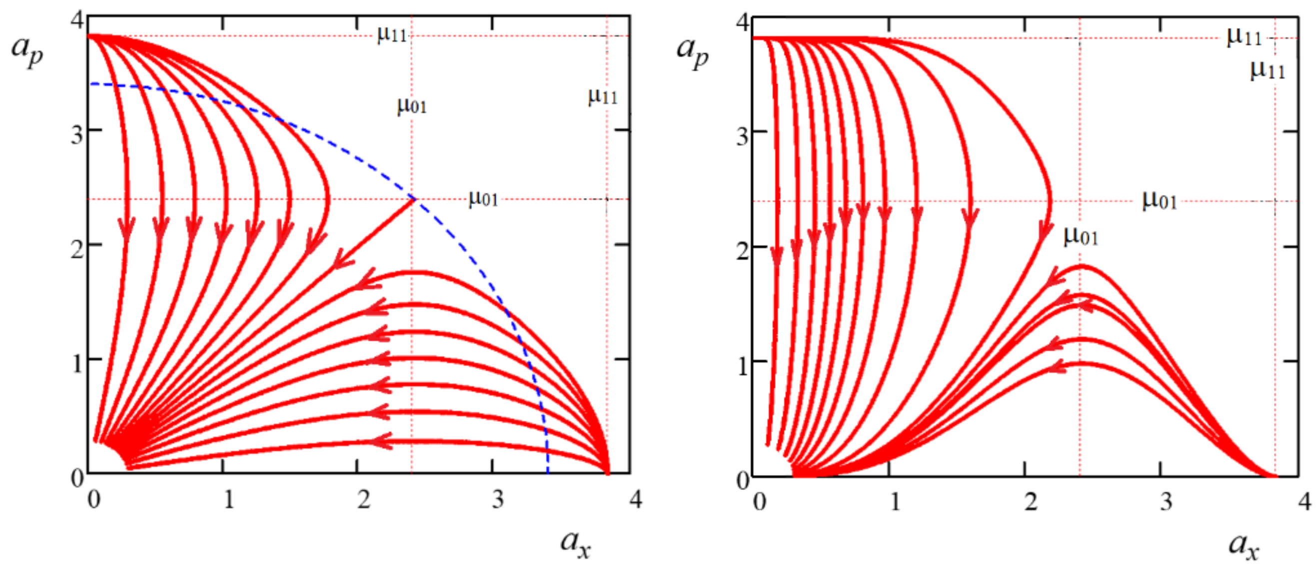
- Performing averaging over betatron and synchrotron oscillations one obtains



$$\lambda_i(a_1,a_2,a_s)=\left(\lambda_i\frac{2}{a_i}J_1(a_i)\right)J_0(a_j)J_0(a_k),$$

$$i=a_1,a_2,a_s,\quad i\neq j\neq k$$

- Cooling may trap large amplitude particles at intermediate amplitudes
- Trap conditions: $a_i\lambda_i=0$ & $d(\lambda_i a_i)/da_i>0$ (for each i)



Amplitude trajectories in the course of OSC cooling;
top – $\lambda_x/\lambda_p=1$;
bottom – $\lambda_x/\lambda_p=0.3$. Blue dashed circle radius - $\sqrt{2}\mu_{01}$.

Beam Energy Calibration – Importance

- OSC is not very sensitive to the beam energy
- However, in the single electron OSC studies the sensitivity of SPAD used for photon registration is greatly reduced if energy is below 100 MeV

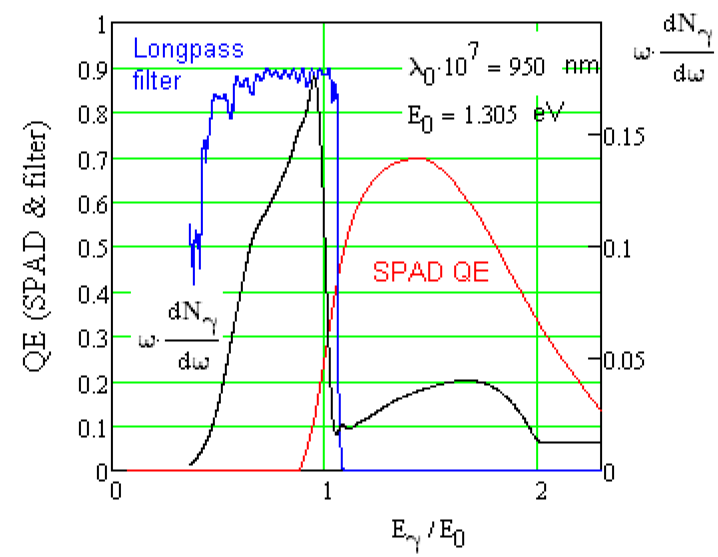
$$\lambda_0 = \frac{\lambda_{und}}{2\gamma^2}\left(1+\frac{K_u^2}{2}\right)$$

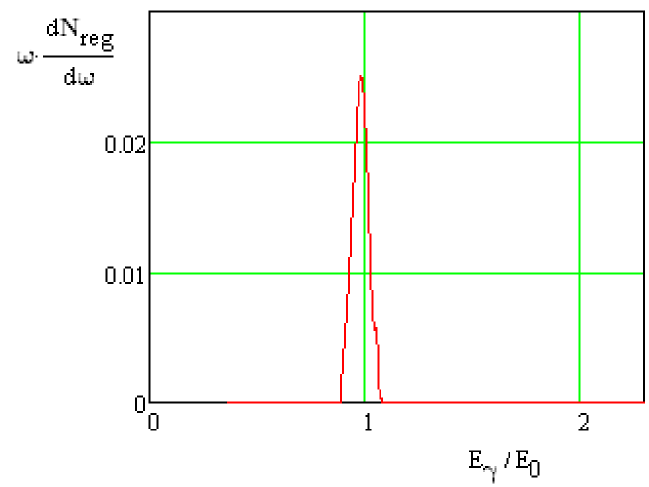
10% energy reduction eliminates all photons in SPAD

- RF voltage calibration is based on beam deceleration due to SR

$$\Delta E = 4\pi e^2 \gamma^4 / (3R_{dip})$$

- 3% energy error generates 12 % error in RF voltage calibration



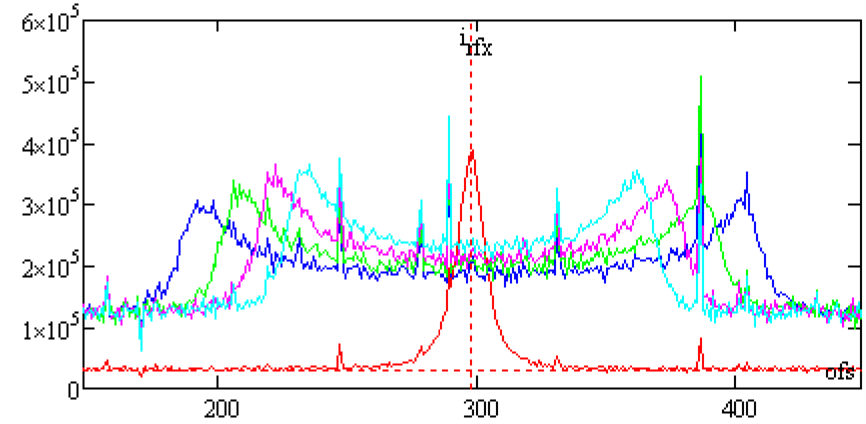


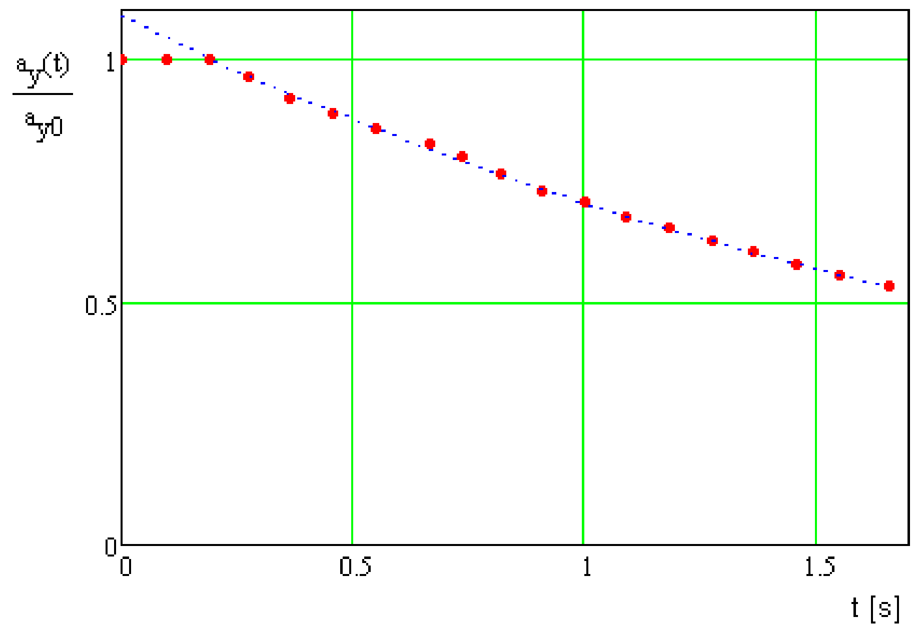
Beam Energy Calibration – Method and Result

- We measured damping rate of vertical betatron oscillations
  - Depends only on the bending radius and beam energy

$$\lambda_y = \frac{1}{3} c r_e \gamma^3 \oint \frac{1}{\rho^2} \frac{ds}{C}$$

  - Damping rate - 2.3 s (~1% accuracy)
  - ⇨ Accuracy of energy measurements (1/3)% + uncertainty of 1/ρ (~1%) *i.e.* ~1%

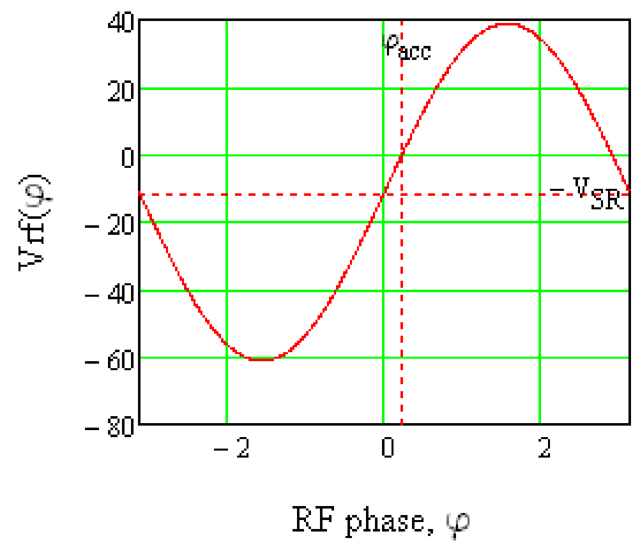




- The beam energy before energy change – 96.9 MeV
- The beam energy after energy change – 101.8 MeV
  - This energy used in all final measurements

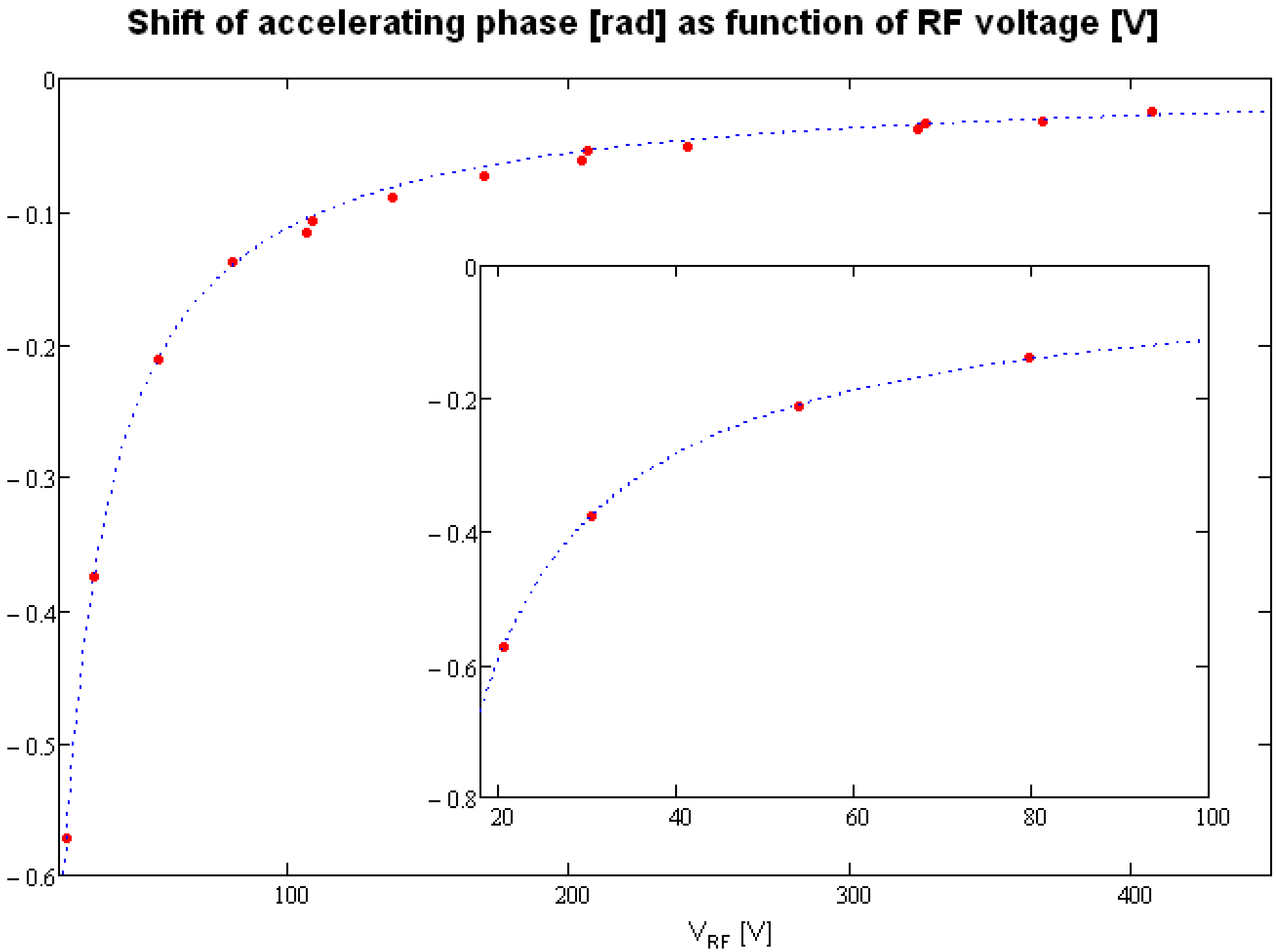
RF Voltage Calibration - Method

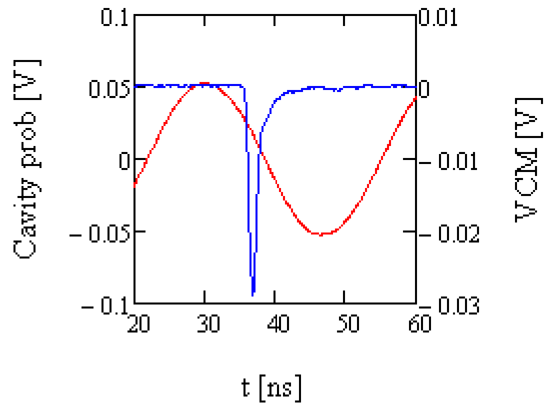
- First accurate RV voltage calibration was done in IOTA Run II
  - Based on Synchrotron frequency. That implied that we know α
- Calibration of the OSC run had two goals:
  (1) Verify the Run II RF voltage calibration
  (2) Measure α (momentum compaction)
  - Computation of α is problematic in the low emittance OSC lattice
- The calibration was based on measuring the beam phase shift relative the RF phase observed at the cavity probe ($\Delta E_{SR} = V_{RF}\sin(\varphi_{acc})$)
  - It uses a known energy loss due to SR ($\Delta E = 4\pi e^2\gamma^4/(3R_{dip})$), and uses energy calibration (see above).
- RF prob voltage and wall current monitor were acquired for different attenuator settings with digital scope (Kermit)
  - Offline analysis yielded $\varphi_{acc}(V_{RF})$

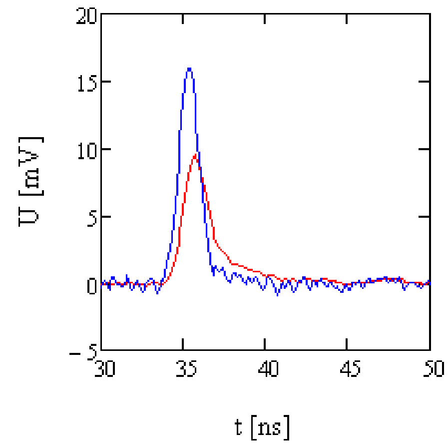


RF Voltage Calibration – Analysis

- It was important to account dispersion in cable

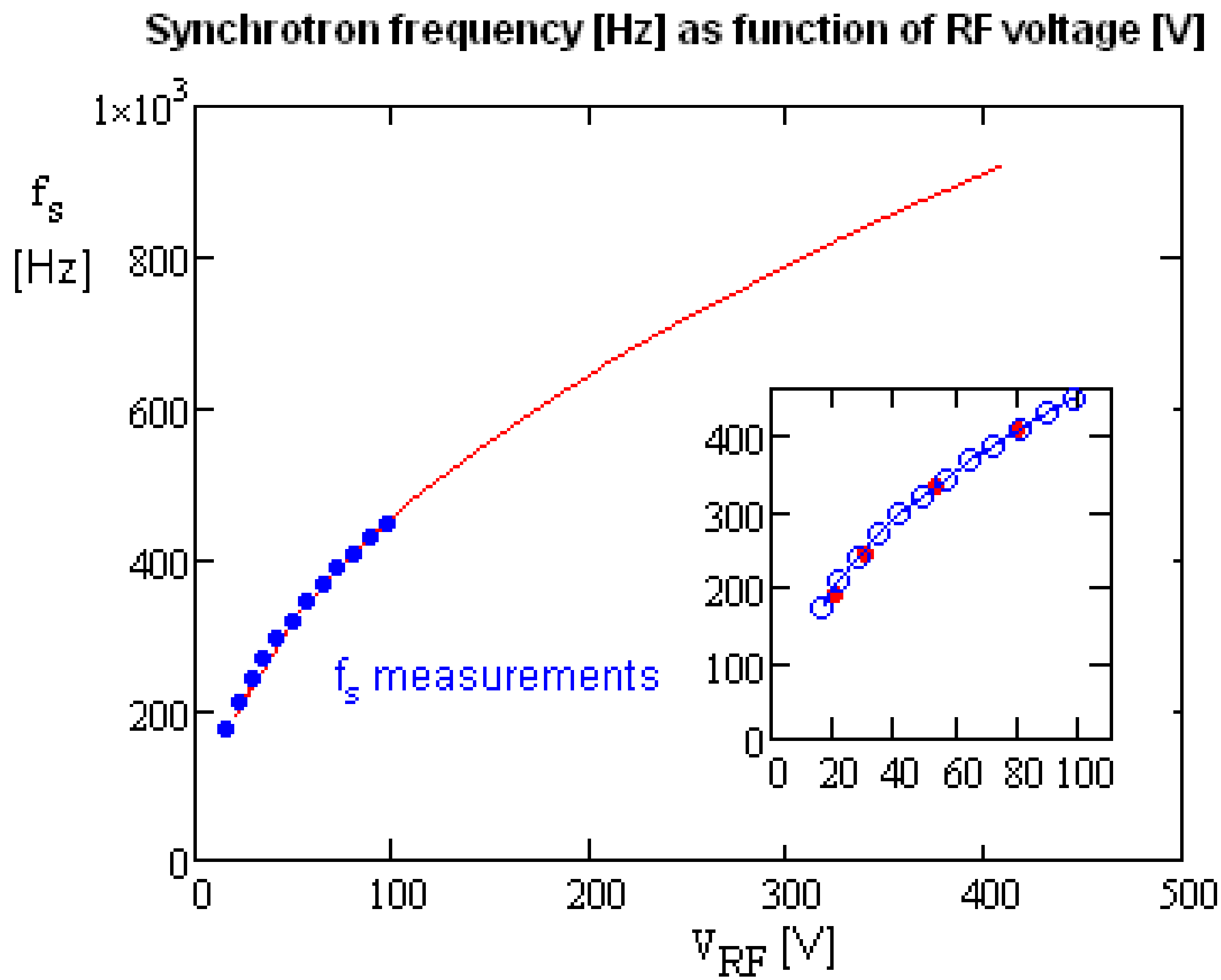






- Calibration result: $V_{RF} = \kappa_{RF} V_{prob}$; $\kappa_{RF} = 7000$
  - Run II calibration $K_{RF} = 7250$.

Measurements of Momentum Compaction

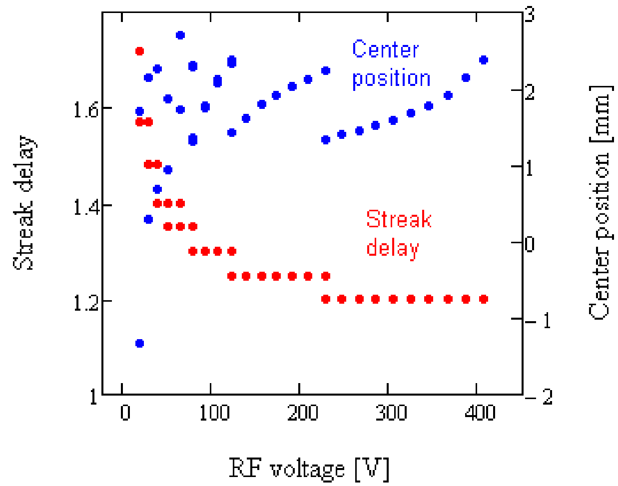


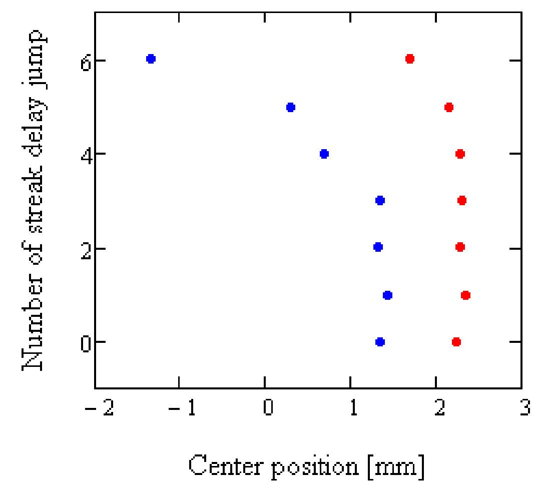
- The only fitting parameter in the plot is the momentum compaction. The measured value is 15% above of computed with optics codes

$$\alpha = 0.00490 \Rightarrow \alpha = 0.00564$$

Calibration of Streak Camera

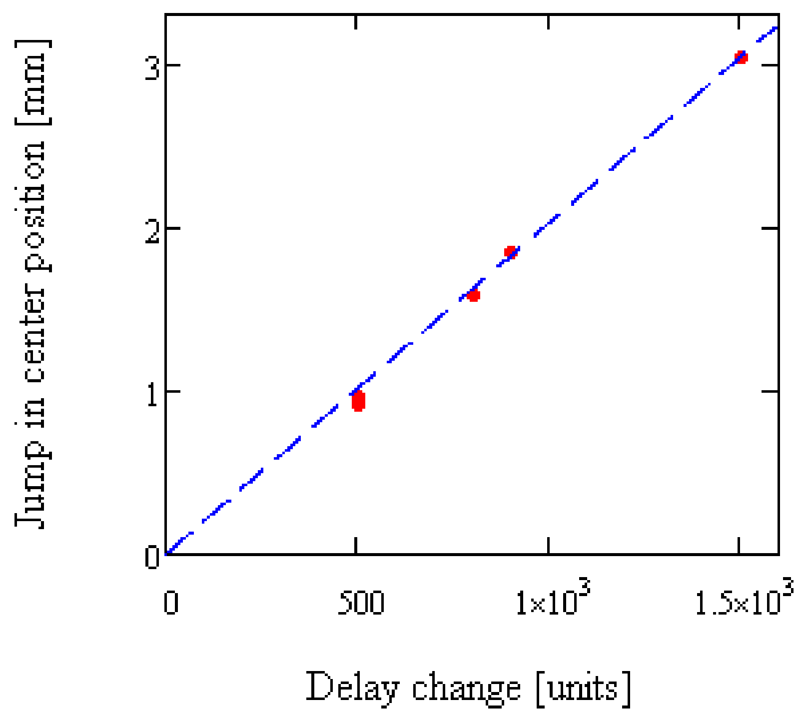
- Calibration done similar to RF voltage calibration
- Change of RF voltage results in a change in bunch phase and, consequently, beam displacement on the CCD screen
  - When the beam is approaching the screen edge a delay in the camera RF (11th harmonic) moves it to other side
    - There is considerable differential nonlinearity (±5%). Origin unknown





- Streak delay calibration(259fs =1step) was performed at May 24/2021

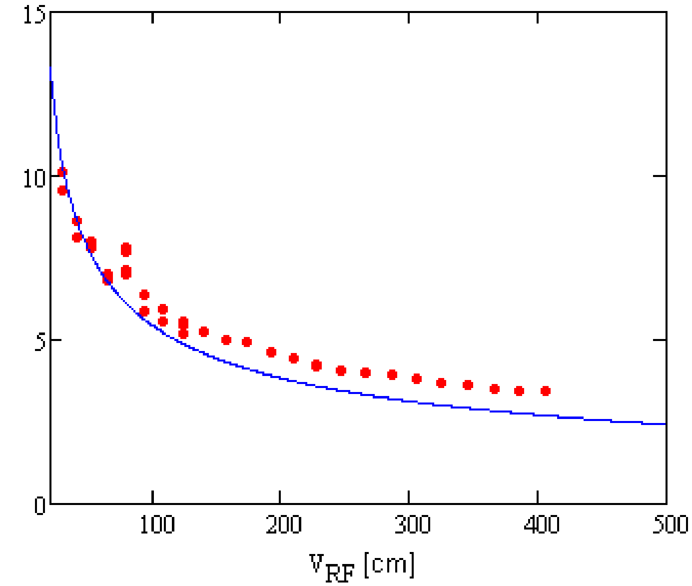
⇨ Calibration of streak camera
1mm(streak)=8.52cm(bunch length)



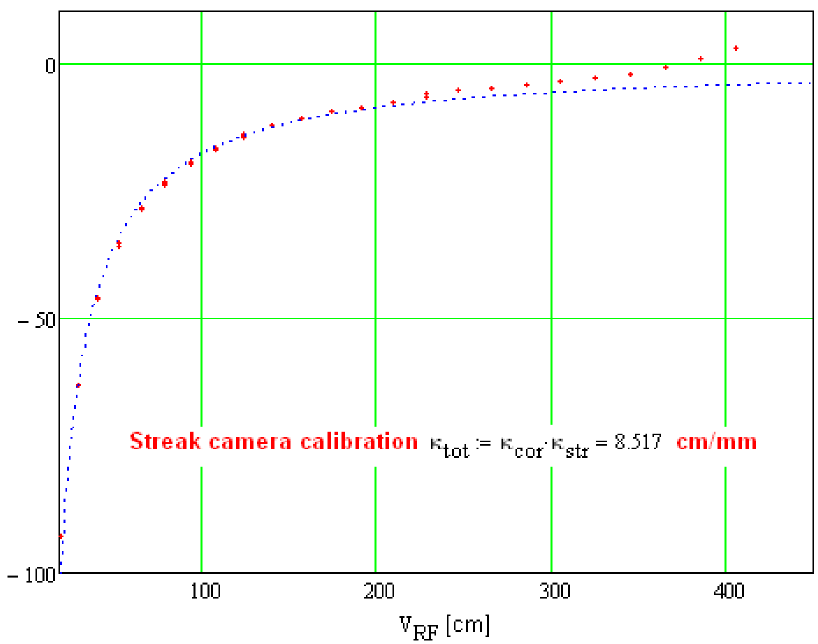
Differential Nonlinearity of Streak Camera Measurements

- There is a decent coincidence in the measured and computed bunch lengths => no big errors in the calibration
- However, the measured (with streak camera) and computed bunch displacements due to RF voltage change are much worse than for the described above WCM measurements acquired with the digital scope
  - ⇨ Considerable differential nonlinearity
    - ♦ Existing data do not allow us to find an origin

Comparison of predicted and measured rms bunch lengths [cm] as functions of RF voltage [V]



Bunch longitudinal displacement [cm] due to deceleration by SR as function of RF voltage [V]



Differential Nonlinearity of Streak Camera

- OSC measurements in antidamping mode show that the maximum non-linearity is at the edges
- The following correction makes the distribution symmetric

$$\begin{cases} x_n = n + \kappa (n - n_c)^5 \\ \rho_n = \dfrac{\rho_{n0}}{1 + 5\kappa (n - n_c)^4} \end{cases}$$

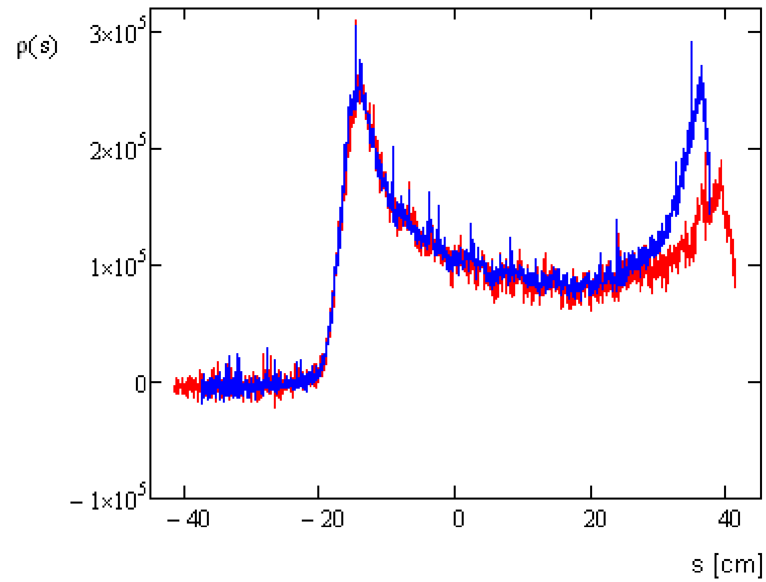
n – pixel number of streak camera

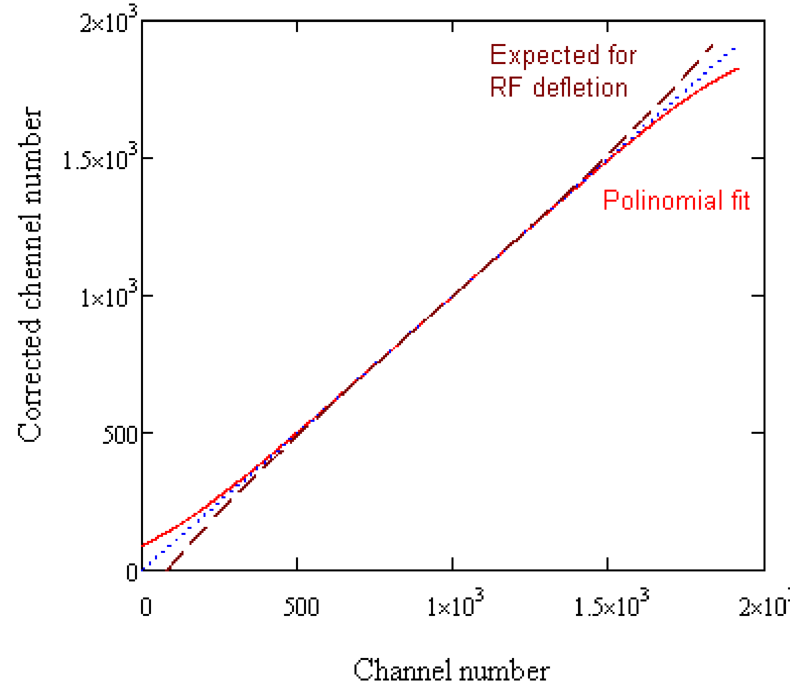
$n_c = 959,\ n \in [0, 2n_c]$

$\kappa = -1.1 \cdot 10^{-13}$

- Questions
  - Why 5th order polynomial (3rd did not work good)
  - Why the sign of correction is different compared to what should be expected for harmonic deflection by 11th harmonic
  - What else contributed to above described problems

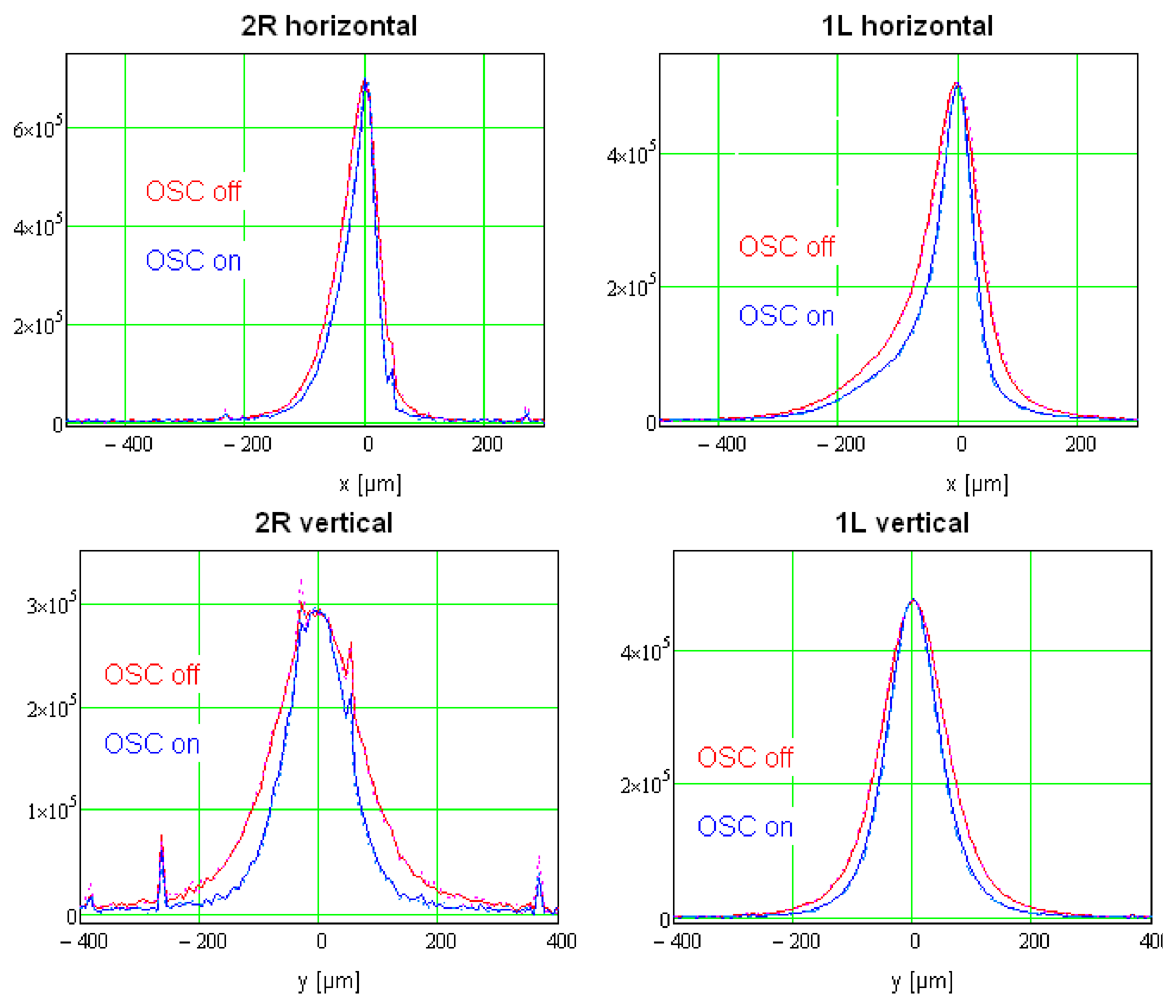
Measured and corrected longitudinal distributions in OSC antidamping mode





Resolution of Sync-light Monitors

- In the OSC measurements we used 2 SL: 2R & 1L
- 1L has larger β_h but is not upgraded for resolution
- For both monitors hor. sizes are dominated by diffraction and are not usable (my be except OSC undamping)
  - Obvious asymmetry
  - Better horizontal resolution of 2R is visible but still insufficient to resolve a smaller beam size (resolution estimate yields twice smaller FWHM)
- 2R vertical sizes are the only trustable values



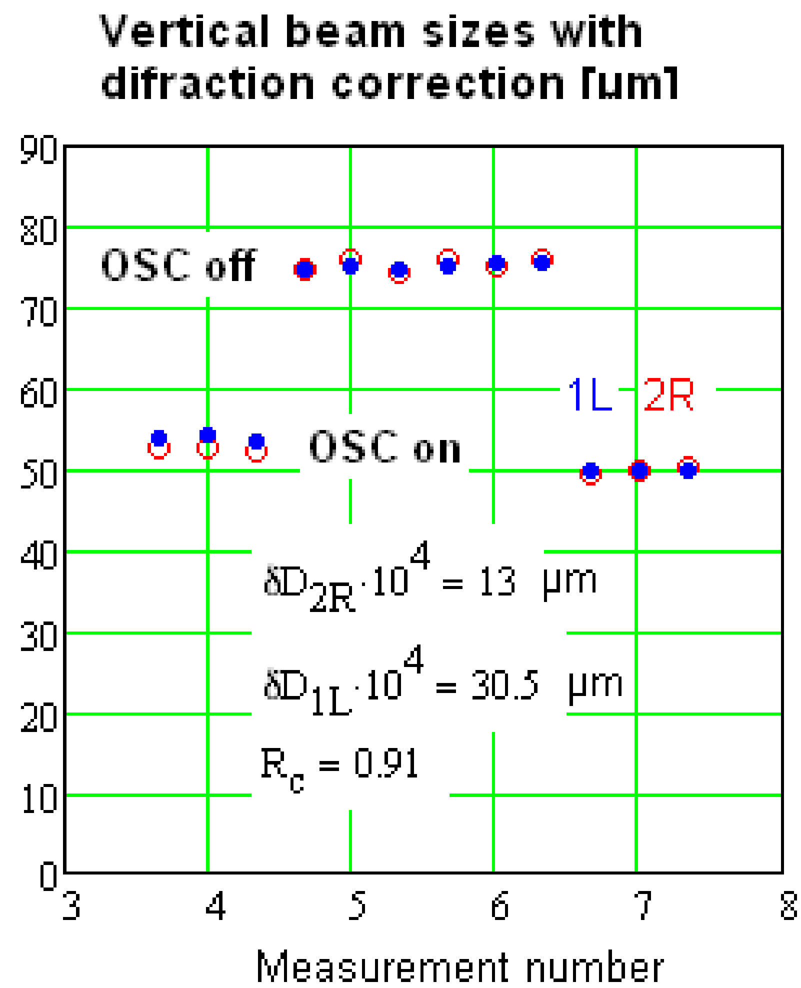
Resolution of Sync-light Monitors (2)

- Diffraction in 2R monitor does not affect its resolution significantly and the theoretical value of 13 μm is used
- It was impossible to match 1L & 2R vertical sizes without changing the ratio of vertical betas by 0.91^2 times relative to theoretical values

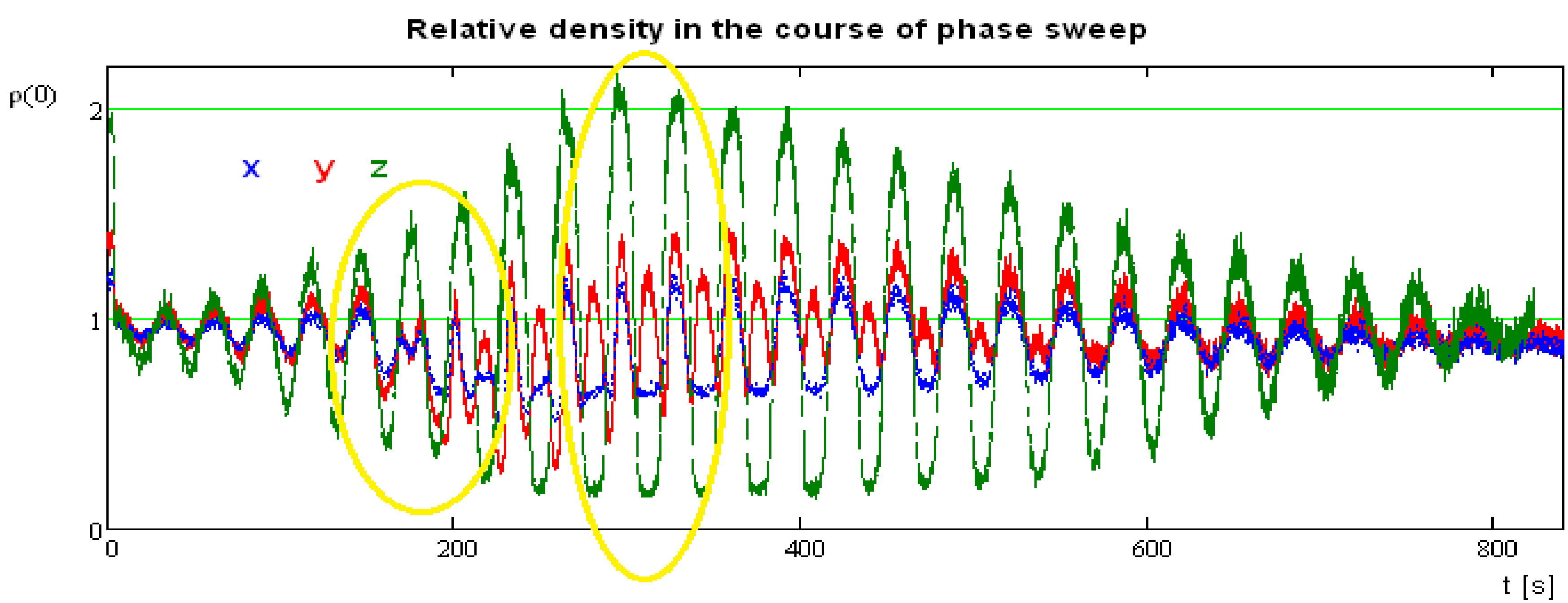
$$\sqrt{\frac{\beta_{1y2R}}{\beta_{1y1L}}} = 1.688 \qquad R_c = 0.91 \quad \Rightarrow \quad R_c \cdot \sqrt{\frac{\beta_{1y2R}}{\beta_{1y1L}}} = 1.536$$

???

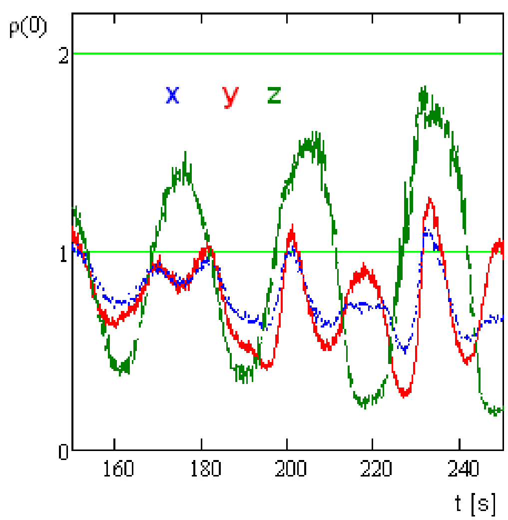
- Diffraction correction of 2L is not sensitive to other parameters and is ~30-33 μm.

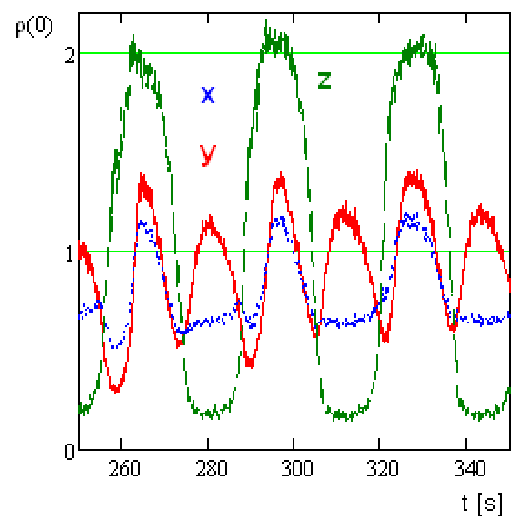


Sweep of Cooling Phase



- Questions
  - Why vertical cooling in anti-cooling phase?
  - What sets the transition to this regime?
  - Is the frequency of oscillations constant?





Maximum OSC Rates

- In absence of IBS the beam sizes and cooling rates are related

$$\frac{d\varepsilon}{dt} = -\lambda\varepsilon + D \xrightarrow{\text{In equilibrium}} \lambda = \frac{D}{\varepsilon} \Rightarrow \quad \frac{\lambda}{\lambda_0} = \frac{\varepsilon_0}{\varepsilon} = \left(\frac{a_0}{a}\right)^2$$

- Data were measured at very small beam current (~60 nA).
  - IBS effect is not visible in data. Theoretical estimate yields ~10% effect on beam sizes but negligible effect on ratio of cooling rates

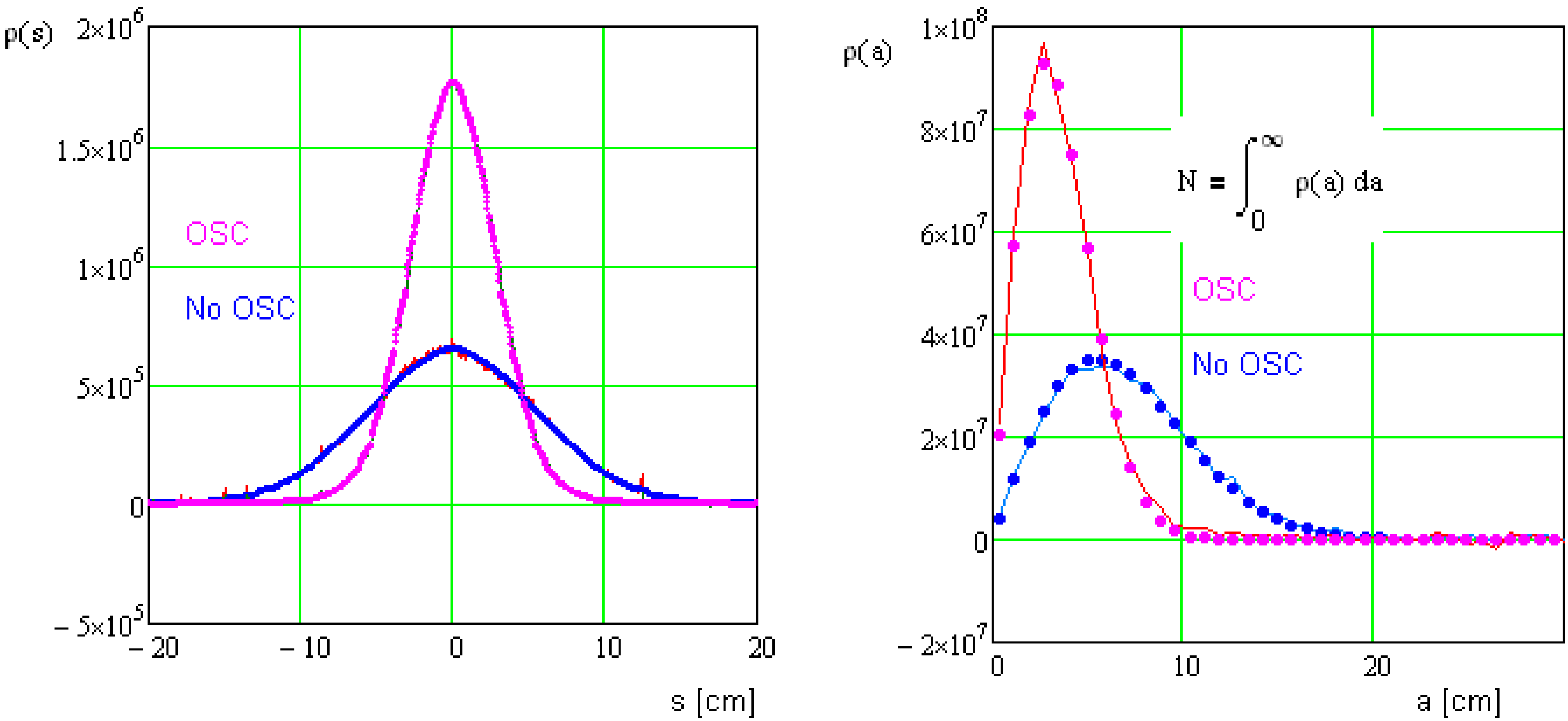
Summary of maximum achieved cooling rates

Transverse SR cooling rate, $\lambda_{1SR} = \lambda_{2SR}$	0.493 s^{-1}
Transverse OSC cooling rate, $\lambda_{1OSC} = \lambda_{2OSC}$	1.180 s^{-1}
Total transverse cooling rate, $\lambda_{1tot} = \lambda_{2tot}$	1.673 s^{-1}
Longitudinal SR cooling rate, λ_{sSR}	1.03 s^{-1}
Longitudinal OSC cooling rate, λ_{sOSC}	6.91 s^{-1}
Total longitudinal cooling rate, λ_{s_tot}	7.93 s^{-1}

*This is amplitude cooling rates. The emittance cooling rates are twice larger.

- Measured cooling rates are about 2 times smaller than predicted???

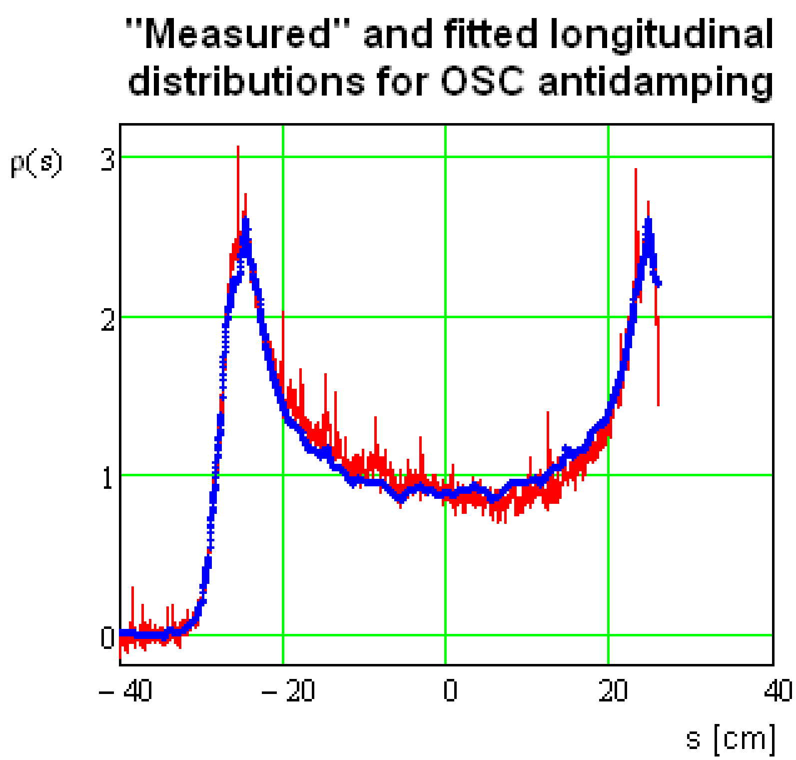
Longitudinal Distribution with and without OSC

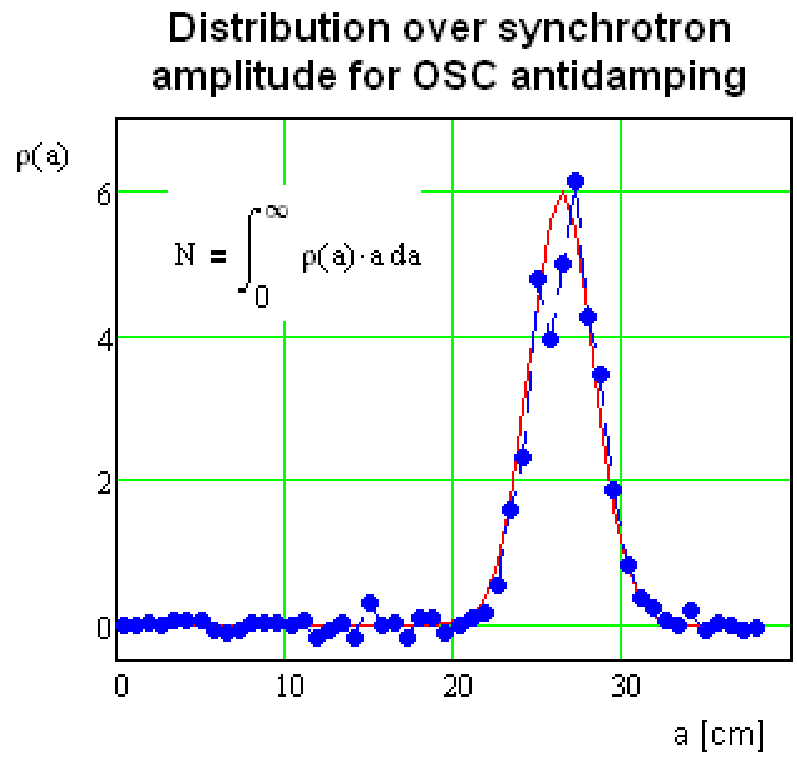


- No deviations from Gaussian distributions

$$\exp\left(-\frac{x^2+p^2}{2\sigma^2}\right)dxdp \to \exp\left(-\frac{I}{\sigma^2}\right)dI \to \exp\left(-\frac{a^2}{2\sigma^2}\right)ada$$

Longitudinal Distribution for Antidamping Phase





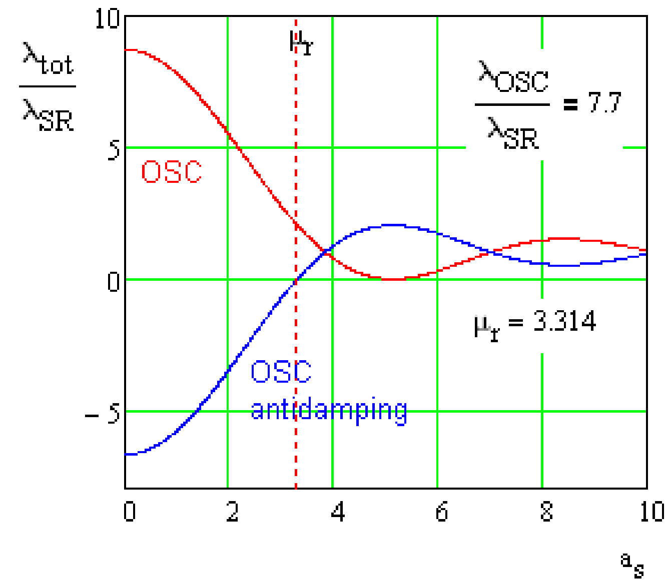
- At maximum antidamping all particles are assembled near new equilibrium point which dimensionless amplitude is expected to be near μ_{11}~3.8

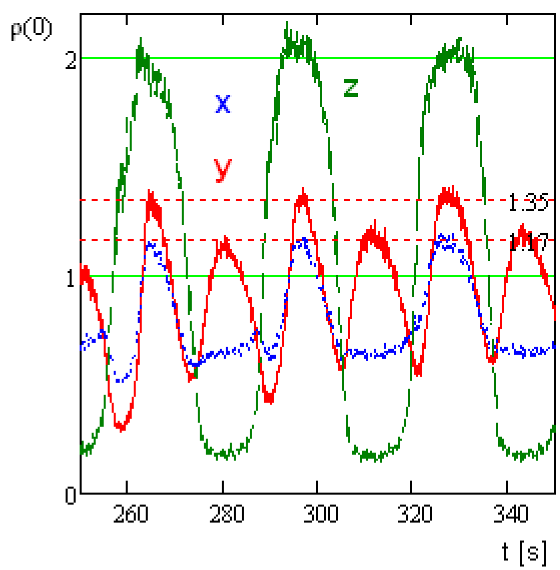
Transverse Cooling for Cooling and Heating Modes

- Transverse cooling rates depend on the dimensionless longitudinal amplitude a_s.
- OSC cooling rate with SR cooling added
  - There is no heating even for the best OSC cooling achieved in our studies
  - SR cooling moves the equilibrium amplitude from a_s=3.83 to $a_s=\mu_r \approx 3.31$.
- OSC transverse cooling rate in antidamping mode: $-\lambda_0 J_0(\mu_r)=0.347\lambda_0$

$$\lambda_1 = \lambda_0^{tr} J_0(a_s) \frac{2J_1(a_1)}{a_1} J_0(a_2)$$

$$\lambda_2 = \lambda_0^{tr} J_0(a_s) \frac{2J_1(a_2)}{a_2} J_0(a_1)$$





- Or the total transverse cooling rate:

  $\lambda_{a\_damp}/\lambda_{damp}=0.572$
- Corresponding ratio of sizes: $\sqrt{\lambda_{a\_damp}/\lambda_{damp}} = 0.76$

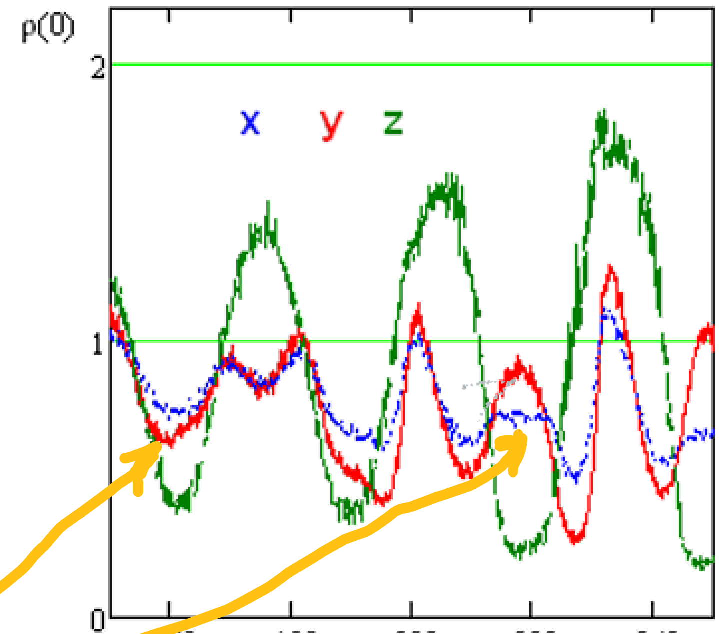
  Measured ratio of 0.87 is closer to 1 because of IBS

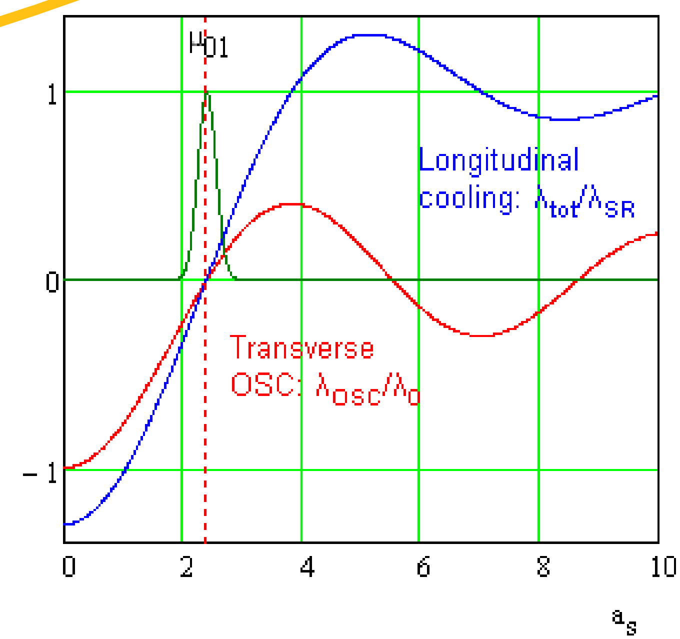
Transition from Cooling to Anticooling at Heating Phases

- The change from ⊥ anticooling to ⊥ cooling happens when the equilibrium dimensionless amplitude crosses μ_{01}=2.405
  - That corresponds to the longitudinal OSC rate - $\lambda_{OSC}/\lambda_{SR}$=2.3

⊥ anticooling

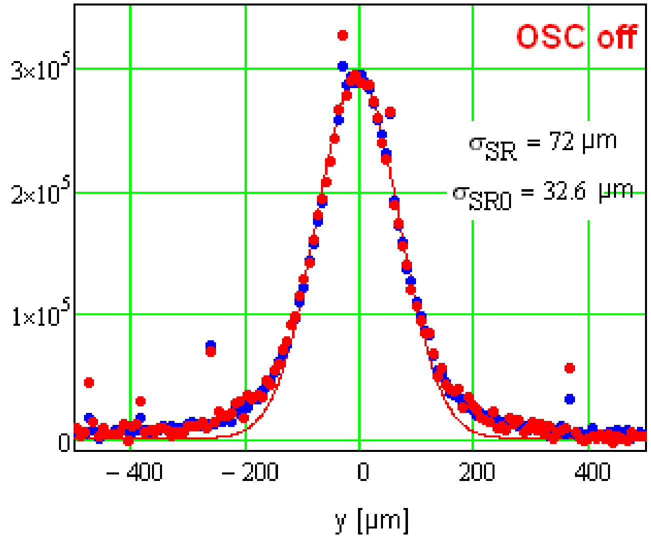
⊥ cooling

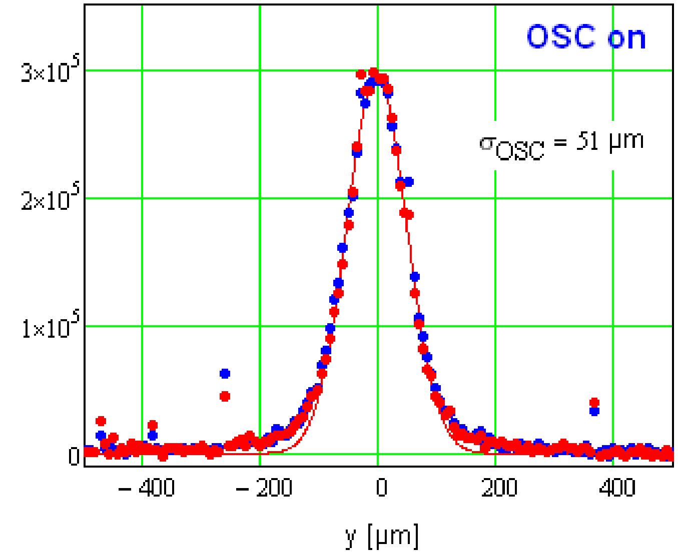




- Horizontal size (blue line) does not show the same behavior as the vertical one because it has large contribution from the momentum spread which is antidamped

Gas Scattering





- Gas scattering creates non-gaussian tails in the transverse beam distribution
- Simple scaling of the design report beam lifetime of 2.9 hour to the measured in the OSC run of 17 minutes yields that vacuum was 10 times worse than the design report request
- Scaling of design report emittance growth rate due to gas scattering yields the emittance growth rate of 5 nm/s and the beam size at 2R of 124 μm while 72 μm was measured.
  - Detailed analysis is required to understand the discrepancy
    - Contribution of tails needs to be subtracted
    - Smaller ring acceptance than assumed in the report quite probable

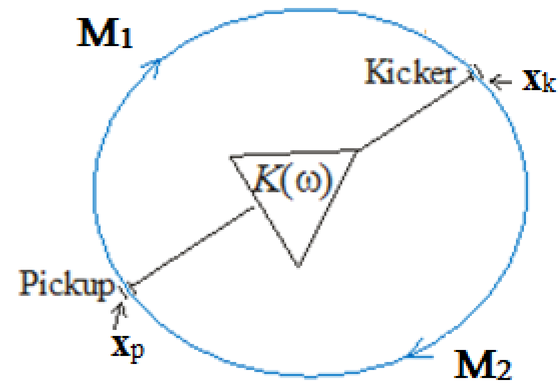
Conclusions

- OSC was demonstrated experimentally
- Data analysis is in advanced state
- We still have unanswered questions
  - Why the measured sum of cooling rates is two times smaller than predicted?
  - Why the ratio of transverse to longitudinal cooling rates are ~3 times weaker than predicted
    - Machine optics, focusing terms in the chicane?
  - What is the transverse cooling range?
- Future steps
  - Passive cooling:
    - Demonstrate effectiveness of sextupoles for an increase of transverse cooling range
    - Answer unanswered questions
  - Active cooling: increase cooling rates by at least 10 times to >100 s^{-1}
- Good study for a student:
  - Detailed simulations of the phase sweep

Backup slides

Small Amplitude Cooling Rates for 3D OSC

One turn map



- OSC maps

$$\mathbf{x}_p = \mathbf{M}_2\mathbf{x}_k\ , \quad \mathbf{x}_k = \mathbf{M}_1\mathbf{x}_p + \mathbf{M}_c\mathbf{x}_p \quad (1)$$

  where $\mathbf{M}_c$ is coupling matrix

- For the chicane located purely in the horizontal plane

$$\frac{\delta p}{p} = -\xi k_0 \left((\mathbf{M}_1)_{51} x + (\mathbf{M}_1)_{52}\theta_x + (\mathbf{M}_1)_{56}\frac{\Delta p}{p} \right)$$

=>

$$\mathbf{M}_c = -k_0\xi \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ (\mathbf{M}_1)_{51} & (\mathbf{M}_1)_{52} & 0 & 0 & 0 & (\mathbf{M}_1)_{56} \end{bmatrix}$$

- Combining Eqs. (1) we obtain one turn map related to kicker position

$$\mathbf{x}_k = \left(\mathbf{M}_1 + \mathbf{M}_c\right)\mathbf{M}_2\mathbf{x}_k$$

$$\Rightarrow \quad \boxed{\mathbf{M}_{tot} = \mathbf{M} + \mathbf{M}_c\mathbf{M}_2\ , \quad \mathbf{M} = \mathbf{M}_1\mathbf{M}_2}$$

  where $\mathbf{M}$ is one-turn matrix without OSC

Small Amplitude Cooling Rates for 3D OSC (2)

Symplectic perturbation theory

- To find the tune shifts we use symplectic perturbation theory [1]

$$\mathbf{M}_{tot} = \left(\mathbf{I} + \mathbf{P}\right)\mathbf{M} \quad => \quad \Delta\mu_i = -\mathbf{v}_i{}^{+}\mathbf{U}\mathbf{P}\mathbf{v}_i / 2$$

where $\mathbf{v}_i$ are the eigen-vectors of unperturbed motion

U – unit symplectic matrix

I – the identity matrix

- Substituting one obtains: $\Delta\mu_i = -\mathbf{v}_{ki}{}^{+}\mathbf{U}\mathbf{M}_c\mathbf{M}_1{}^{-1}\mathbf{v}_{ki} / 2$

- Finally, we can rewrite it through the eigen-vectors in the pickup

$$\Delta\mu_i = -\frac{1}{2}\mathbf{v}_{pi}^{+}\mathbf{M}_1{}^{T}\mathbf{U}\mathbf{M}_c\mathbf{v}_{pi}$$

[1] A. Burov, Phys. Rev. ST-AB, 9, 120101 (2006).

Small Amplitude Cooling Rates for 3D OSC (3)

Sum of cooling rates

$$\sum_{n=1}^{6}\Delta\mu_n = -\frac{1}{2}\sum_{n=1}^{6}(-1)^{n+1}\mathbf{v}_{pn}^{+}\mathbf{M}_1^{\,T}\mathbf{U}\mathbf{M}_c\mathbf{v}_{pn} = -\frac{1}{2}\mathrm{Tr}\left(\mathbf{V}_p^{+}\mathbf{M}_1^{\,T}\mathbf{U}\mathbf{M}_c\mathbf{V}_p\right) = -\frac{1}{2}\mathrm{Tr}\left(\mathbf{V}_k^{+}\mathbf{U}\mathbf{M}_c\mathbf{M}_1^{-1}\mathbf{V}_k\right)$$

where we introduced 6x6 matrix $\mathbf{V}_p = \left[\mathbf{v}_{p1}\quad \mathbf{v}_{p1}^{*}\quad \mathbf{v}_{p2}\quad \mathbf{v}_{p2}^{*}\quad \mathbf{v}_{p3}\quad \mathbf{v}_{p3}^{*}\right]$

which inherits symplectic orthogonality of eigen-vectors:

$(-1)^{n+1}\mathbf{v}_n^{+}\mathbf{U}\mathbf{v}_n = 2i \Rightarrow \mathbf{V}^{+}\mathbf{U}\mathbf{V} = 2i\mathbf{I}$, and accounted that $\mathbf{V}_k = \mathbf{M}_1\mathbf{V}_p$

- Performing cycling permutation inside Tr( ), accounting orthogonality, and that $\mathbf{M}_1^{-1} = -\mathbf{U}\mathbf{M}_1^{\,T}\mathbf{U}, \quad \mathbf{U}\mathbf{U} = -\mathbf{I}$ one obtains

$$\sum_{n=1}^{6}(-1)^{n+1}\Delta\mu_n = -\frac{1}{2}\mathrm{Tr}\left(\mathbf{V}_k^{+}\mathbf{U}\mathbf{V}\mathbf{V}^{-1}\mathbf{M}_c\mathbf{M}_1^{-1}\mathbf{V}_k\right) = i\mathrm{Tr}\left(\mathbf{M}_c\mathbf{M}_1^{-1}\right) = -i\mathrm{Tr}\left(\mathbf{M}_c\mathbf{U}\mathbf{M}_1^{\,T}\mathbf{U}\right)$$

- Account that each pair of eigen-vectors and eigen-numbers are complex conjugated we obtain that the of cooling rates is:

$$\sum_{n=1}^{3}\lambda_n = -\frac{1}{2}\mathrm{Im}\left(\sum_{n=1}^{6}(-1)^{n+1}\Delta\mu_n\right) = \frac{1}{2}\mathrm{Tr}\left(\mathbf{M}_c\mathbf{U}\mathbf{M}_1^{\,T}\mathbf{U}\right)$$

Cooling rates sum does not depend on eigen-vectors. Only on $\mathbf{M}_1$ & $\mathbf{M}_c(\mathbf{M}_1)$

Small Amplitude Cooling Rates for 3D OSC (4)

Sum of cooling rates

- Straightforward matrix multiplications yield $\mathrm{Tr}\left(\mathbf{M}_c \mathbf{U} \mathbf{M}_1{}^T \mathbf{U}\right) = -(\mathbf{M}_1)_{56}$. Consequently, the sum of cooling rates in amplitude per turn:

$$\sum_{n=1}^{3} \lambda_n = \frac{1}{2} \xi k_0 (\mathbf{M}_1)_{56}$$

Longitudinal cooling rate

- Assume an absence of betatron motion but non-zero dispersion
  - ♦ Accounting that $x = D\Delta p / p, \quad \theta_x = D'\Delta p / p$

    one obtains: $$\frac{\delta p}{p} = k_0 \xi \, \Delta s = k_0 \xi \left((\mathbf{M}_1)_{51} D + (\mathbf{M}_1)_{52} D' + (\mathbf{M}_1)_{56} \right) \frac{\Delta p}{p}$$

⇨ $$\lambda_s = \frac{k_0 \xi}{2} \left((\mathbf{M}_1)_{51} D + (\mathbf{M}_1)_{52} D' + (\mathbf{M}_1)_{56} \right)$$

- Consequently, the sum of transverse cooling rates is:

$$\lambda_1 + \lambda_2 = -\frac{k_0 \xi}{2} \left((\mathbf{M}_1)_{51} D + (\mathbf{M}_1)_{52} D' \right)$$

Small Amplitude Cooling Rates for 3D OSC (5)

Transverse cooling rates at coupling resonance

- For operation on coupling resonance a given plan 4D beta-function are equal for both eigen vectors

$$\begin{cases} \mathbf{v}_1 = \left(\sqrt{\beta_{1x}}, -\dfrac{i(1-u)+\alpha_{1x}}{\sqrt{\beta_{1x}}}, \sqrt{\beta_{1y}}e^{i\nu_1}, -\dfrac{iu+\alpha_{1y}}{\sqrt{\beta_{1y}}}e^{i\nu_1} \right)^T, \\ \mathbf{v}_2 = \left(\sqrt{\beta_{2x}}e^{i\nu_2}, -\dfrac{iu+\alpha_{2x}}{\sqrt{\beta_{2x}}}e^{i\nu_2}, \sqrt{\beta_{2y}}, -\dfrac{i(1-u)+\alpha_{2y}}{\sqrt{\beta_{2y}}} \right)^T. \end{cases} \Rightarrow \begin{cases} \mathbf{v}_1 = \left(\sqrt{\dfrac{\beta_x}{2}}, -\dfrac{i+\alpha_x}{\sqrt{2\beta_x}}, \sqrt{\dfrac{\beta_y}{2}}e^{i\nu_1}, -\dfrac{i+\alpha_y}{\sqrt{2\beta_y}}e^{i\nu_1} \right)^T, \\ \mathbf{v}_2 = \left(\sqrt{\dfrac{\beta_x}{2}}e^{i\nu_2}, -\dfrac{i+\alpha_x}{\sqrt{2\beta_x}}e^{i\nu_2}, \sqrt{\dfrac{\beta_y}{2}}, -\dfrac{i+\alpha_y}{\sqrt{2\beta_y}} \right)^T. \end{cases}$$

  where we accounted that 4D β- and α-functions are twice smaller than corresponding functions of uncoupled motion, and $u=1/2$

- Consequently, the cooling rates of both modes are equal

$$\lambda_1 = \lambda_2 = -\frac{k_0\xi}{4}\left((\mathbf{M}_1)_{51} D + (\mathbf{M}_1)_{52} D' \right)$$

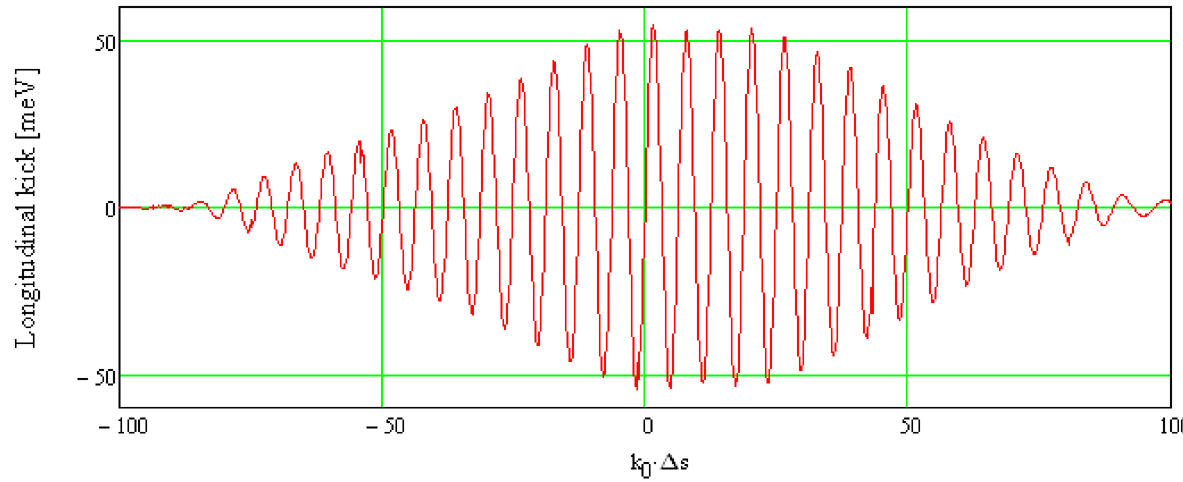
Cooling Rates for Particles with Large Amplitudes

Dimensionless amplitude for longitudinal motion

- The actual cooling force depends on Δs nonlinearly
- In the first approximation the longitudinal kick is

$$\frac{\delta p}{p} = -\xi k_0 \Delta s$$

$$\Rightarrow \quad \frac{\delta p}{p} = -\xi \sin\left(k_0 \Delta s\right)$$



Dependence of long. kick on $k_0\Delta s$ for IOTA passive OSC

- For the longitudinal cooling (no betatron motion)

$$a_s \equiv k_0 \Delta s_{\max} = k_0 \left((\mathbf{M}_1)_{51} D + (\mathbf{M}_1)_{52} D' + (\mathbf{M}_1)_{56} \right) \left.\frac{\Delta p}{p}\right|_{\max}$$

- Consequently, the kick dependence on time is

$$\frac{\delta p}{p} = -\xi \sin\left(a_s \cos(\omega_s t)\right)$$

Cooling Rates for Particles with Large Amplitudes(2)

Dimensionless amplitude for transverse motion

- Amplitude of long. particle displacements due to betatron motion

$$a_i \equiv k_0 \Delta s_{\max} = \max_{over\ \beta-motion} \left(k_0 \left((\mathbf{M}_1)_{51} x + (\mathbf{M}_1)_{52} \theta_x \right) \right), \quad i = 1, 2$$

- Turn by-turn particle coordinates are:

$$\begin{bmatrix} x \\ \theta_x \\ y \\ \theta_y \end{bmatrix}_n = \sqrt{\varepsilon_1} \begin{bmatrix} \sqrt{\beta_{1x}} \\ -\dfrac{(1-u)+\alpha_{1x}}{\sqrt{\beta_{1x}}} \\ \ldots \\ \ldots \end{bmatrix} e^{-i(\mu_1 n + \psi_1)} + e^{i\nu_2} \sqrt{\varepsilon_2} \begin{bmatrix} \sqrt{\beta_{2x}} \\ -\dfrac{(1-u)+\alpha_{2x}}{\sqrt{\beta_{2x}}} \\ \ldots \\ \ldots \end{bmatrix} e^{-i(\mu_1 n + \psi_1)}$$

- Combining we obtain

$$a_1 = k_0 \sqrt{\varepsilon_1 \left((\mathbf{M}_1)_{52}{}^2 \beta_{1x} - 2(\mathbf{M}_1)_{52} (\mathbf{M}_1)_{51}{}^2 \alpha_{1x} + (\mathbf{M}_1)_{52}{}^2 \frac{(1-u)^2 + \alpha_{1x}{}^2}{\beta_{1x}} \right)}$$

$$a_2 = k_0 \sqrt{\varepsilon_2 \left((\mathbf{M}_1)_{52}{}^2 \beta_{2x} - 2(\mathbf{M}_1)_{52} (\mathbf{M}_1)_{51}{}^2 \alpha_{2x} + (\mathbf{M}_1)_{52}{}^2 \frac{u^2 + \alpha_{2x}{}^2}{\beta_{2x}} \right)}$$

- For operation on coupling resonance

$$a_{1,2} = k_0 \sqrt{\frac{\varepsilon_{1,2}}{2} \left((\mathbf{M}_1)_{52}{}^2 \beta_x - 2(\mathbf{M}_1)_{52} (\mathbf{M}_1)_{51}{}^2 \alpha_x + (\mathbf{M}_1)_{52}{}^2 \frac{1 + \alpha_x{}^2}{\beta_{1x}} \right)}$$

$$\varepsilon = \varepsilon_1 = \varepsilon_2, \quad \beta_{1x} = \beta_{2x} = \beta_x / 2, \quad \alpha_{1x} = \alpha_{2x} = \alpha_x / 2, \quad u = 1/2$$

  - ♦ Accounting that $\varepsilon_{1x} = \varepsilon_{2x} = \varepsilon_x/2$ and one obtains $a_1 = a_2 = a_x/2$

Cooling Rates for Particles with Large Amplitudes(3)

- The colling rates for particles with large amplitudes are obtained by averaging over betatron and synchrotron motion

$$k_0 \delta s = a_1 \cos\psi_1 + a_2 \cos\psi_2 + a_s \cos\psi_s$$

- Accounting that $\lambda \propto \int Fvdt$ the averaging for the mode 1 cooling rate may be presented in the following form

$$\lambda_1(a_1, a_2, a_s) = \lambda_1 \frac{\int_0^{2\pi}\int_0^{2\pi}\int_0^{2\pi} \sin(\phi_1+\psi_1)\sin\left[a_1\sin\phi_1 + a_2\sin\phi_2 + a_s\sin\phi_s\right]\frac{d\phi_2 d\phi_s}{(2\pi)^2} d\phi_1}{a_1\int_0^{2\pi}\sin(\phi_1+\psi_1)\sin\left(\sin\phi_1\right)d\phi_1}$$

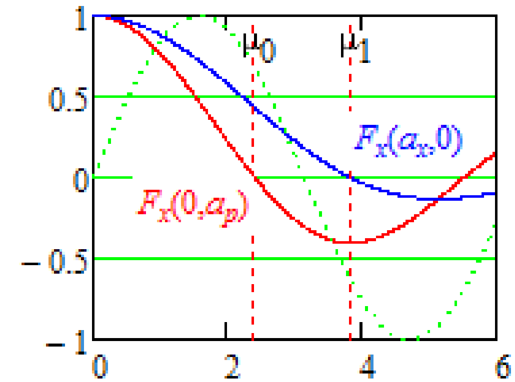
where λ_1 is the cooling rate for small amplitude.

Similar integrals can be written to other "plans"

- Integrating one obtains

$$\lambda_{1,2,s}(a_1, a_2, a_s) = F_{1,2,s}\lambda_{1,2,s}$$

$$F_i = \frac{2}{a_i} J_1(a_i) J_0(a_j) J_0(a_k) , \quad i \neq j \neq k$$

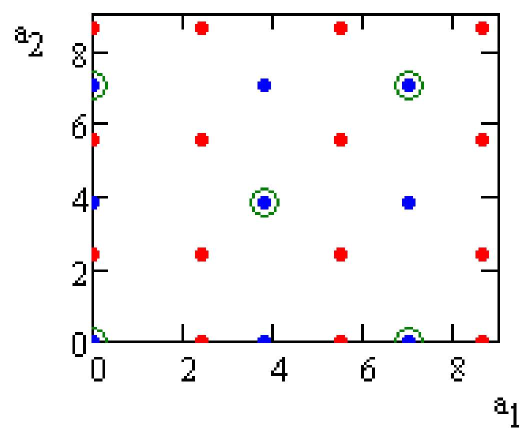


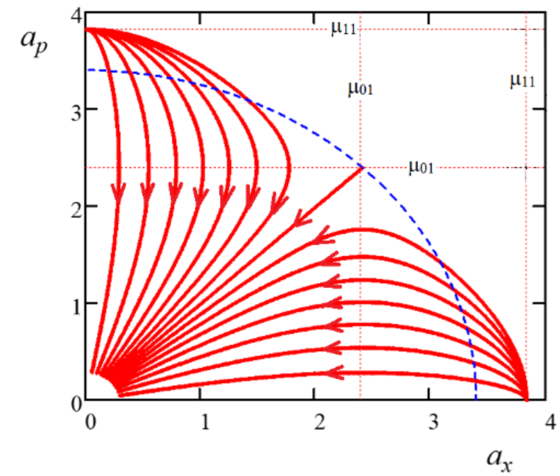
Cooling Rates for Particles with Large Amplitudes(4)

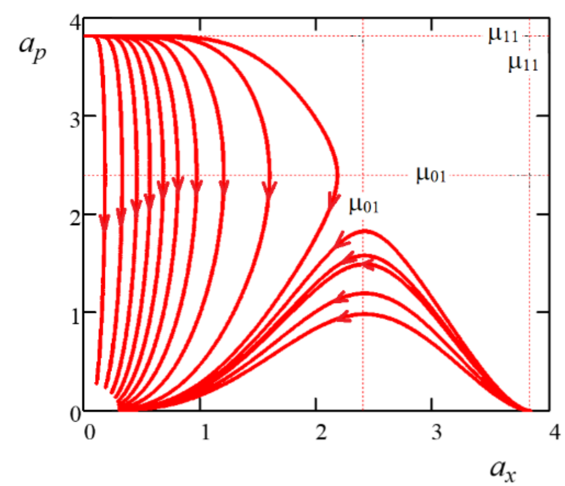
- Cooling may trap large amplitude particles at intermediate amplitudes
- Trap conditions: $a_i\lambda_i=0$ & $d(\lambda_i a_i)/da_i > 0$ (for each i)
  That is equivalent:

$$J_1(a_i)J_0(a_j)J_0(a_k)=0 \quad \& \quad J_0(a_j)J_0(a_k)\frac{d}{da_i}\left(J_1(a_i)\right)\geq 0\,, \quad i\neq j\neq k \qquad (1)$$

  ⇨ Possible trip points: $a_i=\mu_{0n}$, $a_i=\mu_{1n}$
- The lowest order solutions of Eqs. (1) are:
  (μ_{11},μ_{11},0), (μ_{12},0,0), (μ_{12},μ_{12},0), (μ_{12},μ_{12},μ_{12}), (μ_{12},μ_{11},μ_{11})
  \+ all permutations
  Here: (μ_{11}≈3.832, (μ_{12}≈7.016)
- All solutions with μ_{0n} (roots $J_0(x)$) have zero derivatives over a and therefore do not represent centers of stability regions







Amplitude trajectories in the course of OSC cooling; top – λ_x/λ_p=1; bottom – λ_x/λ_p=0.3. Blue dashed circle radius - $\sqrt{2}\mu_{01}$.

Transverse Kicks in OSC

- The OSC theory predicts direct relationship between longitudinal and transverse cooling rates. At coupling resonance

$$\frac{\lambda_x}{\lambda_s} = \frac{\lambda_y}{\lambda_s} = -\frac{1}{2}\frac{(\mathbf{M}_1)_{51} D + (\mathbf{M}_1)_{52} D'}{(\mathbf{M}_1)_{51} D + (\mathbf{M}_1)_{52} D' + (\mathbf{M}_1)_{56}}.$$

  This ratio should not depend on the delay in the OSC chicane. However, it was observed
- This peculiarity forced us to look at possible transverse kicks for the case of not perfectly tuned OSC
- For arbitrary horizontally polarized wave and particle with velocity $\boldsymbol{\beta} = \left[\beta_x, \beta_y, \beta\right]$ one can write

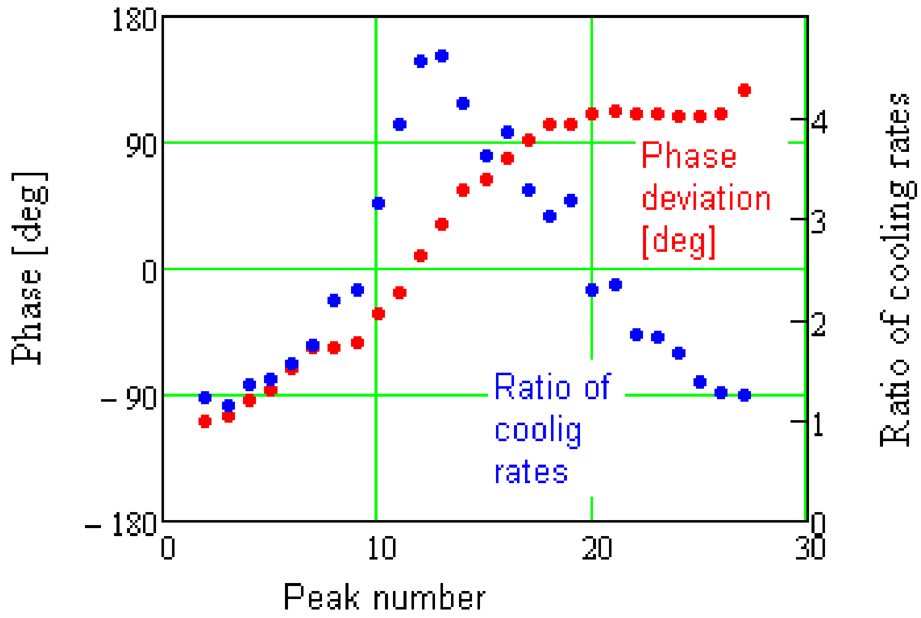
$$\frac{d}{dt}\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \frac{\mathrm{e}}{2\gamma^2} e^{i(\omega t - k_0 z)} \int E_{k_x k_y} \begin{bmatrix} 1 + (\gamma\theta_y)^2 - (\gamma\theta_x)^2 + (\gamma\beta_x)^2 + (\gamma\beta_y)^2 - 2(\gamma\beta_y)(\gamma\theta_y) \\ \gamma\theta_y\left(\gamma\beta_x - \gamma\theta_x\right) \\ \gamma\left(\gamma\beta_x - \gamma\theta_x\right) \end{bmatrix} \frac{dk_x dk_y}{k_0^{\,2}}, \quad \theta_{x,y} = \frac{k_{x,y}}{k_0}$$

- Accounting that in the OSC $\gamma\beta_x, \gamma\beta_y, \gamma\theta_x, \gamma\beta_y \le 1$ we conclude that x and y components of the kick are in γ times smaller than z-component

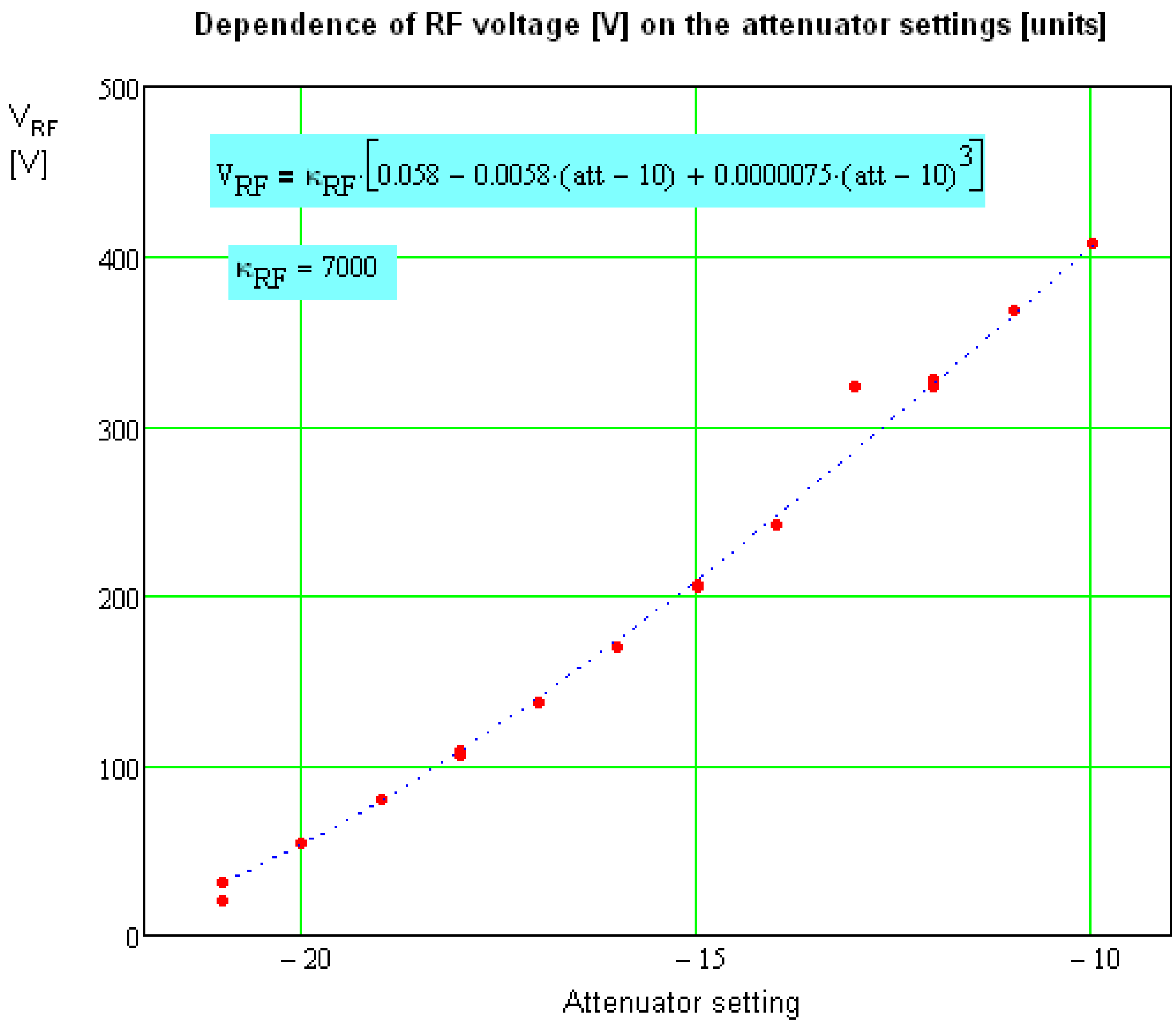
For IOTA it is more than 100 times suppression

Gouy Phase

- Single lens focusing is simple however it results in dephasing of wave and particle in the course of its travel in kicker
- In the course of phase sweep it results in that maximums of OSC cooling do not follow uniformly with delay
  - Corresponding phase shifts have information on details of cooling force and can point out the reason why we are loosing ~2 times in cooling rates
- More accurate data analysis is pending



RF Voltage Calibration – Practical Aspects



- Attenuator setting (N:IRFEAT) is not a logarithmic value (not dB)