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Advanced Scaling FFAG Muon decay ring for Very Low Energy Neutrino Factory (2 GeV)

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1 Introduction

The interest for Very Low Energy Neutrino Factory (VLENF) with a muon decay ring is growing recently for neutrino physics experiment beyond the standard model [1]. A race-track advanced scaling FFAG storage ring is proposed and developped here. The central momentum for this lattice is chosen to be 2 GeV (kinetic energy). Lattice parameters are developed in the first part, then single particle tracking is presented in the second part, and finally multi-particle tracking is shown in the third part.

2 Lattice parameters

The racetrack ring is composed of two types of cell:

- ◇ straight scaling FFAG cell,
- ◇ circular scaling FFAG cell.

There are 18 straight FFAG cells in each long straight section (36 straight FFAG cells for the whole ring).

There are 8 circular FFAG cells in each arc section (16 circular FFAG cells for the whole ring).

2.1 Straight scaling FFAG cell parameters

In the straight scaling FFAG cell, the vertical magnetic field B_{sz} in the median plane follows

$$B_{sz} = B_{0sz} e^{m(x-x_0)} \mathcal{F},$$

with x the horizontal cartesian coordinate, m the normalized field gradient, $\mathcal{F}$ an arbitrary function and $B_{0sz} = B_{sz}(x_0)$. The parameters of the straight scaling FFAG cell are summarized in Tab. 1. The cell is shown in Fig. 1. The red line represents the 2 GeV muon reference trajectory, and its corresponding magnetic field is shown in Fig. 2. Periodic betafunctions are shown in Fig. 3.

Cell type		DFD triplet
Number of cells in the ring		36
Cell length		6 m
x_0		16 m
m-value		$3.9\,\mathrm{m}^{-1}$
Packing factor		0.07
Collimators $(x_{min}, x_{max}, z_{max})$		(15.5 m, 16.5 m, 0.3 m)
Periodic cell dispersion		0.26 m
Horizontal phase advance		13.0 deg.
Vertical phase advance		15.2 deg.
D_1 magnet parameters		
	Magnet center	0.2 m
	Magnet length	0.1 m
	Fringe field fall off	Linear (Length: 0.04 m)
	$B_0(x_0 = 16\ m)$	0.712225 T
F magnet parameters		
	Magnet center	3 m
	Magnet length	0.2 m
	Fringe field fall off	Linear (Length: 0.04 m)
	$B_0(x_0 = 16\ m)$	-0.639761 T
D_2 magnet parameters		
	Magnet center	5.8 m
	Magnet length	0.1 m
	Fringe field fall off	Linear (Length: 0.04 m)
	$B_0(x_0 = 16\ m)$	0.712225 T

Table 1: Parameters of the straight scaling FFAG cell.

2.1.1 Circular scaling FFAG cell parameters

In the circular scaling FFAG cell, the vertical magnetic field B_{cz} in the median plane follows

$$B_{cz} = B_{0cz} \left(\frac{r}{r_0} \right)^k \mathcal{F},$$

with r the radius in the polar coordinate, k the geometrical field index, $\mathcal{F}$ an arbitrary function and $B_{0cz} = B_{cz}(r_0)$. The parameters of the circular scaling FFAG cell are summarized in Tab. 2. The cell is shown in Fig. 4. The red line represents the 2 GeV muon reference trajectory, and its corresponding magnetic field is shown in Fig. 5. Periodic betafunctions are shown in Fig. 6.

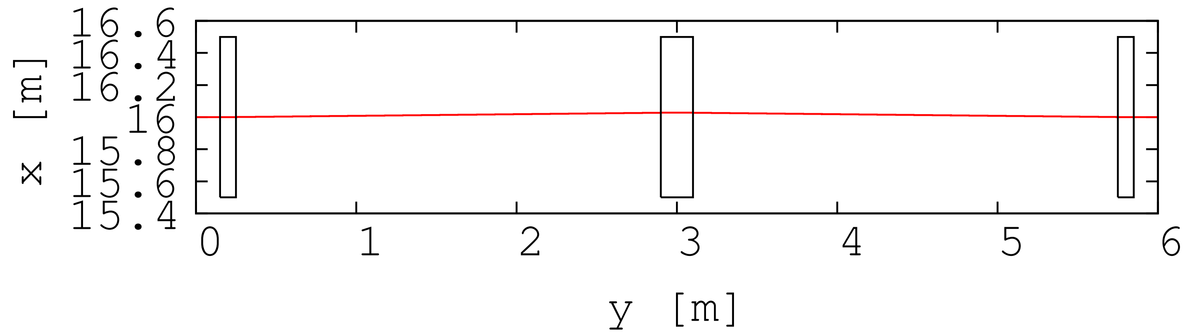

Figure 1: Top view of the straight scaling FFAG cell. 2 GeV muon reference trajectory is shown in red. Effective field boundaries with collimators are shown in black.

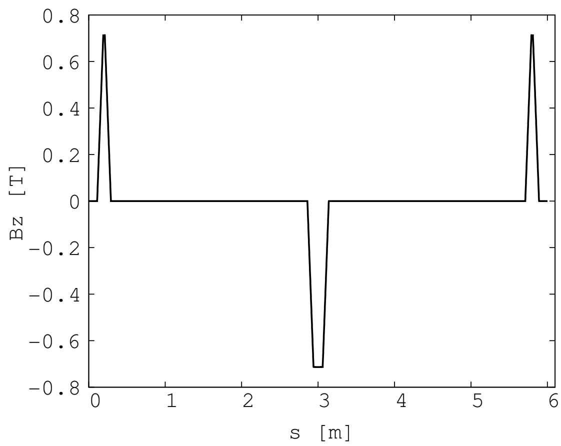

Figure 2: Vertical magnetic field for 2 GeV muon reference trajectory in the straight scaling FFAG cell.

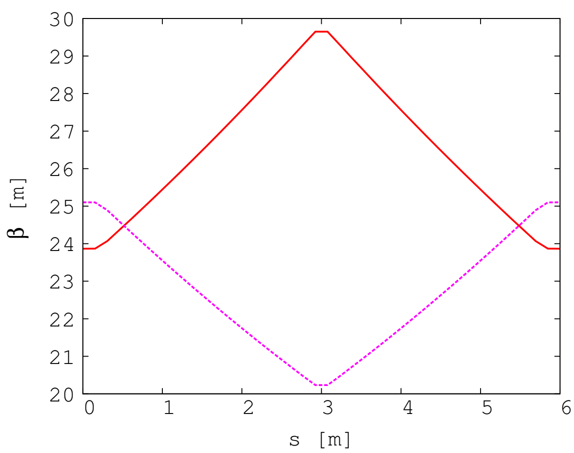

Figure 3: Horizontal (plain red) and vertical (dotted purple) periodic betafunctions of the straight scaling FFAG cell.

3 Single particle tracking

Stepwise tracking using Runge Kutta integration in field model with linear fringe fields has been realized. Interpolation of the magnetic field off the midplane has been done to the first order. Single particle has been tracked first. The particle is muon μ^+. Central momentum p_0 is 2 GeV (kinetic energy), minimum momentum p_{min} 1.66 GeV (kinetic energy), and maximum momentum p_{max} is 2.34 GeV (kinetic energy), so $\Delta p/p_0 = \pm 16\%$.

Tracking stepsize is 1 mm. The exit boundary of a cell is the entrance boundary of the next cell.

Cell type		FDF triplet
Number of cells in the ring		16
Cell opening angle		22.5 deg
r_0		16 m
k-value		10.85
Packing factor		0.9
Collimators $(r_{min}, r_{max}, z_{max})$		(14.5 m, 17.5 m, 0.3 m)
Periodic cell dispersion		1.35 m (at 2 GeV)
Horizontal phase advance		90. deg.
Vertical phase advance		22.5 deg.
F_1 magnet parameters		
	Magnet center	4.1 deg
	Magnet length	6.8 deg
	Fringe field fall off	Linear (Length: 0.1 deg)
	$B_0(r_0 = 16\ m)$	-1.430895 T
D magnet parameters		
	Magnet center	11.25 deg
	Magnet length	6.0 deg
	Fringe field fall off	Linear (Length: 0.1 deg)
	$B_0(r_0 = 16\ m)$	1.866669 T
F_2 magnet parameters		
	Magnet center	18.4 deg
	Magnet length	6.8 deg
	Fringe field fall off	Linear (Length: 0.1 deg)
	$B_0(r_0 = 16\ m)$	-1.430895 T

Table 2: Parameters of the circular scaling FFAG cell.

The ring tune point is (5.30, 2.52) at p_0. Stability of the ring tune has been studied over the momentum range. The tune shift is presented in Fig. 7. The tune point stays within 0.1 shift.

Closed orbits of p_0, p_{min}, and p_{max} particles are shown in Fig. 8. Magnetic field for the p_{max} closed orbit is presented in Fig. 9. Dispersion at p_0 is shown in Fig. 10. Beta-functions of p_0, p_{min}, and p_{max} are plotted in Fig. 11.

Acceptance study at fixed energy has been done. maximum amplitudes with a stable motion at p_0 over 100 turns are shown for horizontal and vertical in Fig. 12 and in Fig. 13, respectively.

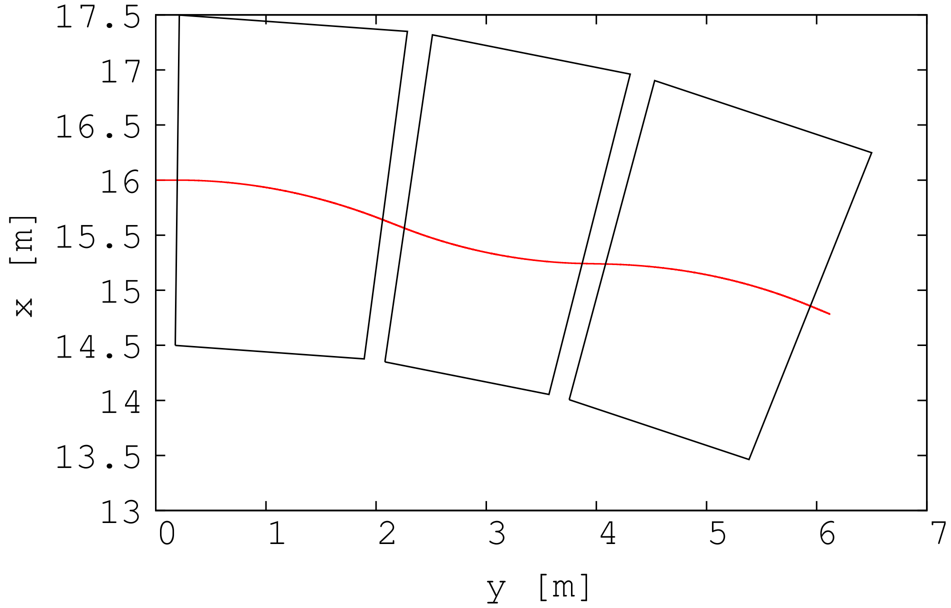

Figure 4: Top view of the circular scaling FFAG cell. 2 GeV muon reference trajectory is shown in red. Effective field boundaries with collimators are shown in black.

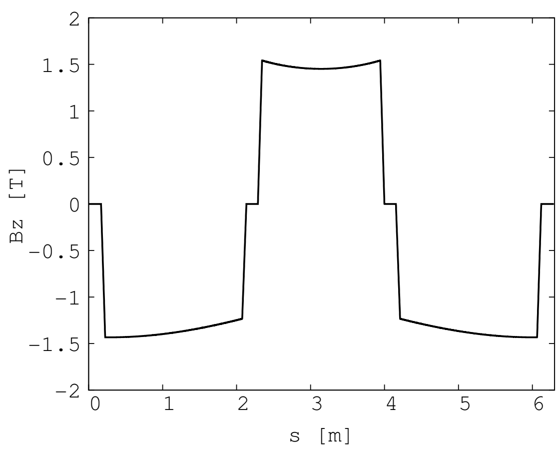

Figure 5: Vertical magnetic field for 2 GeV muon reference trajectory in the circular scaling FFAG cell.

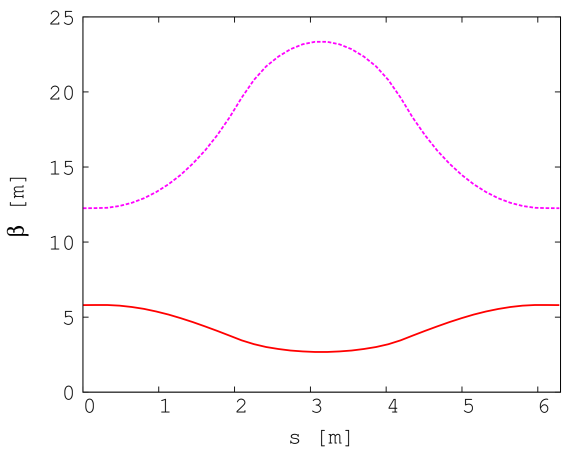

Figure 6: Horizontal (plain red) and vertical (dotted purple) periodic betafunctions of the circular scaling FFAG cell.

4 Multi-particle tracking

A beam composed of 1000 particles has been generated and tracked in the lattice. These particles are uniformly distributed at 2 GeV (kinetic energy) ±16% in momentum and then uniformly distributed inside a transverse 4D ellipsoid (Waterbag distribution). The initial beam size in the horizontal phase space is shown in Fig. 14 and in the vertical phase space in Fig. 15. The emittance is 400π mm.mrad unnormalized both in horizontal and vertical, with a beta-function at the injection of 14 m in horizontal, and 25.5 m in vertical.

The particles do 60 turns (about their lifetime). The results show that 69 particles are lost in the lattice (6.9% of the particles, see Fig. 14 and Fig. 15), the horizontal and

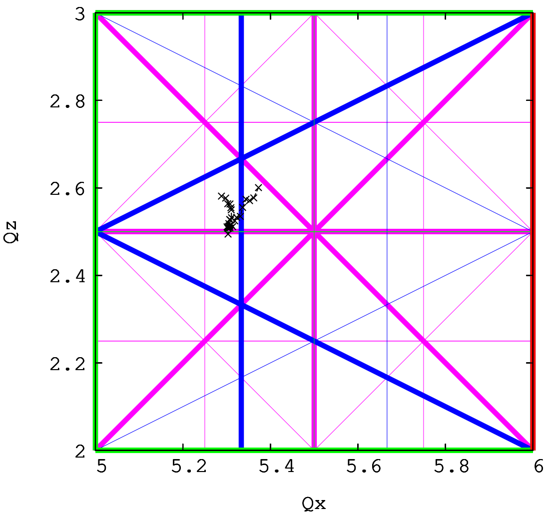

Figure 7: Tune diagram for muons from p_{min} to p_{max} (±16% in momentum around 2.1 GeV/c). Integer (red), half-integer (green), third integer (blue) and fourth integer (purple) normal resonances are plotted. Structural resonances are in bold.

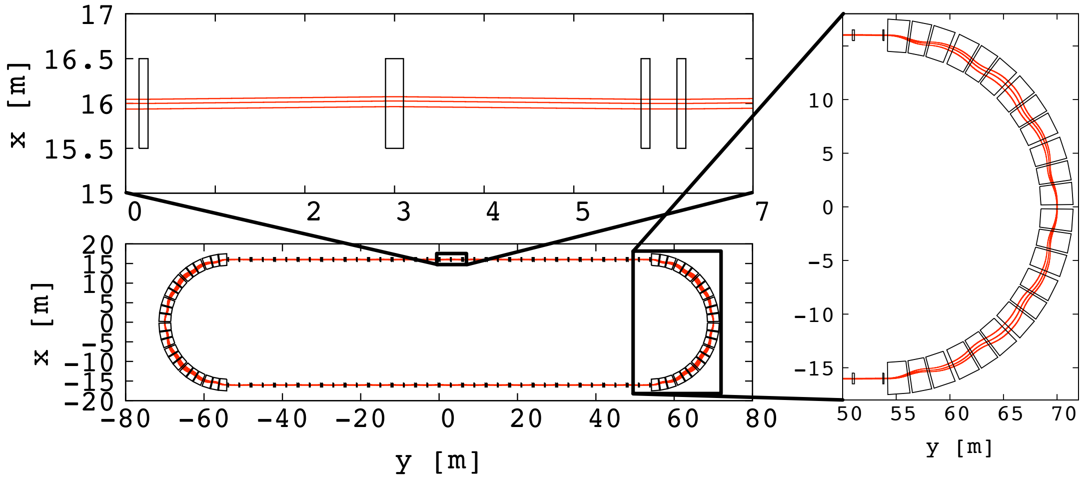

Figure 8: Top view of the racetrack FFAG lattice (bottom left scheme). The top left scheme shows a zoom on the straight section and the right scheme a zoom on the arc section. p_0, p_{min}, and p_{max} muon closed orbits are shown in red. Effective field boundaries with collimators are shown in black.

vertical Poincarre map of the beam after the 60 turns are presented in Fig. 16 and Fig. 17, respectively. The distribution in horizontal angle of the particles is shown in Fig. 18.

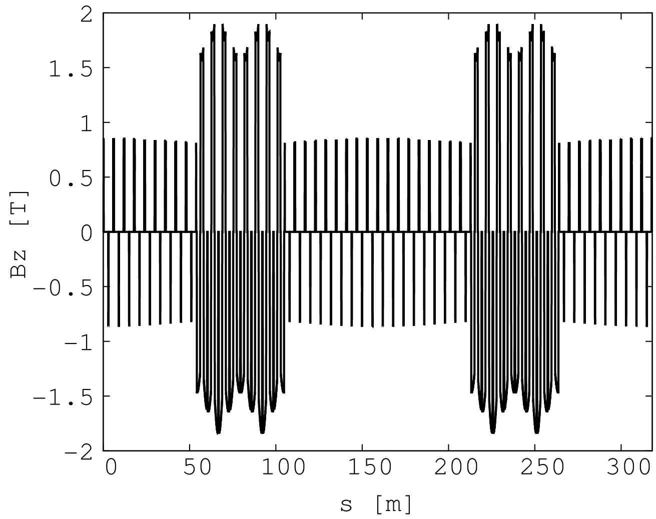

Figure 9: Vertical magnetic field for p_{max} muon closed orbit in the racetrack FFAG ring.

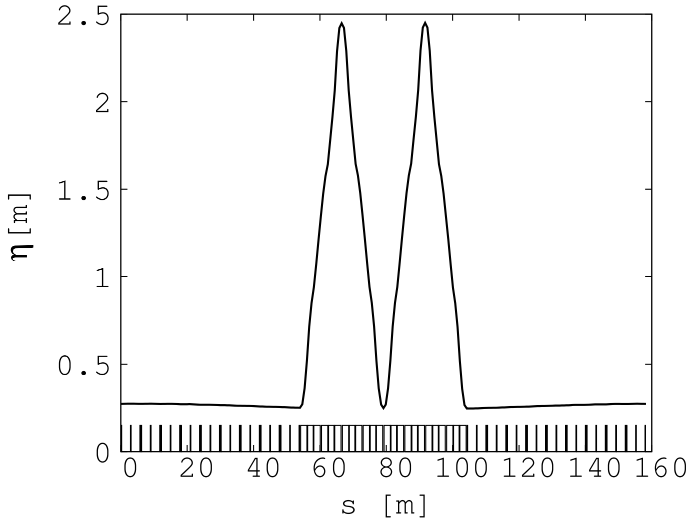

Figure 10: Dispersion function for p_0 in half of the ring. The plot is centered on the arc part.

5 Summary and conclusion

Muon decay ring made of Advanced scaling FFAG cells is presented here for VLENF purpose. The momentum range is ±16% around 2 GeV, and is limited by the dispersion mismatch between the straight part and the arc part. A better dispersion matching could improve it. Multi particle tracking has been done, and 6.9% of the particles are lost in the simulation. The tune working point is very close to the horizontal third-integer resonance. A better choice of the working point could decrease the number of lost particles.

References

[1] A. Bross, "The very Low Energy Neutrino Factory", Nufact'11 slides, (2011), http://indico.cern.ch/getFile.py/access?contribId=136&sessionId=1&resId=2&materialId =slides&confId=114816.

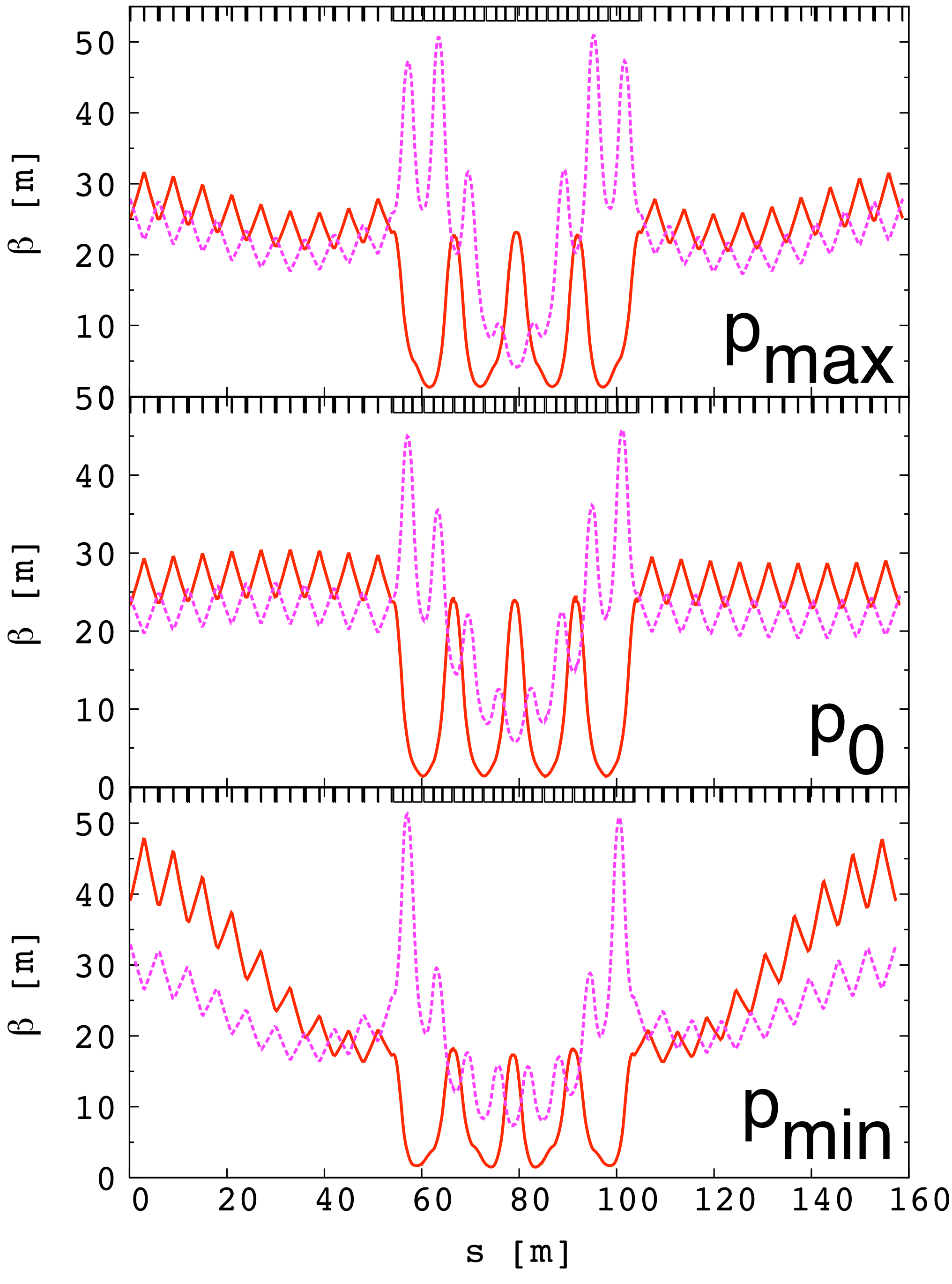

Figure 11: Horizontal (plain red) and vertical (dotted purple) periodic betafunctions of half of the ring for p_{max} (top scheme), p_0 (middle scheme), and p_{min} (bottom scheme). The plot is centered on the arc part.

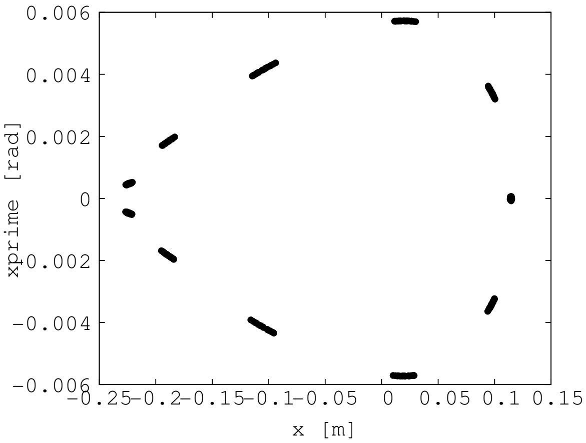

Figure 12: Horizontal Poincarre map showing the maximum amplitude for a stable motion over 100 turns for p_0.

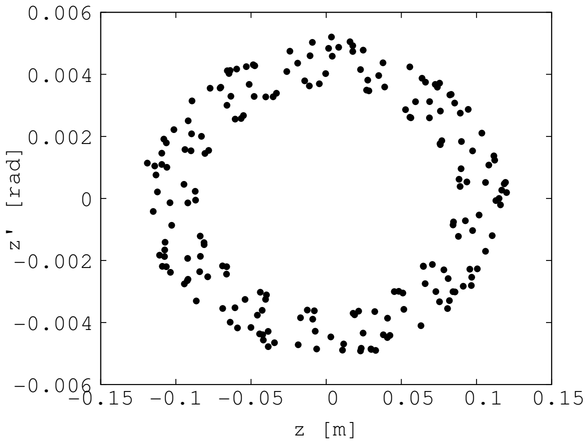

Figure 13: Vertical Poincarre map showing the maximum amplitude for a stable motion over 100 turns for p_0.

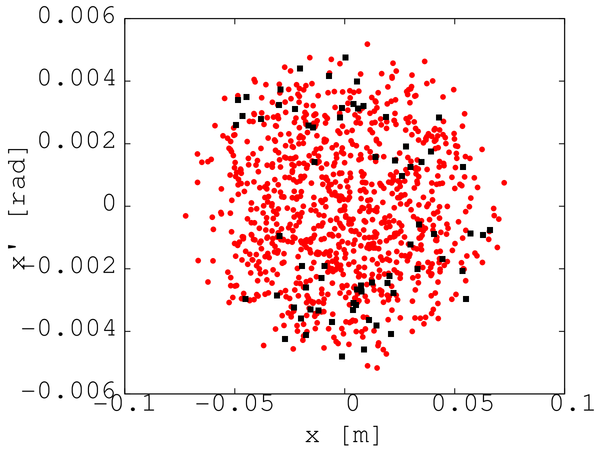

Figure 14: Horizontal Poincarre map showing the 1000 particles distribution at the beginning of the simulation in red circles (∼400 π mm.mrad unormalized emittance). The black squares indicate the injected particles lost during the simulation.

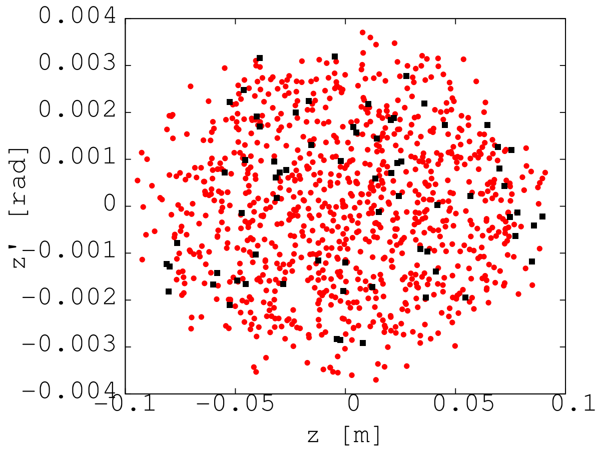

Figure 15: Vertical Poincarre map showing the 1000 particles distribution at the beginning of the simulation in red circles (∼400 π mm.mrad unormalized emittance). The black squares indicate the injected particles lost during the simulation.

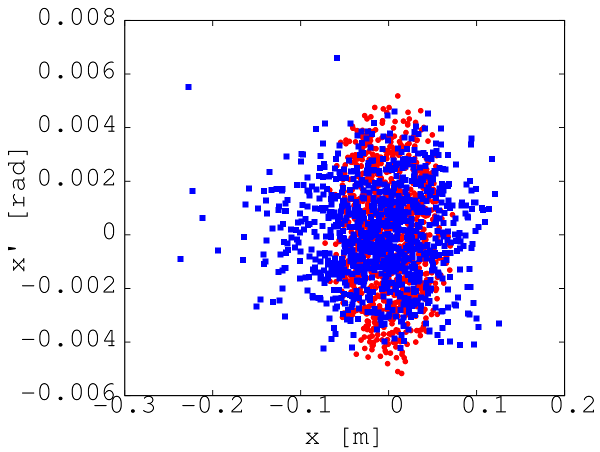

Figure 16: Horizontal Poincarre map showing the 1000 particles distribution at the injection (red circles) and at the end of the simulation (blue squares).

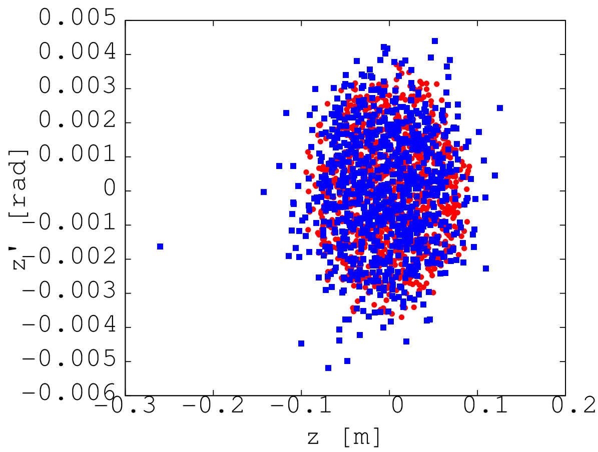

Figure 17: Vertical Poincarre map showing the 1000 particles distribution at the injection (red circles) and at the end of the simulation (blue squares).

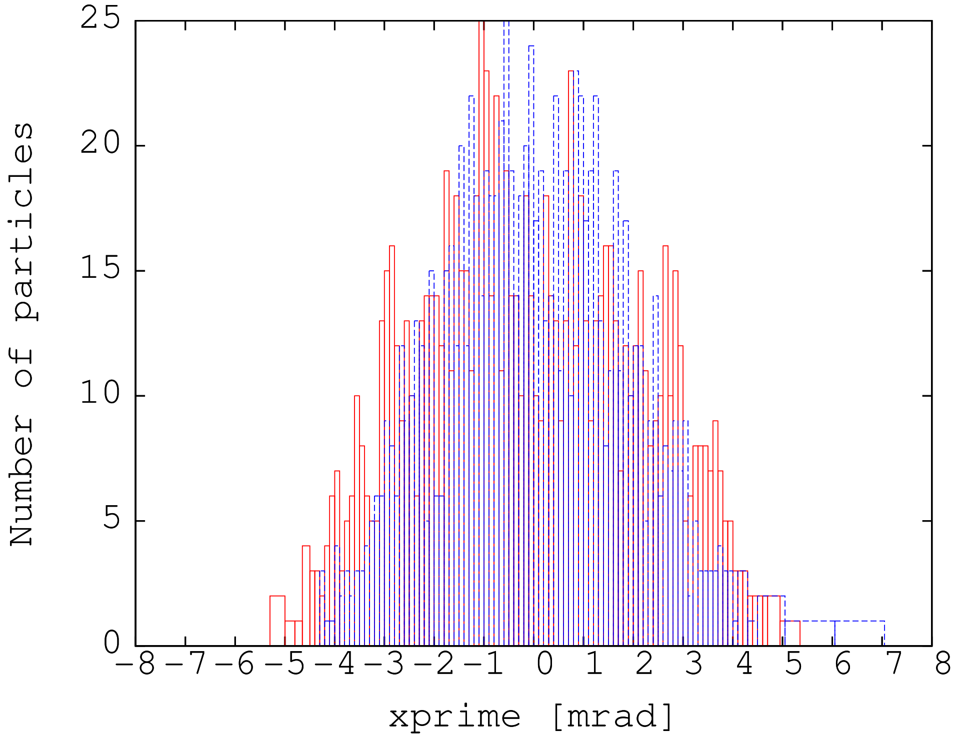

Figure 18: Horizontal distribution in angle of the injected beam (plain red), and at the end of the simulation (dotted blue).