Theory versus neutrino cross section data, overview

Guillermo D. Megias

University of Seville, Spain
RCCN, ICRR, University of Tokyo, Japan

NUSTEC Workshop, 28 March 2022

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No. 839481

Challenge for nuclear models and cross section analysis: How to improve understanding of $\nu - A$ interactions for oscillation analysis?

- Improve theory: FSI, ground states, PB and shell effects (E_b), nuclear potentials, etc.
- Use external constraints (e, e'), $(e, e'p)$ to characterize nuclear effects and improve model selection in ν event generators
- Compare several nuclear models with *e*- and $nu - A$ inclusive (*lepton* detection), semi-inclusive $(l + N,\ l + \pi)$ and more exclusive $(l + p + \pi)$ cross section measurements

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Neutrino- and electron-nucleus scattering are connected (CVC) to each other and a **reliable model** must be able to describe both processes. Neutrinos can probe both V and A nuclear responses, unlike electrons which are only sensitive to the V response.

Differential $\nu - A$ cross section $\quad \chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$

$$\left[\frac{d\sigma}{dk_\mu d\Omega_\mu}\right]_\chi = \sigma_0 \left[V_L R_L + V_T R_T + \chi\,(2 V_{T'} R_{T'})\right]$$

$$V_L R_L = V_{CC} R_{CC} + 2 V_{CL} R_{CL} + V_{LL} R_{LL}$$

$$R_L = R_L^{VV} + R_L^{AA} \; ; \; R_T = R_T^{VV} + R_T^{AA} \; ; \; R_{T'} = R_{T'}^{VA}$$

See M. Barbaro's talk for more details.

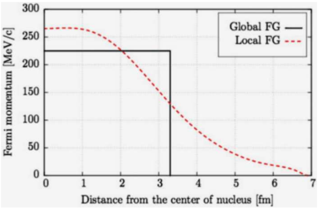
$$\begin{aligned} R_{CC} &= W^{00} \\ R_{CL} &= -\frac{1}{2}\left(W^{03} + W^{30}\right) \\ R_{LL} &= W^{33} \\ R_T &= W^{11} + W^{22} \\ R_{T'} &= -\frac{i}{2}\left(W^{12} - W^{21}\right) \end{aligned}$$

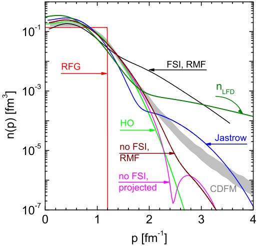
Comparison with (e, e') reactions

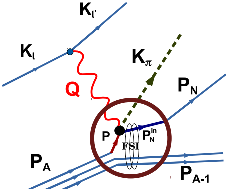
$$\left[\frac{d\sigma}{dk_\mu d\Omega}\right] = \sigma_{Mott}\left(v_L R_L^{VV} + v_T R_T^{VV}\right) \quad ; \quad \sigma_{Mott} = \frac{\alpha^2 \cos^2\theta/2}{4E_i \sin^4\theta/2}$$

Neutrino-Nucleus Interaction Models in the market

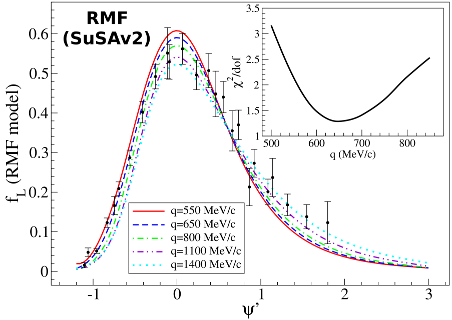
- **Relativistic Fermi Gas (RFG)**: nucleus as a system of non-interacting on-shell nucleons. Bound nucleons below p_F. Too simple. Used for most 2p2h models. Easily extendable to all nuclei.
- **Local Fermi Gas (LFG)**: RFG extension with local density approx ($n(p)$ depends on nucleon position, nuclear finite size effects). Used in NEUT and GENIE. Bad agreement with (e,e') data.
- **Spectral Function (SF)**: based on the factorization ansatz ($\sigma_{\nu-N} \cdot S(p, E_b)$) where S represents the probability of finding a nucleon (p, E_b) within the nucleus. Semiphenomenological based on (e,e'p) data, mean-field calculations and LFG. Shell model. Non relativistic. Implemented in NEUT.
- **Random Phase Approximation (RPA)**: can be added to the top of LFG/SF/HF/MF to incorporate NN correlations. Very accurate description at low q_0, Q^2 but not relativistic.
- **Ab initio calculations**: QMC calculations based on Green's function MC methods. Acurate treatment of nuclear dynamics. Not fully relativistic.
- **Relativistic Mean Field (RMF, ED-RMF, SuSAv2)**: Fully relativistic shell model with accurate description of nuclear dynamics and FSI effects. Bound nucleons: self-consistent Dirac-Hartree solutions, derived within a RMF Lagrangian with local relativistic potentials (S+V) fitted to saturation properties of nuclear matter, radii and nuclear masses. Valid for 1p1h and SPP (π), easily extendable to all nuclei.

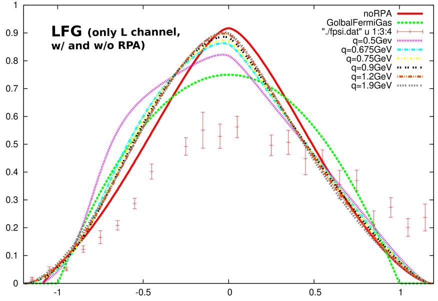






Testing SuperScaling for $^{12}C(e,e')$ in different nuclear models





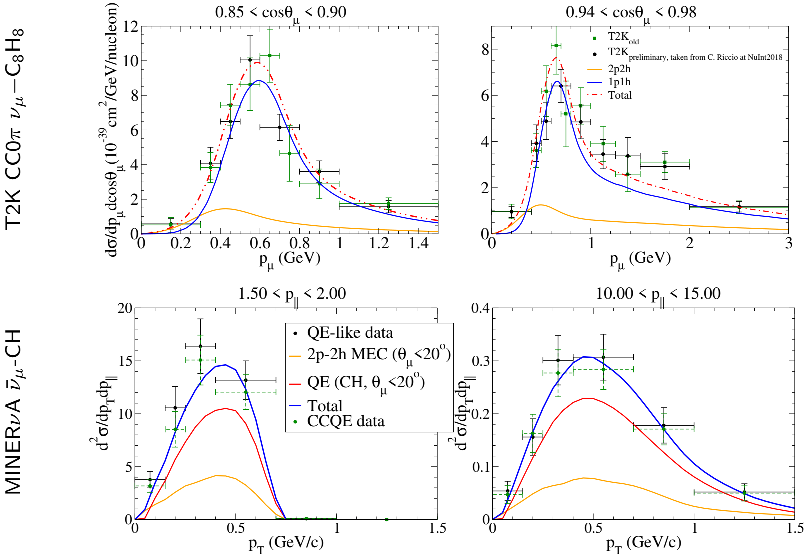
The SuSAv2 model — *PRC90, 035501 (2014)* — *PRD94, 013012 (2016)*

✪ **SuSAv2 model:** lepton-nucleus reactions adressed in the SuperScaling Approach and based on Relativistic Mean Field (RMF) theoretical scaling functions (FSI) to reproduce nuclear dynamics.

✪ RMF: Good description of the QE (e,e') data and superscaling properties $(f^{ee'}_{L,exp})$. RMF predicts $f_T > f_L$ ($\sim$ 20%) as a pure relativistic effect (FSI with the residual nucleus). Strong RMF potentials at high q_3 are corrected by RPWIA and q-dependent blending function.

$$f(\psi) \equiv f(q,\omega) \sim \frac{\sigma_{QE}\left(\substack{\text{nuclear}\\ \text{effects}}\right)}{\sigma_{\text{single nucleon}}\left(\substack{\text{no nuclear}\\ \text{effects}}\right)} \quad ; \quad f(\psi') = k_F \frac{\left(\frac{d^2\sigma}{d\Omega_e d\omega}\right)_{exp}}{\sigma_{Mott}(v_L G_L^{ee'} + v_T G_T^{ee'})}$$

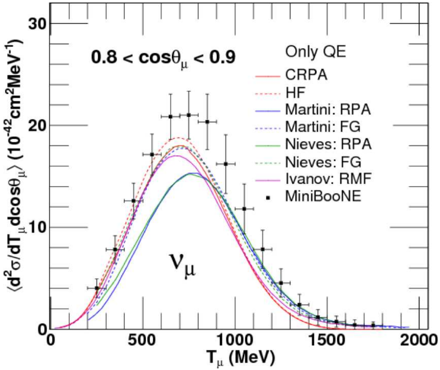
Comparison with CC0π ν_μ-nucleus data



CCQE ν inclusive cross sections with different models

Different models can give similar inclusive CS but different semi-inclusive ones (more sensitive to nuclear-medium effects) $\Rightarrow$ **very different ν oscillation analyses (which relies on semi-inclusive predictions)**

THE FUTURE IS SEMI-INCLUSIVE $\Rightarrow$ Best way to produce consistent theory-vs-data comparison. Less dependency on simulations and deeper analysis of model nuclear effects.



PROBLEM: Current lack of full semi-inclusive models and proper implementation in generators.

Semi-inclusive $\Rightarrow$ Inclusive (but not viceversa) $\Rightarrow$ Factorization approach is questionable.

- QE and 2p2h inclusive: We only need $W^{\mu\nu}(q,\omega)$ or, equivalently, $W^{\mu\nu}(p_\mu, \cos\theta_\mu)$
- QE semi-inclusive : 5D diff. CS (θ_μ, p_μ, p_N, θ_N, ϕ_N) - 2p2h semi-inclusive: 9D diff. CS.

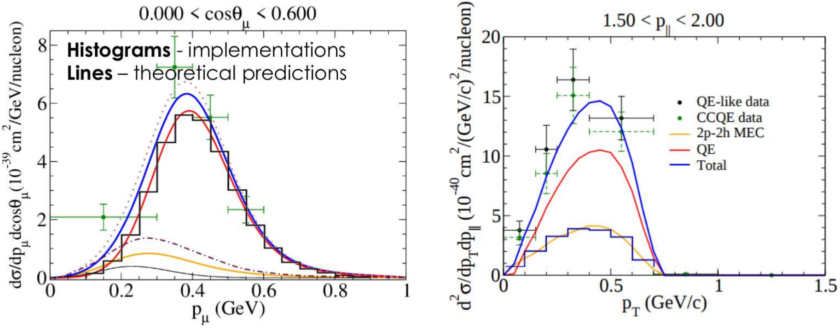
SuSAv2-MEC implementation in MC event generators *arXiv:1905.08556*

1st step: Implemented the SuSAv2 1p1h and 2p2h models in GENIEv3 for both (e, e') and CC ν_μ scattering. **Next step: Implementation in NEUT.**

➲ **New 1p1h and 2p2h model calculated using pre-computed hadron tensors for (e, e') and CC ν reactions**. Global factor / lepton tensor are easily calculated - shared by other models. Use of a GENIE's bilinear interpolation function to evaluate specific q_0, q_3 values. Hadron tensors are initially provided for **a few targets** (C and O so far, may add others). Can easily scale to other nuclei.

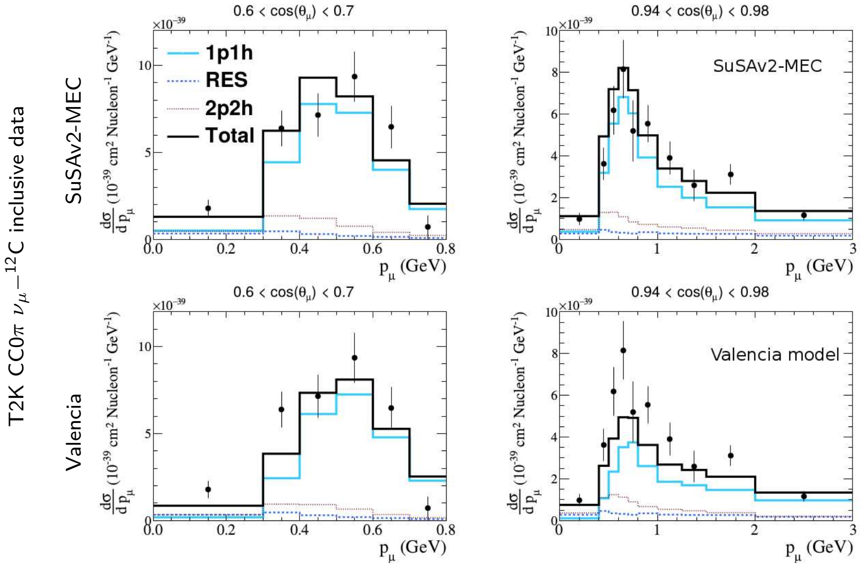
2nd step: Adding SuSAv2 formulas, parameters and parametrization of scaling functions into generators to speed up simulations and to allow reweighting (M_A^{QE}, p_F, E_b, etc.). Introducing RMF nucleon momentum distribution in generators to fully test factorization approach.

3rd step: Implement full RMF semi-inclusive model in generators

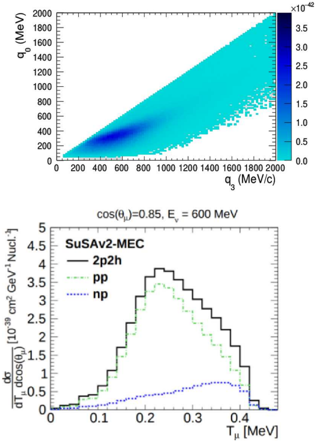


**Adapted from S. Dolan's talk at GENIE Meeting (02/2019)*

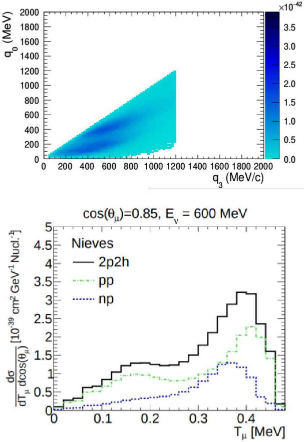
Comparison between 1p1h+2p2h models in generators *arXiv:1905.08556*



New SuSAv2 implementation



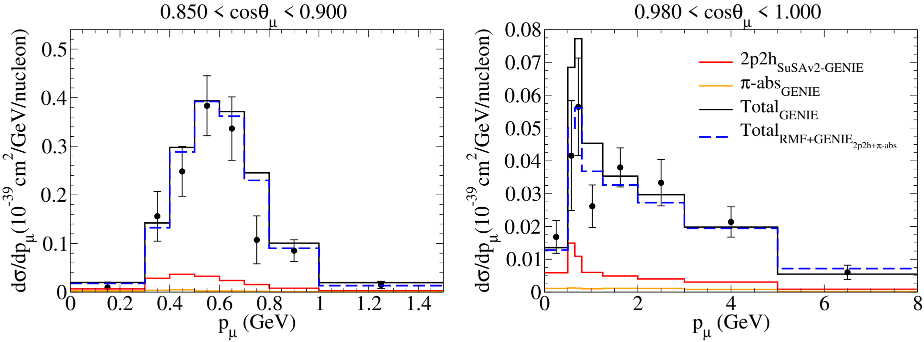
Nieves implementation



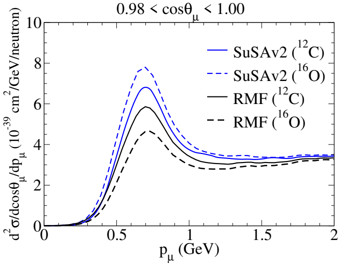
Differences in np/pp separation are mostly related to the treatment of 2p2h direct/exchange interference terms (absent in Nieves model) $\rightarrow$ strongly affects np/pp ratio by a factor $\sim$ 2 (PRC94:054610,2016) $\Rightarrow$ Implications in nucleon multiplicity and hadron E_{reco}

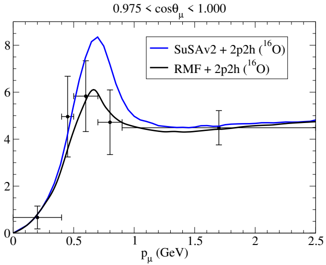
See J.E. Amaro's talk on Wednesday for more details about 2p2h.

Low-energy effects and scaling violations are only appreciable at very forward angles (low q_3, q_0 values). RMF is more accurate than SuSAv2 at these kinematics.

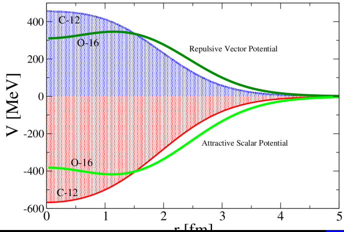


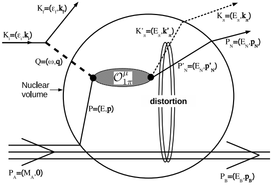
Low-energy nuclear effects and its proper description can have an important effect in the C to O extrapolation, which is essential for T2K and HK.



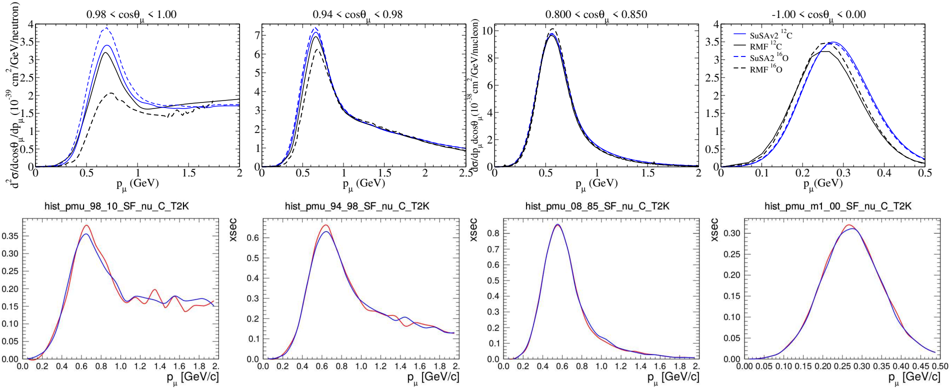


RMF models could reveal C/O differences due to different binding energy and shell effects, mass of the residual nucleus, FSI and Coulomb distortions, etc.





$d^2\sigma/dp_\mu/d\cos\theta_\mu$ vs. p_μ: SuSAv2 and RMF (top) vs NEUT SF (bottom)



Large differences between 12C and 16O emerges at very forward angles (low-energy region) within the **RMF model** due to different nuclear effects (binding energies of the different shells and different S+V nuclear potentials).

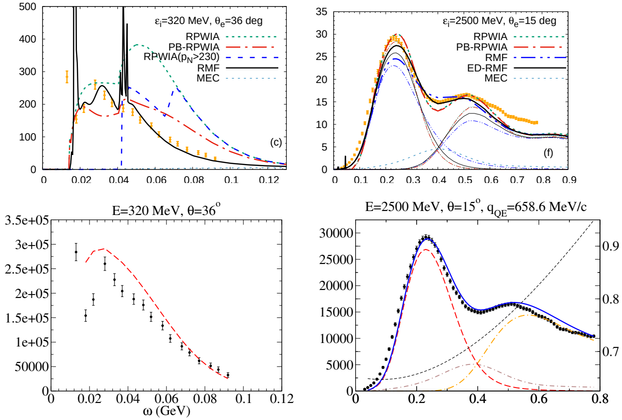
SF (12C) $\sim$ RMF (12C) SF (16O) $>$ RMF (16O) at low-kinematics

SF curves: Red (12C), Blue (16O)

ED-RMF, RMF, SuSAv2 for $(e,e')^{12}C$ $d^2\sigma/d\Omega/d\omega$ vs. ω

RMF, ED-RMF, others

SuSAv2



✪ SuSAv2 is an inclusive model where scaling violations and low-energy effects from RMF (semi-inclusive) are not fully included.

✪ Strong q-dependence of RMF vector and scalar potentials at high kinematics is addressed in SuSAv2 with a blending function to introduce RPWIA (no FSI). *To have a more consistent model and preserve orthogonality, unitarity and dispersion relations* ⇒ **Solution: ED-RMF (both inclusive and semi-inclusive for ^{12}C, ^{16}O, ^{40}Ar, etc.)**

✪ Solution: ED-RMF model (both inclusive and semi-inclusive for ^{12}C, ^{16}O, ^{40}Ar, etc.) introduces an Energy-Dependent potential (based on SuSAv2) to the RMF to keep strength for low p_N while making RMF potential softer for increasing p_N. (PRC 100, 045501 (2019),PRC 101, 015503 (2020))

RMF semi-inclusive formalism vs. NEUT SF

The starting point is the six differential cross section:

$$\frac{d^6\sigma(\varepsilon_i)}{dk_f d\Omega_f dp_N d\Omega_N} = \mathcal{F}\, S(E_m)\frac{k_f^2 p_N^2}{(2\pi)^5}\,\ell_{\mu\nu}H_\kappa^{\mu\nu}\,,$$

The hadron tensor (for a shell κ) is:

$$H_\kappa^{\mu\nu} = \frac{1}{2j+1}\sum_{m_j}\sum_{s_N}[J^{\mu}_{\kappa,m_j,s_N}(Q,P_N)]^* J^{\nu}_{\kappa,m_j,s_N}(Q,P_N)\,,$$

$L_{\mu\nu}H_\kappa^{\mu\nu}$ (RMF): nuclear tensor contraction (nuclear effects included) for each shell (κ)

SF Factorization approach assumes non-relativ. outgoing N with no FSI

$$\frac{d^5\sigma}{d\Omega'_l d\Omega_N dE'_l} \sim S(E_m)L_{\mu\nu}W^{\mu\nu}$$

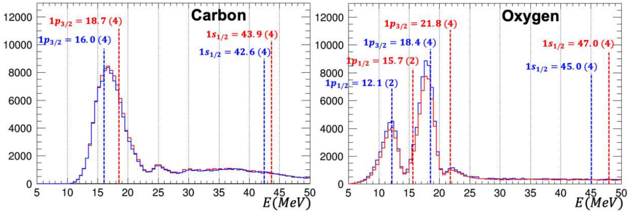
$L_{\mu\nu}W^{\mu\nu}$ (SF): single nucleon tensor contraction (no nuclear effects)

RMF is not based on the SF factorization approach. SF is based on the factorization: single nucleon plane wave (no FSI) CS multiplied by a phenonemological $(e,e'p)$ $S(E_m)$, which can be ok at high energies.

At low energies, it is necessary to consider distorted waves and nuclear effects (FSI, PB, E_b, etc.). These effects can be added on the top of SF using some approaches.

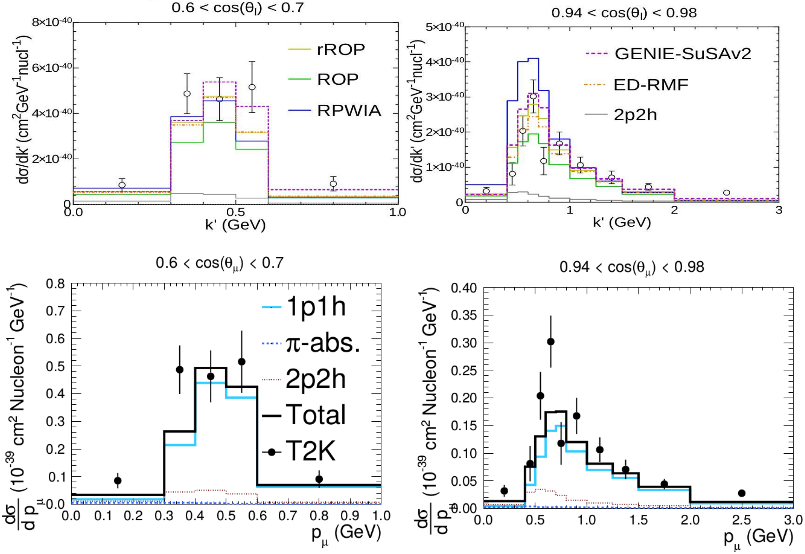
RMF includes all these quantum effects in a natural way as they arises from solving Dirac equation and considering nuclear potentials and nuclear matter properties.

RMF SF $S(E_m)$ can be described just using the fixed E_b values for each shell or via a NEUT-SF like distribution but centered on the RMF E_b values.



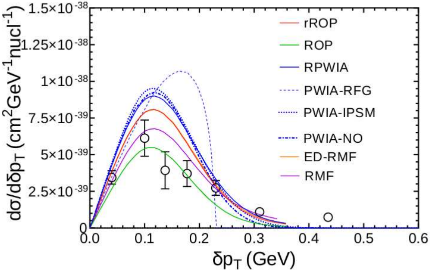
T2K CC0π ν_μ—^{12}C semi-inclusive data (0p>500 MeV)

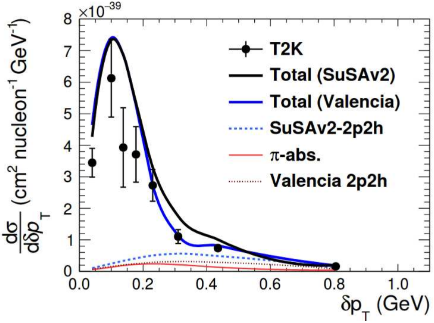
Valencia model

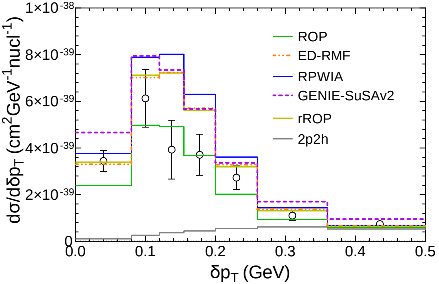


What about more semi-inclusive measurements?

Analysis of FSI effects at T2K CC0π and CC0πNp data (J.F. Franco-Patino et al (in preparation))







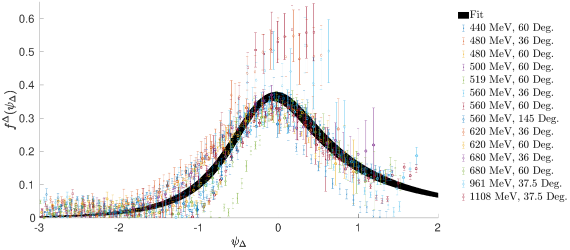
Inclusive total cross section ⇒ Δ-scaling model

Extension of the SuSA approach into the non-QE region, obtained by substracting the QE + 2p-2h MEC + HR and DIS contributions from the total cross section ⇒ assuming that it is dominated by the Δ-resonance.

$$\left(\frac{d^2\sigma}{d\Omega d\omega}\right)^{\mathrm{non-QE}} = \left(\frac{d^2\sigma}{d\Omega d\omega}\right)^{\mathrm{exp}} - \left(\frac{d^2\sigma}{d\Omega d\omega}\right)^{\mathrm{QE,SuSAv2}}_{\mathrm{1p1h}} - \left(\frac{d^2\sigma}{d\Omega d\omega}\right)^{\mathrm{MEC}}_{\mathrm{2p2h}} - \left(\frac{d^2\sigma}{d\Omega d\omega}\right)^{\mathrm{SuSAv2}}_{\mathrm{HR+DIS}}$$

$$f^{\Delta}(\psi_{\Delta}) = k_F \frac{\left(\frac{d^2\sigma}{d\Omega d\omega}\right)^{\mathrm{non-QE}}}{\sigma_M(v_L G_L^{\Delta} + v_T G_T^{\Delta})}$$

Scaling works well up to the center of the Δ peak, $\psi_{\Delta} = 0$, while it breaks at higher energies where other inelastic processes appear ⇒ Error band



J. Gonzalez-Rosa et al. https://arxiv.org/abs/2203.12308

➲ Extension of the SuSAv2 formalism to the complete inelastic spectrum ⇒ resonant (Δ), nonresonant and deep inelastic scattering (DIS).

$$R_{QE}^{L,T} = \frac{\mathcal{N}\xi_F}{\eta_F^3 \kappa m_N} R_{s.n.}^{L,T} f_{SuSAv2}^{L,T}(q_0^{QE}, \psi') \quad ; \quad R_{inel}^{L,T} = \frac{\mathcal{N}\xi_F}{\eta_F^3 \kappa} \int d\mu_X \mu_X R_{inel(s.n.)}^{L,T} f_{SuSAv2}^{L,T}(q_0^{inel}, \psi'_X),$$

$\mu_X = \frac{W_X}{m_N}$: dimensionless invariant mass, ψ'_X: inelastic scaling variable, and $U_{inel}^{L,T}$ depends on the single-nucleon inelastic structure functions W_1, W_2, W_3 ($\equiv F_1, F_2, F_3$), obtained by using:

- Fits of the inelastic structure functions (Bodek-Ritchie, **Bosted-Christy**, ...)

- *PDFs* (GRV98 model, ...)

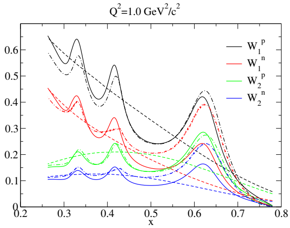
$$F_2^{eN} = \frac{1}{2}\left(F_2^{ep} + F_2^{en}\right) = \frac{5x}{18}\Big(u(x) + \bar{u}(x) + d(x) + \bar{d}(x)\Big).$$

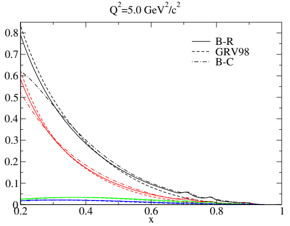
$$F_2^{\nu N} \approx \frac{18}{5} F_2^{eN}. \qquad xF_3^{\nu N} = F_2^{\nu N} - 2\bar{Q}(x)$$

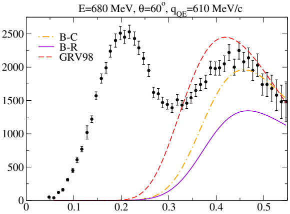
J. Gonzalez-Rosa et al. https://arxiv.org/abs/2203.12308

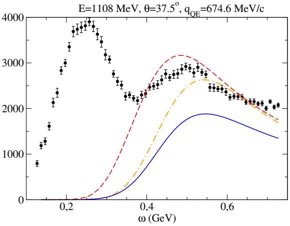
Inelastic structure functions

Inclusive $^{12}C(e,e')$ double differential cross section







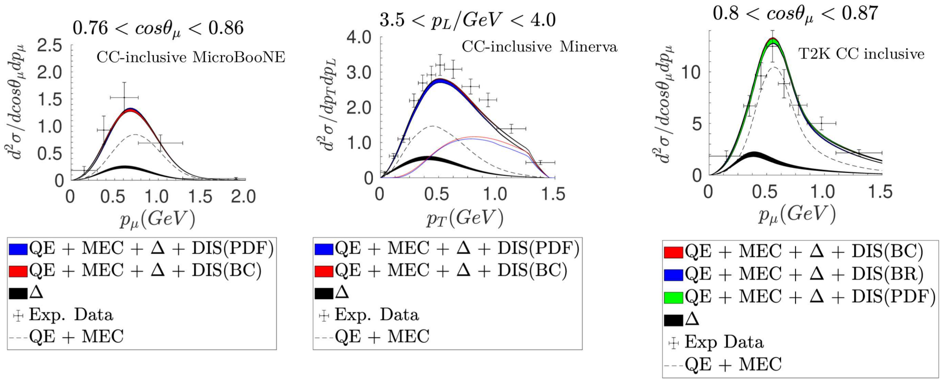


Bodek-Ritchie: poor description of the resonance region.

Bosted-Christy: Good description of the resonant structures observed in (e,e') reactions.

GRV98: No resonant structures (average) and poor description at $Q^2 \lesssim 1$ GeV^2.

SuSAv2-inelastic model for neutrinos



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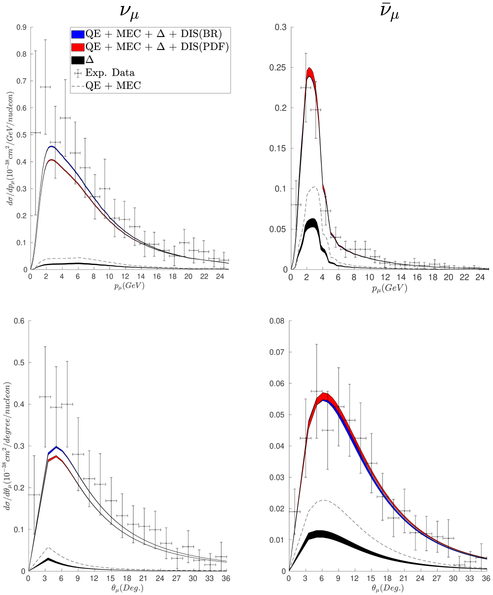
SuSAv2-inelastic model for neutrinos vs. ArgoNEUT



Summary and Conclusions (To be modified)

➲ **Neutrino-nucleus interactions are essential for ν oscillation experiments (T2K, SK)**, being one of the major sources of current systematics.

➲ **The extrapolation to other nuclei (C, O, Ar)** will be essential for future experiments such as HyperK as well as to analyze nuclear-medium uncertainties and inconsistencies between experiments, such as NOvA or MINERvA.

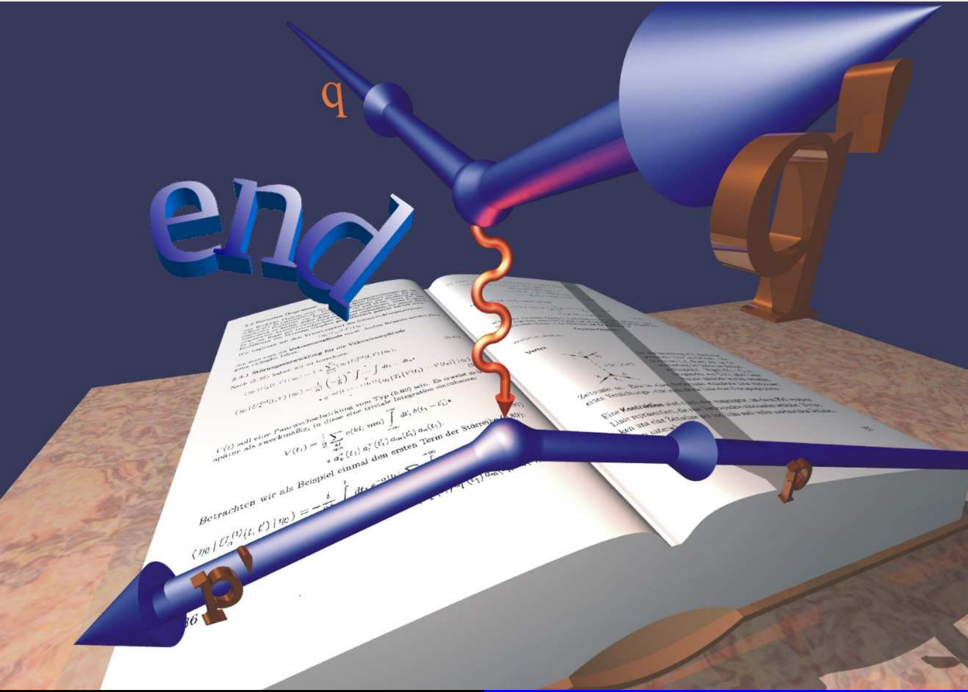
➲ **Forthcoming measurements in water (T2K WAGASCI and NINJA experiments)** will be very useful to validate nuclear models already present in generators.

➲ **Collaboration between experimentalists, generator developers and theorists is essential** to reduce systematics, improve models in MC event generators and gain sensitivity to determination of oscillation parameters, CPV, NMH.

➲ **Validation against (e, e') data is a solid benchmark for nuclear models in ν experiments.** Superscaling is a valuable tool to connect electron and neutrino scattering.

➲ Analysis of semi-inclusive reactions (more sensitive to nuclear model details) is essential for ν oscillation experiments and will help to analyze physics of theoretical models and provide more consistent theory-vs-data comparisons. Different models can give similar inclusive CS but different exclusive ones.

➲ Satisfactory comparison of SuSAv2-MEC and RMF models with (e, e') and (ν, l) inclusive and semi-inclusive data for C, O and other nuclei makes them promising candidate for this purpose.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- SuSAv2 is an inclusive model where scaling violations and low-energy effects present in RMF (semi-inclusive) are not fully included.
- Strong q-dependence of RMF vector and scalar potentials at high kinematics is addressed in SuSAv2 with a blending function to introduce RPWIA (no FSI). *To have a more consistent model and preserve orthogonality, unitarity and dispersion relations* ⇒ **Solution: ED-RMF (both inclusive and semi-inclusive for ^{12}C, ^{16}O, ^{40}Ar, etc.)**
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RMF and ED-RMF are available for 1p1h and SPP, easily extendable to all nuclei. See also PRC100, 045501 (2019), PRC101, 015503 (2020), and A. Nikolakopoulos' talk.

Model	Low kin.	Intermediate kin.	High kin.	(e,e')	CC0pi	CC1pi	Inelastic
RMF	✓	✓	X	✓	✓	✓	X
RPWIA	X	~	✓	✓	✓	✓	X
SuSAv2	~	✓	✓	✓	✓	*	*
ED-RMF	✓	✓	✓	✓	✓	✓	X

*Under some approaches

SuSAv2-MEC 2p2h model is based on RFG microscopic calculations as most 2p2h models (Valencia, Martini, etc.)