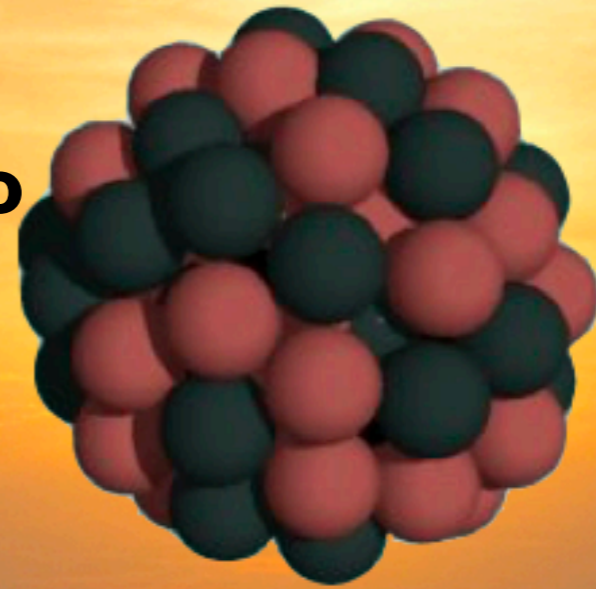


# Electron- versus neutrino-nucleus scattering

**Maria Barbaro**

**Universita' di Torino  
INFN  
IPSA/Paris**



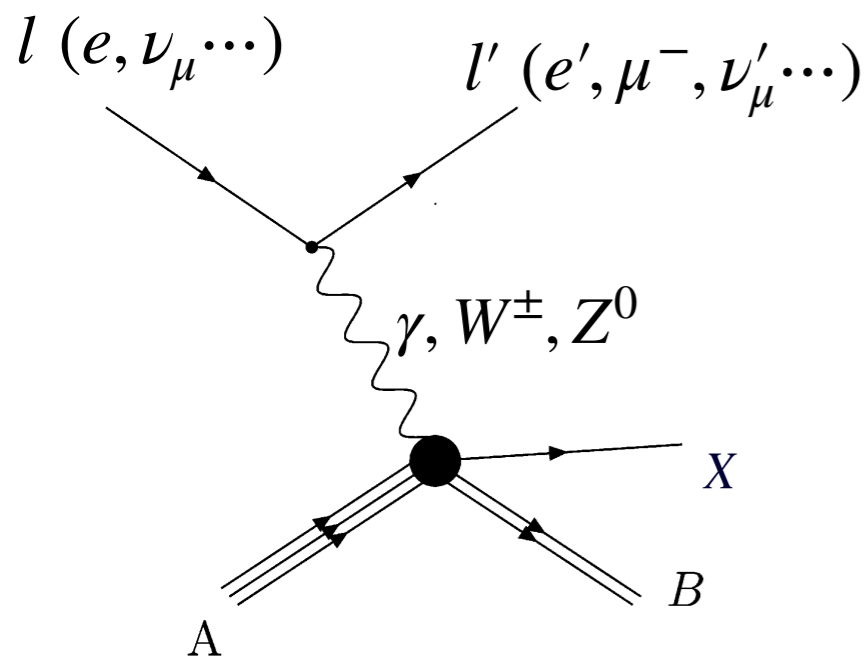
**NuSTEC introducing:  
Expanding our palette**

**Improving the art of neutrino  
nuclei modelling with charged  
lepton scattering data**

**28/3/22 - 1/4/22**

# Similarities and differences between electron and neutrino scattering

- ▶ Nuclear models used in neutrino event generators must be able to describe simultaneously charged and neutral lepton scattering off a nucleus. As a minimal requirement, the model must be tested against electron scattering data before being used to describe neutrino scattering. Pushing it further, electron scattering experimental data can be used to constrain or even predict neutrino cross sections (super-scaling approach).
- ▶ Two sources of difference between e-A and  $\nu$ -A cross sections:
  1. different experimental conditions (monochromatic electron beams versus broadly distributed  $\nu$  beams)
  2. different couplings and currents (weak cross sections have a more complex structure than the electromagnetic ones due to the presence of the axial current).
- ▶ Different kinds of signal may require a different sophistication of nuclear modelling to be properly described



Depending on the detected final state, the measurement can be

- ▶ **Inclusive** ( $l, l'$ ): only the outgoing lepton is detected.
- ▶ **Semi-inclusive** ( $l, l'X$ ): one or more particles are detected in coincidence with the scattered lepton.
- ▶ **Exclusive**: the complete final state, including that of the residual nucleus, is known.

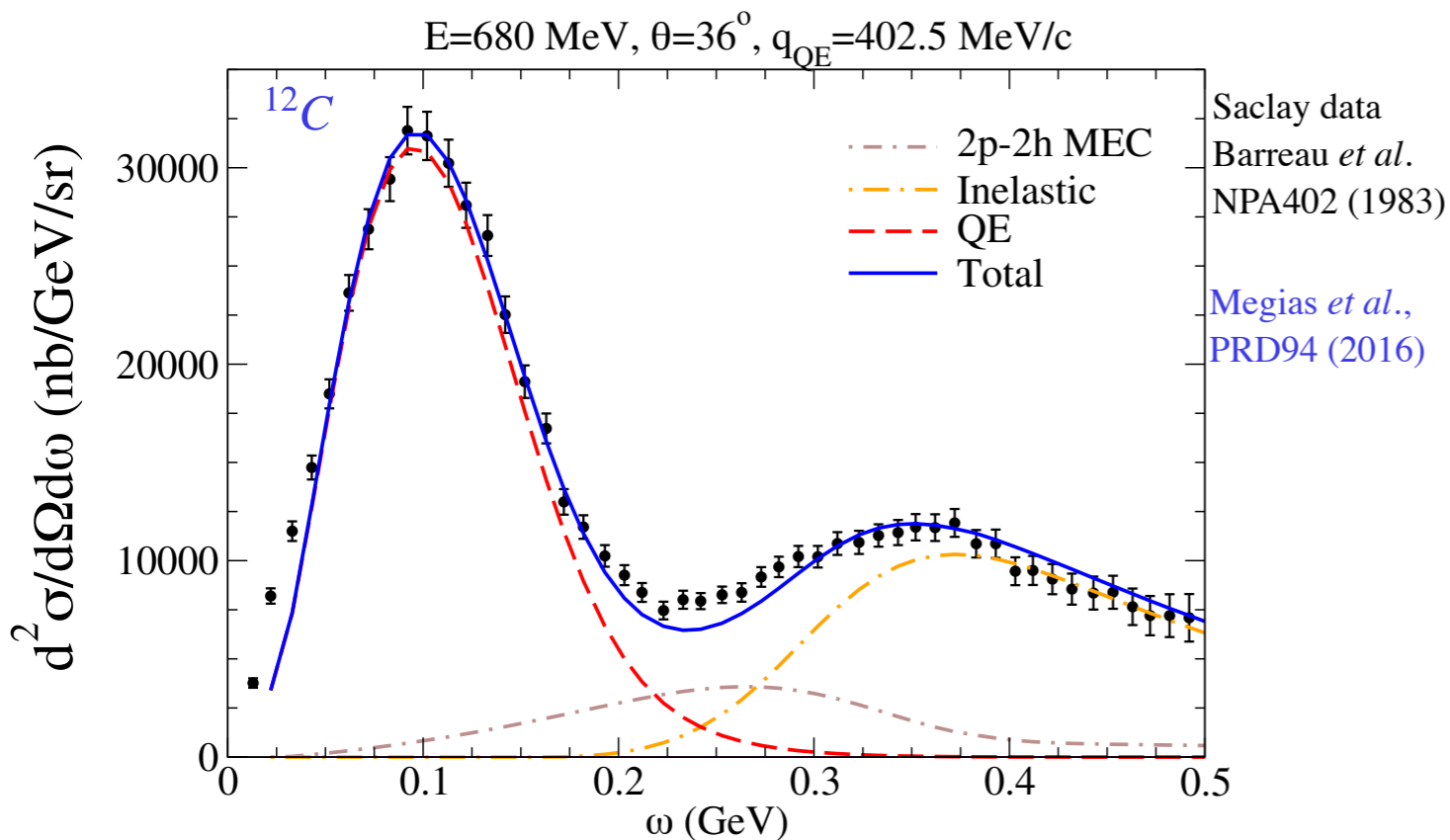
Semi-inclusive predictions require a more detailed description of nuclear effects. A model may describe reasonably well inclusive cross sections and badly fail to agree with semi-inclusive data (typical example the RFG)

# Monochromatic versus broadly distributed incident beams

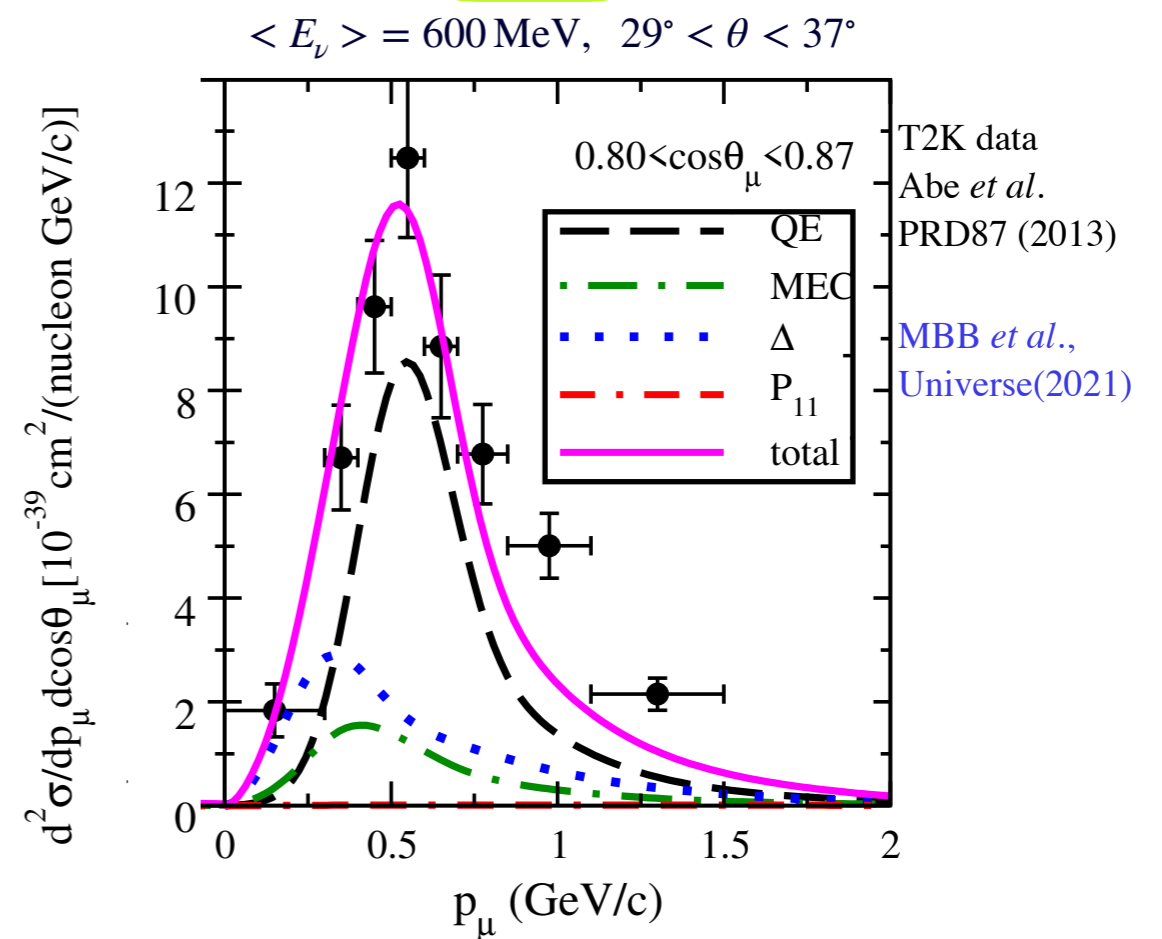
As a consequence of the flux integration, in neutrino experiments different processes occurring in the nucleus cannot be disentangled in the experimental inclusive signal.

## Inclusive cross sections at similar kinematics

$(e, e')$



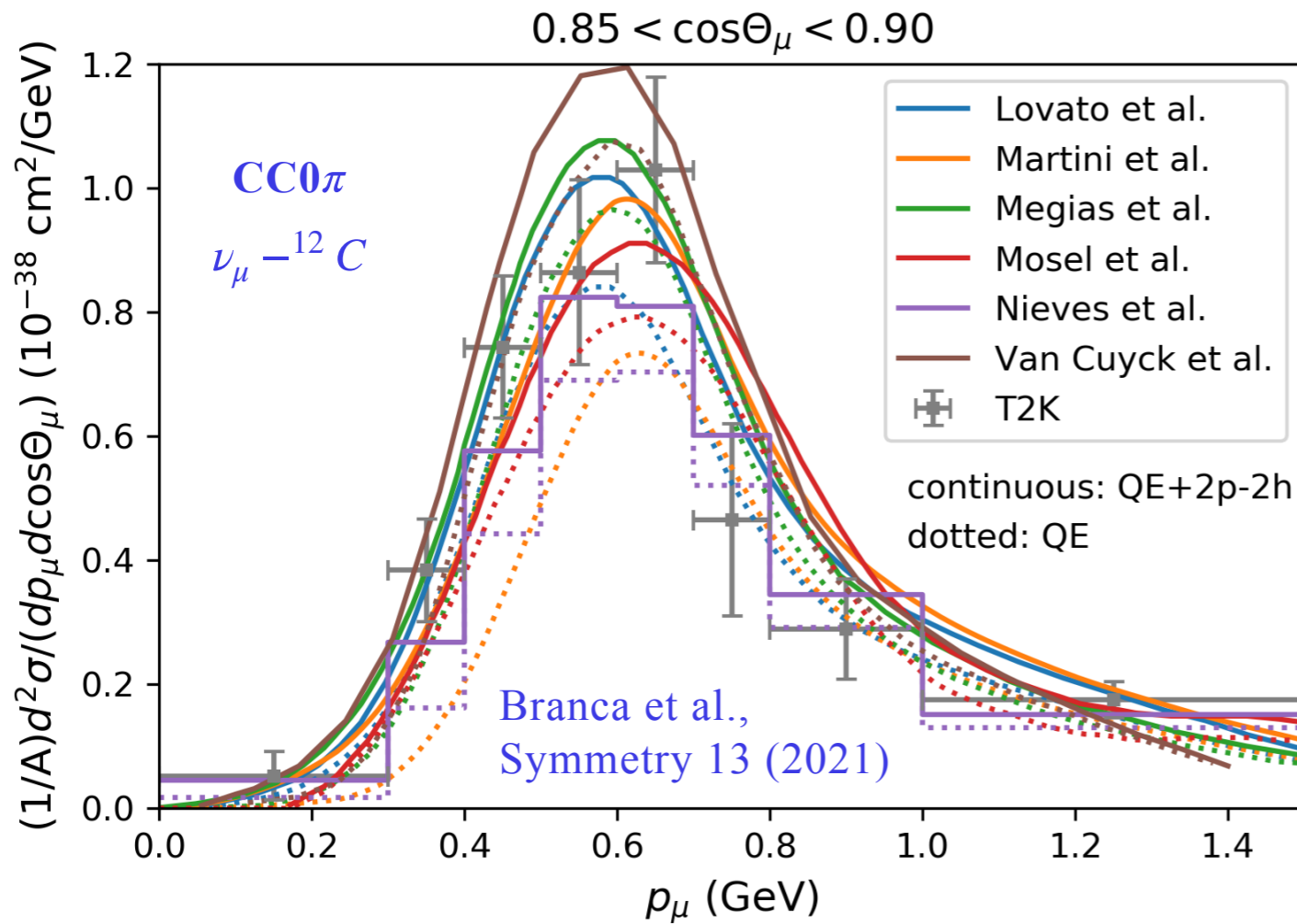
$(\nu_\mu, \mu)$



The quasi-elastic and  $\Delta$  resonance peaks can be separately identified in the  $(e, e')$  spectrum, the 2p2h response filling the dip in  $\omega$  between the two, but in neutrino data the three contributions overlap due to the flux integration.

“Tuning” one of the three contributions to adjust theory to  $\nu$ -A data may destroy the agreement with  $e$ -A data.

# Comparison of different models with CC0 $\pi$ data

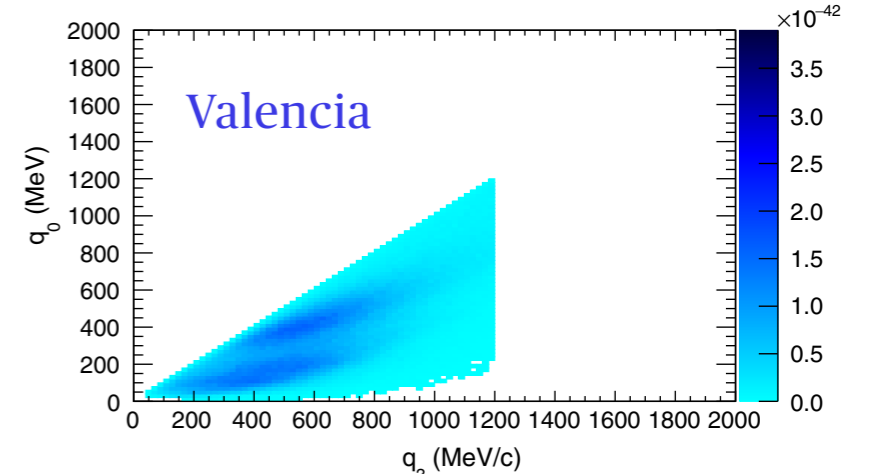
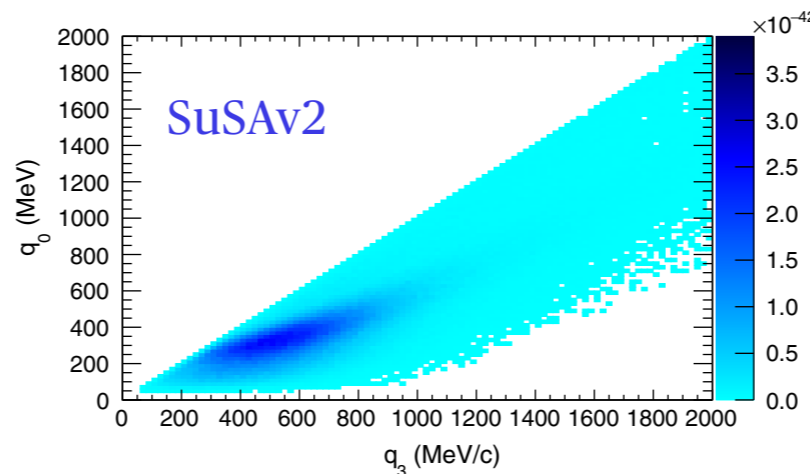
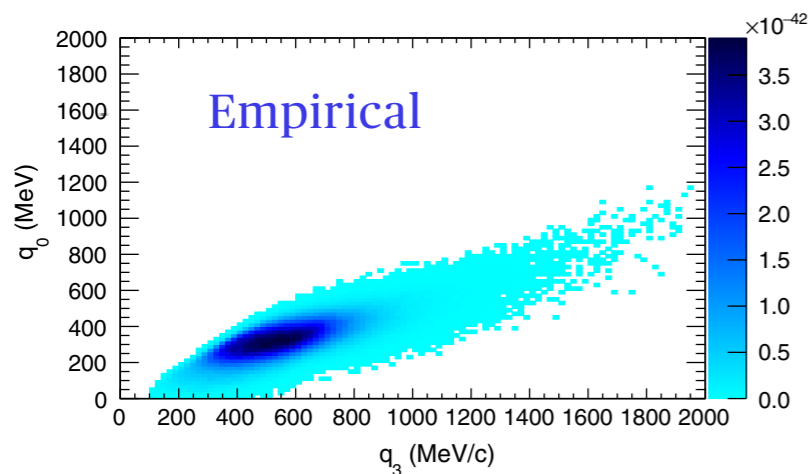


- ▶ Lovato et al.
- ▶ Martini et al.
- ▶ Megias et al.
- ▶ Mosel et al.
- ▶ Nieves et al.
- ▶ Van Cuyck et al.
- ▶ T2K
- ▶ GFMC ab initio
- ▶ RPA/Paris-Lyon
- ▶ SuSAv2
- ▶ GIBUU
- ▶ RPA/Valencia
- ▶ HF-CRPA Ghent

- ▶ Quite large spread between theoretical results
- ▶ Electron scattering ideal tool to discriminate between models.
- ▶ All results agree on the important role of two-body currents (2p2h excitations)

Large differences between 2p2h models presently implemented in GENIE

Dolan et al.,  
PRD101 (2020)



Further constraints can be obtained from:

- ▶ validation versus other data: electron scattering
- ▶ comparison with more exclusive data, involving the final hadrons variables, now available from T2K, MINERvA, MicroBooNE

# Lepton-nucleus cross section

$$d\sigma \sim g^4 D_V^2(Q^2) \eta_{\mu\nu} W^{\mu\nu} = \sigma_0 \mathcal{F}$$

$$\mathcal{F} \equiv \eta_{\mu\nu} W^{\mu\nu}$$

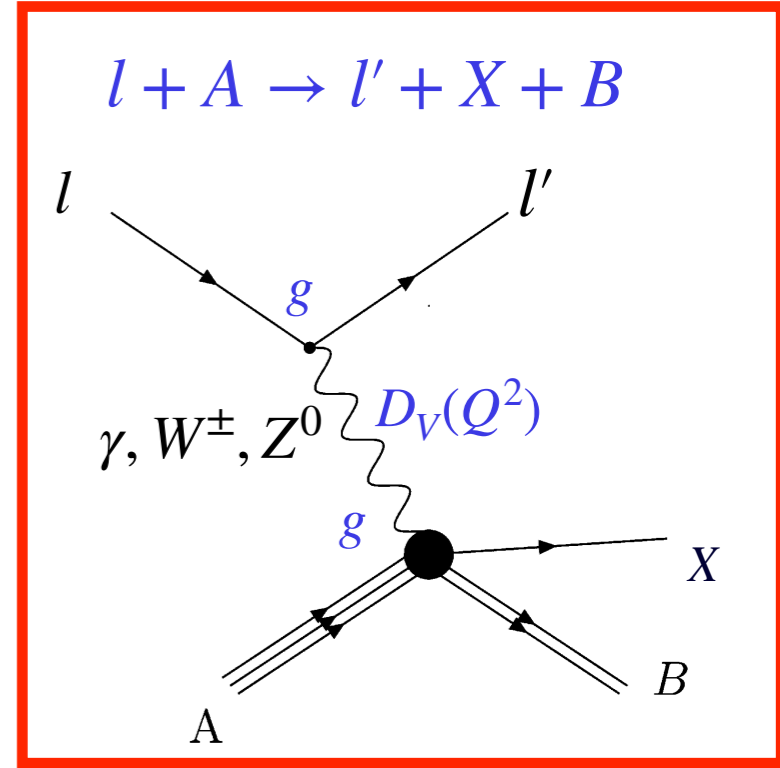
$$\sigma_0 \sim 1/Q^4 \quad \text{em}$$

$$\sigma_0 \sim G_F^2 \quad \text{weak}$$

Leptonic tensor  $\eta_{em}^{\mu\nu(VV)}$  purely vector

$$\eta_{weak}^{\mu\nu} = \eta^{\mu\nu(VV)} + \eta^{\mu\nu(VA)} + \eta^{\mu\nu(AA)} \quad \text{vector and axial}$$

Nuclear tensor  $W^{\mu\nu} = \sum \delta(E_f - E_i - \omega) \langle f | J^\mu(Q) | i \rangle^* \langle f | J^\nu(Q) | i \rangle$



## Inclusive scattering (l, l')

2 variables:  $R_K = R_K(q, \omega)$

### Electron

2 response functions

$$\mathcal{F}_{(e,e')} = V_L R_L + V_T R_T$$

### (Anti-)Neutrino

5 response functions

$$\mathcal{F}_{\pm}^2 = V_{CC} R_{CC} + 2V_{CL} R_{CL} + V_{LL} R_{LL} + V_T R_T \pm V_T' R_T'$$

## Semi-inclusive scattering (l, l'N)

5 variables:  $R_K = R_K(q, \omega, \mathbf{p}_N)$

4 response functions

$$\mathcal{F}_{(e,e'p)}^2 = V_L R_T + V_T R_T + V_{TL} R_{TL} + V_{TT} R_{TT}$$

10 response functions

$$\mathcal{F}_{\pm}^2 = V_{CC} R_{CC} + 2V_{CL} R_{CL} + V_{LL} R_{LL} + V_T R_T \pm V_T' R_T' + V_{TT} R_{TT} + V_{TC} R_{TC} + V_{TL} R_{TL} \pm (V_{TC}' R_{TC}' + V_{TL}' R_{TL}')$$

Inclusive results cannot be used to predict semi-inclusive cross sections

# Super-Scaling

The idea of exploiting  $e$ -scattering for  $\nu$ -scattering studies was proposed in

J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, A. Molinari, I. Sick, PRC71 (2005) 015501

**“Using electron scattering superscaling to predict charged-current neutrino cross sections in nuclei”**

based on the “superscaling” properties of inclusive electron scattering ( $e, e'$ ) data, extensively analysed in the 90s

[Day et al., Ann.Rev.Nucl.Part.Sci.40 (1990); Donnelly and Sick, PRL82; PRC60 (1999)]

## SuperScaling function

$$f(q, \omega; k_F) = k_F \times \frac{[d^2\sigma/d\omega d\Omega]_{exp}^{(e,e')}}{\bar{\sigma}_{eN}} \longrightarrow f(\psi)$$

The **scaling variable**  $\psi \equiv \psi(q, \omega)$  - or  $y(q, \omega)$  - is a combination of  $q$  and  $\omega$  (analogous to  $x$  in DIS) related to the momentum of the bound nucleon

**Superscaling:** in the scaling region (below QEP)  $f$  is

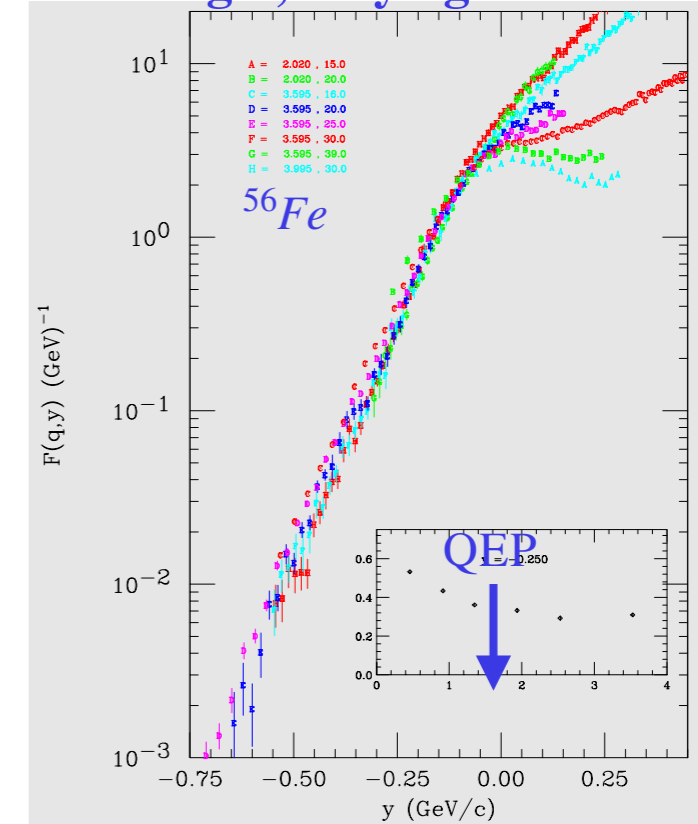
- ▶ independent of the kinematics ( $q$ ) for a given nucleus
- ▶ independent of the nucleus ( $k_F$ ) for given kinematics

$f$  embodies the nuclear dynamics (initial and final state).

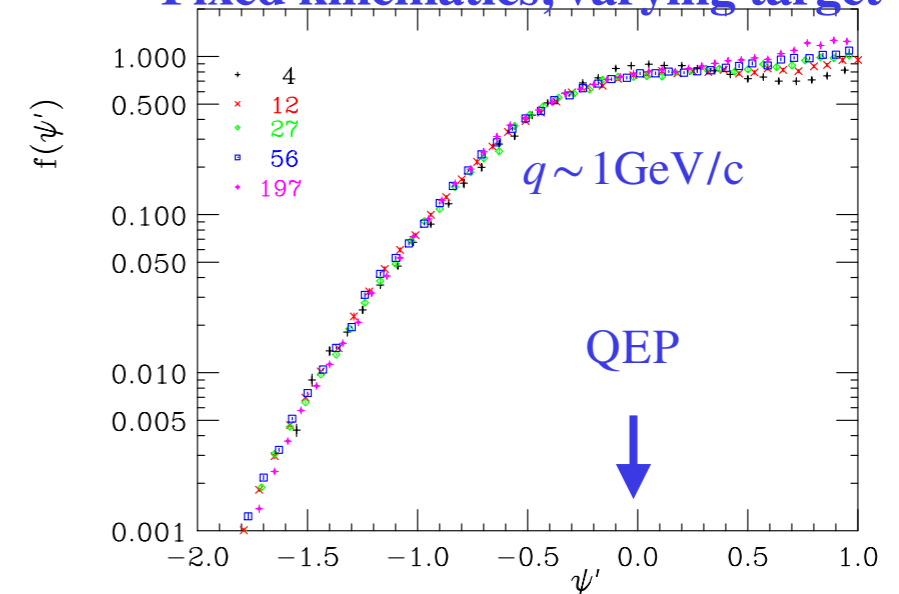
It can be extracted from experiment or calculated within a model and plugged into the neutrino cross section to make predictions at different kinematics and on different nuclei

- ▶ **Scaling violations** occur beyond the QEP mainly in the transverse channel ( $\Delta$ , 2p2h and other inelastic processes)
- ▶ **The approach is valid at high enough momentum transfer** ( $q \gtrsim 300$  MeV/c), where collective effects are absent

## Fixed target, varying kinematics

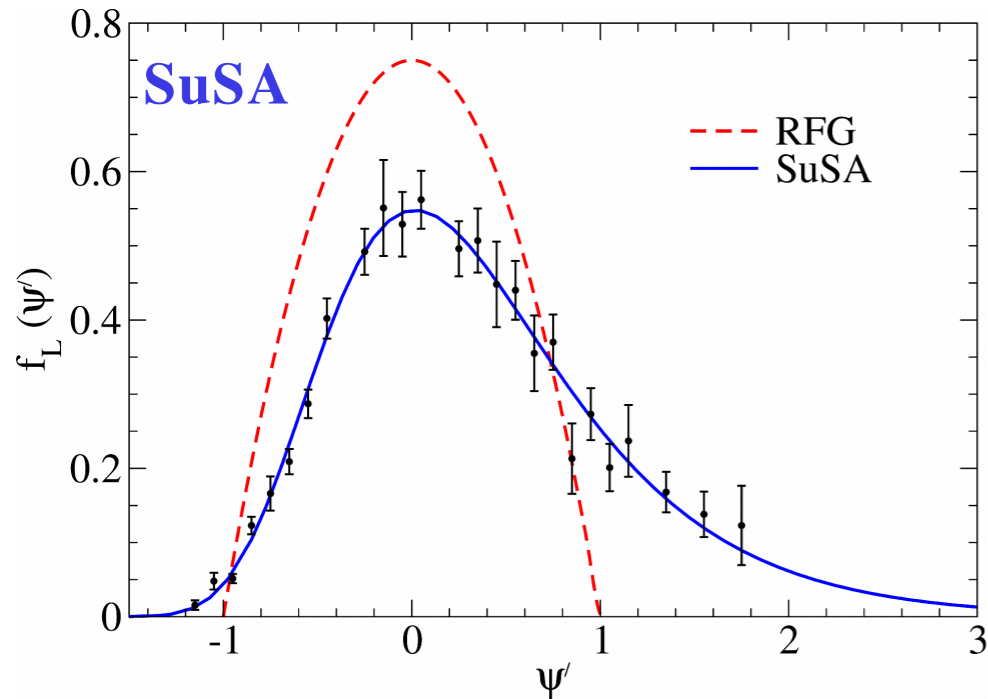


## Fixed kinematics, varying target



# SuSA and SuSAv2 in the quasi-elastic region

Amaro *et al.*, PRC71 (2005)



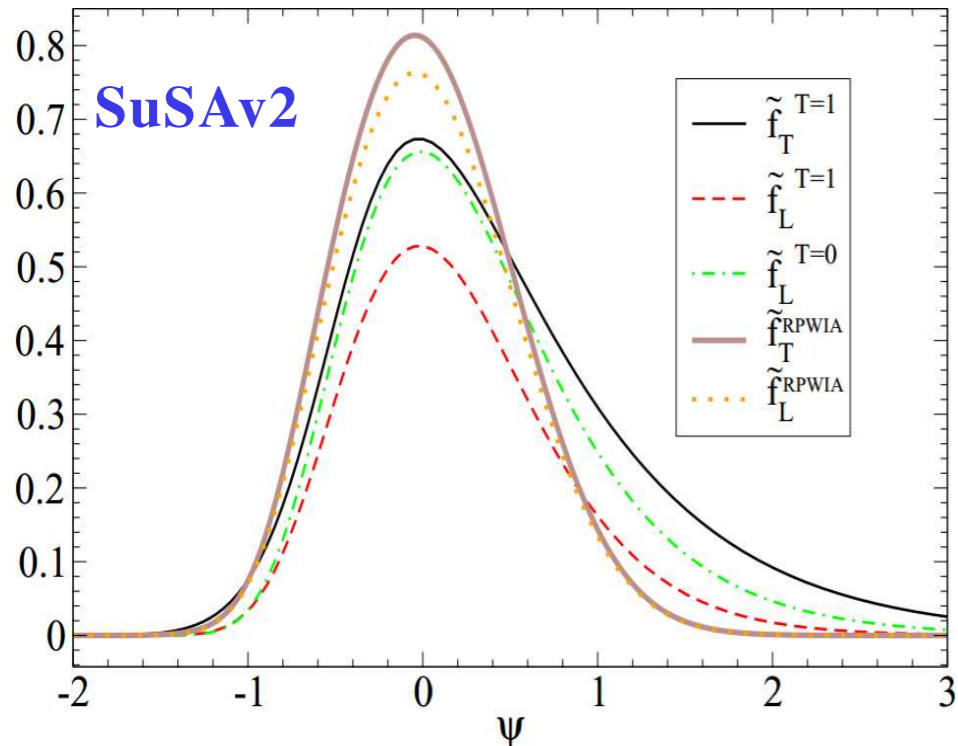
## SuSA

- ▶ phenomenological
- ▶ only one scaling function extracted from longitudinal ( $e, e'$ ) data

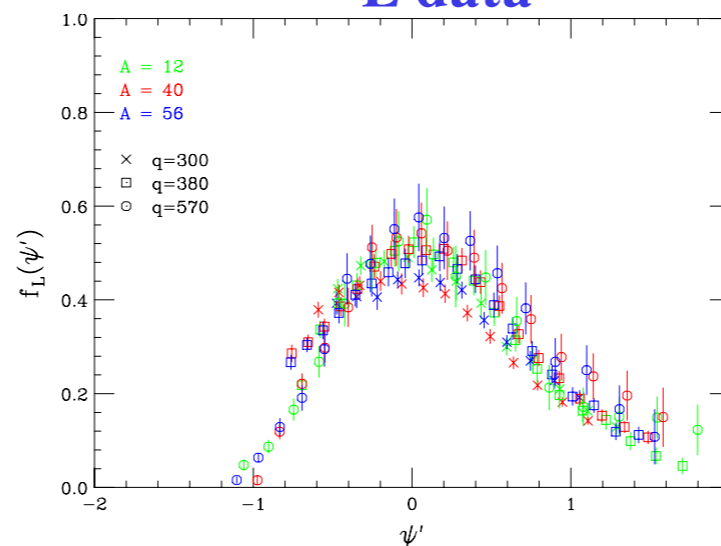
## SuSAv2

- ▶ based on microscopic RMF calculation
- ▶ a set of scaling functions in L, T and isospin channels
- ▶  $f_T > f_L$  in agreement with L/T separated ( $e, e'$ ) data
- ▶ corrects the RMF problem of too strong S and V potentials at high energies recovering RPWIA asymptotic results
- ▶ parameters fitted once and for all to carbon data

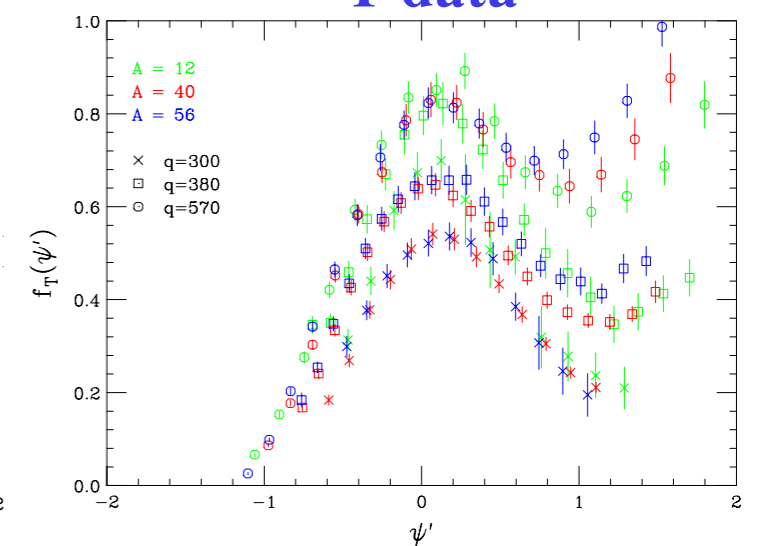
Gonzalez *et al.*, PRC90(2014)



## L data



## T data



The scaling function(s) are used to describe **simultaneously** electron and neutrino scattering

## Beyond the quasi-elastic region

- ▶ **Two-body currents** break scaling and the 2p2h contribution is implemented in the SuSAv2 model within a fully relativistic framework based on the RFG (**see Amaro's talk on Wednesday**)
- ▶ The scaling approach has been extended to the **inelastic region** for both electron and neutrino inclusive scattering.

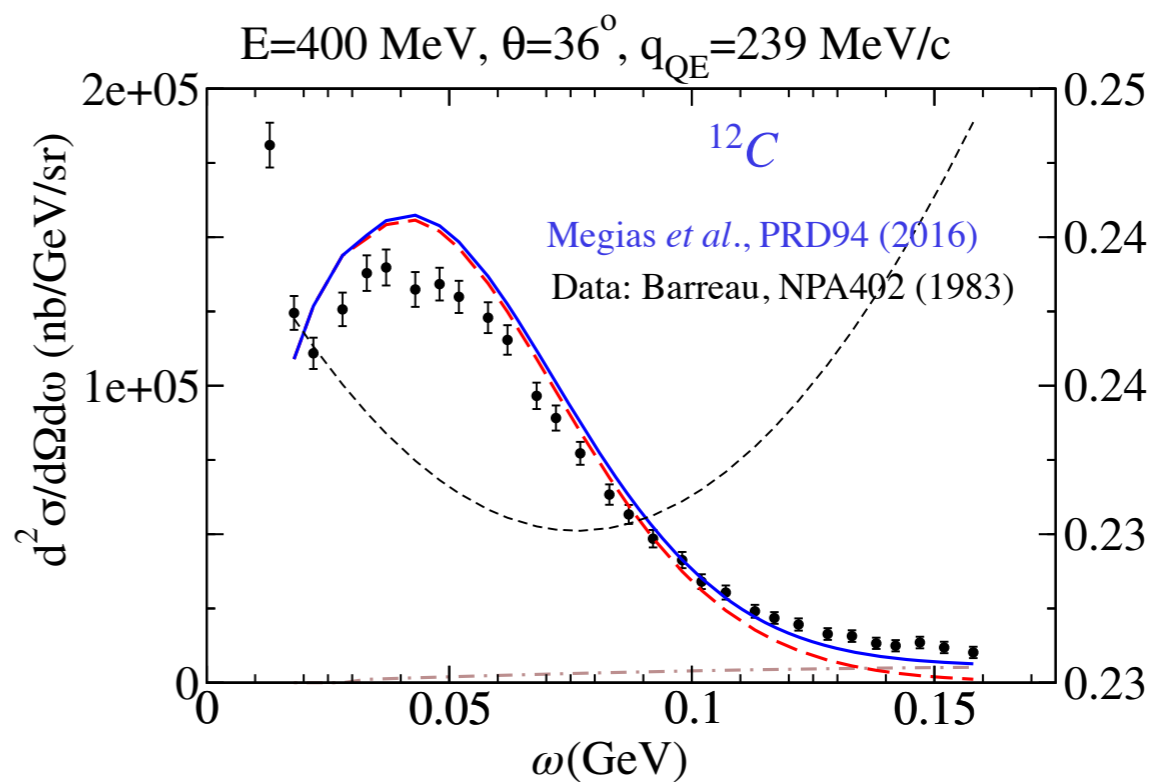
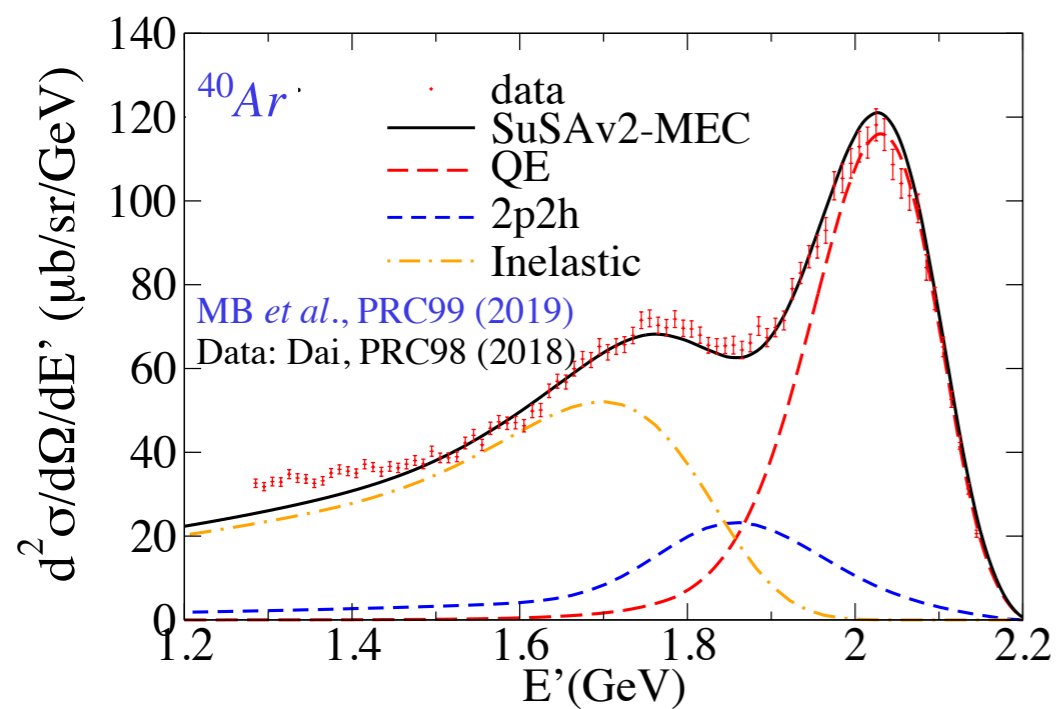
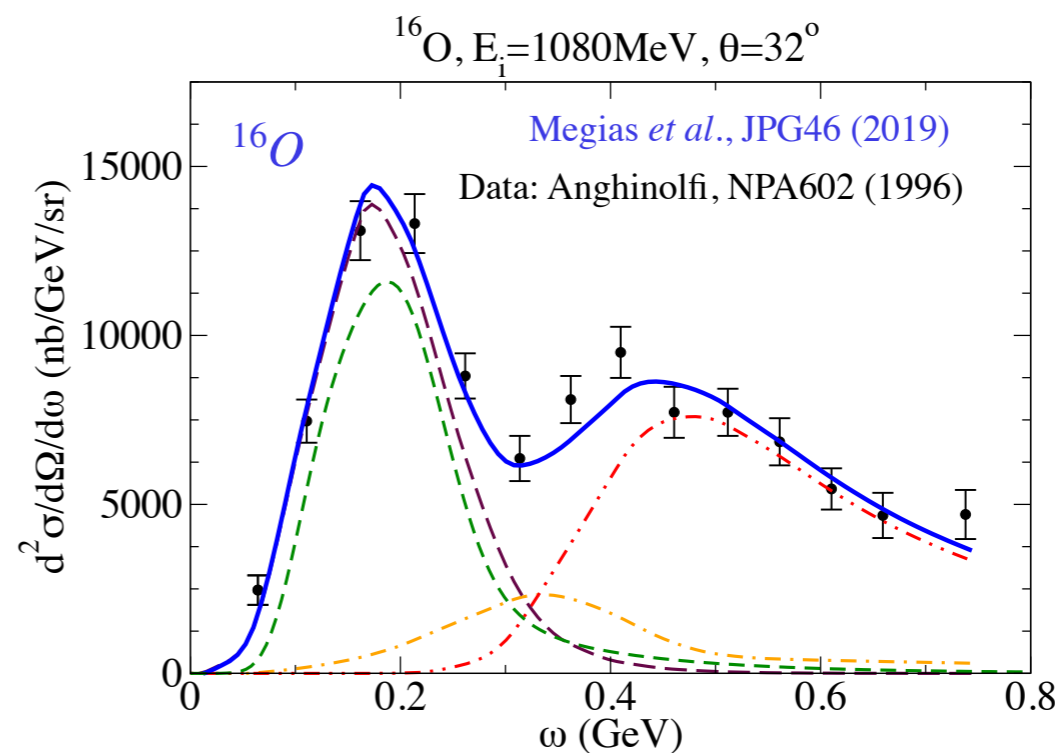
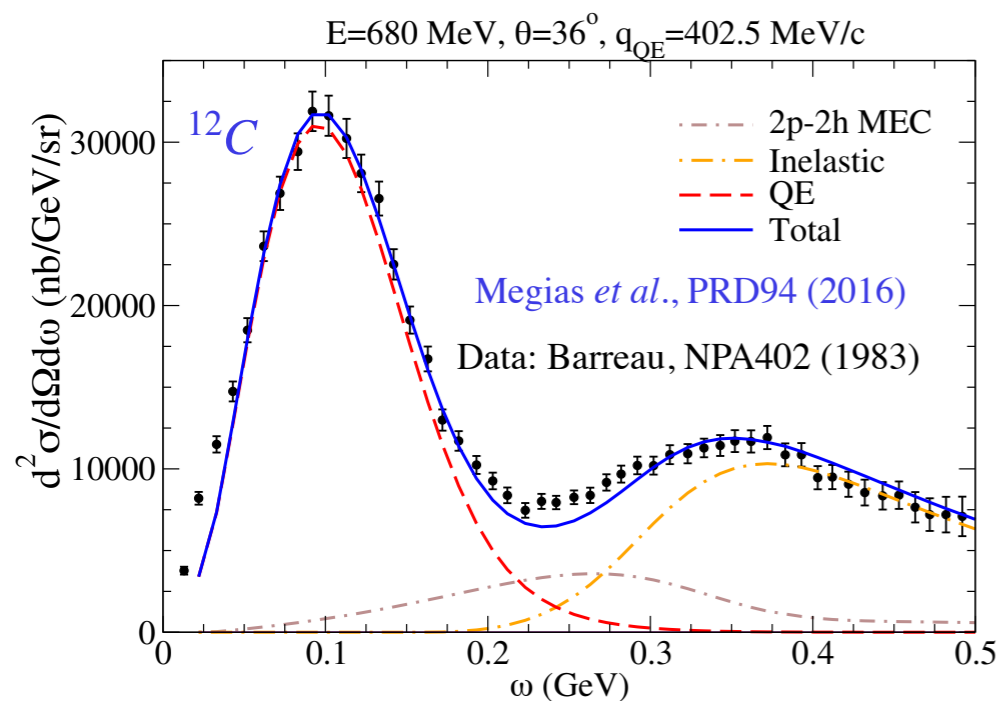
The general strategy consists in introducing a scaling variable  $\psi_X$  for each invariant mass  $W_X$  and fold the elementary response  $G_K^{inel}$  with the scaling variable  $f(\psi_X)$  to obtain the nuclear response  $R_K^{inel}$

$$R_K^{inel}(q, \omega) = N \frac{2m_N T_F}{k_F^3 q} \int_{W^{min}}^{W^{max}} dW_X W_X f^{model}(\psi_X) G_K^{inel}(q, \omega),$$

- ▶ For electron scattering the elementary responses are taken from phenomenological fits of the single nucleon structure functions  $w_1$  and  $w_2$  [Bosted and Christy, PRC77 (2008)&PRC81 (2010)] and the SuSAv2 superscaling function is used.
- ▶ An alternative approach has also been adopted for the  $\Delta$  resonance region, where a scaling function has been extracted from  $(e, e')$  data following the same procedure used for QE, but subtracting from the data the QE and 2p2h contributions.
- ▶ The extension to neutrino scattering is limited by the poor knowledge of the structure function  $w_3$  in the resonance and SIS regions. Different approaches are explored in this recent work:  
J. Gonzalez-Rosa et al., nucl-th/2203.12308 (**see G. Megis' talk**)

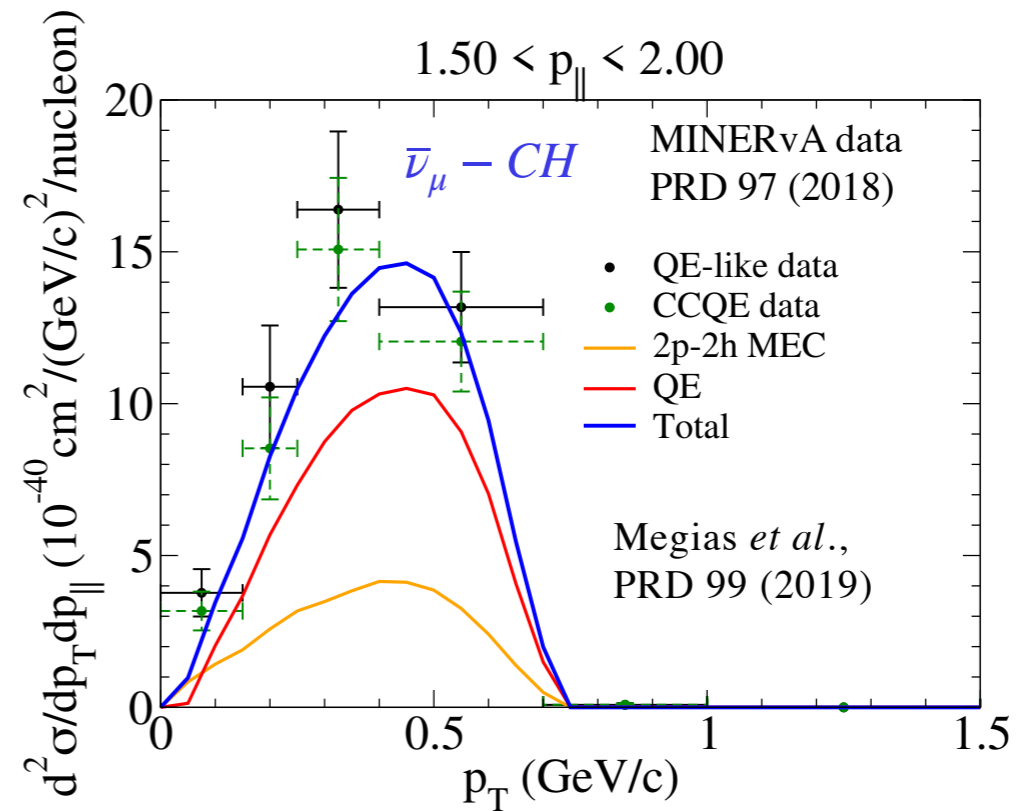
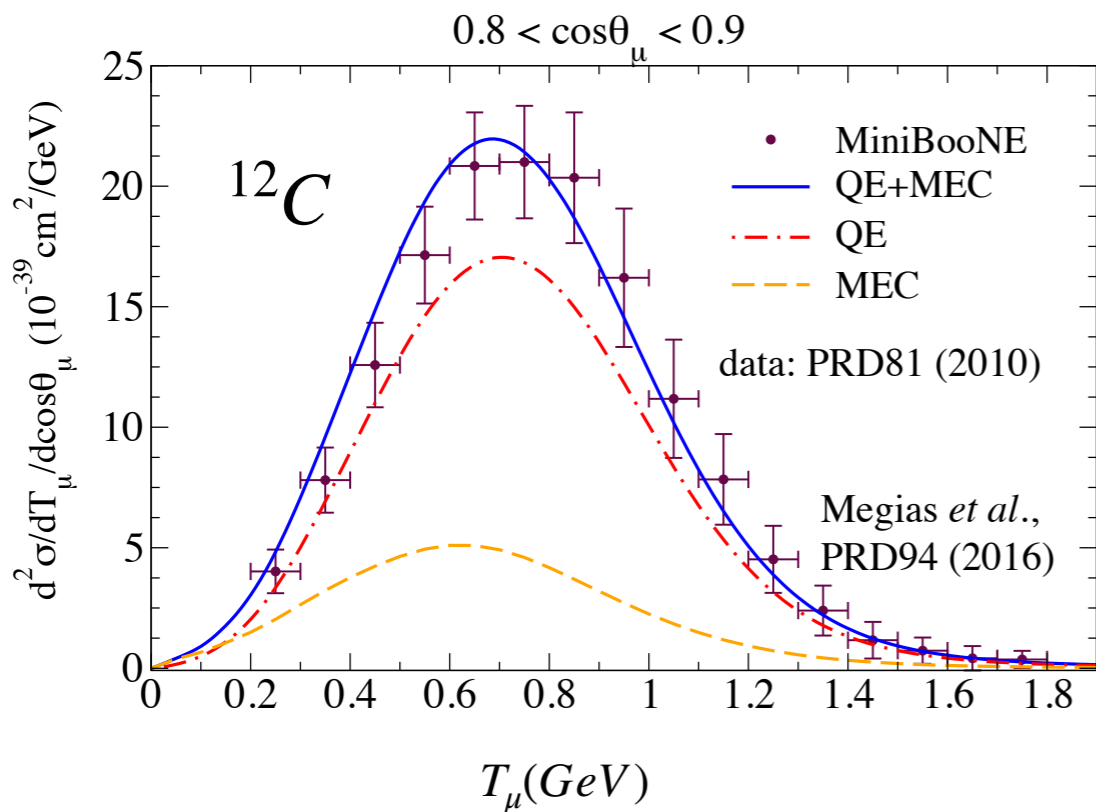
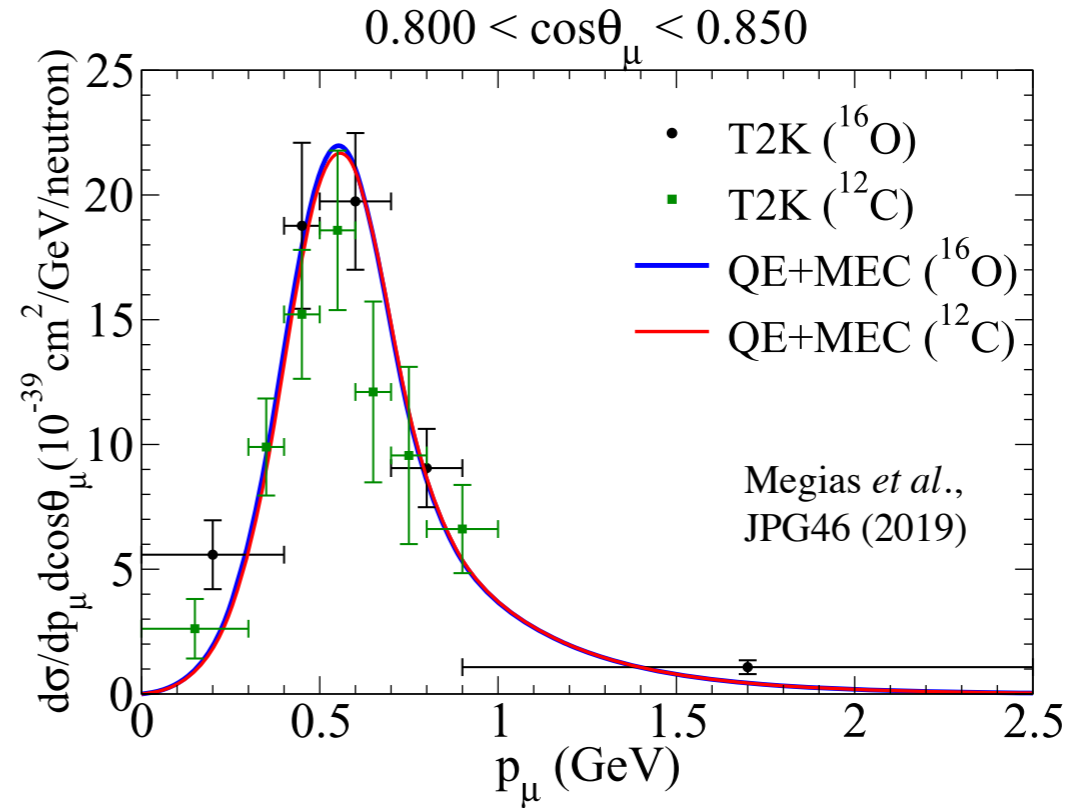
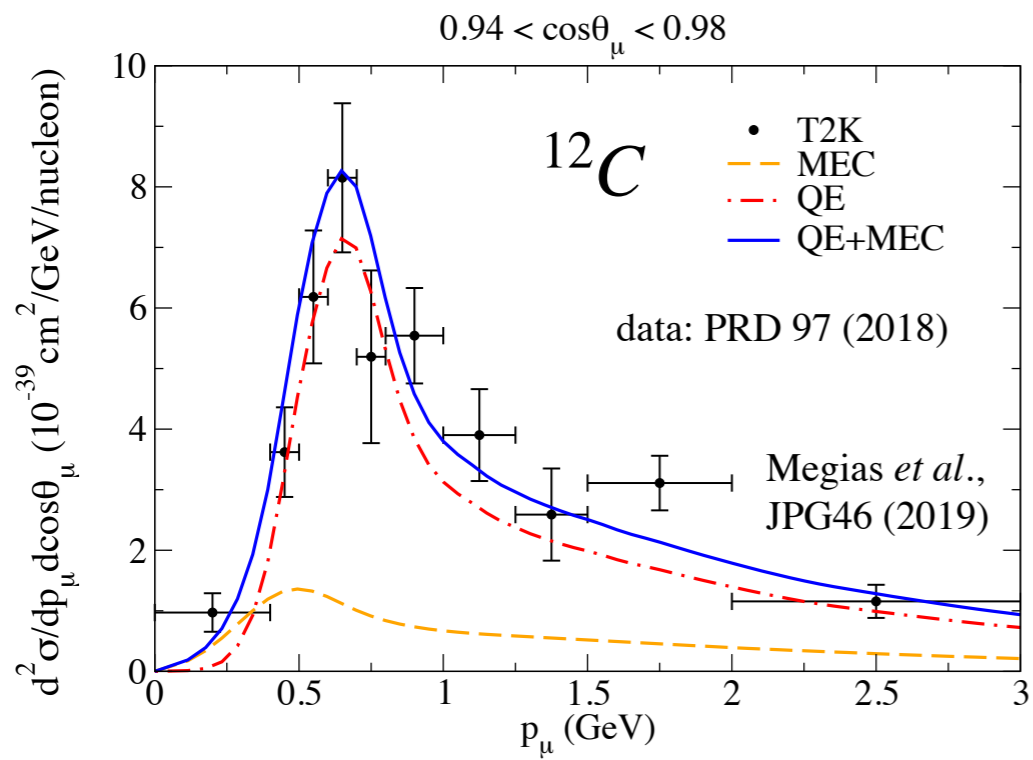


# Validation: SuSAv2 predictions for inclusive ( $e, e'$ )



Good agreement with data for different nuclei in a wide kinematical region, with the exception of the very low  $q$  regime, where the superscaling approach and IA fail and collective effects dominate.

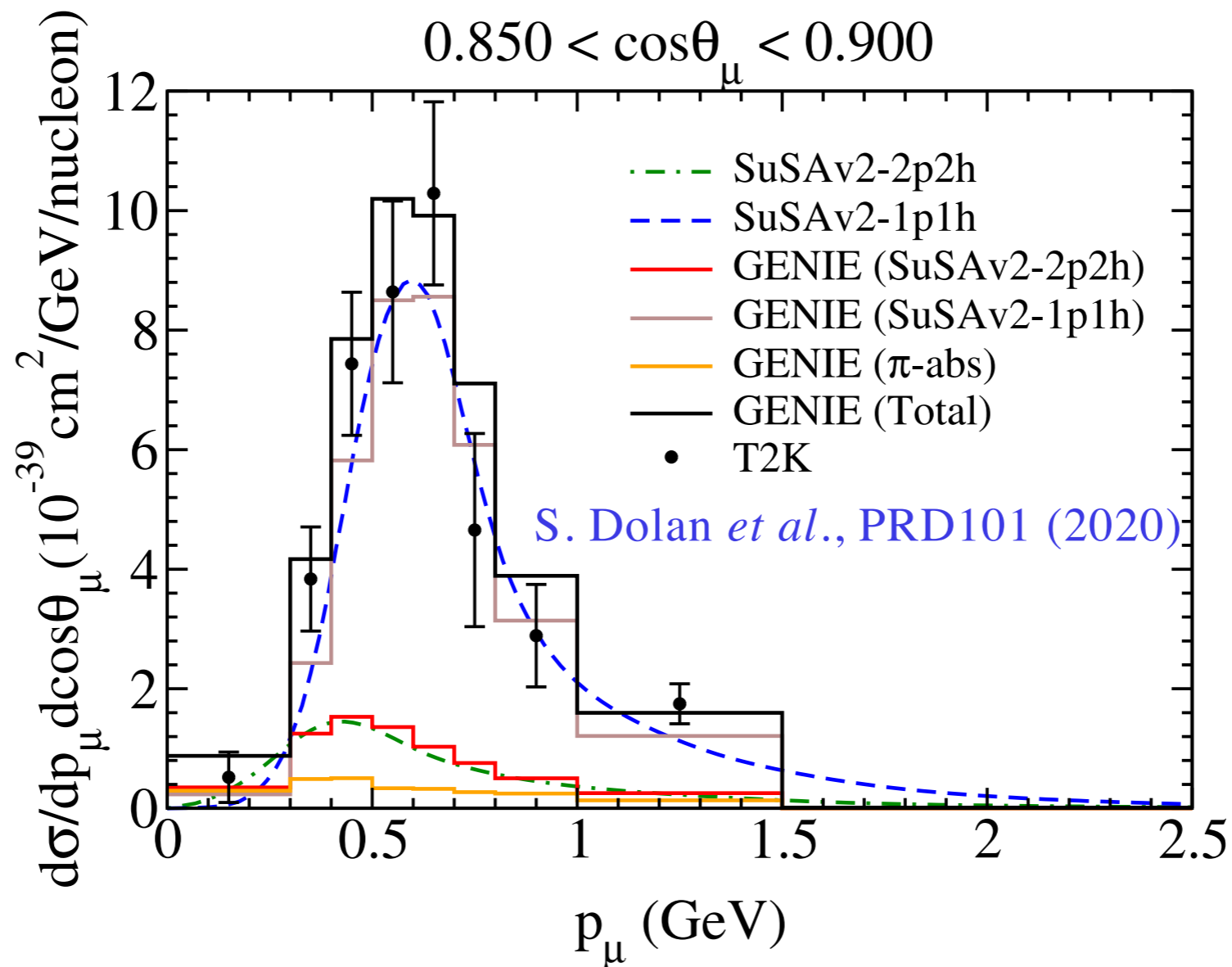
# SuSAv2 predictions for CC0 $\pi$ ( $\nu_\mu, \mu$ )



Good agreement theory/data for different targets and kinematics

## The SuSAv2 model is now implemented in GENIE.

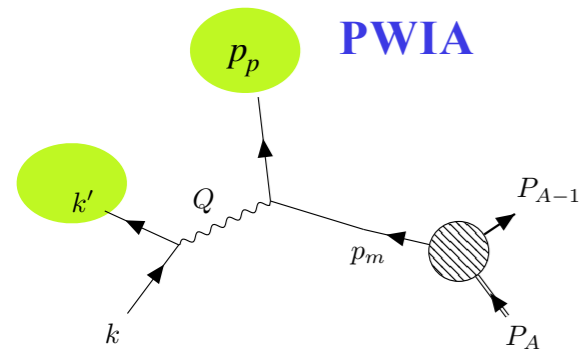
It has been checked that, for the cross section versus the muon variables, the results of the implementation are in good agreement with the original calculation.



However, the model was implemented by using the inclusive hadronic tensor under some assumptions, necessary since **the model is intrinsically inclusive**: the scaling functions only depend on  $q$  and  $\omega$ .

**The implementation can be improved starting from the complete semi-inclusive results.**

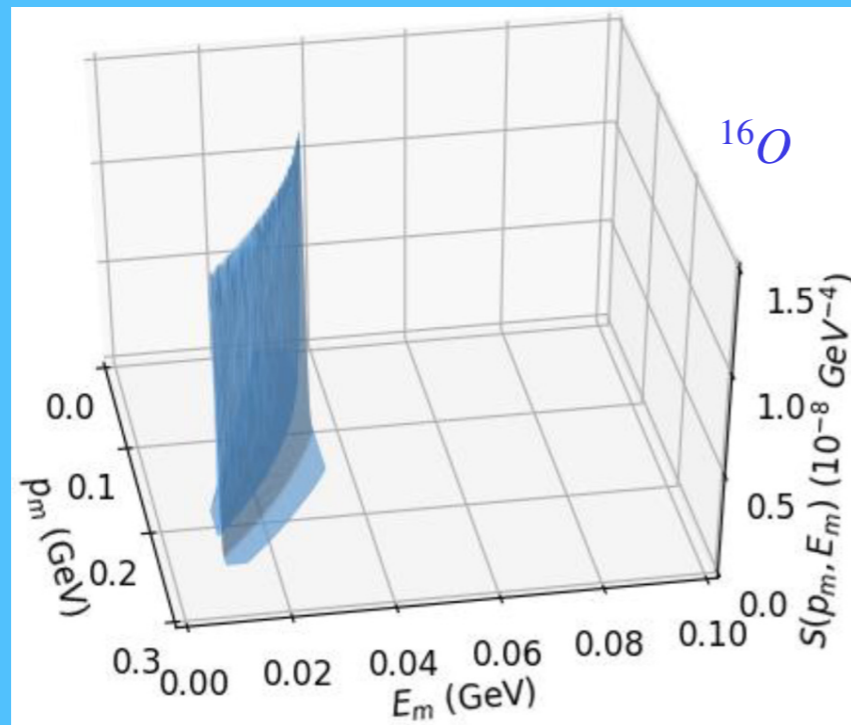
# One nucleon knock-out



$$\left\langle \frac{d^6\sigma}{dk'd\Omega'dp_p d\Omega_p} \right\rangle = \int_0^\infty dk \frac{\Phi_\nu(k)}{k} K_0 S(p_m, E_m) \mathcal{F}^2 \theta(E_m - E_s)$$

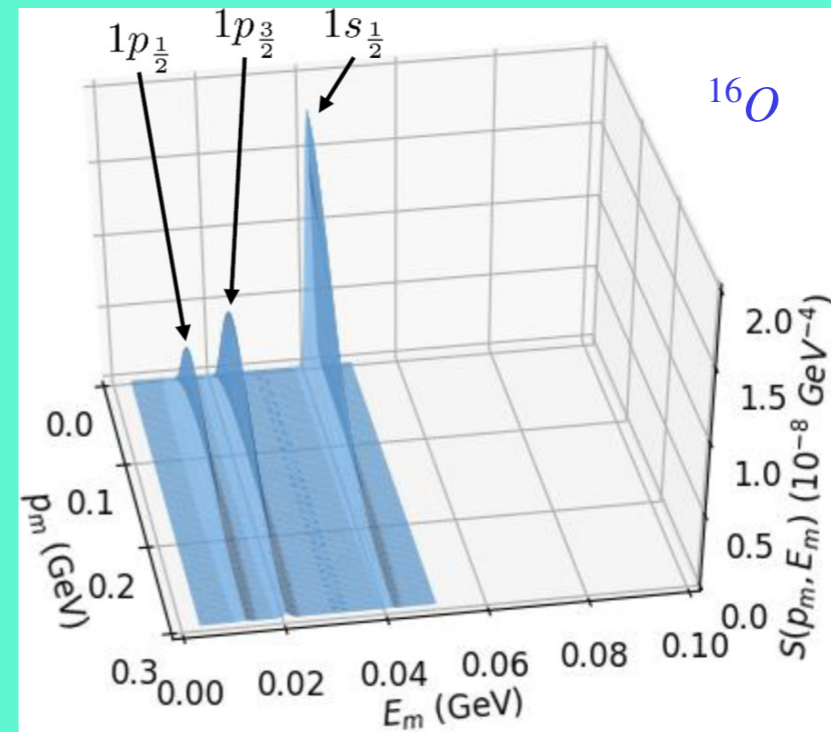
Semi-inclusive reactions are much more sensitive than inclusive ones to **initial state physics**, embodied in the **spectral function S**

## Relativistic Fermi Gas



$$S_{RFG}(p_m, E_m) = \theta(p_F - p_m) \delta\left(E_m - \sqrt{p_m^2 + m_N^2}\right)$$

## Independent Particle Shell Model



$$S_{IPSM}(p_m, E_m) = \sum_{nlj} (2j+1) n_{nlj}(p_m) \delta(E_m - E_{nlj})$$

$$E_m = \omega - T_N - T_{A-1} = \mathcal{E} + E_s$$

missing energy

$$\mathbf{p}_m = \mathbf{q} - \mathbf{p}_N = \mathbf{p}_{A-1}$$

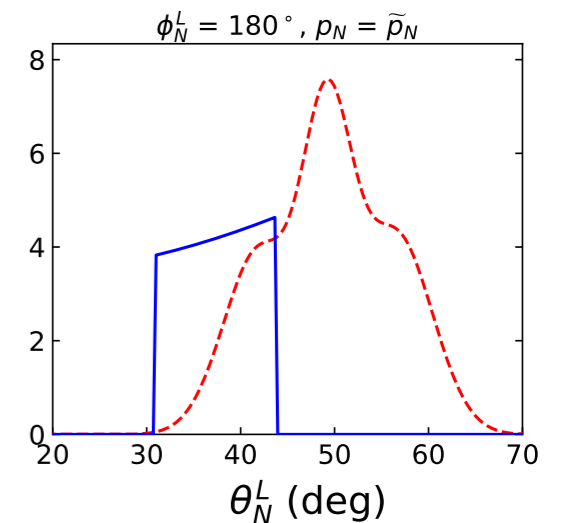
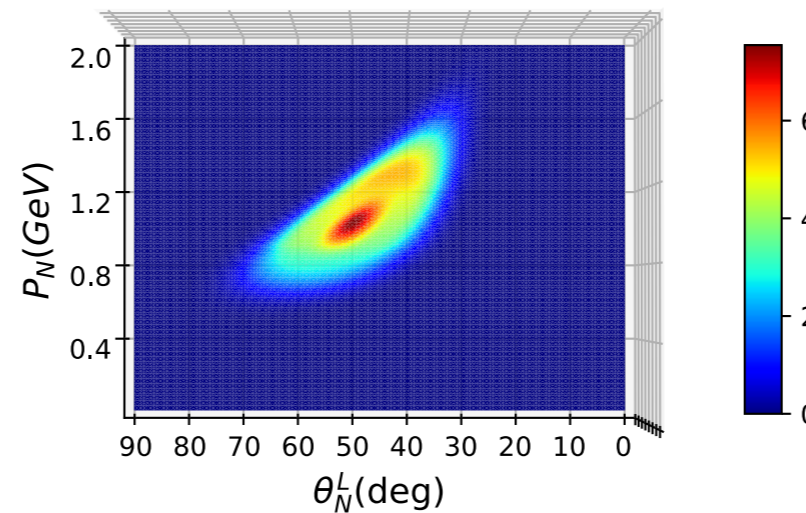
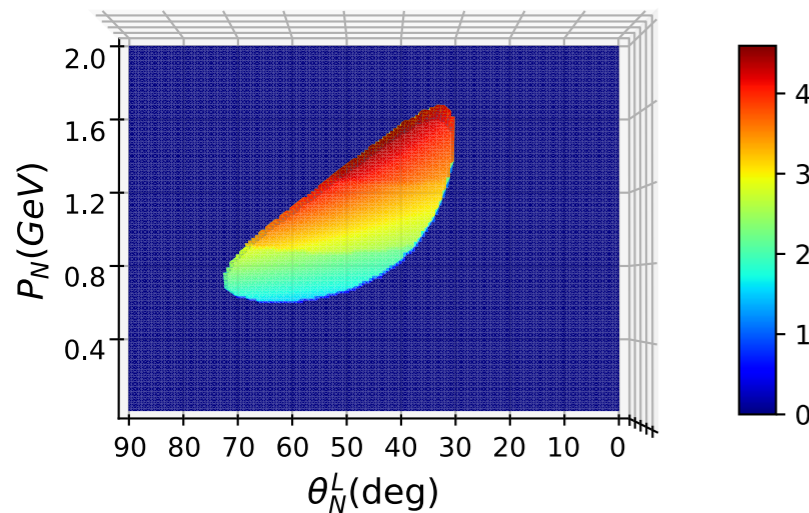
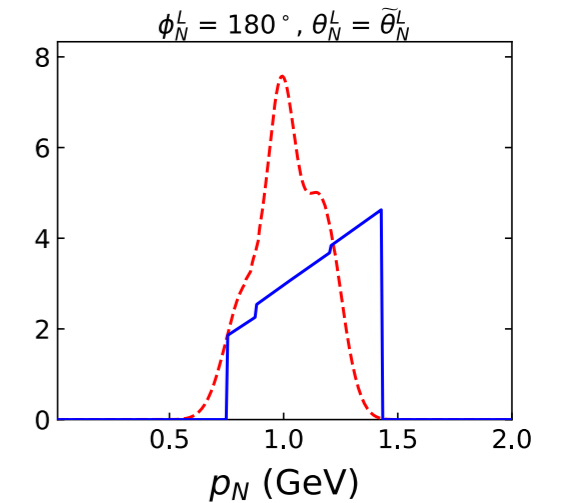
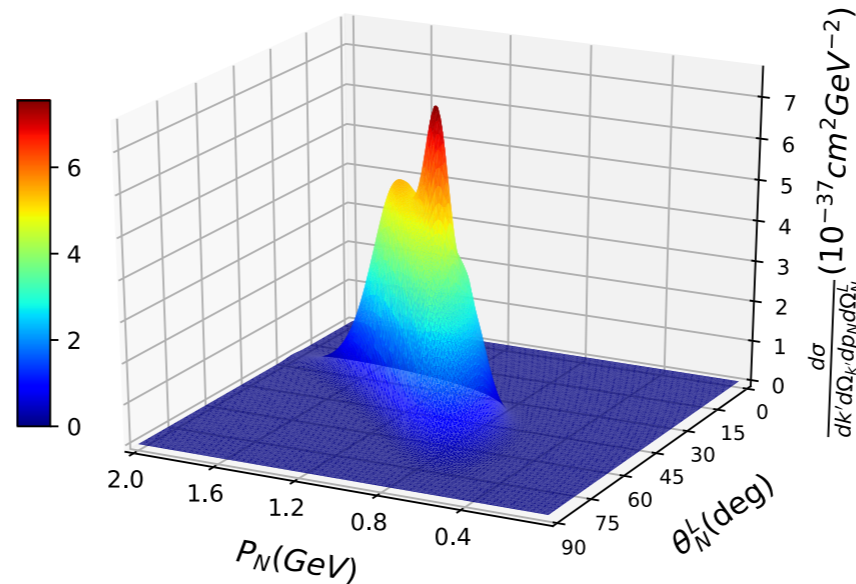
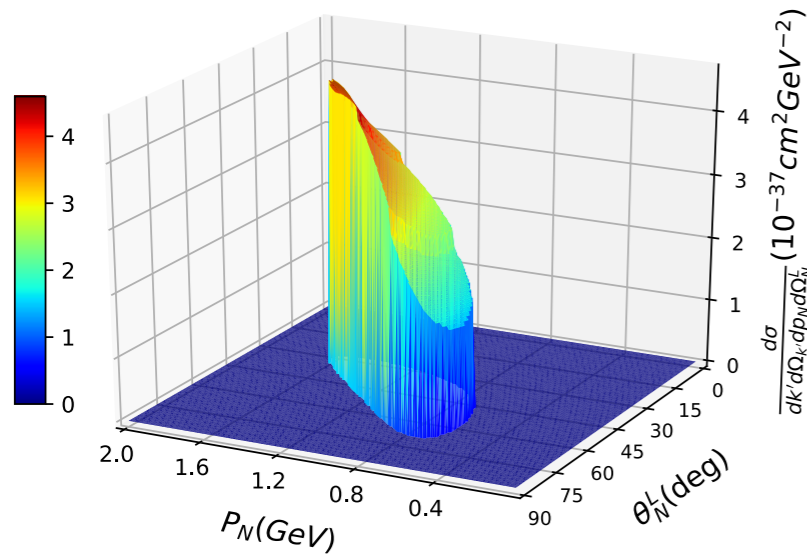
missing momentum

# 6-differential semi-inclusive cross section ( $\nu_\mu, \mu p$ ) nucleon knock-out

$${}^{40}\text{Ar}(\nu_\mu, \mu^- p){}^{39}\text{Cl} \quad \text{DUNE flux} \quad k' = 1.5 \text{ GeV}, \theta_\mu = 30^\circ, \phi_N^L = \pi$$

## Relativistic Fermi Gas

## Independent Particle Shell Model



--- IPSM  
— RFG

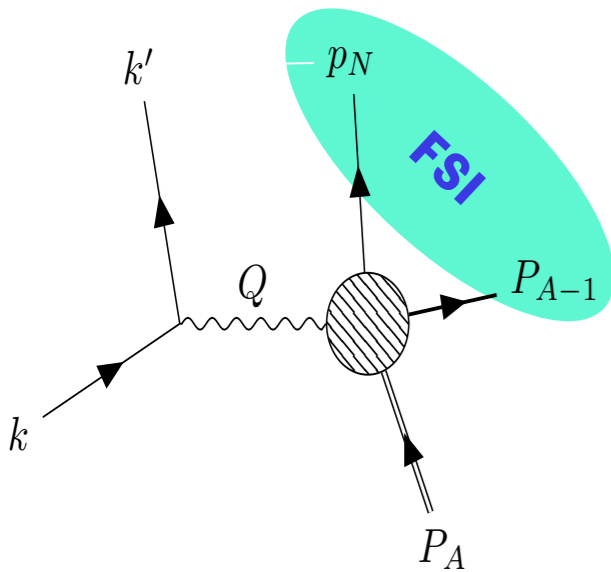
J.M. Franco-Patiño *et al.*, PRC 102, 064626 (2020)

## Relativistic Plane Wave Impulse Approximation (no FSI included)

Striking differences in the cross section due to initial state physics described by different spectral functions. The precise knowledge of the SF is crucial for a reliable modelling of semi-inclusive reactions.

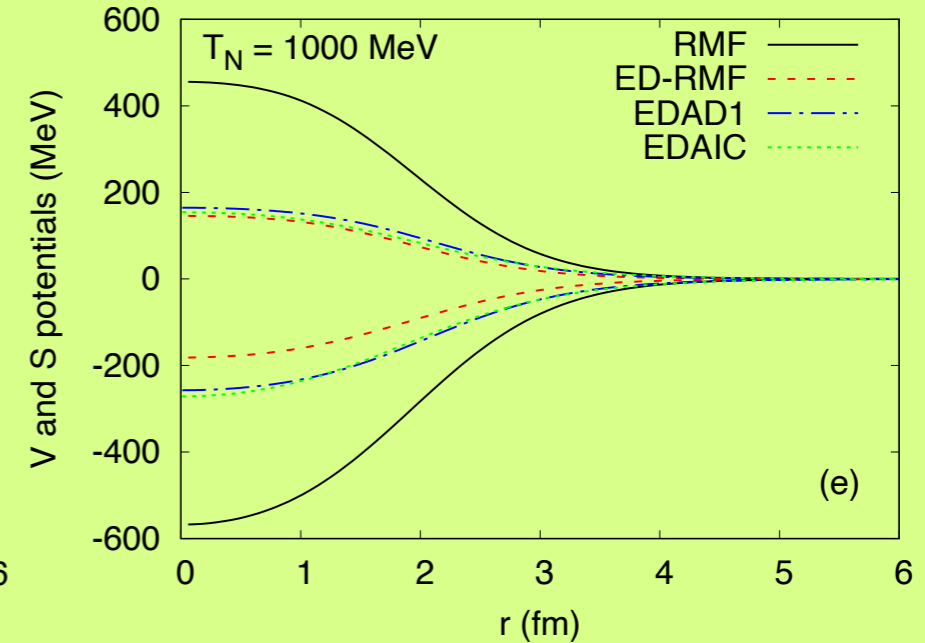
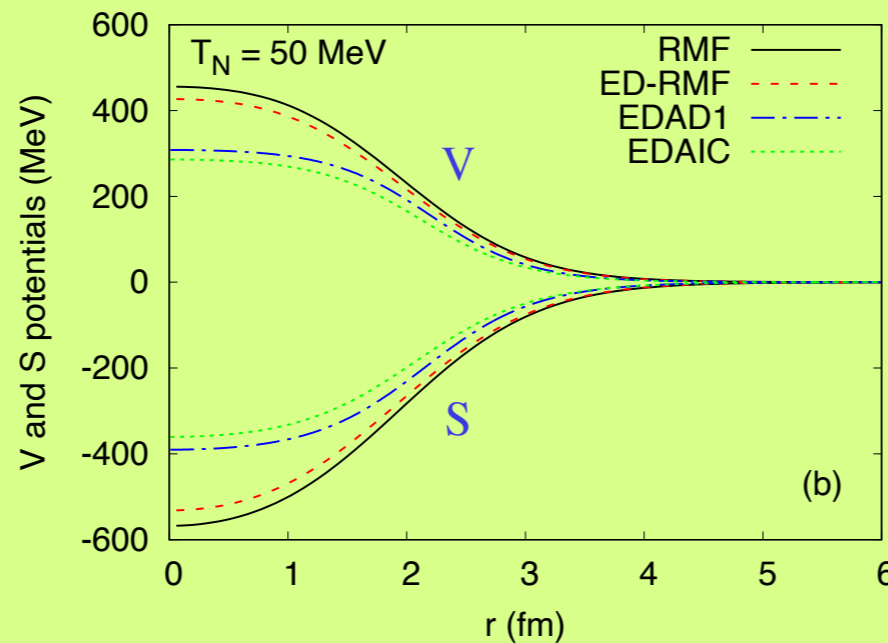
# Final State Interactions

RPWIA  $\rightarrow$  RDWIA



$^{12}\text{C}$

R. Gonzalez-Jimenez *et al.*, Phys.Rev.C 101 (2020) 1, 015503



Note: only the real part of EDAD1 and EDAIC is represented

FSI between the knocked-out nucleon and the residual nucleus can be treated using different approaches:

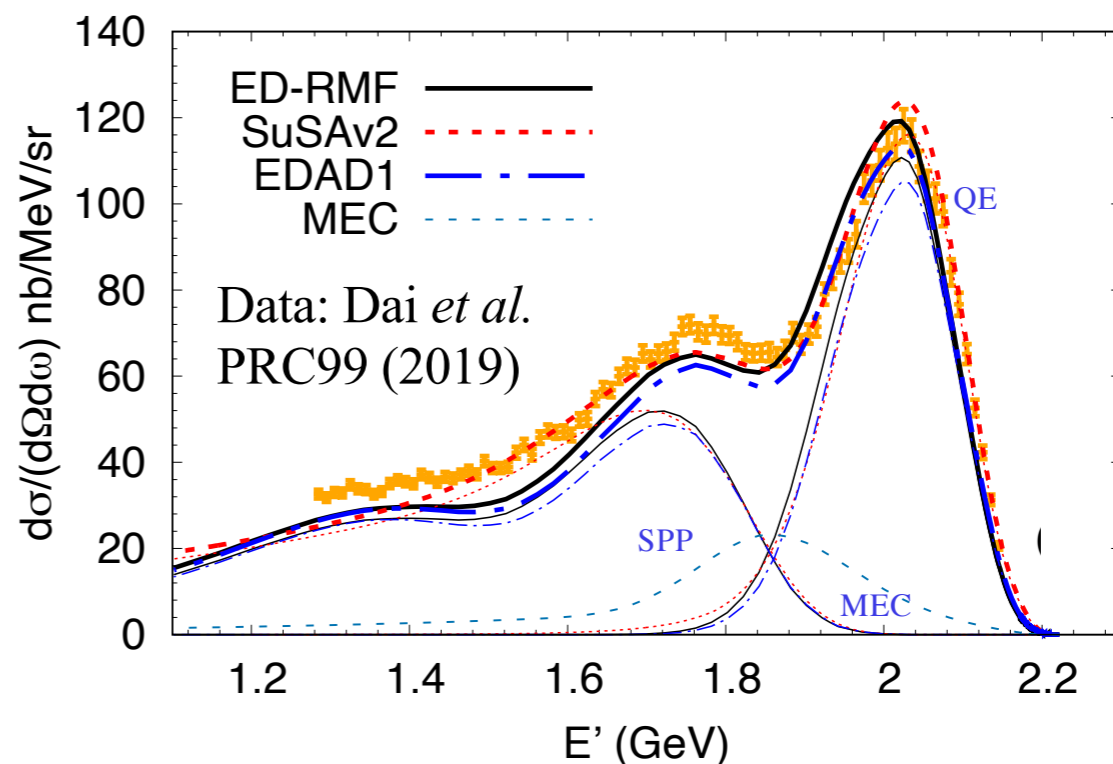
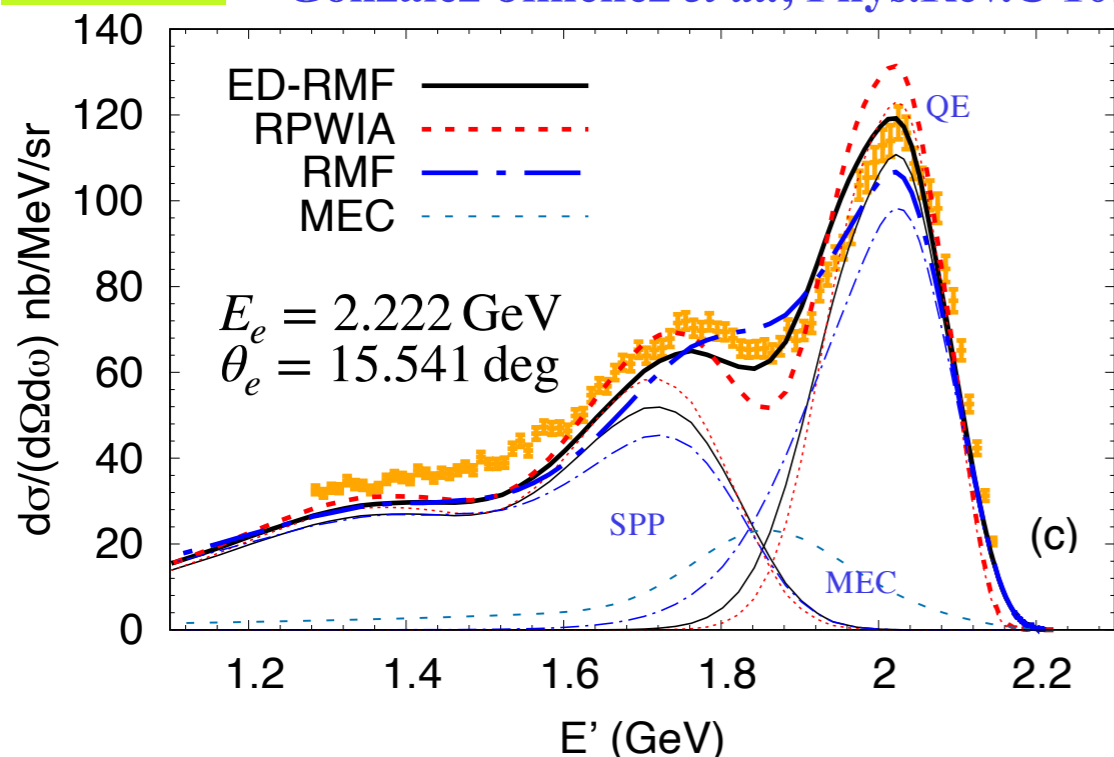
- ▶ **Optical Potentials**, complex energy-dependent A-(in)dependent **EDAD1 (EDAI)**
- ▶ **Relativistic Mean Field (RMF)**, real energy-independent potentials
- ▶ **Energy-Dependent Relativistic Mean Field (ED-RMF)**
- ▶ It has been shown that the **ED-RMF** approach is equivalent to **SuSAv2** for inclusive reactions. However, unlike **SuSAv2**, it is applicable also to the semi-inclusive case.

See **J.M. Udias'** talk on Wednesday

# Final State Interactions in inclusive and semi-inclusive $\nu$ -A scattering

Ar( $e, e'$ )

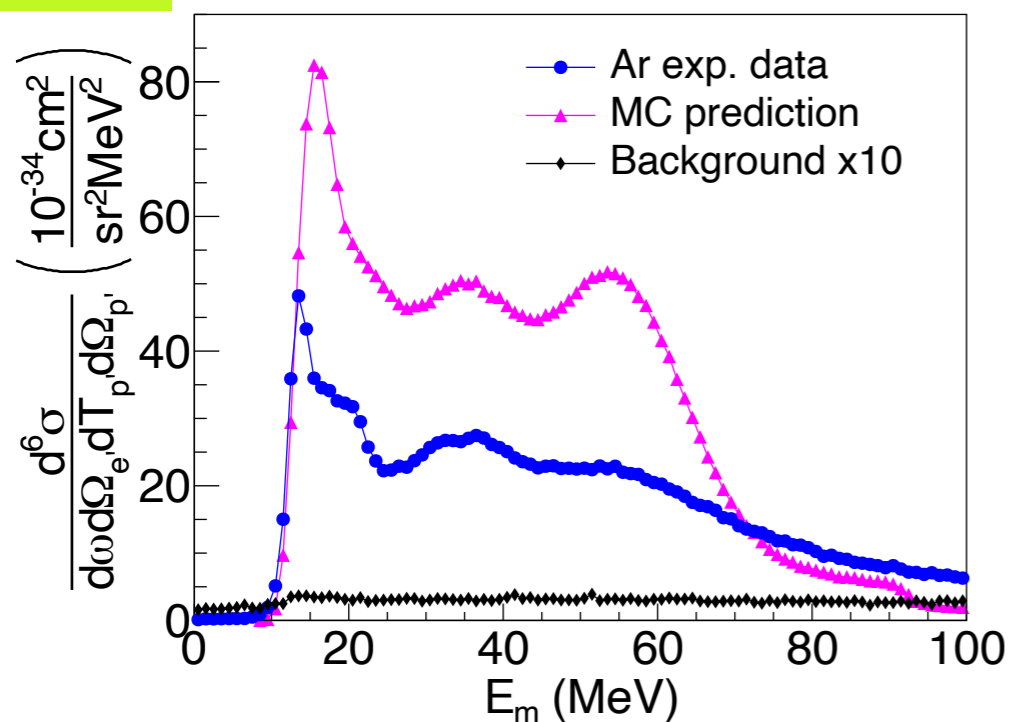
Gonzalez-Jimenez *et al.*, Phys.Rev.C 101 (2020)



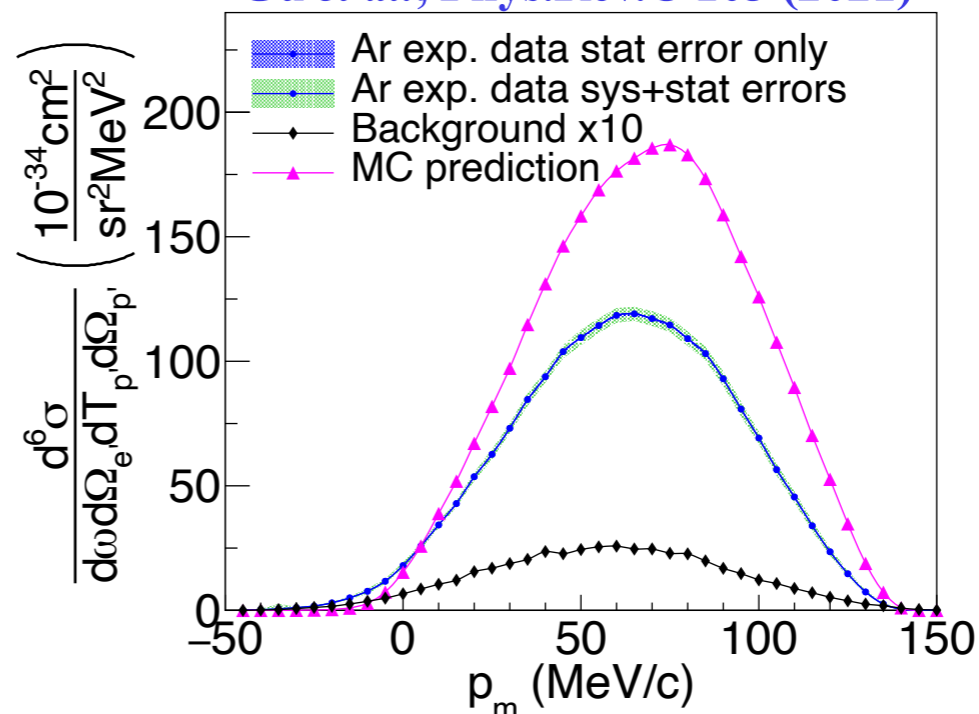
FSI effects are important but not dominant in  $(e, e')$  at typical GeV energies. They essentially re-distribute the strength across the energy spectrum.

On the contrary, the inclusion of FSI effects is extremely important for the description of semi-inclusive data:

Ar( $e, e'p$ )



Gu *et al.*, Phys.Rev.C 103 (2021)



$E_e = 2.222$  GeV

$\theta_e = 21.5^\circ$

$\theta_p = -50^\circ$

parallel kinematics:  $\mathbf{p}_p \parallel \mathbf{q}$

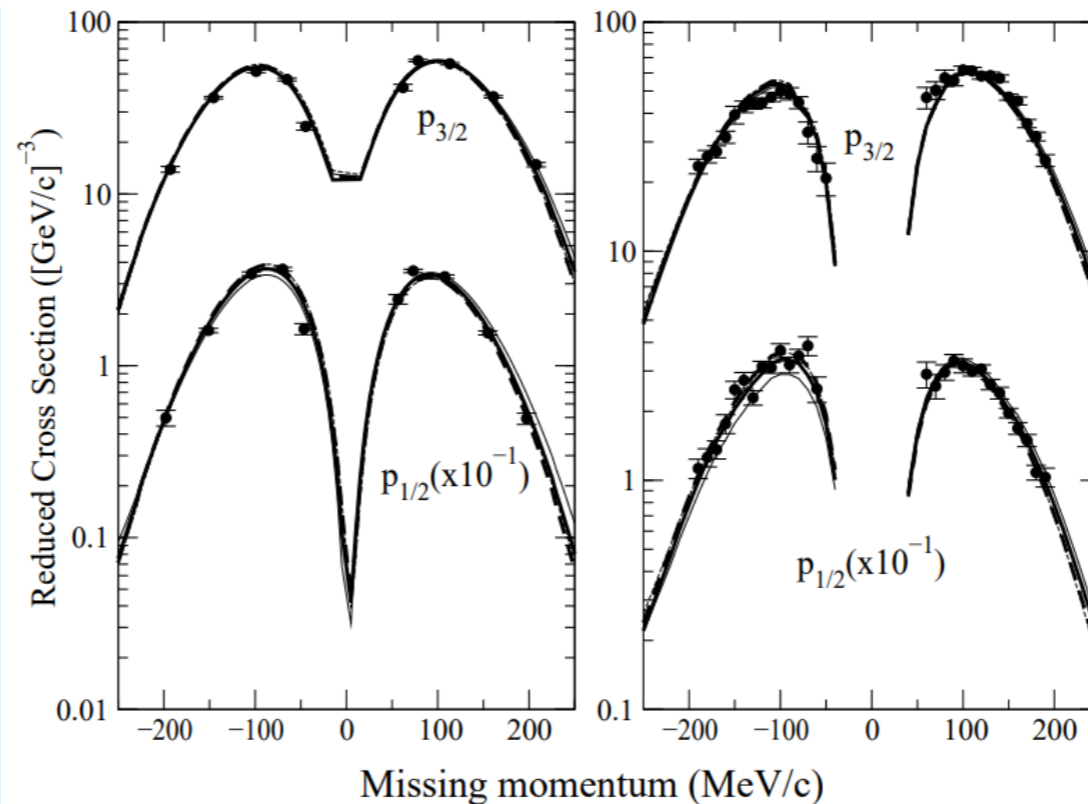
no FSI nor correlations  
in the MC simulation  
→ large overestimation  
of data

# Relativistic Mean Field and $(e,e'p)$ exclusive data

The RMF Distorted Wave Impulse Approximation predictions have been compared successfully in the past with electron scattering exclusive data from Saclay, NIKHEF, MAMI, JLab

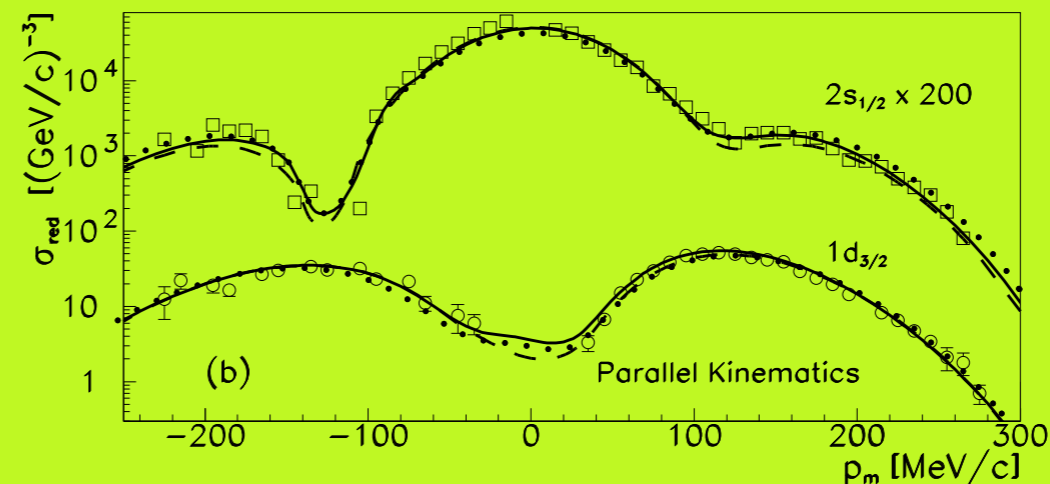
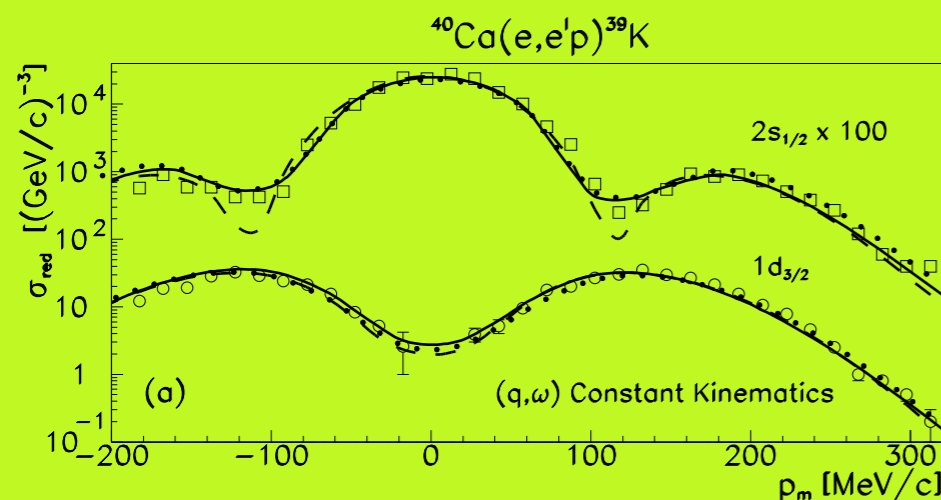
$$^{16}\text{O}(e, e'p)^{15}\text{N}$$

J.M. Udias *et al.*,  
PRC 64, 024614 (2001)



Saclay data (left):  
L. Chinitz *et al.*,  
PRL 67, 568 (1991)

NIKHEF data (right)  
C.M. Spaltro *et al.*,  
PRC48, 2358 (1993)



C. Giusti *et al.*,  
PRC 84, 024615 (2011)

NIKHEF data:  
G.J. Kramer *et al.*,  
PLB 227, 199 (1989)

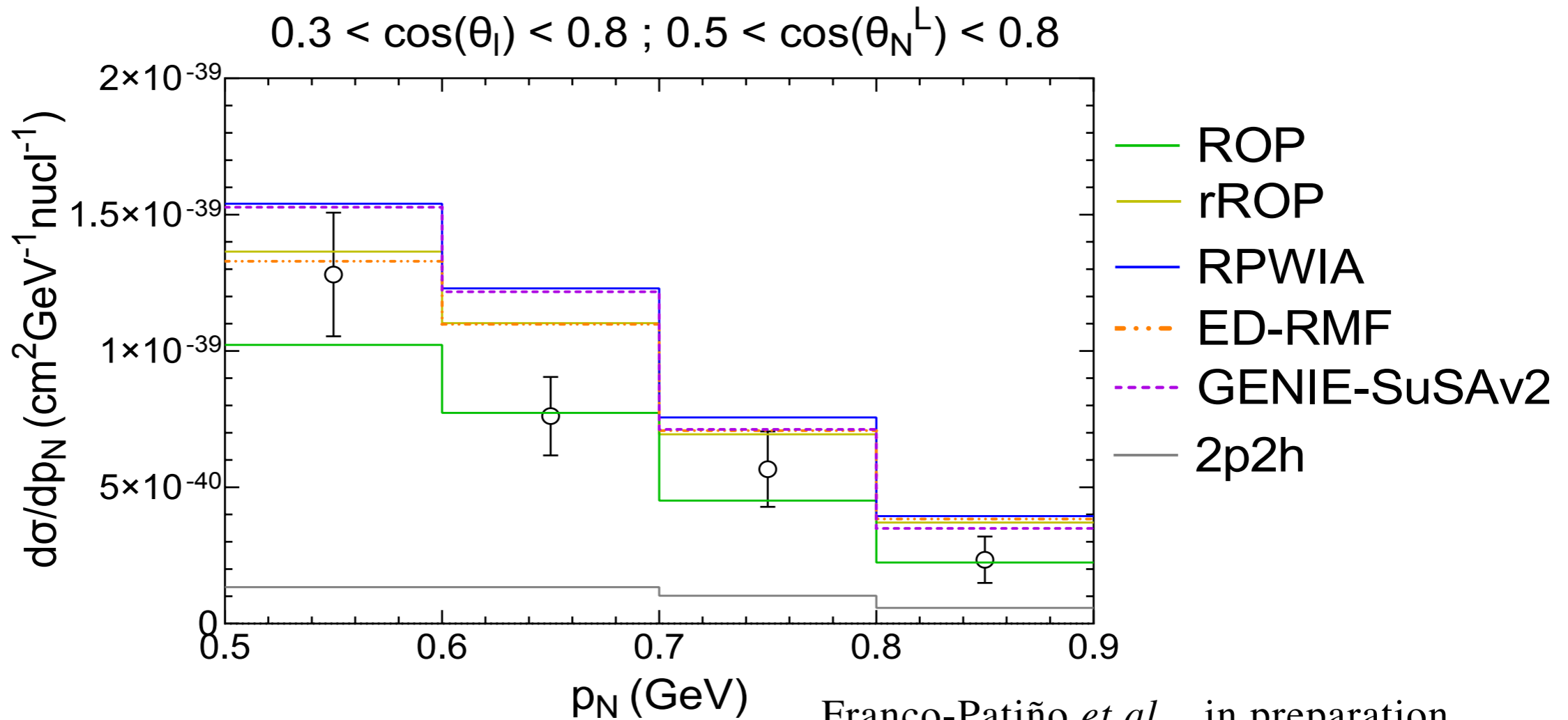
Distorted Wave Impulse Approximation: Woods-Saxon bound state (solid); HF bound state (dotted); RDWIA (dashed). Good description of the shape. To get the right magnitude, a reduction factor (spectroscopic factor) is applied, corresponding to a depletion of quasi-hole states near the Fermi energy of  $\sim 30\text{-}40\%$  from NN correlations.



# Neutrino cross sections versus proton kinematics

T2K data

$\nu_{\mu} - {}^{12}\text{C} \text{ CC}0\pi$   
 $p_N > 500 \text{ MEV}/c$



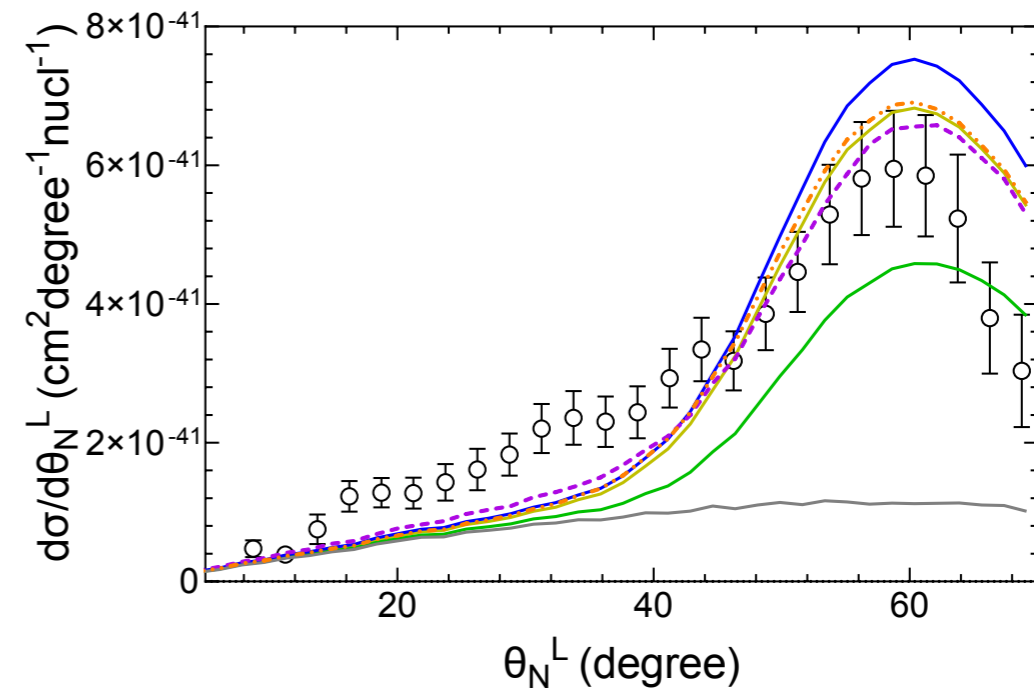
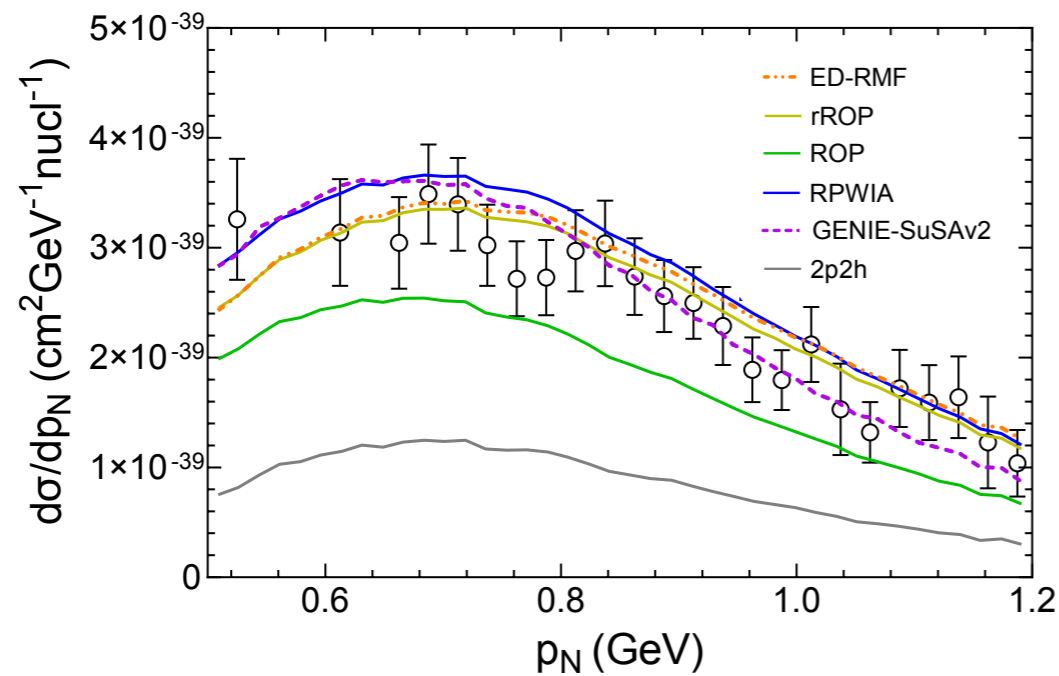
- ▶ FSI implemented with rROP or ED-RMF give very similar results and lower the cross section, improving the agreement with experimental data as compared to the RPWIA prediction.
- ▶ Full complex ROP yields a lower cross section.
- ▶ 2p2h (here implemented with GENIE) give non-negligible contribution.
- ▶ GENIE-SuSAv2 (“inclusive implementation”) gives a larger cross section than the semi-inclusive ED-RMF calculation.

# Neutrino cross sections versus proton kinematics

## MINERvA data

$\nu_\mu - CC0\pi$  at least 1p

Franco-Patiño *et al.*, in preparation



- ▶ rROP and ED-RMF give very similar results
- ▶ Full ROP yields lower cross section.
- ▶ 2p2h are crucial to get good agreement with data
- ▶ GENIE-SuSAv2 differs from full semi-inclusive calculation

# New variables: Single Transverse Kinematic Imbalance (STKI)

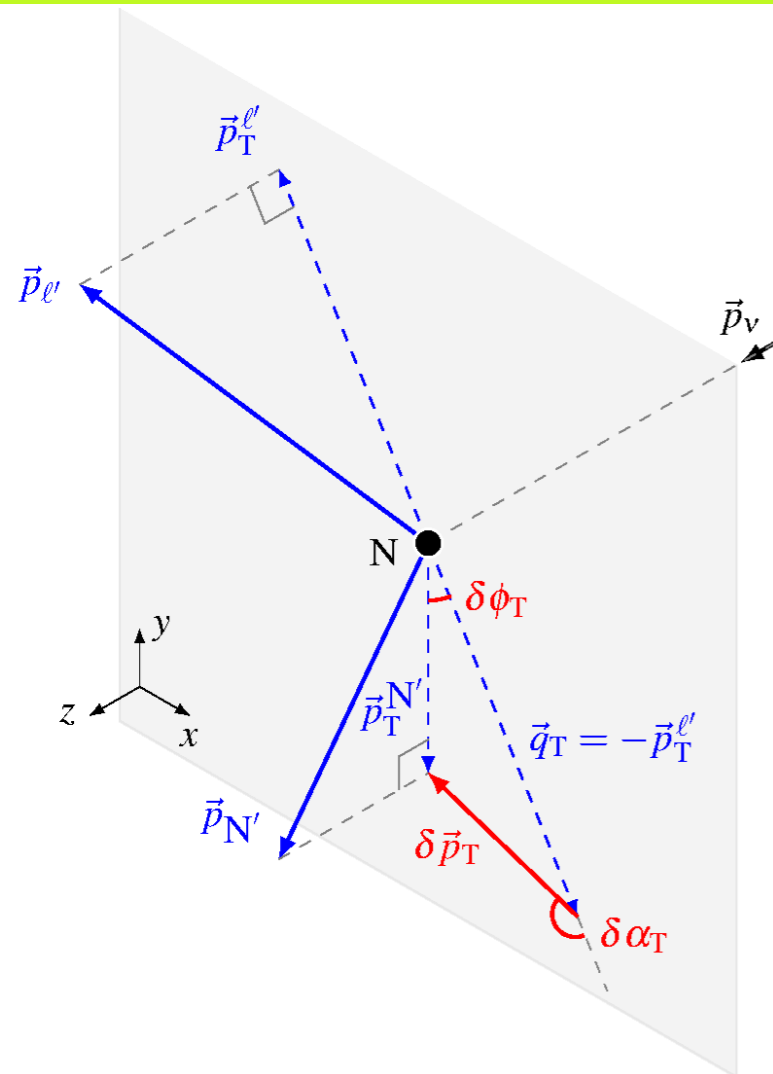
Lu et al., PRC94, 015503 (2016)

$$\delta\vec{p}_T \equiv \vec{p}_T^{\ell'} + \vec{p}_T^{N'}$$

$$\delta\alpha_T \equiv \arccos \frac{-\vec{p}_T^{\ell'} \cdot \delta\vec{p}_T}{p_T^{\ell'} \delta p_T}$$

$$\delta\phi_T \equiv \arccos \frac{-\vec{p}_T^{\ell'} \cdot \vec{p}_T^{N'}}{p_T^{\ell'} p_T^{N'}}$$

new variables devised to enhance sensitivity to nuclear effects



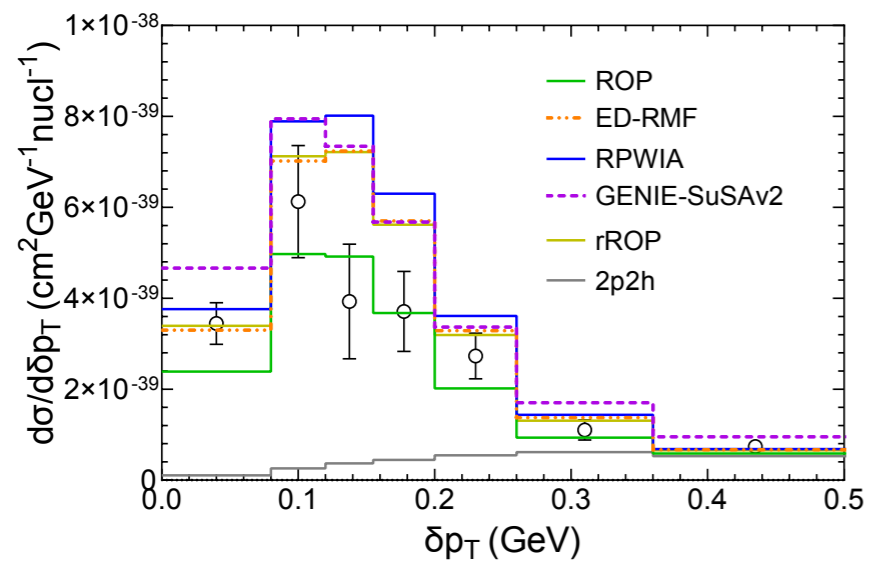
On a free nucleon at rest  $\vec{p}_T^{\ell'} = -\vec{p}_T^{N'}$ :

$\delta p_T = \delta\phi_T = 0 \rightarrow$  peaked distribution

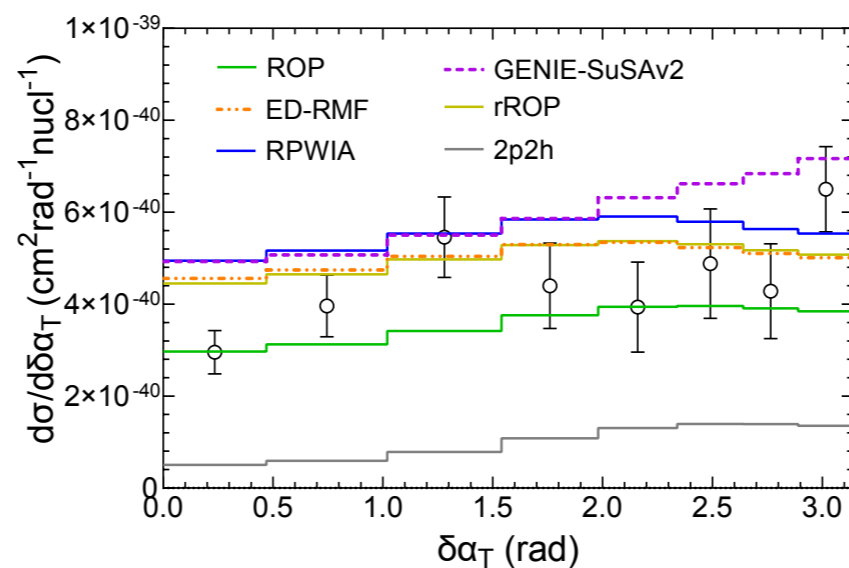
$\delta\alpha_T$  undefined  $\rightarrow$  flat distribution

Deviations from these behaviours “measure” nuclear effects

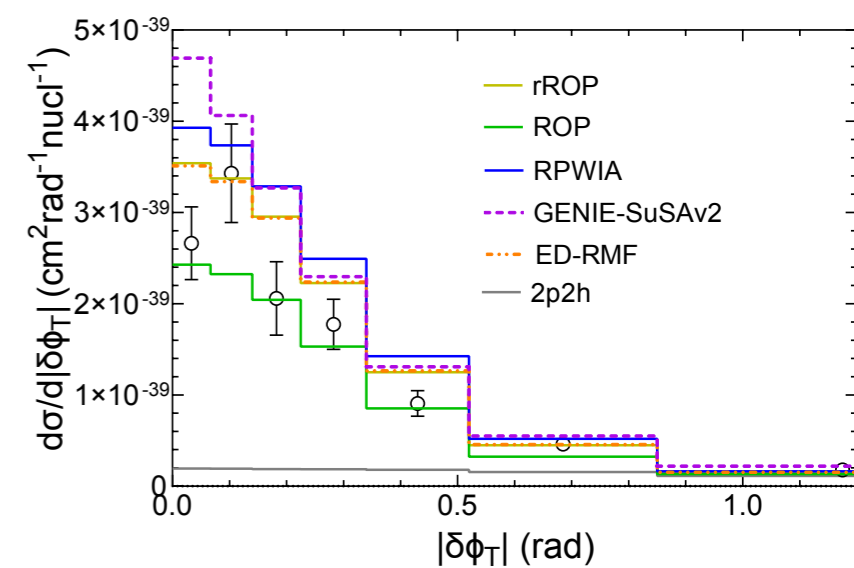
# STKI



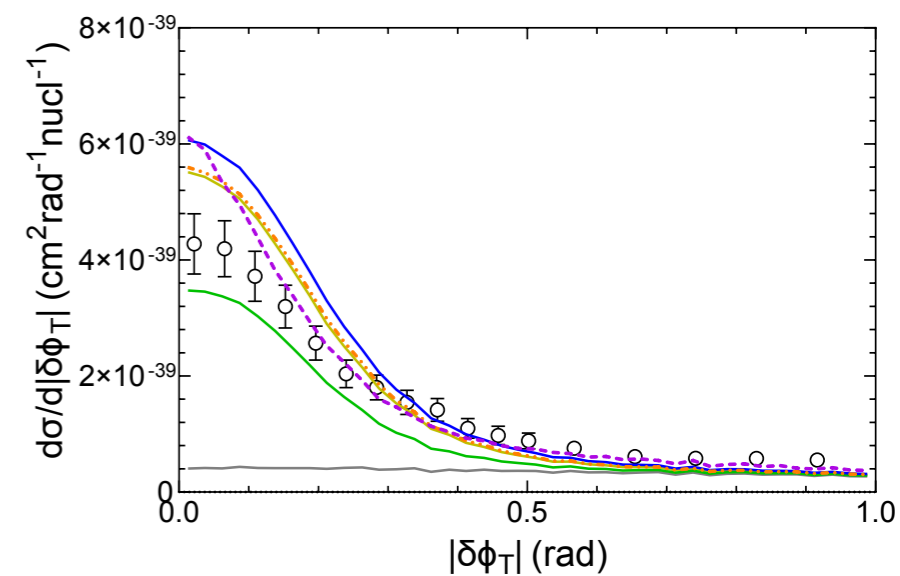
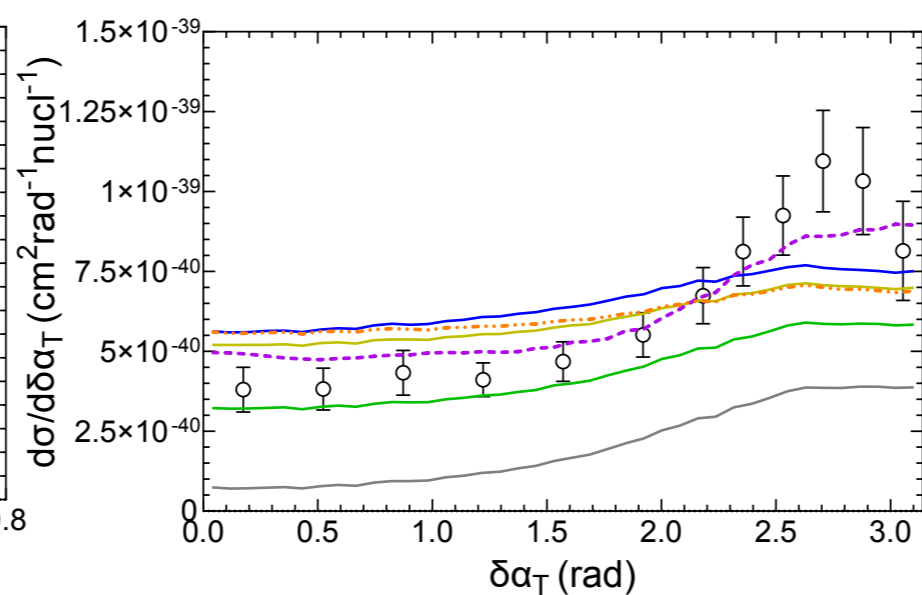
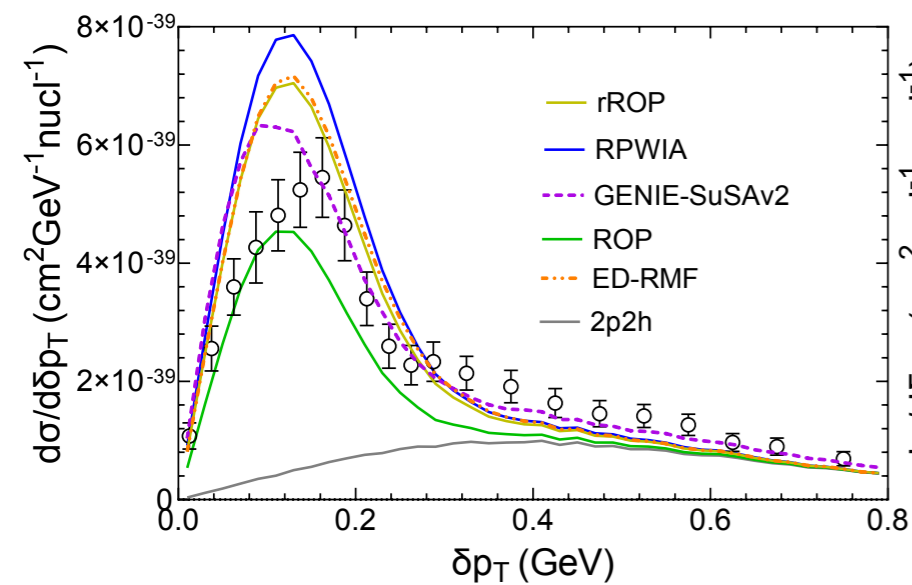
# T2K



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# MINERvA



## Summary

- ▶ Electron scattering data are the best tool to test models for neutrino event generators and discriminate between them.
- ▶ In the Superscaling approach ( $e, e'$ ) data are used to predict neutrino cross sections in the lepton variables in the full energy spectrum (SuSAv2). The agreement with both electron- and neutrino-scattering inclusive data is good.
- ▶ Many recent measurements of semi-inclusive neutrino cross sections help to constrain nuclear models for oscillation experiments. Parallel efforts from the theory side are under way.
- ▶ SuSAv2 is an inclusive model and cannot be used to predict semi-inclusive observables unless assumptions are made, which can be inconsistent with the model. However, the underlying theory (RFM) has been successfully applied in the past to ( $e, e'p$ ) studies and is now being extended to the analysis of semi-inclusive neutrino reactions.
- ▶ Different treatments of Final State Interactions have been analysed and compared to data: FSI in general improve the agreement with data, but further study is needed to assess which FSI model is preferable.