From Langrangian to Predictions:

Novel Methods of Event Generation, and Their Implications

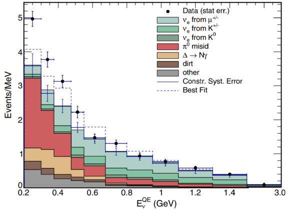
Joshua Isaacson

In Collaboration with Stefan Höche, Diego Lopez Gutierrez, Pedro Machado, and Noemi Rocco

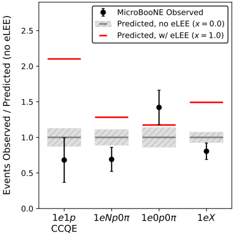
Based on arxiv:2110.15319, arxiv:2204.xxxx

28 March 2022

Motivation: MiniBooNE and MicroBooNE

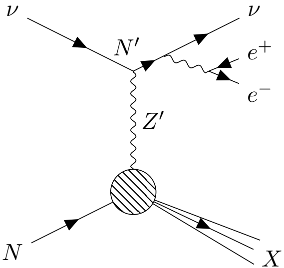


[PRL 121, 221801]



[arXiv:2110.14054]

- MiniBooNE sees excess of events
- MicroBooNE does not see excess of single electron events
- Excess can be from multiple lepton final states
- Event generators can not simulate these processes



Motivation: Theory

Summary of Workshop on Common Neutrino Event Generator Tools

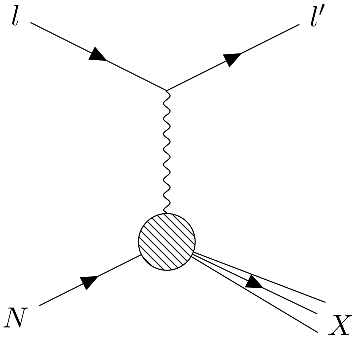
2.5.2 Factors to consider for interface design

While our goal should be to standardize the interface as much as possible, the workshop raised several issues that should be considered when evaluating possible approaches. These issues are summarized below:

4. **Human factors** Our interface should be designed for ease of use, and should consider the skills and limitations of the theorists likely to use it. It was pointed out that many theorists are PhD students or postdoctoral researchers working on limited-term contracts. To fit in with this way of working, it should be possible to develop, implement, and test a model against data on timescales of the order of a year. We must also bear in mind that many theorists are not primarily programmers, and that models may be developed using tools, such as Mathematica, that are not natively compatible with the languages used in generator software. If we restricted ourselves to an interface in a particular programming language, we could severely limit the accessibility of the interface to new models.

[https://arxiv.org/abs/2008.06566]

Leptonic and Hadronic Tensor



Observations:

- Nuclear physics calculations are hard
- Calculating arbitrary perturbative diagrams is a solved problem
- BSM effects of interest only weakly couple to quarks and gluons $\Rightarrow$ no corrections to the nuclear physics

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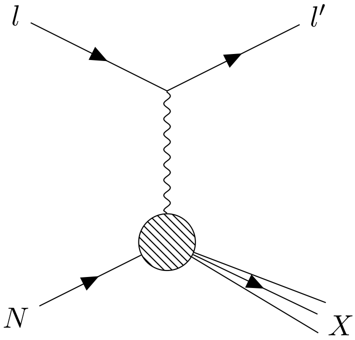

Observations:

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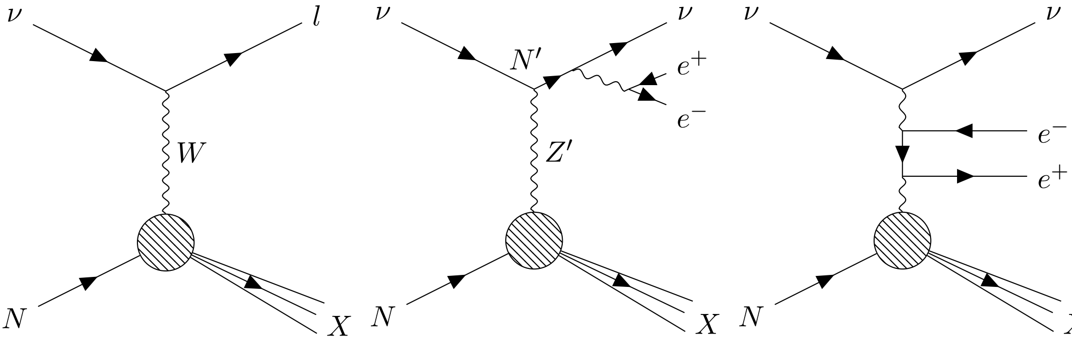
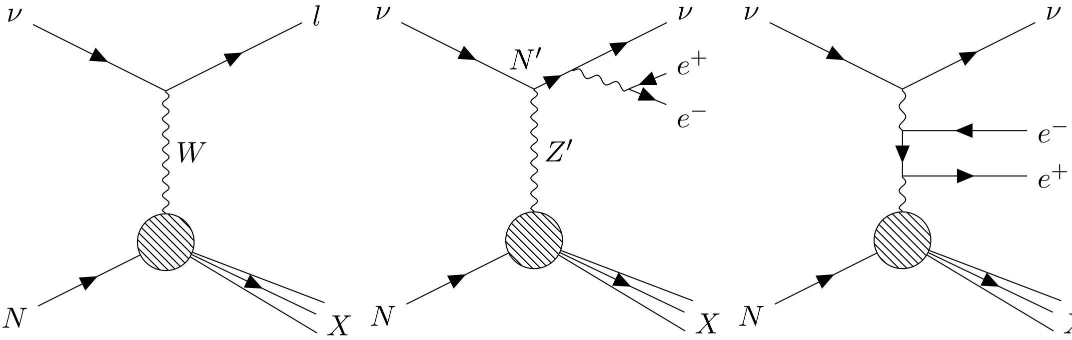
Question

Can we use these observations to automate Beyond the Standard Model Physics?

Leptonic and Hadronic Tensor



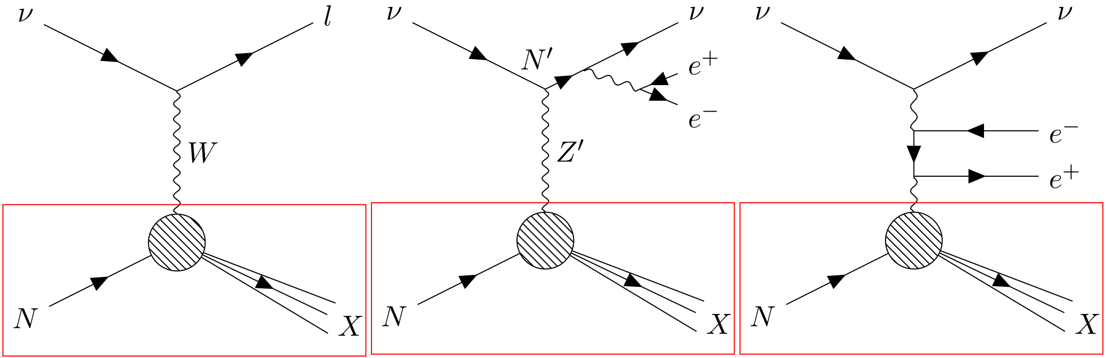
Leptonic and Hadronic Tensor



Question

What do all the diagrams above have in common?

Leptonic and Hadronic Tensor



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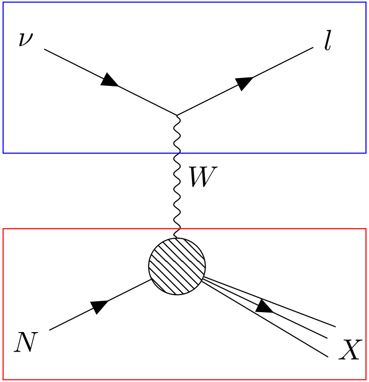
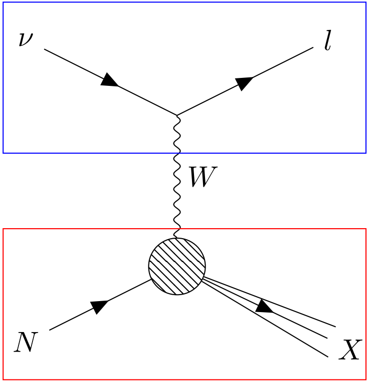


$\longrightarrow$ **Leptonic Tensor** $(L_{\mu\nu})$

$\longrightarrow$ **Hadronic Tensor** $(H^{\mu\nu})$

Leptonic and Hadronic Tensor



Notes:

- Leptonic tensor only contains perturbative physics.
- Can use LHC tools to calculate Leptonic tensor
- Hadronic tensor is difficult, but event generators have these calculations implemented already.

Using Currents

Using tensors:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sum_{i,j} L_{\mu\nu}^{(ij)} W^{(ij)\mu\nu} = L_{\mu\nu}^{(\gamma\gamma)} W^{(\gamma\gamma)\mu\nu} + L_{\mu\nu}^{(\gamma Z)} W^{(\gamma Z)\mu\nu} + L_{\mu\nu}^{(Z\gamma)} W^{(Z\gamma)\mu\nu} + L_{\mu\nu}^{(ZZ)} W^{(ZZ)\mu\nu} + \cdots$$

Using Currents:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left|\sum_{i} L_{\mu}^{(i)} W^{(i)\mu}\right|^2$$

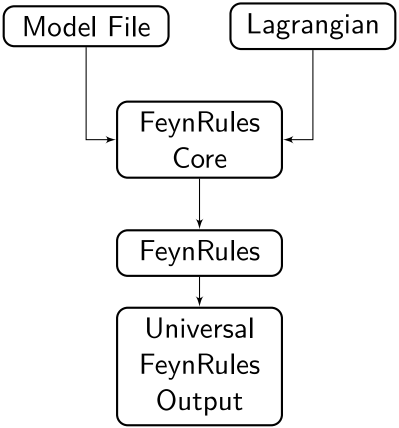
Interferences handled automatically using currents

Interface to tensors provided for nuclear calculations that **must** be expressed using tensors.

FeynRules

- *Mathematica* Program
- Takes model file and Lagrangian as input
- Calculates the Feynman rules
- Outputs in Universal FeynRules Output (UFO) format

[arXiv:0806.4194, arXiv:1310.1921]



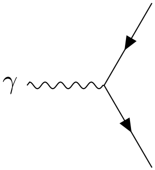
Universal FeynRules Output (UFO)

- Python output files
- Contains model-independent files and model-dependent files
- Contains all information to calculate any tree level matrix element
- Has parameter file to adjust model parameters to scan allowed regions

Example QED ($e^+e^-\gamma$ Vertex):

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(iD^\mu\gamma_\mu - m\right)\psi$$

$$V_{e^+e^-\gamma} = ie\gamma^\mu =$$



[arXiv:1108.2040]

Handling Form Factors

Nuclear one-body current operators:

$$\mathcal{J}^\mu = \left(\mathcal{J}^\mu_V + \mathcal{J}^\mu_A\right)$$
$$\mathcal{J}^\mu_V = \gamma^\mu \mathcal{F}^a_1 + i\sigma^{\mu\nu} q_\nu \frac{\mathcal{F}^a_2}{2M}$$
$$\mathcal{J}^\mu_A = -\gamma^\mu \gamma_5 \mathcal{F}^a_A - q^\mu \gamma_5 \frac{\mathcal{F}^a_P}{M}$$

Coherent Form Factors (spin-0 nucleus):

$$\mathcal{J}^\mu = (p_{\text{in}} + p_{\text{out}})^\mu \mathcal{F}_{\text{coh}}$$

Standard Model Form Factors:

$$\mathcal{F}_i^{\gamma(p,n)} = F_i^{p,n}, \qquad \mathcal{F}_A^\gamma = 0$$
$$\mathcal{F}_i^{W(p,n)} = F_i^p - F_i^n, \qquad \mathcal{F}_A^W = F_A$$
$$\mathcal{F}_i^{Z(p)} = \left(\frac{1}{2} - 2\sin^2\theta_W\right) F_i^p - \frac{1}{2} F_i^n,$$
$$\mathcal{F}_i^{Z(n)} = \left(\frac{1}{2} - 2\sin^2\theta_W\right) F_i^n - \frac{1}{2} F_i^p$$
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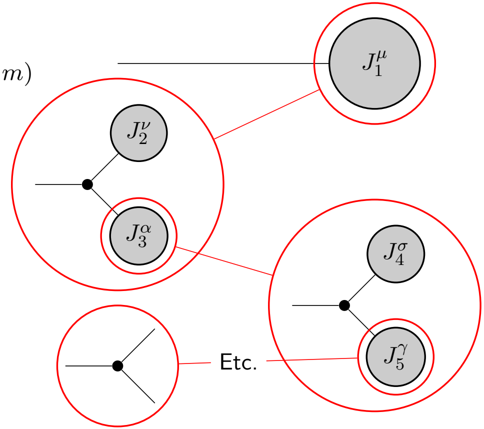
Straight-forward to extend to BSM if CVC is valid

Recursive Matrix Element Generation

$$\mathcal{J}_\alpha(\pi) = P_\alpha(\pi) \sum_{\mathcal{V}_\alpha^{\alpha_1,\alpha_2}} \sum_{\mathcal{P}_2(\pi)} \mathcal{S}(\pi_1,\pi_2) V_\alpha^{\alpha_1,\alpha_2}(\pi_1,\pi_2) \mathcal{J}_{\alpha_1}(\pi_1) \mathcal{J}_{\alpha_2}(\pi_2)$$

$$L_\mu^{(i)}(1,\ldots,m) = \mathcal{J}_\mu^{(i)}(1,\ldots,m)$$

$$L_{\mu\nu}^{(i,j)}(1,\ldots,m) = \mathcal{J}_\mu^{(i)}(1,\ldots,m)\mathcal{J}_\nu^{(j)\dagger}(1,\ldots,m)$$



Berends-Giele Recursion

- Reuse parts of calculation
- Most efficient for high multiplicity
- Reduces computation from $\mathcal{O}(n!)$ to $\mathcal{O}(3^n)$

[Nucl. Phys. B306(1988), 759]

Phase Space Generation

$$d\Phi_n\left(a,b;1,\ldots,n\right)=\delta^{(4)}\left(p_a+p_b-\sum_{i=1}^{n}p_i\right)\left[\prod_{i=1}^{n}\frac{d^4p_i}{(2\pi)^3}\delta\left(p_i^2-m_i^2\right)\Theta\left(p_{i_0}\right)\right]$$

The above phase space definition does not contain the handling of initial states.

Algorithms for n-body phase space generation

- RAMBO [Comput. Phys. Commun. **40**(1986) 359]
- Multi-channel techniques [hep-ph/9405257]
- Recursive Phase Space [arXiv:0808.3674]

Results

Processes Considered:

- Electron-Carbon Scattering
- Neutrino-Carbon Scattering
- Neutrino Tridents
- Dirac/Majorana Heavy Neutral Lepton [1807.09877]

Experimental Setup:

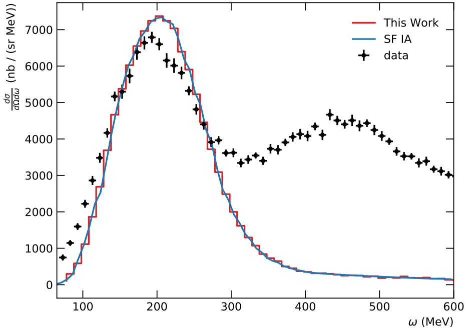
- Target Nucleus: Carbon
- Electron: 961 MeV and 1299 MeV
- Neutrino: 1000 MeV
- Validating beam fluxes

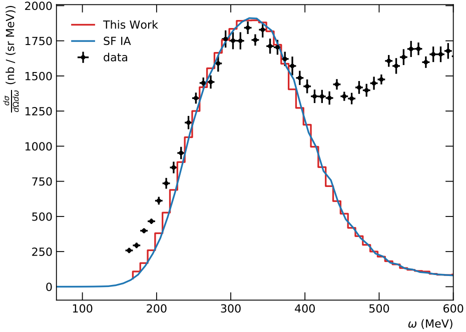
NOTE: All processes are fully differential

Parameters:

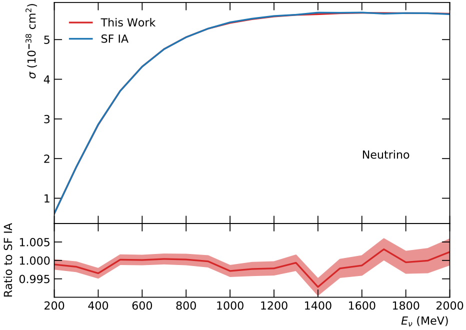
- Only quasielastic scattering (coherent for HNL) is included and no FSI
- EM Form Factors: Kelly [PRC 70, 068202 (2004)]
- Coherent Form Factor: Lovato [1305.6959]
- Axial Form Factor:
  - Dipole
  - $M_A = 1.0$ GeV
  - $g_A = 1.2694$
- $\alpha = 1/137$
- $G_F = 1.16637 \times 10^{-5}$
- $M_Z = 91.1876$ GeV

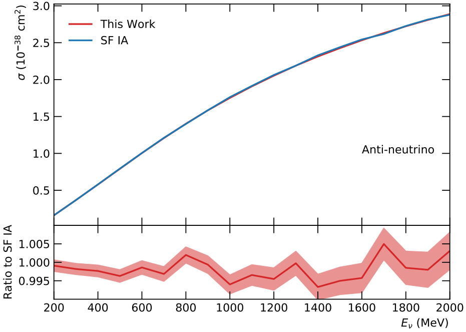
Electron Scattering



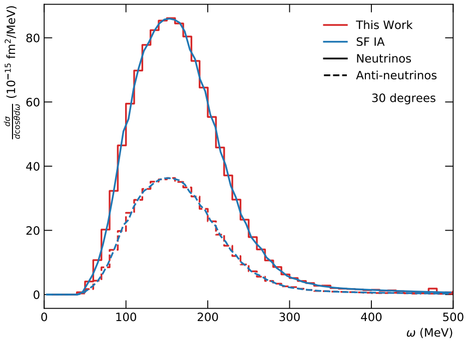


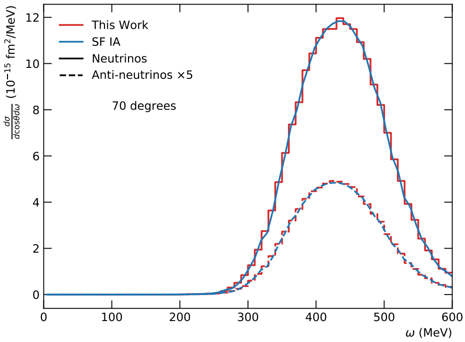
Neutrino Total Cross Section



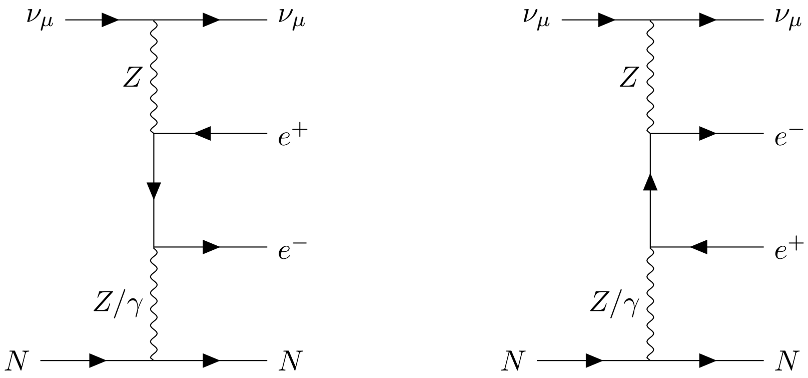


Neutrino Differential Cross Section





Neutrino Tridents



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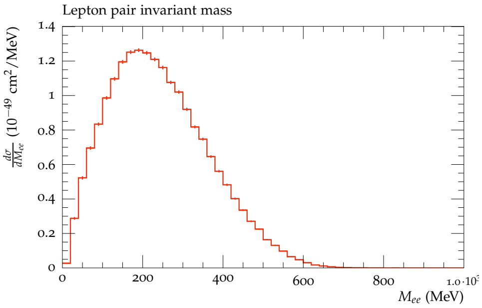
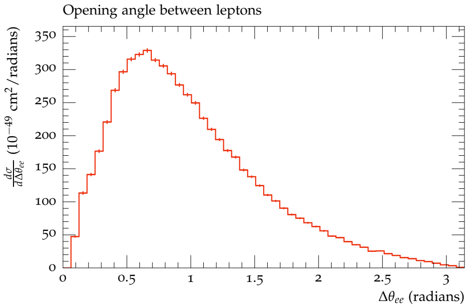
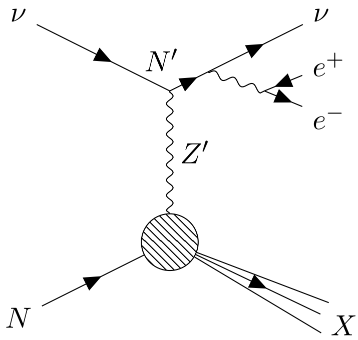




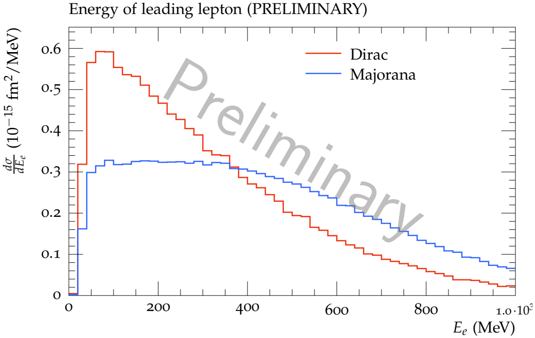
Heavy Neutral Lepton

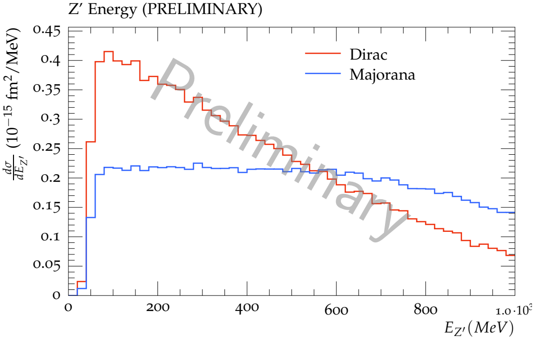


Parameters:

- $m_{N'} = 420$ MeV
- $m_{Z'} = 30$ MeV
- $\alpha_D = 0.25$
- $\alpha\epsilon^2 = 2 \times 10^{-10}$
- $|U_{42}^{\mu}| = 9 \times 10^{-7}$
- Widths of N' and Z' automatically calculated based on input parameters
- Handles both Dirac and Majorana fermions
- Results are flux-averaged over the MiniBooNE / MicroBooNE neutrino flux

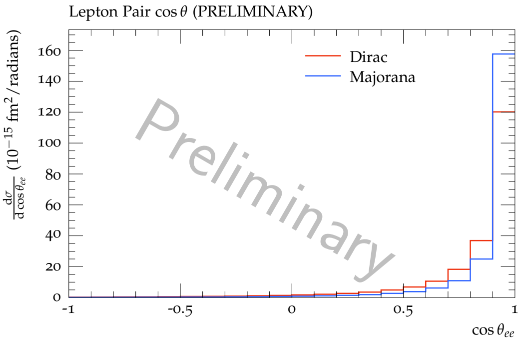
Heavy Neutral Lepton

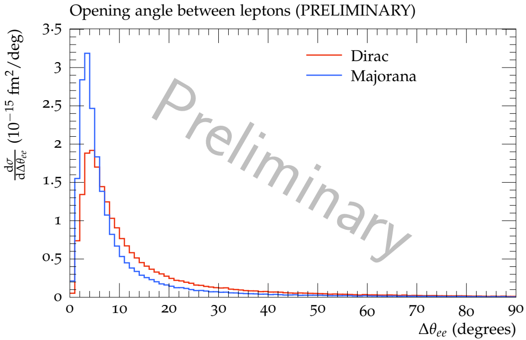




Heavy Neutral Lepton

- No cuts applied yet
- Typical opening angle around 5-6 degrees
- Working on scanning parameter space





- Need to include background to compare to MiniBooNE data
- Simulate possible MicroBooNE limits

Conclusions

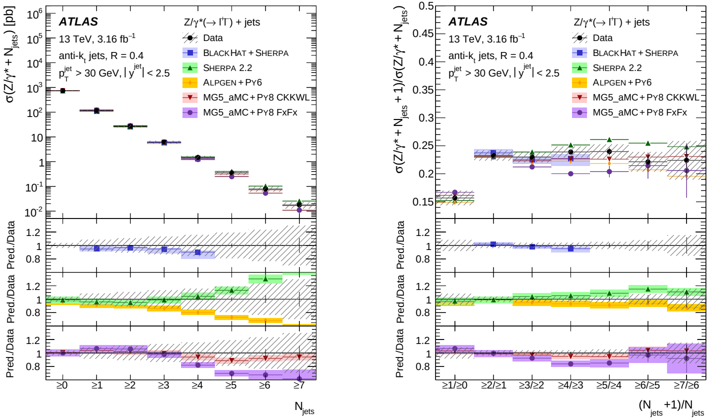
- BSM important for the current and next generation neutrino experiments
- Robust BSM program requires automating theory calculations
- Developed method for handling arbitrary form factors
- Proof of principle for SM processes
- Preliminary results for first BSM process (Heavy Neutral Leptons)
- If you want to test out your model, come talk with me

Universal FeynRules Output (UFO)
Example for photon-electron vertex

```
e__minus__ = Particle(pdg_code=11, name='e-', antiname='e+',
                      spin=2, color=1, mass=Param.ZERO,
                      width=Param.ZERO, texname='e-',
                      antitexname='e+', charge=-1,
                      GhostNumber=0, LeptonNumber=1,
                      Y=0)
V_77 = Vertex(name='V_77',
              particles=[ P.e__plus__, P.e__minus__, P.a ],
              color=[ '1' ], lorentz=[ L.FFV1 ],
              couplings={(0,0):C.GC_3})
FFV1 = Lorentz(name='FFV1', spins=[ 2, 2, 3 ],
               structure='Gamma(3,2,1)')
GC_3 = Coupling(name='GC_3', value='-(ee*complex(0,1))',
                order={'QED':1})
```

Tree Level Matrix Element Generators

- ALPGEN [arXiv:hep-ph/0206293]
- AMEGIC [arXiv:hep-ph/0109036]
- COMIX [arXiv:0808.3674]
- CALCHEP [arXiv:1207.6082]
- HERWIG [arXiv:0803.0883]
- MADGRAPH [arXiv:1405.0301]
- WHIZARD [arXiv:0708.4233]
- etc.



[arXiv:1702.05725]

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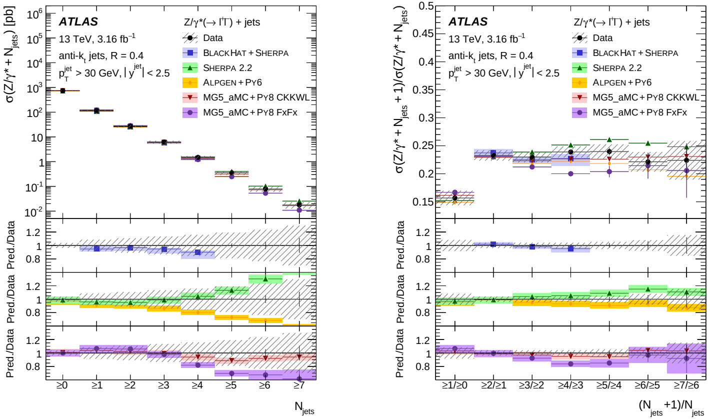

[arXiv:1702.05725]

$2 \to 2$ Phase Space Example

Consider $l + {}^{12}C \to l' + N + X$ in the quasielastic regime.

$$\mathrm{d}\sigma \propto \ \mathrm{d}\Phi_2(a, b; 1, 2) \ \ \mathrm{d}^4 p_a \ \ \mathrm{d}^3 p_b$$

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$$d\sigma \propto d\Phi_2(a, b; 1, 2)\ d^4 p_a\ \ d^3 p_b$$

- Phase space: $d\Phi_2(a, b; 1, 2) = \frac{\lambda(s_{ab}, s_1, s_2)}{16\pi^2 2 s_{ab}} d\cos\theta_1 d\phi_1$

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- Initial nucleon: $\mathrm{d}^4 p_a = |\vec{p}_a|^2 \mathrm{d}p_a \mathrm{d}E_r \mathrm{d}\cos\theta_a \mathrm{d}\phi_a$

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- Initial nucleon: $d^4p_a = |\vec{p}_a|^2 dp_a dE_r d\cos\theta_a d\phi_a$
- Initial lepton (Here only monochromatic): $d^3p_b = \delta^3(p_b - p_{beam})d^3p_b$

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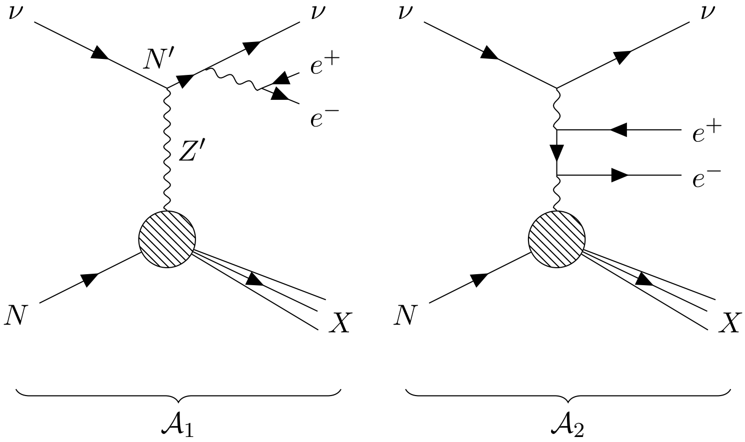
Quasielastic Delta Function: $\delta(E_b - E_1 - E_r + m - E_2)$
Phase Space Delta Function: $\delta(E_a + E_b - E_1 - E_2)$
Define initial nucleon energy as $E_a = m - E_r$. Allows use of phase space tools developed at LHC.

Multi-channel Integration



- Both diagrams contribute to cross section
- They have different pole structures
- Need method to sample these structures efficiently (i.e. $|\mathcal{A}_1 + \mathcal{A}_2|^2$)

Multi-channel Integration

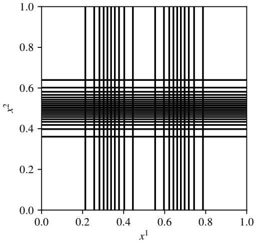
- Generate PS efficiently for $|\mathcal{A}_1|^2$ or $|\mathcal{A}_2|^2$
- Do not know how to efficiently sample $2Re(\mathcal{A}_1\mathcal{A}_2^\dagger)$
- Define channels: C_1 and C_2
- Generate events according to distributions g_i for channel i

$$\int d\vec{x} f(\vec{x}) = \sum_i \alpha_i \int d\vec{x} g_i(\vec{x}) \frac{f(\vec{x})}{g_i(\vec{x})}$$

- Optimize α_i to minimize variance

VEGAS

- Adaptive importance sampling
- Use this to get interference terms more accurately



VEGAS grid for $\int_0^1 d^4x \left(e^{-100(\vec{x}-\vec{r_1})^2} + e^{-100(\vec{x}-\vec{r_2})^2}\right)$

[J.Comput.Phys. 27 (1978) 291, 2009.05112]

Recursive Phase Space Decomposition

Phase space can be decomposed as:

$$\mathrm{d}\Phi_n(a,b;1,\ldots,n) = \mathrm{d}\Phi_{n-m+1}(a,b;m+1,\ldots,n)\frac{\mathrm{d}s_\pi}{2\pi}\mathrm{d}\Phi_m(\pi;1,\ldots,m)$$

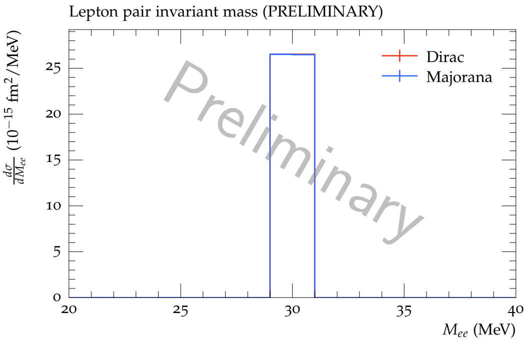
Iterate until only $1 \to 2$ phase spaces remain.
Basic building blocks:

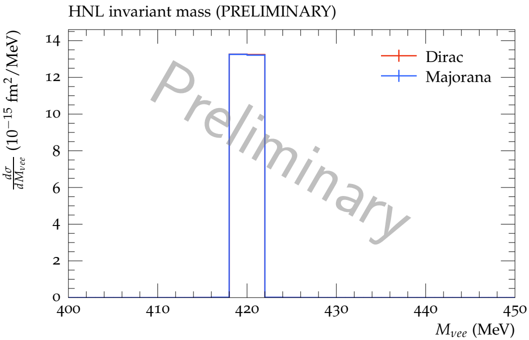
$$S_\pi^{\rho,\pi\setminus\rho} = \frac{\lambda(s_\pi, s_\rho, s_{\pi\setminus\rho})}{16\pi^2\, 2\, s_\pi}\,\mathrm{d}\cos\theta_\rho\,\mathrm{d}\phi_\rho$$

$$T_{\alpha,b}^{\pi,\overline{\alpha b\pi}} = \frac{\lambda(s_{\alpha b}, s_\pi, s_{\overline{\alpha b \pi}})}{16\pi^2\, 2 s_{\alpha b}}\,\mathrm{d}\cos\theta_\pi\,\mathrm{d}\phi_\pi$$

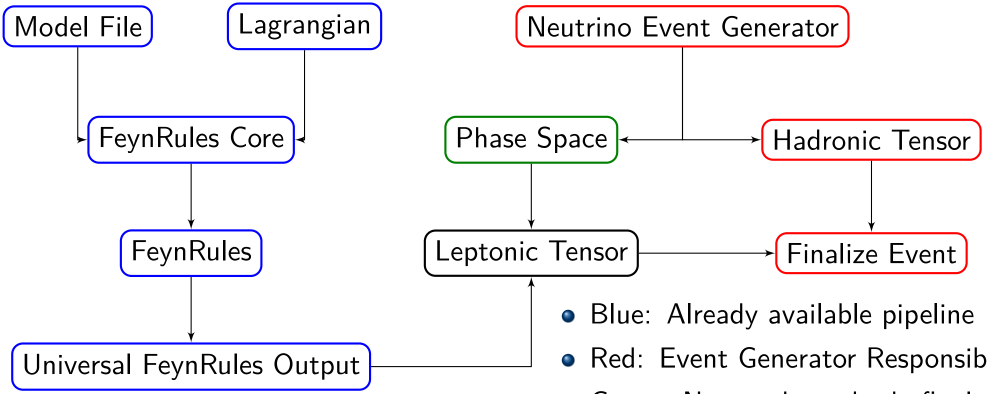
Momentum conservation: $(2\pi)^4\mathrm{d}^4 p_{\overline{\alpha b}}\delta^{(4)}(p_\alpha + p_b - p_{\overline{\alpha b}})$

Additional Heavy Neutral Lepton Plots





Proposed Interface



- Blue: Already available pipeline
- Red: Event Generator Responsibility
- Green: New code: n-body final state event generator: initial state momenta