Initial State Dynamics in Nuclear Interactions

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Outline

- ⋆ Impulse approximation regime and emergence of factorisation
- ⋆ Empirical evidence of factorisation of the electron-nucleus cross-section
- ⋆ The nuclear spectral function
- ⋆ Determination of the target spectral function from the measured $(e, e'p)$ cross section
- ⋆ Outlook

The lepton-nucleus x-section

⋆ Consider, for example, the cross section of the process $\ell + A \to \ell' + X$ at fixed beam energy

$$d\sigma_A \propto L_{\mu\nu} W_A^{\mu\nu}$$

- $L_{\mu\nu}$ is fully specified by the lepton kinematical variables
- The nuclear response tensor

$$W_A^{\mu\nu} = \sum_X \langle 0 | J_A^{\mu\,\dagger} | X \rangle \langle X | J_A^{\nu} | 0 \rangle \delta^{(4)}(P_0 + k - P_X - k')$$

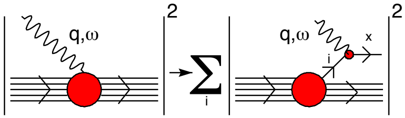
involves

1. the target ground state, $|0\rangle$, largely non relativistic
2. hadronic final state, $|X\rangle$, carrying momentum $q = k - k'$ and possibly involving hadrons other than nucleons → relativistic treatment needed at high q
3. the nuclear current operator, explicitly depending on q → relativistic treatment needed at high q

$$J_A^{\mu} = \sum_i j_i^{\mu} + \sum_{j>i} j_{ij}^{\mu}$$

Impulse approximation and factorisation

- ⋆ for $\lambda \sim 1/|\mathbf{q}| \ll d_{\rm NN} \sim 1.6$ fm, $d_{\rm NN}$ being the average nucleon-nucleon distance in the target nucleus, nuclear scattering reduces to the incoherent sum of scattering processes involving individual nucleons



- ⋆ Basic assumptions
  - ▷ $J^\mu_A(q) \approx \sum_i j^\mu_i(q)$: single-nucleon coupling
  - ▷ $|X\rangle \to |x(\mathbf{p})\rangle \otimes |n_{(A-1)}, \mathbf{p_n}\rangle$: factorisation of the final state

- ⋆ Deviations from the IA, arising from the occurrence of Final State Interactions (FSI) and processes involving two-nucleon currents (MEC) are taken into account as corrections

THE IA CROSS SECTION

- Factorisation allows to rewrite the nuclear transition matrix element as

$$\langle X|J_A^\mu|0\rangle \to \sum_i \int d^3k \; M_n(\mathbf{k})\langle \mathbf{k}+\mathbf{q}|j_i^\mu|\mathbf{k}\rangle$$

  - The nuclear amplitude $M_n = \langle n|a_{\mathbf{k}}|0\rangle$ is independent of momentum transfer. It can be accurately calculated within non relativistic many-body theory
  - The matrix element of the current between free-nucleon states can be computed exactly using the fully relativistic expression

- Nuclear x-section

$$d\sigma_A = \int d^3k dE \; d\sigma_N \; P_h(\mathbf{k}, E)$$

- The lepton-nucleon cross section $d\sigma_N$ can be obtained—at least in principle—from proton and deuteron data, theoretical models, or LQCD
- The spectral function $P_h(\mathbf{k}, E)$ describes the probability of removing a nucleon of momentum $\mathbf{k}$ from the nuclear ground state, leaving the residual system with excitation energy E

EMPIRICAL EVIDENCE OF FACTORISATION: y-SCALING

- Assuming the validity of the IA, and that the elementary scattering process be quasi elastic, from energy and momentum conservation

$$\omega = \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2} + \sqrt{\mathbf{k}^2 + (M_A - m + E)^2}$$

it follows that in the limit $|\mathbf{q}| \to \infty$ the ratio

$$R(\mathbf{q},\omega) = \frac{d\sigma_A}{Z d\sigma_{ep} + (A-Z) d\sigma_{en}} \to F(y) \ ,$$

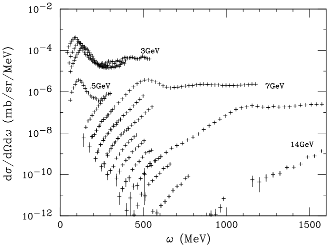
the scaling variable $y = y(|\mathbf{q}|,\omega)$ being trivially related to the longitudinal momentum of the struck nucleon $k_{\parallel} = (\mathbf{k}\cdot\mathbf{q})/|\mathbf{q}|$

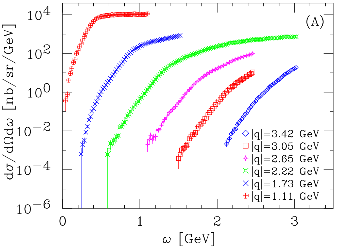
- Remarkable facts about y-scaling

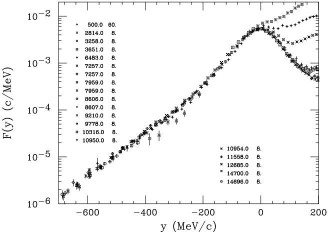
1. The occurrence of scaling provides clearcut evidence of factorisation of the nuclear cross section
2. Scaling reflects the dominance of a reaction mechanism—quasi elastic single-nucleon knock out and absence of FSI—independent of the underlying dynamics
3. Dynamical effects only emerge in scaling violations

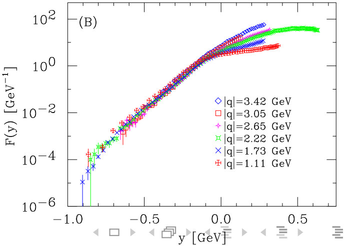
From cross sections to scaling functions

- ^{3}He, D. Day *et al.* PRL, 1987
- ^{56}Fe, J. Arrington *et al.* 1999









THE NUCLEAR SPECTRAL FUNCTION

- The spectral function, being trivially related to the nucleon Green's function through

$$P_h(\mathbf{k}, E) = \frac{1}{\pi} \text{Im}\, G_h(\mathbf{k}, E) = \sum_n |\langle n|a_{\mathbf{k}}|0\rangle|^2 \delta(E - E_0 + E_n)$$

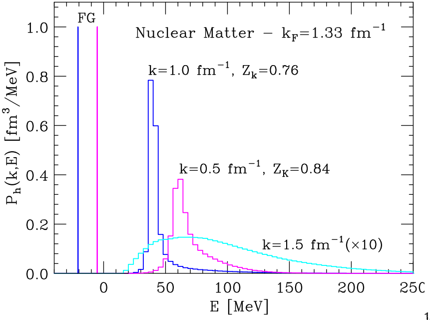
  can be split into pole and smooth contributions

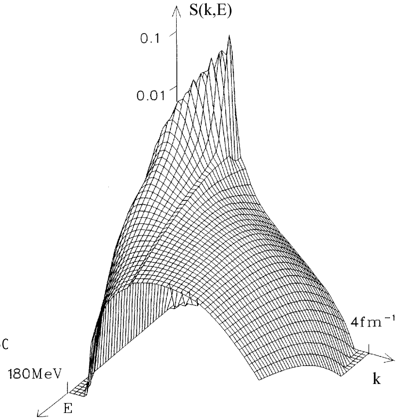
- The smooth component originates from effects beyond the mean field (MF) approximation underlying the nuclear shell model, notably short range correlations

- Prominent correlation effects are
  1. the occurrence of multi-nucleon emission in single-nucleon knockout processes
  2. a corresponding quenching of the strength in the single-nucleon emission sector

Isospin Symmetric Nuclear Matter at Equilibrium

- Calculations carried out using a realistic nuclear Hamiltonian model and the formalism of CBF perturbation theory; O.B. *et al.*, 1989

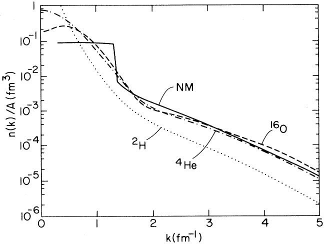




THE LOCAL DENSITY APPROXIMATION (LDA)

$$n(k) = \int dE \; P(\mathbf{k}, E)$$

- ⋆ The tail of the momentum distribution, arising from the continuum contribution to the spectral function, turns out to be largely A-independent for $A > 2$



- ⋆ Spectral functions of complex nuclei can been obtained within LDA

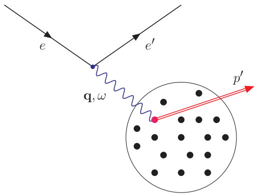
$$P_{\rm LDA}(\mathbf{k}, E) = P_{\rm MF}(\mathbf{k}, E) + \int d^3r \; \rho_A(r) \; P^{NM}_{\rm corr}[\mathbf{k}, E; \rho = \rho_A(r)]$$

using the MF contributions obtained from $(e, e'p)$ data; O.B. *et al.*, 1991

- ⋆ The continuum, or correlation, contribution, $P^{NM}_{corr}(\mathbf{k}, E)$, can be accurately computed in uniform nuclear matter

THE $(e, e'p)$ REACTION

- ⋆ Consider the process $e + A \to e' + p + (A-1)$ in which both the scattering electron and the outgoing proton, carrying momentum p', are detected in coincidence



- ▶ In the absence of FSI, the initial energy and momentum of the knocked out nucleon can be identified with the *measured* missing momentum and missing energy, respectively

$$\mathbf{p}_m = \mathbf{p}' - \mathbf{q}$$

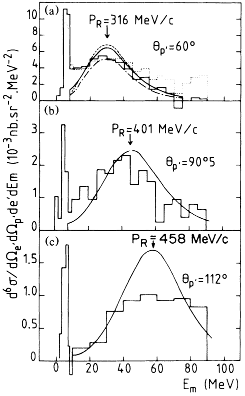
$$E_m = \omega - T_{\mathbf{p}'} - T_{A-1} \approx \omega - T_{\mathbf{p}'}$$

- ⋆ The $(e, e'p)$ cross section provides access to the spectral function

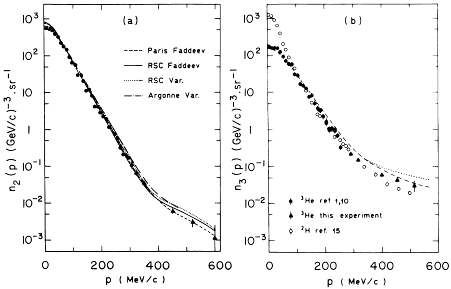
$$\frac{d\sigma_A}{dE_{e'} d\Omega_{e'} dE_p d\Omega_p} \propto \sigma_{ep} P(p_m, E_m)$$

MEASURED $(e, e'p)$ CROSS SECTIONS AND SPECTRAL FUNCTION

- ⋆ The pole and continuum contributions to the spectral function have been clearly observed in ^{3}He$(e, e'p)$; C. Marchand *et al.*, 1988

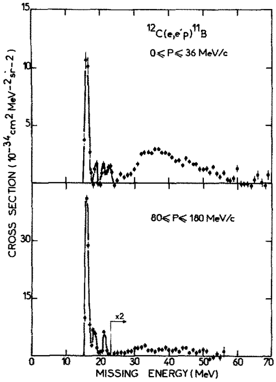


- ▶ Momentum distributions measured in the two- (a) and three-body (b) breakup channels

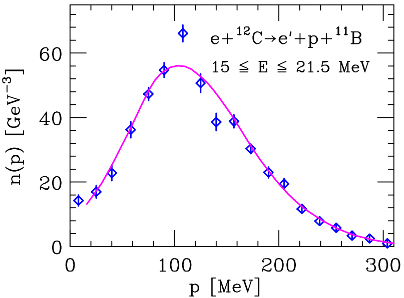


^{12}C$(e, e'p)$ AT MODERATE MISSING ENERGY

- Missing energy spectrum of ^{12}C measured at Saclay;
  J. Mougey, *et al.* (1976)



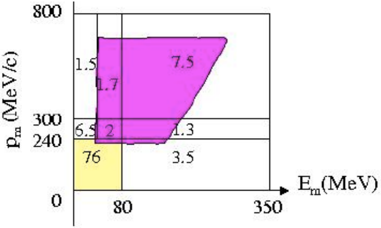
- p-state momentum distribution.

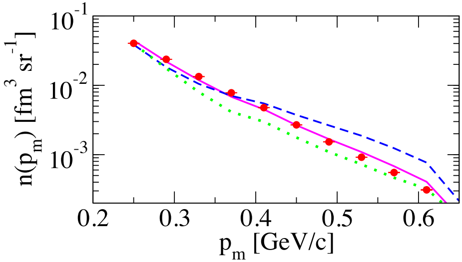


⋆ The occupation probabilities of shell model states turn out to be significantly less than unity: $Z_h \sim 0.6 \div 0.85$

Measured correlation strength in $^{12}\mathrm{C}(e,e'p)$

- The spectral function strength in the continuum region has been measured by the JLAB E97-006 Collaboration using a carbon target; D. Rohe *et al.* (2004).
- strong energy-momentum correlation: $E \sim E_{thr} + \frac{A-2}{A-1}\,\frac{\mathbf{k}^2}{2m}$





- Measured correlation strength 0.61 ± 0.06, to be compared with the theoretical predictions of *ab initio* approaches: 0.46 (GF), 0.61 (SCGF) and 0.64 (CBF)

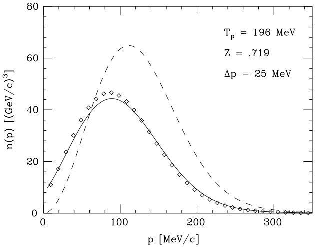
Status of theoretical studies

- ⋆ Accurate calculations of nuclear spectral functions based on phenomenological nuclear Hamiltonian—including both pole and continuum contributions—have been performed for $A = 3$ (H. Mejer-Hajduk *et al.*, 1989) and in the $A \to \infty$ limit (O.B. *et al.*, 1989; A. Ramos *et al.*, 1989).
- ⋆ The LDA scheme has been employed to obtain the proton SF of ^{12}C, ^{16}O, ^{56}Fe, and ^{197}Au (O.B. *et al.*, 1992).
- ⋆ Fully microscopic studies of the SF of complex, non isospin-symmetric, nuclei have been recently carried out using the Self Consistent Green Function formalism (C. Barbieri *et al.*, 2019).
- ⋆ The $^{40}\mathrm{Ar}(e, e'p)$ cross section recently measured at Jefferson Lab, will allow to extend the LDA approach to the nuclear target relevant to neutrino experiments, and provide guidance for the development of accurate microscopic approaches.
- ⋆ A quantitative understanding of FSI corrections to the IA is needed to extract the information on initial state dynamics from the data.

Inclusion of FSI corrections to the IA

- ⋆ The nuclear transparency to the outgoing proton, inferred from $(e, e'p)$ data, provides overwhelming evidence of the significance of FSI

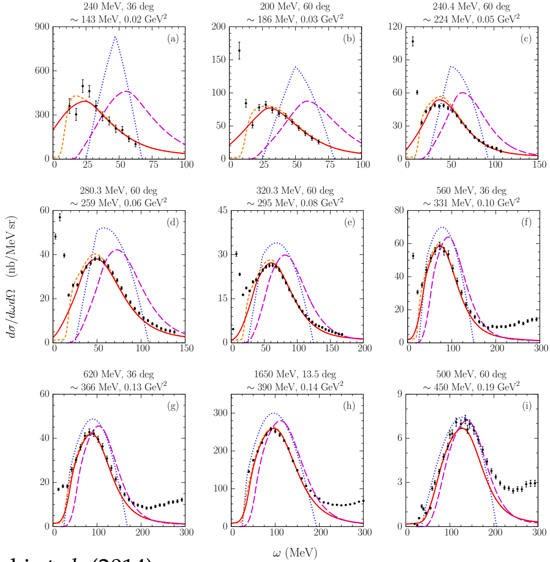
- ▶ prominent effects of FSI
  1. shift of the missing momentum distributions
  2. quenching of the normalisation of the peaks appearing in the missing energy spectra



- ⋆ Departures from factorisation can be minimised by properly choosing the kinematical setup

- ⋆ In the two-body breakup channel, the framework based on the optical potential model has been extensively employed to extract SFs of nuclei ranging from ^{12}C to ^{208}Pb from the cross sections measured at Saclay, NIKHEF-K, and Jefferson Lab

Effects of FSI: $e + {}^{12}\text{C} \to e' + X$ in the QE sector



- Ankowski *et al.* (2014)

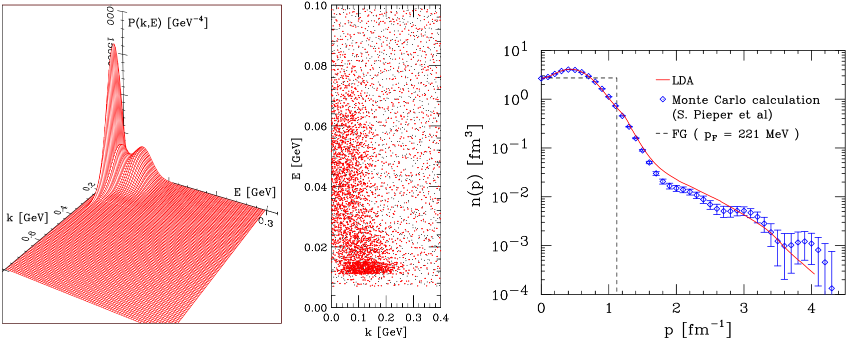
Summary & Outlook

- ⋆ Factorisation of the nuclear cross section in the IA regime is supported by convincing, model independent, empirical evidence.
- ⋆ To the extent to which corrections—arising mainly from FSI—can be reliably taken care of, factorisation can be exploited to extract intrinsic properties of the target from the nuclear cross sections
- ⋆ Most theoretical models of lepton-nucleus scattering involve some level of factorisation. However, to fully exploit its potential, this scheme must be implemented using spectral functions providing an accurate description of the complexity of nuclear dynamics.
- ⋆ The present development of the models employed for the treatment of FSI, while being adequate for two-body breakup sector need to be improved and generalised to treat more complex final states

Thank you!

Backup slides

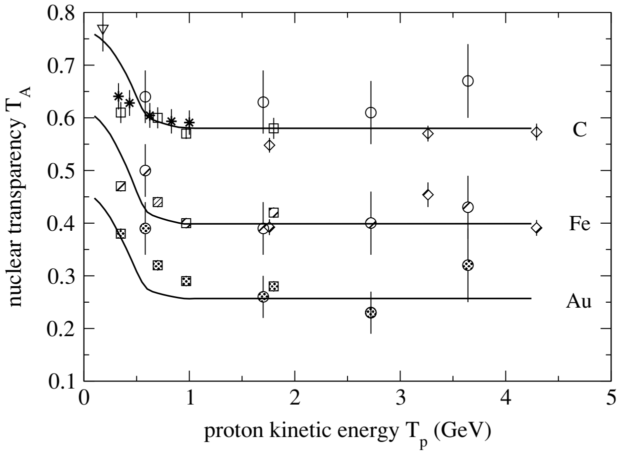
Oxygen Spectral Function



- FG model: $P(\mathbf{p}, E) \propto \theta(p_F - |\mathbf{p}|)\, \delta(E - \sqrt{|\mathbf{p}|^2 + m^2} + \epsilon)$
- shell model states account for $\sim 80\%$ of the strenght
- the remaining $\sim 20\%$, arising from NN correlations, is located at high momentum *and* large removal energy
  ($|\mathbf{p}| \gg p_F \sim 220$ MeV, $E \gg \epsilon$)

GAUGING FSI: NUCLEAR TRANSPARENCY FROM $(e, e'p)$

- Nuclear transparency, measured by the ratio $\sigma_{\rm exp}/\sigma_{\rm IA}$.
  D. Rohe *et al.*, 2005



Kinematic Neutrino Energy Reconstruction

- In the charged current quasi elastic (CCQE) channel, assuming single nucleon single knock, the relevant elementary process is

$$\nu_\ell + n \to \ell^- + p$$

- The *reconstructed* neutrino energy is

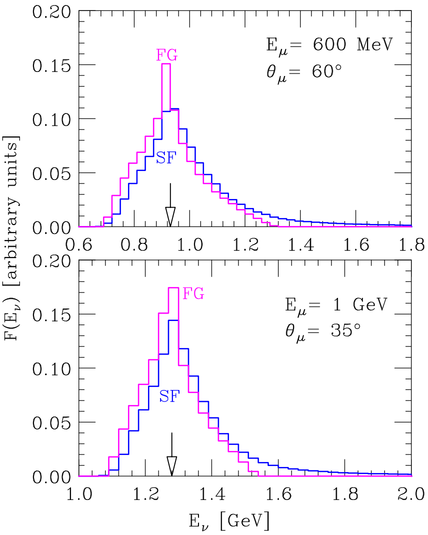
$$E_\nu = \frac{m_p^2 - m_\mu^2 - {E_n}^2 + 2E_\mu E_n - 2\mathbf{k}_\mu \cdot \mathbf{p}_n + |{\mathbf{p}_n}^2|}{2(E_n - E_\mu + |\mathbf{k}_\mu| \cos\theta_\mu - |\mathbf{p}_n| \cos\theta_n)} ,$$

where $|\mathbf{k}_\mu|$ and θ_μ are measured, while $\mathbf{p}_n$ and E_n are the *unknown* momentum and energy of the interacting neutron

- Existing simulation codes routinely use $|\mathbf{p}_n| = 0$, $E_n = m_n - \epsilon$, with $\epsilon \sim 20$ MeV for carbon and oxygen, or the Fermi gas (FG) model

Reconstructed neutrino energy in the CCQE channel

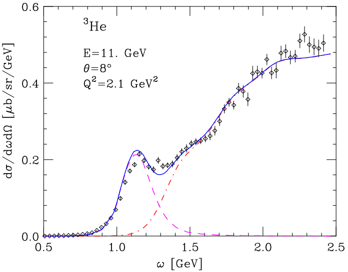
- Neutrino energy reconstructed using 2×10^4 pairs of $(|\mathbf{p}|, E)$ values sampled from realistic (SF) and FG oxygen spectral functions; O.B. and D. Meloni, 2009.
- The average value $\langle E_\nu \rangle$ obtained from the realistic spectral function turns out to be shifted towards larger energy by $\sim$ 70 MeV



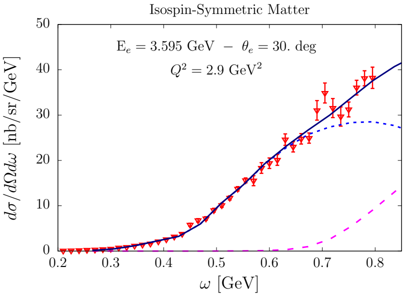
INCLUSIVE ELECTRON-NUCLEUS SCATTERING

- ⋆ elastic and inelastic (RES + DIS) processes consistently taken into account (Bodek & Ritchie parametrisation of SLCA data)
- ⋆ no adjustable parameters involved

- SLAC data; D. Day *et al*, 1979

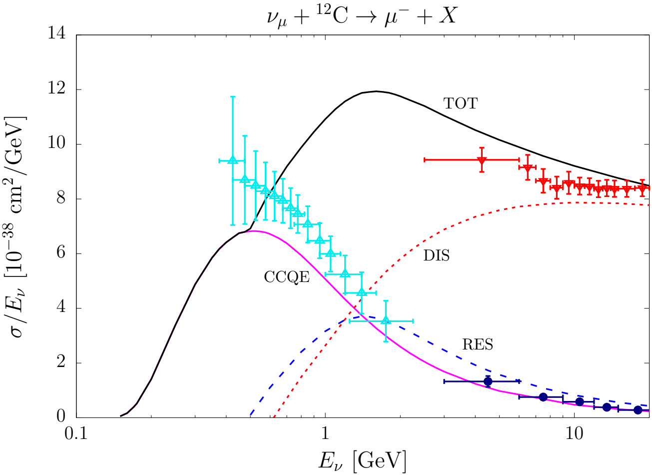


- Extrapolation of SLAC data, taken using targets with $4 \leq A \leq 197$; D. Day *et al*, 1989



Extension to inelastic processes

- Total ν-Carbon Cross Section, E. Vagnoni *et al*, 2017



- σ_{CCQE}: NOMAD, PLB 660, 19 (2008), MiniBooNE, PRD 81, 092005 (2010)
- σ_{TOT}: NOMAD, EPJC 63, 555 (2009)