QMC STA e+^{4}He and e+^{12}C

NuSTEC Workshop on Electron Scattering
29 March 2022

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https://physics.wustl.edu/quantum-monte-carlo-group

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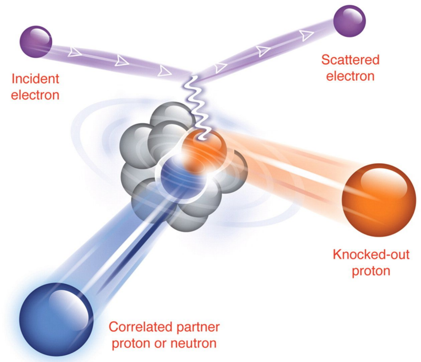
Computational Resources awarded by the DOE ALCC and INCITE programs

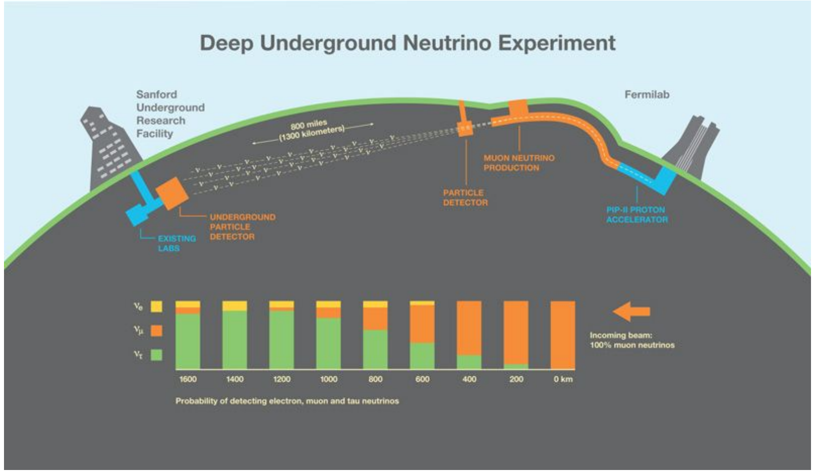
Short-Time-Approximation

Short-Time-Approximation Goals:

- Describe electroweak scattering from A > 12 without losing two-body physics
- Account for exclusive processes
- Incorporate relativistic effects



Subedi et al. Science320(2008)1475



[Stanford Lab article]

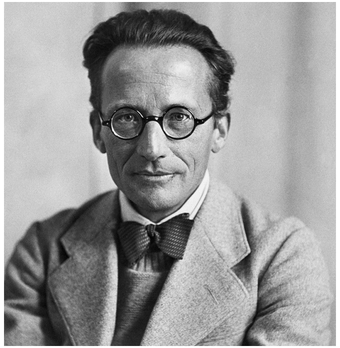
[e4u collaboration]

Microscopic (or *ab initio*) Description of Nuclei

Comprehensive theory that describes quantitatively and predictably nuclear structure and reactions

Requirements:

- Accurate understanding of the interactions/correlations between nucleons in **paris**, **triplets**, ... (**two- and three-nucleon forces**)
- Accurate understanding of the electroweak interactions of external probes (electrons, neutrinos, photons) with nucleons, correlated nucleon-pairs, … (**one- and two-body electroweak currents**)
- **Computational methods** to solve the many-body nuclear problem of strongly interacting particles

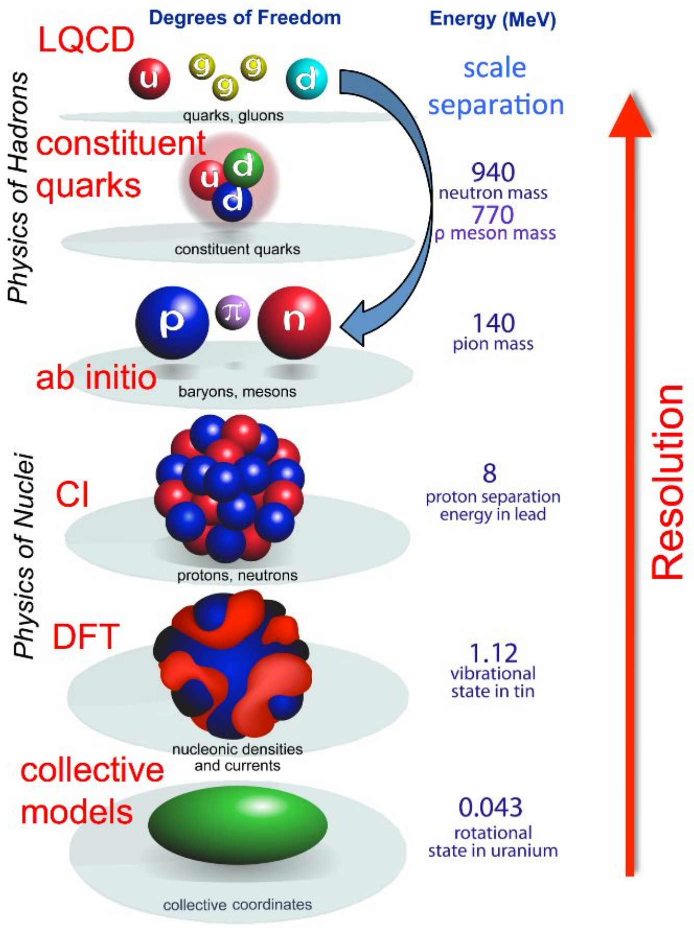


Erwin Schrödinger

$$H\Psi = E\Psi$$

From Quarks to Nuclei

- Nuclei are complex systems made of interacting protons and neutrons, which in turns are composite objects made of interacting constituent quarks.
- All fundamental forces are at play in nuclei.
- **EFTs** low-energy approximations of QCD whose d.o.f. are bound states of QCD (e.g., protons, neutrons, pions, …)
- **EFTs** are used to construct many-nucleon interactions and currents



Many-body Nuclear Problem

Nuclear Many-body Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A, s_1, s_2, \ldots, s_A, t_1, t_2, \ldots, t_A)$$

http://exascaleage.org/np/

Ψ are spin-isospin vectors in 3A dimensions with $2^A \times \frac{A!}{Z!(A-Z)!}$ components

^{4}He : 96
^{6}Li : 1280
^{8}Li : 14336
^{12}C : 540572

Develop Computational Methods to solve (numerically) exactly or within approximations that are under control the many-body nuclear problem

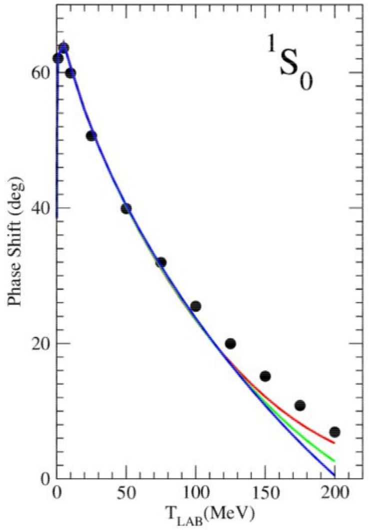
Wide availability of computing is key to critical progress in nuclear physics across all areas.

Many-body Nuclear Interactions

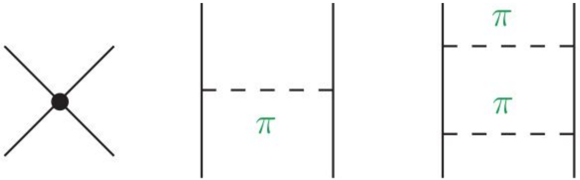
Many-body Nuclear Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

v_{ij} and V_{ijk} are two- and three-nucleon operators based on experimental data fitting; fitted parameters subsume underlying QCD dynamics



SP et al. PRC80(2009)034004

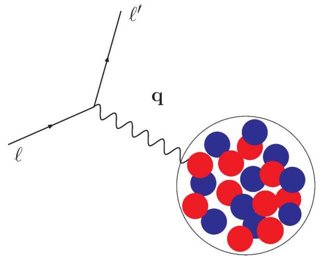


Contact term: short-range
Two-pion range: intermediate-range $r \propto (2\, m_\pi)^{-1}$
One-pion range: long-range $r \propto m_\pi^{-1}$

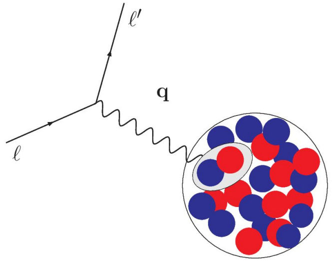
In Quantum Monte Carlo methods we use:

AV18+UIX; AV18+IL7 Wiringa, Schiavilla, Pieper *et al.*
chiral πN N2LO+N2LO Gerzelis, Tews, Lynn *et al.*
chiral πNΔ N3LO+N2LO Piarulli *et al.* Norfolk Models

Many-body Nuclear Electroweak Currents



one-body



two-body

- One-body currents: non-relativistic reduction of covariant nucleons' currents
- Two-body currents are a manifestation of two-nucleon correlations
- Electromagnetic two-body currents are required to satisfy current conservation

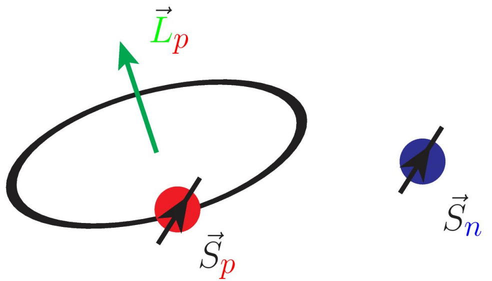
$$\mathbf{q}\cdot\mathbf{j} = [\,H,\,\rho\,] = [t_i + v_{ij} + V_{ijk},\,\rho\,]$$

Nuclear Charge Operator

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots$$

Nuclear (Vector) Current Operator

$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$



Magnetic Moment: Single Particle Picture

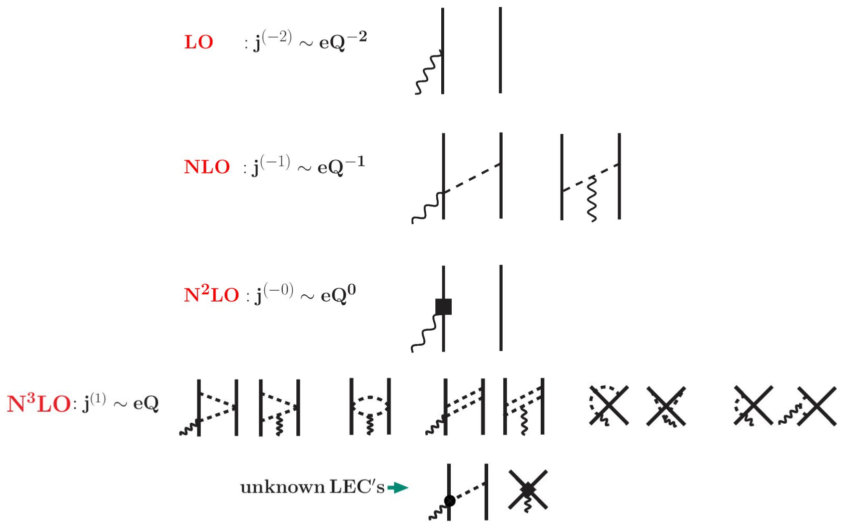
Many-body Currents: Available Models

- **Meson Exchange Currents** (MEC)

Constrain the MEC current operators by imposing that the current conservation relation is satisfied with the given two-body potential

- **Chiral Effective Field Theory Currents**

Are constructed consistently with the two-body chiral potential; Unknown parameters, or Low Energy Constants (LECs), need to be determined by either fits to experimental data or by QCD calculations, as well as nucleonic form factors



Electromagnetic Current Operator

SP *et al.* PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001, PRC87(2013)014006
Park *et al.* NPA596(1996)515, Phillips (2005)
Kölling *et al.* PRC80(2009)045502 & PRC84(2011)054008

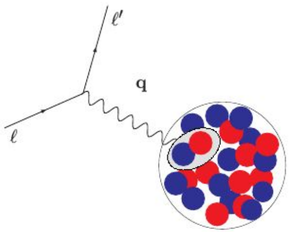
Lepton-Nucleus scattering: Inclusive Processes

Electromagnetic Nuclear Response Functions

$$R_\alpha(q,\omega) = \sum_f \delta\left(\omega + E_0 - E_f\right) |\langle f|O_\alpha(\mathbf{q})|0\rangle|^2$$

Longitudinal response induced by the charge operator $O_L = \rho$
Transverse response induced by the current operator $O_T = \boldsymbol{j}$
5 Responses in neutrino-nucleus scattering

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M \left[v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)\right]$$

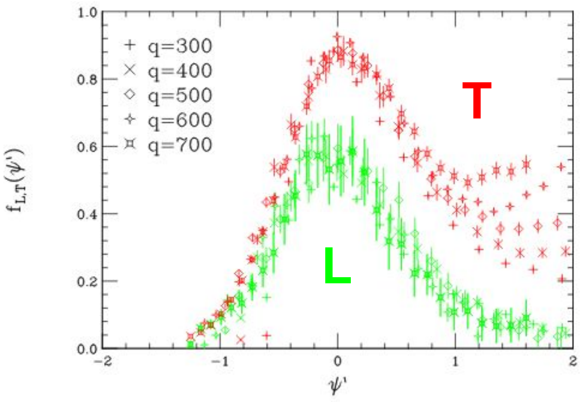


For a recent review on QMC, SF methods see
Rocco *Front. In Phys.*8 (2020)116

Lepton-Nucleus scattering: Data

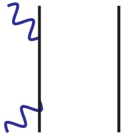
Transverse Sum Rule

$$S_T(q) \propto \langle 0|\mathbf{j}^\dagger\ \mathbf{j}|0\rangle \propto \langle 0|\mathbf{j}_{1b}{}^\dagger\ \mathbf{j}_{1b}|0\rangle + \langle 0|\mathbf{j}_{1b}{}^\dagger\ \mathbf{j}_{2b}|0\rangle + \ldots$$



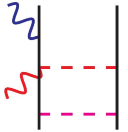
^{4}He Electromagnetic Data
Carlson *et al.* PRC65(2002)024002

Observed transverse enhancement explained by the combined effect of two-body correlations and currents in the interference term



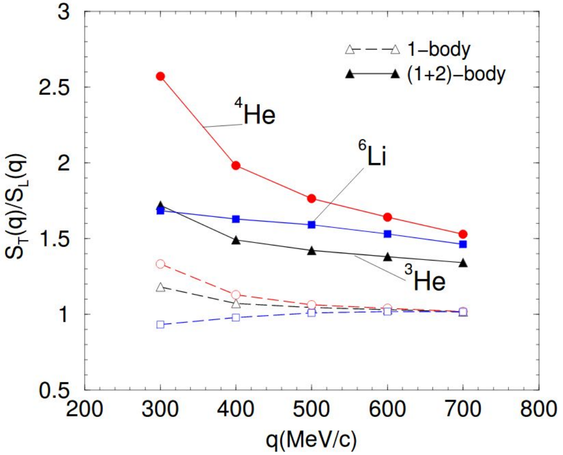
$\langle \mathbf{j}_{1b}^\dagger\ \ \mathbf{j}_{1b}\rangle\ > 0$

Leading one-body term



$\langle \mathbf{j}_{1b}^\dagger\ \mathbf{j}_{2b}\ v_\pi\rangle \propto \langle v_\pi^2\rangle\ \ > 0$

Interference term

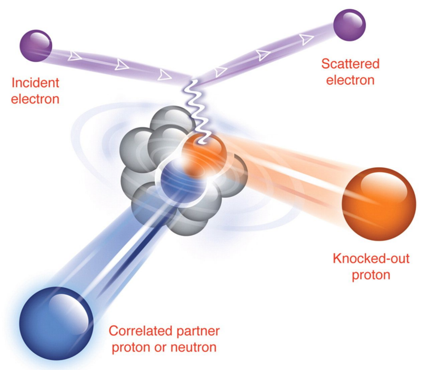


Transverse/Longitudinal Sum Rule
Carlson *et al.* PRC65(2002)024002

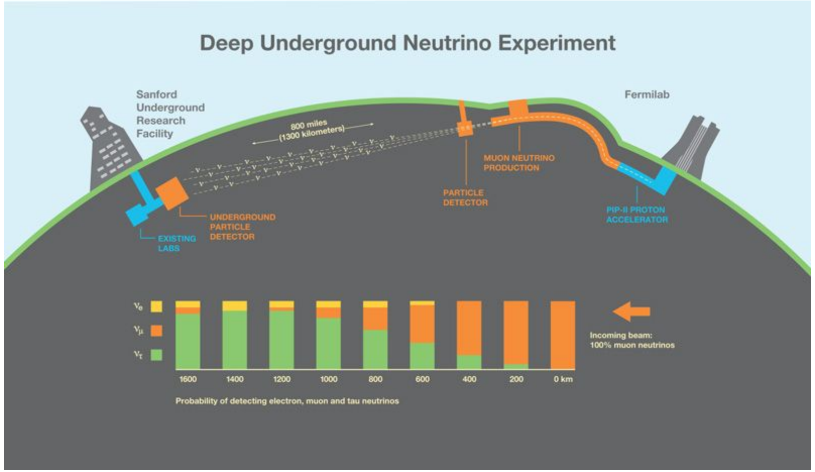
Beyond Inclusive: Short-Time-Approximation

Short-Time-Approximation Goals:

- Describe electroweak scattering from A > 12 without losing two-body physics
- Account for exclusive processes
- Incorporate relativistic effects



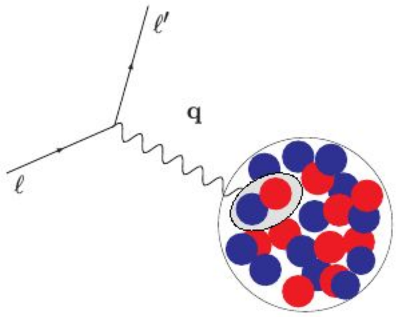
Subedi et al. Science320(2008)1475



[Stanford Lab article]

[e4u collaboration]

Short-Time-Approximation



Short-Time-Approximation:

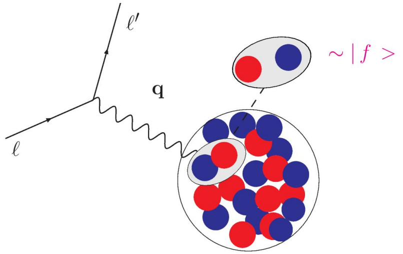
- Based on Factorization
- Retain two-body physics
- Correctly accounts for interference

$$R(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} \mathrm{e}^{i(\omega+E_0)t} \langle 0|O^\dagger \mathrm{e}^{-iHt} O|0\rangle$$

$$O_i^\dagger \mathrm{e}^{-iHt} O_i + O_i^\dagger \mathrm{e}^{-iHt} O_j + O_i^\dagger \mathrm{e}^{-iHt} O_{ij} + O_{ij}^\dagger \mathrm{e}^{-iHt} O_{ij}$$

$$H \sim \sum_i t_i + \sum_{i<j} v_{ij}$$

Short-Time-Approximation



Short-Time-Approximation:

- Based on Factorization
- **Retains two-body physics**
- Response functions are given by the **scattering from pairs of fully interacting nucleons** that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides **"more" exclusive information in terms of nucleon-pair kinematics via the Response Densities**

Response Functions ∝ Cross Sections

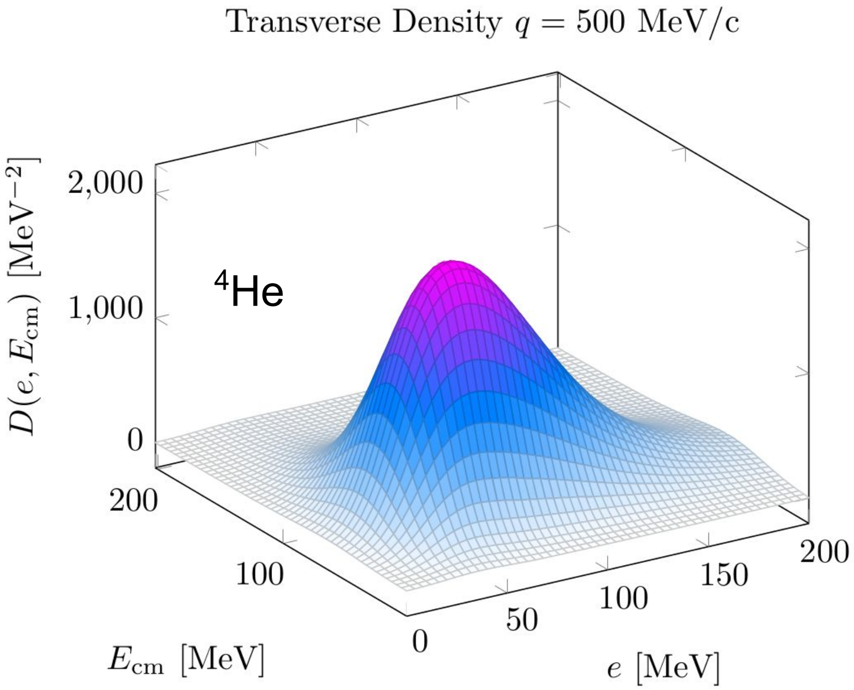
$$R_{\alpha}(q,\omega) = \sum_{f} \delta\left(\omega + E_0 - E_f\right) |\langle f|O_{\alpha}(\mathbf{q})|0\rangle|^2$$

Response ***Densities***

$$R(q,\omega) \sim \int \delta\left(\omega + E_0 - E_f\right)\, dP'\, dp' \mathcal{D}(p', P'; q)$$

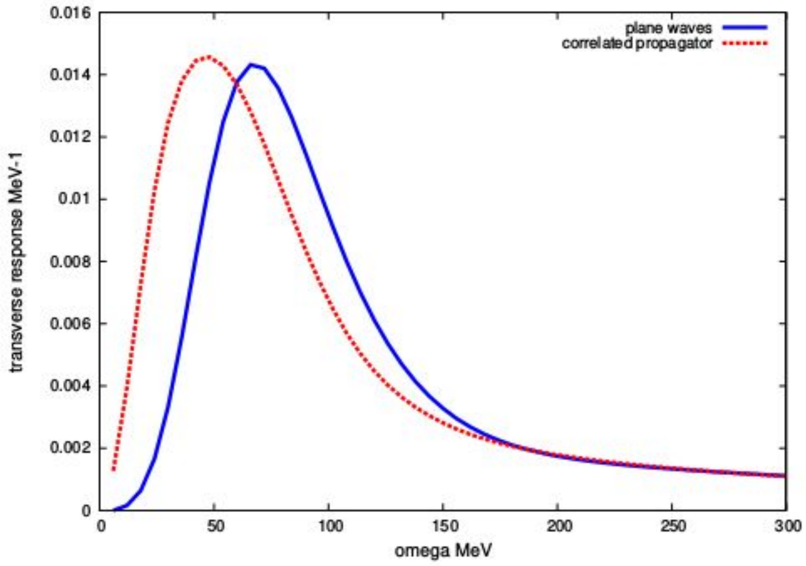
P' and *p'* are the CM and relative momenta of the struck nucleon pair

Transverse Response Density: e-^{4}He scattering



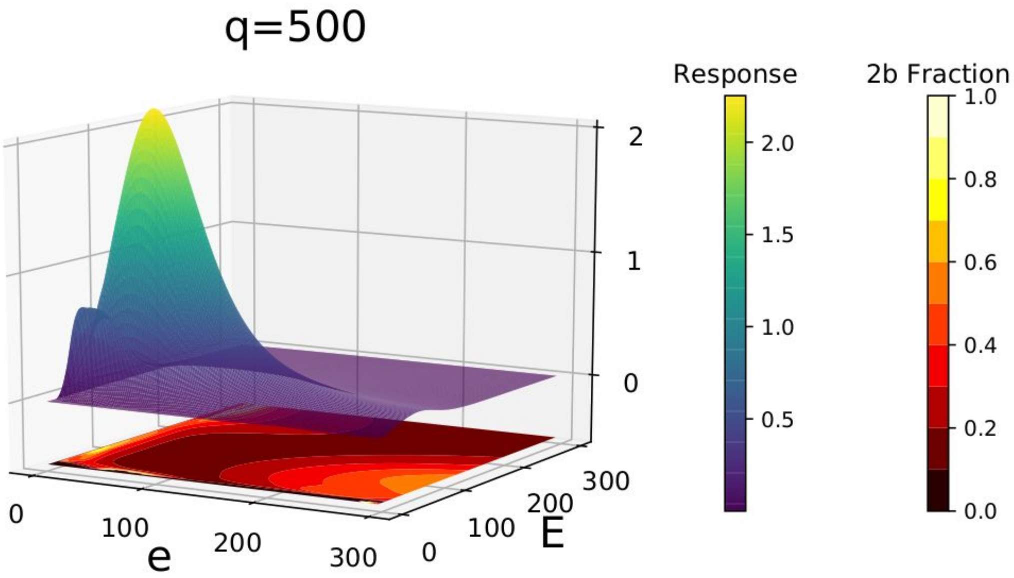
SP *et al.* PRC101(2020)044612

Correlated pairs vs uncorrelated pairs



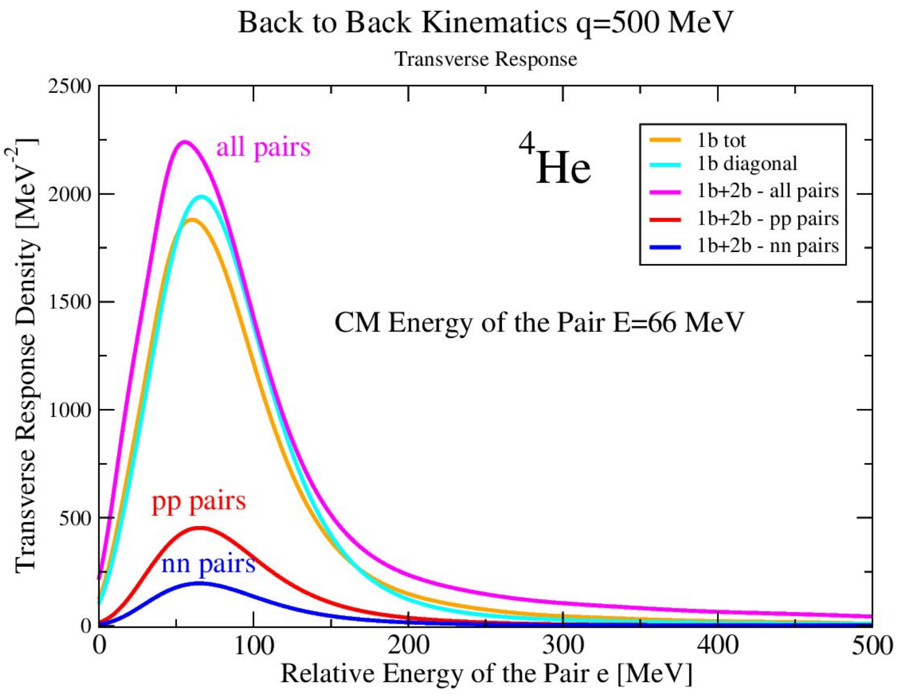
Scattering from uncorrelated vs correlated nucleon pairs

Transverse Response Density: two-body physics



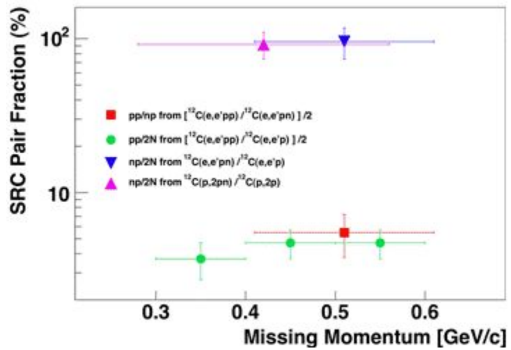
SP *et al.* PRC101(2020)044612

e-^{4}He scattering in the back-to-back kinematic



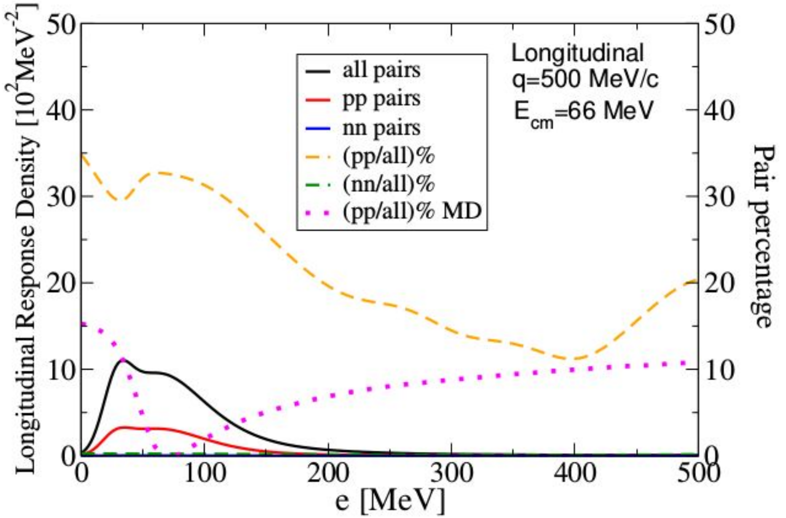
SP *et al.* PRC101(2020)044612

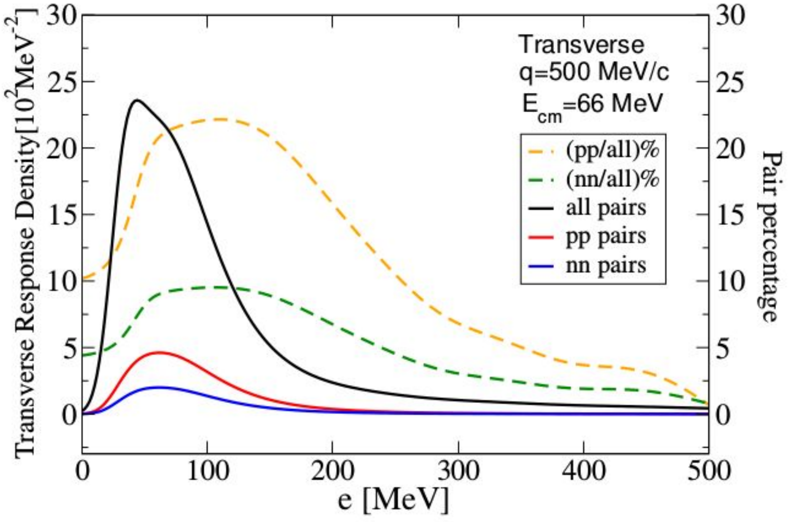
- pp pairs
- nn pairs
- all pairs 1body
- all pairs tot



Subedi et al. Science320(2008)1475

Back to back scattering and particle identity

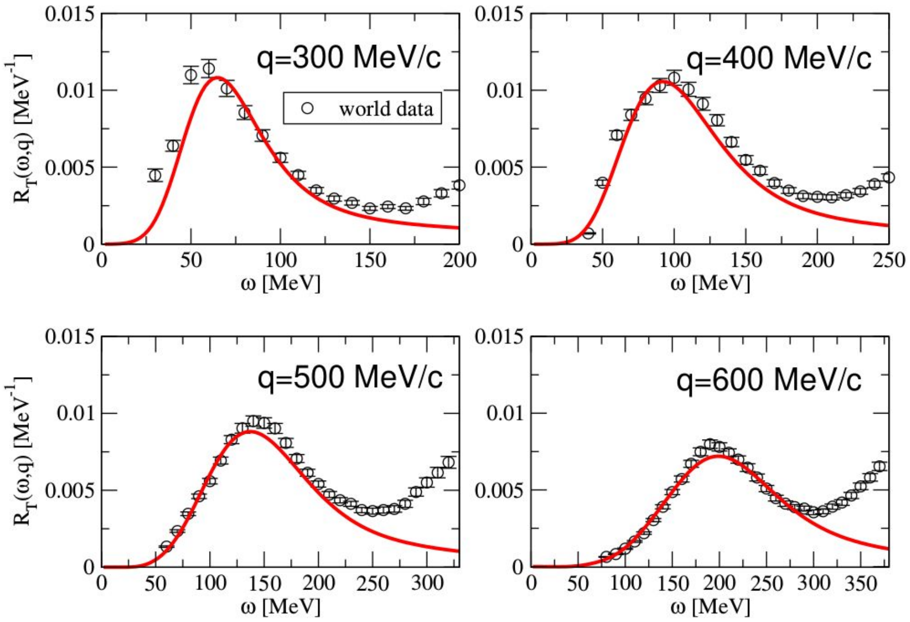


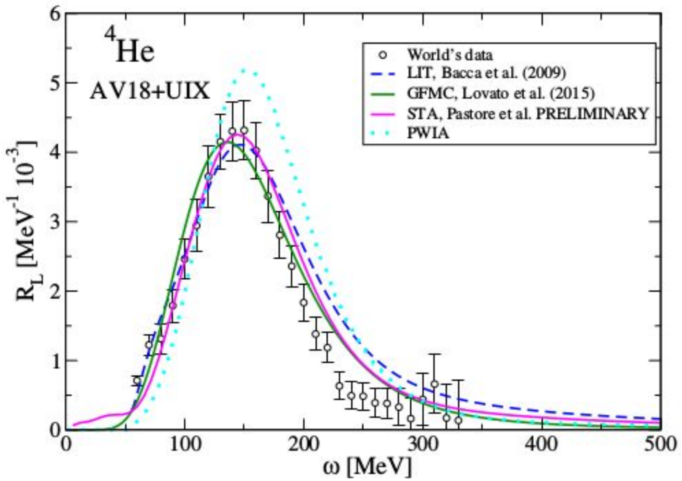


tot
pp nn
pp/all % pp/all % from momentum distributions
nn/all %

SP *et al.* PRC101(2020)044612

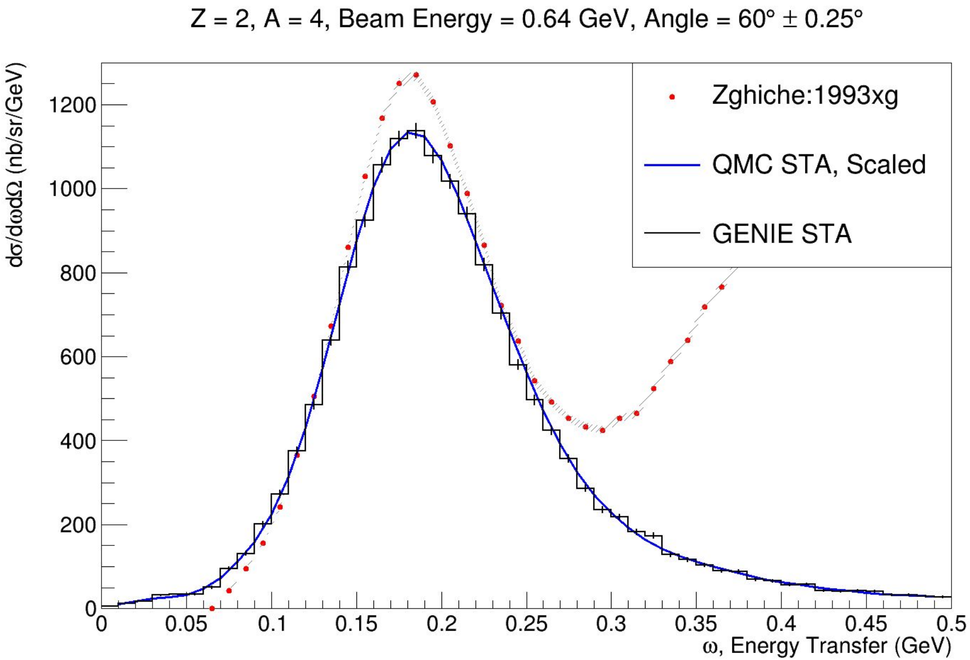
Helium-4 comparison with the data





SP *et al.* PRC101(2020)044612

GENIE validation using e-scattering



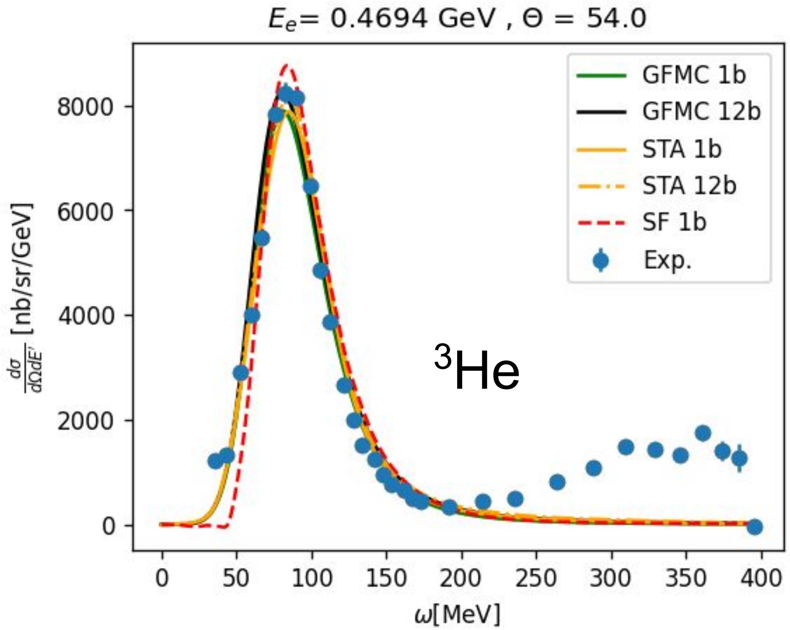
Barrow Gardiner Betancourt SP *et al.*
PRD 103 (2021) 5, 052001

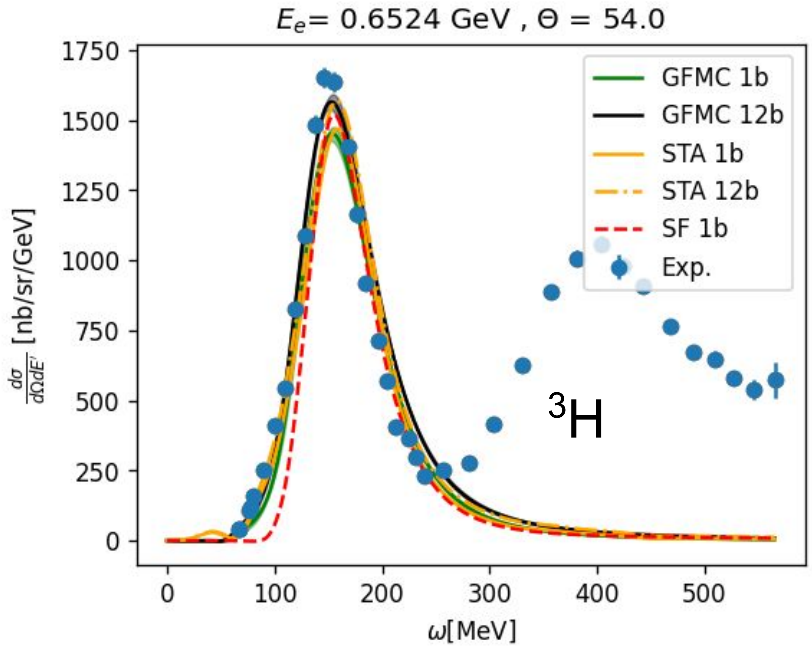
Ongoing work

- Implementation of moment-morphin interpolation techniques
- Implementations of response ***Densities*** in GENIE
- ^{12}C response densities with Lorenzo Andreoli

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M \left[v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)\right]$$

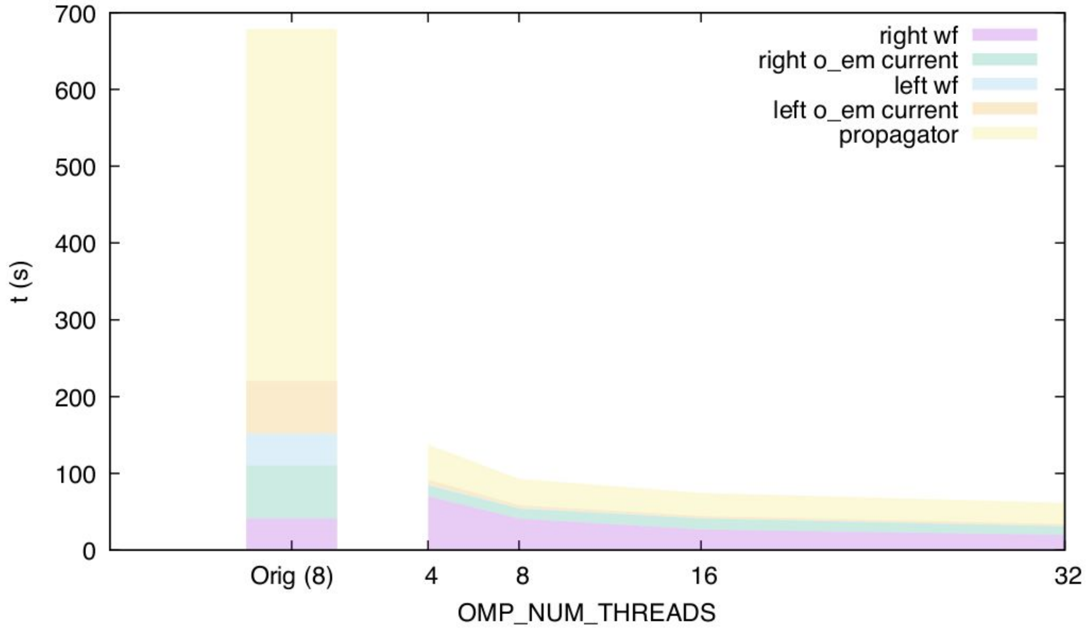
GFMC SF STA: Benchmark & error estimate





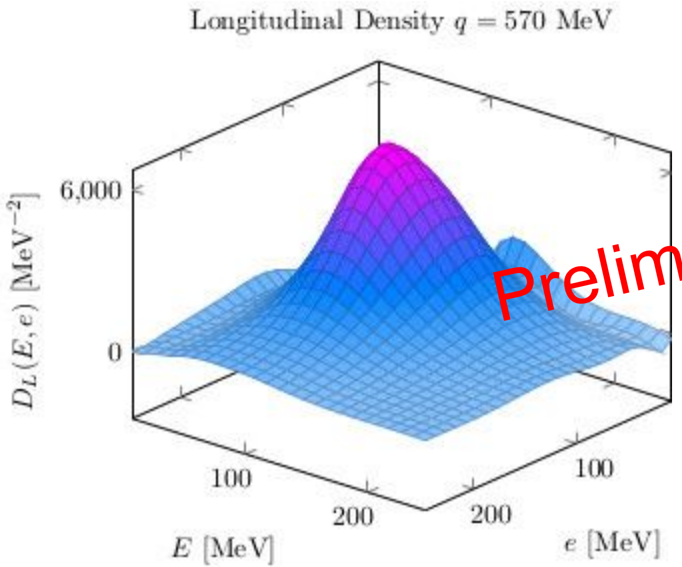
Andreoli Carlson Lovato SP Rocco
Phys.Rev.C 105 (2022) 1, 014002
PRC Editors' suggestion

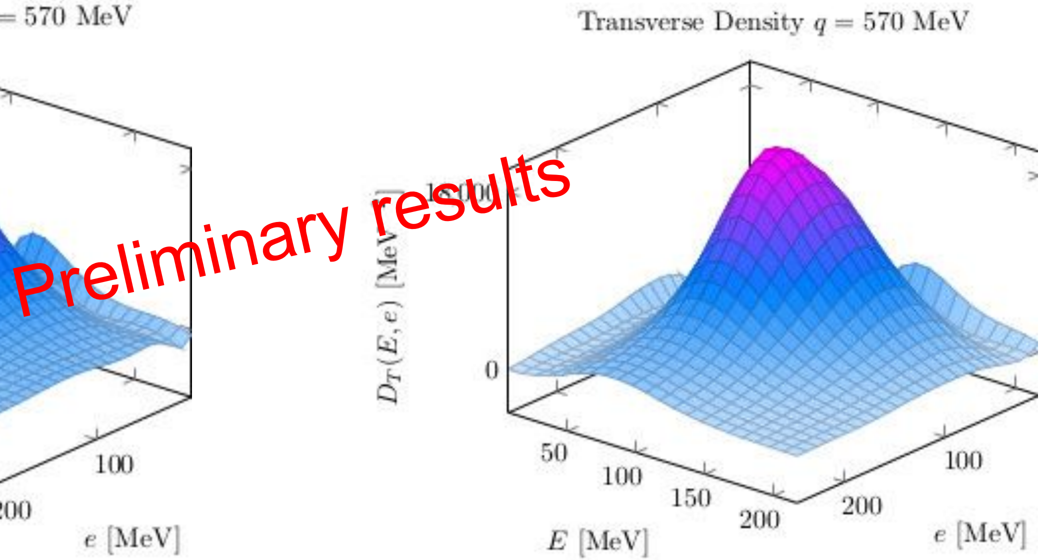
Work in progress: A=12



Lorenzo Andreoli *et al.* in preparation

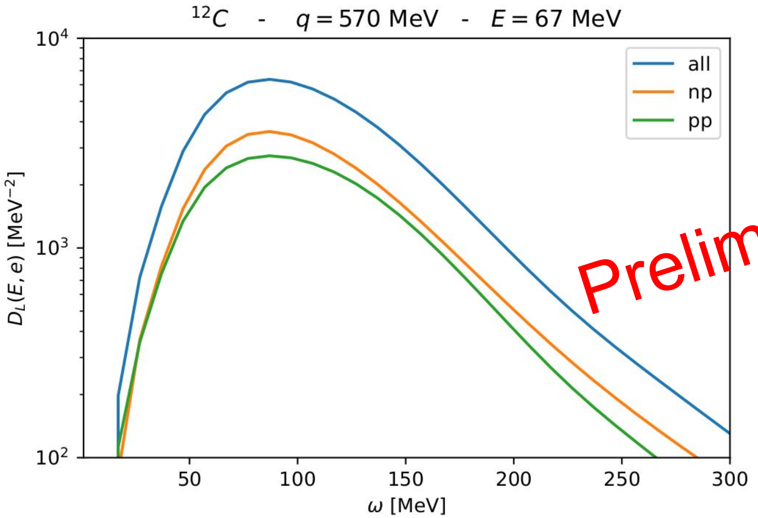
STA for Carbon 12: Response densities

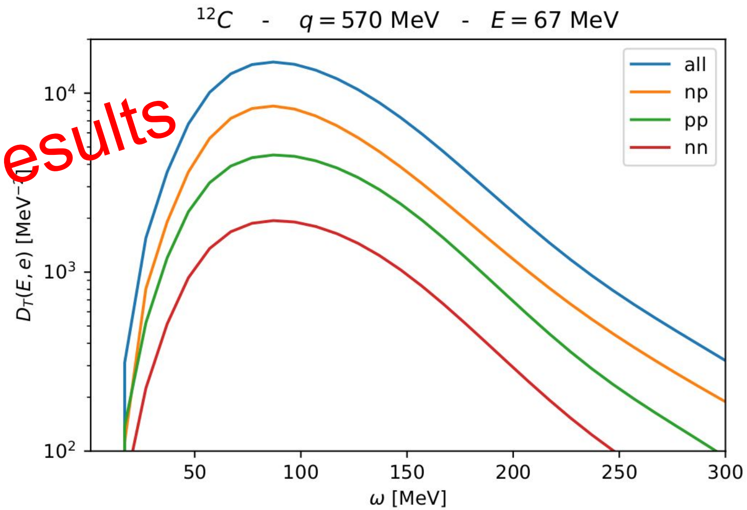




Andreoli *et al.* in preparation

STA for Carbon 12: Particle identity





Preliminary results

Andreoli *et al.* in preparation

Outlook

- Complete calculation on A=12
- Implement response densities in event generators (e.g., GENIE); leading force Barrow & Gardiner *et al.*
- Implement electroweak currents to study neutrino scattering processes
- Implement STA in other QMC methods for medium mass nuclei

Collaborators

WashU: **Andreoli** **Bub** **King** Piarulli
LANL: Carlson Cirigliano Gandolfi Hayes Mereghetti
JLab+ODU: Schiavilla
ANL: Lovato Rocco Wiringa
MIT/TAU: **Barrow**
FNAL: Betancourt **Gardiner**
UW: Dekens
Pisa U/INFN: Kievsky Marcucci Viviani
Salento U: Girlanda
Huzhou U: Dong Wang

Washington University in St. Louis
Arts & Sciences

Theory Alliance
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Quantum Monte Carlo Methods

Minimize the expectation value of the nuclear Hamiltonian: H = T +v_{ij} + V_{ijk}

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

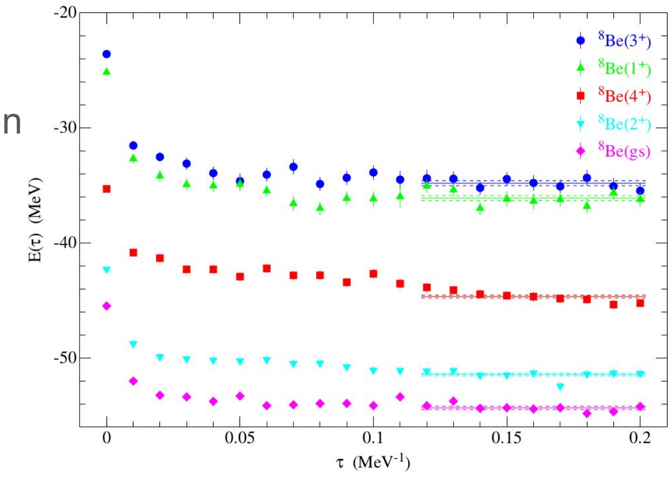
using the trial wave function:

$$|\Psi_V\rangle = \left[\mathcal{S} \prod_{i<j} (1 + U_{ij} + \sum_{k \neq i,j} U_{ijk})\right] \left[\prod_{i<j} f_c(r_{ij})\right] |\Phi_A(JMTT_3)\rangle$$

Further improve the trial wave function by eliminating spurious contaminations via a Green's Function Monte Carlo propagation in imaginary time

$$\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau] a_n \psi_n$$

$$\Psi(\tau \to \infty) = a_0 \psi_0$$



Carlson, Wiringa, Pieper *et al.*