Meson Exchange Currents - Semiempirical formula for 2p2h response functions in neutrino scattering

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Contents

Recent works on MEC in the Relativistic Mean Fiel (RMF) model of nucler matter and SuperScaling with relativistic effective mass.

- V.L. Martinez-Consentino, J.E. Amaro, I. Ruiz Simo **Semiempirical formula for electroweak response functions in the two-nucleon emission channel in neutrino-nucleus scattering**, Phys.Rev.D 104 (2021) 11, 113006
- V.L. Martinez-Consentino, J.E. Amaro, I. Ruiz Simo **Meson-exchange currents and superscaling analysis with relativistic effective mass of quasielastic electron scattering from C12**, Phys.Rev.C 104 (2021) 2, 025501

Quasielastic CC neutrino scattering (ν_μ, μ^-) and $(\overline{\nu}_\mu, \mu^+)$

Neutrino energy: $\epsilon = E_\nu$,
Muon energy: $\epsilon' = m_\mu + T_\mu$,
Lepton momenta are $\mathbf{k}$ and $\mathbf{k}'$.
Four-momentum transfer $k^\mu - k'^\mu = Q^\mu = (\omega, \mathbf{q})$,
$(Q^2 = \omega^2 - q^2 < 0)$.
Lepton scattering angle, θ, the angle between $\mathbf{k}$ and $\mathbf{k}'$.

Inclusive cross section cross section

$$\frac{d^2\sigma}{dT_\mu d\cos\theta} = \frac{G^2\cos^2\theta_c}{4\pi}\frac{k'}{\epsilon}v_0 \times \left[V_{CC}R_{CC} + 2V_{CL}R_{CL} + V_{LL}R_{LL} + V_T R_T \pm 2V_{T'}R_{T'}\right].$$

$(+)$ is for neutrinos and $(-)$ is for antineutrinos.
Weak Fermi constant, $G = 1.166 \times 10^{-11} \quad \mathrm{MeV}^{-2}$
Cabibbo angle, $\cos\theta_c = 0.975$,
$v_0 = (\epsilon + \epsilon')^2 - q^2$.

Leptonic coefficients and nuclear response functions

V_K coefficients depend only on the lepton kinematics

$$
\begin{aligned}
V_{CC} &= 1 + \delta^2 \frac{Q^2}{v_0} \\
V_{CL} &= \frac{\omega}{q} - \frac{\delta^2}{\rho'} \frac{Q^2}{v_0} \\
V_{LL} &= \frac{\omega^2}{q^2} - \left(1 + \frac{2\omega}{q\rho'} + \rho\delta^2\right) \delta^2 \frac{Q^2}{v_0} \\
V_T &= -\frac{Q^2}{v_0} + \frac{\rho}{2} + \frac{\delta^2}{\rho'} \left(\frac{\omega}{q} + \frac{1}{2}\rho\rho'\delta^2\right) \frac{Q^2}{v_0} \\
V_{T'} &= -\frac{1}{\rho'} \left(1 - \frac{\omega\rho'}{q}\delta^2\right) \frac{Q^2}{v_0},
\end{aligned}
$$

$\delta = m_\mu/\sqrt{|Q^2|}$,
$\rho = |Q^2|/q^2$,
$\rho' = q/(\epsilon + \epsilon')$.

The response functions are combinations of the hadronic tensor, $W^{\mu\nu}(q,\omega)$,
The z axis points along $\mathbf{q}$,

$$
\begin{aligned}
R^{CC} &= W^{00} \\
R^{CL} &= -\frac{1}{2}\left(W^{03} + W^{30}\right) \\
R^{LL} &= W^{33} \\
R^{T} &= W^{11} + W^{22} \\
R^{T'} &= -\frac{i}{2}\left(W^{12} - W^{21}\right) .
\end{aligned}
$$

Quasielastic electron scattering

$$\frac{d\sigma}{d\Omega d\epsilon'} = \sigma_{\mathrm{Mott}}(v_L R^L_{em} + v_T R^T_{em}).$$

where σ_{Mott} is the Mott cross section, v_L and v_T are kinematic factors

$$v_L = \frac{Q^4}{q^4} \quad (1)$$

$$v_T = \tan^2\frac{\theta}{2} - \frac{Q^2}{2q^2}. \quad (2)$$

The electromagnetic longitudinal and transverse response functions, $R^L_{em}(q,\omega)$ and $R^T_{em}(q,\omega)$, are

$$R^L_{em} = W^{00}_{em} \quad (3)$$

$$R^T_{em} = W^{11}_{em} + W^{22}_{em} \quad (4)$$

Hadronic tensor

Weak nuclear current operator $J^\mu(Q)$

$$W^{\mu\nu} = \sum_f \overline{\sum_i} \langle f|J^\mu(Q)|i\rangle^* \langle f|J^\nu(Q)|i\rangle \delta(E_i + \omega - E_f).$$

Sum over emission channels

$$W^{\mu\nu} = W^{\mu\nu}_{\rm 1p1h} + W^{\mu\nu}_{\rm 2p2h} + \cdots$$

We focus in the 2p2h channel within the relativistic mean field (RMF) model of nuclear matter.
We have computed the 1p1h channel within the Super Scaling approach with relativistic effective mass (**SuSAM***) in Phys.Rev.C 104 (2021) 2, 025501

Relativistic Mean Field theory of nuclear matter

Relativistic Mean field approximation of quantum hadrodynamics (QHD).
Walecka model of nuclear matter (σ-ω exchange).
The single particle wave functions are plane waves with momentum $\mathbf{p}$,
Dirac equation with strong scalar and vector potentials

$$[\boldsymbol{\alpha} \ \cdot \mathbf{p} + \beta(m_N - g_\sigma S)]\, u(\mathbf{p}) = (E_{RMF} - g_\omega V)u(\mathbf{p})$$

Equivalent to the free Dirac equation with a effective mass

$$m_N^* = m_N - g_\sigma S$$

and effective energy

$$E = E_{RMF} - g_\omega V = E_{RMF} - E_V$$

Energy-momentum relation

$$E = \sqrt{\mathbf{p}^2 + m_N^{*2}}$$

We use values of k_F, M^* and E_v that are obtained phenomenologically from the data of (e, e') for ^{12}C.

$$M^* = m_N^*/m_N = 0.8, \qquad E_v = 141 MeV$$

2p2h hadronic tensor

$$
\begin{aligned}
W^{\mu\nu}_{\rm 2p2h} \quad &= \frac{V}{(2\pi)^9}\int d^3p'_1 d^3p'_2 d^3h_1 d^3h_2 \frac{(m_N^*)^4}{E_1E_2E'_1E'_2} w^{\mu\nu}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2) \\
&\times\theta(p'_1-k_F)\theta(k_F-h_1)\theta(p'_2-k_F)\theta(k_F-h_2) \\
&\times\delta(E'_1+E'_2-E_1-E_2-\omega)\delta(\mathbf{p}'_1+\mathbf{p}'_2-\mathbf{q}-\mathbf{h}_1-\mathbf{h}_2)
\end{aligned}
$$

$V/(2\pi)^3 = Z/(\frac{8}{3}\pi k_F^3)$ for symmetric nuclear matter.
Hadron tensor for single 2p2h transitions

$$
w^{\mu\nu}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2) = \frac{1}{4}\sum_{s_1s_2s'_1s'_2}\sum_{t_1t_2t'_1t'_2} j^\mu(1',2',1,2)^*_A j^\nu(1',2',1,2)_A
$$

The factor $1/4$ accounts for the anti-symmetry of the wave function.
The two-body current matrix elements are antisymetrized

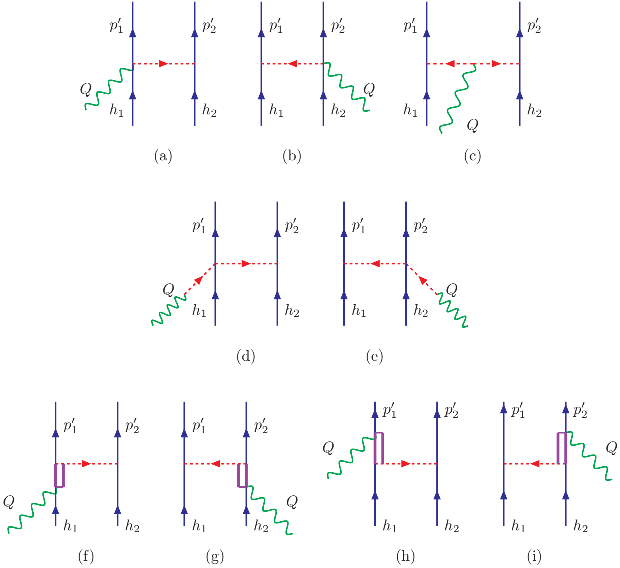
$$
j^\mu(1',2',1,2)_A \equiv j^\mu(1',2',1,2) - j^\mu(1',2',2,1)\,.
$$

2p2h response functions

$$
\begin{aligned}
R^{K}_{\rm 2p2h} &= \frac{V}{(2\pi)^9}\int d^3p'_1 d^3h_1 d^3h_2 \frac{(m^*_N)^4}{E_1E_2E'_1E'_2} r^K(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2) \\
&\times\theta(p'_1-k_F)\theta(k_F-h_1)\theta(p'_2-k_F)\theta(k_F-h_2) \\
&\times\delta(E'_1+E'_2-E_1-E_2-\omega),
\end{aligned}
$$

where $\mathbf{p'_2} = \mathbf{h_1} + \mathbf{h_2} + \mathbf{q} - \mathbf{p'_1}$.
$r^K(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2)$ are the elementary response functions for a 2p2h excitation, $K = CC, CL, LL, T, T'$.
Seven dimensional integral that we compute numerically.
The numerical calculation not practical for neutrino cross sections (additional flux integration) $\longrightarrow$ PARAMETRIZATION NEEDED
$\longrightarrow$ **SEMIEMPIRICAL FORMULA**

MEC DIAGRAMS



Seagull (a,b), pion in flight (c), pion-pole (d,e) and Δ(1232) excitation forward (f,g), and backward (h,i).

MEC matrix elements

$$j^\mu_{\rm sea} = \left[I_V^{\pm}\right]_{1'2',12} \frac{f^2_{\pi NN}}{m^2_\pi} V^{s'_1 s_1}_{\pi NN}(\mathbf{p}'_1, \mathbf{h}_1) F_{\pi NN}(k_2^2) \bar{u}_{s'_2}(\mathbf{p}'_2)[F_1^V(Q^2)\gamma_5\gamma^\mu + \frac{F_\rho\left(k_2^2\right)}{g_A}\gamma^\mu] u_{s_2}(\mathbf{h}_2) + (1 \leftrightarrow 2)$$

$$j^\mu_\pi = \left[I_V^{\pm}\right]_{1'2',12} \frac{f^2_{\pi NN}}{m^2_\pi} F_1^V(Q^2) V^{s'_1 s_1}_{\pi NN}(\mathbf{p}'_1, \mathbf{h}_1) V^{s'_2 s_2}_{\pi NN}(\mathbf{p}'_2, \mathbf{h}_2) \left(k_1^\mu - k_2^\mu\right)$$

$$j^\mu_{\rm pole} = \left[I_V^{\pm}\right]_{1'2',12} \frac{f^2_{\pi NN}}{m^2_\pi} \frac{F_\rho\left(k_1^2\right)}{g_A} F_{\pi NN}(k_2^2) \frac{Q^\mu \bar{u}_{s'_1}(\mathbf{p}'_1) \not{Q} u_{s_1}(\mathbf{h}_1)}{Q^2 - m^2_\pi} V^{s'_2 s_2}_{\pi NN}(\mathbf{p}'_2, \mathbf{h}_2) + (1 \leftrightarrow 2)$$

$$j^\mu_{\Delta F} = \left[U_{\rm F}^{\pm}\right]_{1'2',12} \frac{f^* f_{\pi NN}}{m^2_\pi} V^{s'_2 s_2}_{\pi NN}(\mathbf{p}'_2, \mathbf{h}_2) F_{\pi N\Delta}(k_2^2) \bar{u}_{s'_1}(\mathbf{p}'_1) k_2^\alpha G_{\alpha\beta}(h_1 + Q) \Gamma^{\beta\mu}(Q) u_{s_1}(\mathbf{h}_1) + (1 \leftrightarrow 2)$$

$$j^\mu_{\Delta B} = \left[U_{\rm B}^{\pm}\right]_{1'2',12} \frac{f^* f_{\pi NN}}{m^2_\pi} V^{s'_2 s_2}_{\pi NN}(\mathbf{p}'_2, \mathbf{h}_2) F_{\pi N\Delta}(k_2^2) \bar{u}_{s'_1}(\mathbf{p}'_1) k_2^\beta \hat{\Gamma}^{\mu\alpha}(Q) G_{\alpha\beta}(p'_1 - Q) u_{s_1}(\mathbf{h}_1) + (1 \leftrightarrow 2)$$

$k_i^\mu = (p'_i - h_i)^\mu$: four momentum transferred to the i-th nucleon.

Pion vertex and propagator: $V^{s'_1 s_1}_{\pi NN}(\mathbf{p}'_1, \mathbf{h}_1) \equiv F_{\pi NN}(k_1^2) \frac{\bar{u}_{s'_1}(\mathbf{p}'_1)\,\gamma_5 \not{k}_1\, u_{s_1}(\mathbf{h}_1)}{k_1^2 - m_\pi^2}$.

Strong form factor $F_{\pi NN}(k_1^2) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - k_1^2}$

MEC

Isospin operators ($\vec{T}$ is the transition operator from isospin $\frac{3}{2}$ to $\frac{1}{2}$)

$$\begin{aligned} \boldsymbol{I}_V &= i\,[\boldsymbol{\tau}(1)\times\boldsymbol{\tau}(2)] & I_V^{\pm} &= (I_V)_x \pm i(I_V)_y \\ (U_F)_j &= \sqrt{\frac{3}{2}}\sum_{i=1}^{3}\left(T_i T_j^{\dagger}\right)\otimes\tau_i & U_F^{\pm} &= (U_F)_x \pm i(U_F)_y \\ (U_B)_j &= \sqrt{\frac{3}{2}}\sum_{i=1}^{3}\left(T_j\, T_i^{\dagger}\right)\otimes\tau_i, & U_B^{\pm} &= (U_B)_x \pm i(U_B)_y \end{aligned}$$

$N \to \Delta$ transition vertex in the forward current

$$\Gamma^{\beta\mu}(Q) = \frac{C_3^V}{m_N}\left(g^{\beta\mu}\ \not{Q} - Q^{\beta}\gamma^{\mu}\right)\gamma_5 + C_5^A g^{\beta\mu},$$

backward current

$$\hat{\Gamma}^{\mu\alpha}(Q) = \gamma^0\,[\Gamma^{\alpha\mu}(-Q)]^{\dagger}\,\gamma^0\,.$$

Δ-propagator,

$$G_{\alpha\beta}(P) = \frac{\mathcal{P}_{\alpha\beta}(P)}{P^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta(P^2) + \frac{\Gamma_\Delta(P^2)^2}{4}}$$

Projector spin-$\frac{3}{2}$ particles

$$\mathcal{P}_{\alpha\beta}(P) = -(\not{P} + M_\Delta)\left[g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3}\frac{P_\alpha P_\beta}{M_\Delta^2} + \frac{1}{3}\frac{P_\alpha\gamma_\beta - P_\beta\gamma_\alpha}{M_\Delta}\right]$$

Semiempirical MEC: (1) the phase space integral

The phase space determines the global behavior of the 2p2h responses,
Proportional to the number of 2p2h states that can be excited while conserving energy and momentum (for r^K = constant)

$$\frac{V}{(2\pi)^9}F(q,\omega) = \frac{V}{(2\pi)^9}\int d^3p'_1 d^3h_1 d^3h_2 \frac{(m_N^*)^4}{E_1E_2E'_1E'_2}\delta(E'_1+E'_2-E_1-E_2-\omega). \times\theta(p'_1-k_F)\theta(k_F-h_1)\theta(p'_2-k_F)\theta(k_F-h_2)$$

$\mathbf{p'_2} = \mathbf{h_1} + \mathbf{h_2} + \mathbf{q} - \mathbf{p'_1}$.
Frozen nucleon approximation, $\mathbf{h}_1 \simeq \mathbf{h}_2 \simeq 0$, & Neglect Pauli Blocking

$$F(q,\omega) \simeq F(q,\omega)_{\rm frozen} \simeq 4\pi\left(\frac{4}{3}\pi k_F^3\right)^2 \frac{m_N^{*2}}{2}\sqrt{1-\frac{4m_N^{*2}}{(2m_N^*+\omega)^2-q^2}},$$

The minimum ω to excite a 2p2h state for fixed q in this approximation.

$$\omega_{\rm min} = \sqrt{4m_N^{*2}+q^2} - 2m_N^*$$

The phase space integral is zero below this value.

Semiempirical MEC: (2) The Averaged Δ propagator

The dominant contribution to the MEC responses comes from the peak of the denominator in the the Δ propagator for the forward diagrams

$$G_\Delta(H+Q) \equiv \frac{1}{(H+Q)^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta + \frac{\Gamma_\Delta^2}{4}} \simeq \frac{1}{a - 2\,\mathbf{h}\cdot\mathbf{q} + ib}\,,$$

where $H^\mu = (E_{RMF}, \mathbf{h})$ is the four-momentum of the hole.
Can be replaced by an average of the propagator over the Fermi gas

$$G_{\rm av}(Q) = G_{\rm av}(q,\omega) = \frac{1}{\frac{4}{3}\pi k_F^3}\int \frac{d^3h\,\theta(k_F - |\mathbf{h}|)}{a - 2\,\mathbf{h}\cdot\mathbf{q} + ib}\,,$$

$$= \frac{1}{\frac{4}{3}\pi k_F^3}\frac{\pi}{q}\left\{\frac{(a+ib)\,k_F}{2q} + \frac{4q^2k_F^2 - (a+ib)^2}{8q^2}\ln\left[\frac{a+2k_Fq+ib}{a-2k_Fq+ib}\right]\right\}$$

$$a \equiv m_N^{*2} + (\omega + E_v)^2 - q^2 + 2m_N^*(\omega + E_v + \Sigma) - M_\Delta^2 + \frac{\Gamma^2}{4} \qquad (5)$$

$$b \equiv M_\Delta\Gamma\,. \qquad (6)$$

The Smeared shift and width parameters $\Sigma(q)$, $\Gamma(q)$ in the semiempirical formula are fitted to the exact responses for each value of q.

2p2h MEC subresponses

MEC expansion

$$j^\mu = j^\mu_{SP} + j^\mu_{\Delta F} + j^\mu_{\Delta B}, \qquad j^\mu_{SP} = j^\mu_{\rm sea} + j^\mu_\pi + j^\mu_{\rm pole}$$

In the hadronic tensor we deal with products of the kind

$$\begin{aligned} j^{\mu*} j^\nu &= j^{\mu*}_{SP} j^\nu_{SP} + j^{\mu*}_{\Delta F} j^\nu_{\Delta F} + j^{\mu*}_{\Delta B} j^\nu_{\Delta B} + \\ &\quad + j^{\mu*}_{SP} j^\nu_{\Delta F} + j^{\mu*}_{\Delta F} j^\nu_{SP} + j^{\mu*}_{SP} j^\nu_{\Delta B} + j^{\mu*}_{\Delta B} j^\nu_{SP} + j^{\mu*}_{\Delta F} j^\nu_{\Delta B} + j^{\mu*}_{\Delta B} j^\nu_{\Delta F} \end{aligned}$$

Separate vector and axial responses

$$R^K(q,\omega) = R^{K,VV} + R^{K,AA}, \qquad K = CC, CL, LL, T$$

Expansion in subresponses

$$\begin{aligned} R^{K,VV} &= R^{K,VV}_{SP} + R^{K,VV}_{\Delta F} + R^{K,VV}_{\Delta B} + R^{K,VV}_{\Delta F-SP} + R^{K,VV}_{\Delta B-SP} + R^{K,VV}_{\Delta F-\Delta B} \\ R^{K,AA} &= R^{K,AA}_{SP} + R^{K,AA}_{\Delta F} + R^{K,AA}_{\Delta B} + R^{K,AA}_{\Delta F-SP} + R^{K,AA}_{\Delta B-SP} + R^{K,AA}_{\Delta F-\Delta B} \end{aligned}$$

In the case of the T' response, only the vector-axial product contributes,

$$R^{T'} = R^{T',VA} = R^{T',VA}_{SP} + R^{T',VA}_{\Delta F} + R^{T',VA}_{\Delta B} + R^{T',VA}_{\Delta F-SP} + R^{T',VA}_{\Delta B-SP} + R^{T',VA}_{\Delta F-\Delta B}$$

54 Subresponses = 24 vector-vector, $R^{K,VV}_{I,J}$ + 24 axial-axial, $R^{K,AA}_{I,J}$ + 6 vector-axial, $R^{T',VA}_{I,J}$, with $I, J = SP, \Delta F, \Delta B$.

Semiempirical formulas of MEC subresponses

For each subresponse we factorize

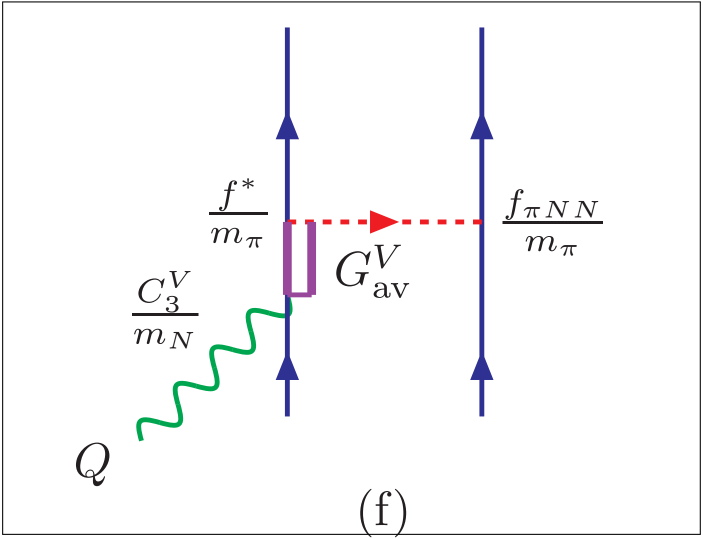
1. the phase-space $F(q, \omega)$
2. the coupling constants
3. the electroweak form factors,
4. the averaged Δ propagator $G_{\rm av}(q, \omega)$ in the ΔF current

$$
\begin{aligned}
R_i(q,\omega) \quad = \quad & [\text{phase-space}] \\
& \times[\text{coupling constants}] \\
& \times[\text{form factors}] \\
& \times[\text{averaged } \Delta \text{ propagators}] \\
& \times \tilde{C}_i(q)
\end{aligned}
$$

The coefficients $\tilde{C}_i(q)$ are fitted to the exact subresponses for each q.
The coefficients represent nuclear averages of spin-isospin contributions for each subresponse in a 2p2h excitation.
For better results Some of the subresponses require two coefficients of some additional ω dependence.

Example: Δ-forward - vector current

"Feymman" rules for the semiempirical formula



Response R_T^{VV}

$$R_{\Delta F}^{T,VV} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2}\right)^2 \left(\frac{C_3^V}{m_N}\right)^2 \left[\tilde{C}_{1,V1}(Re(G_{\rm av}^V))^2 + \tilde{C}_{1,V2}(Im(G_{\rm av}^V))^2\right] (m_N^4)$$

$$R_{SP}^{T,VV} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f_{\pi NN}^2}{m_\pi^2}\right)^2 \left(F_1^V\right)^2 (\tilde{C}_{2,V} \cdot m_N^{-2}) \left[1 - \frac{\omega - 0.7q}{m_N}\right]^2$$

$$R_{\Delta B}^{T,VV} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2}\right)^2 \left(\frac{C_3^V}{m_N}\right)^2 (\tilde{C}_{3,V})$$

$$R_{\Delta F-SP}^{T,VV} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}^3}{m_\pi^4}\right) \left(\frac{C_3^V}{m_N}\right) \left(F_1^V\right) \left[\tilde{C}_{4,V1}(Re(G_{\rm av}^V)) + \tilde{C}_{4,V2}(Im(G_{\rm av}^V))\right] m_N$$

$$R_{\Delta F-\Delta B}^{T,VV} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2}\right)^2 \left(\frac{C_3^V}{m_N}\right)^2 \left[\tilde{C}_{5,V1}(Re(G_{\rm av}^V)) + \tilde{C}_{5,V2}(Im(G_{\rm av}^V))\right] (m_N^2)$$

$$R_{\Delta B-SP}^{T,VV} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}^3}{m_\pi^4}\right) \left(\frac{C_3^V}{m_N}\right) \left(F_1^V\right) (\tilde{C}_{6,V} \cdot m_N^{-1})$$

Response R_T^{AA}

$$R_{\Delta F}^{T,AA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2} \right)^2 \left(C_A^5 \right)^2 \left[\tilde{C}_{1,A1} (Re(G_{\rm av}^A))^2 + \tilde{C}_{1,A2} (Im(G_{\rm av}^A))^2 \right] (m_N^2)$$

$$R_{SP}^{T,AA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f_{\pi NN}^2}{m_\pi^2} \right)^2 \left(\frac{1}{g_A} \right)^2 (\tilde{C}_{2,A} \cdot m_N^{-2})$$

$$R_{\Delta B}^{T,AA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2} \right)^2 \left(C_5^A \right)^2 (\tilde{C}_{3,A} \cdot m_N^{-2})$$

$$R_{\Delta F-SP}^{T,AA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}^3}{m_\pi^4} \right) \left(C_5^A \right) \left(\frac{1}{g_A} \right) \left[\tilde{C}_{4,A1} (Re(G_{\rm av}^A)) + \tilde{C}_{4,A2} (Im(G_{\rm av}^A)) \right]$$

$$R_{\Delta F-\Delta B}^{T,AA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2} \right)^2 \left(C_5^A \right)^2 \left[\tilde{C}_{5,A1} (Re(G_{\rm av}^A)) + \tilde{C}_{5,A2} (Im(G_{\rm av}^A)) \right]$$

$$R_{\Delta B-SP}^{T,AA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}^3}{m_\pi^4} \right) \left(C_5^A \right) \left(\frac{1}{g_A} \right) (\tilde{C}_{6,A} \cdot m_N^{-2})$$

Response $R_{T'}^{VA}$

$$R_{\Delta F}^{T',VA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2}\right)^2 \left(\frac{C_3^V}{m_N}\right) C_5^A \left[\tilde{C}_{1,VA1}(Re(G_{\text{av}}^{VA}))^2 + \tilde{C}_{1,VA2}(Im(G_{\text{av}}^{VA}))^2\right] m_N^3$$

$$R_{SP}^{T',VA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f_{\pi NN}^2}{m_\pi^2}\right)^2 \left(F_1^V\right) \left(\frac{1}{g_A}\right) (\tilde{C}_{2,VA} \cdot m_N^{-2}) \left[1 - \frac{\omega - 0.7q}{m_N}\right]^2$$

$$R_{\Delta B}^{T',VA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2}\right)^2 \left(\frac{C_3^V}{m_N}\right) \left(C_5^A\right) (\tilde{C}_{3,VA} \cdot m_N^{-1})$$

$$R_{\Delta F-SP}^{T',VA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}^3}{m_\pi^4}\right) \left|G_{\text{av}}^{VA}\right| \left[\frac{C_3^V}{m_N} \left(\frac{1}{g_A}\right) \tilde{C}_{4,VA1} \cdot m_N + C_5^A \left(F_1^V\right) \tilde{C}_{4,VA2}\right]$$

$$R_{\Delta F-\Delta B}^{T',VA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2}\right)^2 \frac{C_3^V}{m_N} C_5^A \left[\tilde{C}_{5,VA1}(Re(G_{\text{av}}^{VA})) + \tilde{C}_{5,VA2}(Im(G_{\text{av}}^{VA}))\right] 2m_N$$

$$R_{\Delta B-SP}^{T',VA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}^3}{m_\pi^4}\right) \left[\frac{C_3^V}{m_N} \left(\frac{1}{g_A}\right) \tilde{C}_{6,VA} \cdot m_N^{-1} + C_5^A \left(F_1^V\right) \tilde{C}_{6,AV} \cdot m_N^{-2}\right]$$

Response R_{CC}^{VV}

$$R_{\Delta F}^{CC,VV} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2}\right)^2 \left(\frac{C_3^V}{m_N}\right)^2 \left[\tilde{C}_{1,V1}(Re(G_{\mathrm{av}}^V))^2 + \tilde{C}_{1,V2}(Im(G_{\mathrm{av}}^V))^2\right] (m_N^4)$$

$$R_{SP}^{CC,VV} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f_{\pi NN}^2}{m_\pi^2}\right)^2 \left(F_1^V\right)^2 (\tilde{C}_{2,V} \cdot m_N^{-2})$$

$$R_{\Delta B}^{CC,VV} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2}\right)^2 \left(\frac{C_3^V}{m_N}\right)^2 (\tilde{C}_{3,V})$$

$$R_{\Delta F-SP}^{CC,VV} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}^3}{m_\pi^4}\right) \left(\frac{C_3^V}{m_N}\right) F_1^V \left[\tilde{C}_{4,V1}(Re(G_{\mathrm{av}}^V)) + \tilde{C}_{4,V2}(Im(G_{\mathrm{av}}^V))\right] m_N$$

$$R_{\Delta F-\Delta B}^{CC,VV} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2}\right)^2 \left(\frac{C_3^V}{m_N}\right)^2 \left[\tilde{C}_{5,V1}(Re(G_{\mathrm{av}}^V)) + \tilde{C}_{5,V2}(Im(G_{\mathrm{av}}^V))\right] (m_N^2)$$

$$R_{\Delta B-SP}^{CC,VV} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}^3}{m_\pi^4}\right) \left(\frac{C_3^V}{m_N}\right) \left(F_1^V\right) (\tilde{C}_{6,V} \cdot m_N^{-1})$$

Response R_{CC}^{AA}

$$R_{\Delta F}^{CC,AA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2}\right)^2 \left(C_A^5\right)^2 \left[\tilde{C}_{1,A1}(Re(G_{\rm av}^A))^2 + \tilde{C}_{1,A2}(Im(G_{\rm av}^A))^2\right] (m_N^2)$$

$$R_{SP}^{CC,AA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f_{\pi NN}^2}{m_\pi^2}\right)^2 \left(\frac{1}{g_A}\right)^2 \left[\tilde{C}_{2,A1} + \tilde{C}_{2,A2}\left(\frac{\omega \cdot m_N}{Q^2 - m_\pi^2}\right)^2\right] (m_N^{-2})$$

$$R_{\Delta B}^{CC,AA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2}\right)^2 \left(C_5^A\right)^2 (\tilde{C}_{3,A} \cdot m_N^{-2})$$

$$R_{\Delta F-SP}^{CC,AA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}^3}{m_\pi^4}\right) \left(C_5^A\right) \left(\frac{1}{g_A}\right) \left[\tilde{C}_{4,A1}(Re(G_{\rm av}^A)) + \tilde{C}_{4,A2}(Im(G_{\rm av}^A))\right]$$

$$R_{\Delta F-\Delta B}^{CC,AA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}}{m_\pi^2}\right)^2 \left(C_5^A\right)^2 \left[\tilde{C}_{5,A1}(Re(G_{\rm av}^A)) + \tilde{C}_{5,A2}(Im(G_{\rm av}^A))\right]$$

$$R_{\Delta B-SP}^{CC,AA} = \frac{V}{(2\pi)^9} F(q,\omega) \left(\frac{f^* f_{\pi NN}^3}{m_\pi^4}\right) \left(C_5^A\right) \left(\frac{1}{g_A}\right) (\tilde{C}_{6,A} \cdot m_N^{-2})$$

Responses R_{CL}^{VV} and R_{LL}^{VV}

These responses are computed assuming conservation of the vector current

$$R^{CL,VV} = -\frac{\omega}{q} R^{CC,VV} \tag{7}$$

$$R^{LL,VV} = \frac{\omega^2}{q^2} R^{CC,VV} \tag{8}$$

Responses R^{AA}_{CL} and R^{AA}_{LL}

The semi-empirical formulas for R^{AA}_{CL} and R^{AA}_{LL} are similar to the R^{AA}_{CC}. Only the numerical values of the coefficients $\tilde{C}_i$ change.

Electromagnetic responses R^L_{em} and R^T_{em}

For symmetric nuclear matter, the electromagnetic 2p2h responses are one half of the VV weak responses

$$R^L_{em} = \frac{1}{2} R^{CC,VV} \tag{9}$$

$$R^T_{em} = \frac{1}{2} R^{T,VV} \tag{10}$$

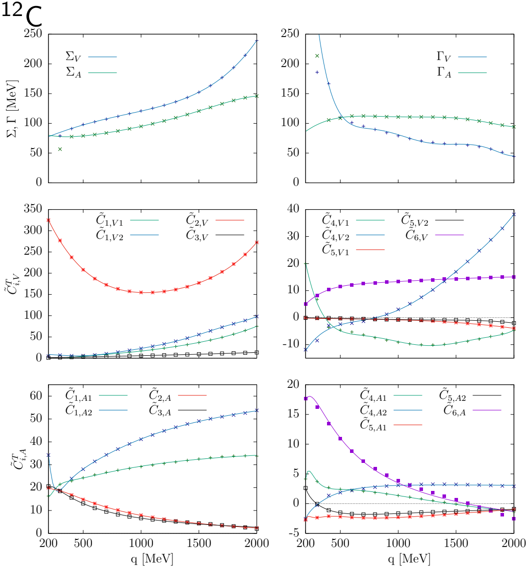
$$\tag{11}$$

The same semi-empirical formulas for the VV responses apply for the electromagnetic responses with a factor $1/2$.

Properties of semi-empirical formulas

- Explicit dependence of the responses on physical magnitudes —form factors, coupling constants, Fermi momentum, effective mass, ...
- All ω-dependente is analytical, from phase space, electroweak form factors and averaged Δ propagator.
- Exceptions are $R_{SP}^{T,VV}$, and $R_{SP}^{T',VA}$ sub-responses, which include an additional ω-dependent factor.
- The coefficients $\tilde{C}_i$ are dimensionless.
- The phase space $F(q,\omega)$ is computed analytically using the frozen nucleon approximation
- The responses are proportional to $V = (2\pi)^3 Z/(\frac{8}{3}\pi k_F^3)$ for symmetric nuclear matter $Z = N$.
- The averaged Δ propagator appears separated in real and imaginary parts. There are three versions: $G_{\rm av}^{V}$ for the VV responses, $G_{\rm av}^{A}$ for AA and $G_{\rm av}^{VA}$ for the T', responses. They differ in the values of the effective width, Γ, and shift, Σ, of the Δ propagator.
- 54 subresponses are described with 73 parameters.

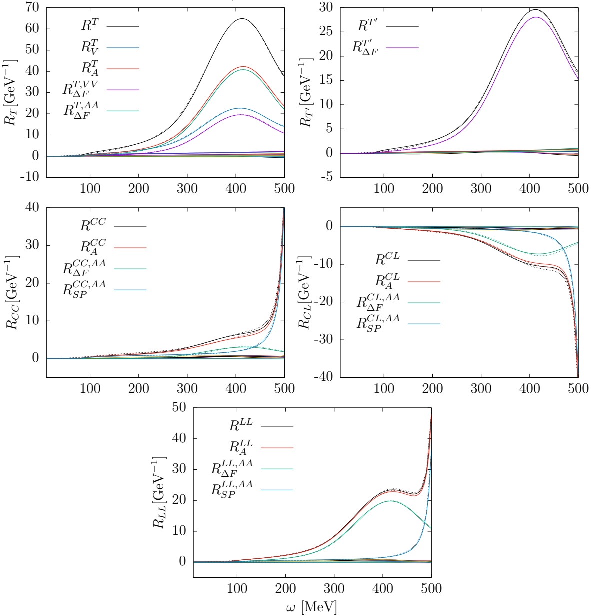
Parameters of the semiempirical formula for R^T



Tables and parametrizations in Phys.Rev.D 104 (2021) 11, 113006

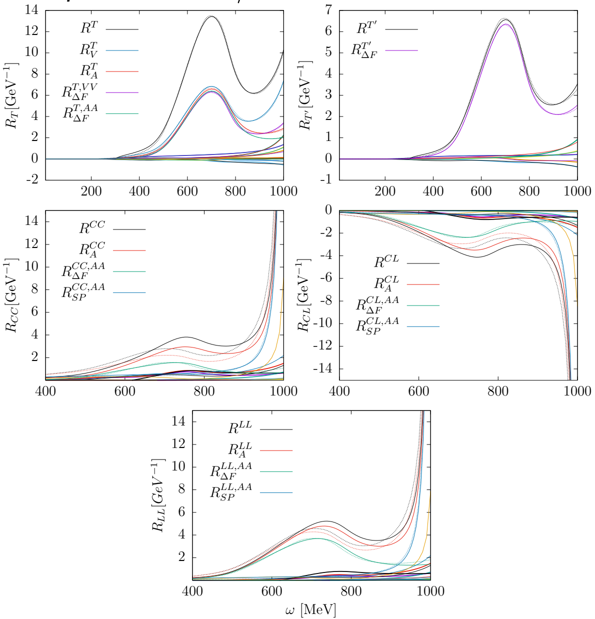
Comparison between semiempirical and exact subresponses

^{12}C, $q = 500$ MeV/c



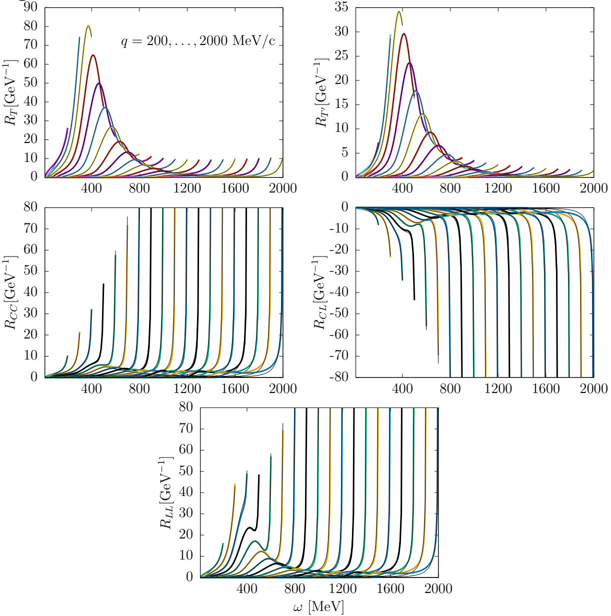
Comparison between semiempirical and exact subresponses

^{12}C $q = 1000$ MeV/c.

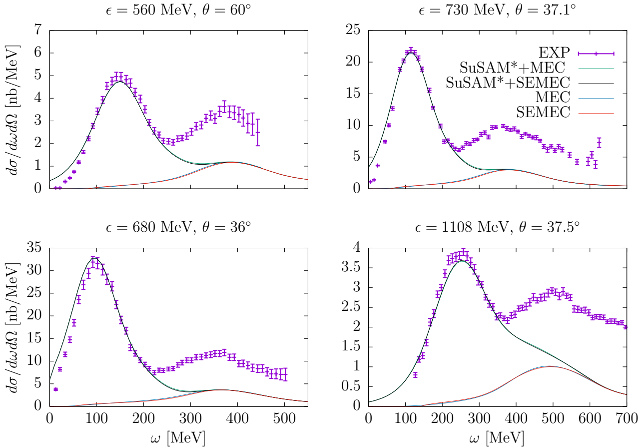


Semiempirical compared to exact 2p2h responses

^{12}C



Semiempirical 2p2h cross section for $^{12}C(e, e')$

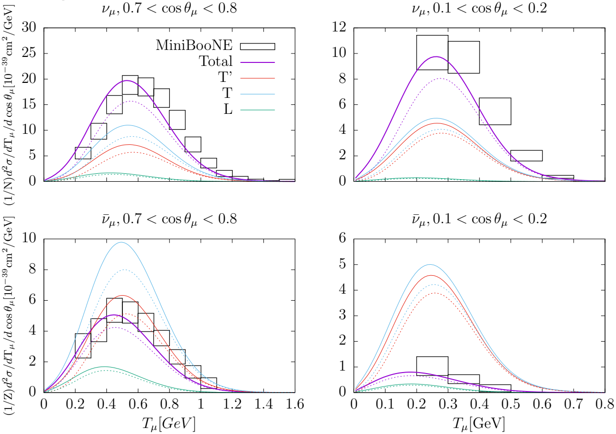


1p1h cross section computed with the SuSAM* model [Phys.Rev.C 104 (2021) 2, 025501]

2p2h cross section calculated in the RMF model of nuclear matter, and with the semi-empirical formula (SE-MEC)

MiniBooNE neutrino cross section with and without 2p2h

Semiempirical MEC



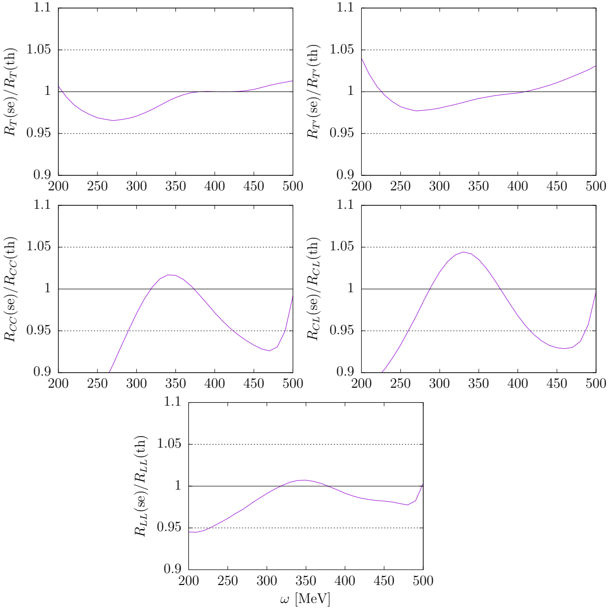
Quasielastic neutrino and antineutrino, differential cross section integrated over the neutrino flux

We show the separate T, T' and longitudinal responses (L = CC + CL + LL) with and without 2p2h

1p1h (dotted lines) computed in the SuSAM* model [Phys.Rev.C 104 (2021) 2, 025501]

Semiempirical divided by exact 2p2h responses

For $q = 500$ MeV/c as a function of ω



CONCLUSIONS

- 2p2h nuclear responses in the Relativistic Mean Field model of nuclear matter
- Relativistic model of MEC with seagull, pionic and Δ contributions
- Semiempirical formula for (54) MEC subresponses as products of 2p2h phase space, form factors, couplig constants and averaged Δ propagator
- Explicit dependence on k_F and relativistic effective mass M^*
- Parameters $\tilde{C}_i$, and the effective witdth, Γ and shift Σ of the averaged Δ propagator in the VV, AA and VA channels. Fitted to each subresponse as a function of q.
- The semiempirical formula allows introducing efective mass and vector energy of interacting Δ's in the medium
- Can be extended to separate PN, NN, and PP responses and other nuclei with $N \neq Z$
- The parameters can also be fitted to other models or to experimental data