Spin correlation in EEC-type observable

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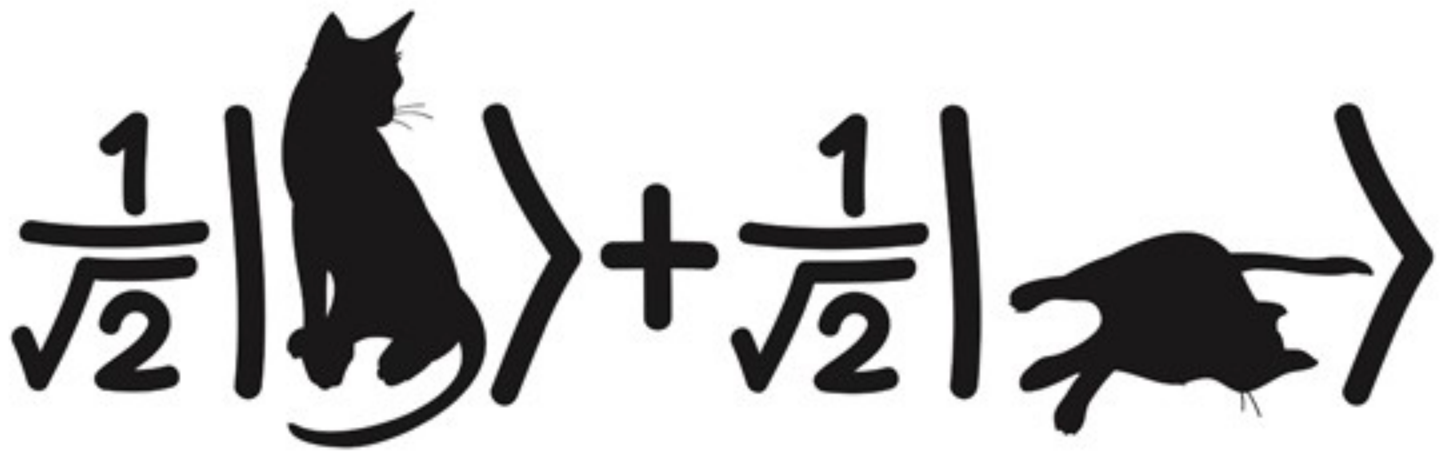
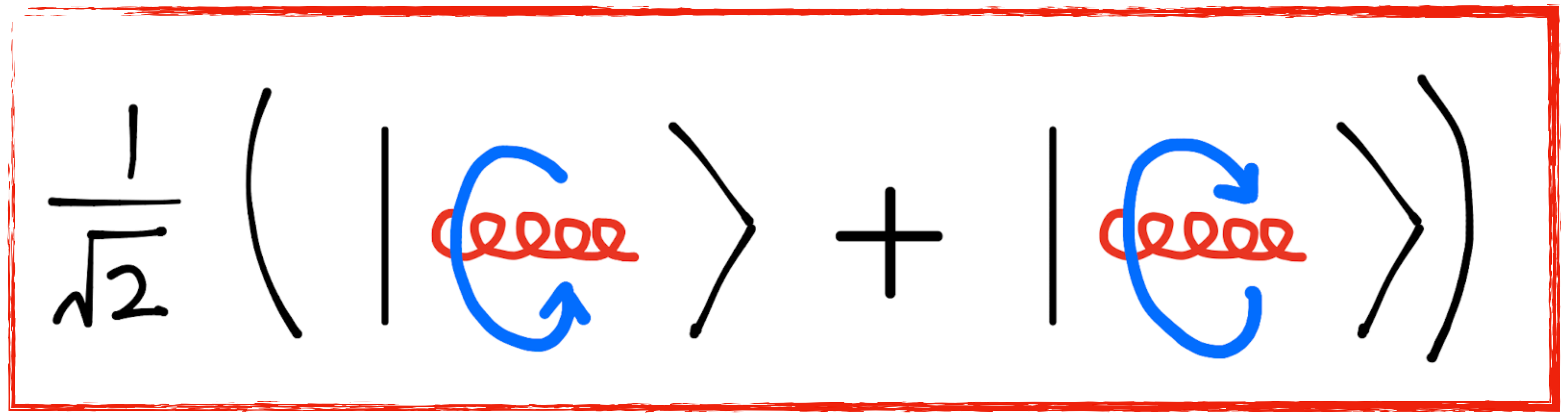
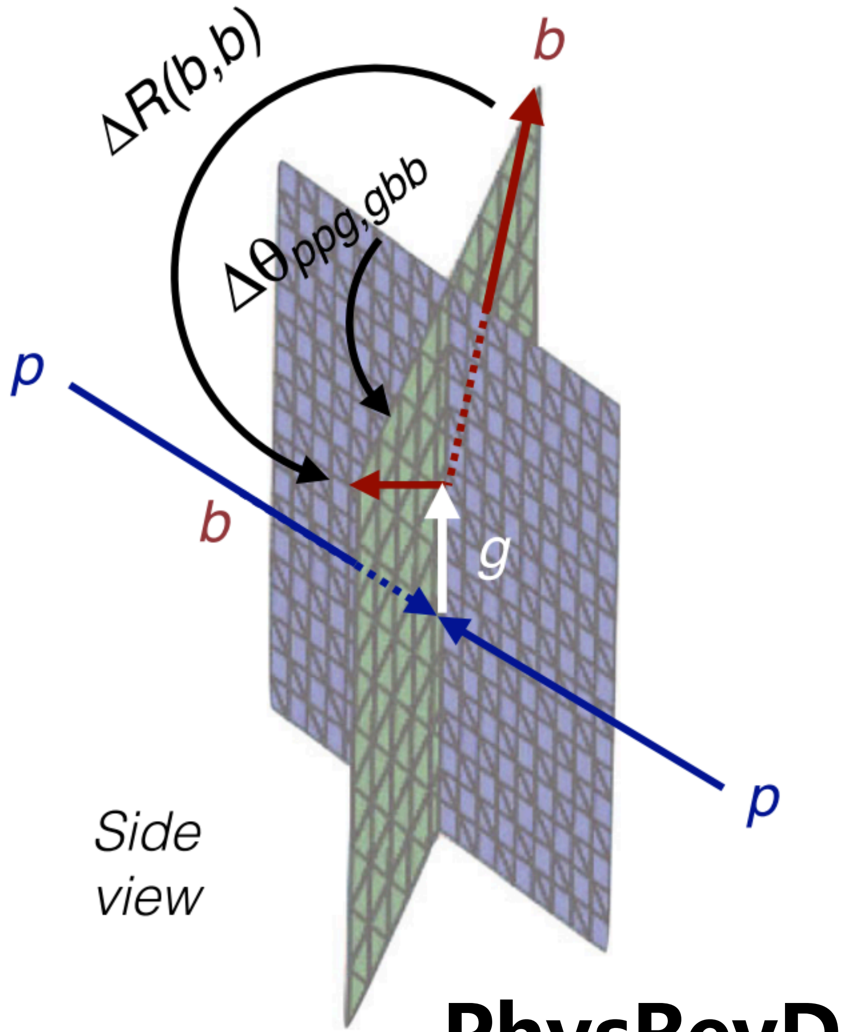
Quantum superposition with gluon

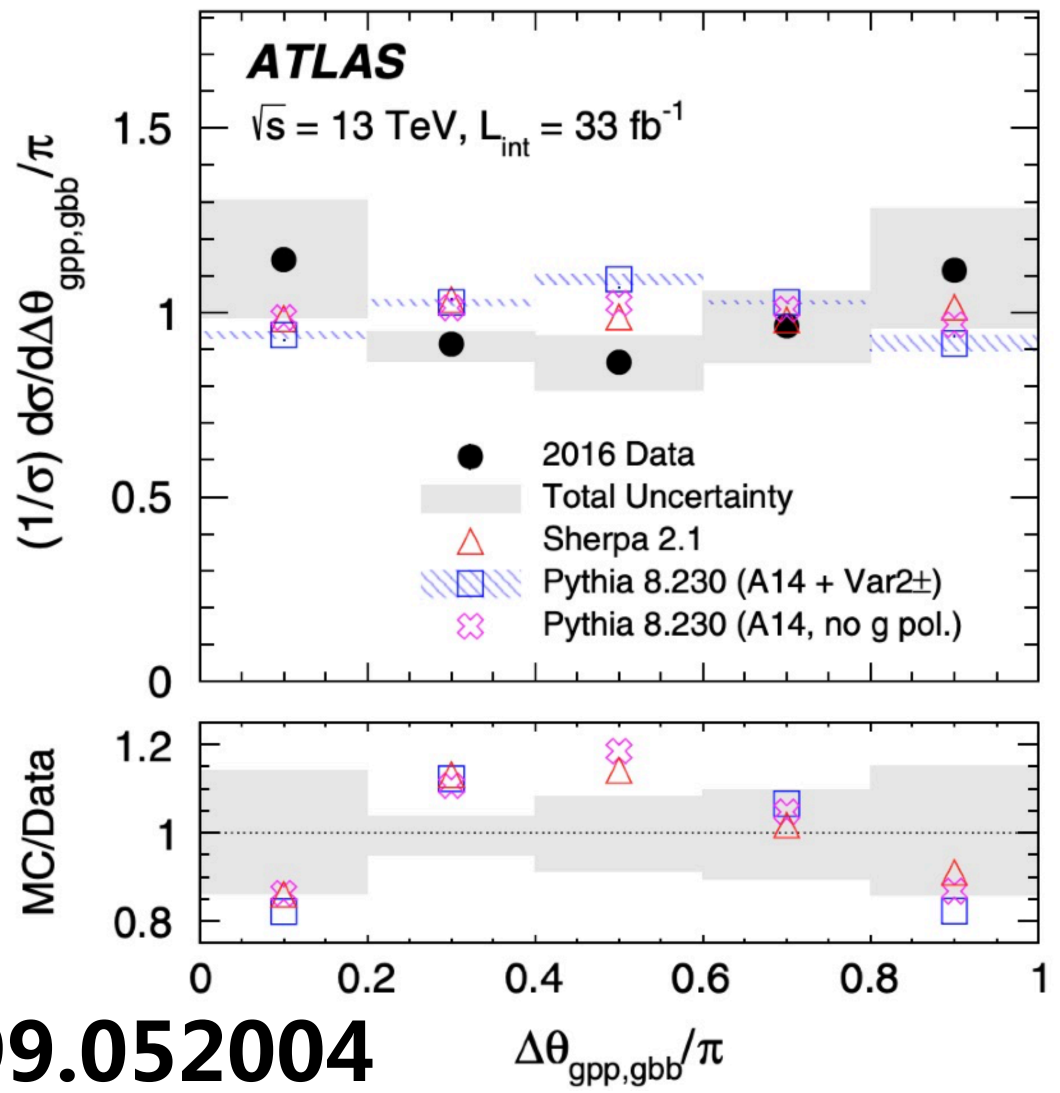
Quantum superposition is a fundamental principle, and one of the most publicized concept of quantum mechanics.
Very well tested using photon, atom, ion, molecule, or even top quark.



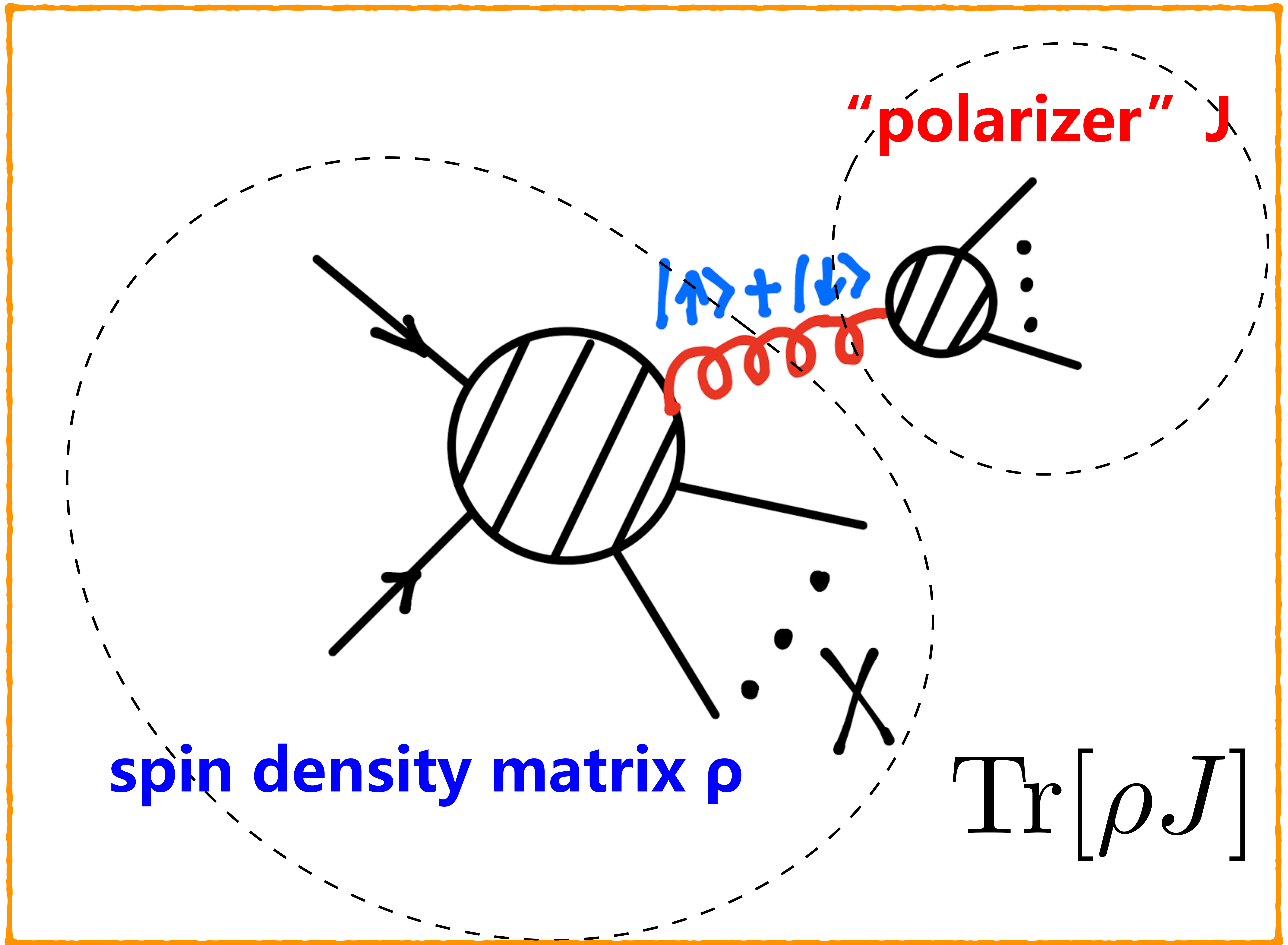
How do we observe the quantum nature of gluon?



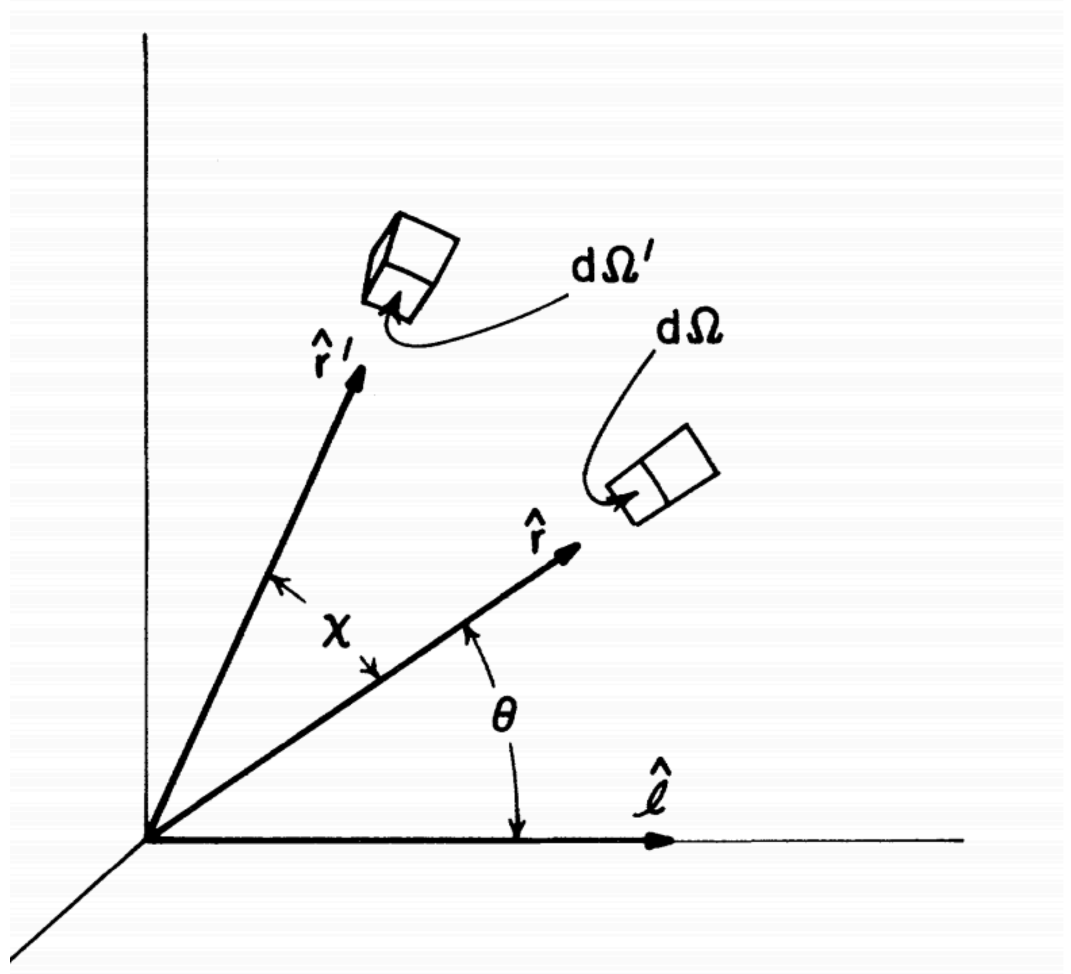




PhysRevD.99.052004



A quick intro to EEC and its multipoint generalization



Basham, Brown, Ellis, Love, 1978

$$\mathrm{EEC}(\chi) = \frac{1}{\sigma_{\mathrm{tot}}} \sum_{i,j} \int d\sigma_{e^+e^- \to i,j+X} \frac{E_i E_j}{E_{\mathrm{tot}}^2} \delta(\chi - \chi_{ij})$$

EEC: Two-particle angular correlation wighted by energies

Was a classic observable for e+e- colliders, drop out of experimental interest during the late stage of LEP

Resurgent of interests in recent years

- **Connection to OPE of lightray operator in conformal field theory**
- **Analytic calculation with CFT or amplitude methods**
- **Improved understanding of collinear limit in QCD**
- **Connection to transverse-momentum dependent physics**
- **Generalization to multipoint**
- **Generalization to pp, DIS**
- **Incorporation of tracks**

Ali, Belitsky, C.H. Chang, H. Chen, Chicherin, Dixon, Ebert, A.J. Gao, J. Gao, Henn, Hofman, Hohenegger, G. Li, H.T. Li, Y.B. Li, M.X. Luo, Kologlu, Korchemsky, Kravchuk, Makris, Maldacena, Mistlberger, Moult, Simmons-Duffin, Shtabovenko, Sokatchev, van Velzen, Vita, Vitev, Waalewijn, W. Wang, Z.P. Xing, K. Yan, T.Z. Yang, X.Y. Zhang, Zhiboedov, HXZ, Y.J. Zhu ...

Energy flow operator (lightray operator)

Tkachov; Korchemsky, Oderda, Sterman; Hofman, Maldacena; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov

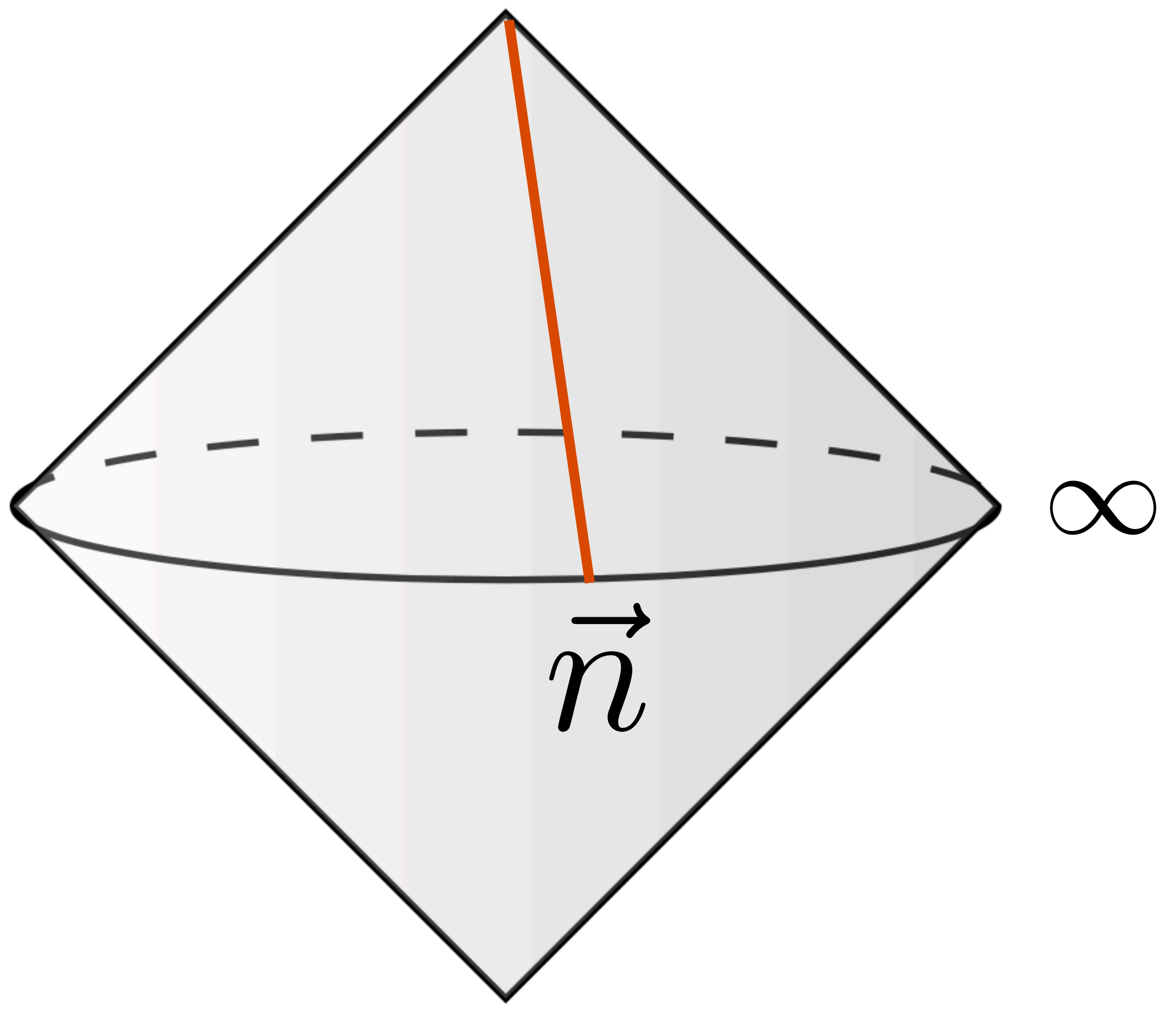
What we learn about QCD is mostly from the energy flow recorded by the detector. How do we characterize energy flow in QFT?

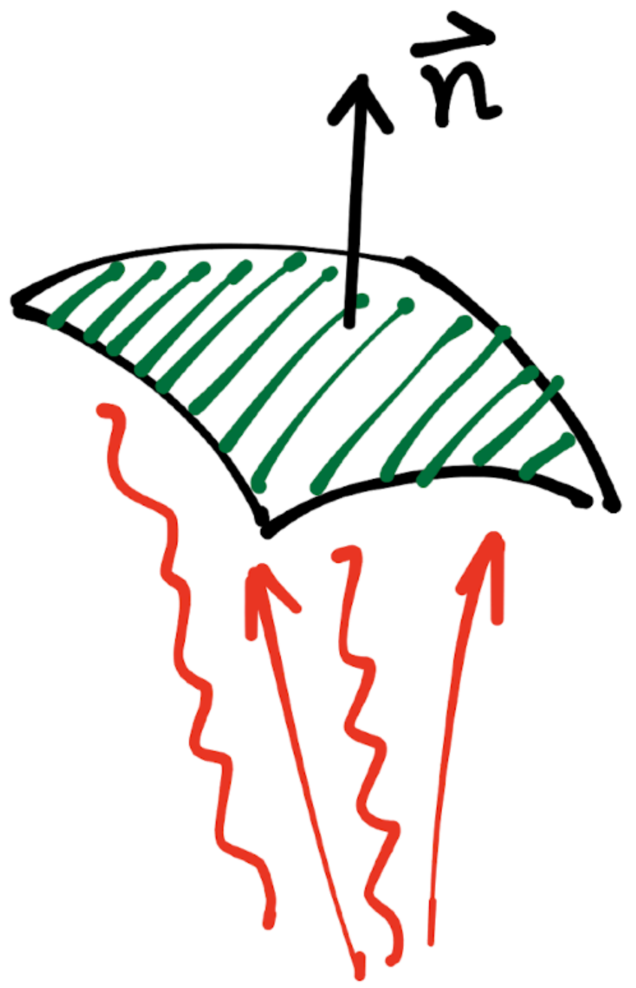
local twist operator

$$\mathbb{O}(\vec{n}) = \lim_{r\to\infty} r^{\Delta-J} \int_0^\infty dt\, O^{\mu_1\ldots\mu_J}(t, r\vec{n})\bar{n}_{\mu_1}\ldots\bar{n}_{\mu_J}$$

$(J-1, \Delta-1)$ (Δ, J)

(dim, spin)





$$\mathbb{O}_{(J-1,\Delta-1)}(\vec{n}) \sim E_{\vec{n}}^{J-1}$$

An insertion of a lightray operator of dimension J-1 equivalent to weighting the cross section by E^{J-1}

In this sense, correlation function of lightray operator is like taking the Mellin moment of calorimeter cell

Special case: energy flow operator

$$\mathcal{E}_{(1,3)}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt T^{\mu\nu}(t, r\vec{n})\bar{n}_\mu\bar{n}_\nu$$

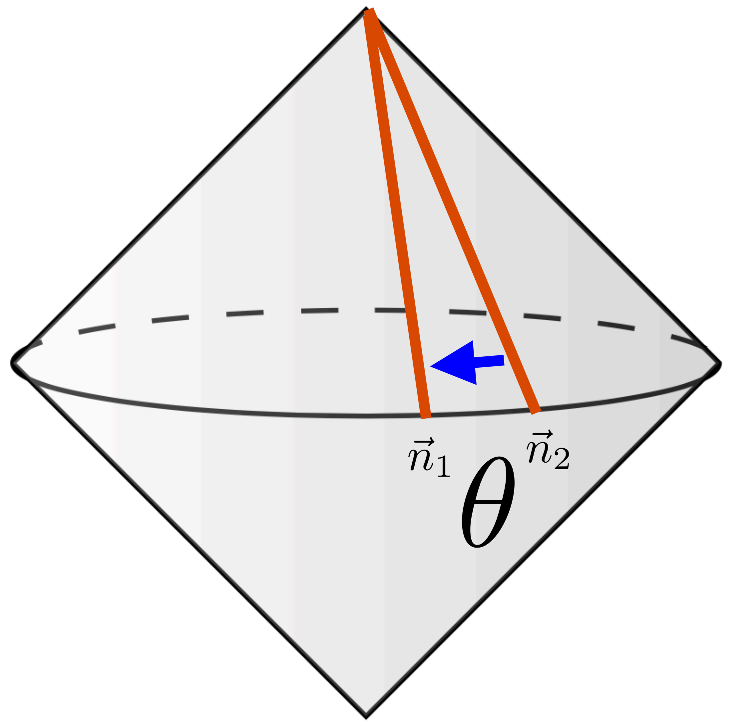
Conserved current

Collinear safety

Lightray OPE without transverse spin

Hofman, Maldacena; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov

Energy insertion within a jet corresponds to taking OPE limit of energy flow operator



$$\lim_{\vec{n}_2 \to \vec{n}_1} \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) = \sum_i \theta_{12}^{\gamma_i} \mathbb{O}_i(\vec{n}_1)$$

dimension	$1 + 1$	$=$	0	$+$	$(J-1)$	J=3
collinear spin	$3 + 3$	$=$	$-\gamma_i$	$+$	$(\Delta - 1)$	

$$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) \sim \sum_i c_i \theta^{\Delta-7} \mathbb{O}_i(\vec{n}_2)$$

$c_i \to \dfrac{\alpha_S(\mu)}{\alpha_S(\theta\mu)}$ **in QCD**

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J} \bar{\psi} \gamma^+ (iD^+)^{J-1} \psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J} F_a^{\mu+} (iD^+)^{J-2} F_a^{\mu+}$$

$$\hat{\gamma}(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f \gamma_{qg}(J) \\ \gamma_{gq}(J) & \gamma_{gg}(J) \end{pmatrix}$$

$$\gamma_{ij} = -\int_0^1 dx\, x^{J-1} P_{ij}(x)$$

equivalent language in terms of splitting functions

Small angle expansion controlled by anomalous dimension of local twist operator.

Jet substructure in the language of OPE.

Konishi, Ukawa, Veneziano; Dixon, Moult, HXZ

Lightray OPE with transverse spin

H. Chen, Moult, HXZ; C.H. Chang, Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov

transverse spin-0

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi$$

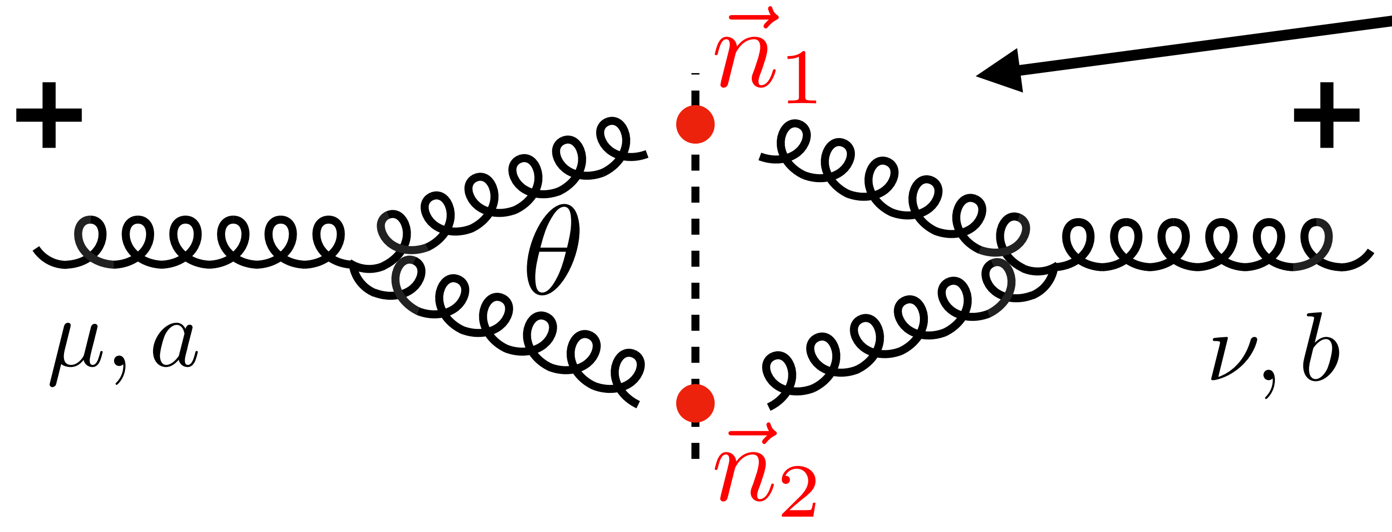
$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+}$$

transverse spin-2

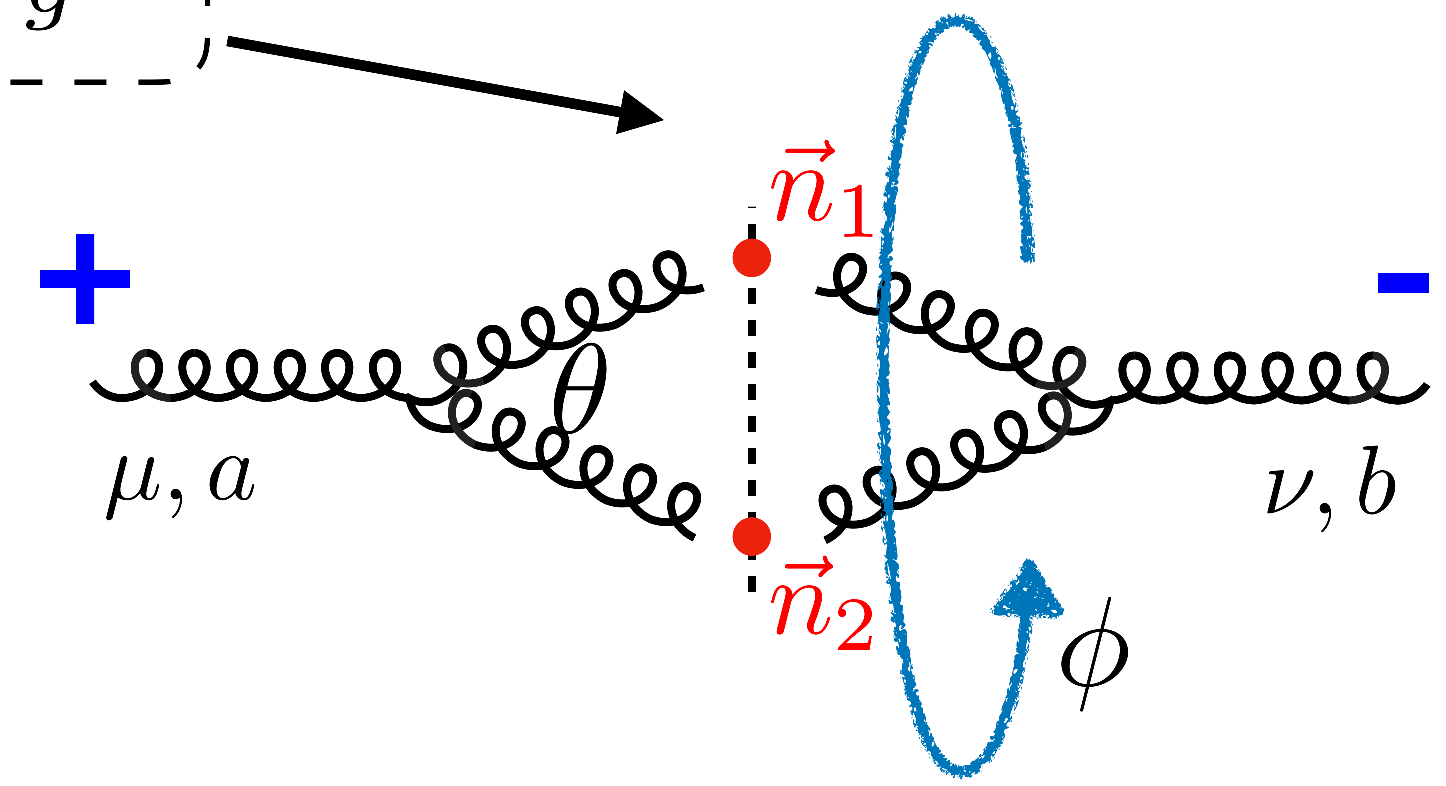
$$\mathcal{O}_{\tilde{g}\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

helicity $\pm$

$$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) \sim C_q\mathbb{O}_q^{J=3} + C_g\mathbb{O}_g^{J=3} + C_{\tilde{g}}\mathbb{O}_{\tilde{g}}^{J=3} + \cdots$$



$$\langle A_\perp^\mu|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)|A_\perp^\nu\rangle$$



$$C_g \sim \frac{1}{\theta^2}\int_0^1 dx\, x^{J-1}(P_{qg}(x) + P_{gg}(x))$$

$$C_{\tilde{g}} \sim \frac{\cos(2\phi)}{\theta^2}\int_0^1 dx\, x^{J-1}(P_{q\tilde{g}}(x) + P_{g\tilde{g}}(x))$$

Transverse modulation can play the role of polarizer for gluon!

$$P_{q\tilde{g}}(x) = -4T_R z(1-z)$$

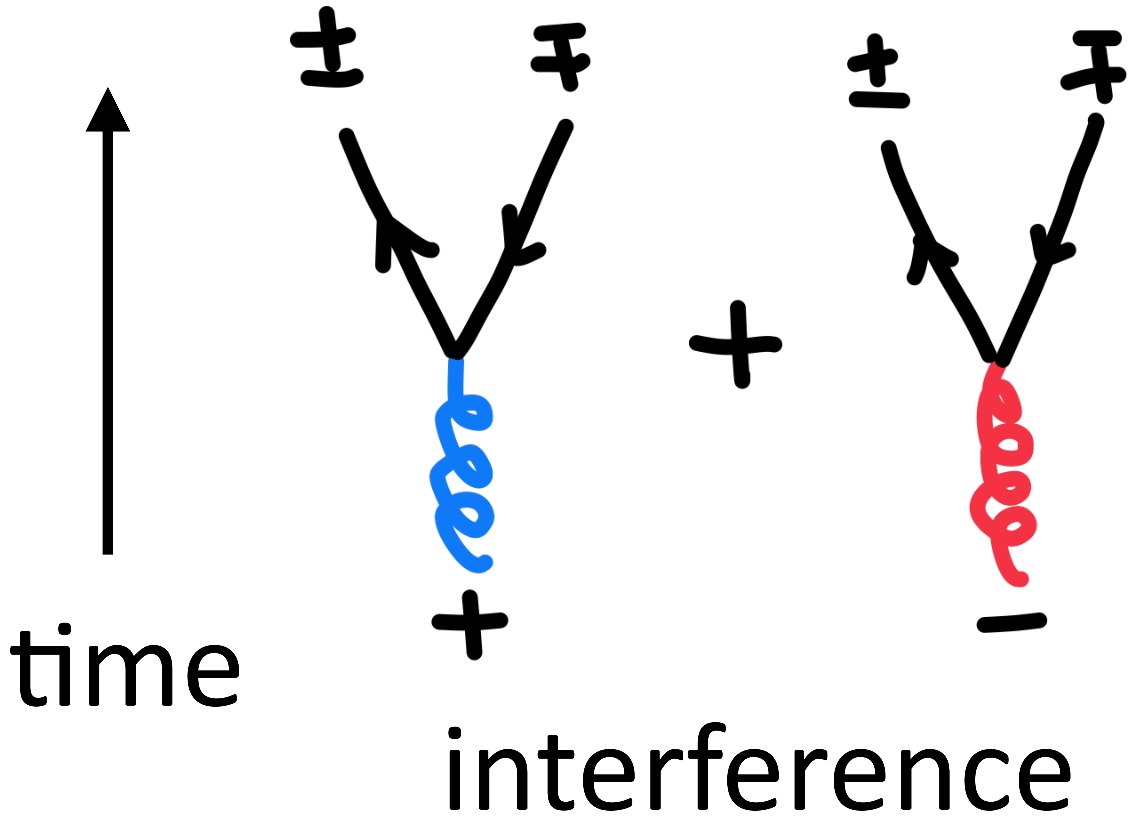
$$P_{g\tilde{g}}(x) = -4C_A z(1-z)$$

QCD evolution of transverse spin 2 part

H. Chen, Moult, HXZ

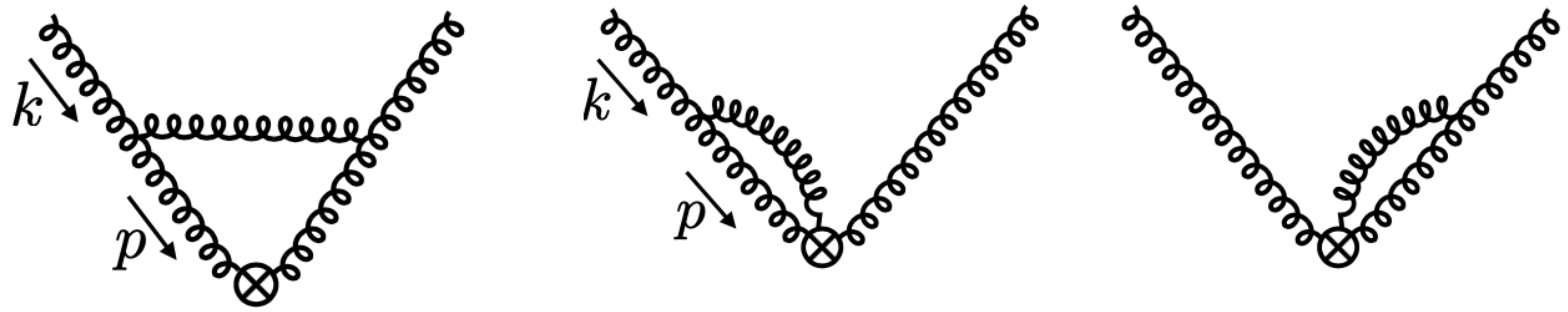
$$C_g \sim \frac{1}{\theta^2}\int_0^1 dx\, x^{J-1}(P_{qg}(x)+P_{gg}(x))$$

$$C_{\tilde{g}} \sim \frac{\cos(2\phi)}{\theta^2}\int_0^1 dx\, x^{J-1}(P_{q\tilde{g}}(x)+P_{g\tilde{g}}(x))$$



RG for $\begin{pmatrix} \mathbb{O}_q^J \\ \mathbb{O}_g^J \\ \mathbb{O}_{\tilde{g}}^J \end{pmatrix}$

$$\widehat{\gamma}(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 0 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & 0 \\ 0 & 0 & \gamma_{\tilde{g}\tilde{g}}(J)\mathbf{1} \end{pmatrix}$$



$$\langle k,+|O_{J,\lambda}|k,-\rangle \quad \text{or} \quad \langle k,-|O_{J,\lambda}|k,+\rangle\,,$$

$$O_{J,\lambda} = \epsilon_{\lambda,i}\epsilon_{\lambda,j}O_J^{ij} = \epsilon_{\lambda,i}\epsilon_{\lambda,j}F^{+i}(iD^+)^{J-2}F^{+j}$$

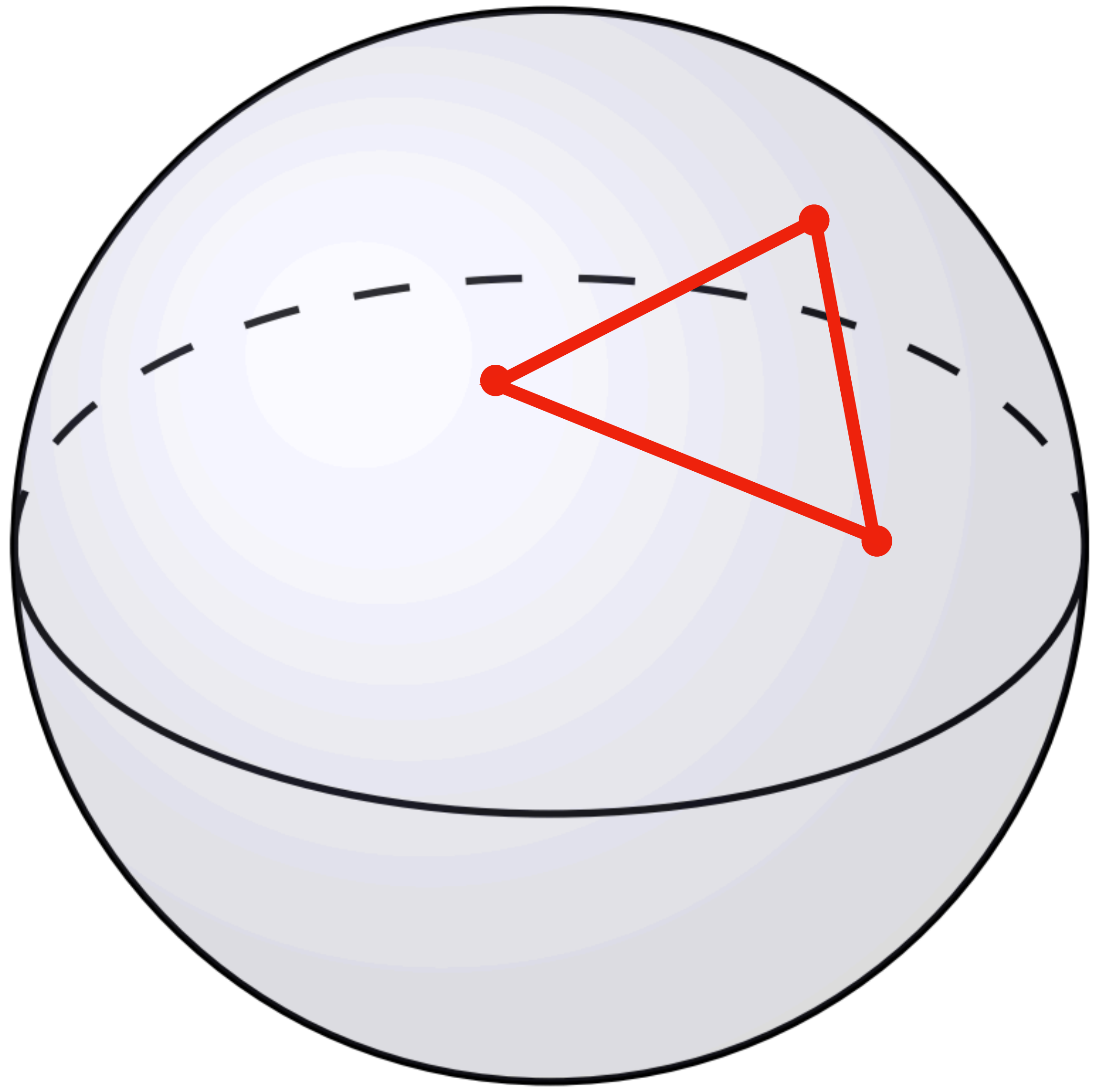
$$\gamma_{\tilde{g}\tilde{g}}(J) = \frac{g^2}{(4\pi)^2}\left(4C_A\sum_{j=1}^{J}\frac{1}{j} - \beta_0\right)$$

$$\boxed{P_{\tilde{g}\tilde{g}}(z) = 2C_A\left(\frac{1}{[1-z]_+} - 1\right) + \frac{\beta_0}{2}\delta(1-z)}$$

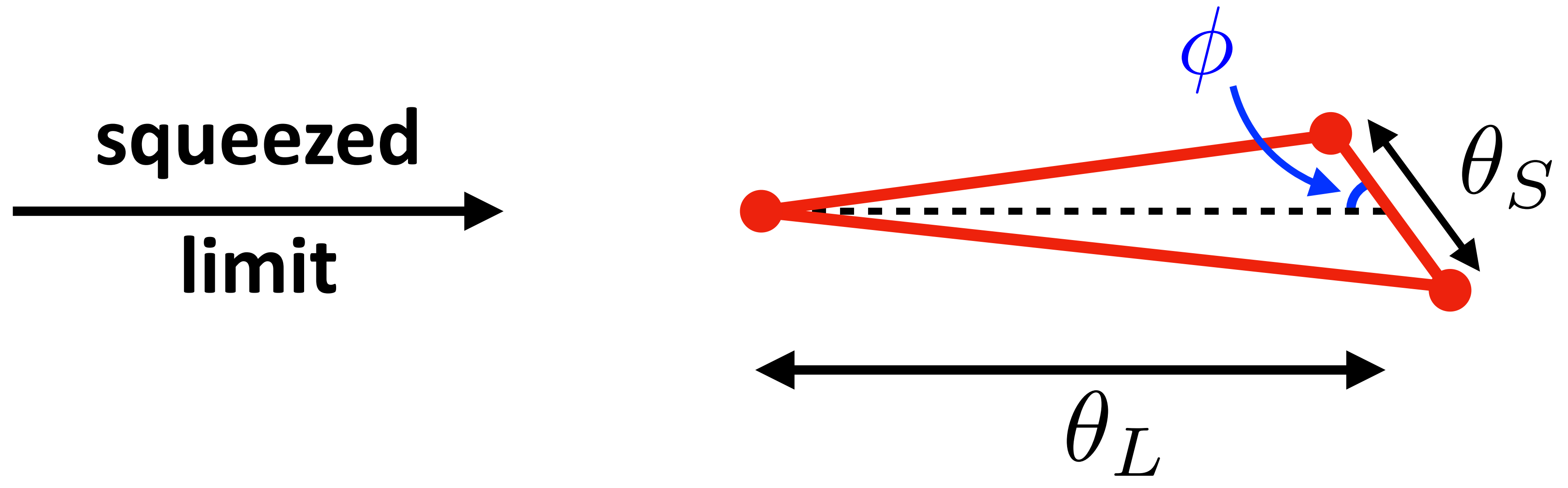
Would be interesting to how how this kernel is connected to shower evolution

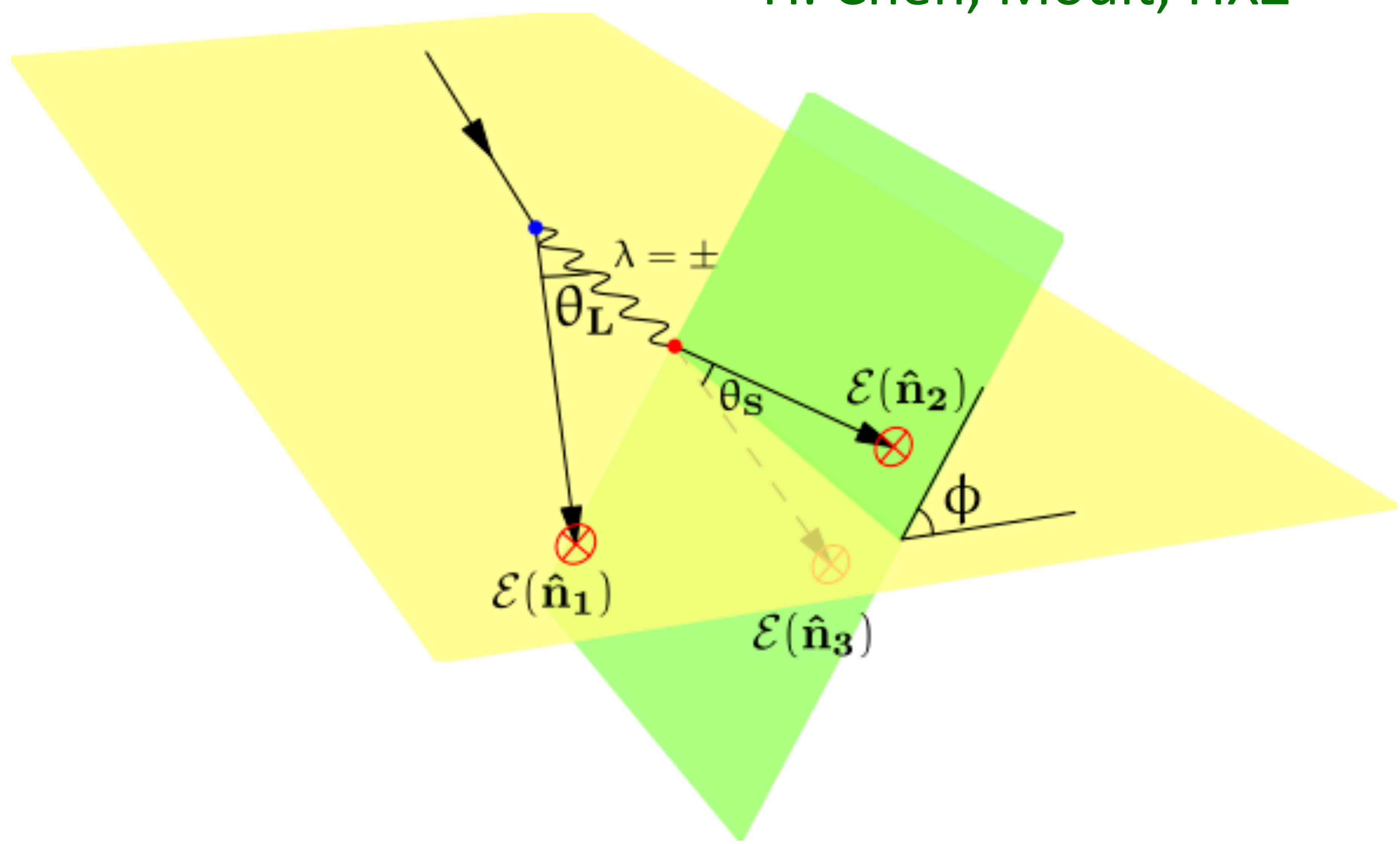
density matrix I: collinear EEEC in squeeze limit

H. Chen, Moult, HXZ

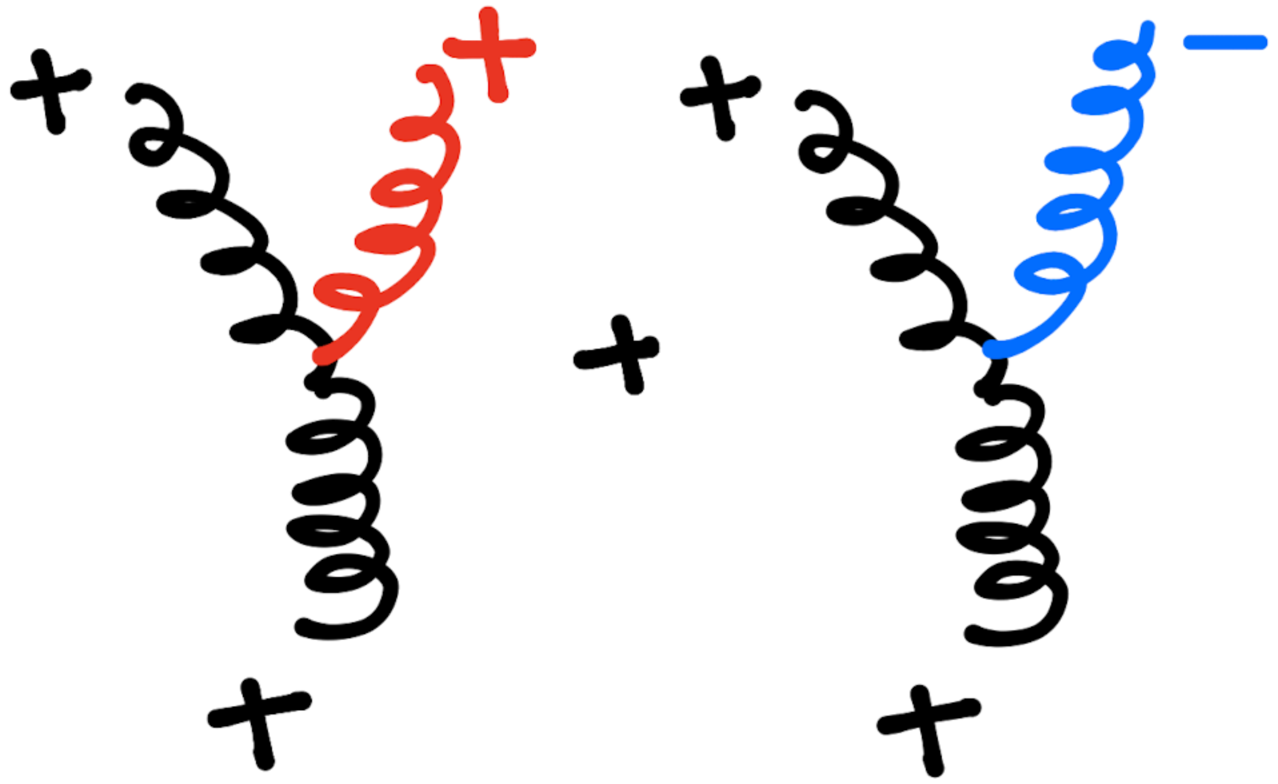


squeezed limit





$$\frac{d^3\Sigma_i}{d\theta_L^2\, d\theta_S^2\, d\phi} \simeq \frac{1}{\pi}\left(\frac{\alpha_s}{4\pi}\right)^2 \frac{\mathrm{Sq}_i^{(0)}(\phi)}{\theta_L^2\theta_S^2} + \cdots$$



$$\mathrm{Split}_-^{g\to g_1 g_2,\,\mathrm{tree}}(1^+,2^+) = \frac{1}{\sqrt{z(1-z)}\,\langle 1\,2\rangle},$$

$$\mathrm{Split}_-^{g\to g_1 g_2,\,\mathrm{tree}}(1^-,2^+) = \frac{-(1-z)^2}{\sqrt{z(1-z)}\,[1\,2]}.$$

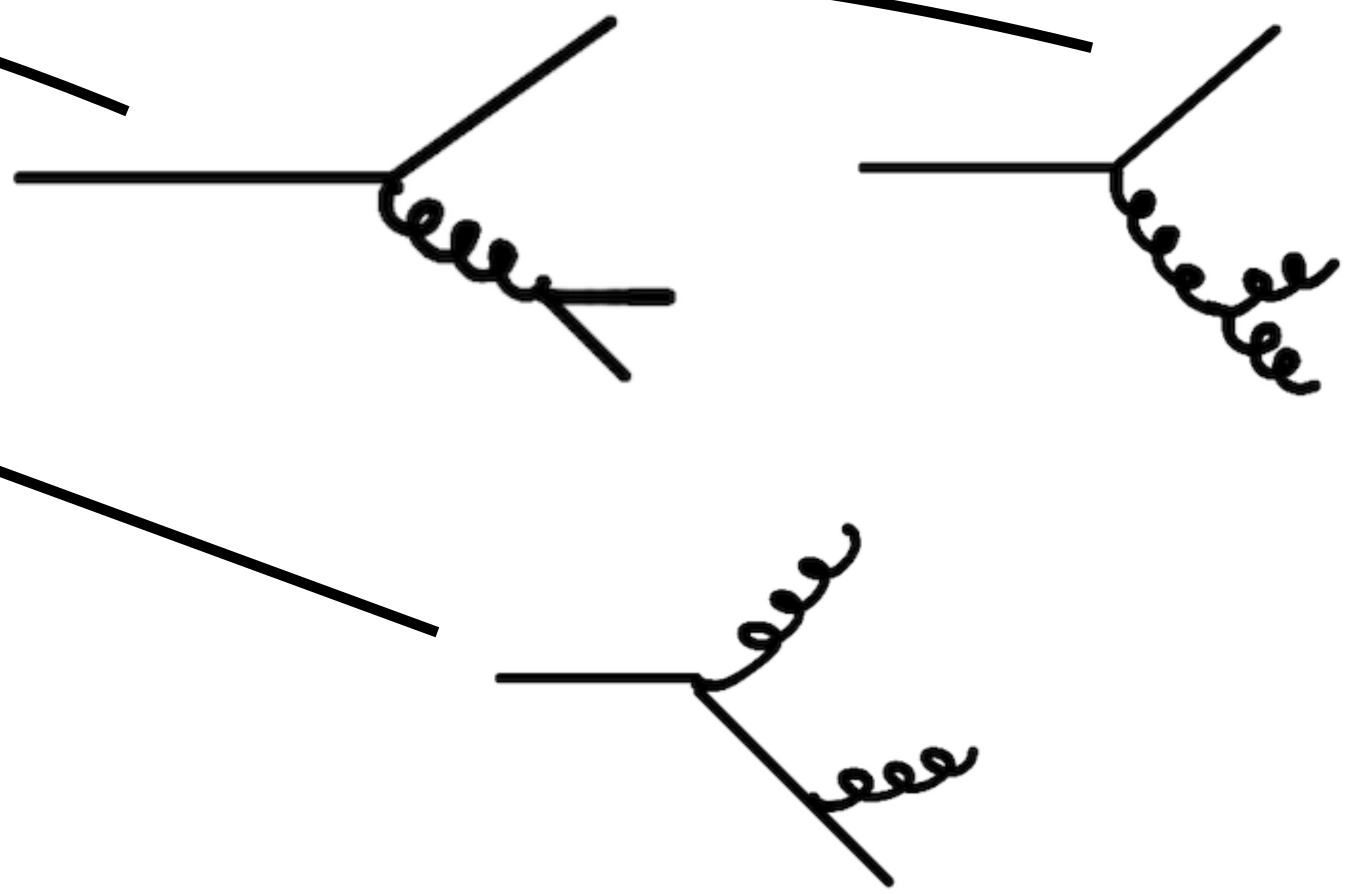
$$\rho = \begin{pmatrix} \mathrm{Sp}_{g^+\to g^+g^+}\mathrm{Sp}^*_{g^+\to g^+g^+} & \mathrm{Sp}_{g^+\to g^+g^+}\mathrm{Sp}^*_{g^+\to g^-g^+} \\ \mathrm{Sp}_{g^+\to g^-g^+}\mathrm{Sp}^*_{g^+\to g^+g^+} & \mathrm{Sp}_{g^+\to g^-g^+}\mathrm{Sp}^*_{g^+\to g^-g^+} \end{pmatrix}$$

density matrix I: collinear EEEC

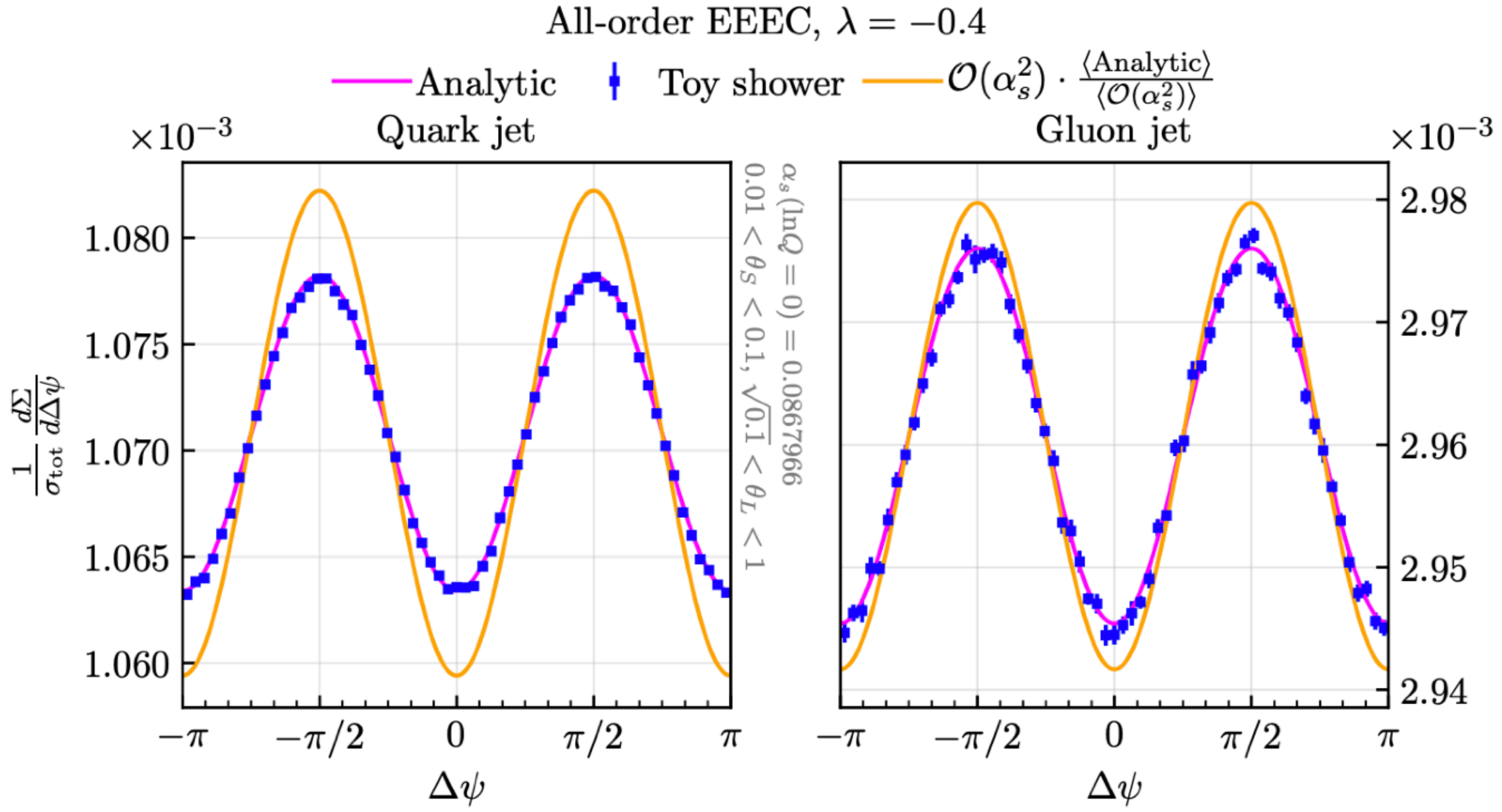
H. Chen, Moult, HXZ

$$\mathrm{Sq}_q^{(0)}(\phi) = C_F n_f T_F \left(\frac{39 - 20\cos(2\phi)}{225}\right) + C_F C_A \left(\frac{273 + 10\cos(2\phi)}{225}\right) + C_F^2 \frac{16}{5}$$

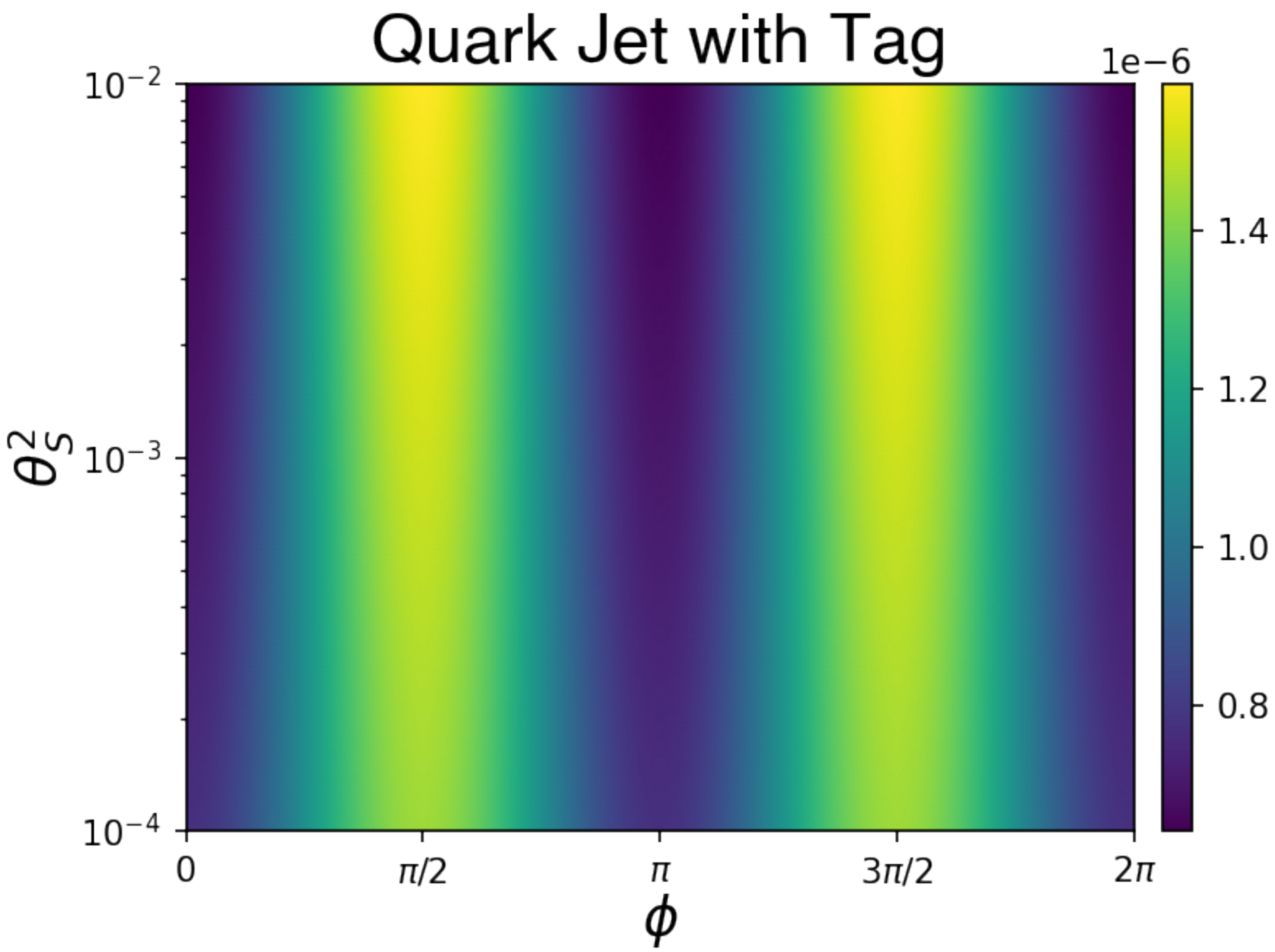
$$\mathrm{Sq}_g^{(0)}(\phi) = C_A n_f T_F \left(\frac{126 - 20\cos(2\phi)}{225}\right) + C_A^2 \left(\frac{882 + 10\cos(2\phi)}{225}\right) + C_F n_f T_F \frac{3}{5}$$



Large cancellation between boson and fermion

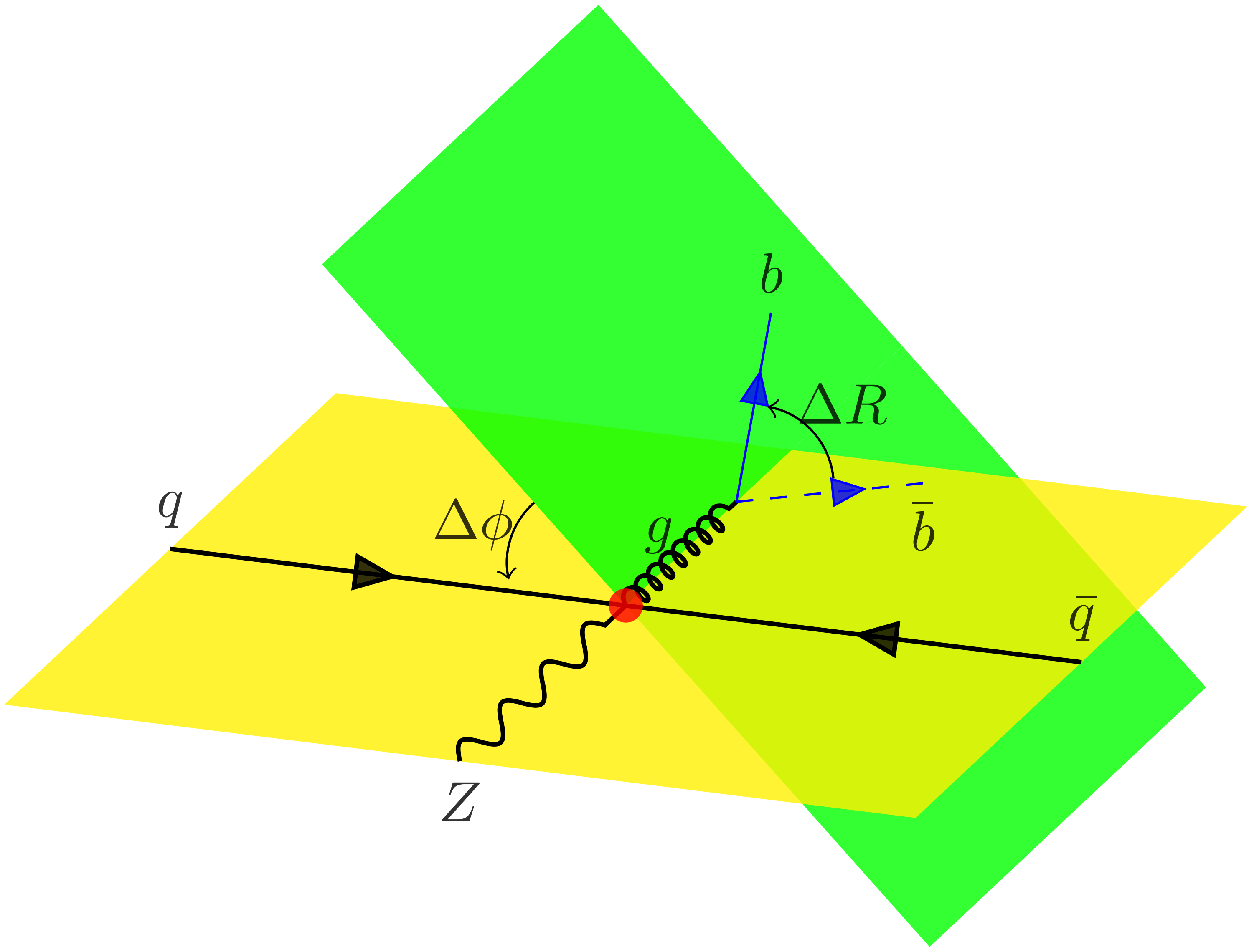


Karlberg, Salam, Scyboz, Verheyen
see also enhancement using Lund plane



density matrix II: Z + jet production

Instead of multi-prong jet substructure, one can also employ the hard scattering for generating the spin density matrix. Consider Z+jet: (the other process worth considering is 3jet production)

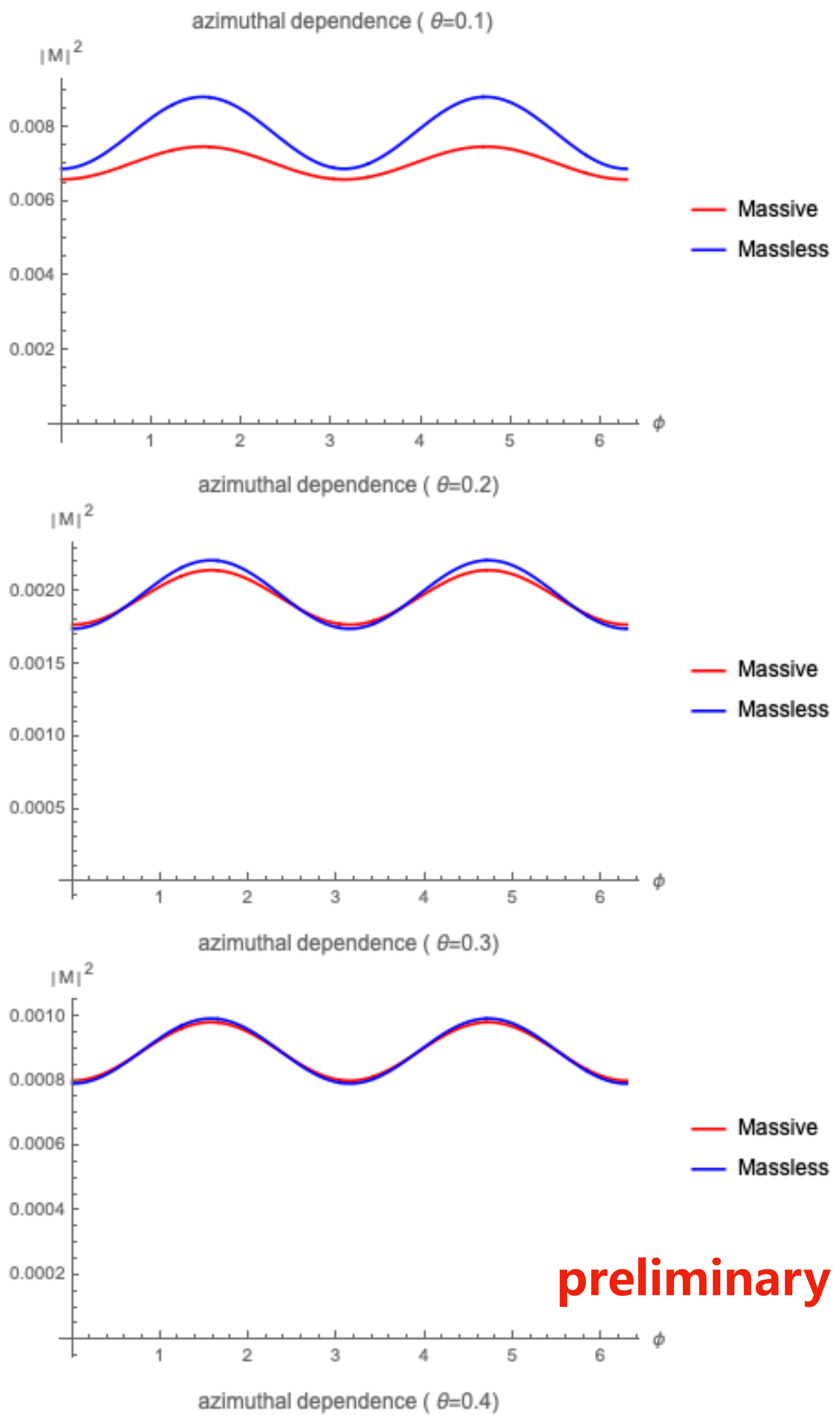


The matrix element for the off-diagonal term non vanishing

$$M_{1^+2^-\to Z+g^+} = \frac{2ieg_s}{\sin 2\theta_w}(1-2Q_u\sin^2\theta_w)\frac{\langle 2\,\mathbf{3}\rangle\langle 2\,\mathbf{3}\rangle}{\langle 1\,g\rangle\langle 2\,g\rangle}$$

$$M_{1^+2^-\to Z+g^-} = \frac{2ieg_s}{\sin 2\theta_w}(1-2Q_u\sin^2\theta_w)\frac{[1\,\mathbf{3}][1\,\mathbf{3}]}{[1\,g][2\,g]}$$

H. Chen, J. Gao, Moult, HXZ, in progress



Outlook

- Quantum supposition of gluon reflected by spin correlation effects.
- Lightray OPE suitable for analysis of transverse spin structure within jet.
- EEEC and Z+jet as two examples providing non-trivial spin density matrix.
- Direct observation challenging, requires enhancement. b-tagging? charge?
- Interesting theory structure beyond collinear approximation. Connection to conformal block.