# Validation of pyORBIT for IOTA Coasting Beam Simulation

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1

# Previous Simulation of IOTA Bunched Beam

- Tests without space charge

  - Symplecticity test using linear IOTA lattice or IOTA lattice with octupole insert are in close agreement with MADX
  - The size and shape of dynamic aperture with octupoles obtained by pyORBIT agree with results from MADX
  - Tune footprint plot agrees with results from MADX
  - The scale of observed single particle Hamiltonian invariant fluctuations is consistent with results obtained with MADX

- Tests with space charge

  - The bunch is initialized with transverse gaussian and longitudinal waterbag distribution
  - We use #MPs = $5*10^5$ , grid size 128x128x5, and #betatron kicks per wavelength = 63 on the basis of convergence tests.
  - A slow initialization procedure is performed such that the bunch gradually reaches full intensity over 40 turns
  - Zero-amplitude tune shift is close to analytically calculated results
  - Tune footprint agrees well with analytical results at intensity $10^{10}$ , but it does not agree well with analytical results at full intensity intensity $9*10^{10}$ , since tunes were calculated without averaging over test particles

- All these results are included in the report FERMILAB-TM-2753-AD available at [https://arxiv.org/abs/2106.03327](https://arxiv.org/abs/2106.03327)

# Outline

- Simulations with large apertures and no octupoles
  - Lattice info and coasting beam preparation
  - Periodic boundary condition
  - Emittance growth (simulation vs theory)
  - 0 amplitude tune shift obtained from test particles with small amplitude
  - Tune footprint

- Simulations with real aperture and no octupoles
  - Aperture info and convergence test
  - Emittance and particle distribution evolution during tracking

- Simulations with real aperture and octupoles
  - Dynamic aperture size
  - Emittance variation and change in particle distribution during tracking

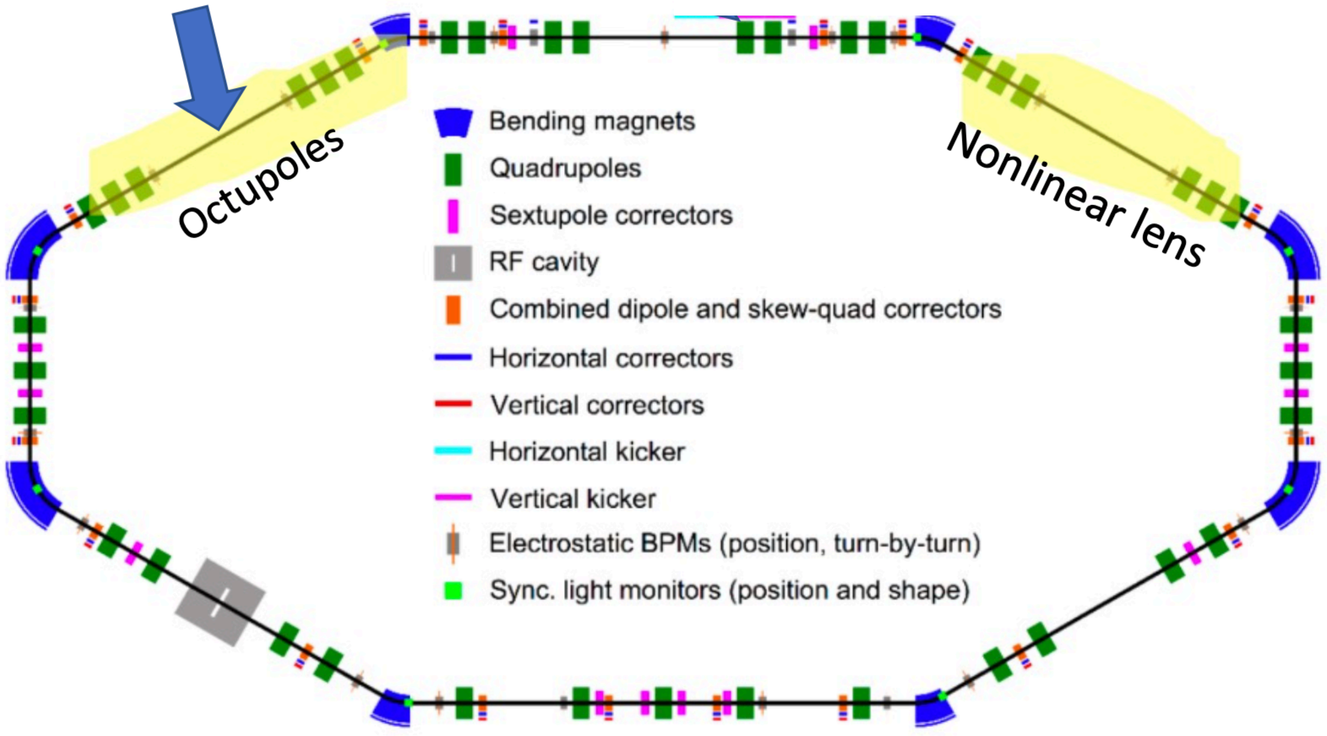
- Benchmarking pyORBIT with ImpactZ with coasting beam

# PART I.
# Simulations with Large Apertures and no Octupoles

# IOTA Proton Beam Parameters

- Simulation injection point: middle of octupole insert region, where $\beta_x = \beta_y$ and $D_x = D_y = \alpha_x = \alpha_y = 0$
- Nonlinearity in all elements has been removed, including octupoles, nonlinear lens, sextupoles and nonlinear transitions in dipoles and quadrupoles
- Physical apertures are set at 0.1m to prevent particle loss

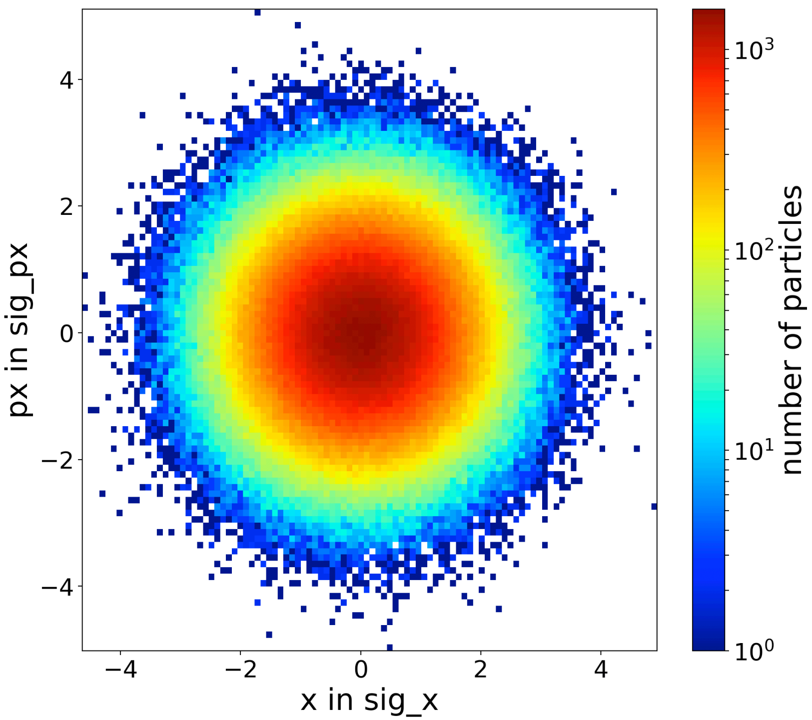
$$\beta_x = \beta_y$$
$$0 = \alpha_x = \alpha_y = D_x = D'_x$$

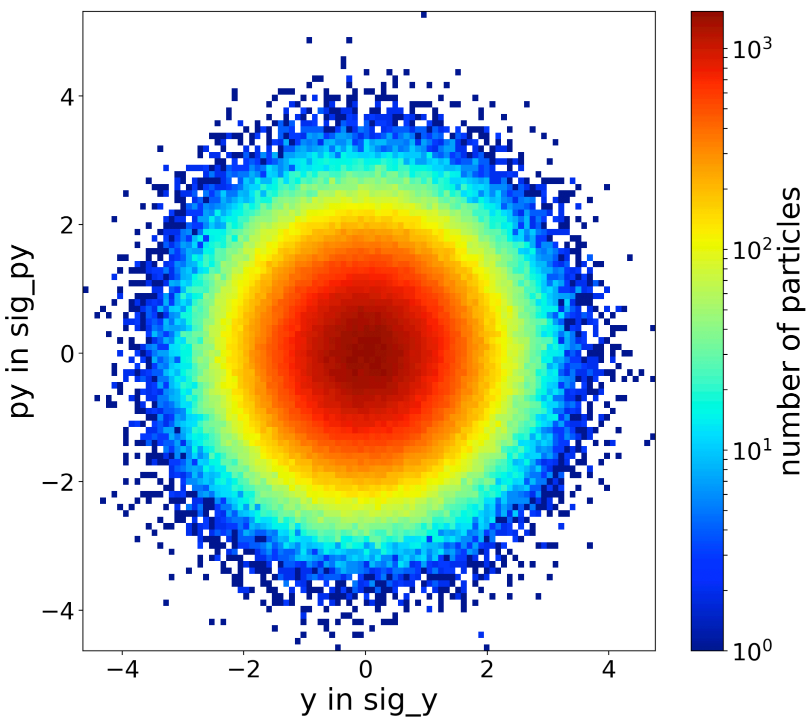


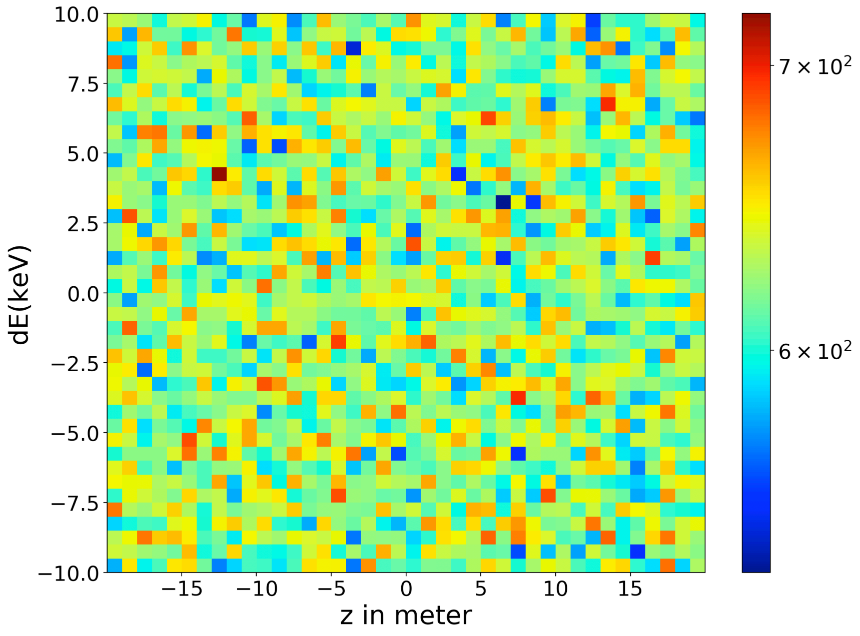

| IOTA proton parameters | |
|---|---|
| Circumference | 39.97 [m] |
| Kinetic Energy | 2.5 [MeV] |
| Maximum bunch intensity /current | $9\times10^{10}$ / 8 [mA] |
| Transverse normalized rms emittance | (0.3, 0.3) [mm-mrad] |
| Betatron tunes | (5.3, 5.3) |
| Natural chromaticities | (-8.2, -8.1) |
| Average transverse beam sizes (rms) | (2.22, 2.22) [mm] |
| Kinematic $\gamma$ / Transition $\gamma_T$ | 1.003 / 3.75 |
| Rf voltage | 400 [V] |
| Rf frequency / harmonic number | 2.2 [MHz] / 4 |
| Bucket wavelength | $\sim$ 10 [m] |
| Bucket half height in $\Delta p/p$ | $3.72\times10^{-3}$ |
| rms bunch length | 1.7 [m] |
| rms energy /momentum spread | $1.05\times10^{-5}$ / $1.99\times10^{-3}$ |
| Beam pipe radius | 25 [mm] |
| Bunch density | $6.9\times10^{14}$ [m$^{-3}$] |
| Plasma period $\tau_p$ | 0.18 [$\mu$-sec] / 0.1 [turns] |
| Average Debye length $\lambda_D$ | 559 [$\mu$m] |

# Coasting beam setup

- Transverse gaussian distribution with normalized emittance 0.3um
- Longitudinal coasting beam with dE uniformly in +-10keV and z uniformly in +-lattice length / 2
- RF Cavity has been removed
- All tracking uses pyORBIT 2D space charge model with
  - 1M macro particles
  - grid size 128x128x1
  - 63 SC kicks per betatron wavelength

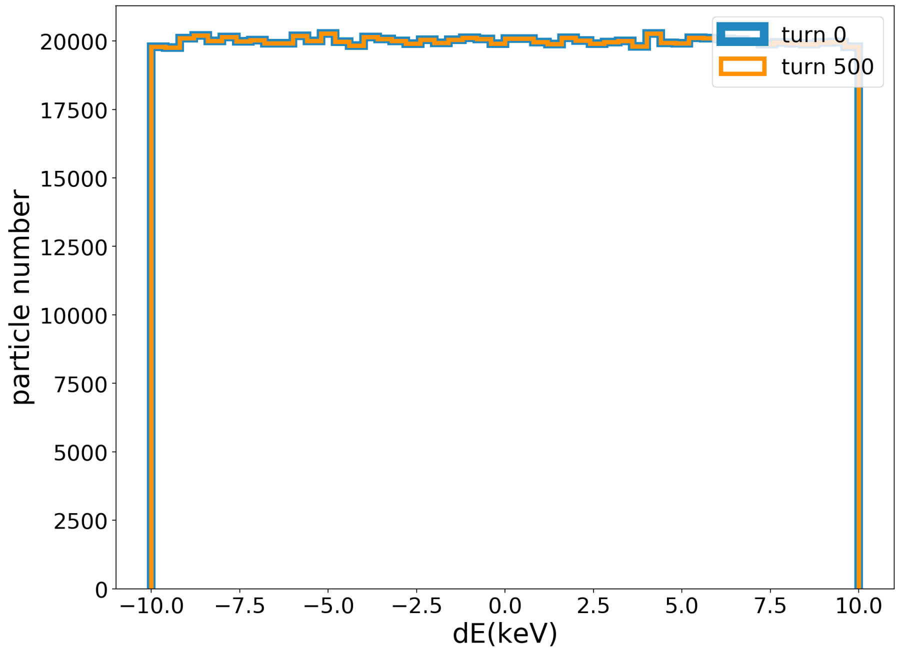


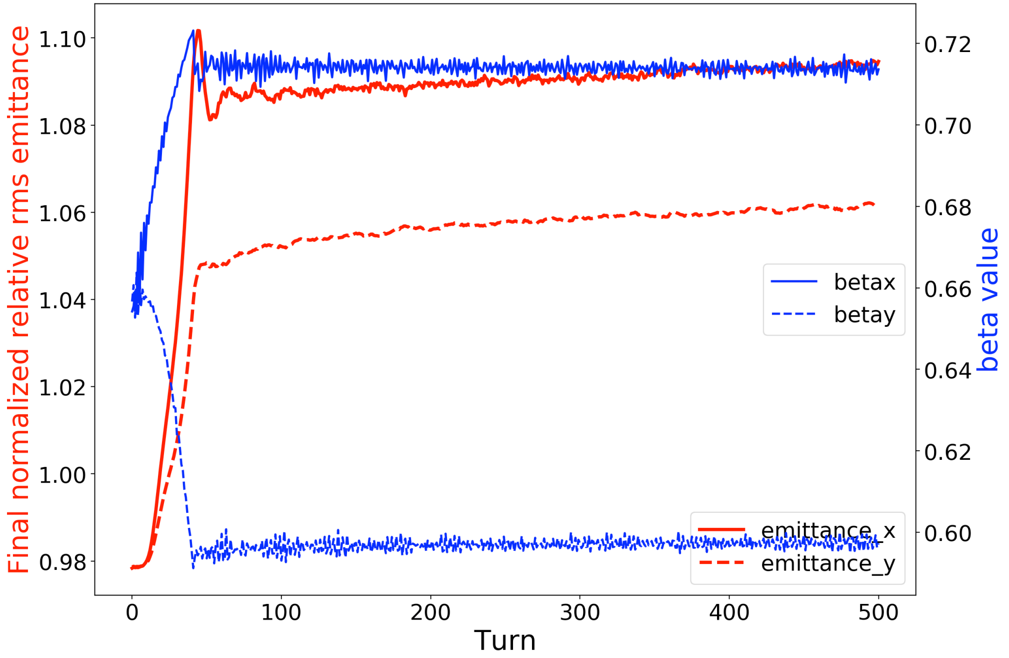



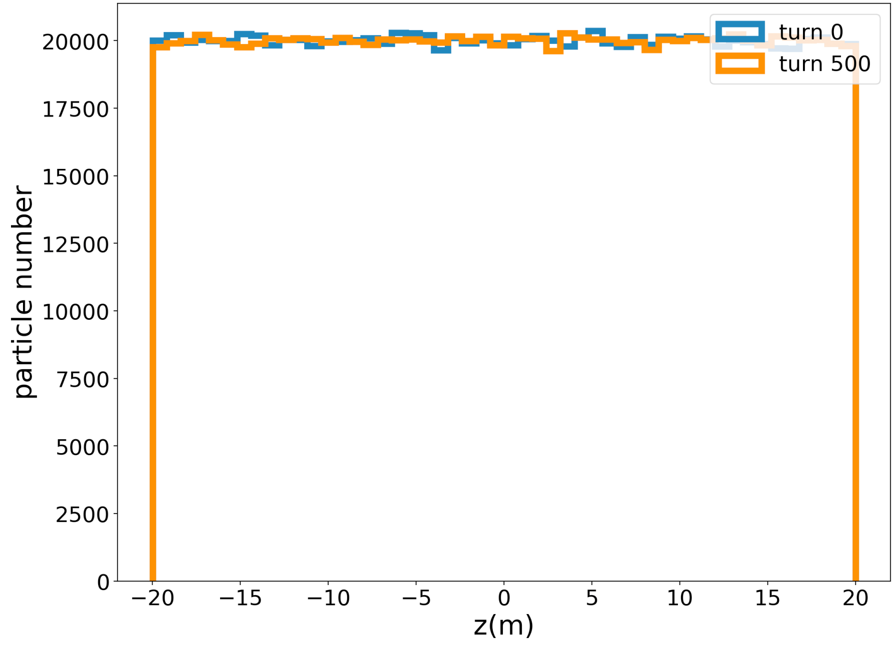



# Periodic Boundary Condition and Tracking Test

- A longitudinal periodic boundary condition is applied. At the start of each turn particles are moved so that -C/2 < z < C/2
- With a 40 turns slow initialization and aperture size 0.1m, the bunch is tracked for 500 turns. The initial and final z and dE distributions remain uniform as shown on the right plots.
- The bottom plot shows the evolutions of beta and relative emittances (5% in emit_y and 10% in emit_x)
- Particle loss is 0



# Coasting Beam Emittance Growth

- A beam is matched if its emittance is stationary. There is a perfect balance between the external focusing force, the space charge force, and the emittance term, shown in the envelope equation

$$a = 2x_{rms} \quad k_0 = Q/R$$

$$\begin{aligned} K_{sc} &= \frac{e\lambda_L}{2\pi\epsilon_0\beta\gamma^2 pc} \\ \lambda_L &= \frac{N}{2\sqrt{2\pi}\sigma_z}, \quad \text{Bunched beam} \\ \lambda_L &= \frac{N}{C}, \quad \text{Coasting beam} \end{aligned}$$

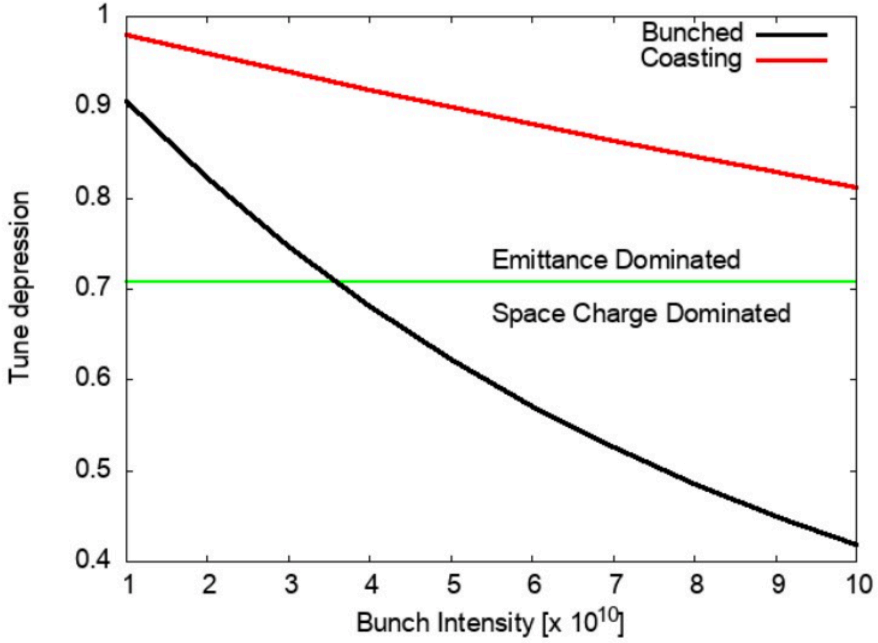
$$k_0^2 a - \frac{K_{sc}}{a} - \frac{\epsilon^2}{a^3} = 0$$

- If the beam is mismatched, there will be increased field energy, and the emittance will evolve. If the space charge (second) term dominates (greater than the third emittance term), the beam can grow without bound.

- Transition between emittance dominated and space charge dominated behavior occurs at a tune depression (tune with space charge/tune without space charge) of ~ 0.7

$$\begin{aligned} K_{sc} &= k_i^2 a^2 \Rightarrow a^2(k_0^2 - k_i^2) = k_i^2 a^2 \\ \frac{k_i}{k_0} &= \sqrt{\frac{1}{2}} \approx 0.707 \end{aligned}$$

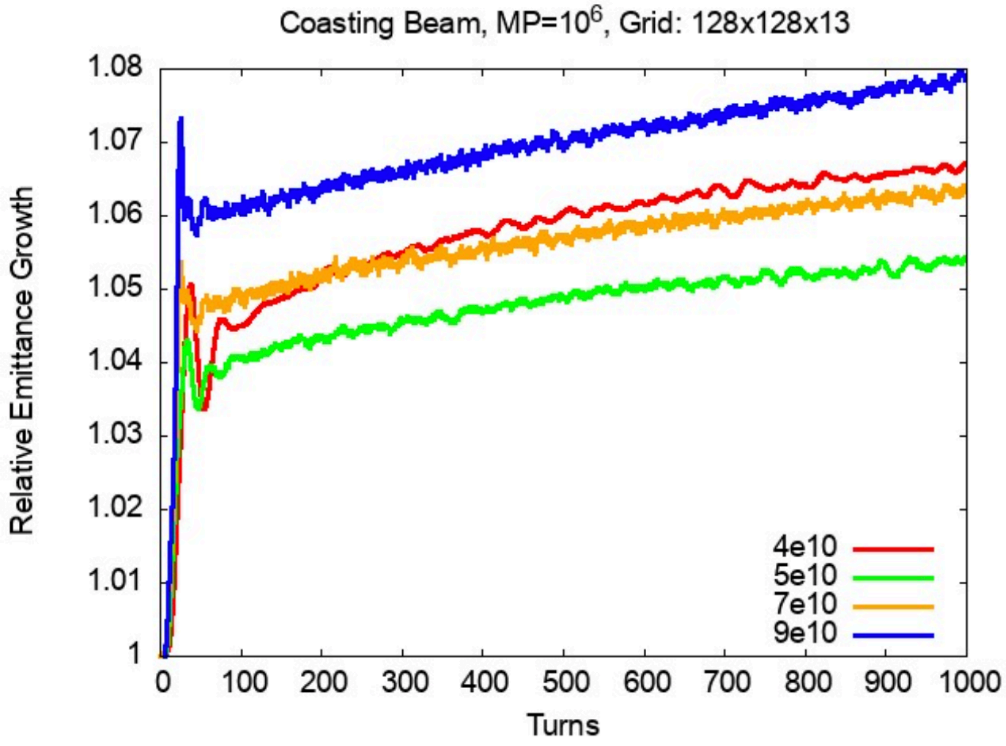
- Coasting beams in IOTA are not space charge dominated even at full intensity, and bunched beams become space charge dominated at intensities > 4 x $10^{10}$

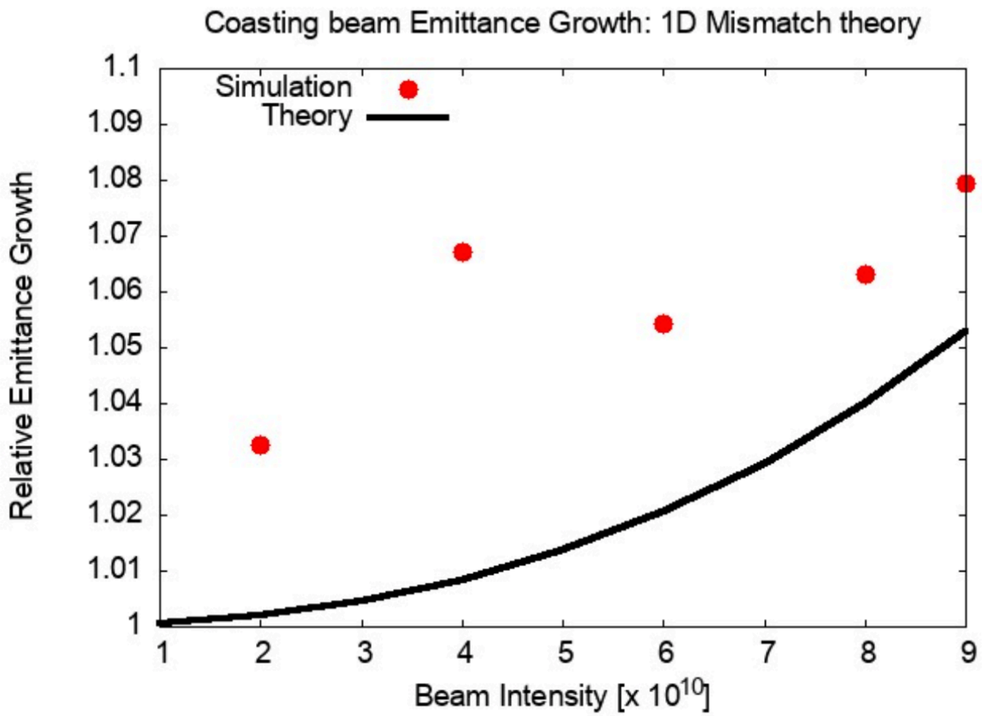


- Assume smooth focusing and perfect axial symmetry in the x-y plane, if the beam has an RMS size $a_0$ that is different from the matched beam size $a_i$ for the same emittance, using energy conservation in the transverse planes, we obtain a relation for the change in emittance*

$$\frac{\epsilon_f}{\epsilon_i} = \frac{a_f}{a_i}\left[1+\frac{k_0^2}{k_i^2}\{(\frac{a_f}{a_i})^2-1\}\right]^{1/2}$$

- The simulated emittance growth for a coasting beam agrees well with theory, with discrepancy ~2%






*M. Reiser, *J. App. Phys.*, 70, 1919 (1991)

# 0 Amplitude Tune Shift

Expected 0 amplitude tune shift of coasting beam is (from http://cds.cern.ch/record/941316):

$$\Delta Q_y = -\frac{r_0 I R}{e c \beta^3 \gamma^3} \left\langle \frac{\beta_y}{2\sigma^2} \right\rangle = -\frac{r_0 N}{2\pi \beta^2 \gamma^3} \frac{2}{E_y} \text{ for } r \ll \sigma \quad (30)$$

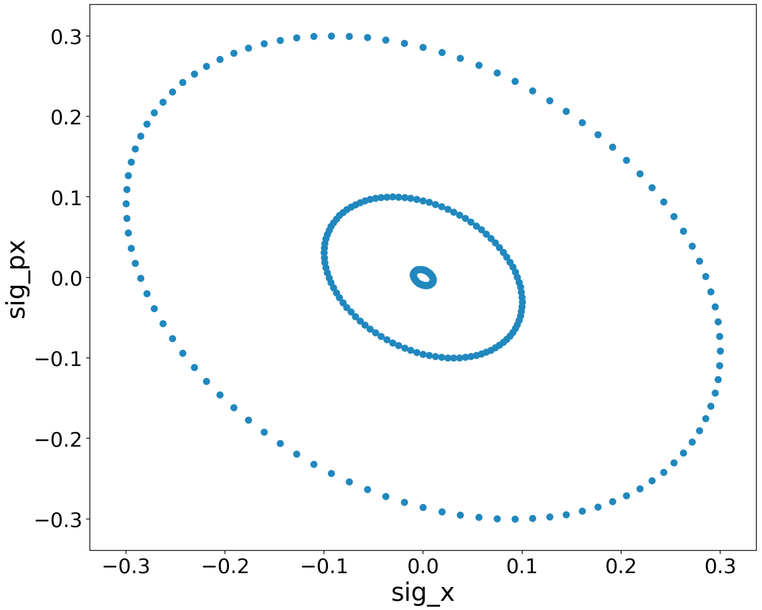
with $E_y = 4\sigma^2/\beta_y$, the 95% emittance.

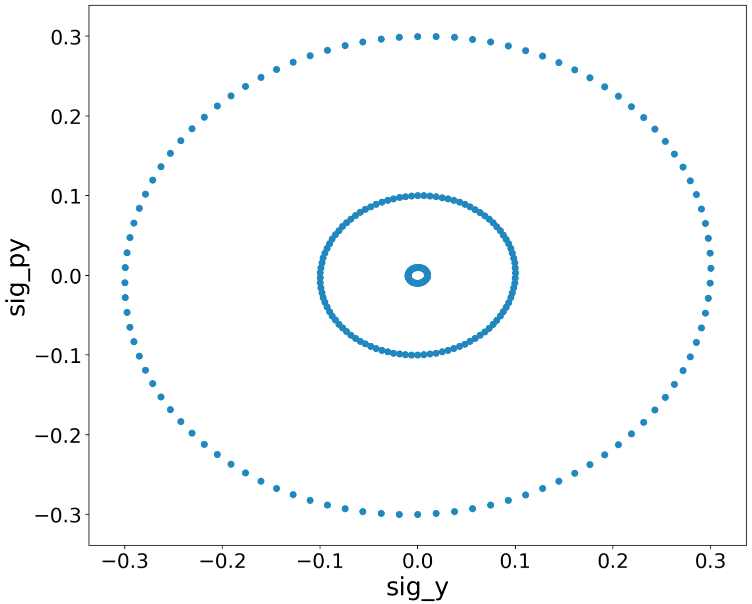
The bunch reaches stable normalized rms emittances $\varepsilon_x$ = 0.327mm-mrad and $\varepsilon_y$ = 0.318mm-mrad.

dQx = -0.455, Qx = 4.85

dQy = -0.469, Qy = 4.83

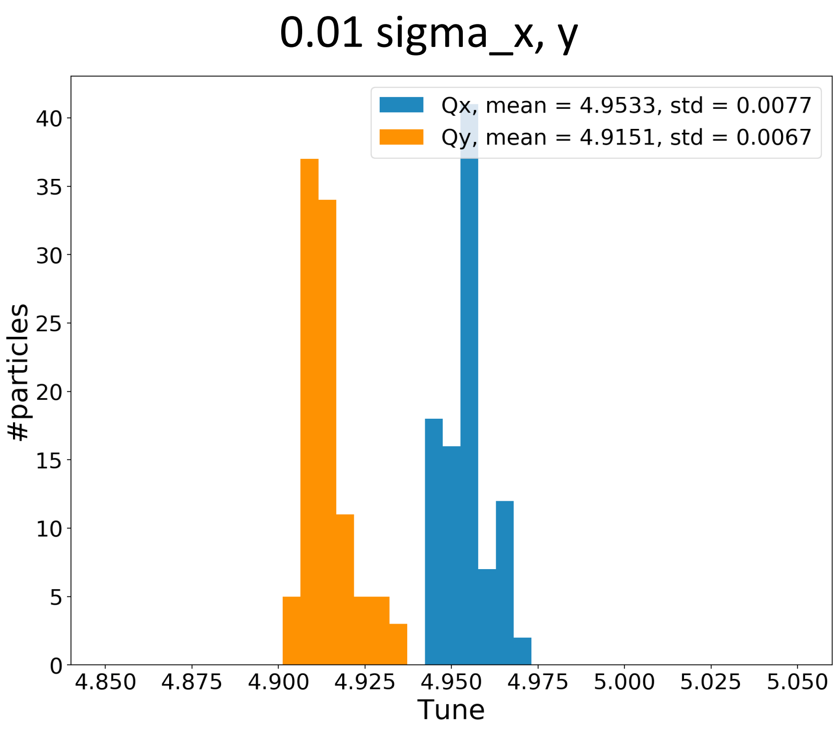
300 test particles are added into the stable bunch at 3 emittance levels. Tunes are calculated from betatron oscillations over 100 turns.

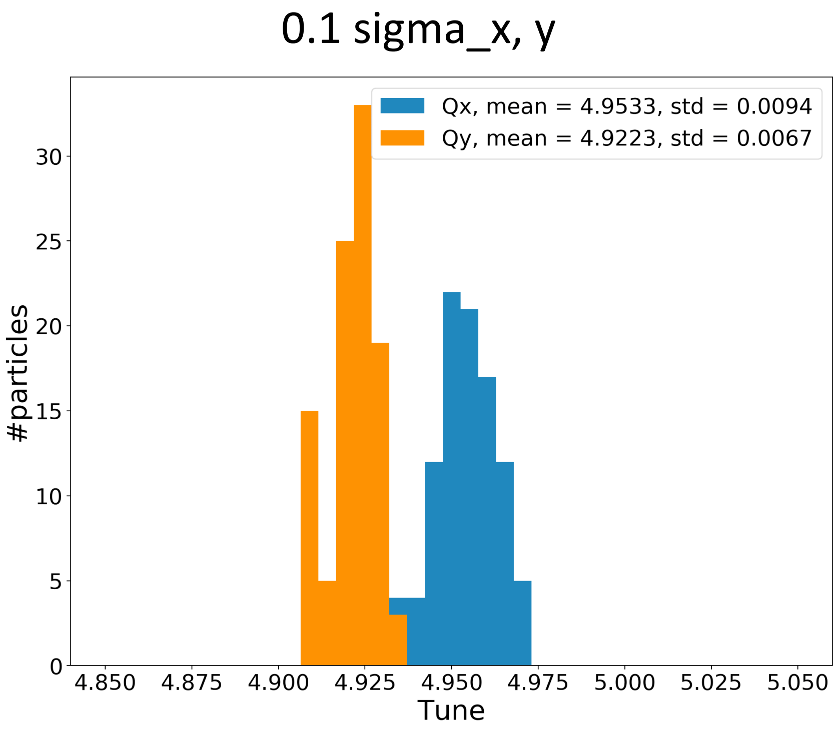


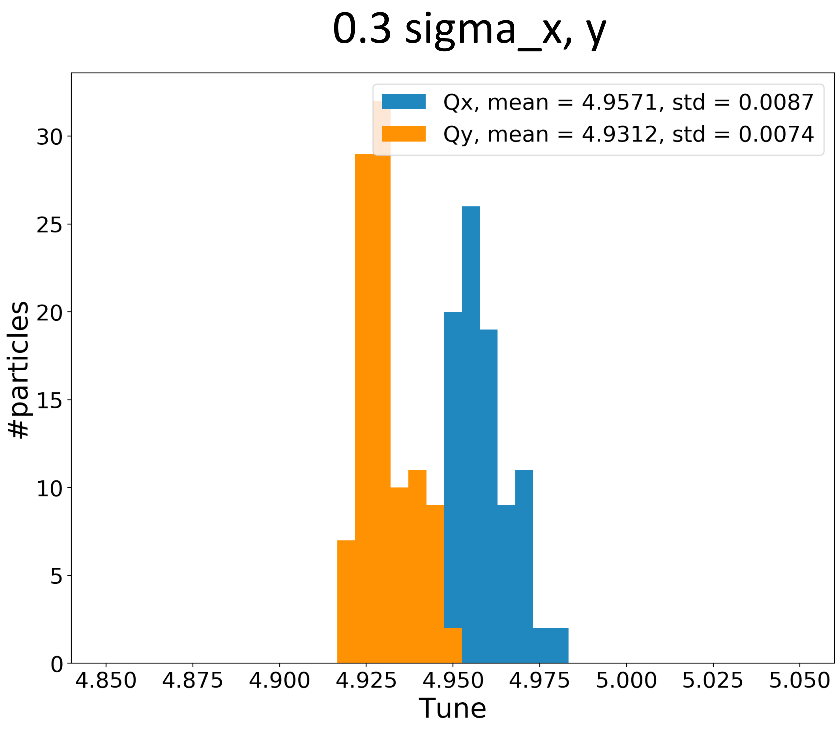




Initial distribution of test particles

# 0 Amplitude Tune Shift



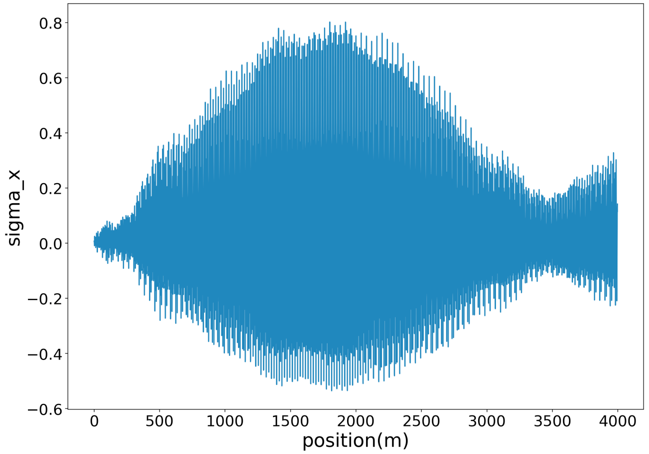




- Qx is larger than Qy; since horizontal emittance is largest, horizontal SC detuning is smallest
- Tune spread is small, and tune separation is consistent with theory
- Peaks do not match the analytically calculated value, which is most likely due to the chaotic nature of small amplitude particles trajectories

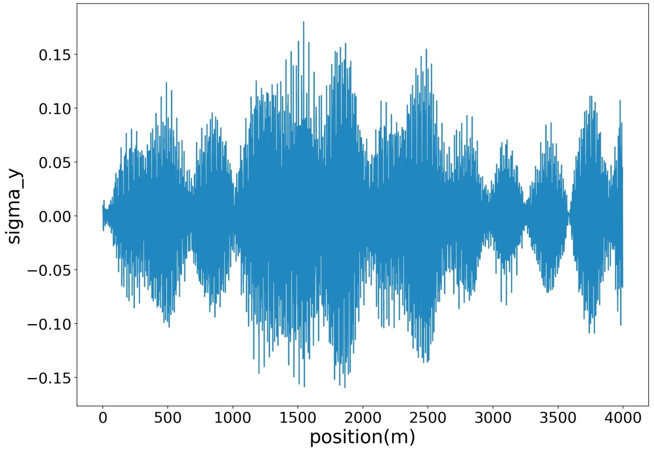
# Sample Trajectories of Small Amplitude Test Particles

0.01 sigma_x,y




oscillate between

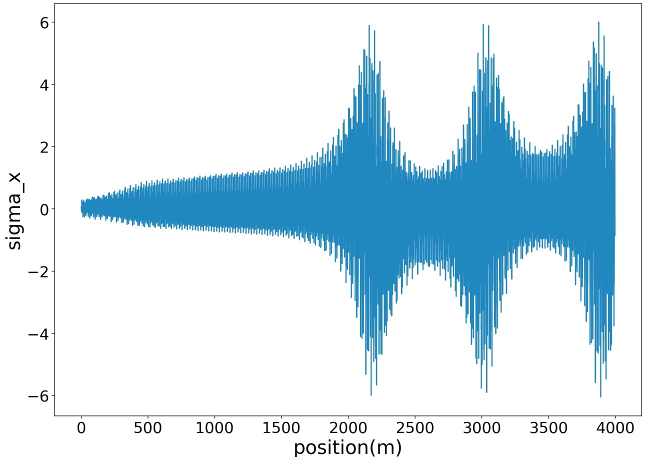
-0.6 to 0.8 sigma



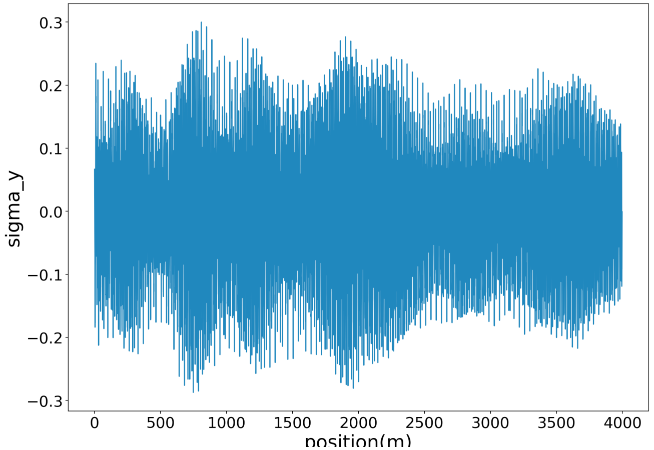

oscillate between

-0.15 to 0.15 sigma

0.1 sigma_x,y

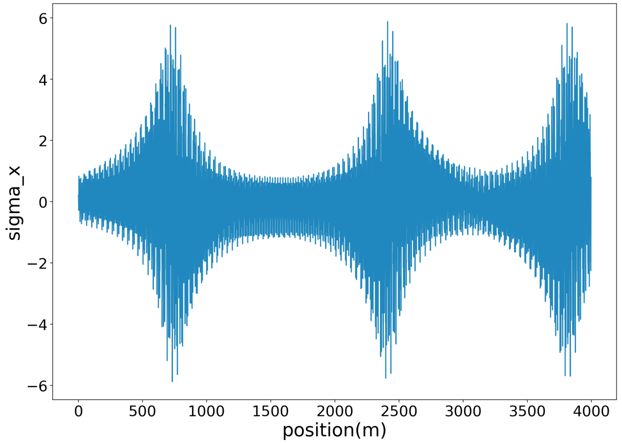



-6 to 6 sigma

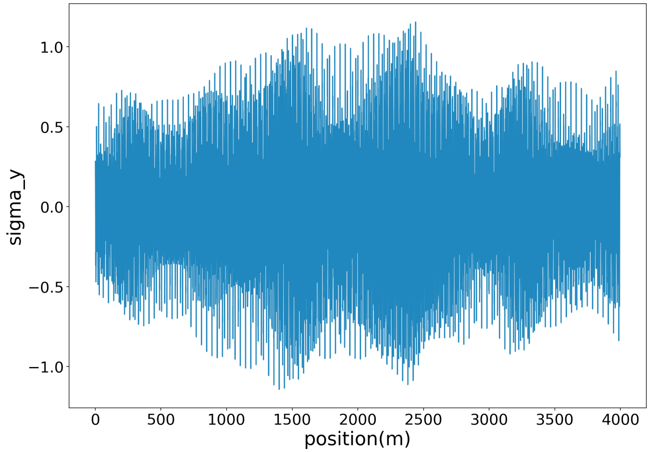



-0.3 to 0.3 sigma

0.3 sigma_x,y



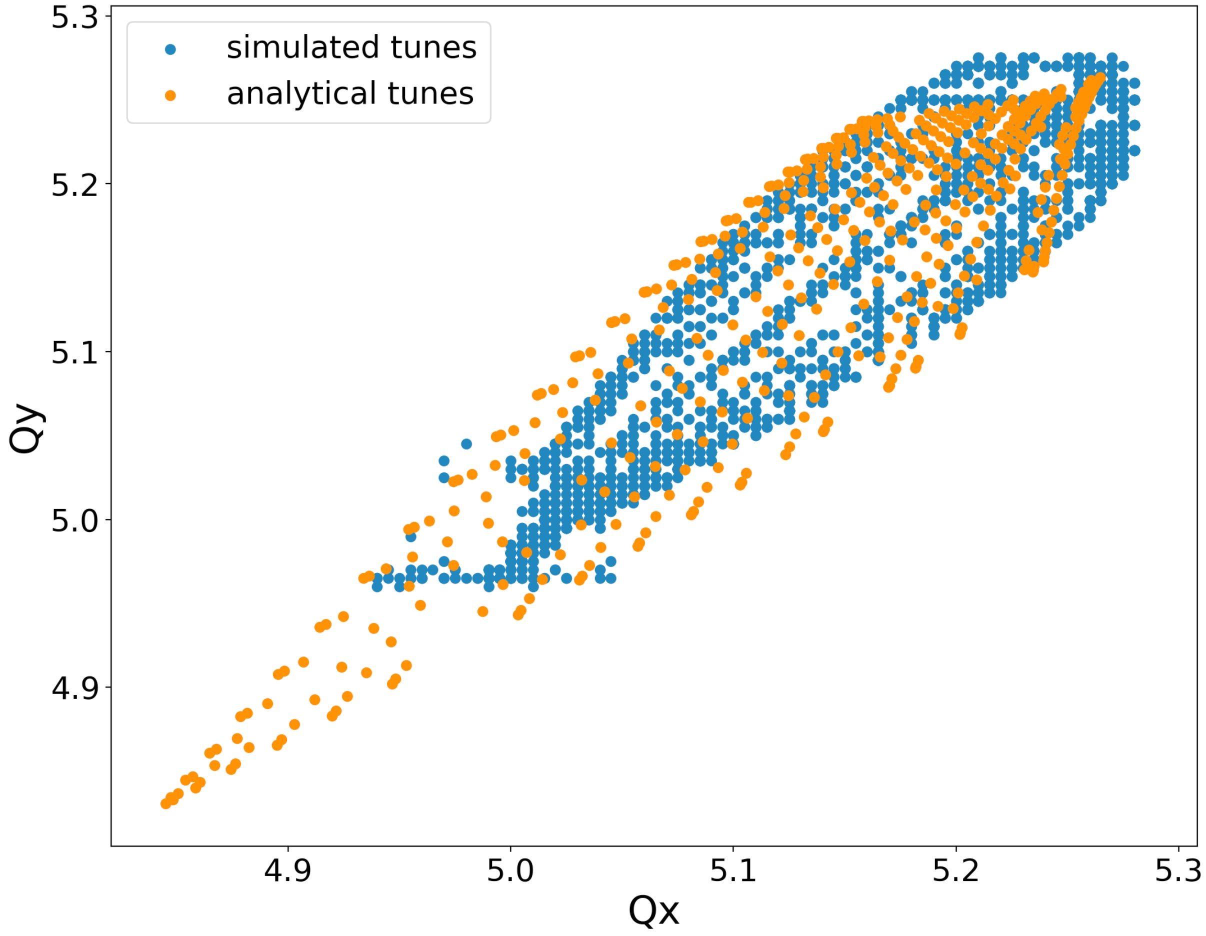

-6 to 6 sigma




-1.1 to 1.1 sigma

In this test, after slow initialization and stabilization, test particles are injected with initial positions on semi-circular arcs of radius (0 - 8)σ in *x*−*y* space. These test particles are tracked for 100 turns.

Just as we observed with the small amplitude tune test, we observe that the tune shifts for test particles with near 0 amplitude are not well behaved.

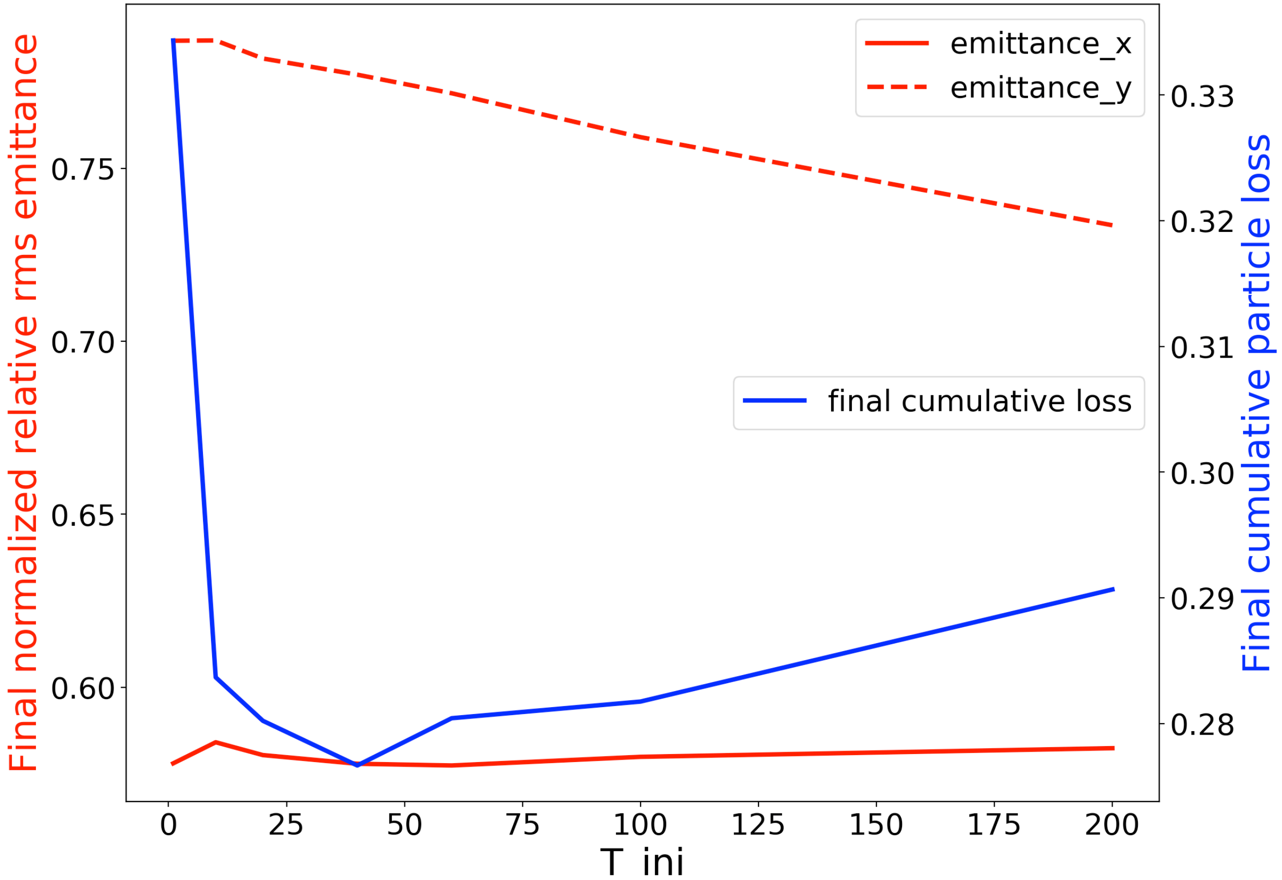


# PART II.
# Simulations with Real Apertures and no Octupoles

# Introducing Real-sized Apertures: Convergence Test

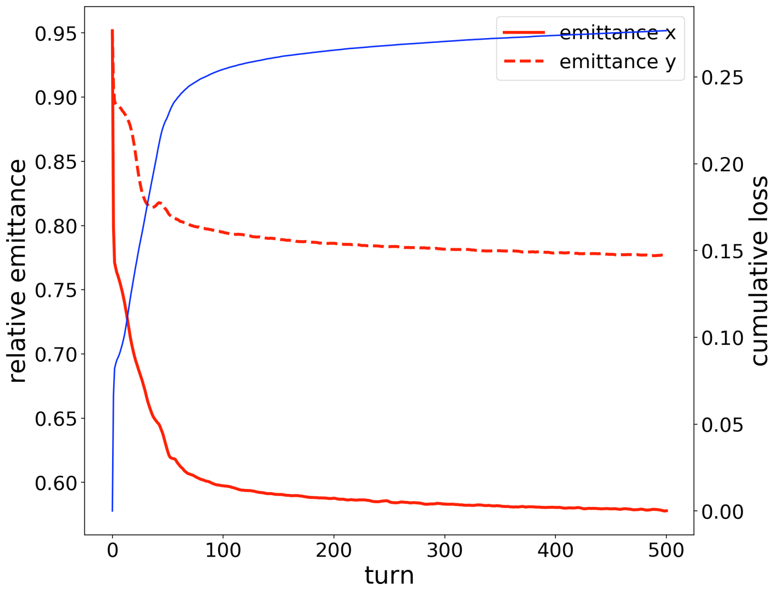
Adding realistic apertures which is 25mm everywhere in the ring except in the insert region (Min aperture size x = 3.94mm y = 5.26mm).

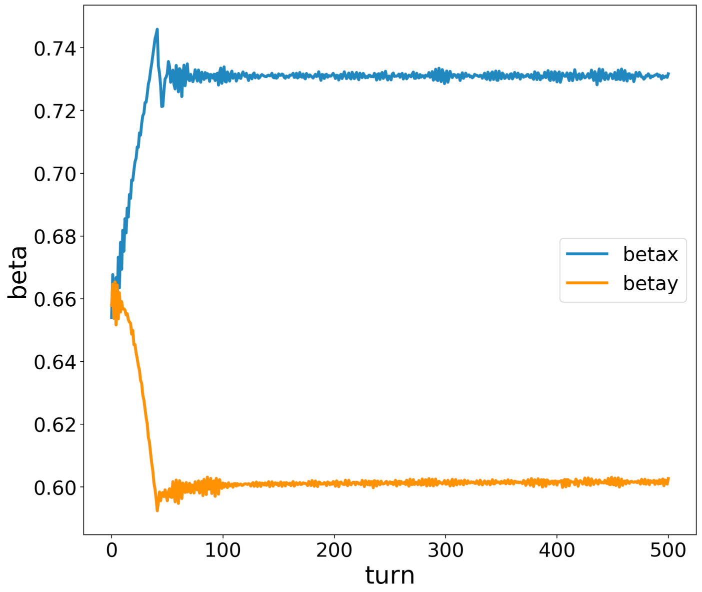
Track the bunch for 500 turns, including T_ini turns of slow initialization to reach full intensity of 8mA.



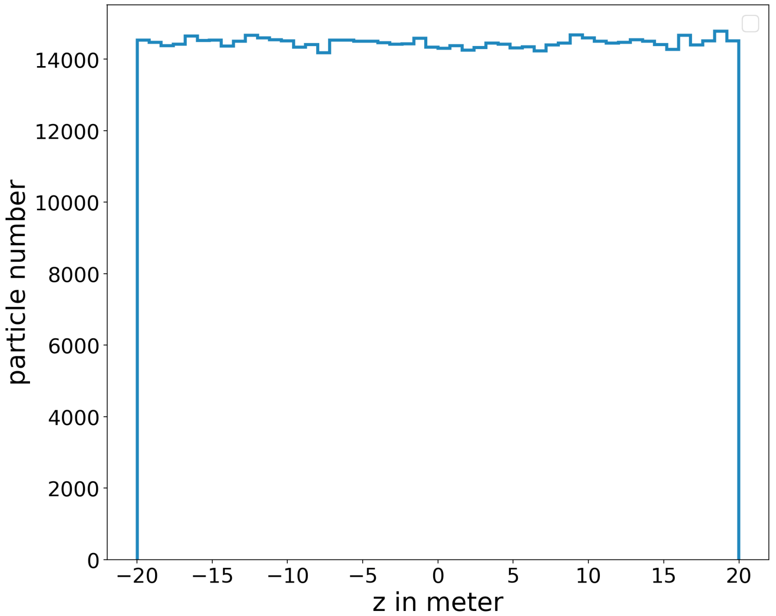

T_ini is varied from 1 to 200 turns. We look for a minimum in cumulative particle loss.
This minimum occurs for T_ini =40 turns.

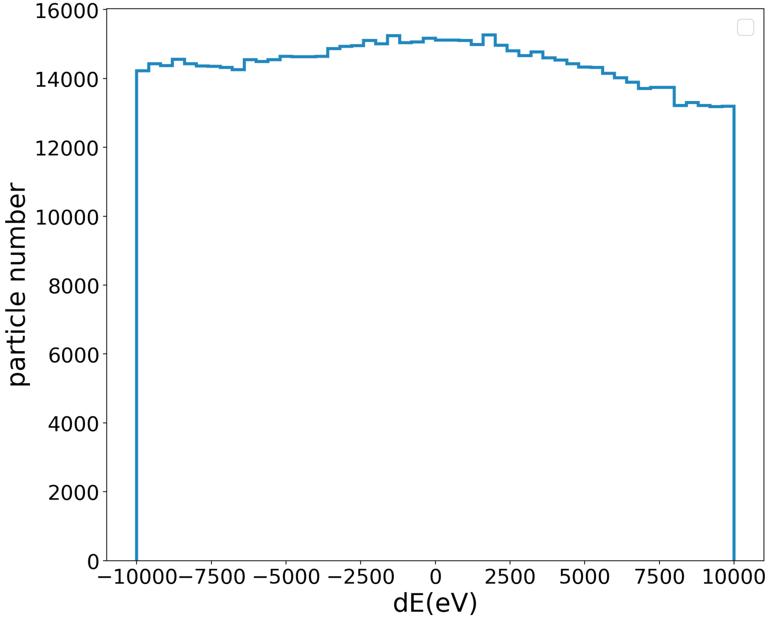
# Emittance and Particle Distribution Evolutions during Tracking




The dE distribution is not as uniform as we observed in the large aperture case. This is likely due to horizontal dispersion (see backup slides).

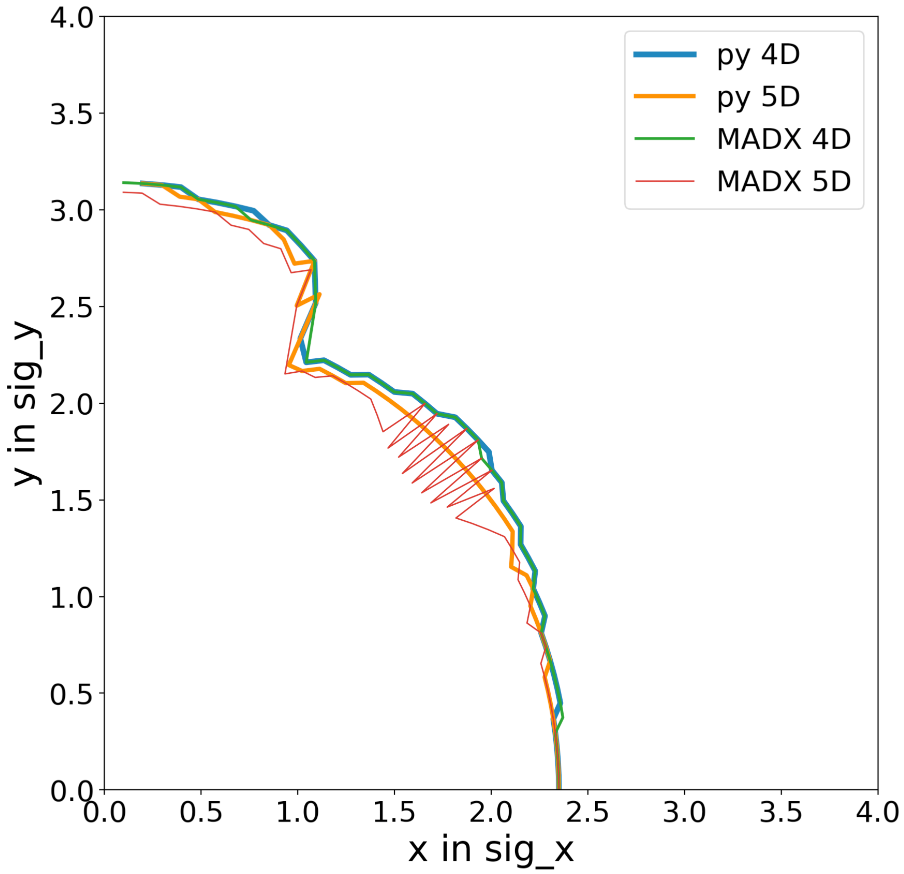


# PART III.
# Simulations with Real Apertures and Octupoles

Dynamic Aperture with Octupoles

- In this step, octupoles are added. Realistic apertures are used.
- The dynamic aperture is obtained by initializing 5000 test particles at (xi, 0, yi, 0, 0, dEi), tracking for 5000 turns, and taking the the largest excursions of the surviving particles. dEi is 0 for 4D test and 10keV for 5D test, however, since in our setup rf is turned off and the smallest apertures are located at 0 dispersion region, the effect of dispersion here is negligible.
- The dynamic aperture obtained from MADX and pyORBIT are similar, and they indicate that the bunch should be cut at around 2sigma



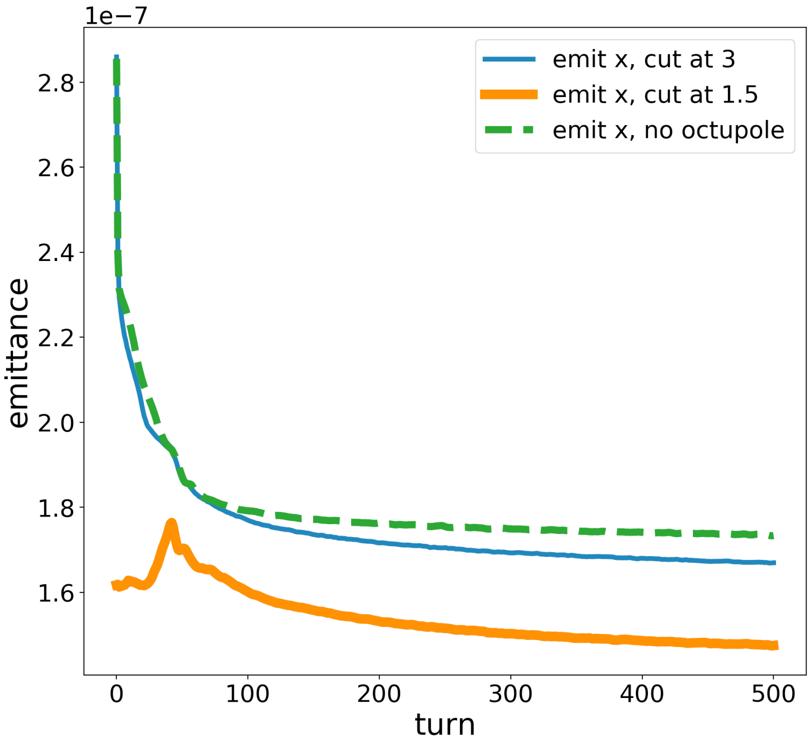

| | Min Aperture Size in sigma | Avg Aperture Size in sigma |
|---|---|---|
| 4D pyORBIT | 2.35 | 2.62 |
| 4D MADX | 2.35 | 2.58 |
| 5D pyORBIT | 2.35 | 2.64 |
| 5D MADX | 2.25 | 2.53 |

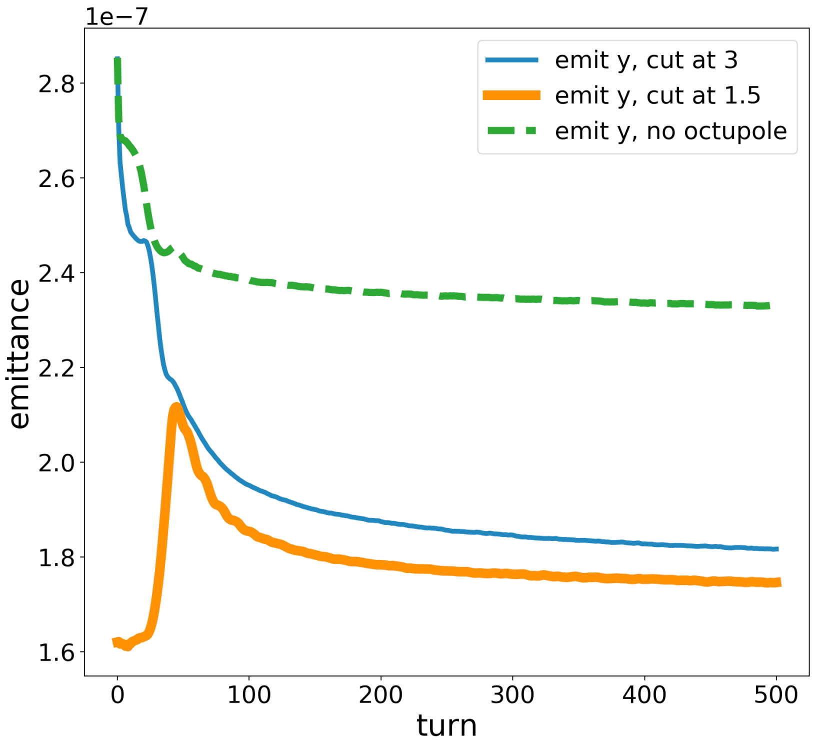
Smallest physical aperture corresponds to size 2.5 sigma (Min aperture size x = 3.94mm rms sigma = sqrt(4.1um * 0.65) = 1.6mm)

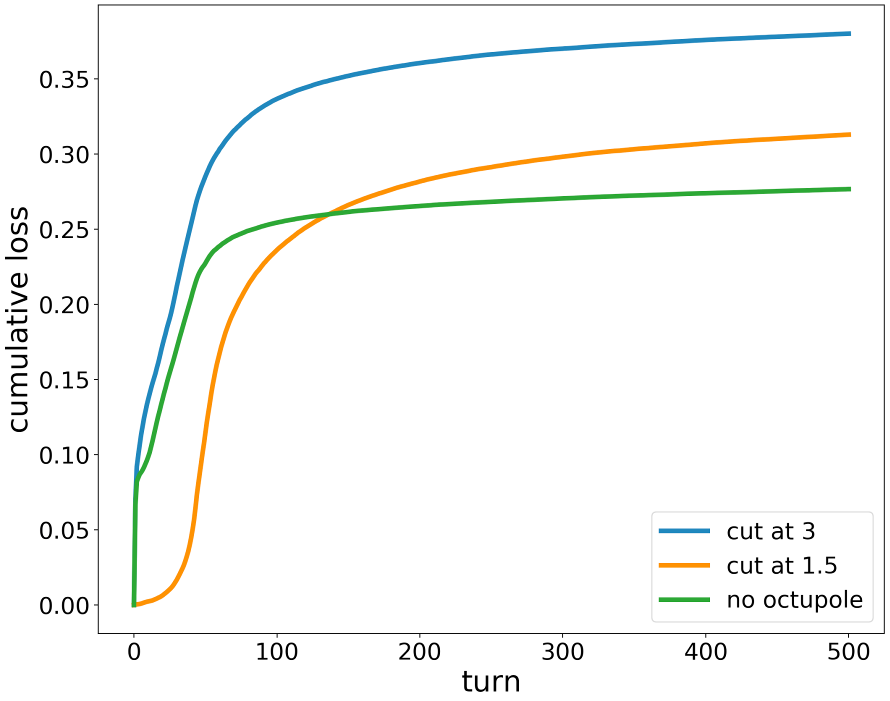
# Emittance Evolution and Particle Loss with Realistic Apertures and Octupoles

Emittance growth and particle loss for:

- No octupole, no truncation
- Octupole with initial distribution cut at 3 sigma
- Octupole with initial distribution cut at 1.5 sigma



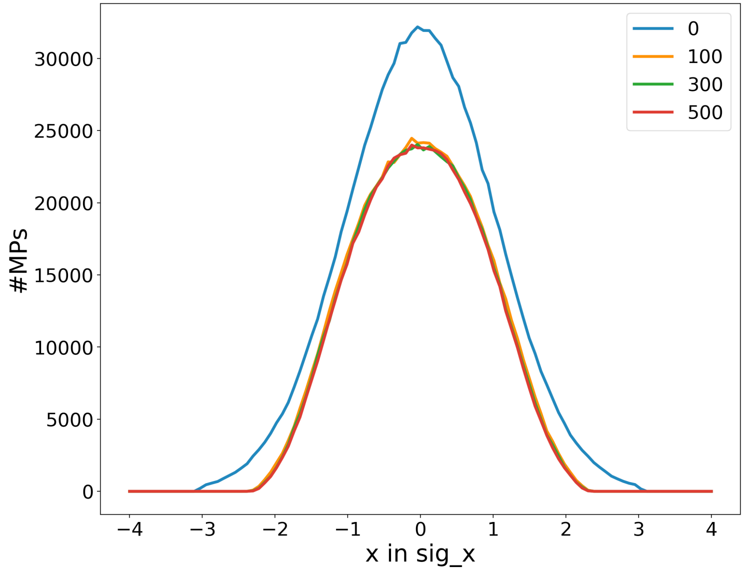

After trunacting at 1.5 sigma, emittance grows sharply until the bunch size reaches the physical aperture. It then decays due to particle losses and eventually stabilizes.

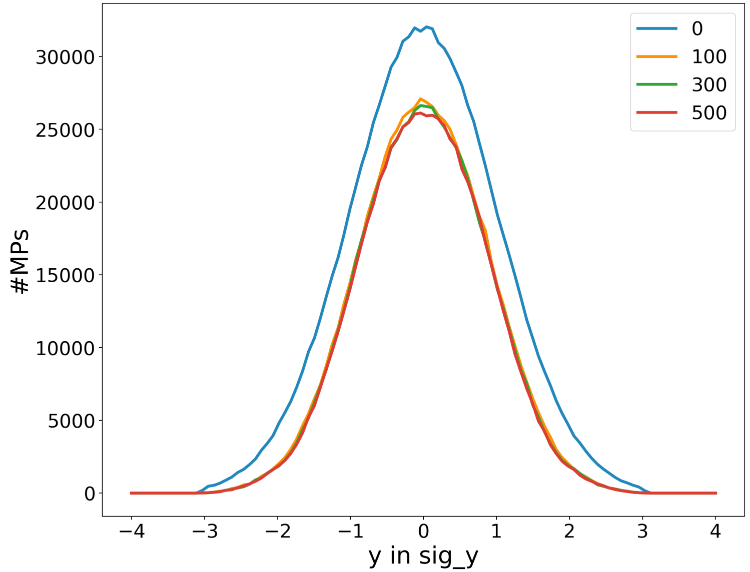
Truncating the initial distribution at 1.5 sigma results in reduced overall particle losses.

# Beam evolution at turn 0, 100, 300, 500 for 3 cases

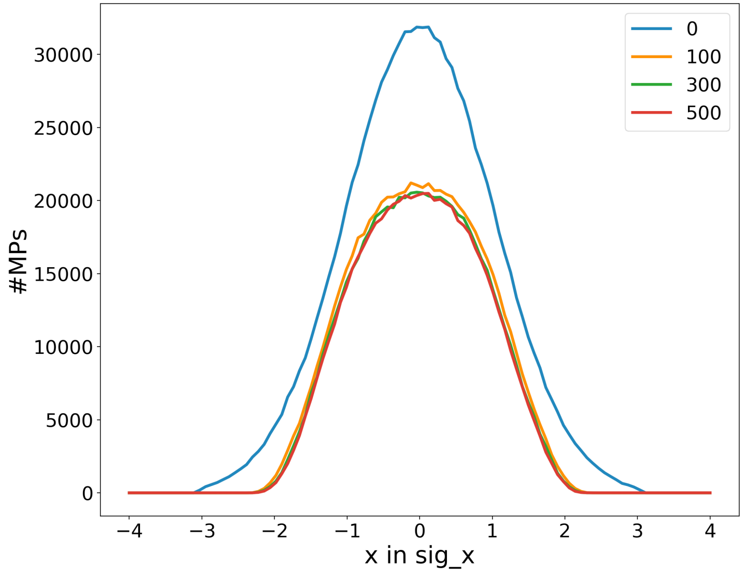
In all 3 cases, the particle distribution change is most significant during the first 100 turns due to slow initialization.

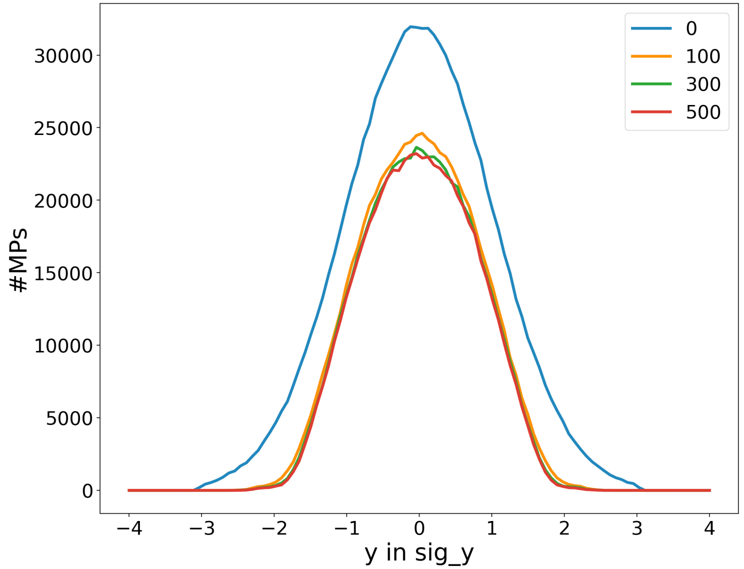
no octupole



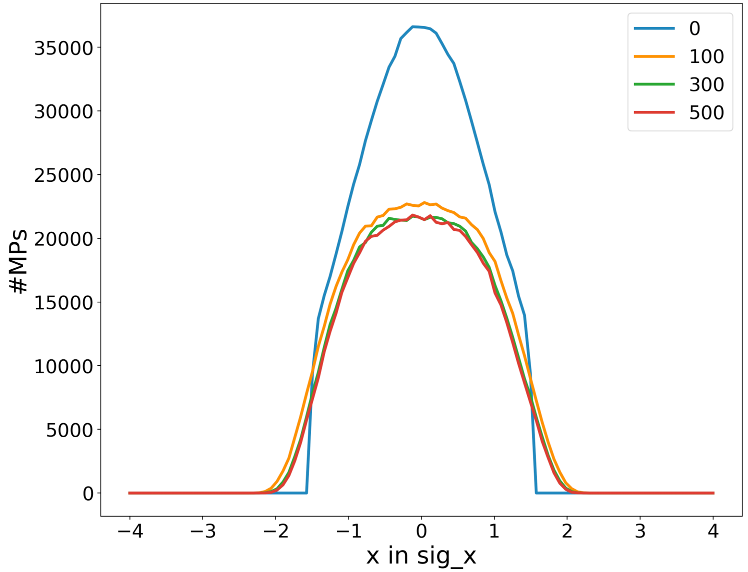



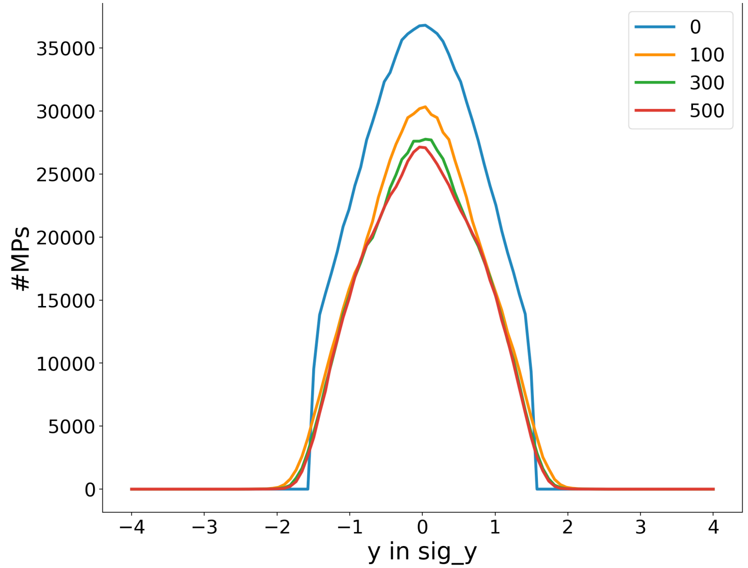

with octupole, cut at 3sigma




with octupole, cut at 1.5sigma

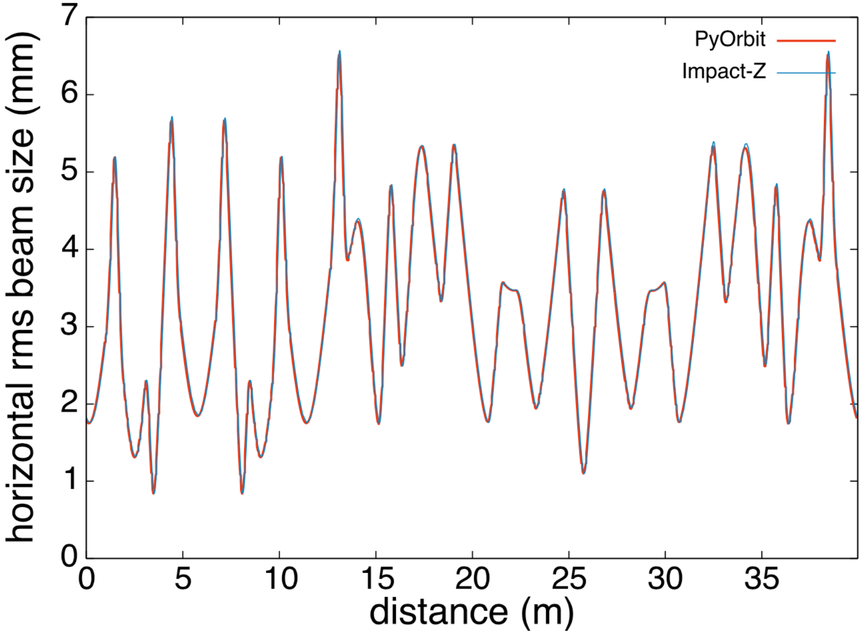


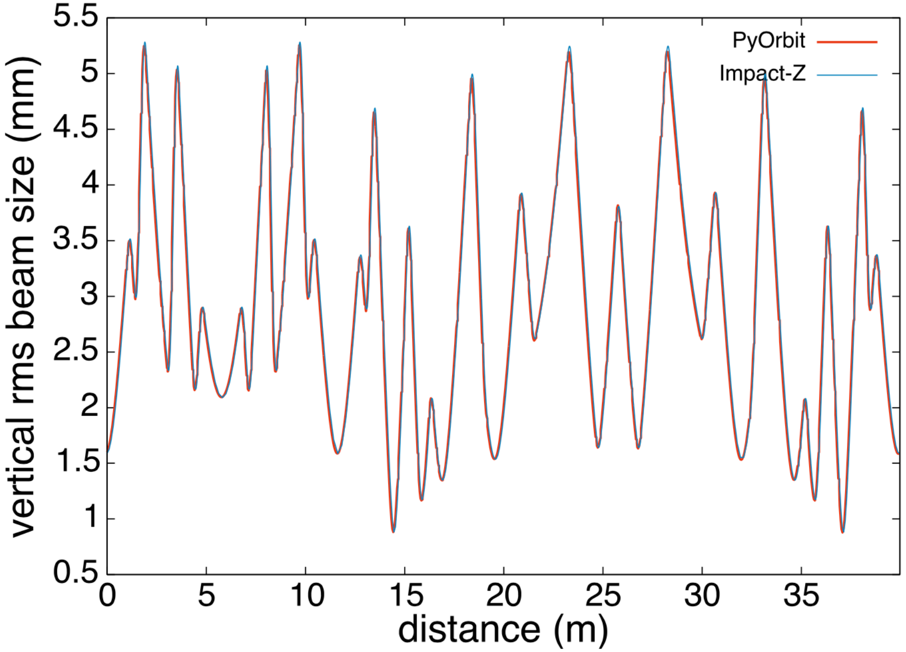



# PART IV.
# Benchmarking pyORBIT tracking with ImpactZ

Crosscheck between ImpactZ and pyORBIT by Tracking Coasting Beam

- The same initial distribution is used, which is generated by pyORBIT
- The distribution is tracked for 10 turn with impactZ (Spectral 2D SC Solver) by Chad and with pyORBIT (FFT 2.5D Solver) by me. The H/V bunch sizes (in units of sigma is) are recorded at every location around the ring during the final turn.




- More tests will be performed, including:
  - Trajectories of small amplitude particles and associated tunes
  - Initial emittance growth and particle loss
  - Initial particle distribution evolution

# Conclusions

- Simulations with large apertures and no octupoles
  - The injection point of this simulation is the middle of octupole insert region, and a coasting beam is used
  - A longitudinal periodic boundary condition is enforced at the beginning of every turn
  - Simulated emittance growth matches well with analytical result
  - 0 amplitude tune shift obtained from test particles with small amplitude does not match the analytical result, which is most likely due to the chaotic nature of small amplitude particles trajectories
  - Small amplitude test particles trajectories ocillations are chaotic and reach large amplitudes
  - Tune footprint for test particles with near 0 amplitude are not well behaved for the same reason

- Simulations with real aperture and no octupoles
  - Convergence test shows minimum particle loss at T_ini = 40
  - Due to horizontal dispersion, the dE distribution is not as uniform as we observed in the large aperture case

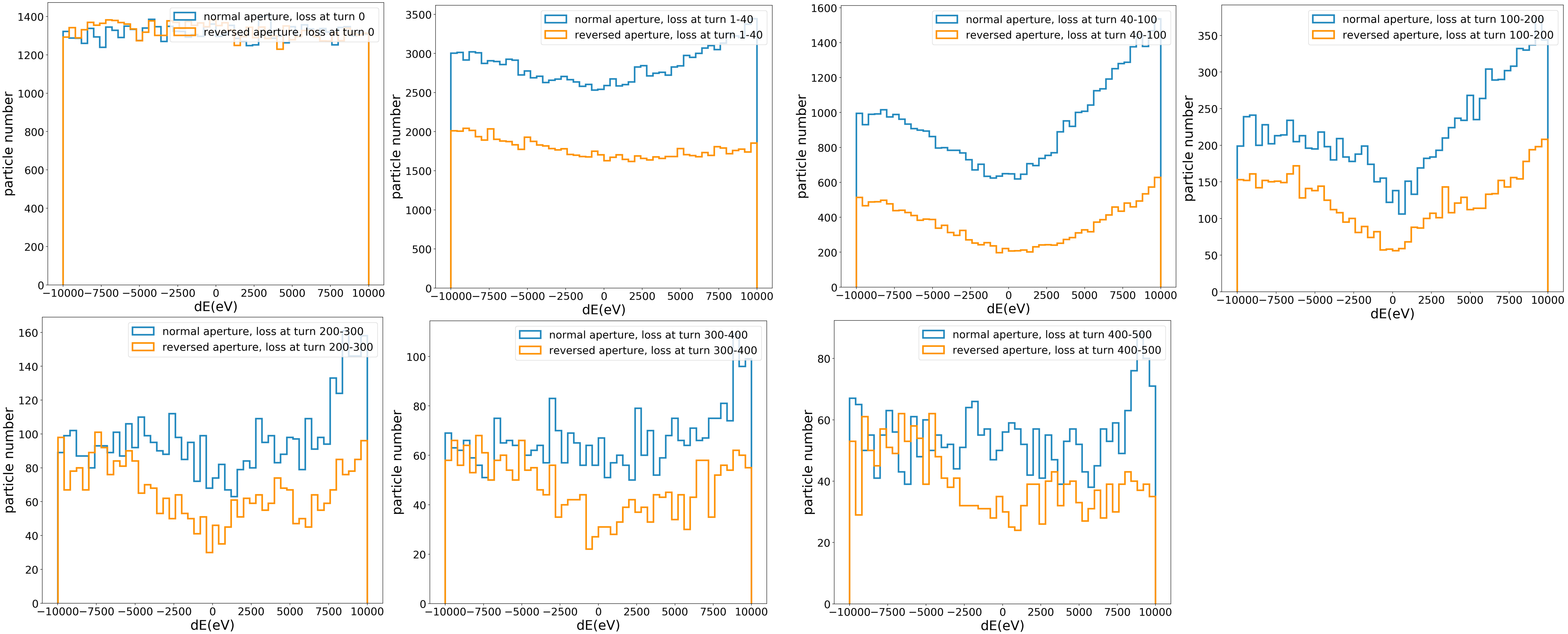
- Simulations with real aperture and octupoles
  - Dynamic aperture size obtained from pyORBIT is similar with results from MADX, about 2.4 sigma
  - Truncating the transverse distribution at 1.5 sigma reduces overall particle losses
  - The bulk of the particle distribution evolution occurs during the first 100 turns. The distribution is stable thereafter.

- Beam sizes predicted by pyORBIT and ImpactZ in simulations where space charge is accounted for are in excellent agreement after 10 turns; more tests will be performed.

# Backup Slides

# Emittance and Particle Distribution Evolution during Tracking

The non-uniform dE distribution is likely due to the dispersion effect, as we see in the comparison of loss distribution between realistic aperture (elliptical aperture with x size < y size) and reversed aperture (reversed elliptical aperture with x size > y size)




Due to the fact that after reversing most of the loss happens due to y position instead of x, and given that the extend of asymmetry and non-uniformity of loss distribution in dE is smaller after the reversion, we conclude that, **although we are in the octupole insert region where dispersion is supposed to be close to 0**, there is some dispersive effect in x that brings the non-uniform dE loss distribution