Status of muon g-2 theory

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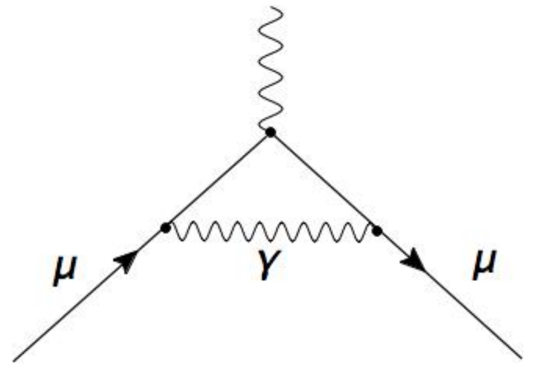
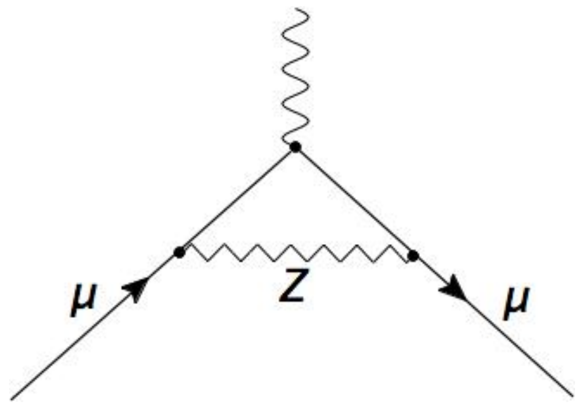
Fermilab Public PAC Meeting
16-18 November 2021

Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

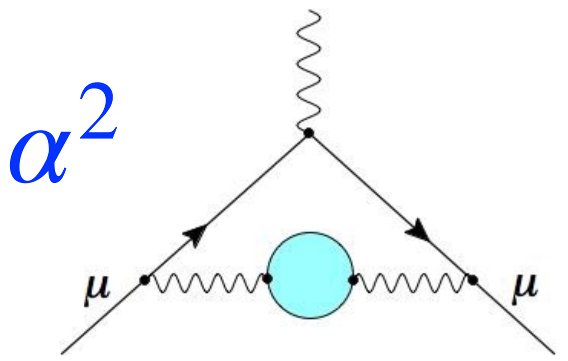
QED 

$+\ldots$ $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

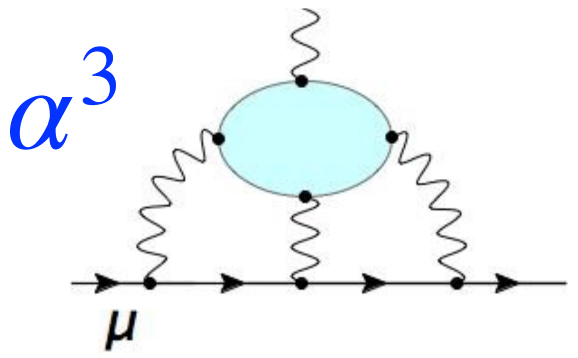
EW 

$+\ldots$ $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic corrections

HVP α^2 

$+\ldots$ $6845\,(40) \times 10^{-11}$ [0.6%] 0.34 ppm

HLbL α^3 

$+\ldots$ $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

Muon g-2 Theory Initiative

- Maximize the impact of the Fermilab and J-PARC experiments
  ➟ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Lattice HVP at high precision workshop (virtual): 16-20 November 2020
  Fourth plenary workshop @ KEK (virtual): 28 June - 02 July 2021
  Fifth plenary workshop @ Higgs Centre (Edinburgh): 5-9 September 2022
- 1st White Paper published in 2020 (132 authors, 82 institutions, 21 countries)
  [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]
- 2nd White Paper (~2023): First discussions @ KEK meeting in June 2021
  expect to develop a concrete plan (outline, authors) @ Higgs Centre workshop

Muon g-2 Theory Initiative

Steering Committee

Simon Eidelman

[photo by Hartmut Wittig]

(1948-2021)

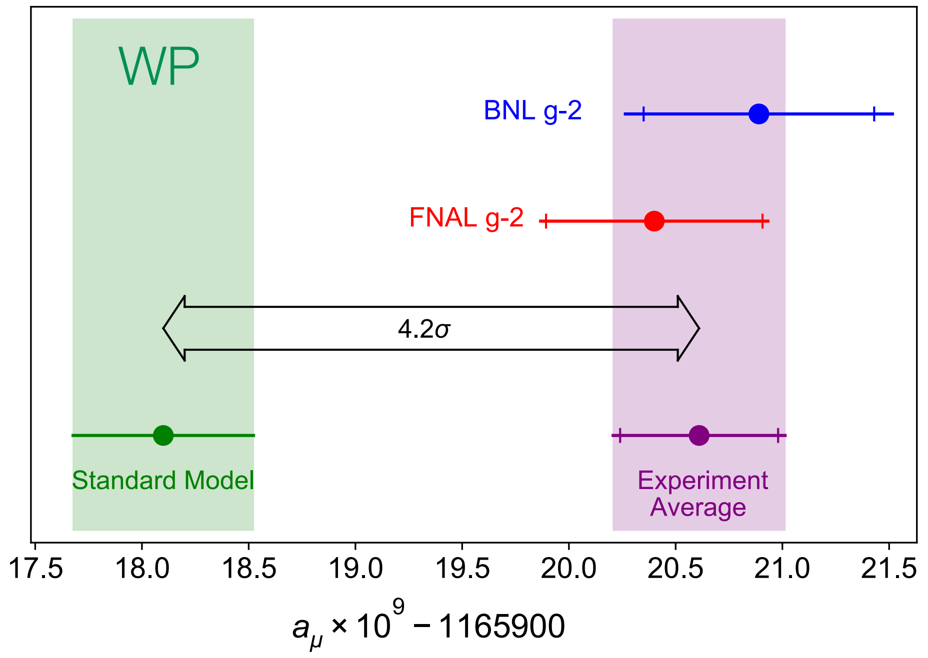
- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University & BNL) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK) J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston) Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

website: https://muon-gm2-theory.illinois.edu

Timeline


Physics Reports

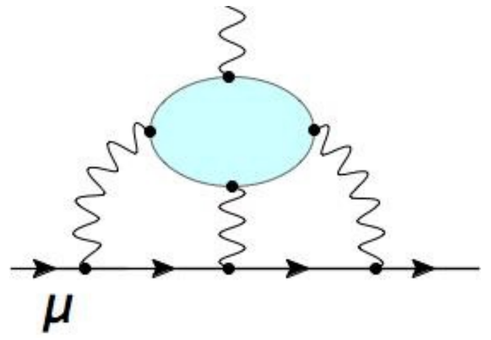
The anomalous magnetic moment of the muon in the Standard Model



Theory Initiative:

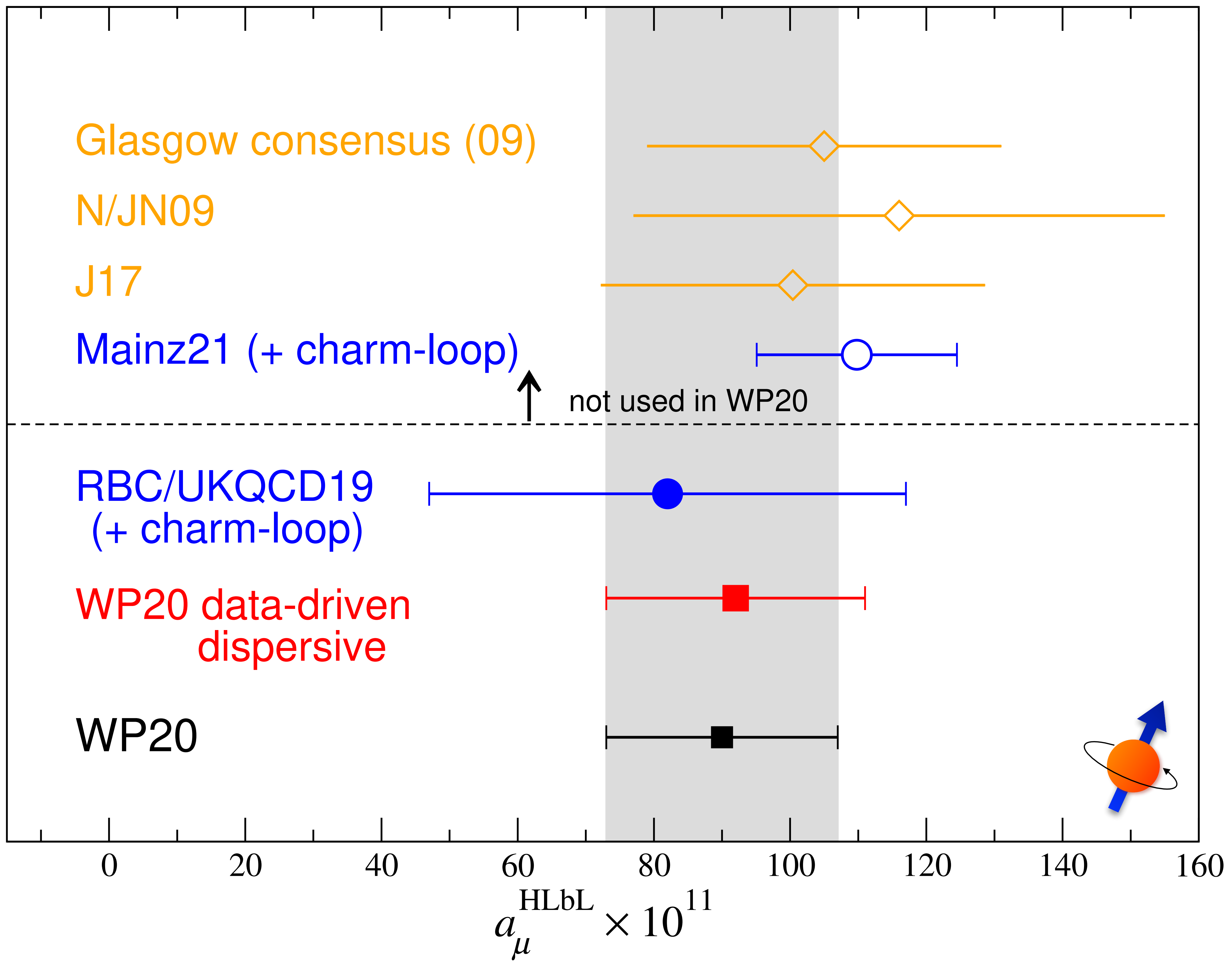
- ☆ ongoing activities: develop method average for Lattice HVP
- ☆ plan to update WP with new SM predictions (~ 2023)

★ TI workshops: Jun 2021 @ KEK (virtual)
Sep 2022 @ Higgscentre



HLbL: Comparison

$$a_\mu^{\rm HLbL}$$

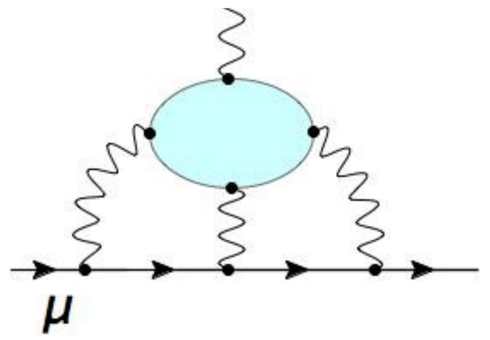


models of QCD +EFT, large N_c

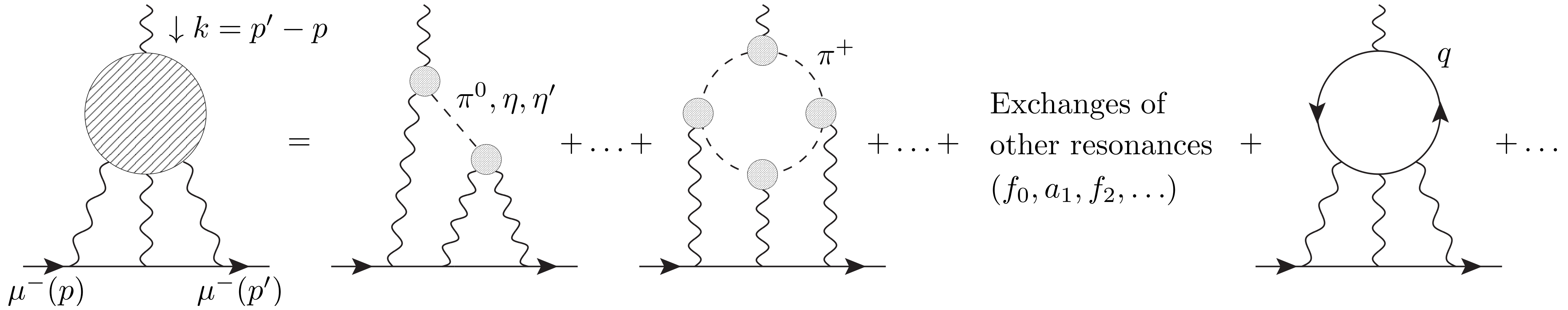
Lattice QCD + QED

data + dispersive approach

Now well-determined in two independent approaches, systematically improvable



Hadronic Light-by-light

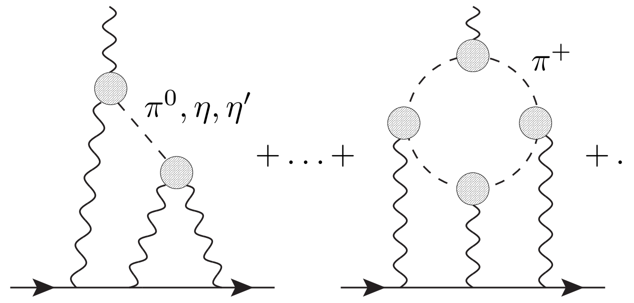


Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

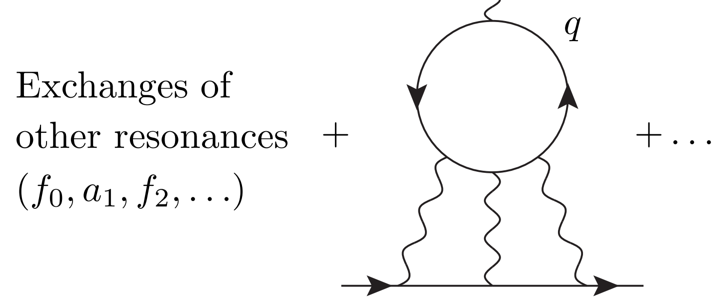
- model independent
- significantly more complicated than for HVP
- provides a framework for data-driven evaluations
- ➠ can also use lattice results as inputs

Dominant contributions ($\approx 75\,\%$ of total):



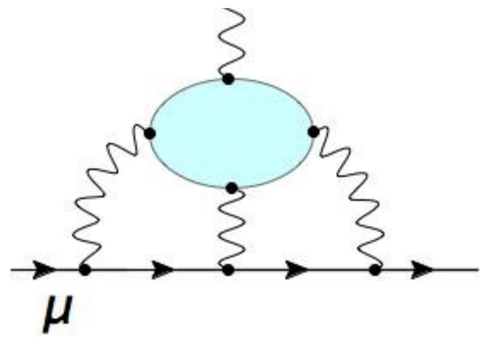
- Well quantified with $\approx 6\,\%$ uncertainty
- η, η' pole contributions: Canterbury approximants only
- Ongoing work: consolidation of η, η' pole contributions using disp. relations and LQCD

Subleading contributions ($\approx 15\,\%$ of total):



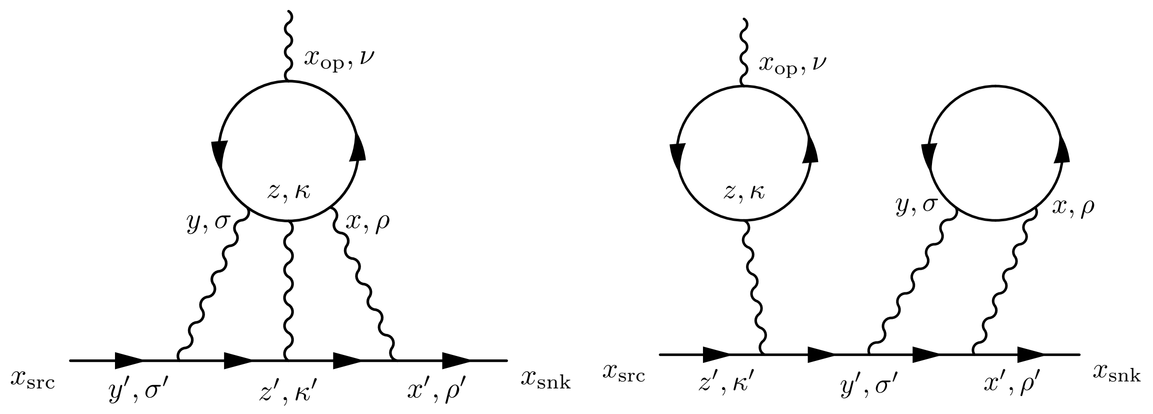
- Not yet well known
  ➠ dominant contribution to total uncertainty
- Ongoing work:
  - Implementation of short-distance constraints (now at 2-loop)
  - DR implementation for axial vector contributions
  - BESIII ramping up $\gamma^{(*)}\gamma^*$ program

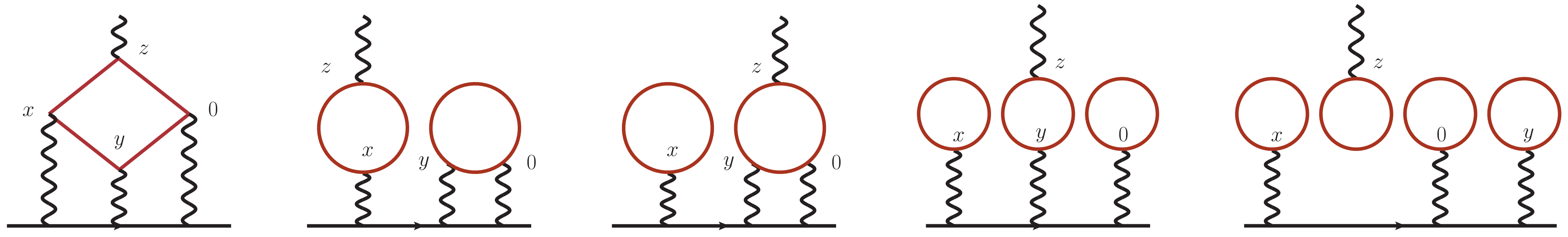
Dispersive, data-driven evaluation of HLbL with $\leq 10\,\%$ total uncertainty feasible by ~2025.



Hadronic Light-by-light

Lattice QCD+QED: Two independent and complete direct calculations of a_μ^{HLbL}





- ✦ RBC/UKQCD
  [T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123, 2020 PRL]
- ✦ QCD + $\mathrm{QED_L}$ (finite volume)

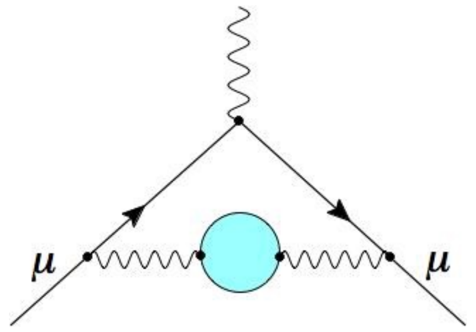
  DWF ensembles at/near phys mass,
  $a \approx 0.08 - 0.2\,\mathrm{fm},\ L \sim 4.5 - 9.3\,\mathrm{fm}$

- ✦ Mainz group
  [E. Chao et al, arXiv:2104.02632]
- ✦ QCD + QED (infinite volume & continuum)

  CLS (2+1 Wilson-clover) ensembles
  $m_\pi \sim 200 - 430\ \mathrm{MeV},\ a \approx 0.05 - 0.1\,\mathrm{fm},\ m_\pi L > 4$

- ✦ Cross checks between RBC/UKQCD & Mainz approaches in White Paper at unphysical pion mass
- ✦ Both groups will continue to improve their calculations, adding more statistics, lattice spacings, physical mass ensemble (Mainz)

Lattice HLbL results with $10\,\%$ total uncertainty feasible by ~2025



HVP: Comparison

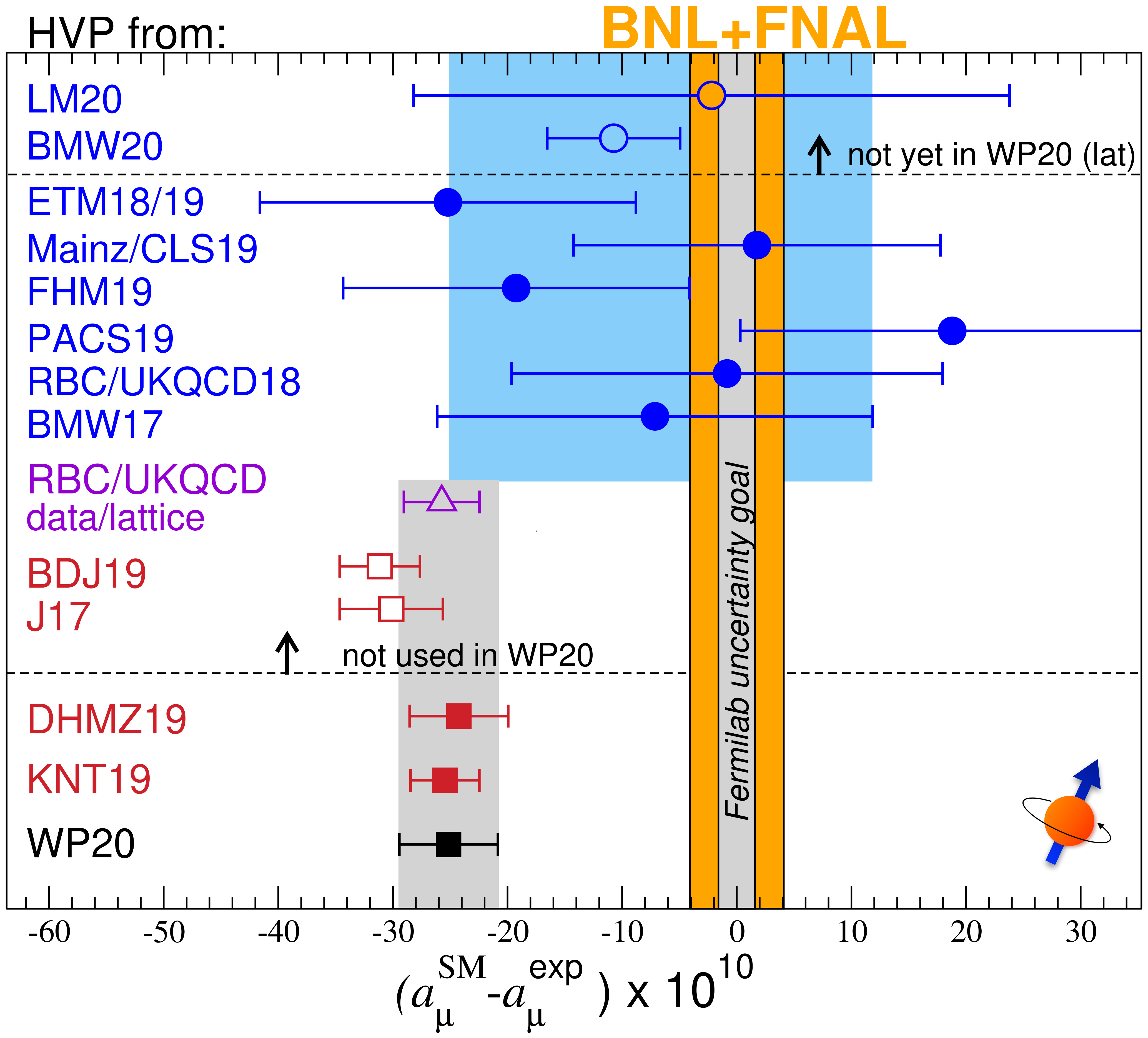
$$a_\mu^{\rm SM} < a_\mu^{\rm HVP} + \left[a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HLbL}\right]$$

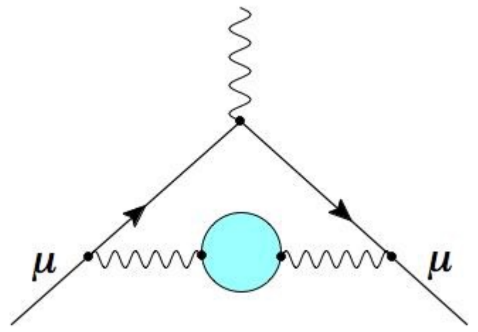
Lattice QCD + QED

hybrid: combine data & lattice

data driven

+ unitarity/analyticity constraints





HVP: data-driven

In 2020 WP:

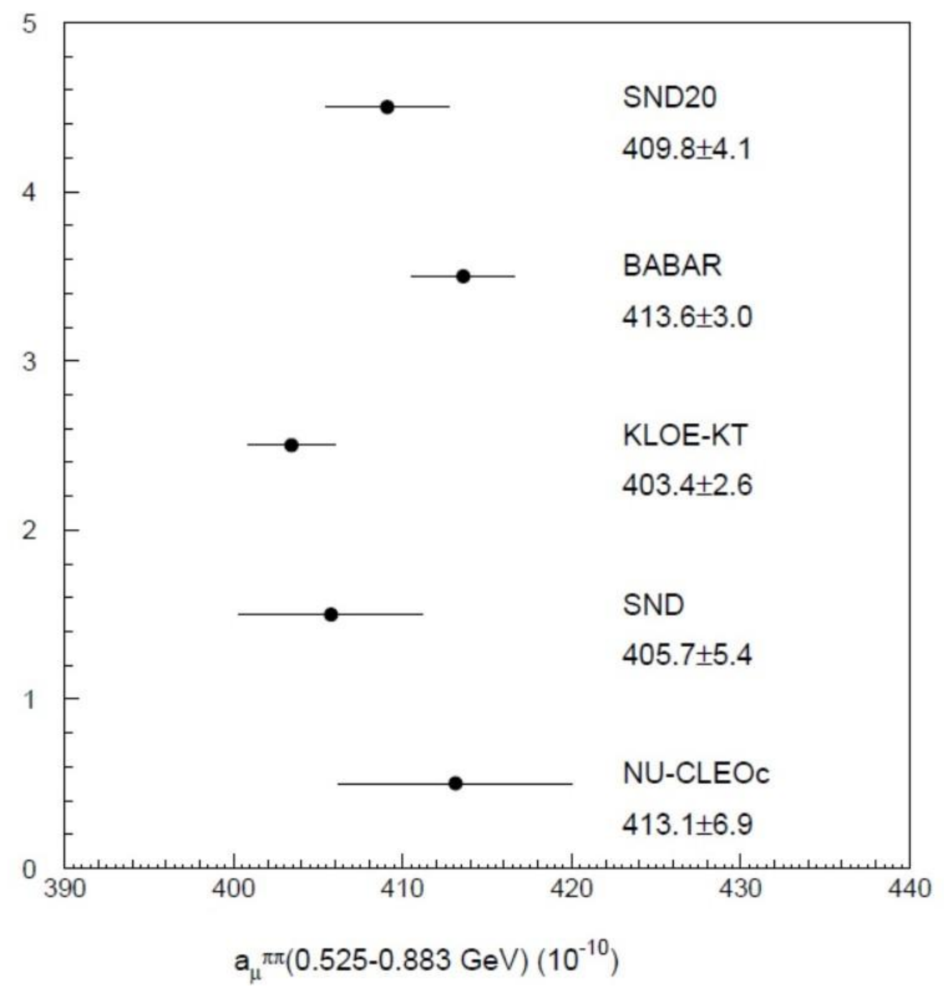
Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs
- include correlations between systematic errors
- cross checks from unitarity & analyticity constraints [Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections [Campanario et al, 2019

$$a_\mu^{\rm HVP,LO} = 693.1\,(2.8)_{\rm exp}\,(0.7)_{\rm DV+pQCD}\,(2.8)_{\rm BaBar-KLOE} \times 10^{-10}$$

$$= 693.1\,(4.0) \times 10^{-10}$$

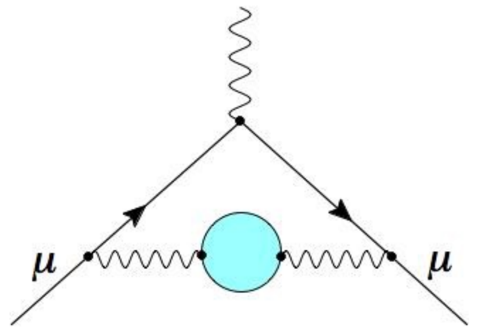
[M. Davier @ KEK workshop]



Ongoing work:

- BaBar: new analysis of large (7×) data set in $\pi\pi$ channel (1-2 years), also $\pi\pi\pi$, other channels
- SND: new results for $\pi\pi$ channel, other channels in progress
- CMD-3: ongoing analyses for $\pi\pi$ and other channels
- BESIII: new results in 2021 for $\pi\pi$ channel, continued analysis also for $\pi\pi\pi$, other channels
- Belle II: will have high-statistics data for low-energy cross sections.
- Need blind analyses to resolve the tensions (esp. for $\pi\pi$ channel)
- Developing NNLO Monte Carlo generators (STRONG 2020 workshop next week https://agenda.infn.it/event/28089/)

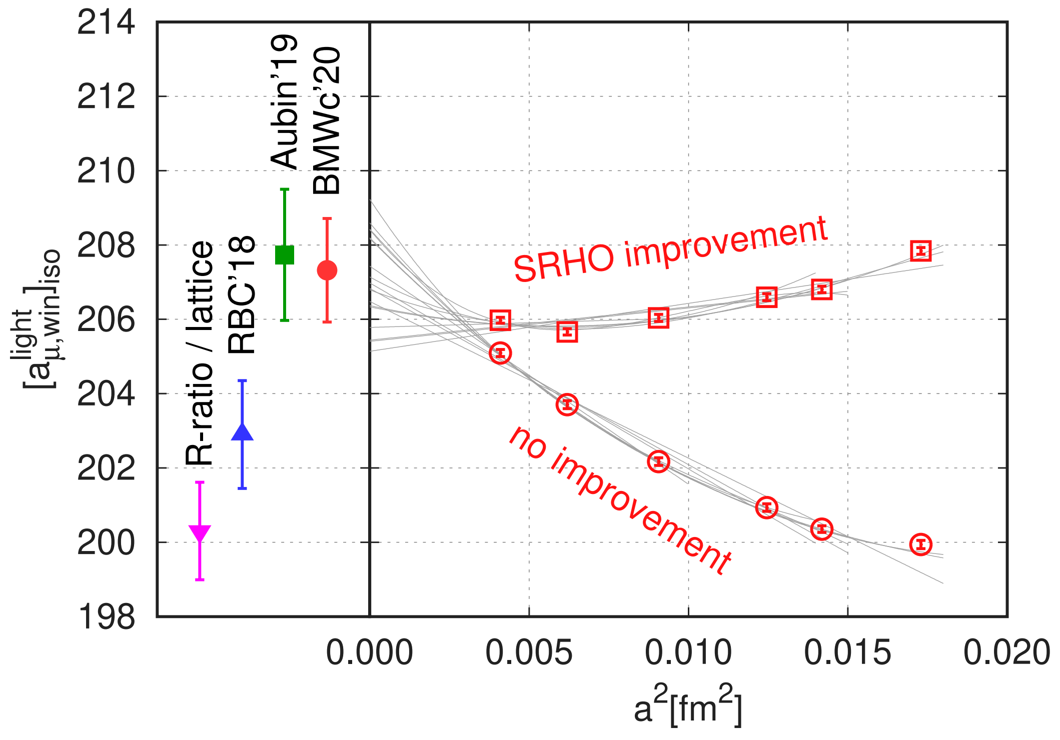
Data-driven evaluations of HVP with $\sim 0.3\,\%$ feasible by ~2025, if tensions between experiments are resolved.



HVP: lattice

In 2020 WP:

- Lattice HVP average at $2.6\,\%$ total uncertainty: $a_\mu^{\mathrm{HVP,LO}} = 711.6\,(18.4) \times 10^{10}$
- BMW 20 (published in 2021)
  first LQCD calculation with sub-percent $(0.8\,\%)$ error
  but in tension with data-driven HVP (2.1σ)
- Further tensions for intermediate window:



-3.7σ tension with data-driven evaluation

-2.2σ tension with RBC/UKQCD18

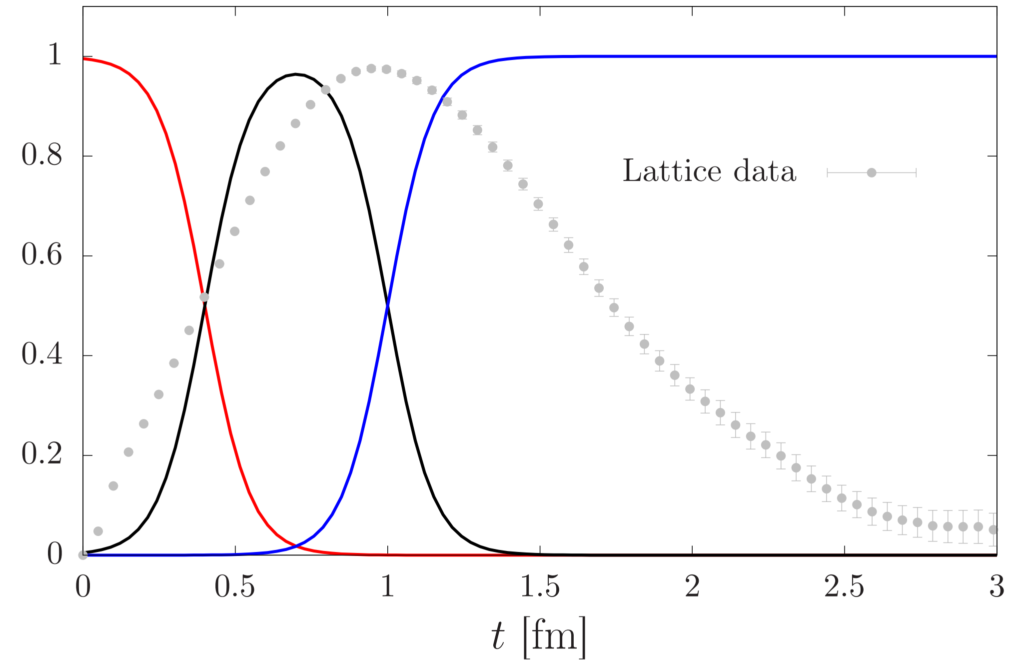
$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

- Use windows in Euclidean time to consider the different time regions separately. [T. Blum et al, arXiv:1801.07224, 2018 PRL]

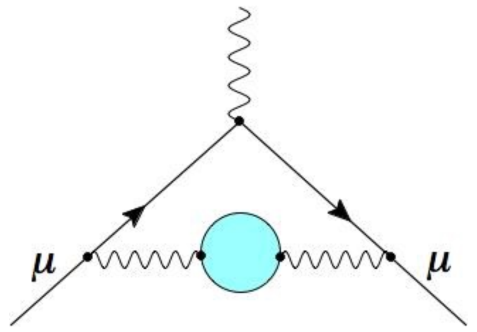
Short Distance (SD)	$t : 0 \to t_0$
Intermediate (W)	$t : t_0 \to t_1$
Long Distance (LD)	$t : t_1 \to \infty$

$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}$

H. Wittig @ Lattice 2021



- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD & using disperse approach:
- Internal cross check:
  Compute each window separately (in continuum, infinite volume limits,…) and combine: $a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$



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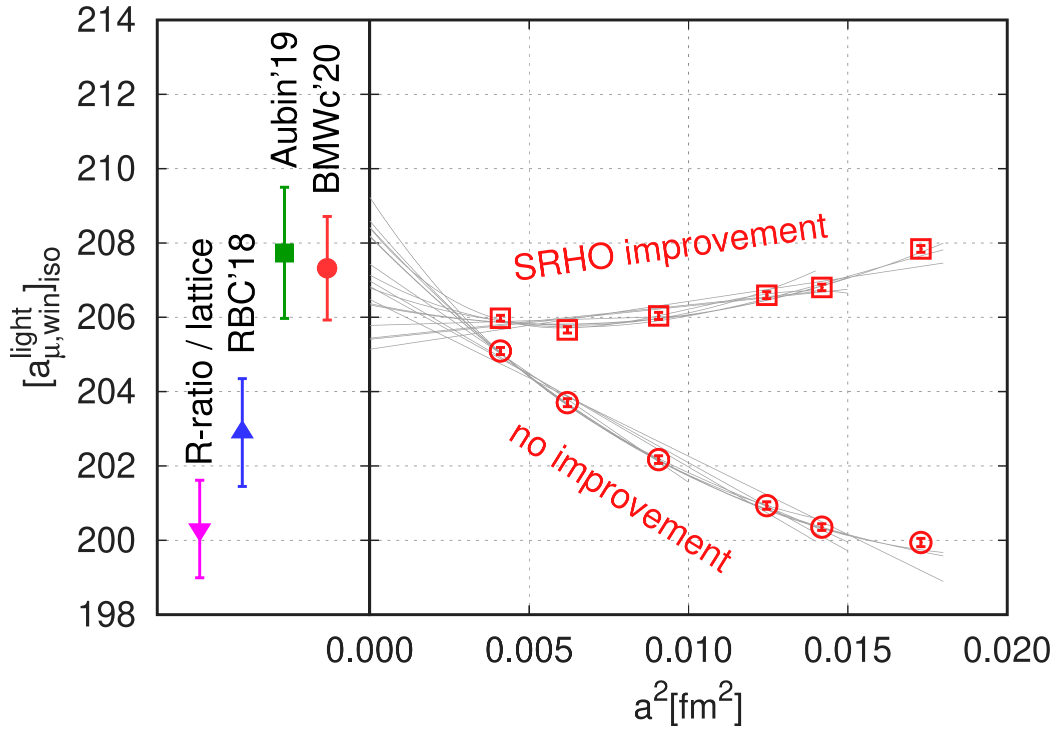
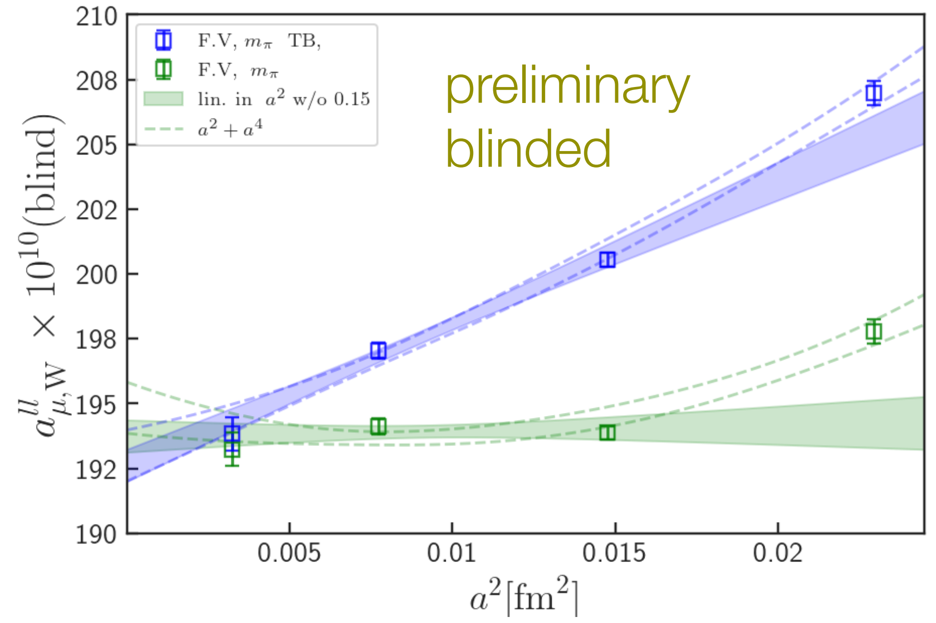


-3.7σ tension with data-driven evaluation
-2.2σ tension with RBC/UKQCD18

Ongoing work:

- Expect new results from RBC/UKQCD and FNAL/MILC (and likely other lattice groups) in the coming months:

S. Lahert (FNAL/MILC)



- need blind analyses (already being done in FNAL/MILC and RBC/UKQCD)
- Including $\pi\pi$ states for refined long-distance computation
  (Mainz, RBC/UKQCD, FNAL/MILC)
- Developing method average for lattice HVP — started at KEK workshop (June 2021), based on detailed comparisons
  - list of sub quantities (and their definitions)
  - common prescription for separating QCD & QED
  - quality criteria for inclusion
- Most groups plan to include smaller lattice spacings to test continuum extrapolations (needs adequate computational resources)

If results are consistent, Lattice HVP (average) with $\lesssim 0.5\,\%$ errors feasible by 2025

Summary and Outlook

- Theory Initiative:
  - WP update ~2023 will include any new available results and a method average for lattice HVP, HLbL
  - Concrete plans for writing WP update (outline, authors,…) @ next workshop
- Progams and plans in place to improve:
  - data-driven HVP $\sim 0.3\,\%$
  - lattice HVP $\lesssim 0.5\,\%$
  - dispersive HLbL and lattice HLbL: $\sim 10\,\%$

  … assuming tensions are resolved.
- If tensions between data-driven HVP and lattice HVP are resolved, SM predictions will likely reach desired precision
- Beyond 2025: MUonE (space-like momentum measurement of $\Delta\alpha$) will provide more information/cross checks.

➠Next workshop of the Muon g-2 Theory Initiative: 5-9 Sep 2022



Thank you!





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Thank you!

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SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

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Appendix

Updated WP Summary Table

Contribution	Value $\times 10^{11}$	References
Experimental average (E989+E821)	116592061(41)	Phys.Rev.Lett. 124, 141801
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, uds)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)	

website: https://muon-gm2-theory.illinois.edu

Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

QED

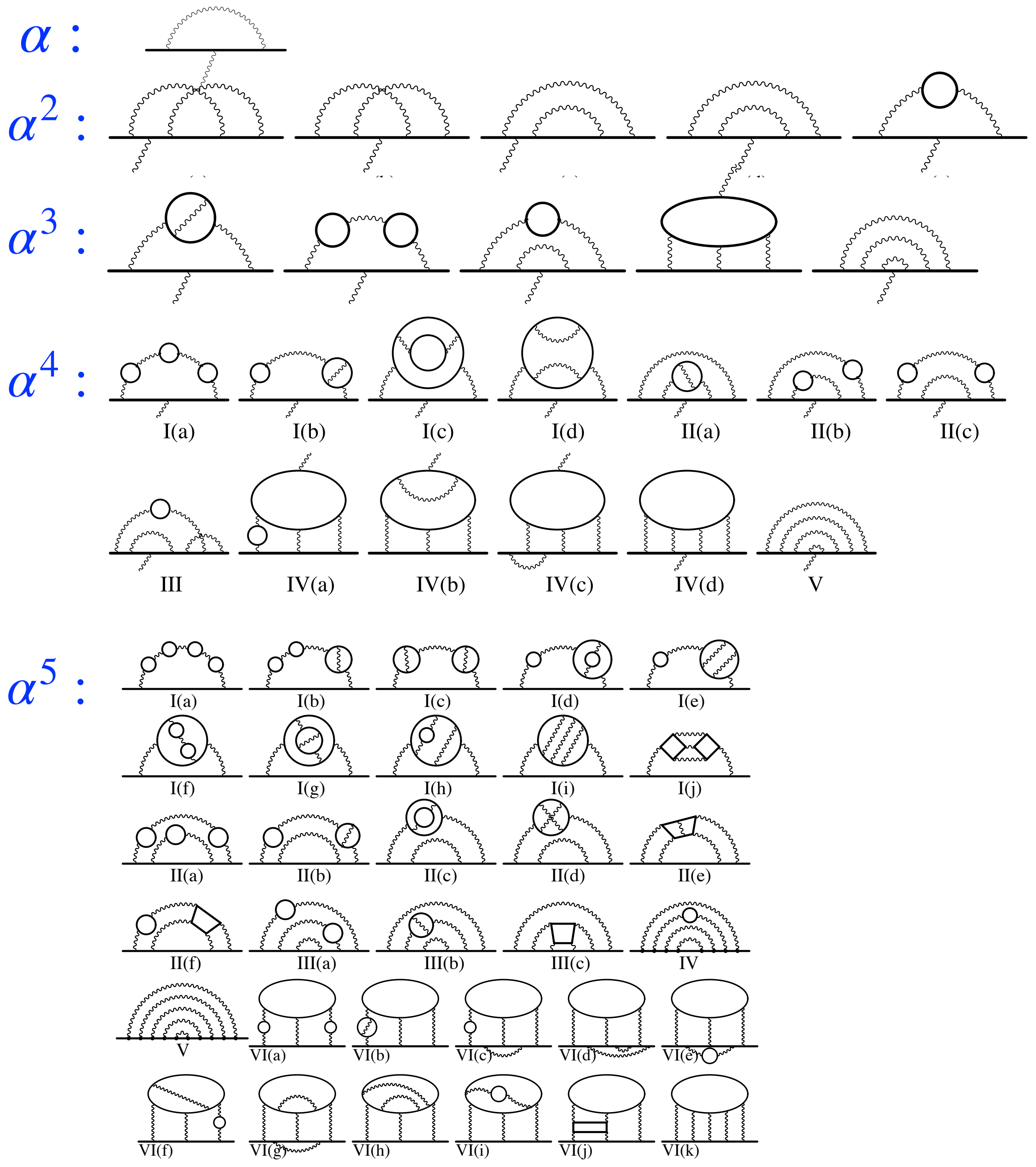
$$a_\mu(\text{QED}) = A_1 + A_2\left(\frac{m_\mu}{m_e}\right) + A_2\left(\frac{m_\mu}{m_\tau}\right) + A_3\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)$$

$$A_i = \sum_{n=0}\left(\frac{\alpha}{\pi}\right)^n A_i^{2n}$$

n	# of diagrams	Contribution x 10^{11}
1	1	116140973.32
2	7	413 217.63
3	71	30141.90
4	891	381.00
5	12672	5.08

$$a_\mu(\text{QED}) = 116\,584\,718.9\,(1) \times 10^{-11}$$

[T. Aoyama et al, arXiv:1205.5370, PRL;
T. Aoyama, T. Kinoshita, M. Nio, Atoms 7 (1) (2019) 28]

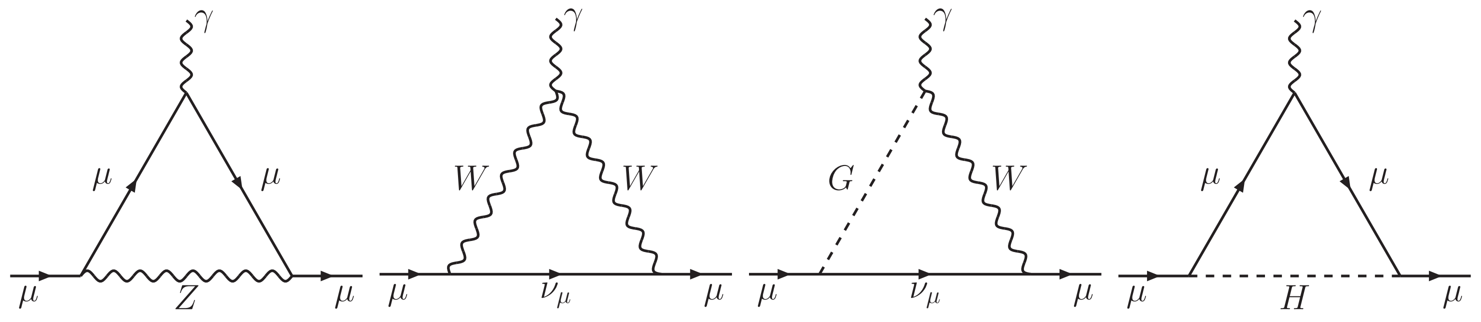


Muon g-2: SM contributions

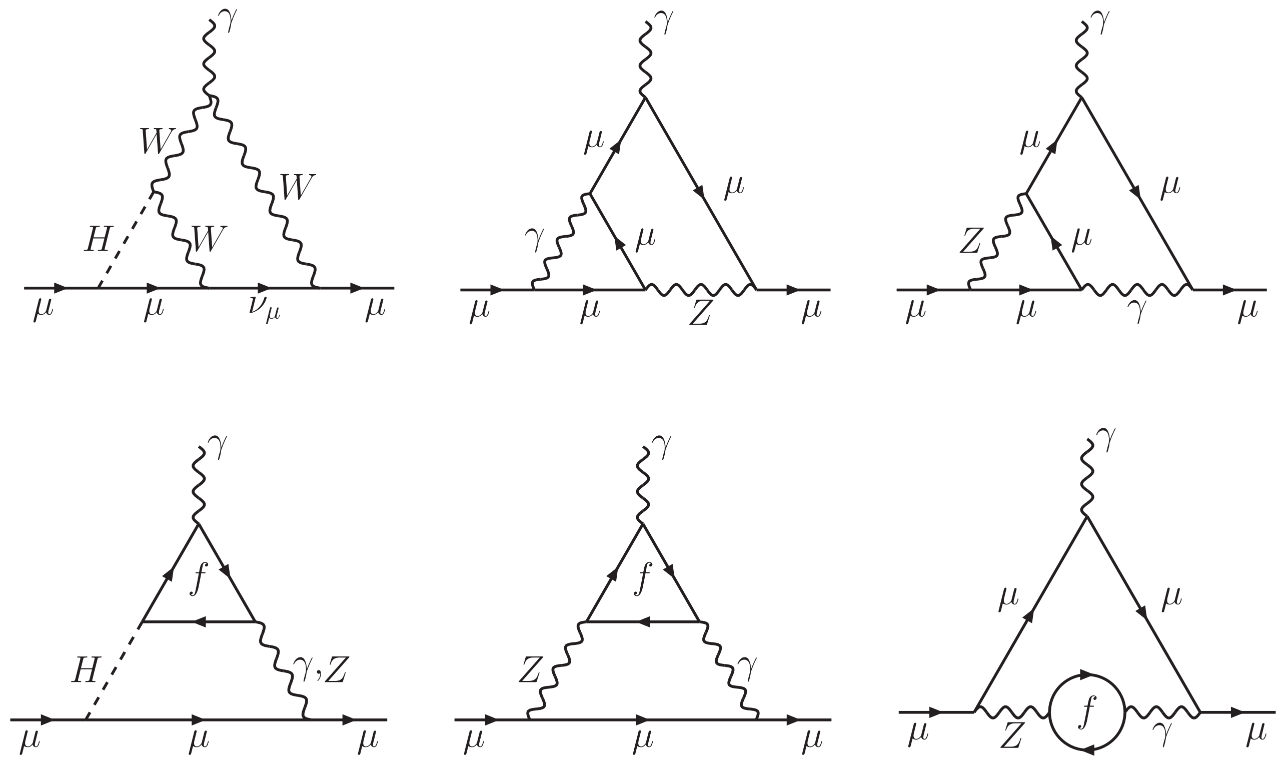
$$a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})$$

Electroweak
(contributions from W,Z,H,.. bosons)

1-loop



2-loop



$$a_\mu(\mathrm{EW}) = 153.6\,(1.0) \times 10^{-11}$$

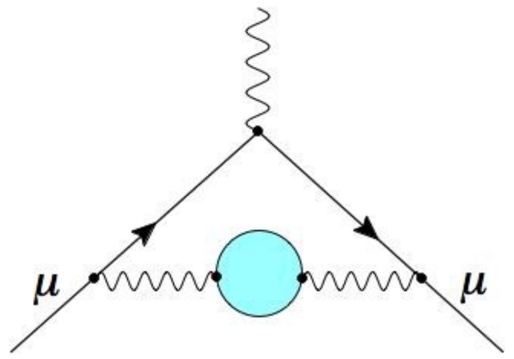
[A. Czarnecki et al, hep-ph/0212229, PRD;
C. Gnendinger et al, arXiv:1306.5546, PRD]

Muon g-2: SM contributions

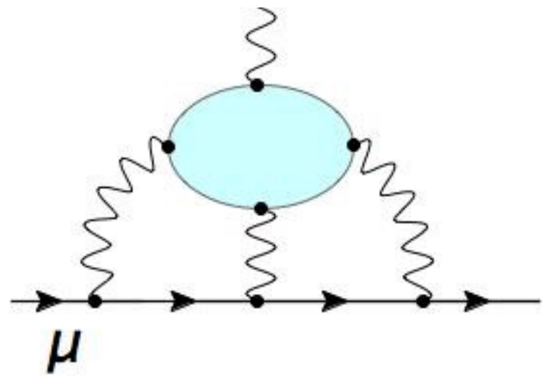
$$a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})$$

leading hadronic

α^2



α^3

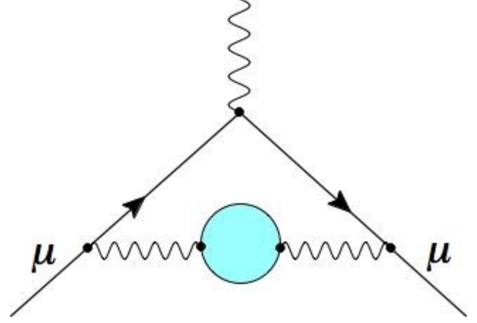


✦ The hadronic contributions are written as:

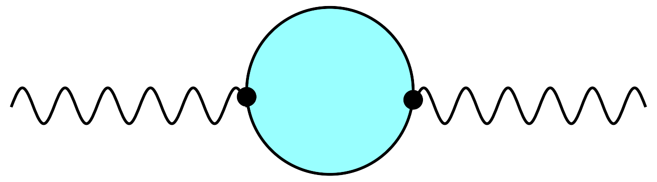
$$a_\ell(\mathrm{hadronic}) = a_\ell^{\mathrm{HVP,\,LO}} + a_\ell^{\mathrm{HVP,\,NLO}} + a_\ell^{\mathrm{HVP,\,NNLO}} + \ldots$$
$$+ a_\ell^{\mathrm{HLbL}} + a_\ell^{\mathrm{HLbL,\,NLO}} + \ldots$$

α^2 α^3 α^4

$\sim \quad 10^{-7}$



Hadronic vacuum polarization



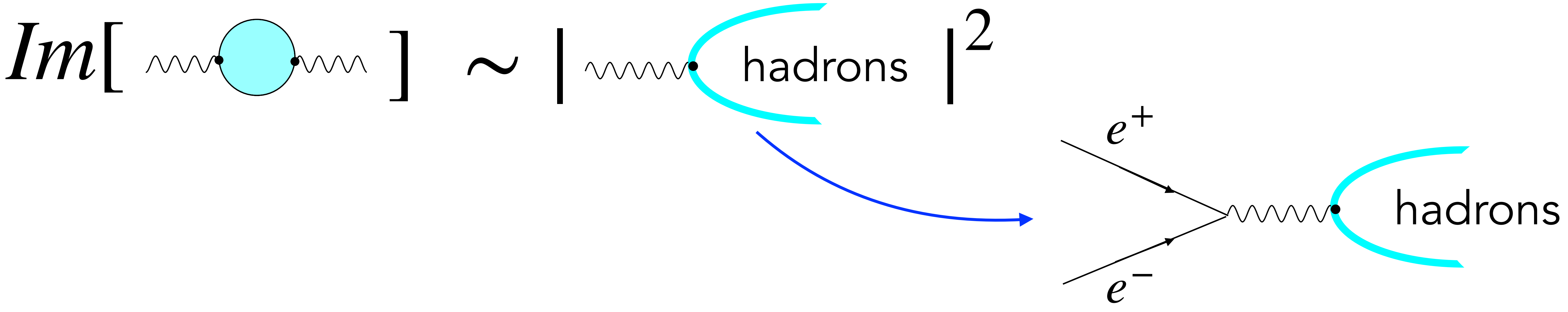
$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

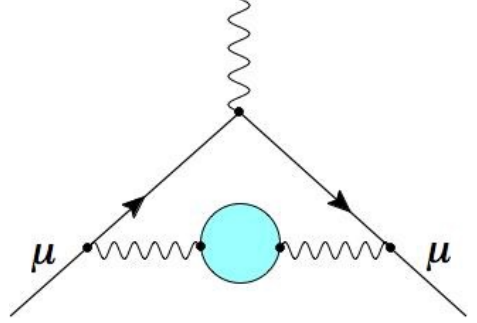
$$\Pi_{\mu\nu} = \int d^4x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

Leading order HVP correction:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

- Use optical theorem and dispersion relation to rewrite the integral in terms of the hadronic e^+e^- cross section:





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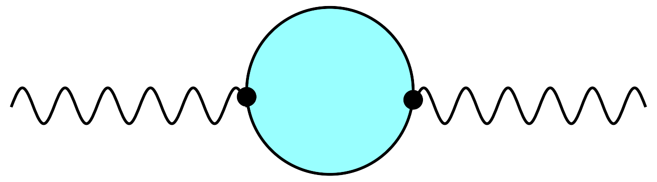

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Leading order HVP correction: $$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

- Use optical theorem and dispersion relation to rewrite the integral in terms of the hadronic e^+e^- cross section:

$$a_\mu^{\mathrm{HVP,LO}} = \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\mathrm{exp}}(s)$$

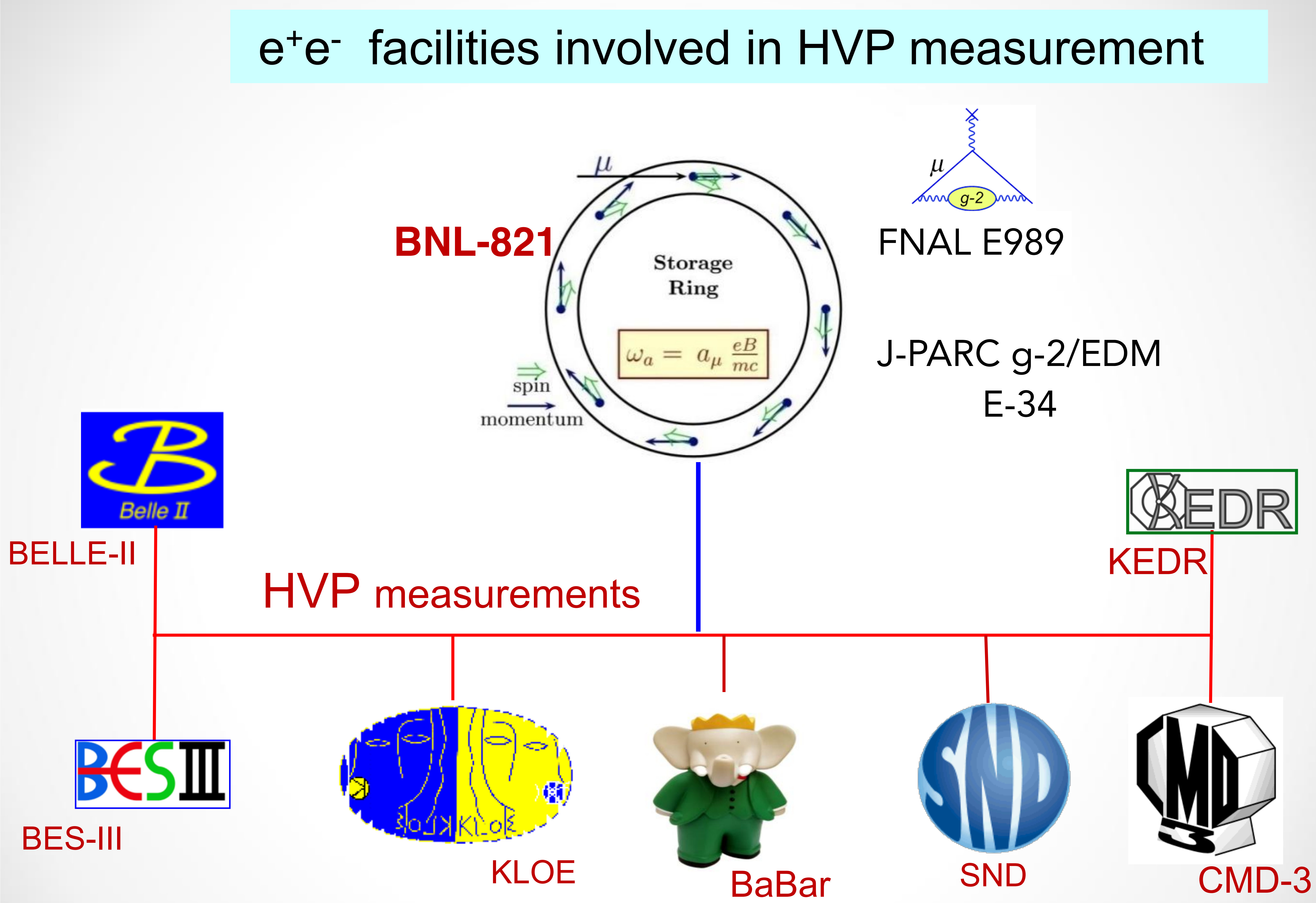
Dominant contributions from low energies

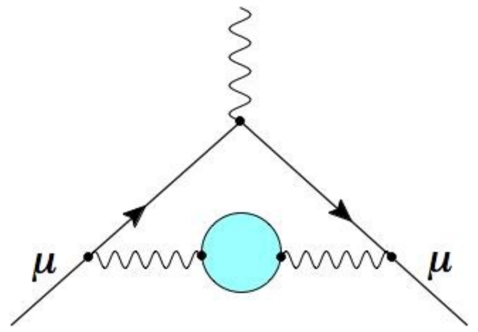
$\pi^+\pi^-$ channel: 73% of total

- Use direct integration method, summing up cross sections for all possible hadronic channels up to ~ 2 GeV

Experimental Inputs to HVP

S. Serednyakov (for SND) @ HVP KEK workshop





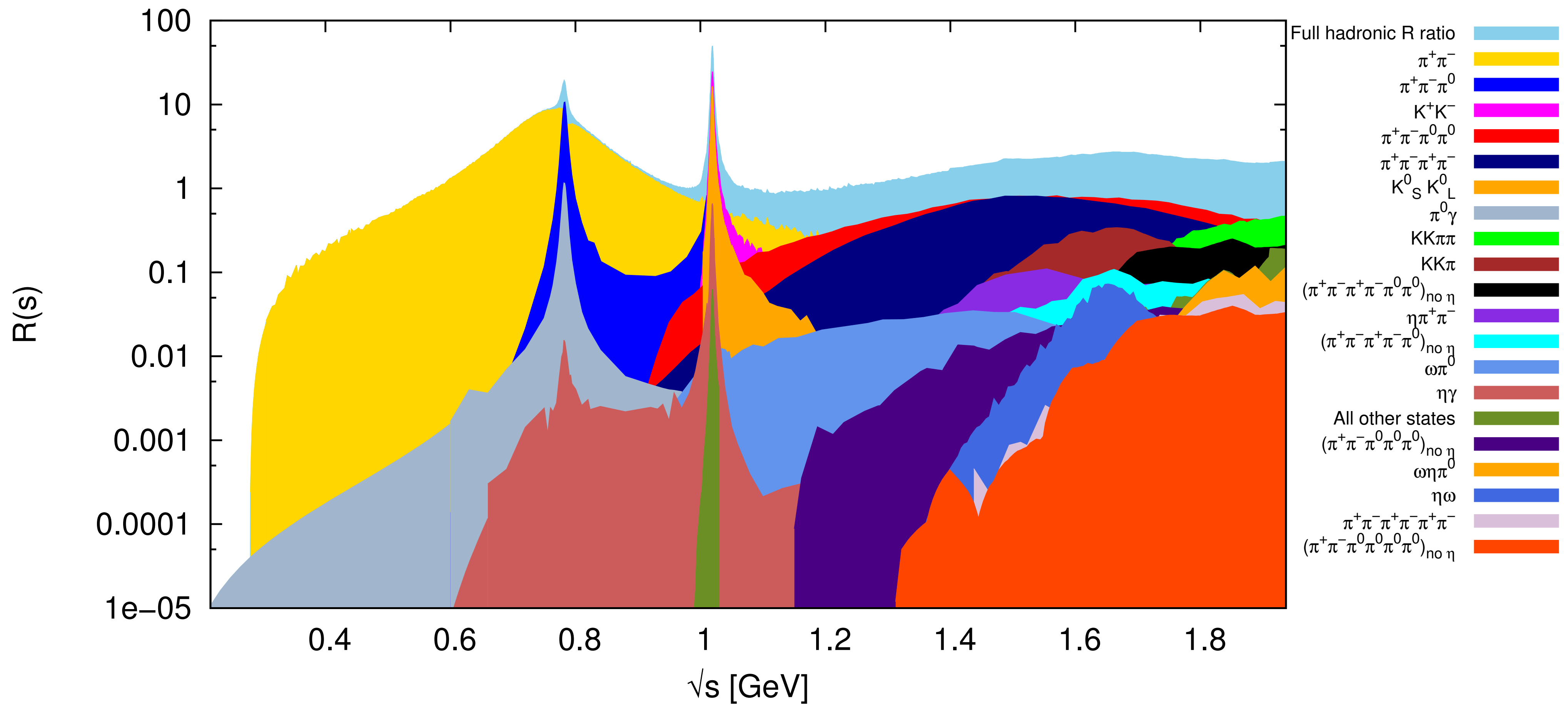
HVP: data-driven

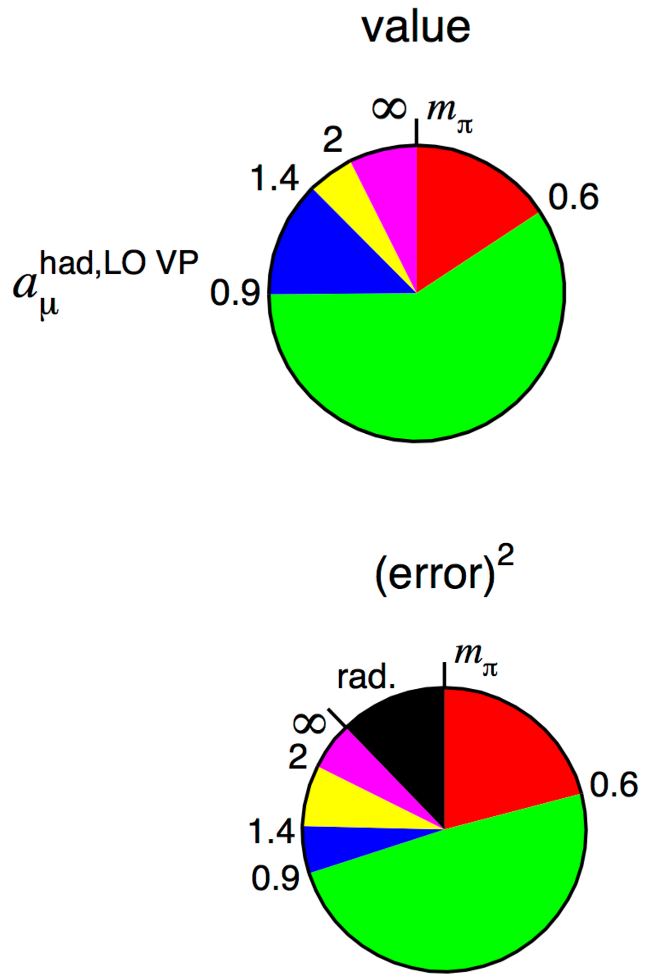
Z. Zhang for DHMZ @ INT g-2 workshop

[M. Davier et al, arXiv:1908.00921]

Channel	$a_\mu^{\text{had, LO}}$ $[10^{-10}]$
$\pi^0\gamma$	$4.29 \pm 0.06 \pm 0.04 \pm 0.07$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$
$\pi^+\pi^-$	$507.80 \pm 0.83 \pm 3.19 \pm 0.60$
$\pi^+\pi^-\pi^0$	$46.20 \pm 0.40 \pm 1.10 \pm 0.86$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$
$\omega(\pi\pi)^0$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$
ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$
$K\overline{K}3\pi$ (estimate)	$-0.02 \pm 0.01 \pm 0.01 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$
J/ψ (BW integral)	6.28 ± 0.07
$\psi(2S)$ (BW integral)	1.57 ± 0.03
R data $[3.7 - 5.0]$ GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$
R_{QCD} $[1.8 - 3.7$ GeV$]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\text{dual}}$
R_{QCD} $[5.0 - 9.3$ GeV$]_{udsc}$	6.86 ± 0.04
R_{QCD} $[9.3 - 12.0$ GeV$]_{udscb}$	1.21 ± 0.01
R_{QCD} $[12.0 - 40.0$ GeV$]_{udscb}$	1.64 ± 0.00
R_{QCD} $[> 40.0$ GeV$]_{udscb}$	0.16 ± 0.00
R_{QCD} $[> 40.0$ GeV$]_{t}$	0.00 ± 0.00
Sum	$693.9 \pm 1.0 \pm 3.4 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\text{QCD}}$

[A. Keshavarzi et al, arXiv:1802.02995]

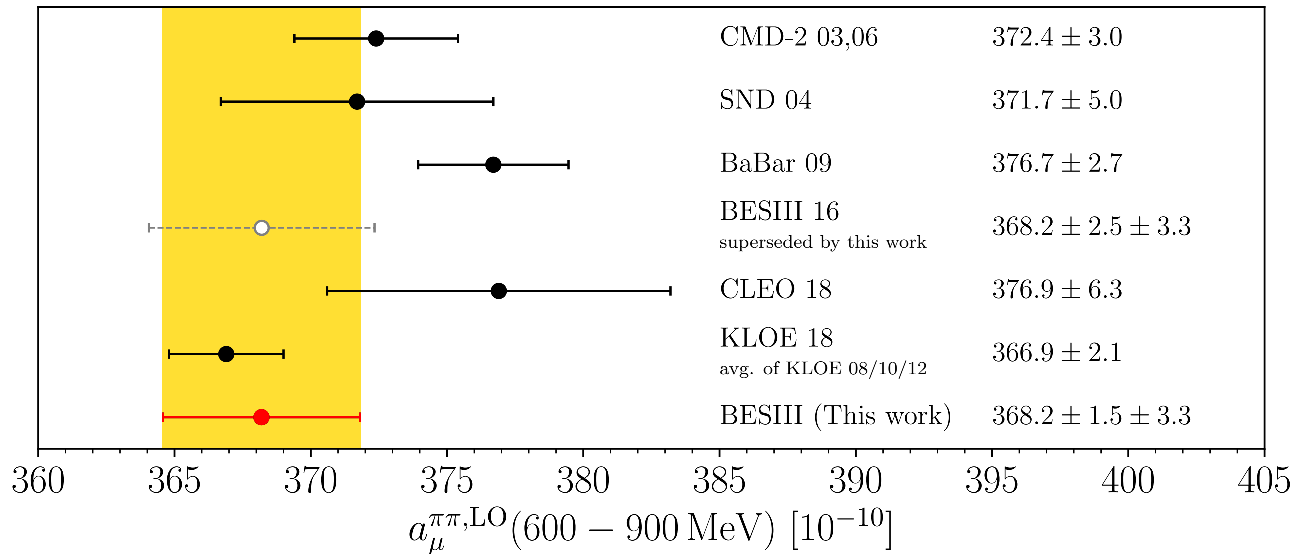




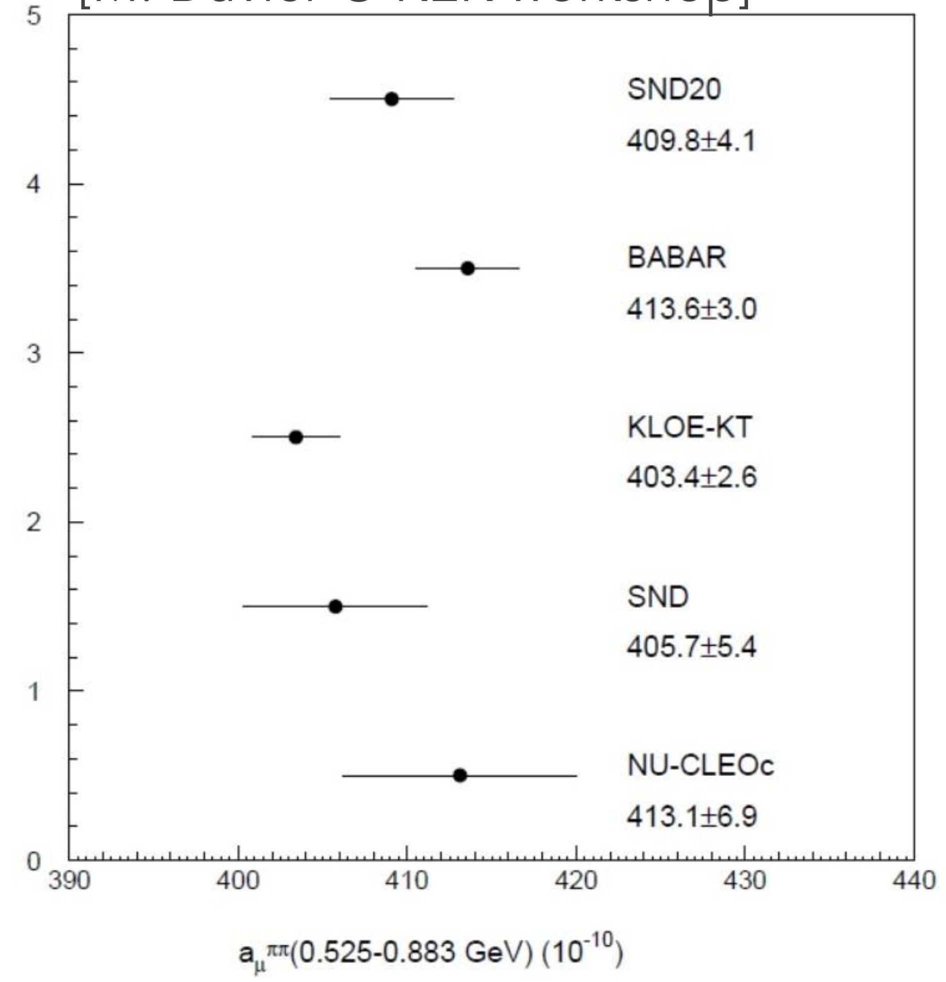
Tensions between BaBar and KLOE data sets:

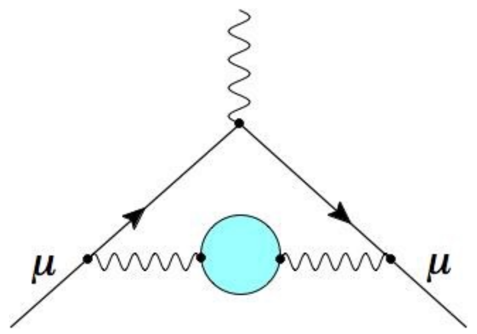
- Cross checks using analyticity and unitarity relating pion form factor to $\pi\pi$ scattering
- Combinations of data sets affected by tensions
  ➠ conservative merging procedure

[M. Ablikim et al (BES III), arXiv:2009.05011]



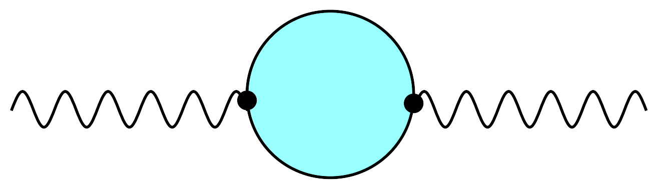
[M. Davier @ KEK workshop]





Lattice HVP: Introduction

[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972; E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]



$\hat{\Pi}(q^2)$

Leading order HVP correction: $$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

- Calculate $a_\mu^{\mathrm{HVP,LO}}$ in Lattice QCD

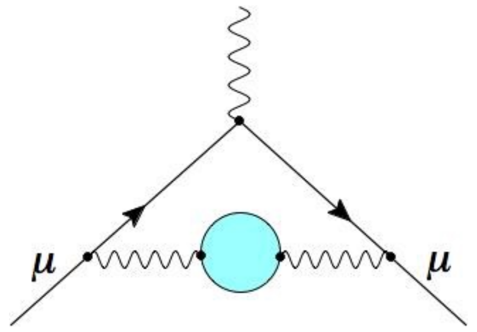
  Compute correlation function: $$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i(x,t) j_i(0,0) \rangle$$

  and $$\hat{\Pi}(Q^2) = 4\pi^2 \int_0^\infty dt\, C(t) \left[t^2 - \frac{4}{Q^2} \sin^2\left(\frac{Qt}{2}\right) \right]$$

  [D. Bernecker and H. Meyer, arXiv:1107.4388, EPJA 2011]

Obtain $a_\mu^{\mathrm{HVP,LO}}$ from an integral over Euclidean time:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

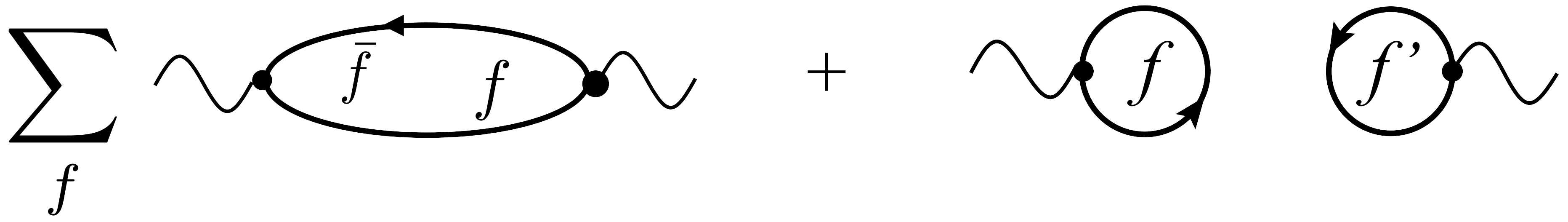


Lattice HVP: Introduction

Calculate a_μ^{HVP} in Lattice QCD:

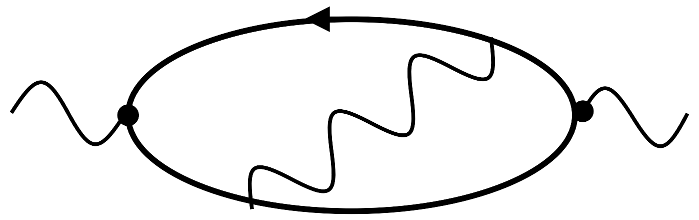
$$a_\mu^{\mathrm{HVP,LO}} = \sum_f a_{\mu,f}^{\mathrm{HVP,LO}} + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$$

- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)
  Note: almost always $m_u = m_d$



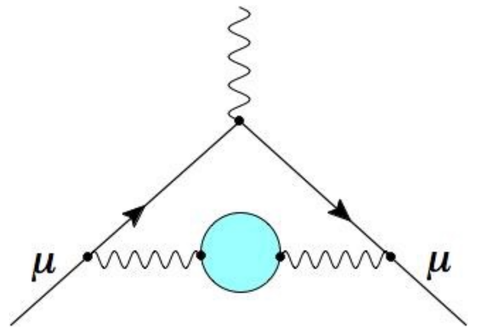
f= ud, s, c, b

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:



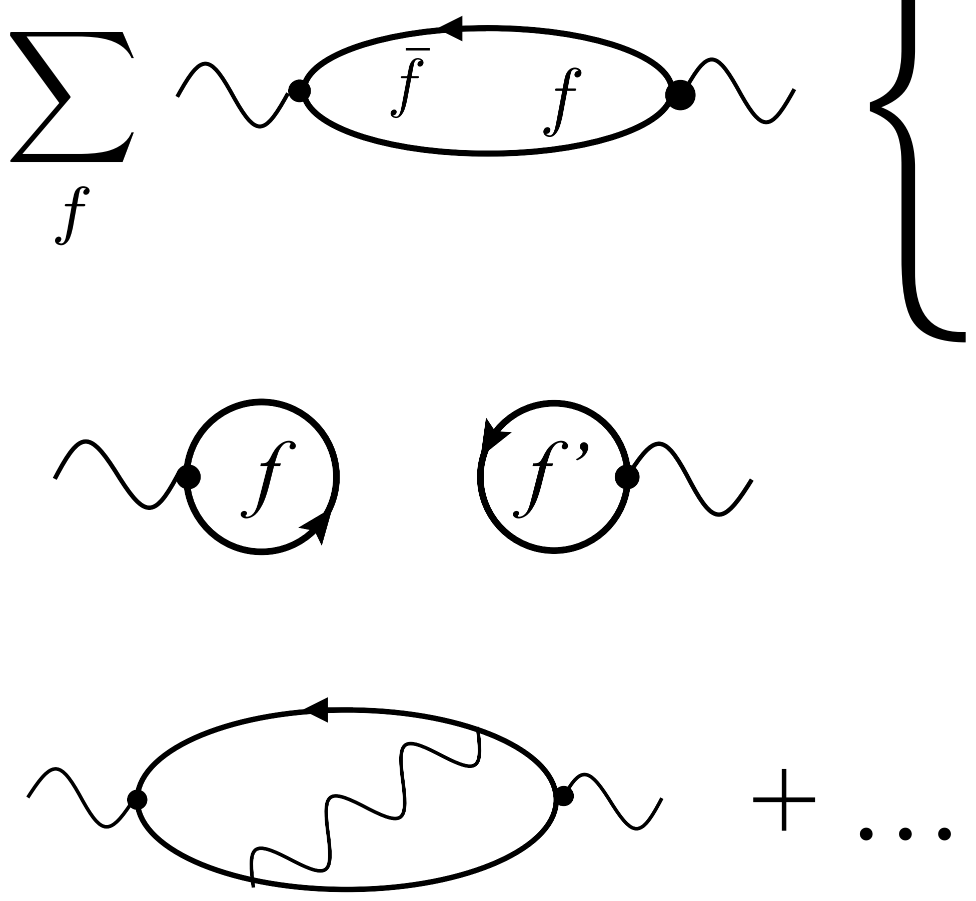
$+ \ldots$

- either perturbatively on isospin symmetric QCD background
- or by using QCD + QED ensembles with $m_u \neq m_d$



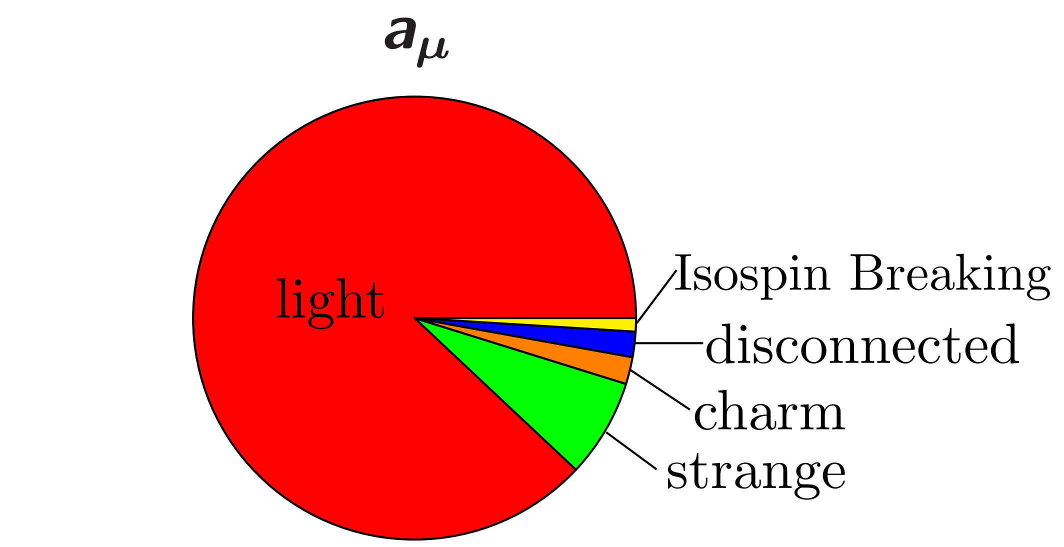
Lattice HVP: Introduction

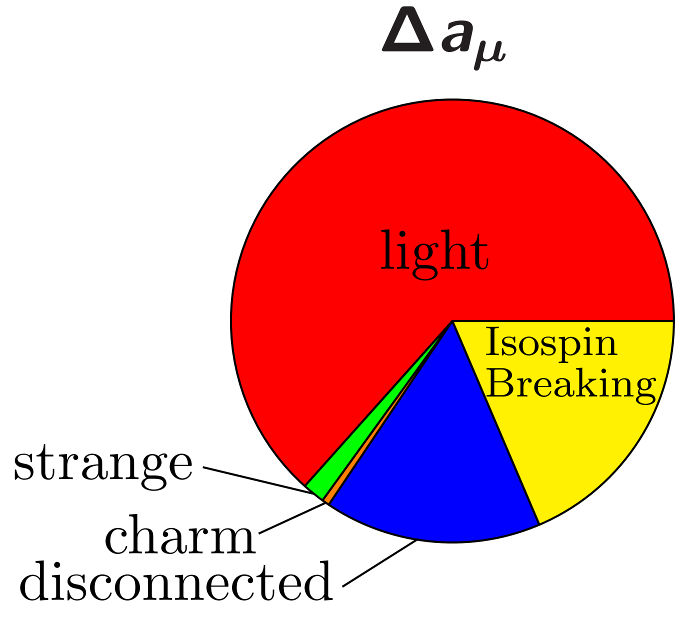
- Target: ~ 0.2% total error



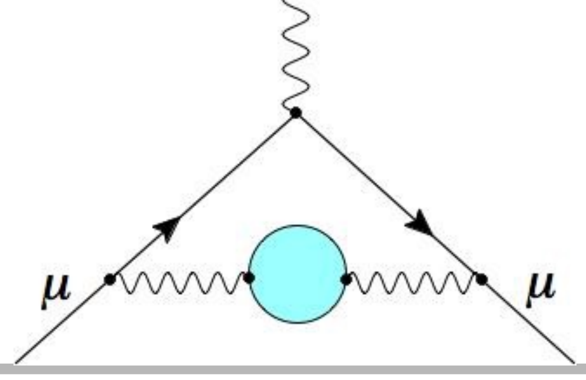
- light-quark connected contribution:
  $a_\mu^{\mathrm{HVP,LO}}(ud)$ ~90% of total
- s,c,b-quark contributions
  $a_\mu^{\mathrm{HVP,LO}}(s,c,b)$ ~8%, 2%, 0.05% of total
- disconnected contribution:
  $a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$ ~2% of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  $\delta a_\mu^{\mathrm{HVP,LO}}$ ~1% of total

V. Gülpers @ Lattice HVP workshop



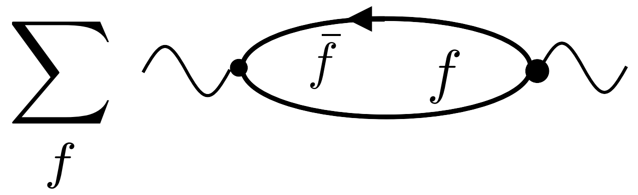


$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}} + \delta a_\mu^{\mathrm{HVP,LO}}$$

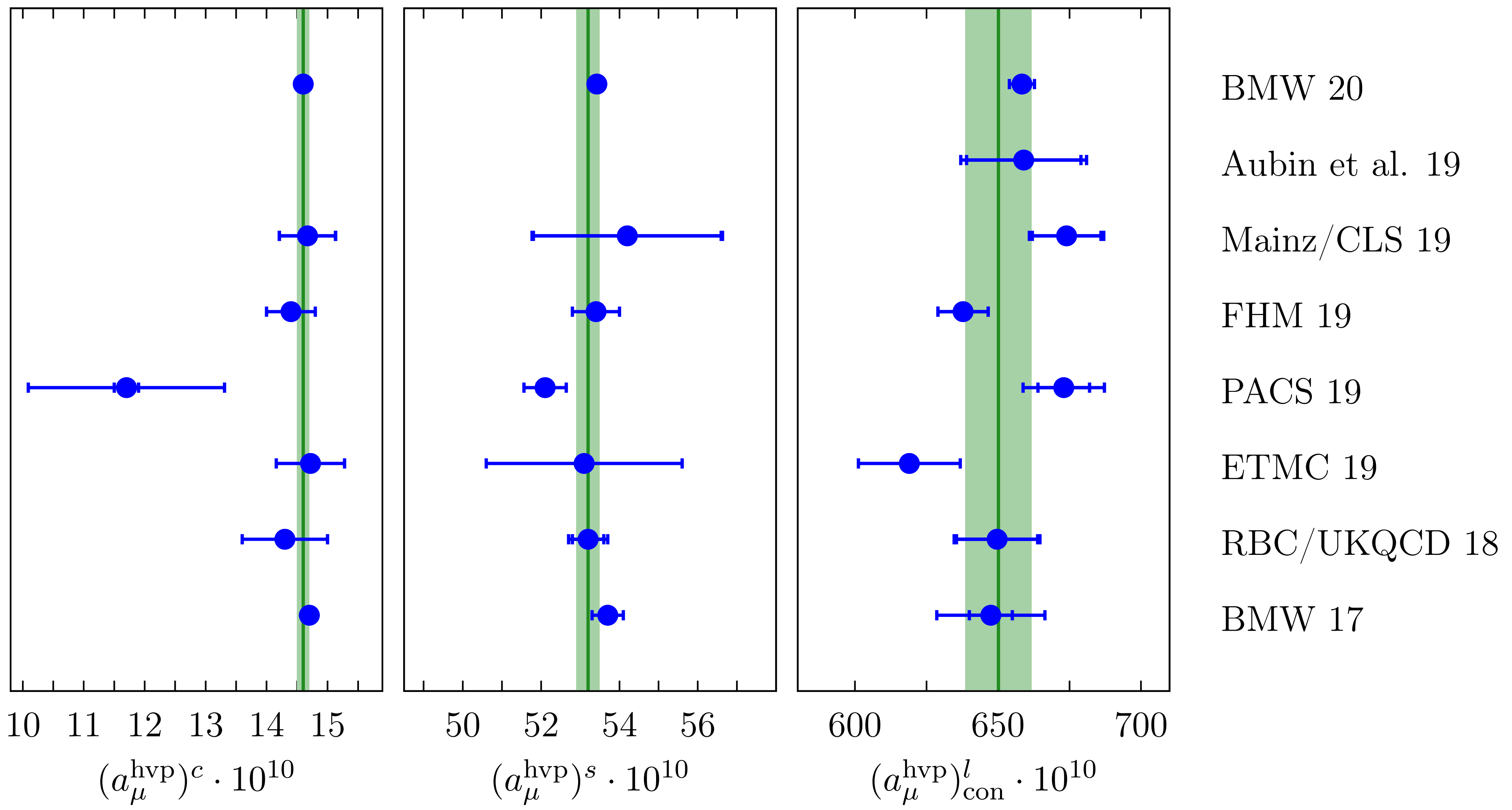


Lattice HVP: Isospin corrections

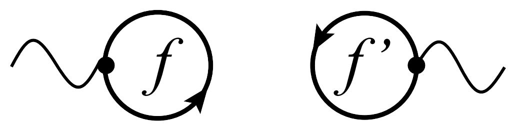
H. Wittig @ Lattice HVP workshop



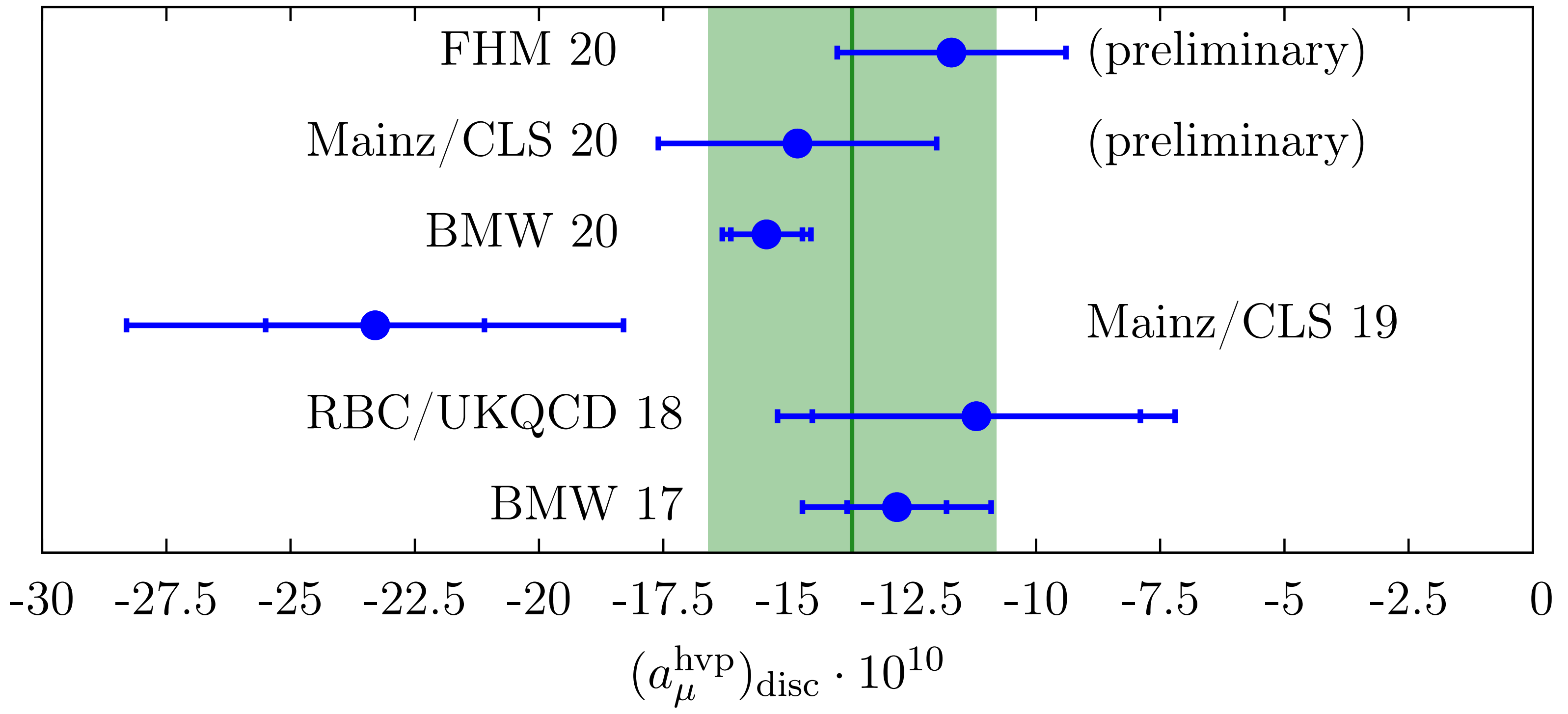
- Charm, strange contributions already well determined.
- Mild tensions for light contribution



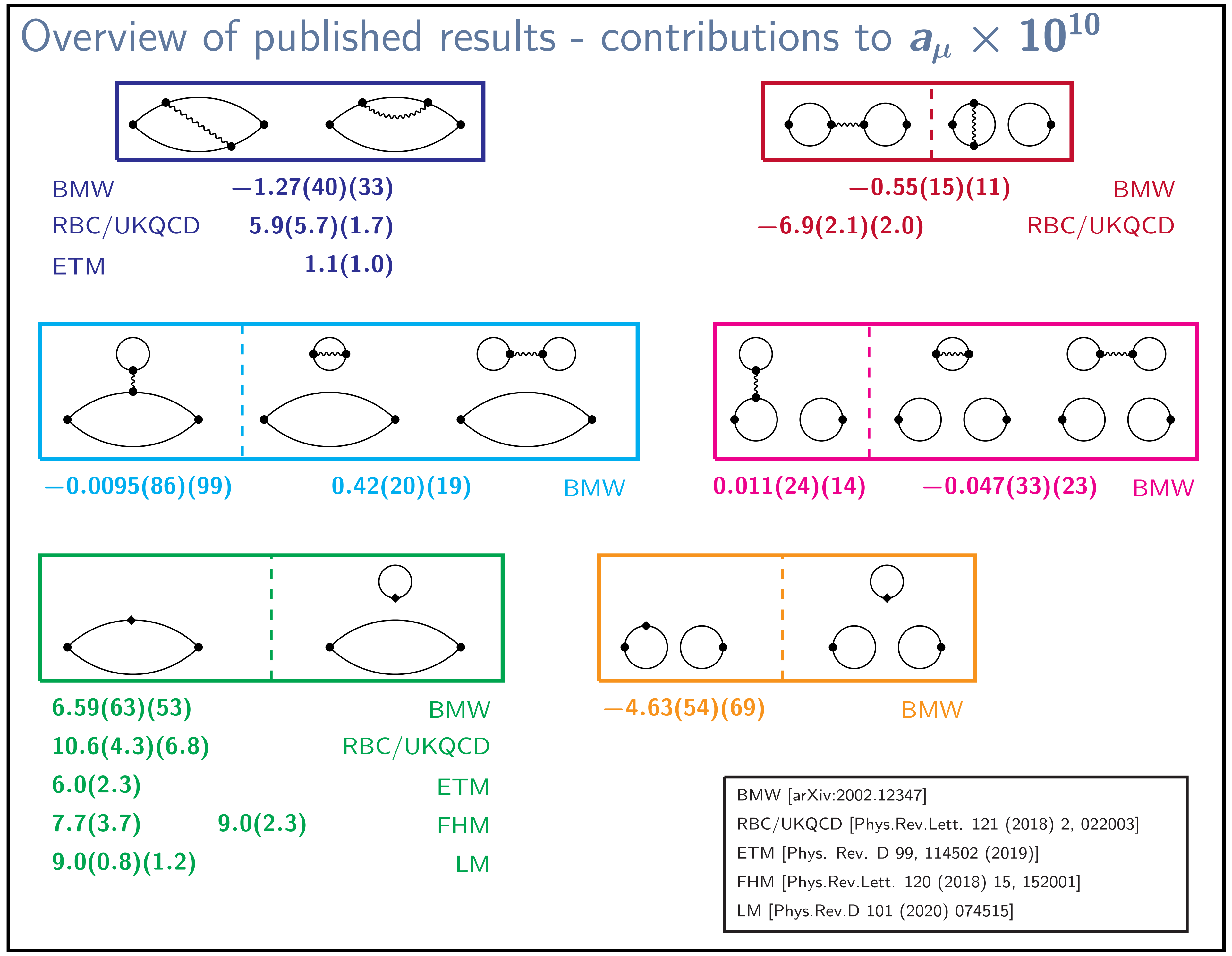
Ongoing efforts by FNAL-HPQCD-MILC RBC/UKQCD, Mainz



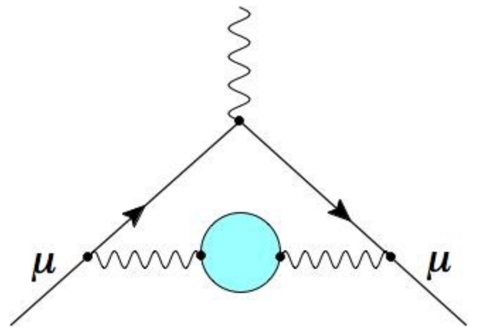
Consistent results with increasing precision



V. Gülpers @ Lattice HVP workshop



- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions: $\delta a_\mu^{\rm IB} \lesssim 1\,\%$



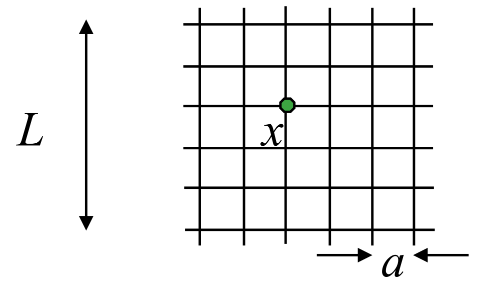
Lattice HVP: Introduction

- Target: ~ 0.2% total error

- Challenges:
  - ✓needs ensembles with (light sea) quark masses at their physical values
  - ✓ finite volume corrections
  - growth of statistical errors at long-distances
  - Continuum extrapolation
  - scale setting
  - disconnected contribution
  - QED and strong isospin breaking corrections ($m_u \neq m_d$)

- Focus on windows in Euclidean times [T. Blum et al, arXiv:1801.07224, 2018 PRL]
  - disentangle systematics/statistics from long distance/FV and discretization effects
    ➠ valuable cross checks
  - intermediate window easy to compute & compare with disperse methods

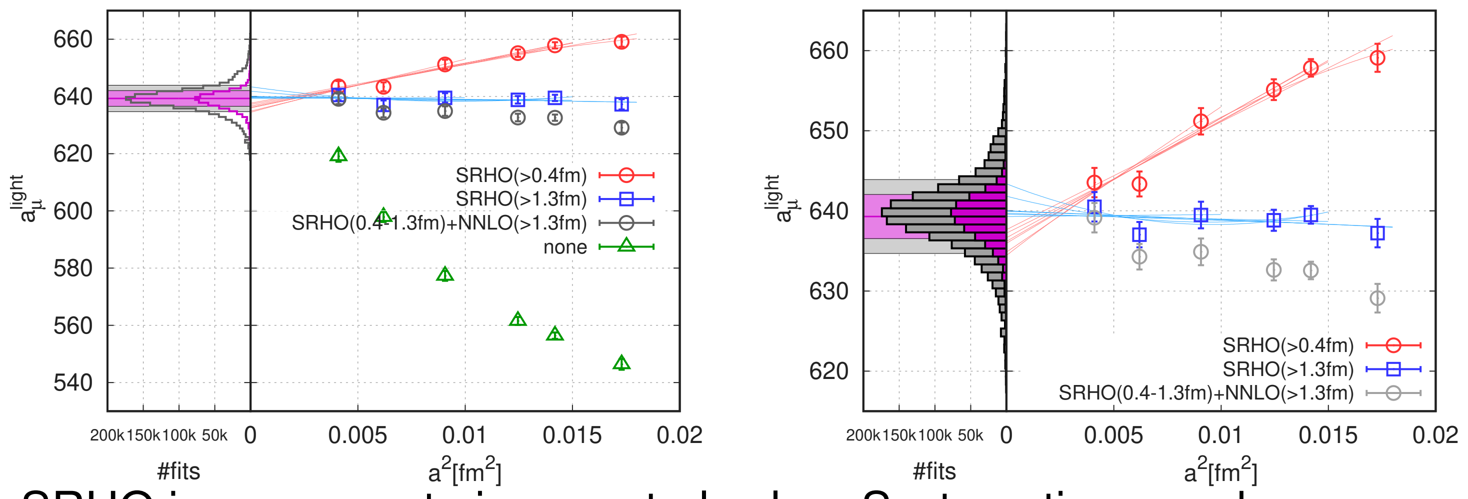
Continuum extrapolation



Kalman Szabo (BMWc) @ Lattice 2021

Taste improvement II

- $a_\mu(a) \to a_\mu(a) - a_\mu^{\rm SRHO}(a) + a_\mu^{\rm RHO}$
- reduces lattice artefact, also makes a^2 dependence linear



SRHO improvement gives central value. Systematic errors by:

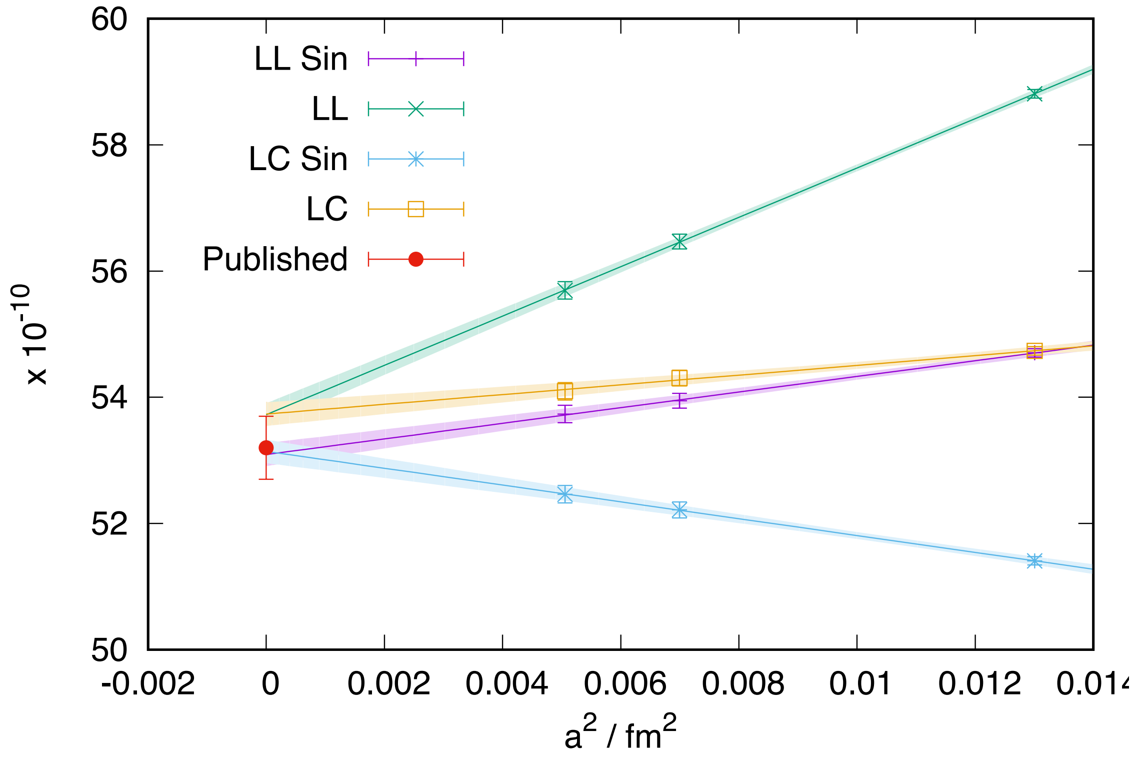
1. change starting point of improvement $t = 0.4 \to 1.3$ fm
2. skip coarse lattices
3. change $\Gamma = 0$ and $\Gamma = 3$
4. replace SRHO by NNLO SXPT above 1.3 fm

- Large taste-breaking effects with BMW set-up ➠ uncorrected data not easily fit to power series, i.e.

1. $A_0 + A_1\left[a^2\right] + A_2\left[a^2\right]^2$
2. $A_0 + A_1\left[a^2\alpha_s^3(\frac{1}{a})\right] + A_2\left[a^2\alpha_s^3(\frac{1}{a})\right]^2$

Christoph Lehner (RBC/UKQCD) @ Lattice 2021

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):

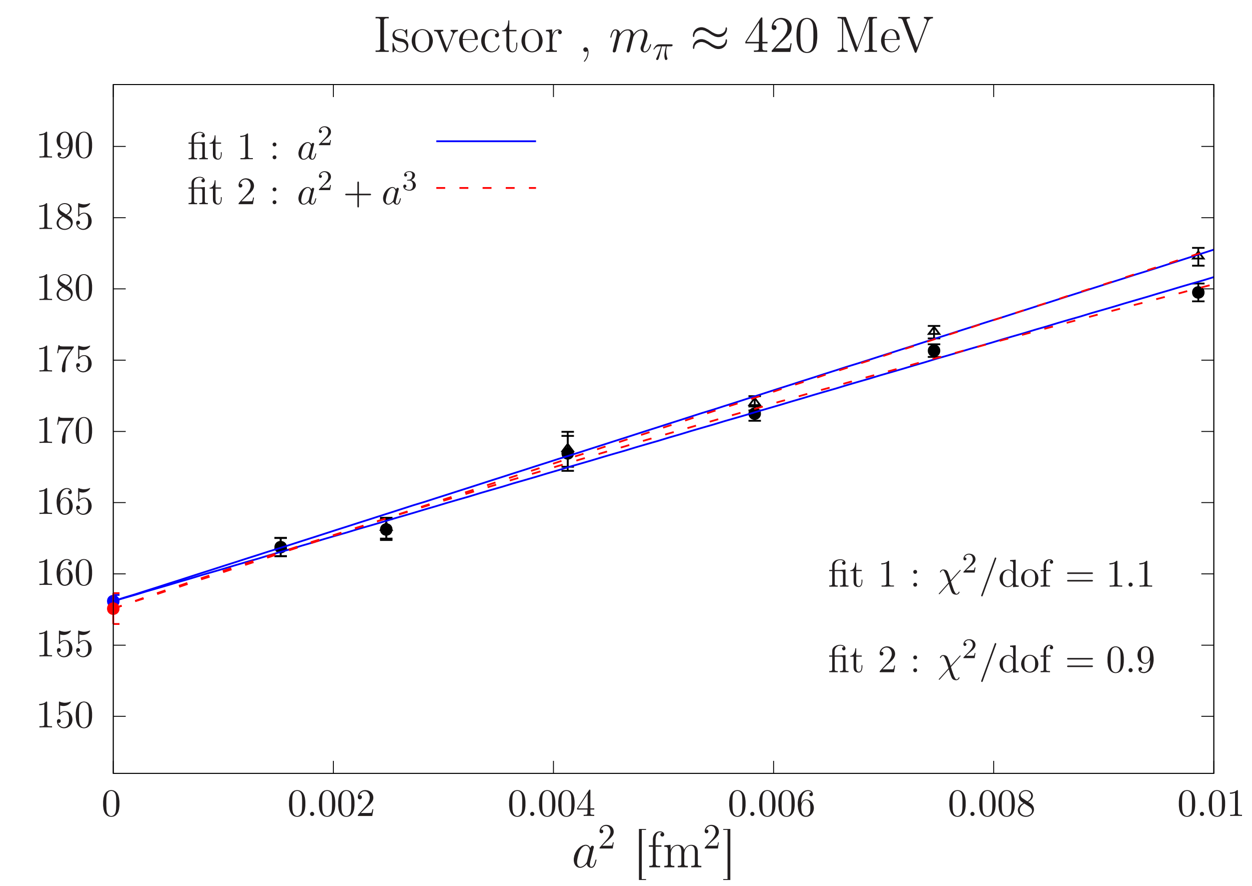


- For light quark use new 96I ensemble at physical pion mass. Data still being generated on Summit in USA and Booster in Germany ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV)

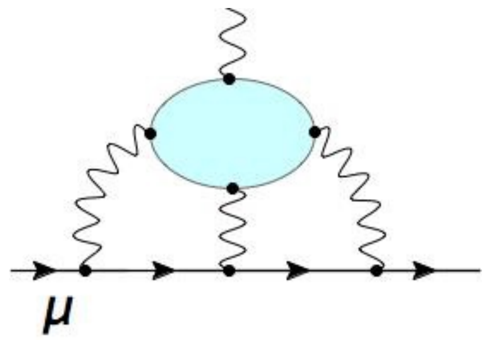
- RBC:
  Currently adding add a third lattice spacing

- Fermilab-HPQCD-MILC:
  planning to add a 5th lattice spacing (0.042 fm).

Hartmut Wittig (Mainz) @ Lattice 2021



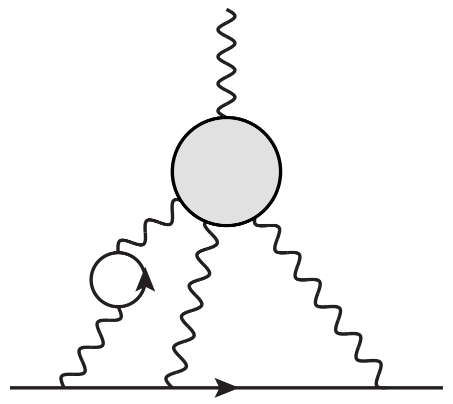
- Mainz and ETMc perform combined chair and continuum extrapolaiton



HLbL: dispersive

Comparison:

Contribution	PdRV(09) [471]	N/JN(09) [472, 573]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3)
tensors	–	–	1.1(1)	}
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)



NLO HLbL contribution:

$$a_\mu^{\text{HLbL,NLO}} = 2\,(1) \times 10^{-11}$$

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.

- a shift in a_μ^{HVP} also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits
  [Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]
  If the shift is due to differences in the low ($\lesssim 2\,\text{GeV}$) energy region, the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.

- A shift in a_μ^{HVP} from low ($\lesssim 2\,\text{GeV}$) energies ➠ $\sigma(e^+e^- \to \pi\pi)$
  must satisfy unitarity & analyticity constraints ➠ $F_\pi^V(s)$
  can be tested with lattice calculations
  [Colangelo, Hoferichter, Stoffer 2021]

Connections

Martin Hoferichter @ Lattice HVP workshop

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) \times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

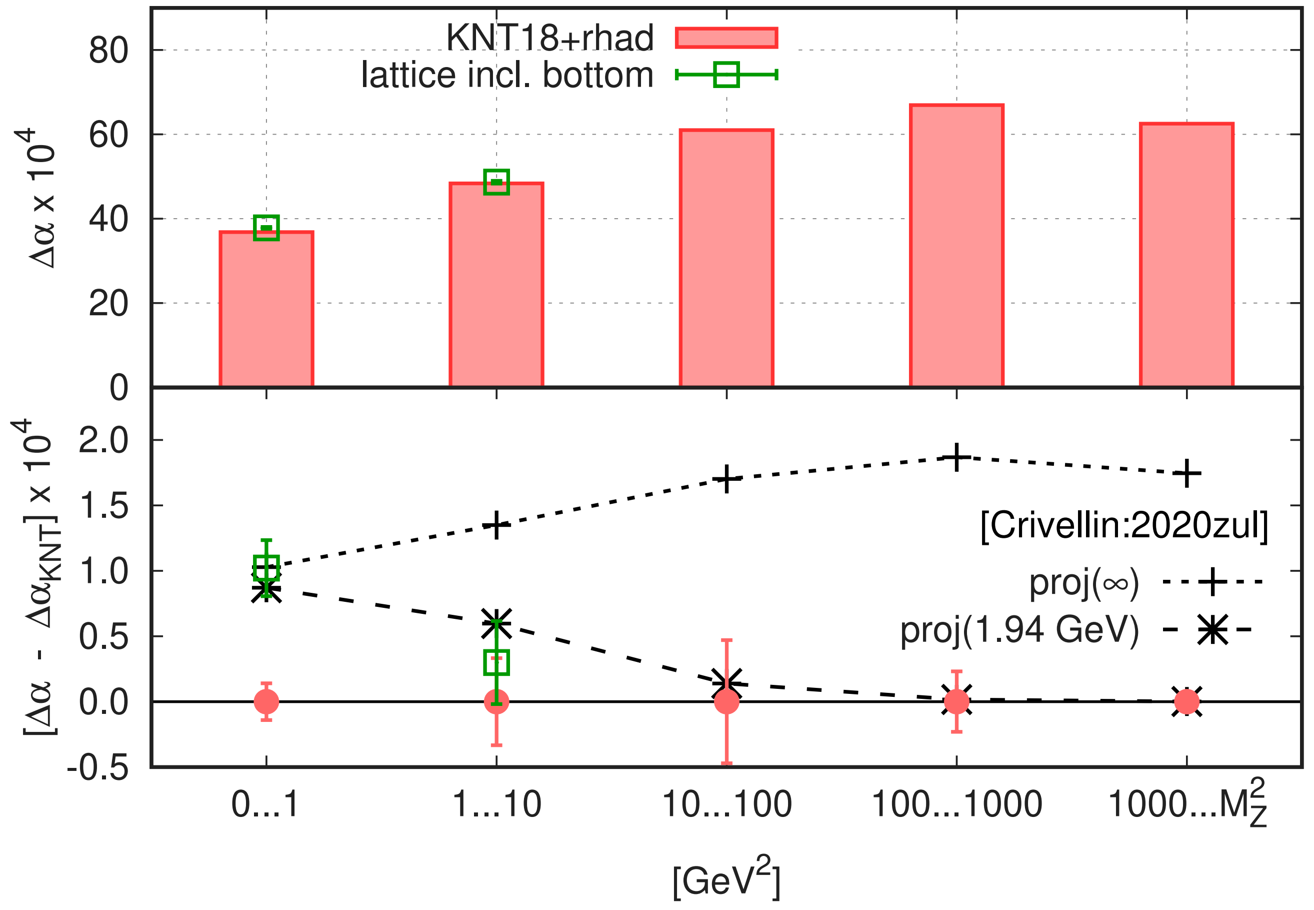
- **Time-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- **Space-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi}\left(\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2)\right)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020

More in talks by M. Passera, B. Malaescu (phenomenology) and K. Miura, T. San José (lattice)

Connections

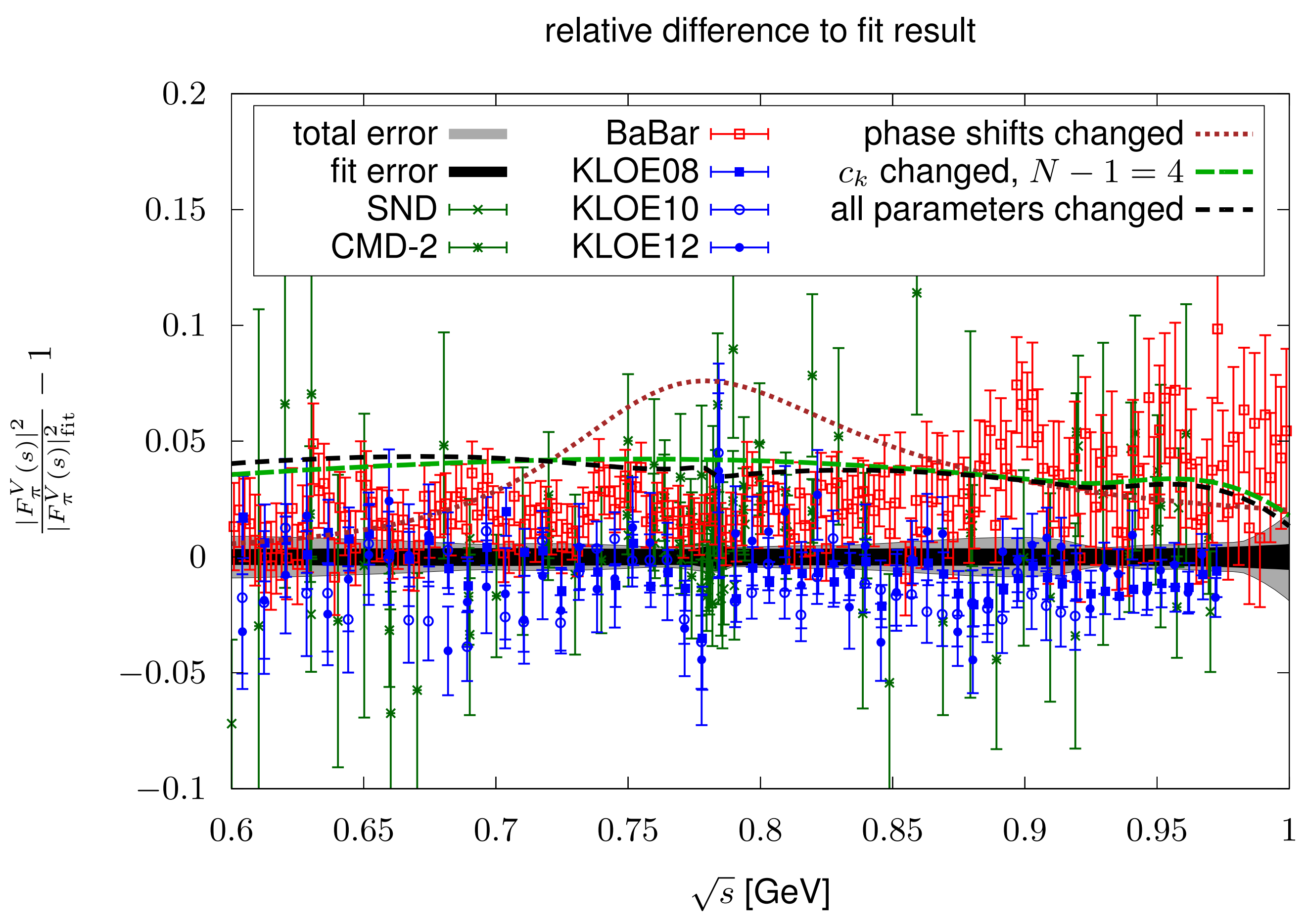
Peter Stoffer @ Lattice HVP workshop

Constraints on the two-pion contribution to HVP arXiv:2010.07943 [hep-ph]

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

- "low-energy" scenario: local changes in cross section of $\sim 8\%$ **around ρ**
- "high-energy" scenario: impact on **pion charge radius** and space-like VFF $\Rightarrow$ chance for **independent lattice-QCD checks**
- requires **factor ~ 3 improvement** over χQCD result: $\langle r_\pi^2 \rangle = 0.433(9)(13)\,\text{fm}^2$

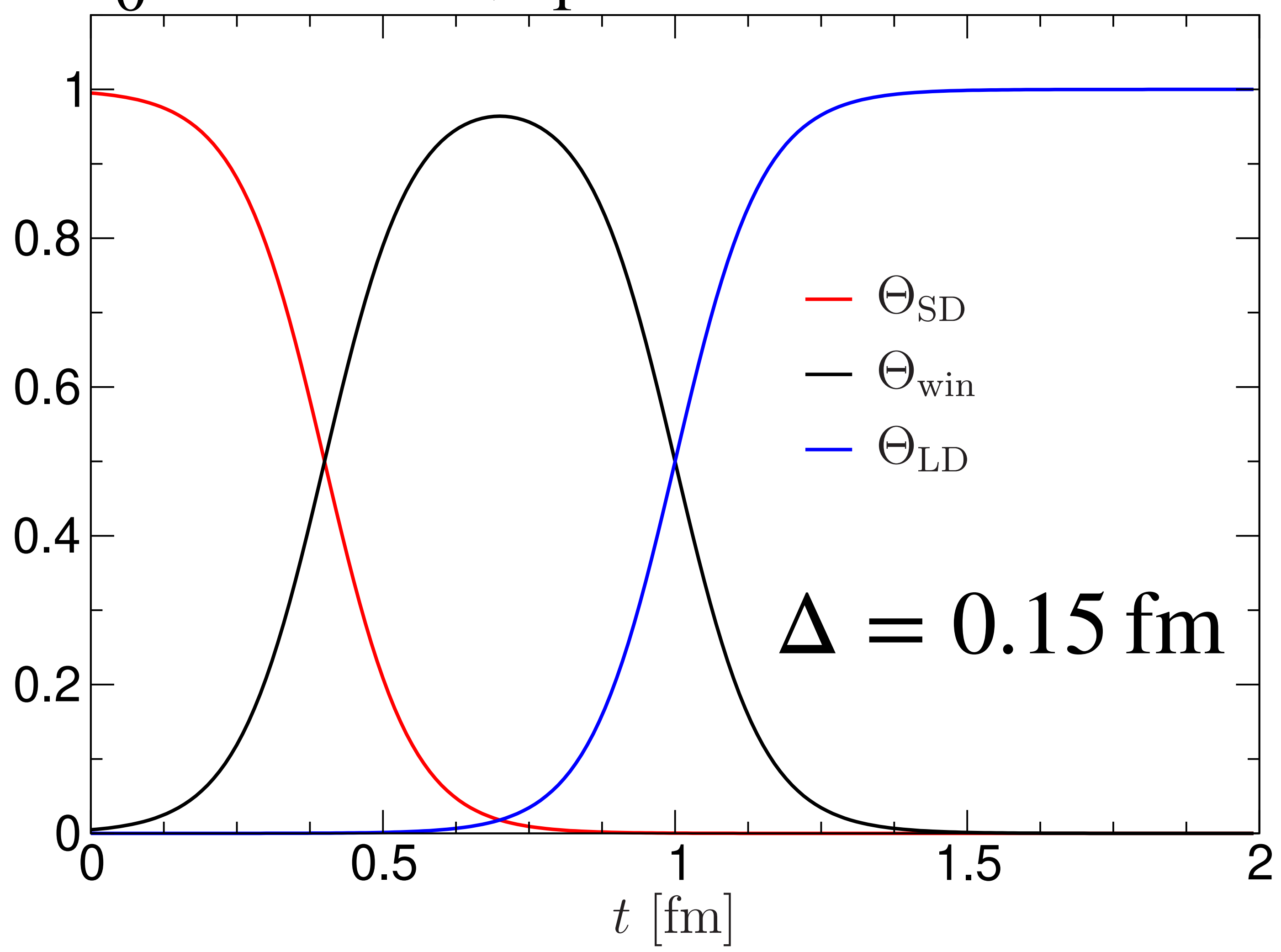
  $\to$ arXiv:2006.05431 [hep-ph]

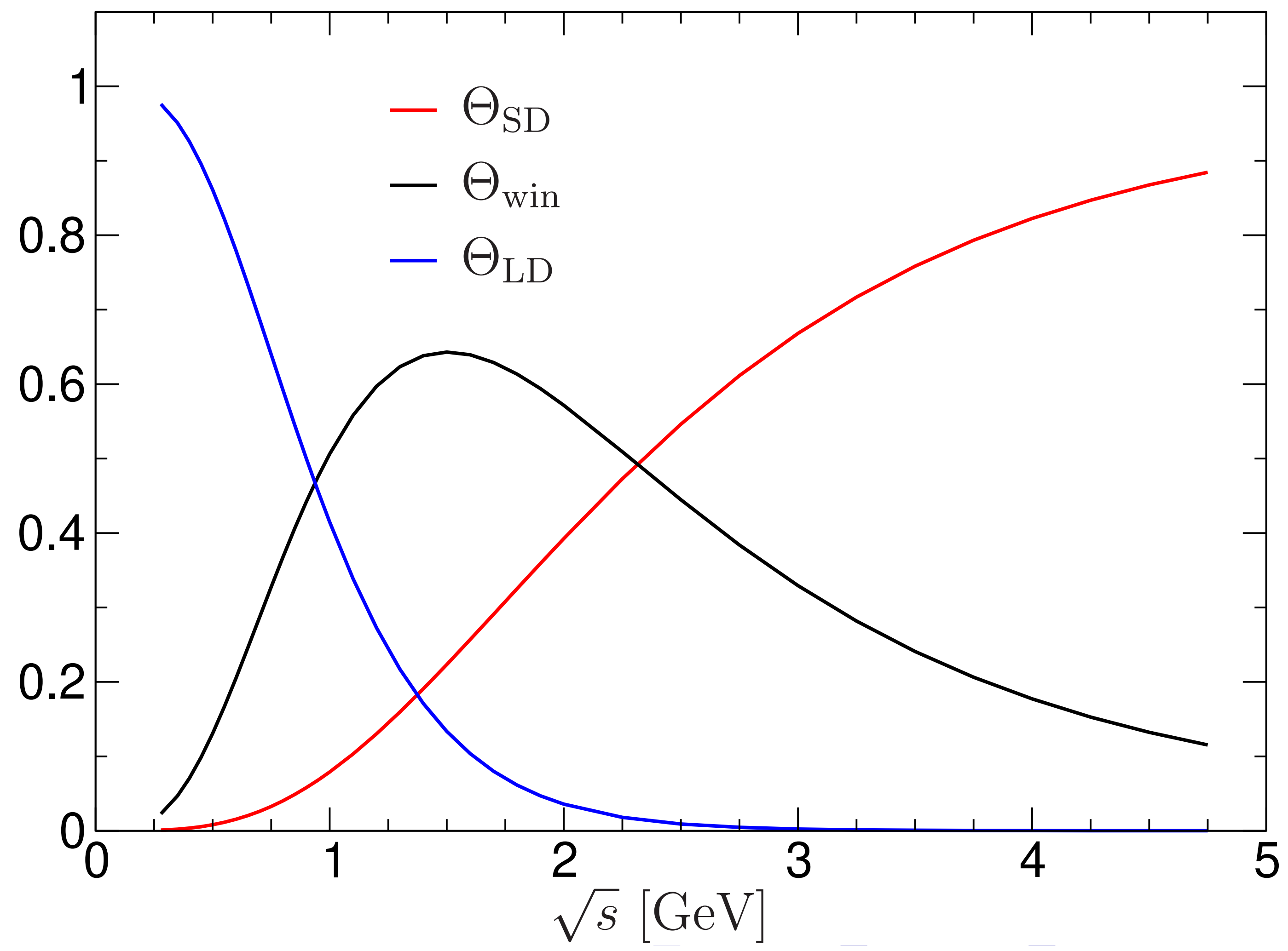


Windows: Euclidean time vs $\sqrt{s}$

Martin Hoferichter @ Lattice HVP workshop

$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}$





$[t_0, t_1]$ intermediate window	percentage captured of $\pi\pi$ channel $\leq$ 1 GeV		
	SD	intermediate	LD
[0.4, 1.0] fm	3	28	69
[1.0, 2.0] fm	31	51	18
[1.0, 2.5] fm	31	61	9
[1.0, 3.0] fm	31	65	4

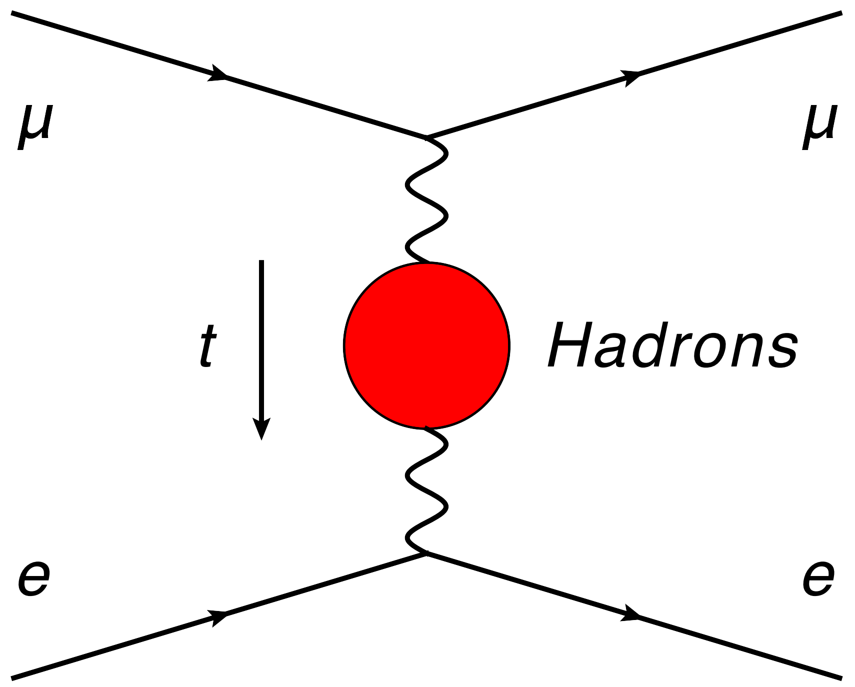
SD: $[0, t_0]$
LD: $[t_1, \infty]$
intermediate: $[t_0, t_1]$

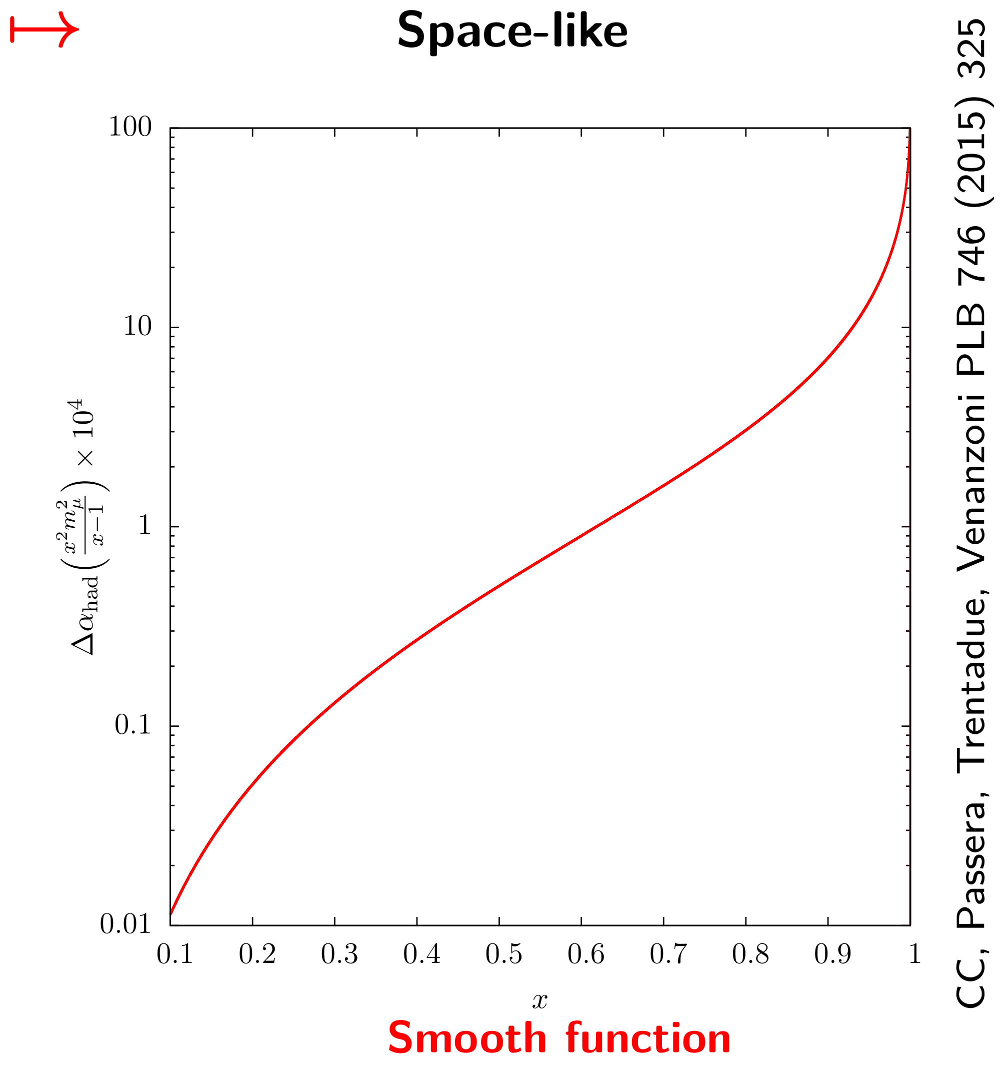
For intermediate window:
~30% from $\sigma(\pi\pi) \lesssim 1\,\mathrm{GeV}$

Hadronic vacuum polarization

μ-e elastic scattering to measure a_μ^{HVP}

LOI June 2019 [P. Banerjeei et al, arXiv:2004.13663, Eur.Phys.J.C 80 (2020)]





- use CERN M2 muon beam (150 GeV)
- Physics beyond colliders program @ CERN
- LOI June 2019
- pilot run in 2021
- full apparatus in 2023-2024

MUonE