The Muon g-2 and the B Anomalies

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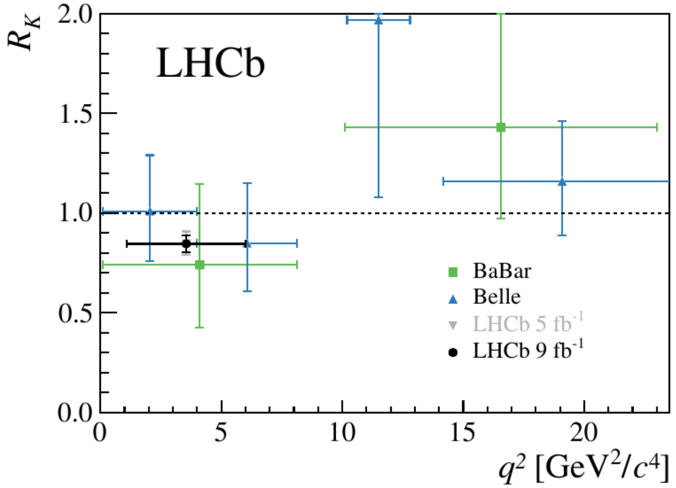
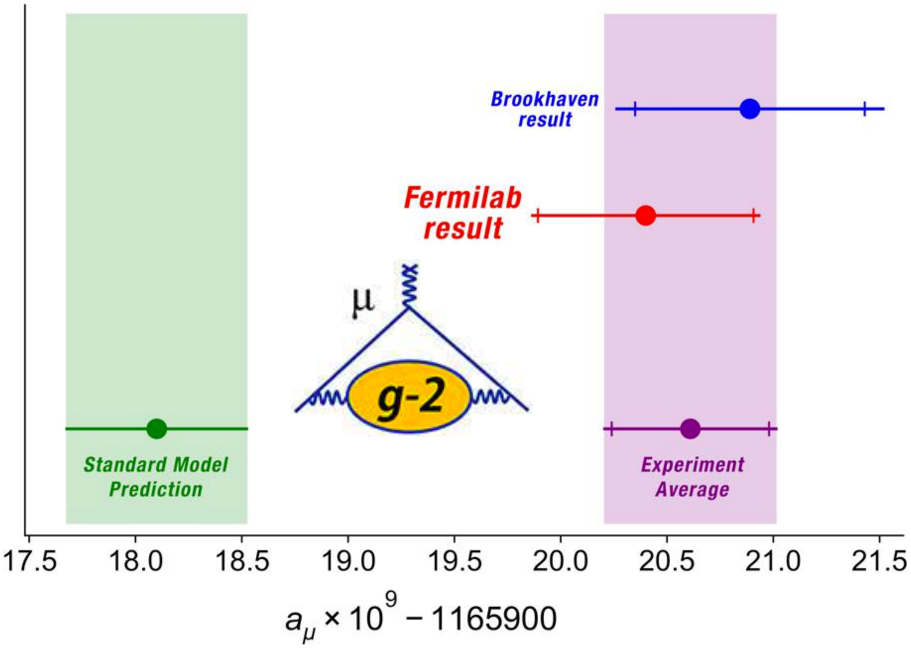
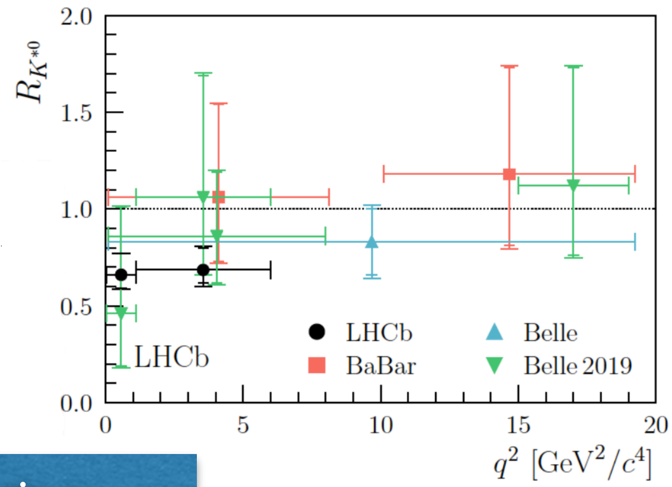




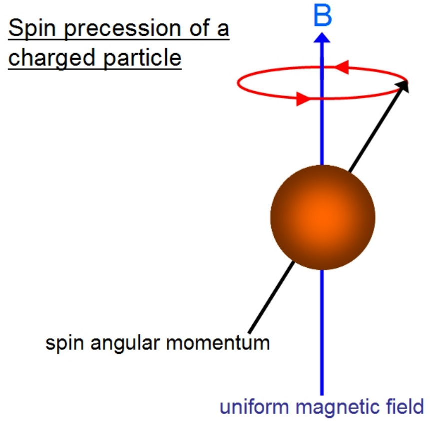


Fermilab PAC Meeting
Wednesday, November 17, 2021

Precision Tests of QED : g-2

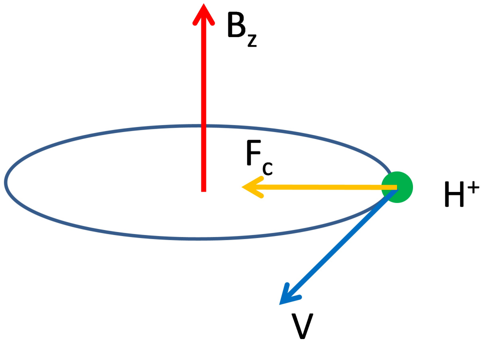
- The precession frequency of the lepton spin in a magnetic field is controlled by the so-called g-factor ($g \simeq 2$)

$$\vec{\omega}_S = -\frac{q\vec{B}}{m\gamma} - \frac{q\vec{B}}{2m}(g-2)$$



- That can be compared with the cyclotron frequency

$$\vec{\omega}_C = -\frac{q\vec{B}}{m\gamma}.$$



$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

- Hence,

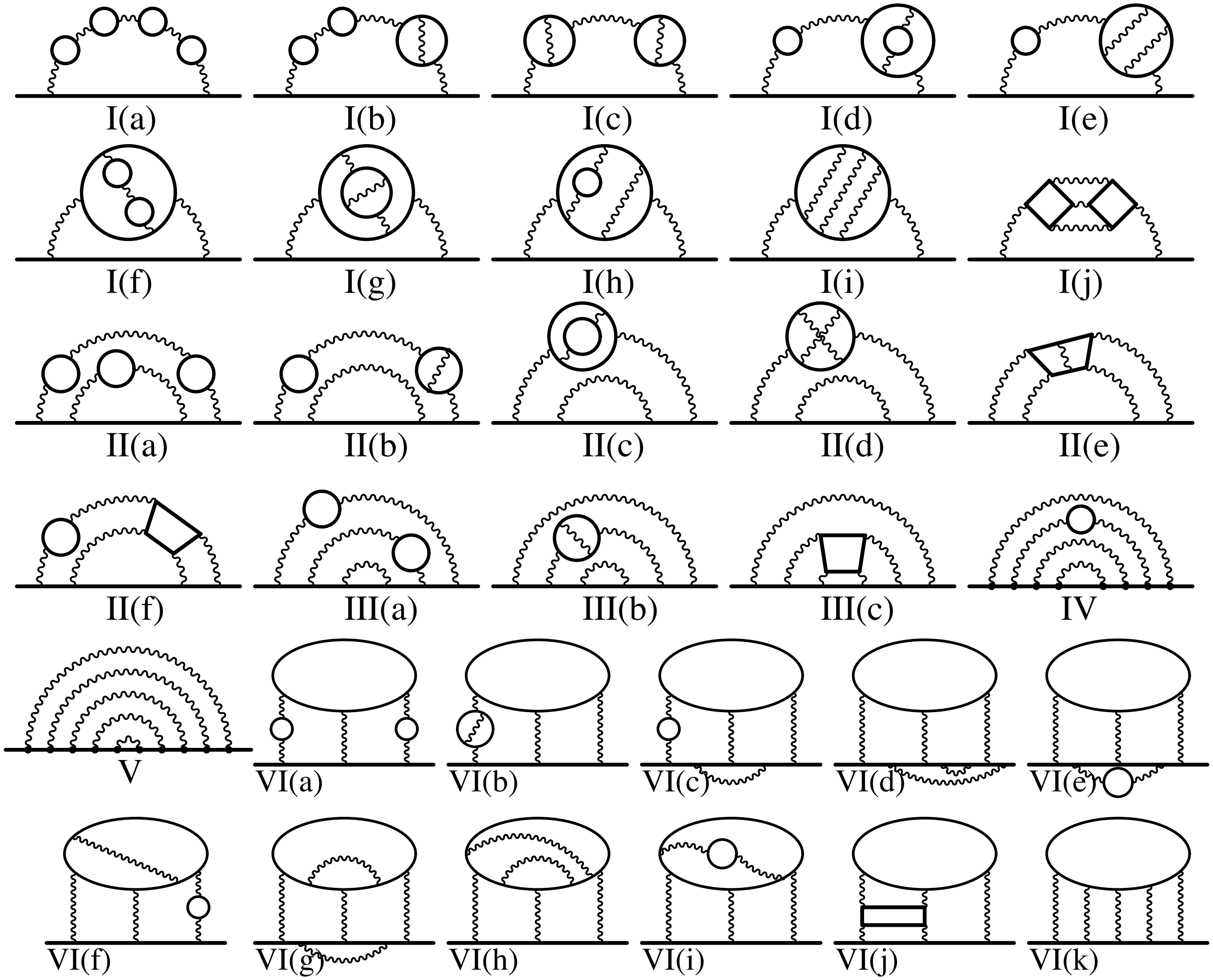
$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\frac{q\vec{B}}{m}\left(\frac{g-2}{2}\right)$$

$$\boxed{a_l = \left(\frac{g-2}{2}\right) = \frac{\alpha}{2\pi} + \dots}$$

- Precise measurement of g-2 is based on a clever way of measuring these frequency difference in a uniform magnetic field.

See, for example, Aoyama, Kinoshita, Nio'17,

5 QED Loop Contributions



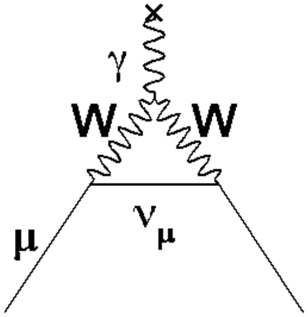
$$a_\mu^{\mathrm{QED}}(\alpha(\mathrm{Cs})) = 116\,584\,718.931(7)(17)(6)(100)(23)[104] \times 10^{-11}\,,$$

$$a_\mu^{\mathrm{QED}}(\alpha(a_e)) = 116\,584\,718.842(7)(17)(6)(100)(28)[106] \times 10^{-11}\,,$$

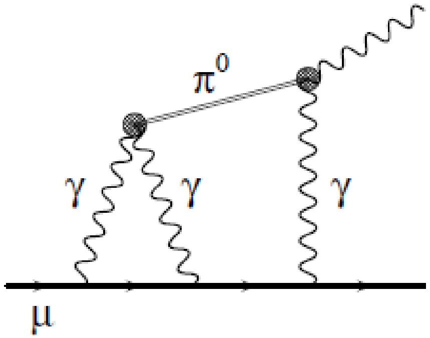
Muon g-2 factor

- The muon is a heavier cousin of the electron with a mass that is about 200 times larger.
- The muon g-2 factor is affected by the same corrections as the electron one, but also by the contribution of weak gauge bosons and heavy mesons in QCD,

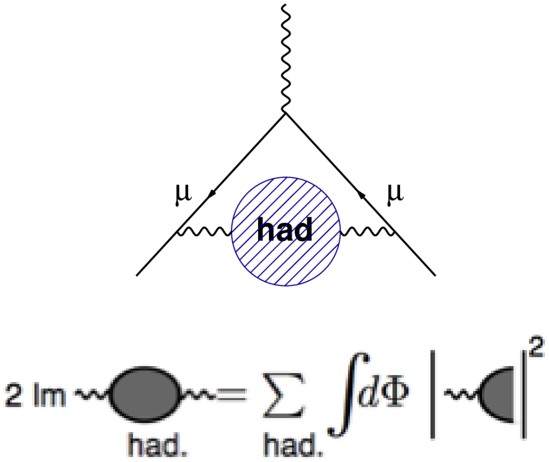
$$\Delta a_l \propto \left(\frac{m_l}{m_{\text{heavy}}}\right)^2$$



$$a_\mu^{\rm EW} = 153.6(1.0)\times 10^{-11}\,,$$



$$a_\mu^{LBL} = 92(19)\times 10^{-11}$$

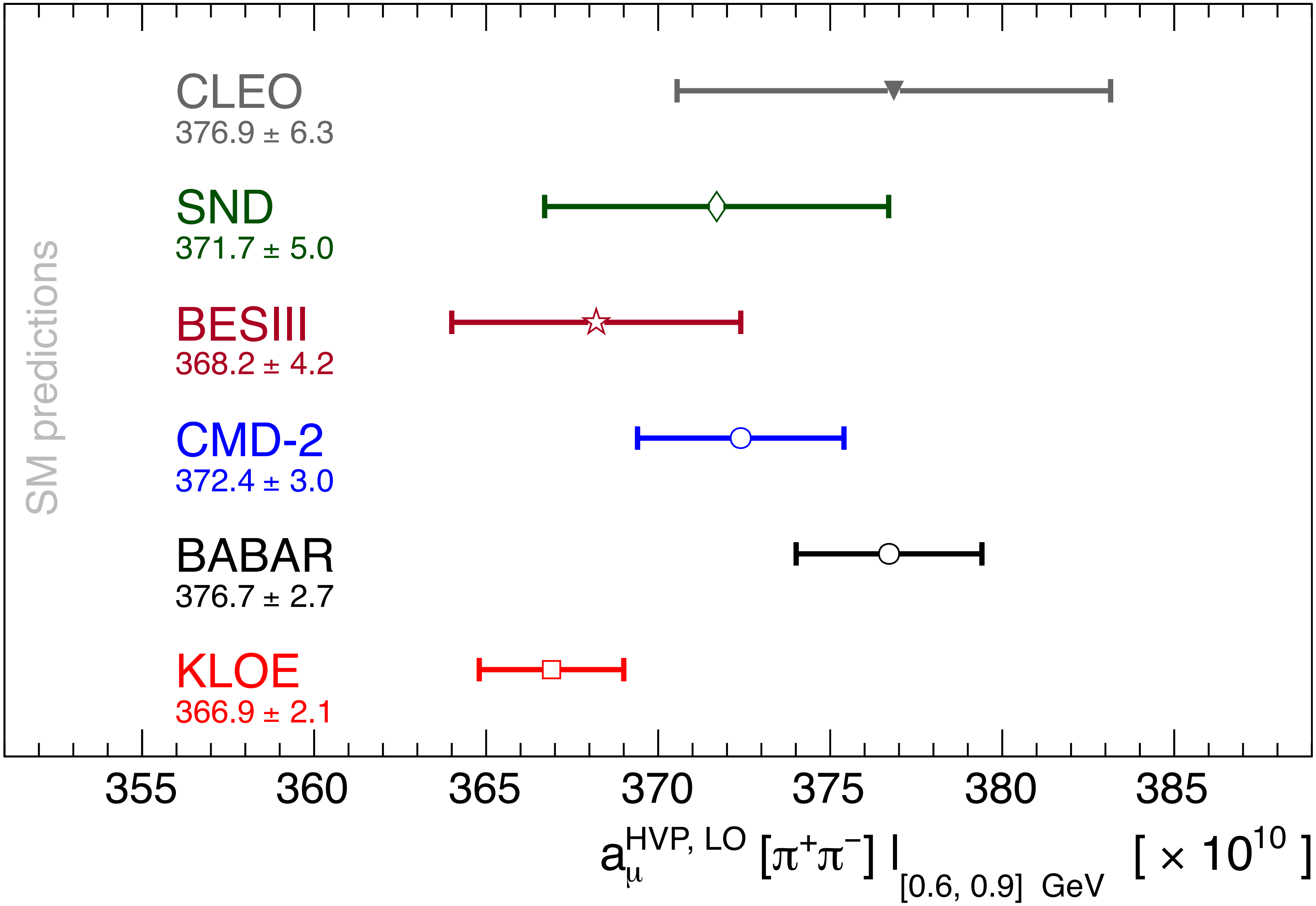


Vacuum polarization contributions computed using hadron cross section data and dispersion relations (optical theorem)

arXiv:2006.04822

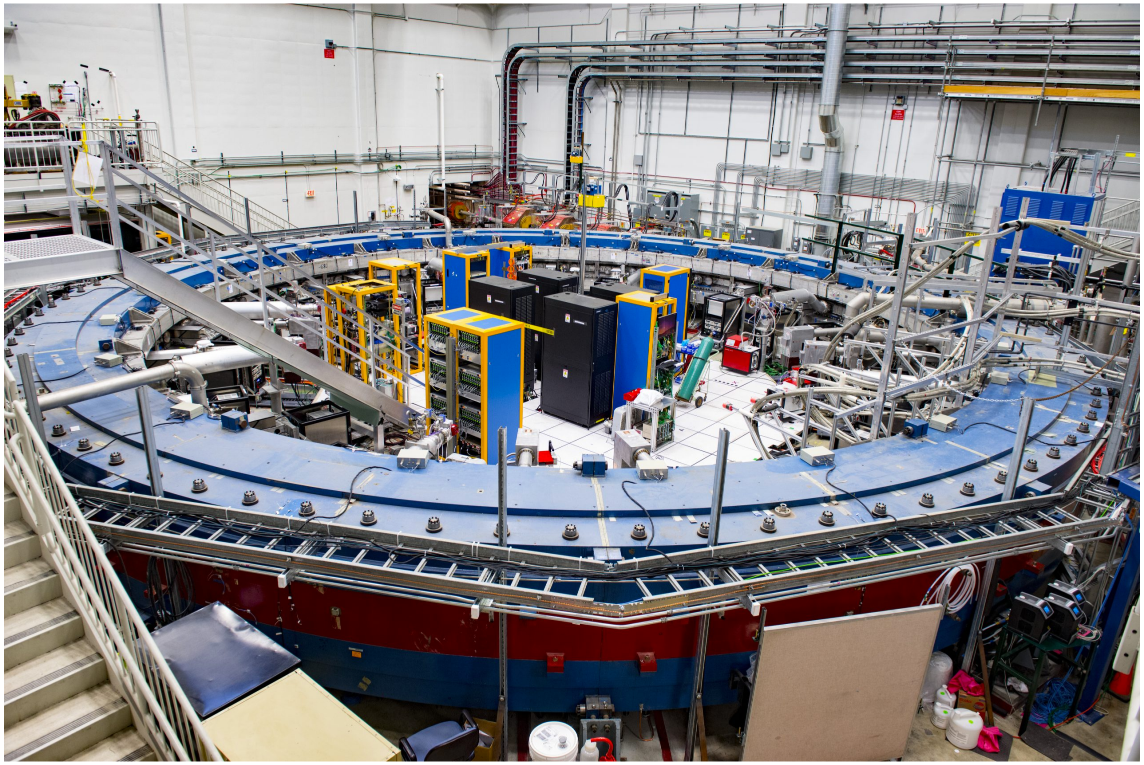
Hadronic Vacuum Polarization Contributions based on Data Driven Methods

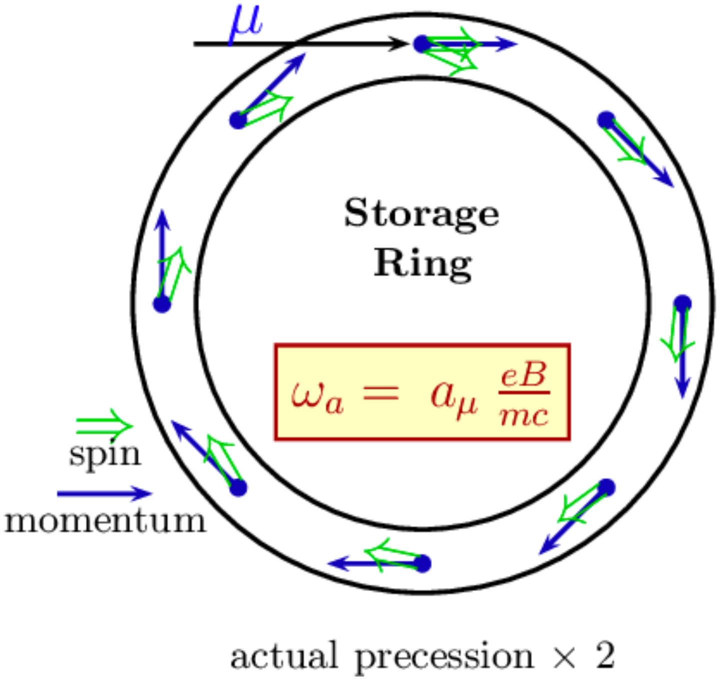
e+e- hadronic cross section + dispersion relations



Fermilab g-2 Experiment

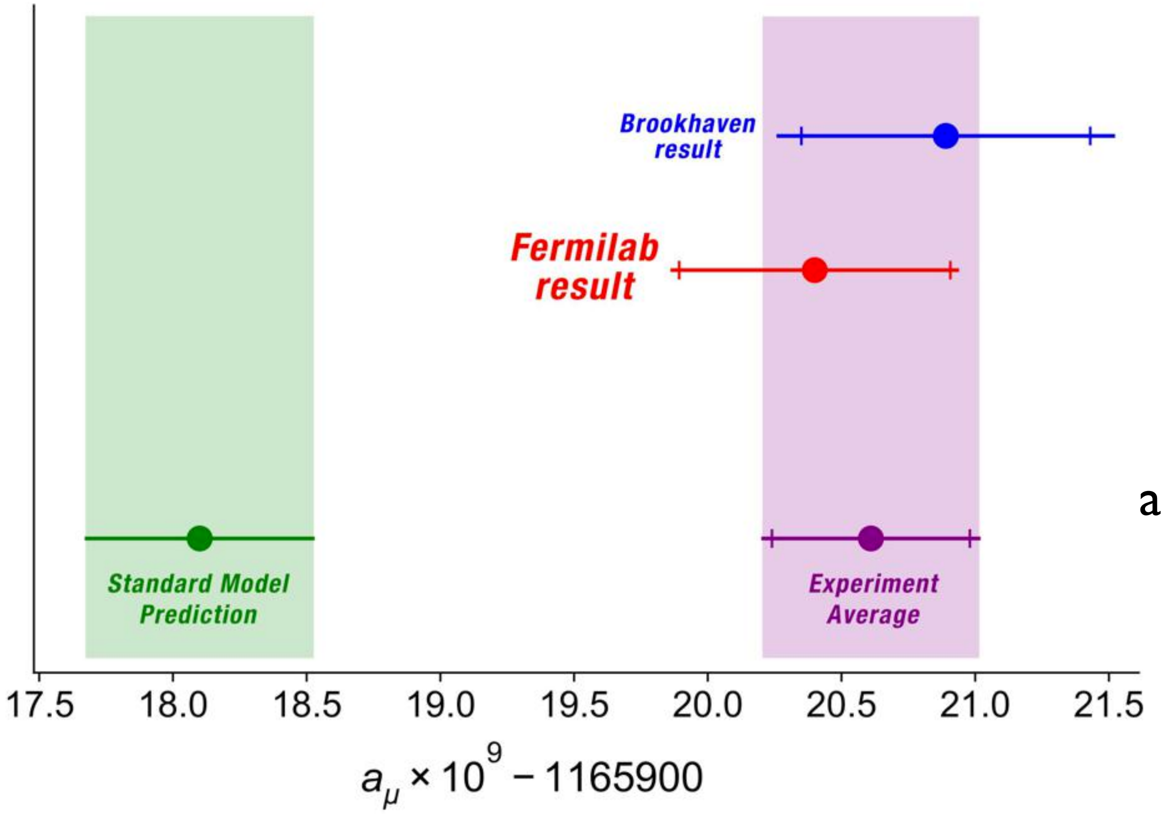
Accurate determination of muon Spin precession in a delicately uniform magnetic field

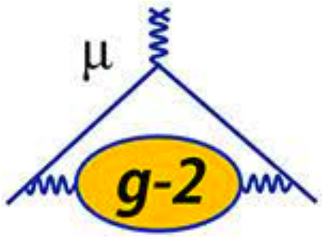




The muon g-2 collaboration confirms the Brookhaven result.
Deviation of 4.2 standard deviations from SM Expectations.
A very important result, that will be further tested in the coming years.

Observe that the g-2 errors are mainly statistical ones.





arXiv:2104.03281

$$a_\mu^{\rm SM} = 116\ 591\ 810(43) \times 10^{-11}$$

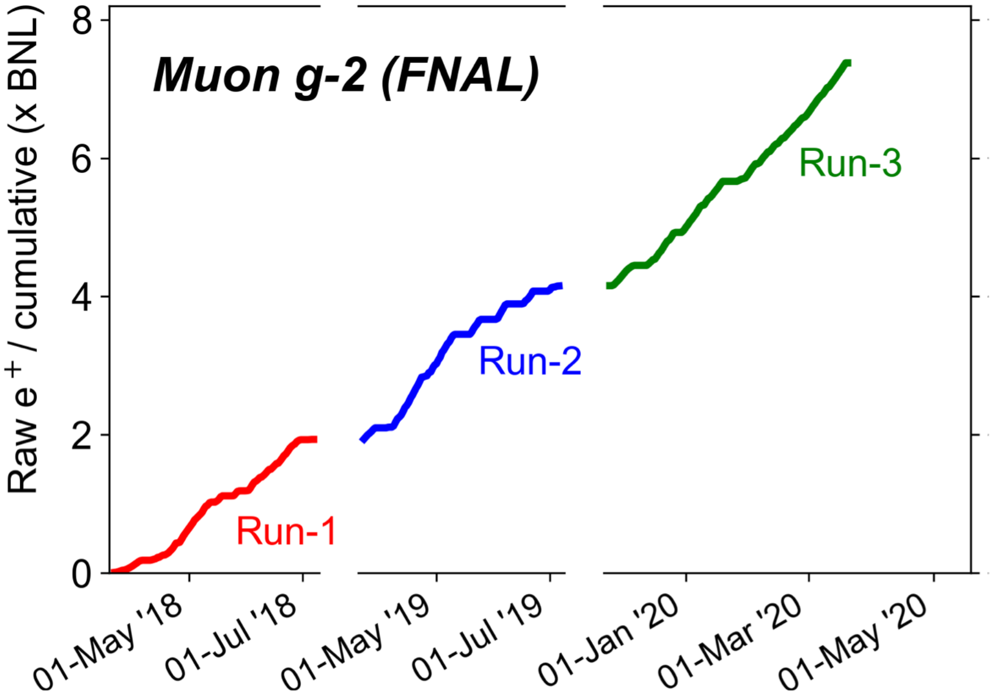
arXiv:2006.04822

$$a_\mu^{\rm exp} = 116\ 592\ 061(41) \times 10^{-11}$$

$$\Delta a_\mu \equiv (a_\mu^{\rm exp} - a_\mu^{\rm SM}) = (251 \pm 59) \times 10^{-11}$$

Data accumulated so far

- Accumulated 7.4xBNL through run 3
- Full run 1 has ~1.2xBNL after Data Quality Cuts
- Improvements between run1 and run 2/3 for:
  - Better beam dynamics
  - Reduced muon loss
  - More stable temperature



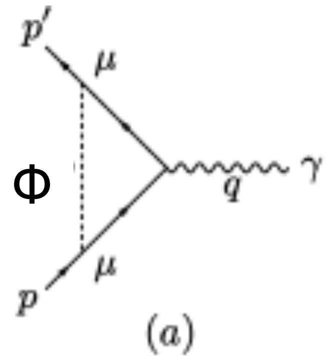
RUN1: March-July 2018, ~2x BNL (→ 1.2 xBNL after data quality)
RUN2: March 2019 – July 2019 ~2x BNL
RUN3: Nov 2019 – March 2020 ~3.2 x BNL

New physics ? Too many possibilities.

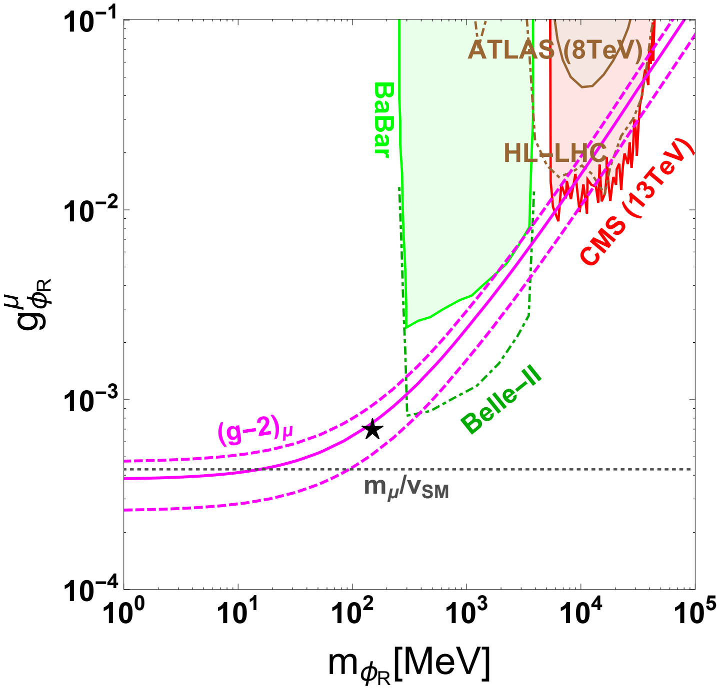
Marciano and Czarnecki, hep-ph/ 0102122

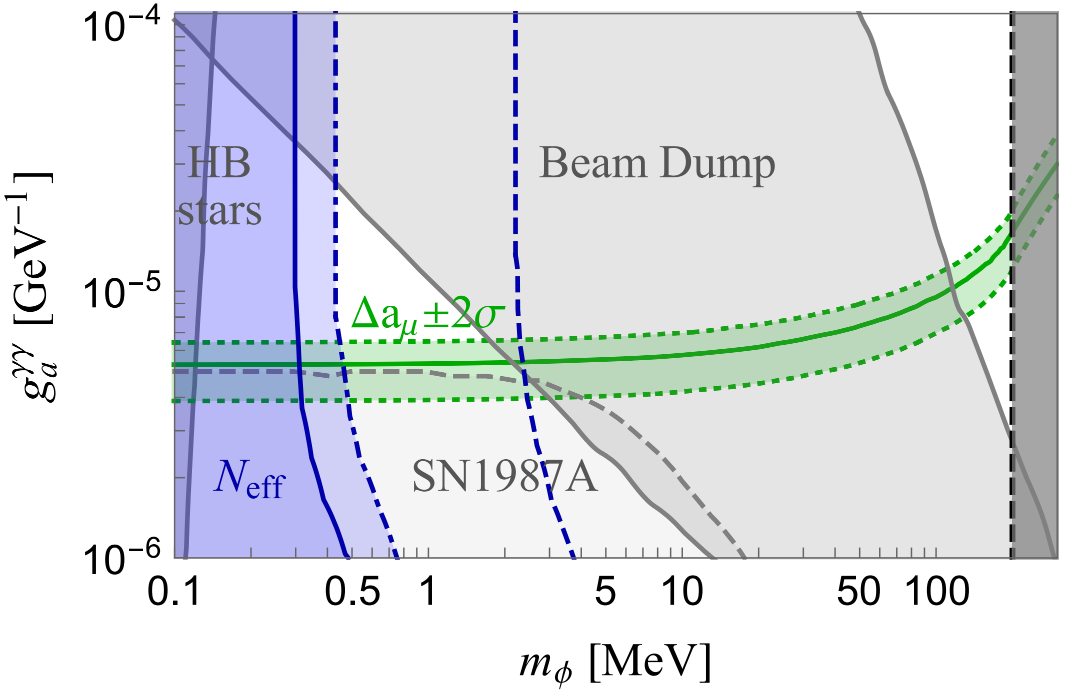
Scalar that couples to muons which induces a photon coupling. Cosmological bound in the 1MeV region may be avoided if ϕ is the source of neutrino masses

$$\Delta a_\ell = \frac{1}{8\pi^2}\int_0^1 dx \frac{(1-x)^2\left((1+x)g_R^2-(1-x)g_I^2\right)}{(1-x)^2+x\left(m_S/m_\ell\right)^2}$$

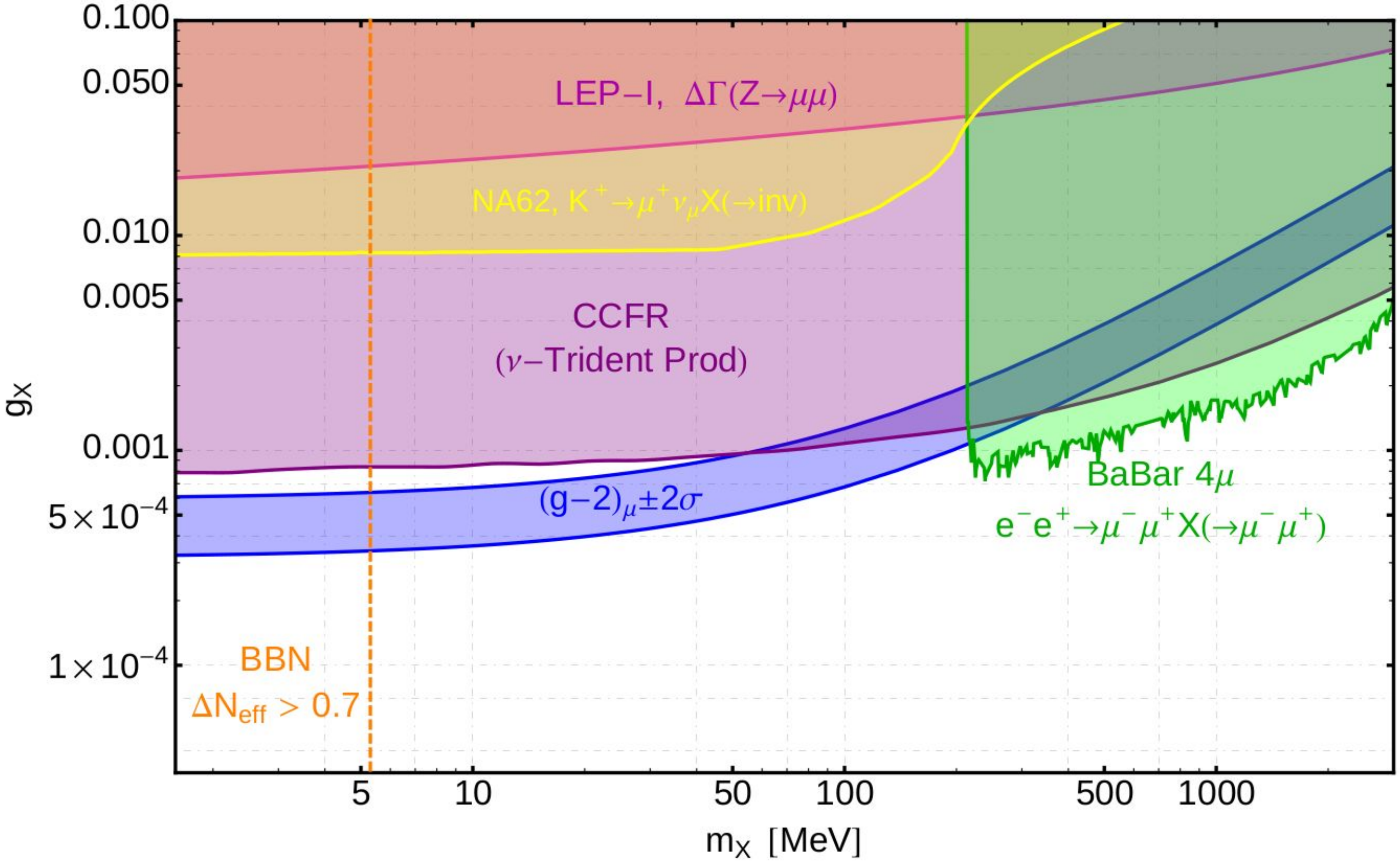


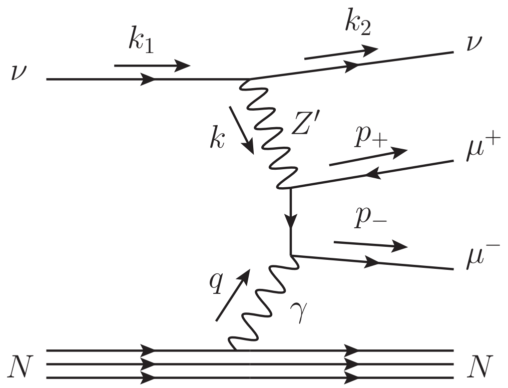
J. Liu, N. McGinnis, X. Wang, arXiv:1810.11028, 2110.14665





Bounds on gauge bosons coupled to muons but not electrons or quarks





Altmannshofer, Gori, Pospelov, Yavin'14

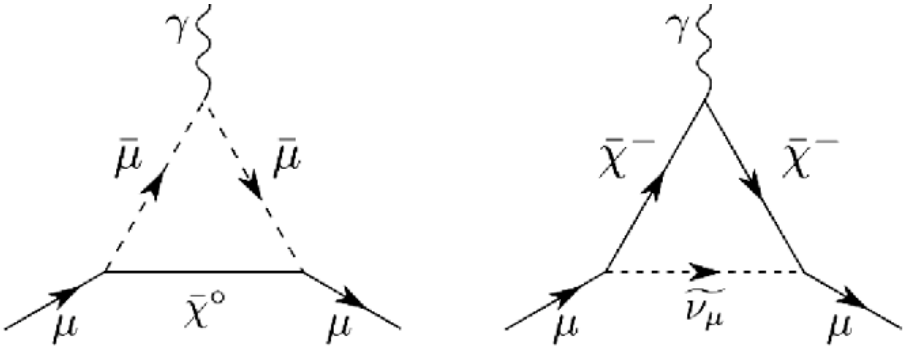
Babar (and similar LHC) constraints may be avoided for large masses, but the trident constraints remain powerful.

Supersymmetry

Barbieri, Maiani'82, Ellis et al'82, Grifols and Mendez'82
Moroi'95, Carena, Giudice, CW'95, Martin and Wells'00…

$$a_\mu^{\widetilde{\chi}^\pm-\widetilde{\nu}_\mu} \simeq \frac{\alpha m_\mu^2 \mu M_2 \tan\beta}{4\pi \sin^2\theta_W m_{\widetilde{\nu}_\mu}^2} \left[\frac{f_{\chi^\pm}\left(M_2^2/m_{\widetilde{\nu}_\mu}^2\right) - f_{\chi^\pm}\left(\mu^2/m_{\widetilde{\nu}_\mu}^2\right)}{M_2^2-\mu^2} \right] ,$$

$$a_\mu^{\widetilde{\chi}^0-\widetilde{\mu}} \simeq \frac{\alpha m_\mu^2 M_1 \left(\mu\tan\beta - A_\mu\right)}{4\pi\cos^2\theta_W \left(m_{\widetilde{\mu}_R}^2 - m_{\widetilde{\mu}_L}^2\right)} \left[\frac{f_{\chi^0}\left(M_1^2/m_{\widetilde{\mu}_R}^2\right)}{m_{\widetilde{\mu}_R}^2} - \frac{f_{\chi^0}\left(M_1^2/m_{\widetilde{\mu}_L}^2\right)}{m_{\widetilde{\mu}_L}^2} \right]$$



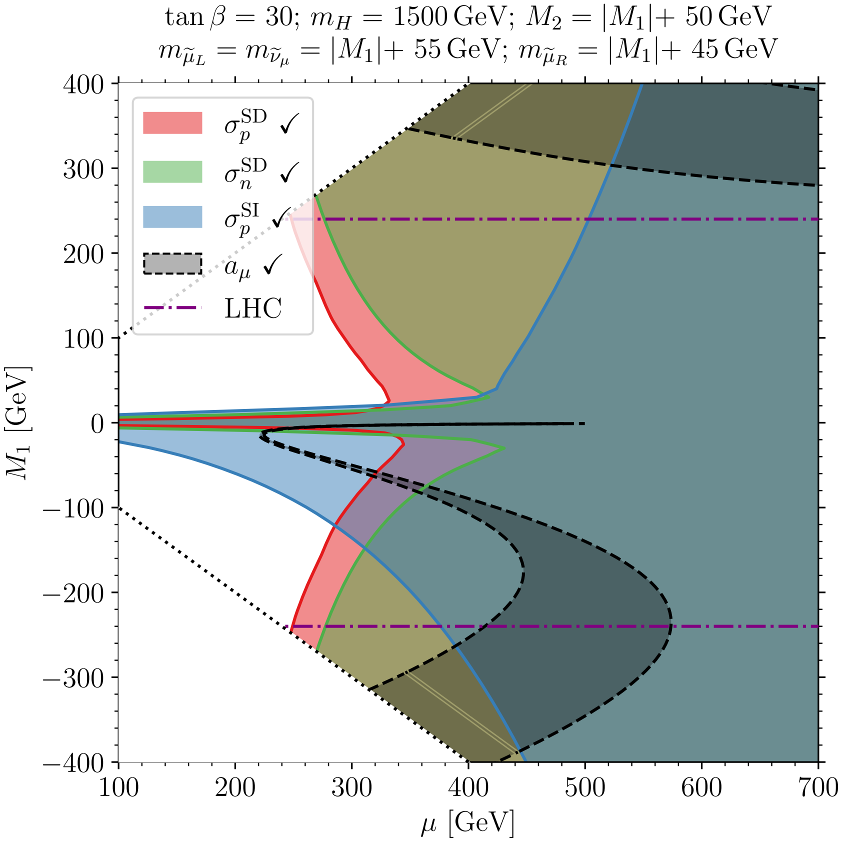
$$f_{\chi^\pm}(x) = \frac{x^2-4x+3+2\ln(x)}{(1-x)^3} ,$$

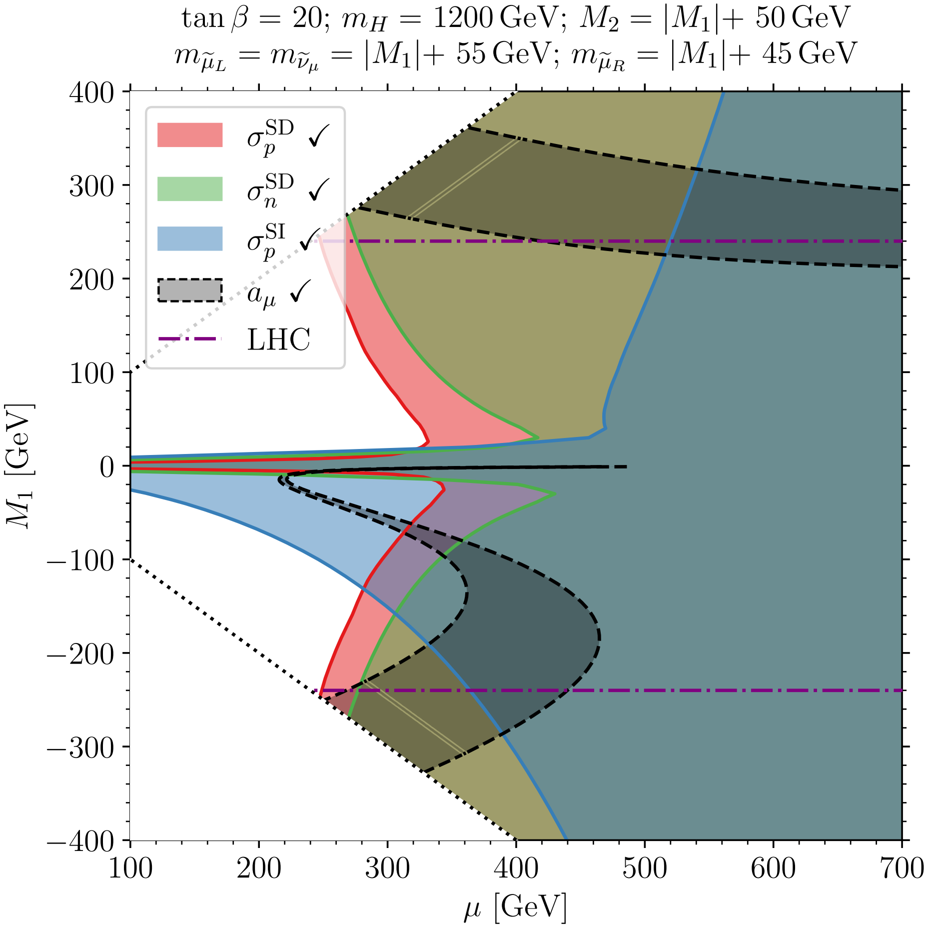
$$f_{\chi^0}(x) = \frac{x^2-1-2x\ln(x)}{(1-x)^3} ;$$

Baum, Carena, Shah and C.W., arXiv:2104.03302

Compatibility of Dark Matter and g-2 Constraints for a representative example of a compressed spectrum. Stau co-annihilation is assumed

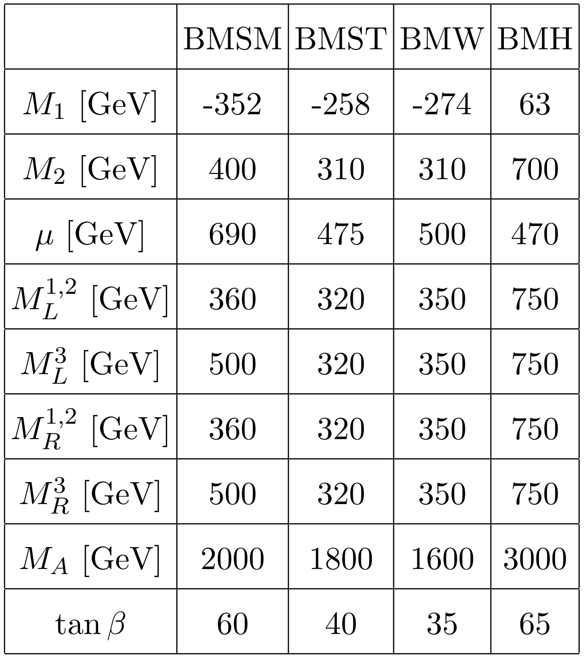
Large hierarchy of values of μ between positive and negative values of the Bino mass parameter is observed.



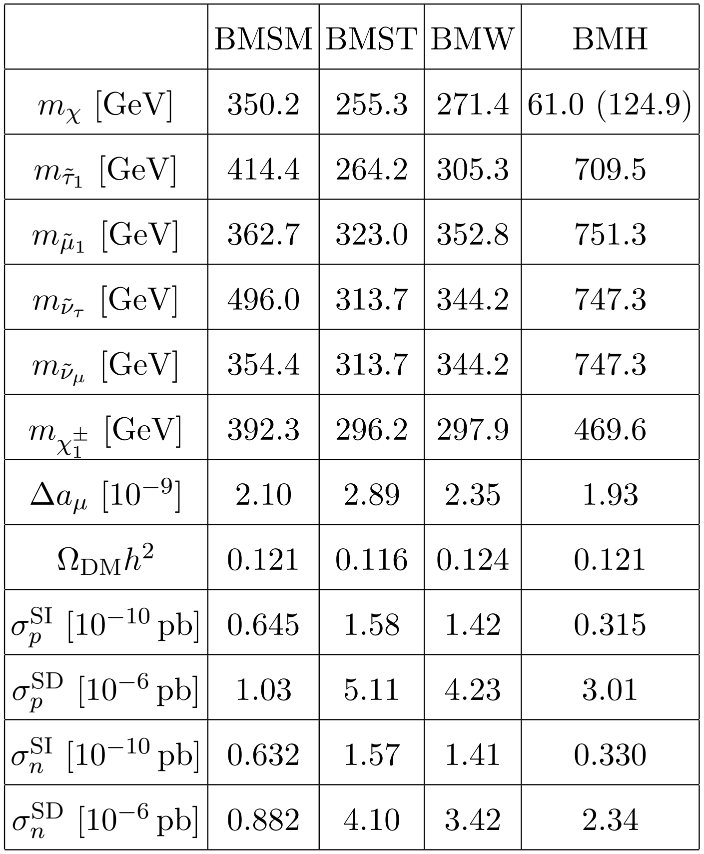


Baum, Carena, Shah and C.W., arXiv:2104.03302

Benchmark Scenarios for negative $\mu \times M_1$



	BMSM	BMST	BMW	BMH
M_1 [GeV]	-352	-258	-274	63
M_2 [GeV]	400	310	310	700
μ [GeV]	690	475	500	470
$M_L^{1,2}$ [GeV]	360	320	350	750
M_L^3 [GeV]	500	320	350	750
$M_R^{1,2}$ [GeV]	360	320	350	750
M_R^3 [GeV]	500	320	350	750
M_A [GeV]	2000	1800	1600	3000
$\tan\beta$	60	40	35	65



	BMSM	BMST	BMW	BMH
m_χ [GeV]	350.2	255.3	271.4	61.0 (124.9)
$m_{\tilde{\tau}_1}$ [GeV]	414.4	264.2	305.3	709.5
$m_{\tilde{\mu}_1}$ [GeV]	362.7	323.0	352.8	751.3
$m_{\tilde{\nu}_\tau}$ [GeV]	496.0	313.7	344.2	747.3
$m_{\tilde{\nu}_\mu}$ [GeV]	354.4	313.7	344.2	747.3
$m_{\chi_1^\pm}$ [GeV]	392.3	296.2	297.9	469.6
Δa_μ [10^{-9}]	2.10	2.89	2.35	1.93
$\Omega_{\rm DM}h^2$	0.121	0.116	0.124	0.121
$\sigma_p^{\rm SI}$ [10^{-10} pb]	0.645	1.58	1.42	0.315
$\sigma_p^{\rm SD}$ [10^{-6} pb]	1.03	5.11	4.23	3.01
$\sigma_n^{\rm SI}$ [10^{-10} pb]	0.632	1.57	1.41	0.330
$\sigma_n^{\rm SD}$ [10^{-6} pb]	0.882	4.10	3.42	2.34

BMSM: Muon co-annihilation,

BMST: Stau co-annihilation

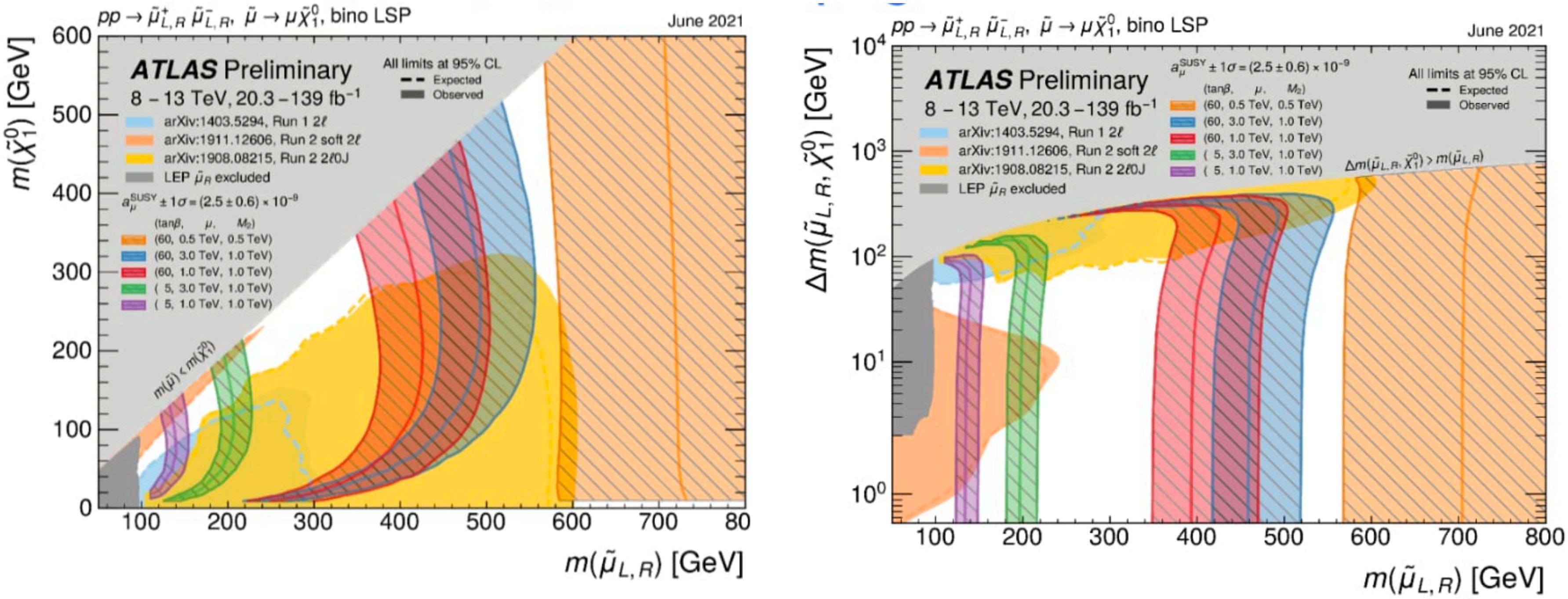
BMW : Wino co-annihilation,
a

BMH ; Resonant s-channel annihilation via the lightest Higgs

All this region of parameters may be tested by DM Direct Detection, Higgs and SUSY particle searches. Region of positive μ admits solution with large Higgs and Higgsino mass parameters and remains more challenging.

Solutions for large μ

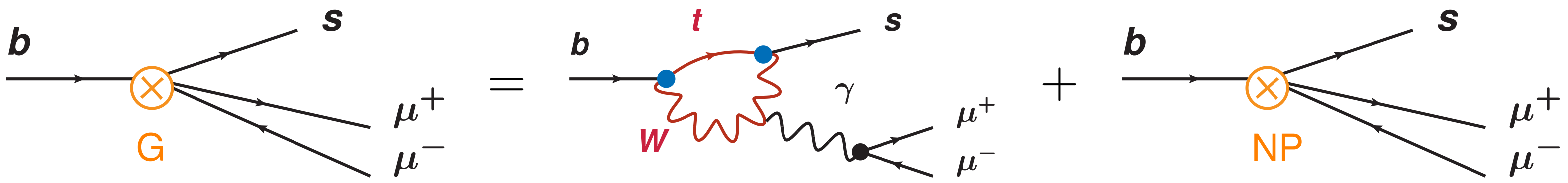
See, for example, Chakraborty, Heinemeyer, Saha, 2104.03287
Cox et al.'2104.03290, Ahmed et al, 2104.03491…



Comparison of ATLAS slepton mass limits with g-2 preferred regions

B-Anomalies

Relevant B transition amplitudes



$$G \sim \frac{1}{16\pi^2}\frac{g^4}{m_W^2}\frac{m_t^2}{m_W^2}V_{tb}V_{ts}^* + \frac{C_{NP}}{\Lambda_{NP}^2}$$

measure precisely

calculate precisely the SM contribution

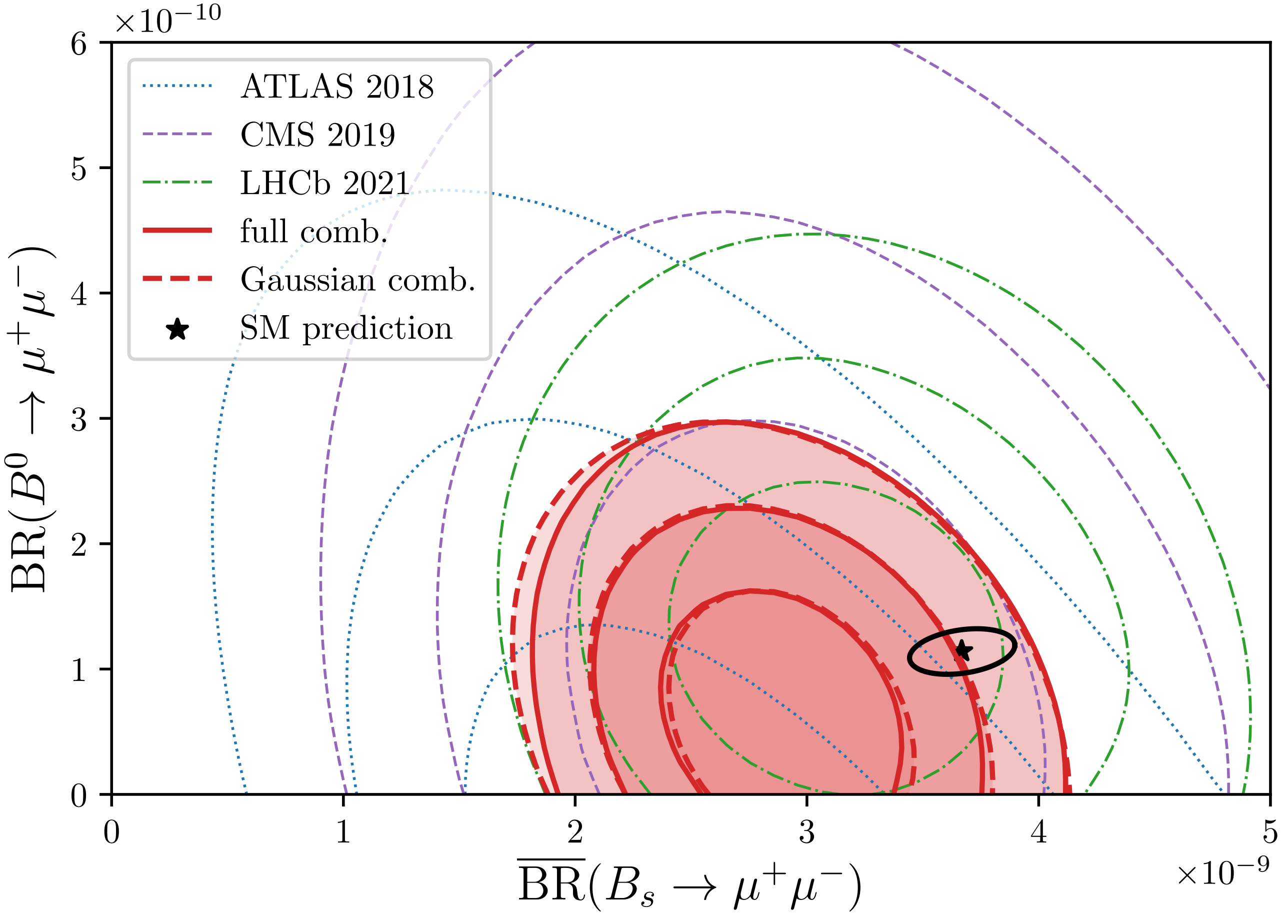
get information on NP coupling and scale

Anomalies in rare b decays could establish
a new scale in particle physics

$$\Lambda_{NP} \sim \frac{4\pi v}{\sqrt{|V_{tb}V_{ts}^*|}} \sim O(10\mathrm{TeV})$$

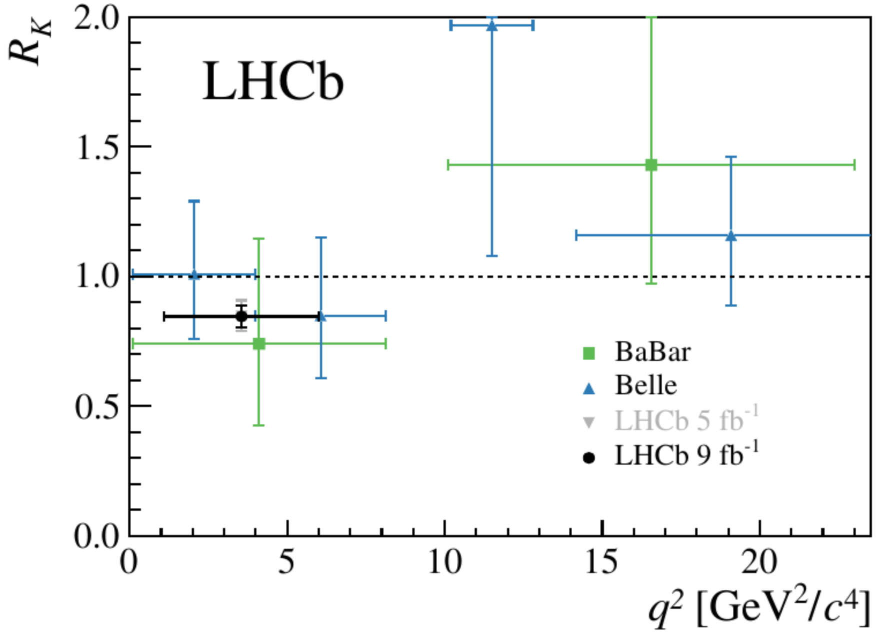
Altmannshofer'21

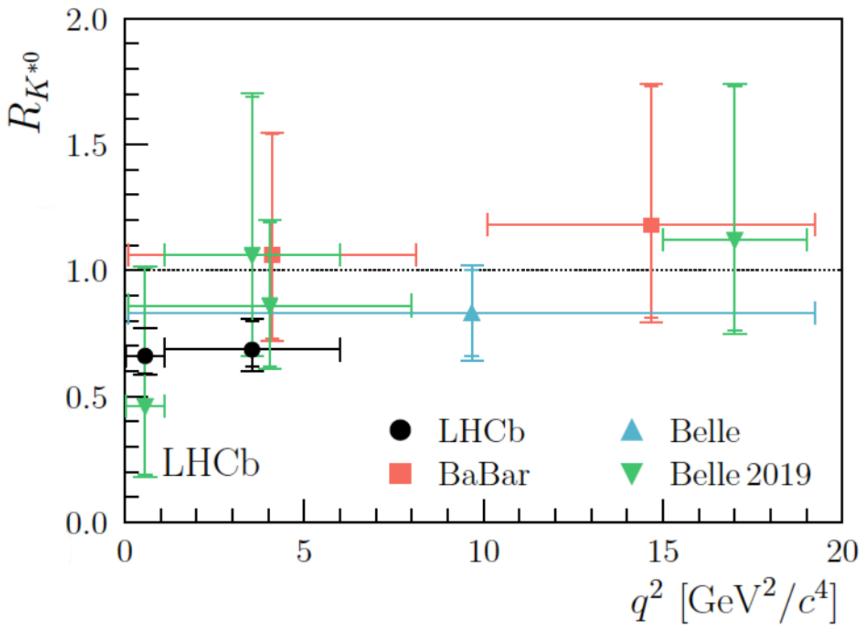
Altmannshofer, Stangl, arXiv: 2103.13370
Combination of LHCb 2108.09284, CMS 1910.12127, ATLAS 1812.03017



$\sim 2\sigma$ tension between SM and experiment

Tests of Lepton Flavor Universality





$$R_K = \frac{BR(B^+ \to K^+ \mu\mu)}{BR(B^+ \to K^+ ee)}$$

$$R_{K^*} = \frac{BR(B^0 \to K^{*0} \mu\mu)}{BR(B^0 \to K^{*0} ee)}$$

$$R_K^{[1,6]} = 0.846^{+0.042\,+0.013}_{-0.039\,-0.012}$$

$$R_{K^*}^{[0.045,1.1]} = 0.66^{+0.11}_{-0.07} \pm 0.03$$

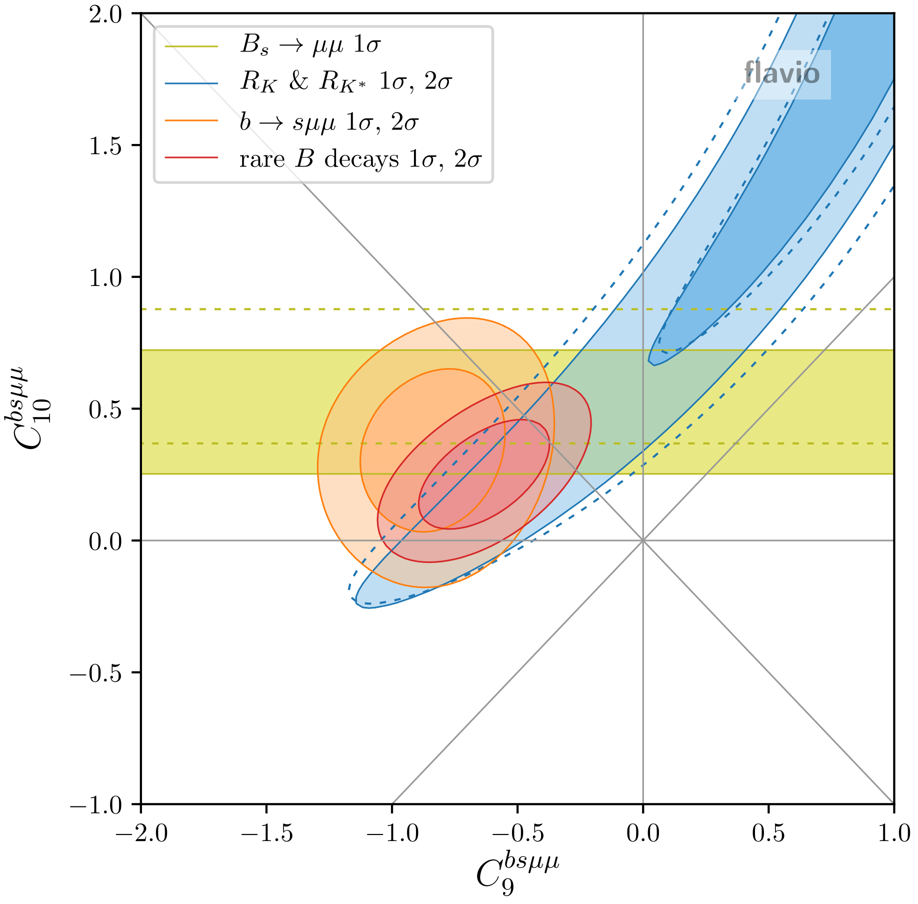
$$R_{K^*}^{[1.1,6]} = 0.69^{+0.11}_{-0.07} \pm 0.05$$

Observed deviations
of the order of 2 to 3 σ
from SM predictions,

$$R_K^{[1,6]} \sim R_{K^*}^{[1,6]} \sim 1.0$$

$$R_{K^*}^{[0.045,1]} \sim 0.9$$

Altmannshofer, Stangl, arXiv: 2103.13370



$$C_9^{bs\mu\mu}(\bar{s}\gamma_\alpha P_L b)(\bar{\mu}\gamma^\alpha \mu)$$

$$C_{10}^{bs\mu\mu}(\bar{s}\gamma_\alpha P_L b)(\bar{\mu}\gamma^\alpha \gamma_5 \mu)$$

- LFU ratios prefer non-standard C_{10}, but large degeneracy
- $B_s \to \mu^+\mu^-$ branching ratio shows slight preference for non-standard C_{10}
- $b \to s\mu\mu$ observables prefer non-standard C_9
- best fit point

$$C_9^{bs\mu\mu} \simeq -0.63$$

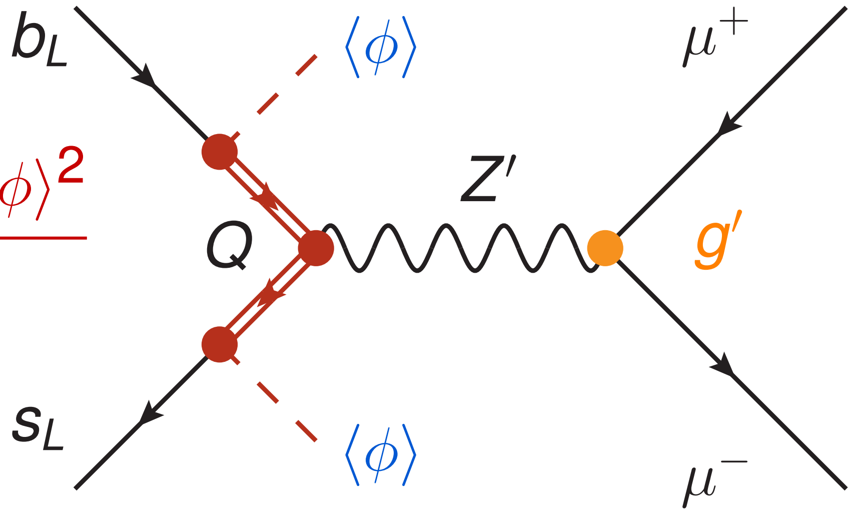
$$C_{10}^{bs\mu\mu} \simeq +0.25$$

Gauge Extension of the SM

Z' based on gauging $L_\mu - L_\tau$ (He, Joshi, Lew, Volkas PRD 43, 22-24)
with effective flavor violating couplings to quarks

Altmannshofer, Gori, Pospelov, Yavin, arXiv:1403.1269

$$\frac{g' Y_{Qb} Y^*_{Qs} \langle\phi\rangle^2}{2 m_Q^2}$$



predicted Lepton
Universality Violation!

Q: heavy vectorlike fermions with mass $\sim 1 - 10$ TeV
ϕ: scalar that breaks $L_\mu - L_\tau$

This new scalar may be visible at the LHC, two photons decays
Liu, Wang, C.W. arXiv: 1805.01476

Ballett et al, arXiv:1902.08579

Altmannshofer, Gori, Martin-Albo, Sousa, Wallbank, 1902.06765

Neutrino Tridents

B_s mixing

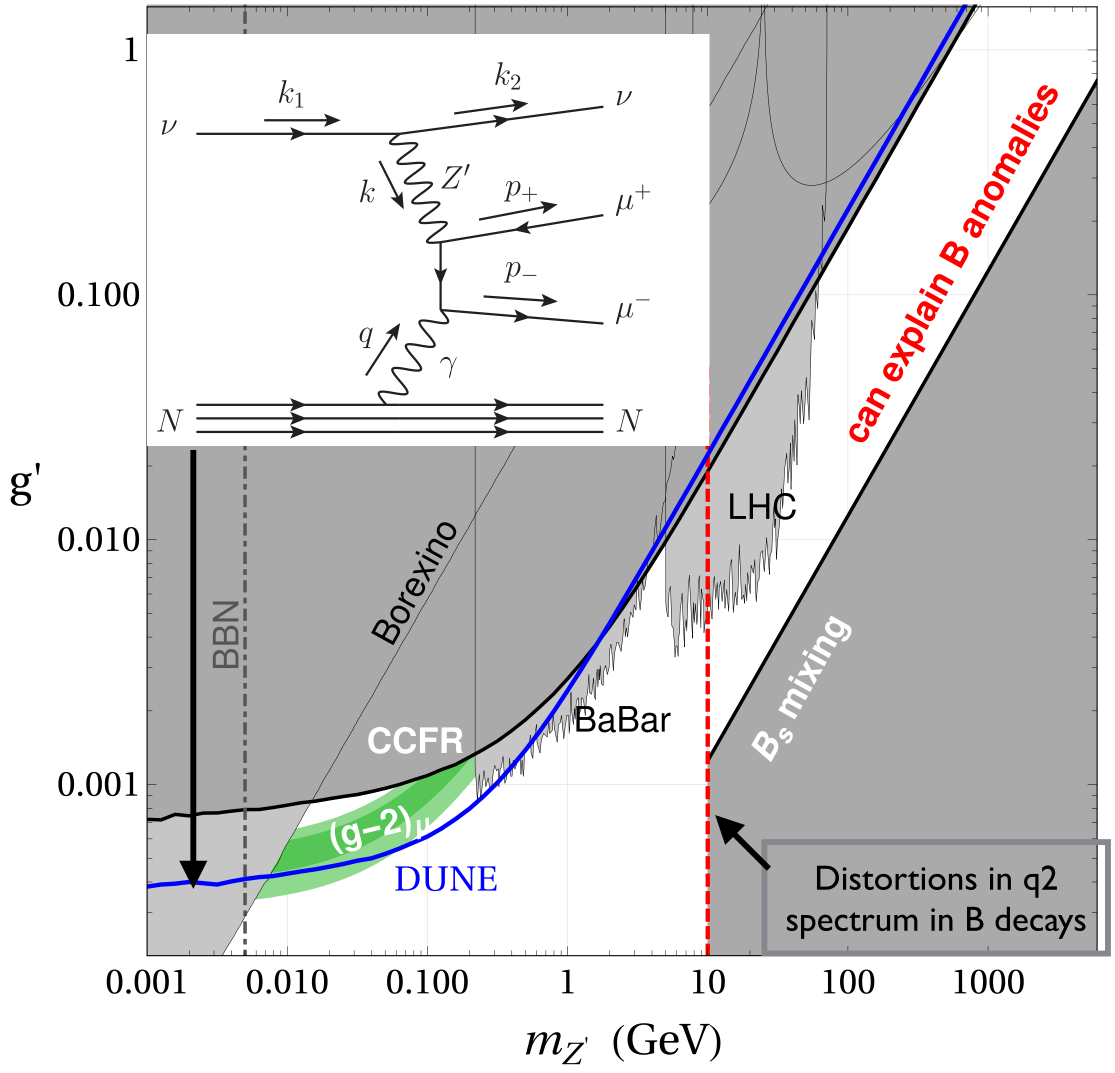
$(g-2)_\mu$

νe scattering

$Z \to \ell\ell$

$Z \to 4\mu$

$e^+e^- \to 4\mu$

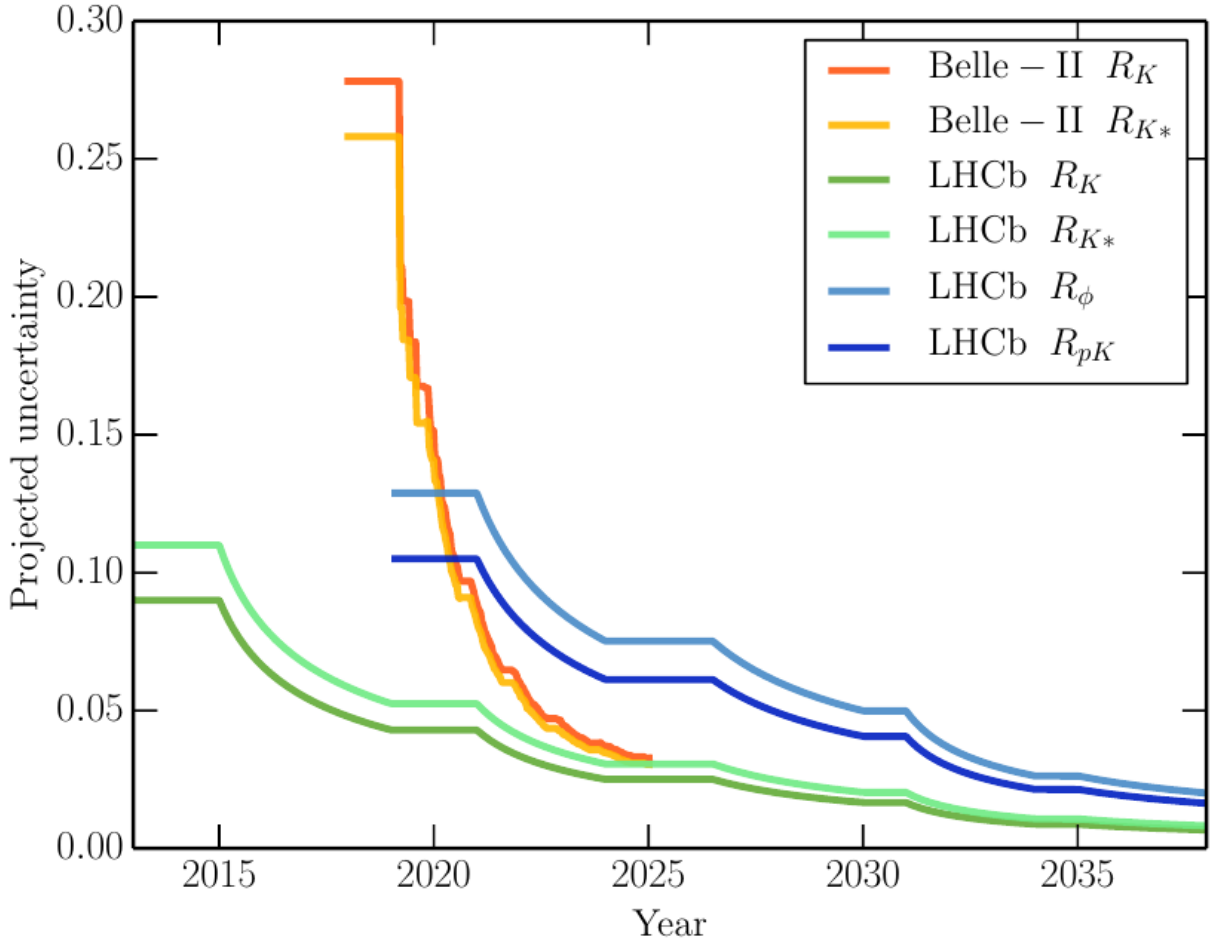


Distortions in q2 spectrum in B decays

The two anomalies cannot be explained simultaneously in this vanilla scenario

Future Projections

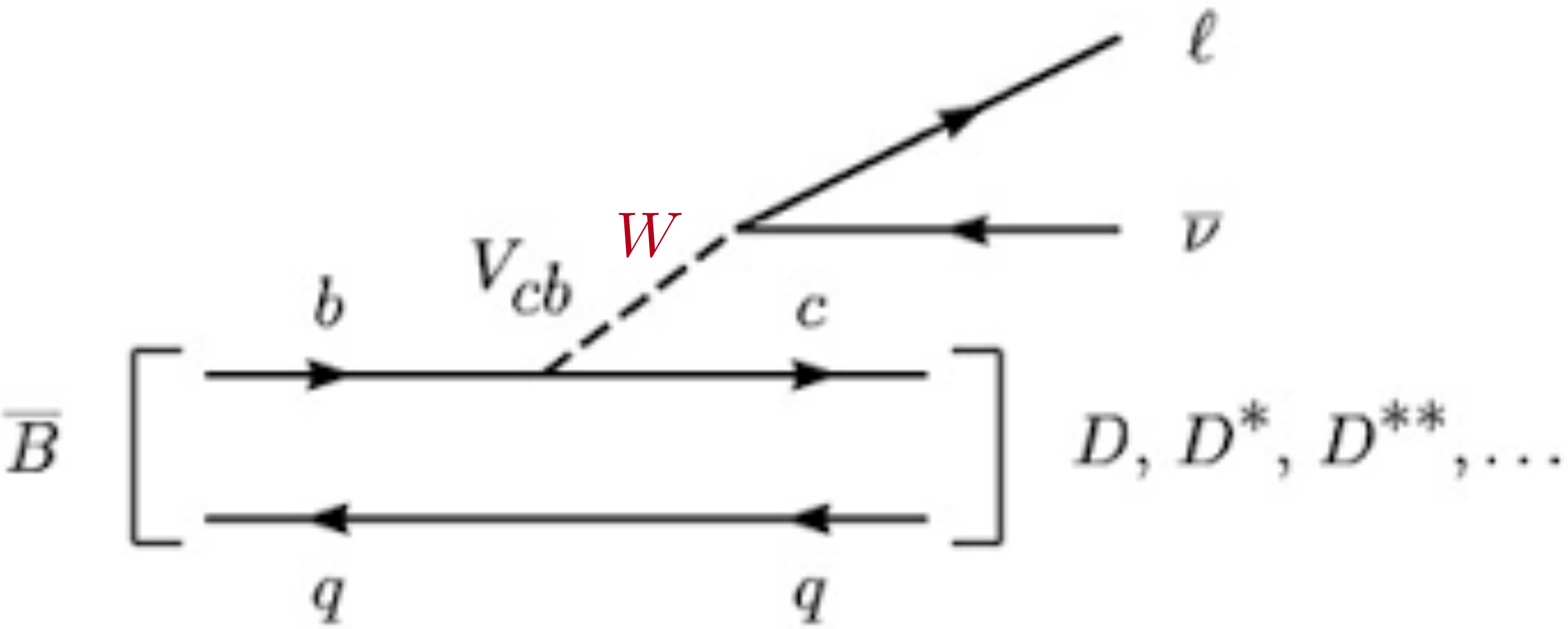
- LHCb and Belle II can push uncertainties down to few percent
- (can ATLAS and CMS say something?)
- with sufficient statistics, LFU of angular distrib. can be tested



Bifani et al. 1809.06229

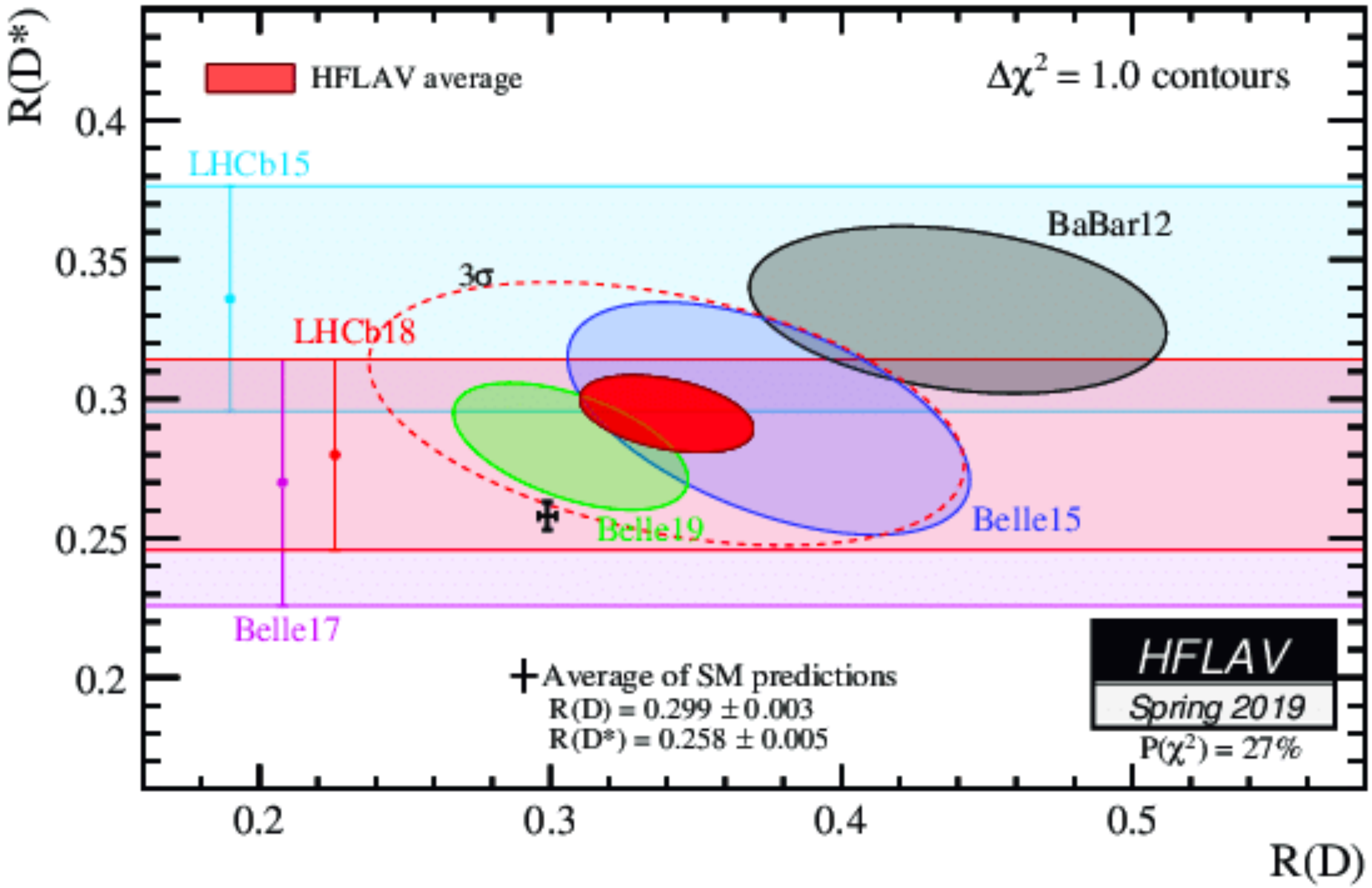
$$R(D^{(*)}) = \frac{\mathcal{B}(D^{(*)}\tau\nu_\tau)}{\mathcal{B}(D^{(*)}l\nu_l)}$$

In the Standard Model, this is a tree-level process



Surprisingly large deviation of the experimental values with respect to theoretical determinations

See also Bazavov et al, arXiv:2105.14019



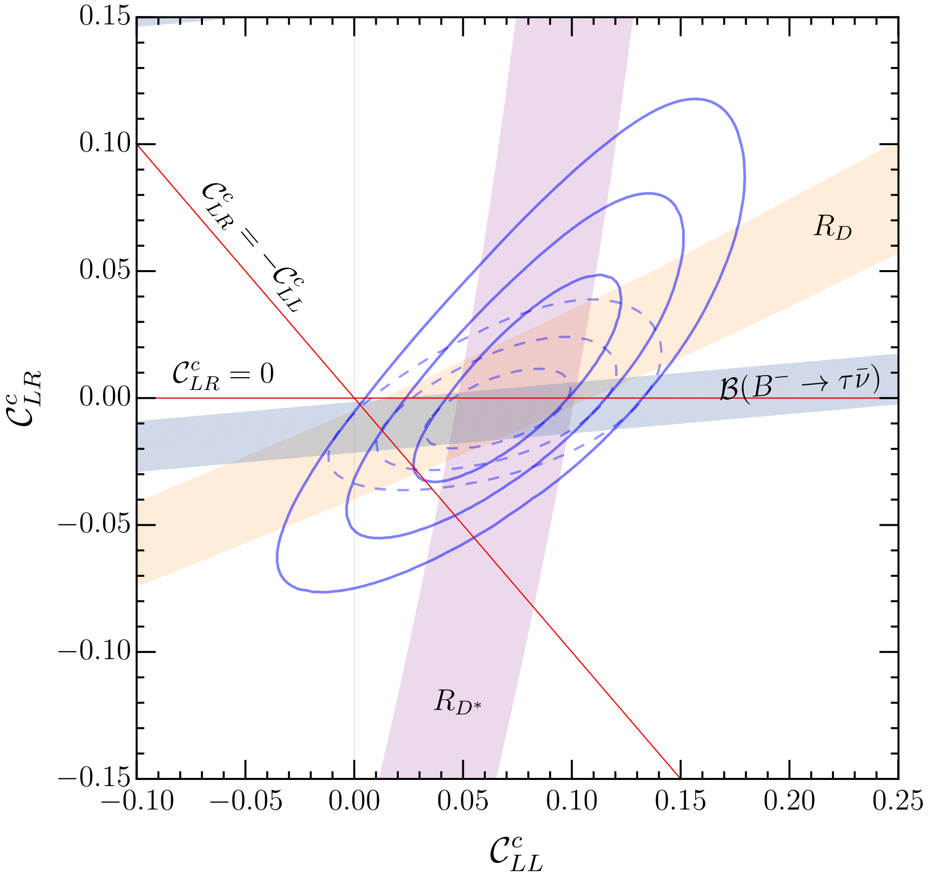
$$\mathcal{O}_{LL}^{ij\alpha\beta} = (\bar{q}_L^i \gamma_\mu \ell_L^\alpha)(\bar{\ell}_L^\beta \gamma^\mu q_L^j)$$

$$\mathcal{O}_{LR}^{ij\alpha\beta} = (\bar{q}_L^i \gamma_\mu \ell_L^\alpha)(\bar{e}_R^\beta \gamma^\mu d_R^j)$$

$$\mathcal{O}_{RR}^{ij\alpha\beta} = (\bar{d}_R^i \gamma_\mu e_R^\alpha)(\bar{e}_R^\beta \gamma^\mu d_R^j)$$

$$\mathcal{L}_{b\to u_i\tau\nu} = -\frac{4G_F}{\sqrt{2}} \sum_{i=1,2} V_{ib} \left[\left(1 + \mathcal{C}_{LL}^{u_i}\right)(\bar{u}_L^i \gamma_\mu b_L)(\bar{\tau}_L \gamma^\mu \nu_L) - 2\,\mathcal{C}_{LR}^{u_i}\,(\bar{u}_L^i b_R)(\bar{\tau}_R\,\nu_L) \right]$$

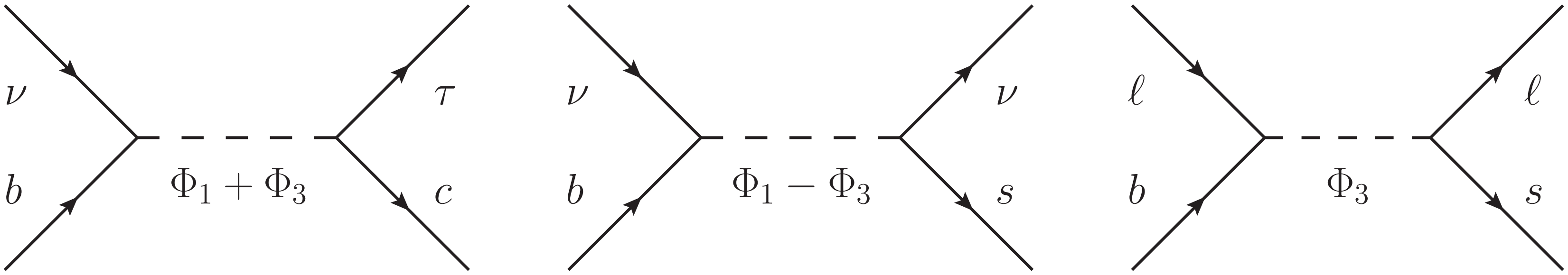
Cornella, Faroughy, Fuentes-Martin, Isidori, Neubert, arXiv:2103.16558



Common Solution to the B-Anomalies ?

- Leptoquarks may provide appropriate solutions to the B-Anomalies.
- For instance, the addition of an SU(2) triplet and singlet of charge -1/3 can contribute to these processes through the following amplitudes

Crivellin, Mueller, Ota, arXiv:1703.09226



(the minus and plus sign indicate destructive/constructive interference)

Alternative explanations

- An SU(2) singlet vector like leptoquark, with hipercharge 4/3 can lead to the common explanation of the B-anomalies.

See, for instance, Kumar, London, Watanabe, arXiv:1806.07403

- A model based on a custodial gauge extension of the SM,

$$SU(3)_c \times SU(2)_L \times SU(2)_R$$

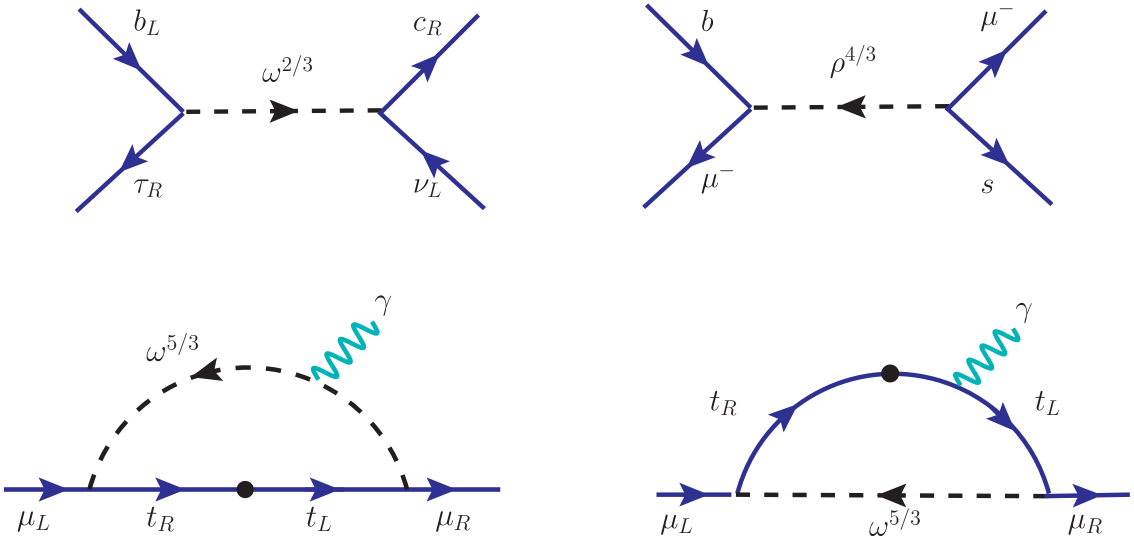
in warped extra dimensions, can also explain all B anomalies

Carena, Meguias, Quiros, CW, arXiv:1809.01107

Common Solution to the B-Anomalies and g-2 ?

Babu, Buphal Dev, Jana, Thapa, arXiv:2009.01771

- It can also be done within leptoquark models.
- There is a second option of leptoquarks, with charges (3, 2, 7/6), (3, 3, -1/3). The first of these can couple to both right and left handed muons and contribute to g-2.



Typical leptoquark masses of the order of a few TeV, that can be searched for at the HL-LHC

Conclusions

- The g-2 experiment at Fermilab and Flavor physics experiments, in particular LHCb have observed intriguing deviations of the SM predictions.
- Although these measurement are unrelated, they both involve muons.
- The g-2 anomaly may be explained in simple extensions of the Standard Model, including light scalars, vector bosons and Supersymmetry.
- The B physics anomalies demand a more complex field theoretical realization, but can be explained, for instance, within extended flavor dependent gauge sectors or leptoquarks.
- Leptoquarks can provide an explanation of all observed anomalies. However, their couplings must be carefully chosen.
- These anomalies will be checked in the coming few years. Let's hope that, if verified, their resolution leads to advances in our understanding of Nature.

Backup Slides

Comments on the current g-2 Anomaly

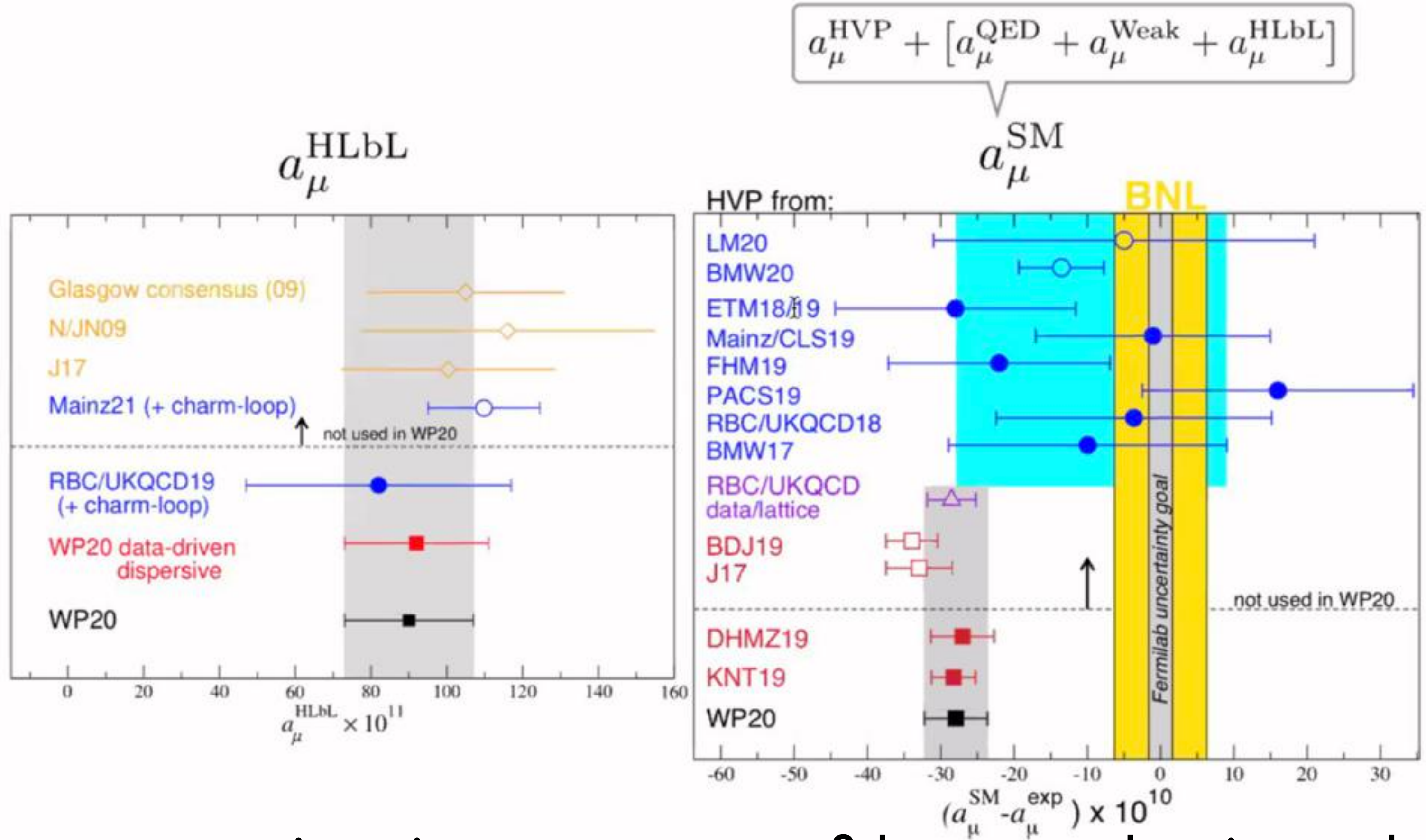
- In a sense, the current discrepancy is between the experimental determination of g-2, supported by the Brookhaven and the Fermilab g-2 experiments, and the e+e- hadronic cross section data.

- All other factors are, I believe, under good control and the uncertainties are small.

- In that sense, this anomaly should be taken very seriously. It is difficult to imagine where something could have gone wrong, even taken into account the current tension in the hadronic cross section data (KLOE vs BABAR), that cannot lead to an explanation of the measured anomaly, and has already been taken into account in the systematic errors.

- The good thing is that the g-2 collaboration will reduce the error by a factor 2 by next summer and there will be further work on the theoretical estimates.

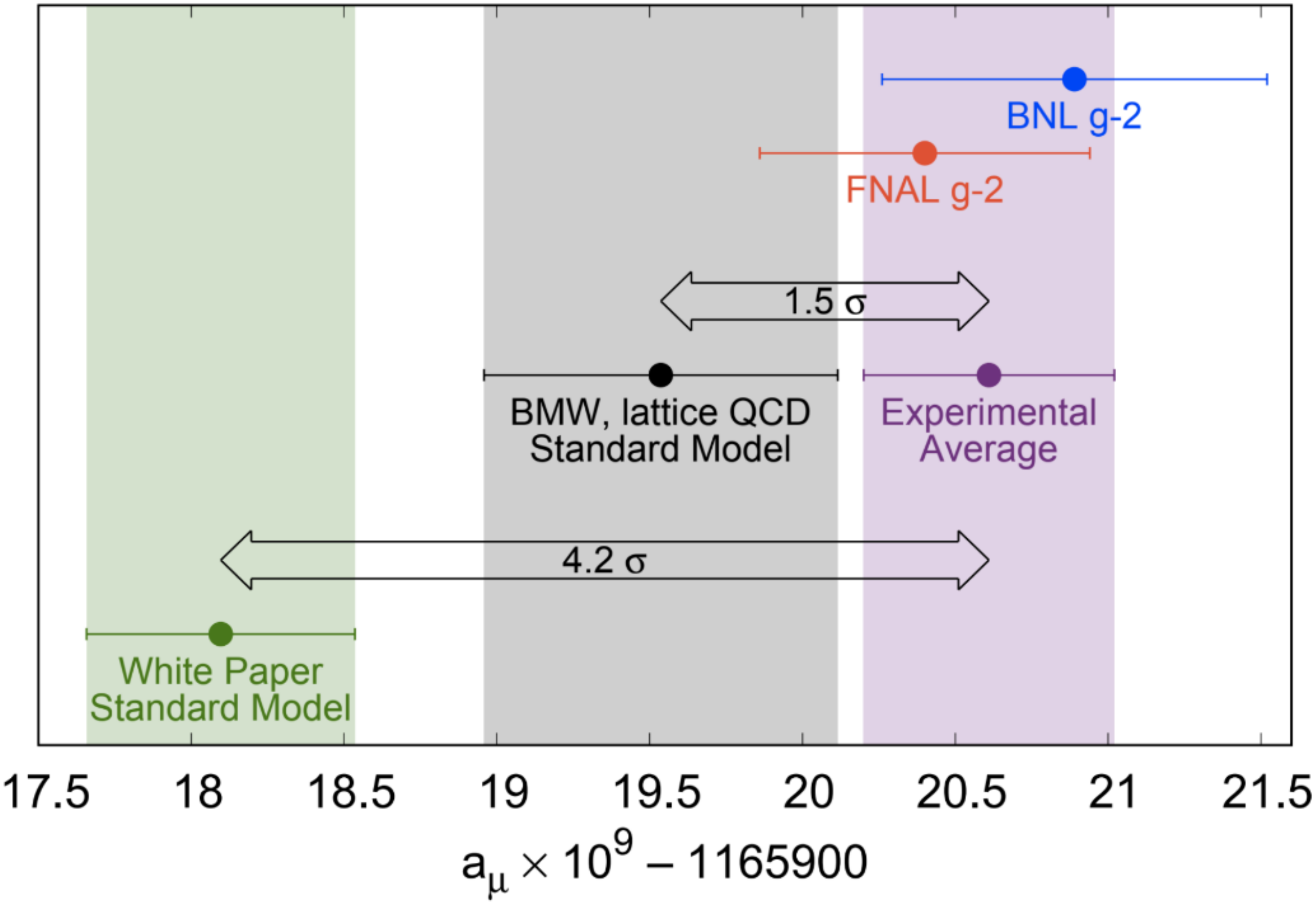
Lattice Computations



1. Lattice computations increase our confidence on the size and magnitude of the light by light contributions
2. In the computation of the hadronic vacuum polarization contributions, the BMW lattice collaboration finds results that reduce the tension with the g-2 experimental data. These results are hence in some tension with data driven evaluations.

Comparison of BMW lattice computation with data driven methods

Z. Fodor ‘ 21



Comments on the Lattice Evaluation

- The Lattice results should be taking seriously and are a triumph of physics.

- We should clearly wait for other lattice groups to corroborate the BMW result.

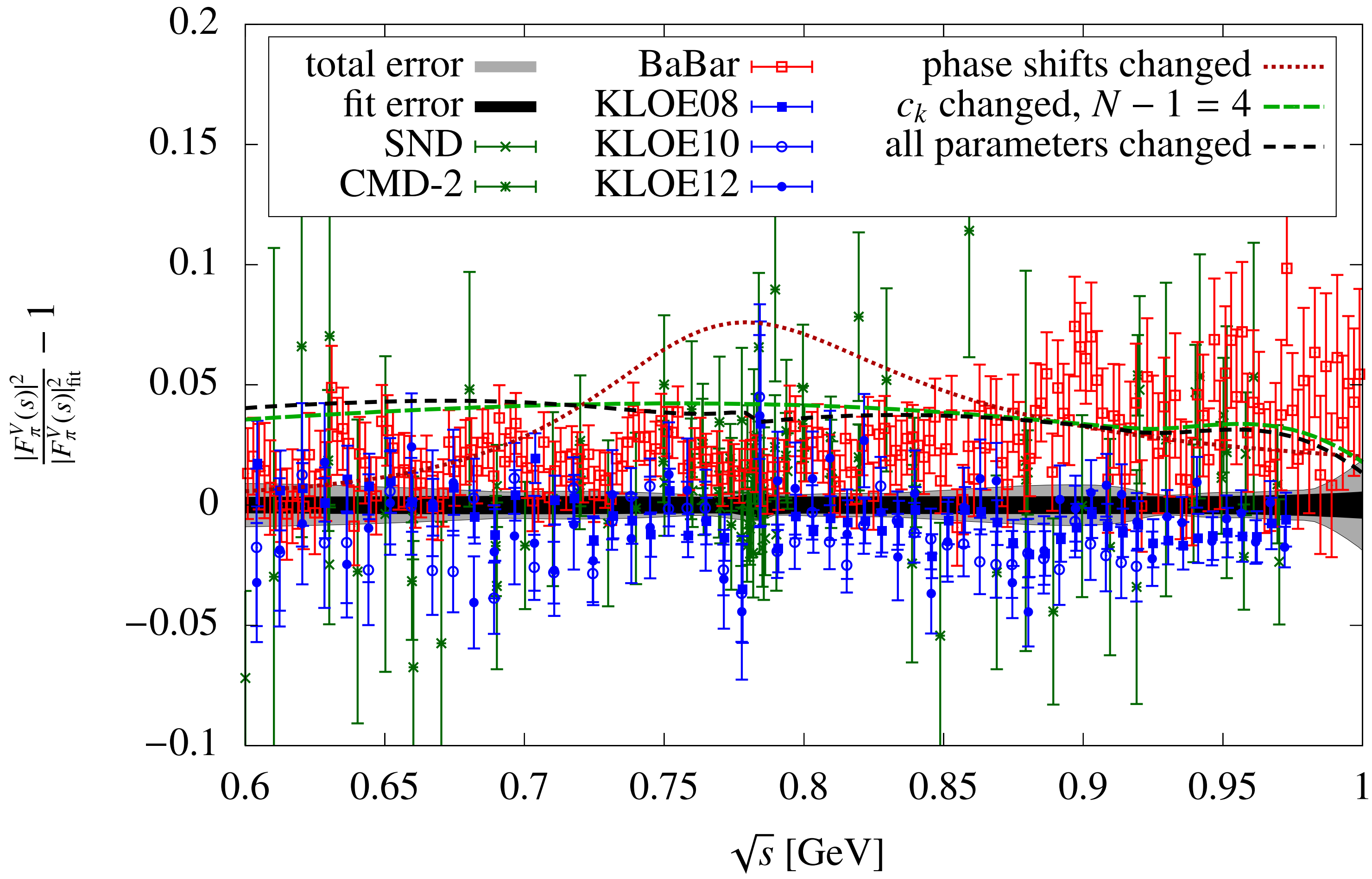
- HPV effects would have an impact on the variation of the fine structure constant, affecting precision measurements at Mz, and any correction from the current values should be limited to energies below 0.9 GeV, something that seems to be confirmed by the BMW study.

Crivellin et al, 2003.04886; Kezhavarzi, Marciano, Pasera, Sirlin, arXiv: 2006.12666

- Tension with data could be resolved by a large systematic error in the cross sections evaluation or by new physics contributing to them. Both possibilities look unlikely, but certainly not impossible.

- It could also be resolved by some unaccounted systematic error in the lattice evaluations. BMW provides a detailed account of their error estimates and it could be therefore double checked by other lattice groups.

What would be the value of the hadronic cross sections necessary for compatibility with lattice values ?



arXiv:2010.07943

Many other Solutions

- Axion light particles (beyond the naive one loop solution)
- Leptoquarks, for suitable arrangement of couplings
- Two Higgs doublet models, for certain arrangement of the Higgs mass splittings…

- Are any of these theories connected to a further understanding of physics at high energies ?

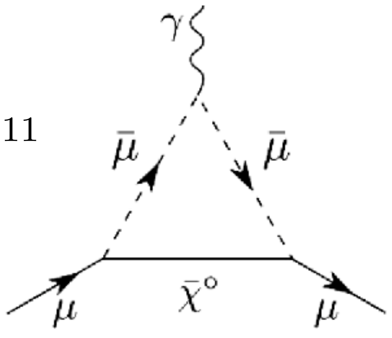
Rough Approximation

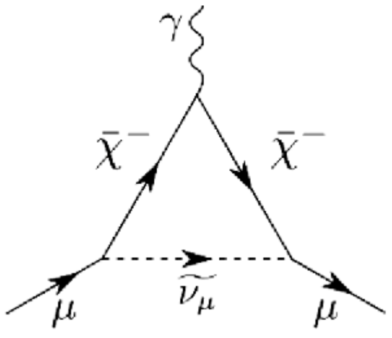
- If all weakly interacting supersymmetric particle masses were the same, and the gaugino masses had the same sign, then

$$(\Delta a_\mu)^{\mathrm{SUSY}} \simeq 150 \times 10^{-11} \left(\frac{100\ \mathrm{GeV}}{m_{\mathrm{SUSY}}}\right)^2 \tan\beta$$

- This implies that, for tanβ = 10, particle masses of order 250 GeV could explain the anomaly, while for values of tanβ = 60 (consistent with the unification of the top and bottom Yukawa) these particle masses could be of order 700 GeV.

$$\Delta a_\mu \equiv (a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}) = (251 \pm 59) \times 10^{-11}$$





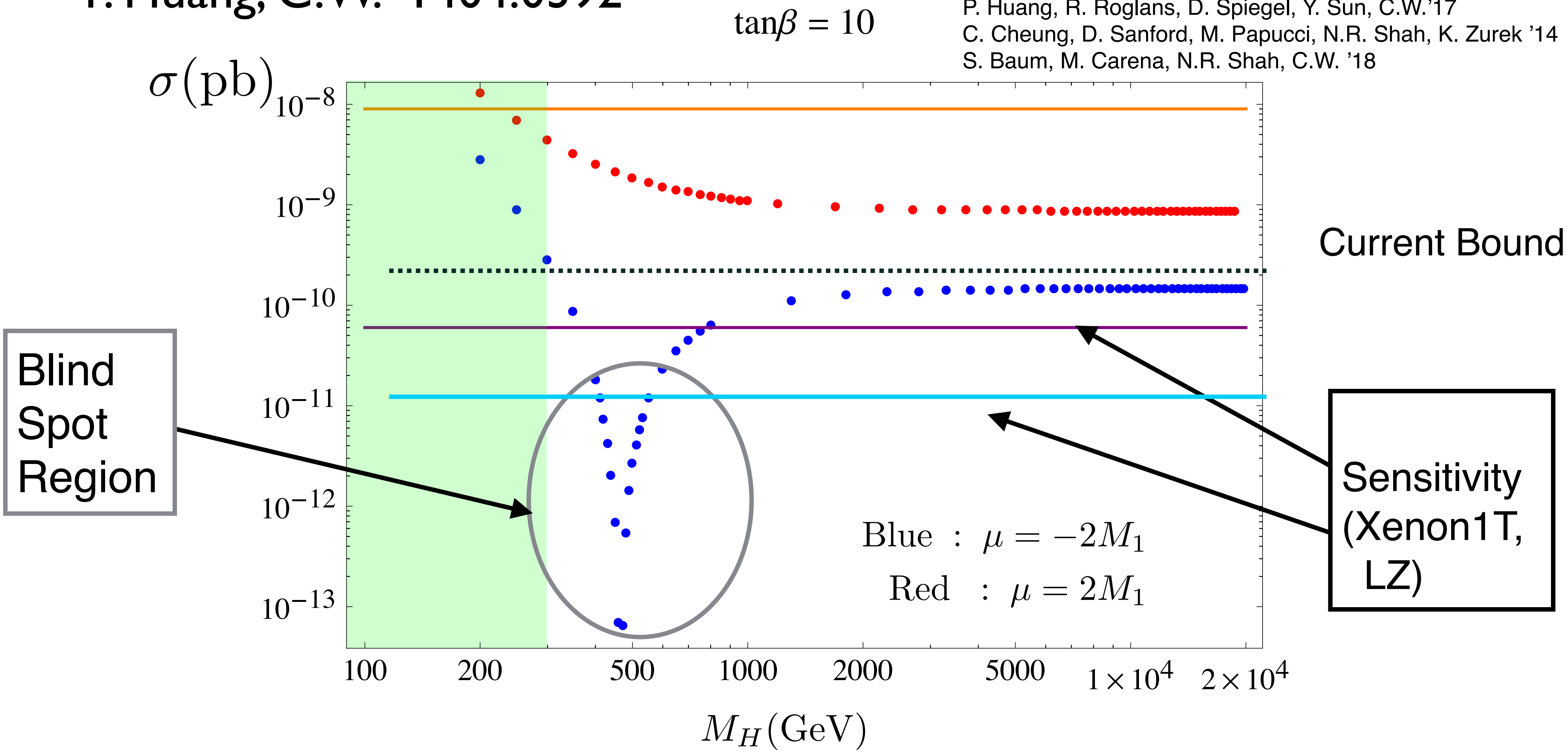
Dependence of the cross section on the heavy Higgs mass

Negative values of μ : Much weaker direct spin-independent detection bounds

Blind Spots : $$2\left(m_{\chi^0} + \mu \sin 2\beta\right)\frac{1}{m_h^2} = -\mu \tan\beta \frac{1}{M_H^2}$$

H. Baer, A. Mustafayev, E.K. Park, X. Tata'07
C. Cheung, L. Hall, D. Pinner, J. Ruderman'12
P. Huang, C.W.'14
P. Huang, R. Roglans, D. Spiegel, Y. Sun, C.W.'17
C. Cheung, D. Sanford, M. Papucci, N.R. Shah, K. Zurek '14
S. Baum, M. Carena, N.R. Shah, C.W. '18

P. Huang, C.W. 1404.0392



$$m_{\chi^\pm} = |\mu|, \quad m_\chi^0 = M_1, \qquad M_1 = 200\ \mathrm{GeV}$$

$$R_{K^{(*)}} = 1 + \mathcal{O}\left(\frac{m_\mu^2}{q^2}\right) \times \left(1 + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right) + \mathcal{O}\left(\alpha_s\right)\right) + \mathcal{O}\left(\frac{\alpha_{\rm em}}{\pi}\log^2\left(\frac{m_e^2}{m_\mu^2}\right)\right)$$

phase space
(tiny effect)

hadronic corrections
(tiny effect)

QED corrections
(soft and collinear
photon emission)

- QED corrections seem to be under control at the level of the total rate, given the experimental cuts on e.g. the reconstructed B meson mass

  Bordone, Isidori, Pattori 1605.07633, Isidori, Nabeebaccus, Zwicky 2009.00929

$$R_K^{[1,6]} = 1.00 \pm 0.01 \;, \quad R_{K^*}^{[1.1,6]} = 1.00 \pm 0.01 \;, \quad R_{K^*}^{[0.045,1.1]} = 0.91 \pm 0.03$$

- potentially larger QED effects at the differential level

New measurements of R_K and R_{K^*} in the decays
$B^0 \to K_S \ell^+ \ell^-$ and $B^+ \to K^{*+} (\to \pi^+ K_S) \ell^+ \ell^-$

Measurements are much more difficult because the K_S is hard to detect

$$R_{K_S} = 0.66^{+0.20}_{-0.14}{}^{+0.02}_{-0.04}$$

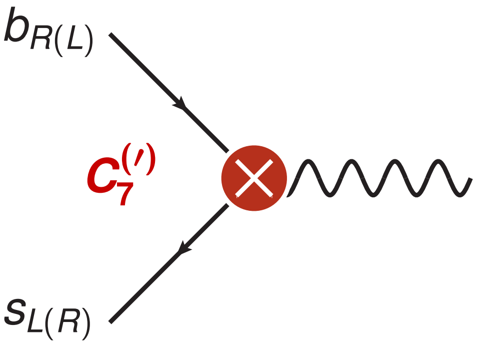
$$R_{K^{*+}} = 0.70^{+0.18}_{-0.13}{}^{+0.03}_{-0.04}$$

Large uncertainties, but both are $\sim 1.5\sigma$ below the SM prediction.

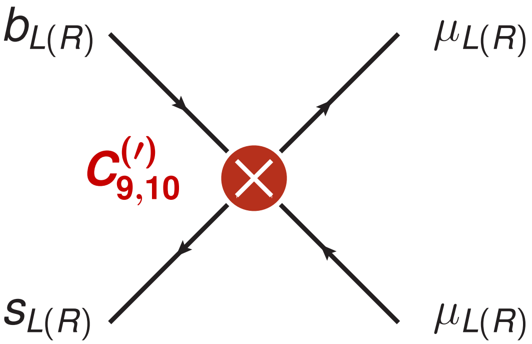
LHCb 2110.09501

$$\mathcal{H}_{\text{eff}}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_i \left(C_i \mathcal{O}_i + C_i' \mathcal{O}_i' \right)$$

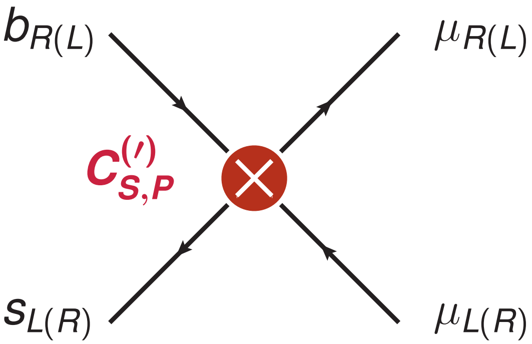
magnetic dipole operators



semileptonic operators



scalar operators



$$C_7^{(\prime)} (\bar{s}\sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu} \quad , \quad C_9^{(\prime)} (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\mu}\gamma^\mu \mu) \quad , \quad C_S^{(\prime)} (\bar{s} P_{R(L)} b)(\bar{\mu} P_{L(R)} \mu)$$

$$C_{10}^{(\prime)} (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\mu}\gamma^\mu \gamma_5 \mu)$$

Main Contribution of Different Operators

Altmannshofer'21

	$C_7,\ C_7'$	$C_9,\ C_9'$	$C_{10},\ C_{10}'$	$C_S,\ C_S'$
$B \to (X_s, K^*)\gamma$	★			
$B_s \to \phi\gamma$	★			
$B \to (X_s, K, K^*)\ \mu^+\mu^-$	★	★	★	⋆
$B_s \to \phi\ \mu^+\mu^-$	★	★	★	⋆
$\Lambda_b \to \Lambda\ \mu^+\mu^-$	★	★	★	⋆
$B_s \to \mu^+\mu^-$			★	★