

Study of A_{LR} using radiative return events at ILC250

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Physics at ILC 250

- Primary physics aim of ILC 250: measuring the **coupling constants between the Higgs boson and various other standard model (SM) particles**
 - The coupling constants can deviate from their SM values because of possible Beyond the Standard Model (BSM) effects.
 - The size of corrections to the SM predictions for the Higgs boson couplings:
$$a \frac{m_H^2}{M^2}$$

M : new particle mass [GeV]
 m_H : 125 [GeV]
 a : coefficients of order 1
- Need to measure the couplings with the precision $\sim 1\%$

SM effective field theory (SMEFT)

- Lorentz invariant and $SU(2) \times U(1)$ gauge invariant dim-6 operators

$$\begin{aligned}
 \Delta\mathcal{L} = & \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \overleftrightarrow{D}^\mu \Phi) (\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3 \\
 & + \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger \Phi W_{\mu\nu}^a W^{a\mu\nu} + \frac{4gg' c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W_{\mu\nu}^a B^{\mu\nu} \\
 & + \frac{g'^2 c_{BB}}{m_W^2} \Phi^\dagger \Phi B_{\mu\nu} B^{\mu\nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{abc} W_{\mu\nu}^a W^{b\nu\rho} W^{c\rho\mu} \\
 & + i \frac{c_{HL}}{v^2} (\Phi^\dagger \overleftrightarrow{D}^\mu \Phi) (\bar{L} \gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \overleftrightarrow{D}^\mu \Phi) (\bar{L} \gamma_\mu t^a L) \\
 & + i \frac{c_{HE}}{v^2} (\Phi^\dagger \overleftrightarrow{D}^\mu \Phi) (\bar{e} \gamma_\mu e)
 \end{aligned}$$

ILC, Higgs related part

(CP-Conserving)

- The useful observables for the SMEFT global fit include not only those from the reactions that directly involve the Higgs boson, but also those from Electroweak Precision Observables (EWPOs) for W and Z bosons.

For example, left-right polarization asymmetry A_{LR} of the Z-pole cross section is important.

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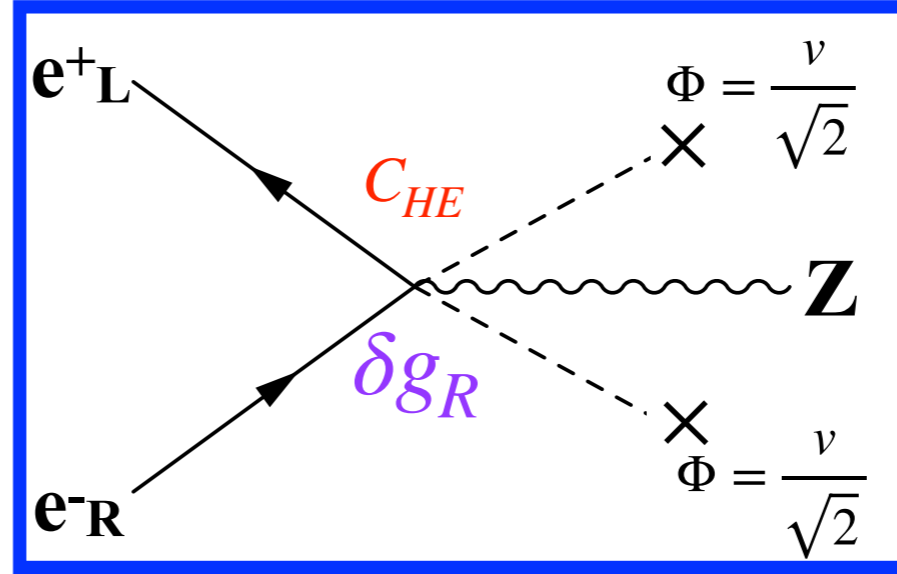
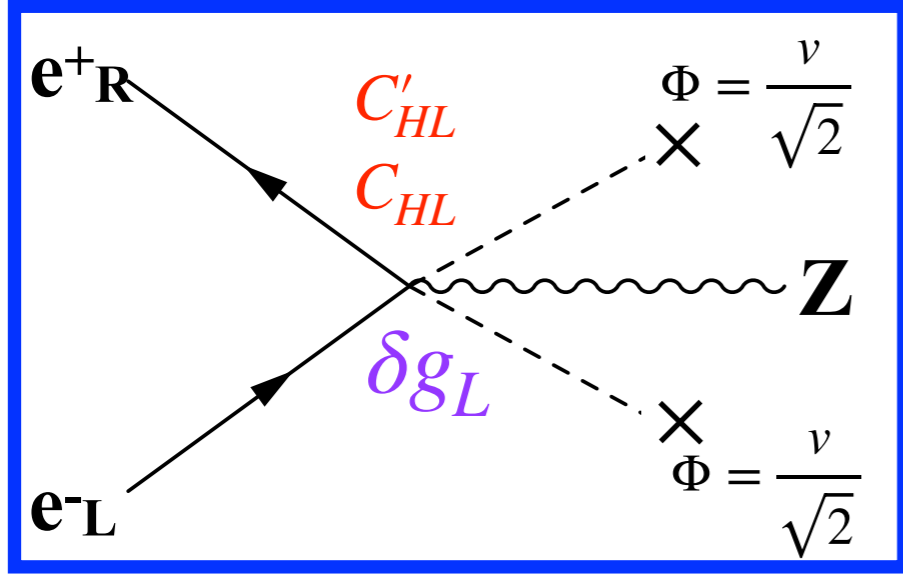
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For example, left-right polarization asymmetry A_{LR} of the Z-pole cross section is important.

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$$(vZ^\mu\nu) \quad (vt^a Z^\mu\nu)$$

$$(vZ^\mu\nu)$$



$$g_L = \frac{g}{c_W} \left[\left(-\frac{1}{2} + s_W^2 \right) \left(1 + \frac{1}{2} \delta Z_Z \right) - \frac{1}{2} (c_{HL} + c'_{HL}) - s_W c_W \delta Z_{AZ} \right]$$

$$g_R = \frac{g}{c_W} \left[\left(+s_W^2 \right) \left(1 + \frac{1}{2} \delta Z_Z \right) - \frac{1}{2} c_{HE} - s_W c_W \delta Z_{AZ} \right]$$

$$\delta Z_Z = c_W^2 (\delta c_{WW}) + 2s_W^2 (\delta c_{WB}) + \frac{s_W^4}{c_W^2} (\delta c_{BB})$$

$$\delta Z_{AZ} = s_W c_W \left((\delta c_{WW}) - \left(1 - \frac{s_W}{c_W^2} \right) (\delta c_{WB}) - \frac{s_W}{c_W^2} (\delta c_{BB}) \right)$$

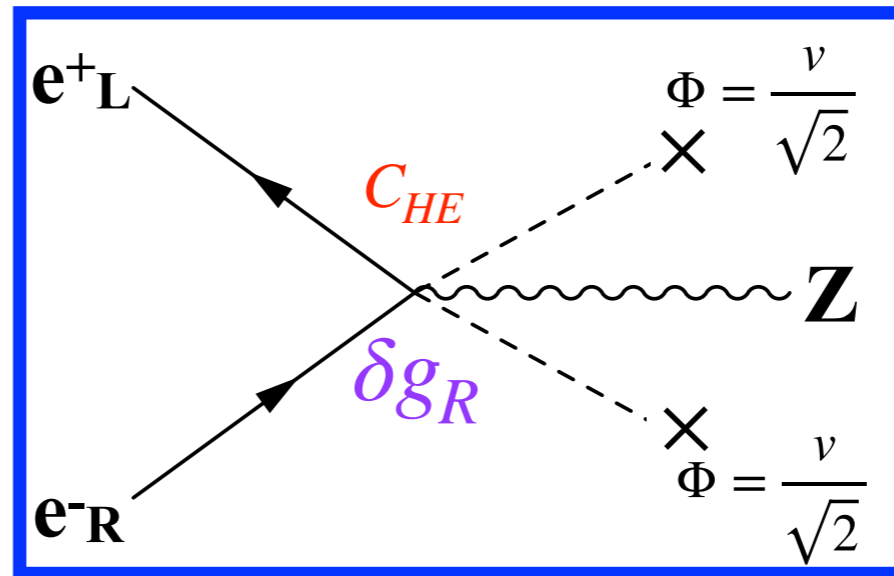
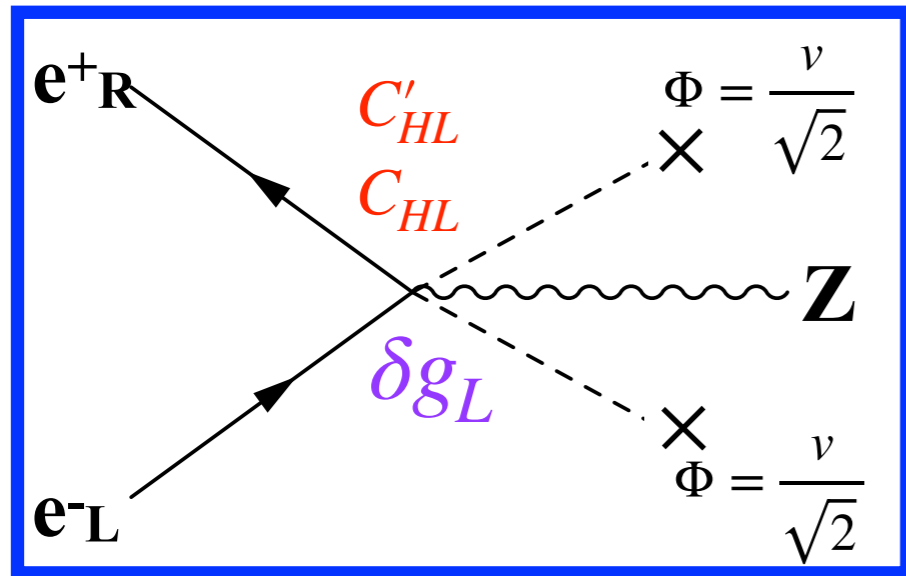
$$A_{LR} = A_e = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2}$$

$$\delta A_e = \frac{4g_{Le}^2 g_{Re}^2 (\delta g_{Le} - \delta g_{Re})}{g_{Le}^4 - g_{Re}^4}$$

$$\Delta\mathcal{L} = i\frac{C_{HL}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\bar{L}\gamma_\mu L) + 4i\frac{C'_{HL}}{v^2}(\Phi^\dagger t^a \overleftrightarrow{D}^\mu \Phi)(\bar{L}\gamma_\mu t^a L) + i\frac{\bar{C}_{HE}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\bar{e}\gamma_\mu e)$$

$$(vZ^\mu\nu) \quad (vt^a Z^\mu\nu)$$

$$(vZ^\mu\nu)$$



Current best measurement

$$A_{LR} = 0.1514 \pm 0.0019 \text{ (statistic error)} \pm 0.0011 \text{ (systematic error)}$$

Need full detector simulation of radiative return events to check how much error we have at ILC 250

$$A_{LR} = A_e = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2}$$

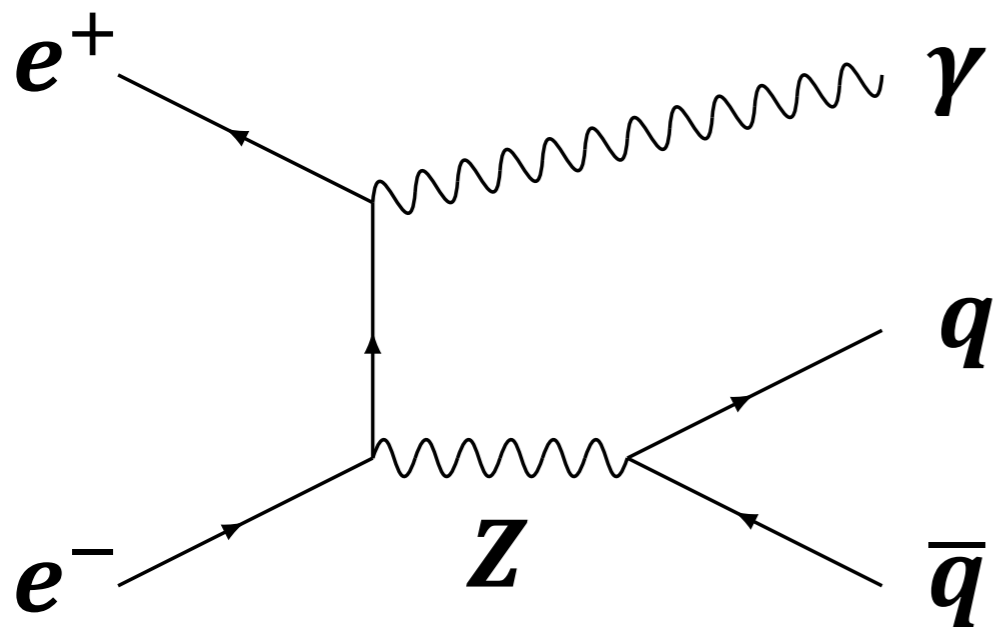
$$\delta A_e = \frac{4g_{Le}^2 g_{Re}^2 (\delta g_{Le} - \delta g_{Re})}{g_{Le}^4 - g_{Re}^4}$$

Simulation Setup

Full simulation

- $E_{\text{CM}} = 250 \text{ GeV}$
- $\int \mathcal{L} dt = 900 \text{ fb}^{-1}$ for each of the 2 polarization combinations
- $\sin \theta_W = 0.22225$ which is equivalent to $A_{\text{LR}} = 0.219298$.

Signal: $e^+e^- \rightarrow Z\gamma \rightarrow 2 \text{ jets} + \gamma$

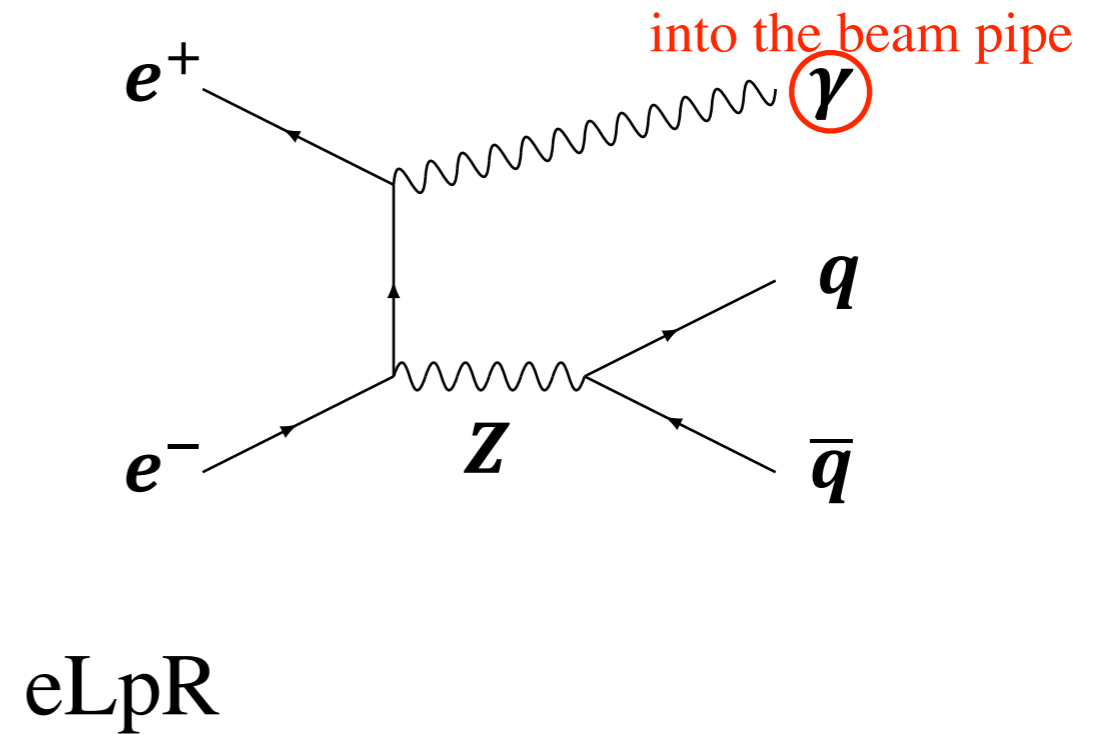
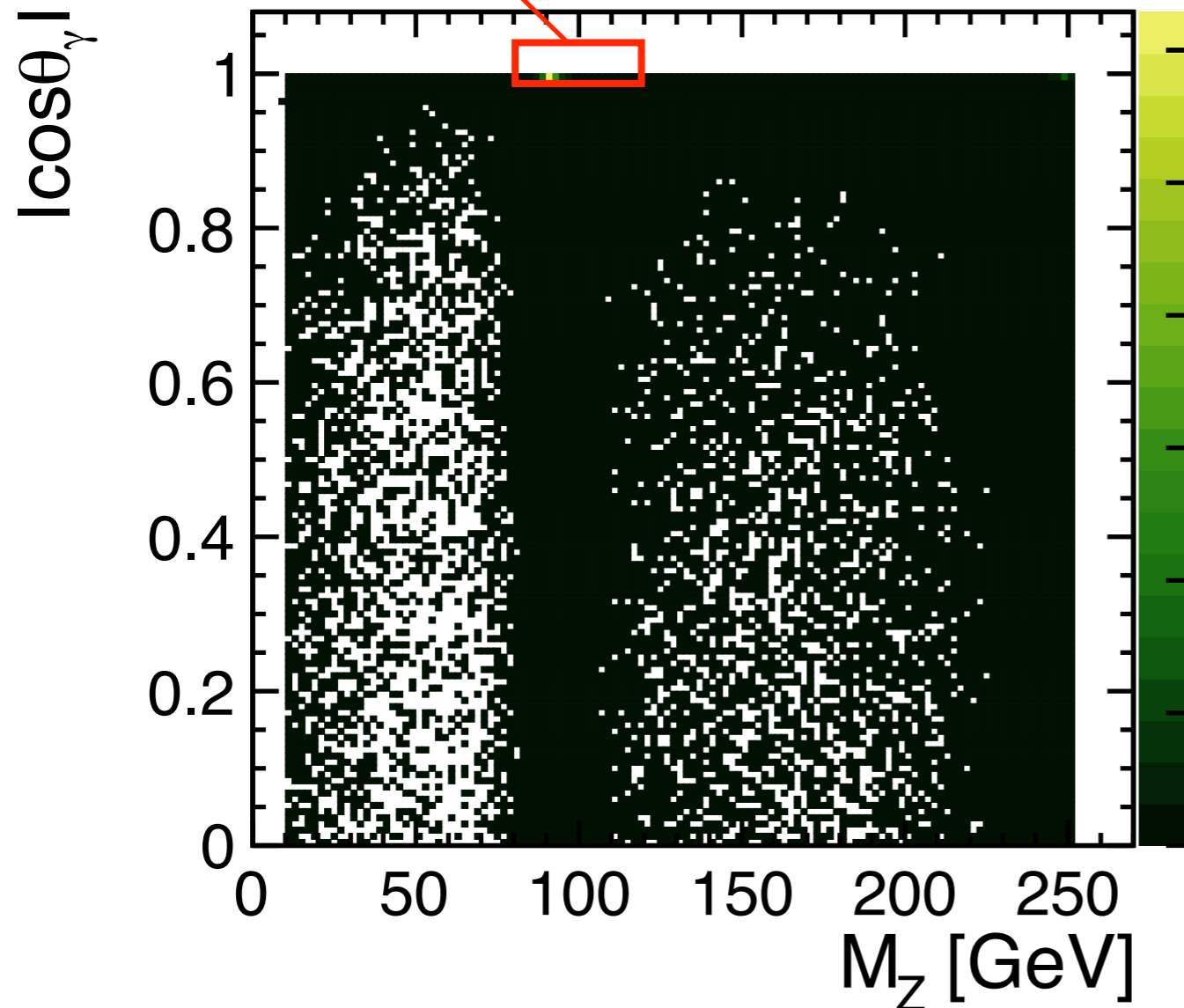


Background: All the events only with 2 jets in the final state

Signal event definition

Signal event: radiative return with photon escaping into beam pipe

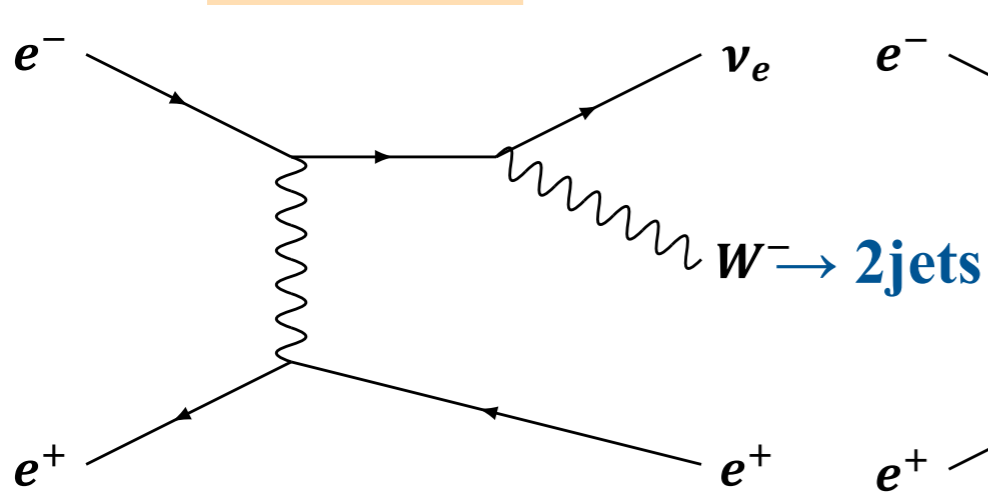
- A. $80 \text{ GeV} < M_{Z(\text{truth})} < 120 \text{ GeV}$
- B. $|\cos\theta_{\gamma(\text{truth})}| > 0.999$



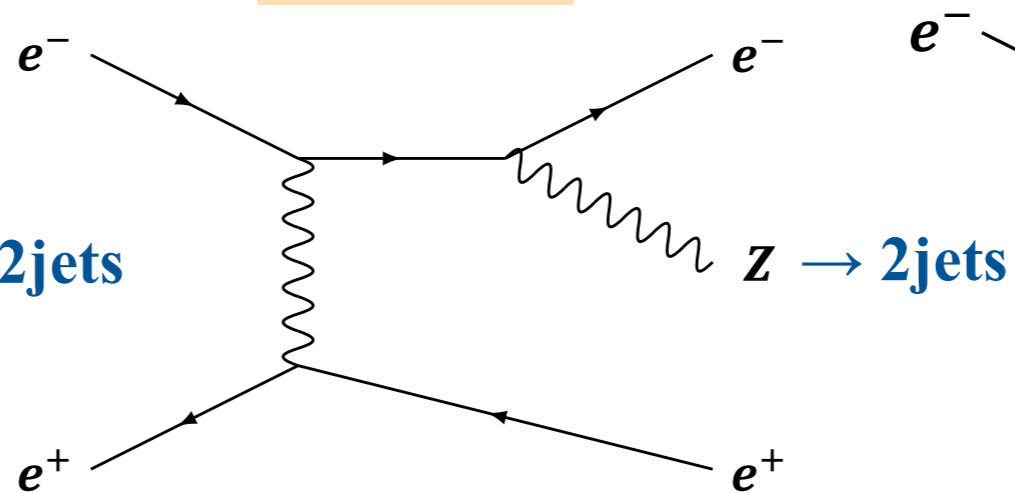
eLpR

Major backgrounds

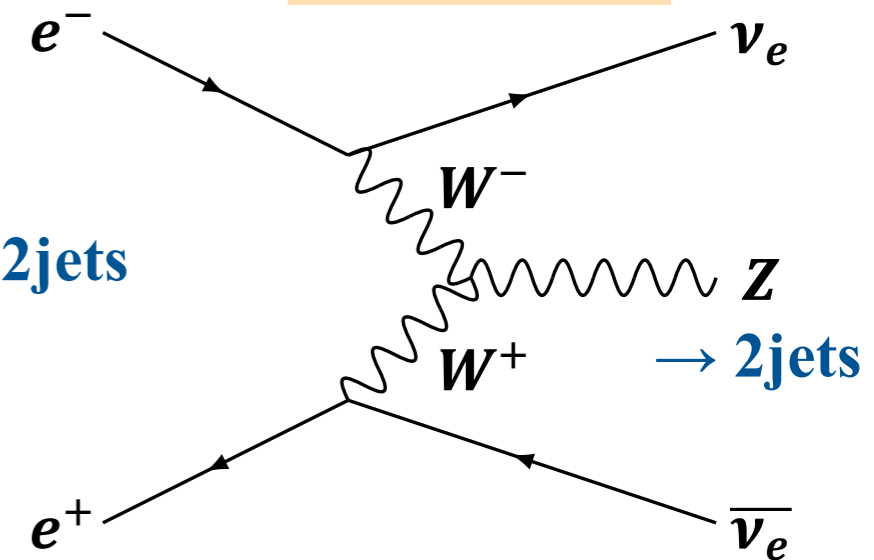
4f_sw_sl



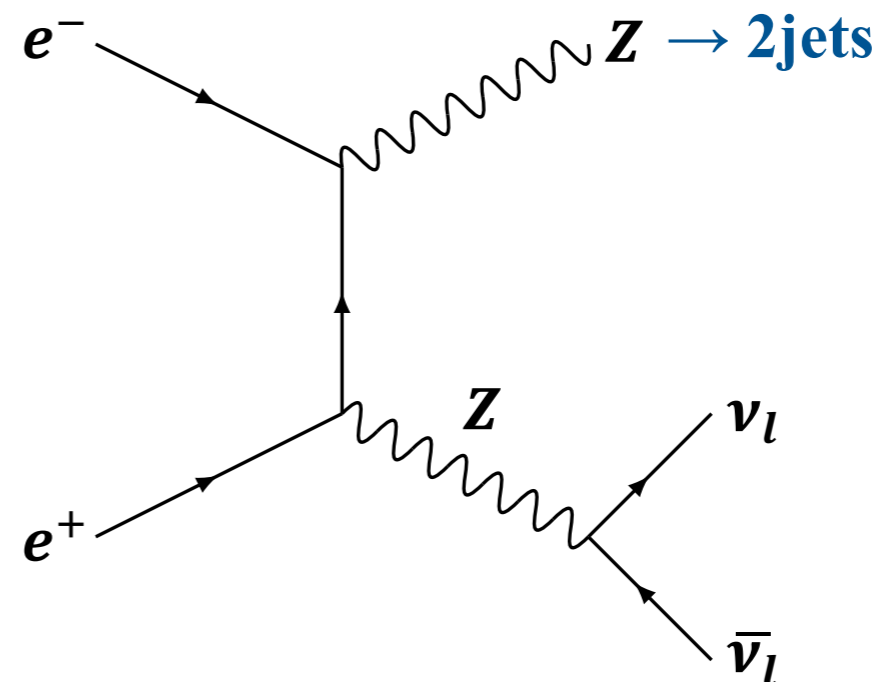
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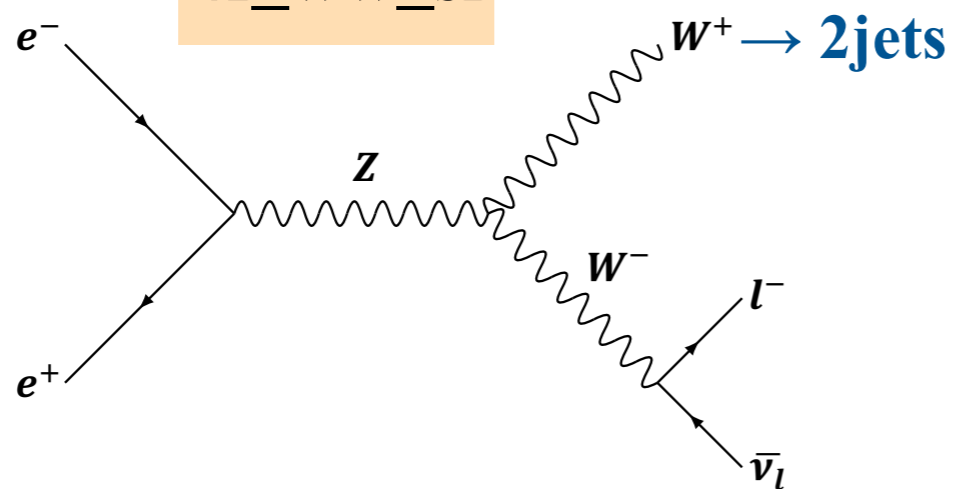
4f_sznu_sl



4f_zz_sl



4f_ww_sl



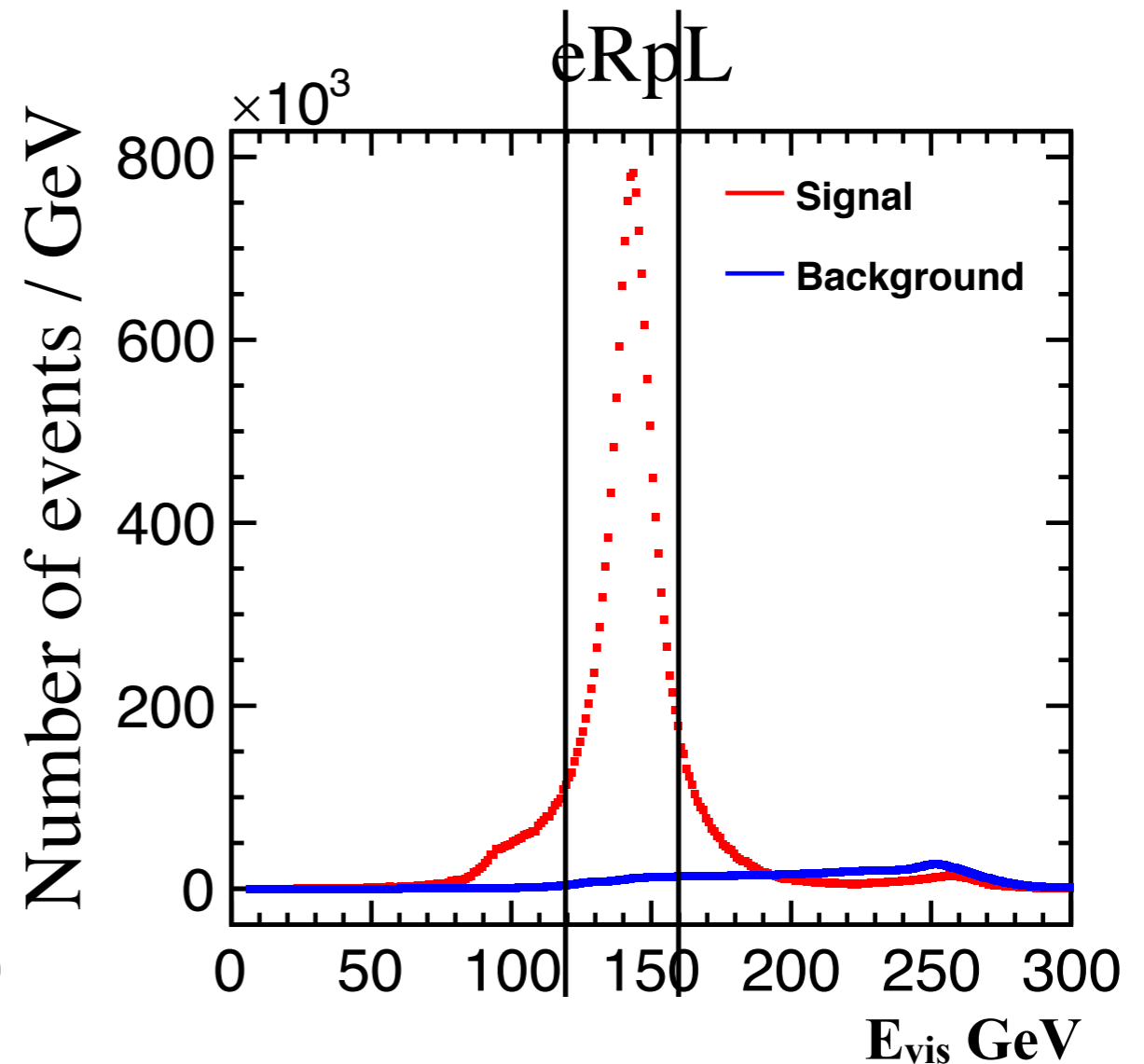
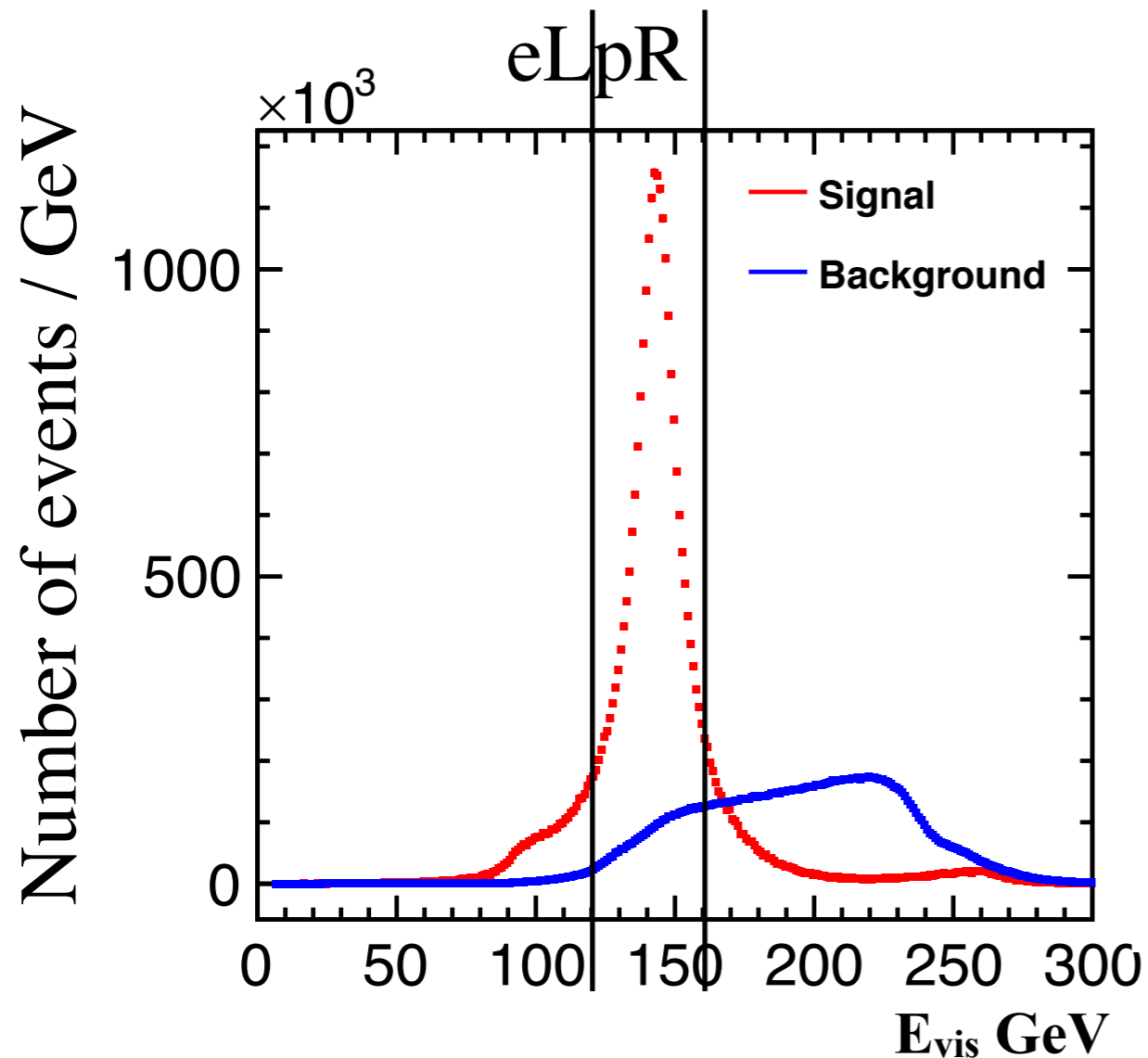
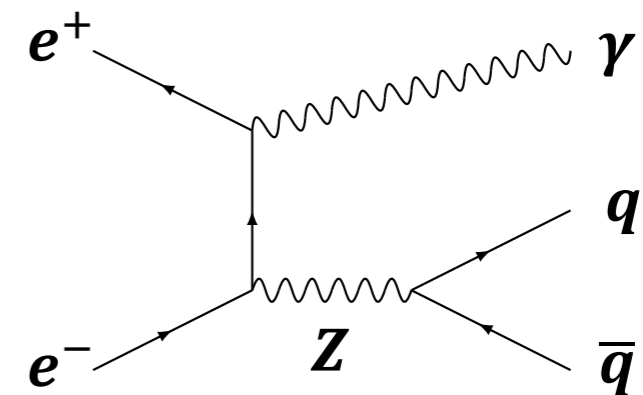
Event selection

Cut 1 $N_{\gamma(E>50\text{ GeV})} = 0$

Cut 2 $120\text{ GeV} < E_{vis} < 160\text{ GeV}$

Cut 3 $|\cos\theta_{2j}| > 0.95$

Cut 4 $50\text{ GeV} < M_{2j} < 160\text{ GeV}$



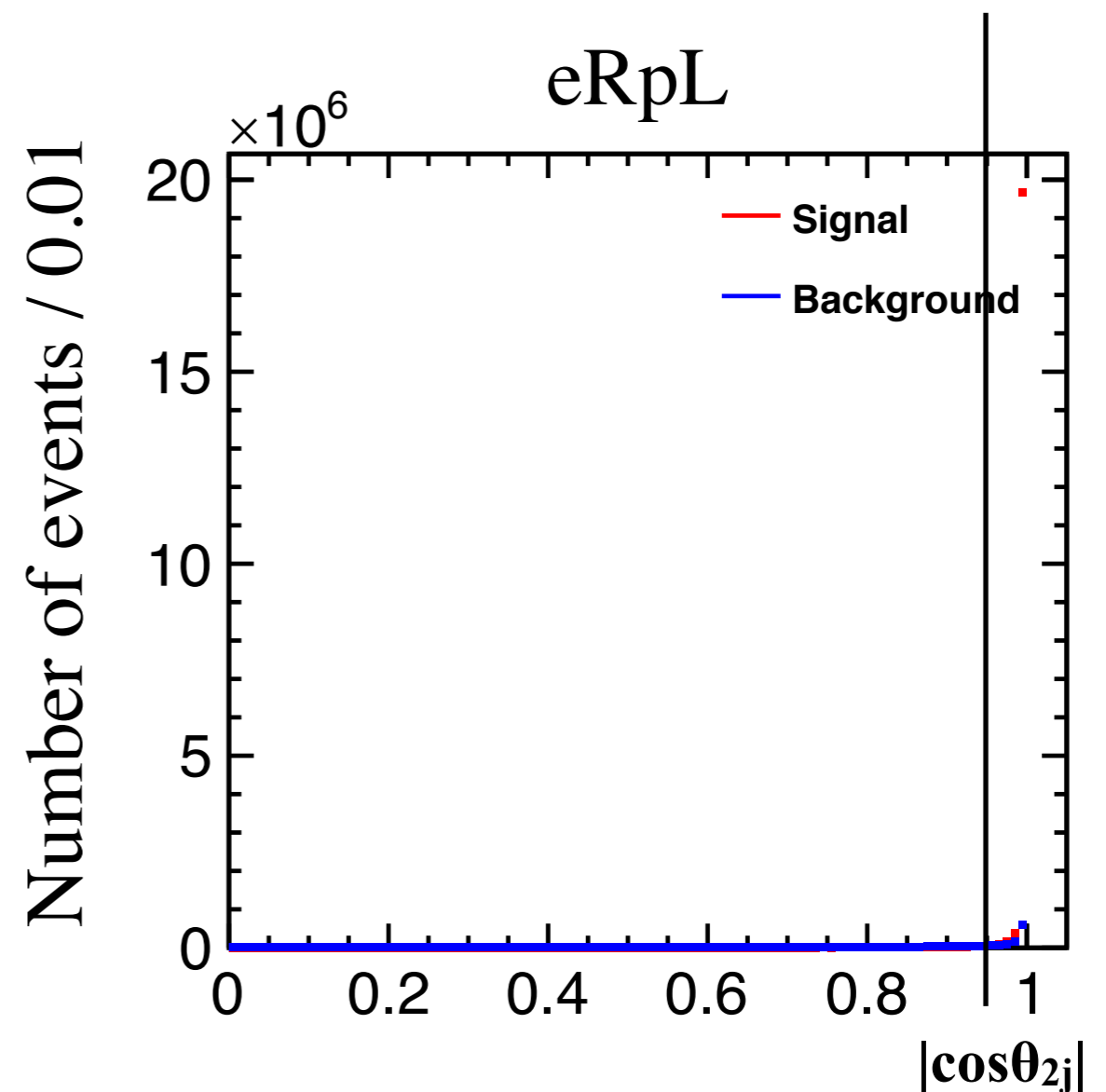
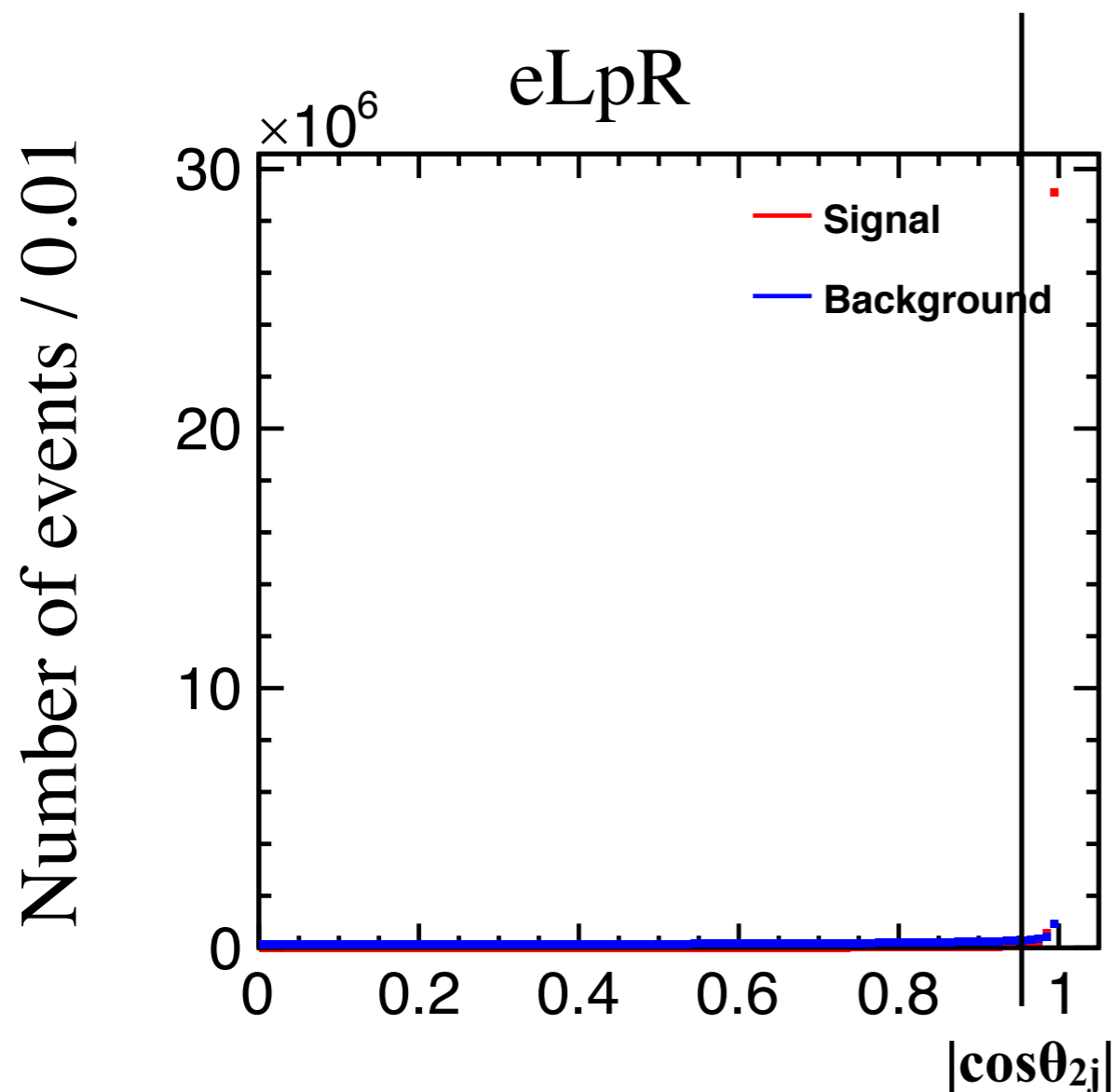
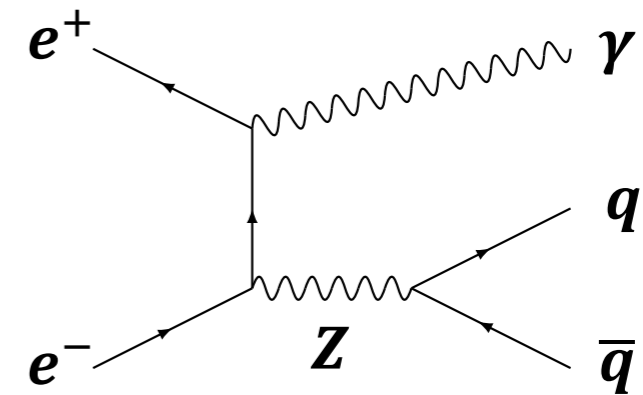
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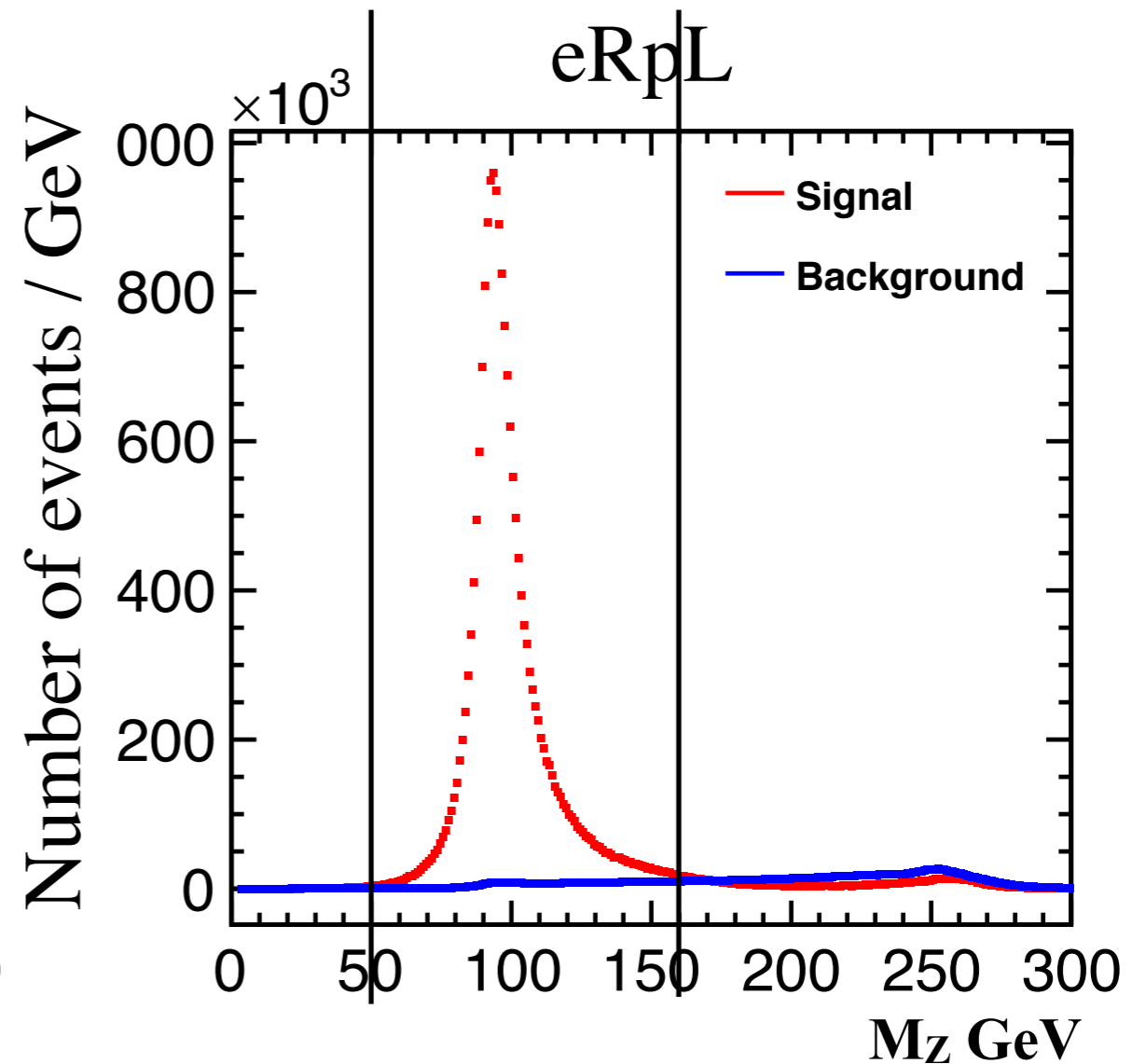
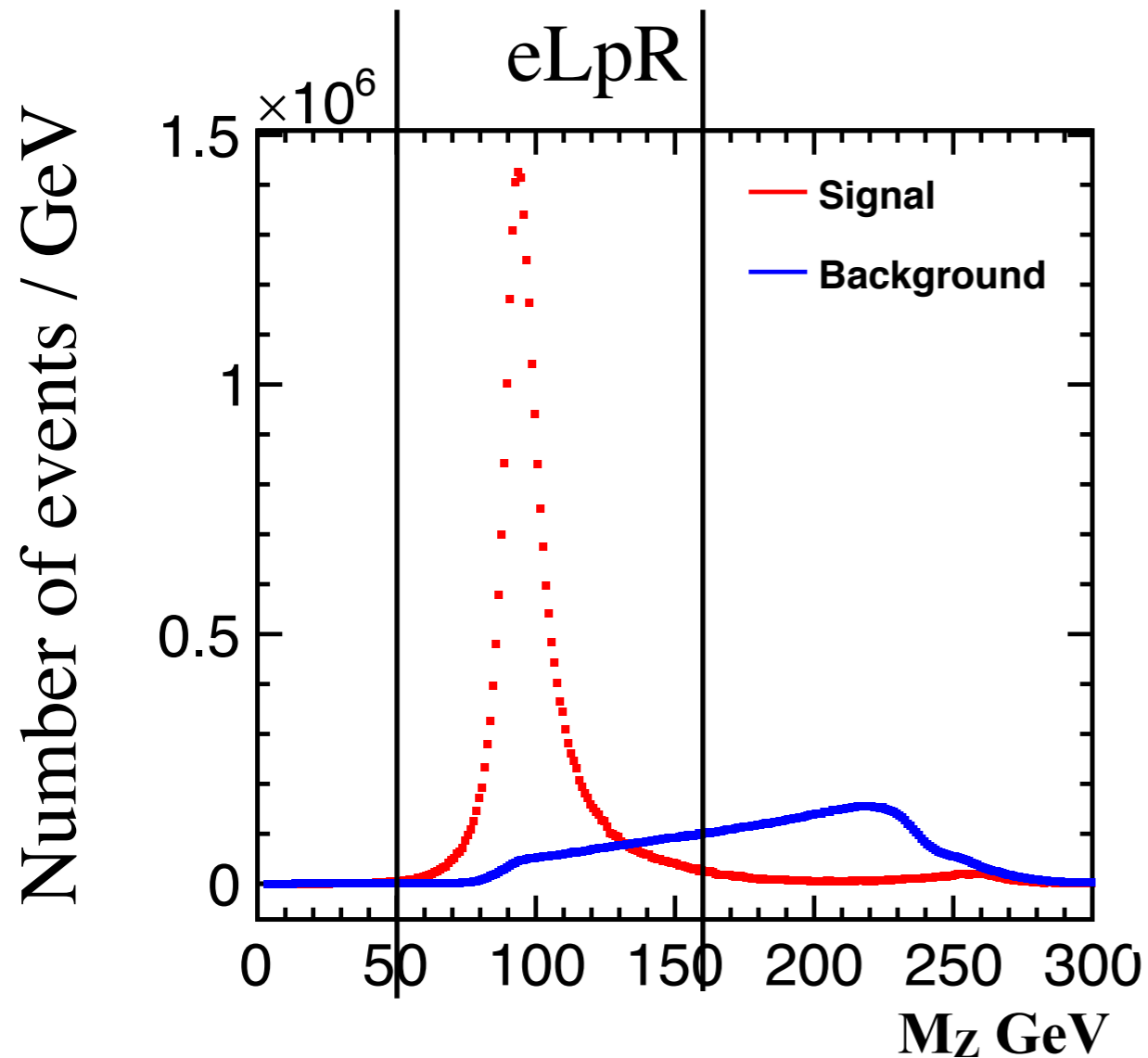
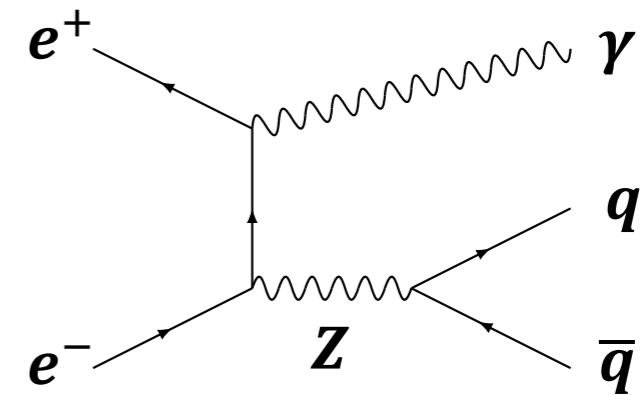
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Cut Table

Number of events with $(P_{e^-}, P_{e^+}) = (-0.8, +0.3)$ polarization

Process	Signal	4f_sw_sl	4f_size_sl	4f_sznu_sl	4f_ww	4f_zz	Background	Total
Expected	3.25017e+07	5.4719e+06	1.18316e+06	243096	9.89268e+06	455954	1.72468e+07	
Cut 1	3.10963e+07	5.10134e+06	534339	241228	9.64957e+06	432962	1.59594e+07	
Cut 2	2.44416e+07	566437	25287.2	140890	2.40774e+06	75801.6	3.21616e+06	
Cut 3	2.44199e+07	61265.7	23784.1	18126.3	362353	60237.6	525767	
Cut 4	2.44198e+07	61190	23730.7	18022.4	362353	60061.6	525358	

Efficiency = 0.75134 ±0.00028

B/S= 0.02151 for (-0.8, +0.3)

Number of events with $(P_{e^-}, P_{e^+}) = (+0.8, -0.3)$ polarization

Process	Signal	4f_sw_sl	4f_size_sl	4f_sznu_sl	4f_ww	4f_zz	Background	Total
Expected	2.15581e+07	434154	1.08253e+06	83384	682874	272178	2.55512e+06	
Cut 1	2.06154e+07	399858	455053	82533.5	666727	257213	1.86138e+06	
Cut 2	1.62129e+07	66637.4	24657.1	54821.6	173510	50281.4	369907	
Cut 3	1.61991e+07	11216.1	23192.1	7382.85	23112.3	41545.2	106448	
Cut 4	1.6199e+07	11189.3	23132	7314.02	23112	41358.8	106106	

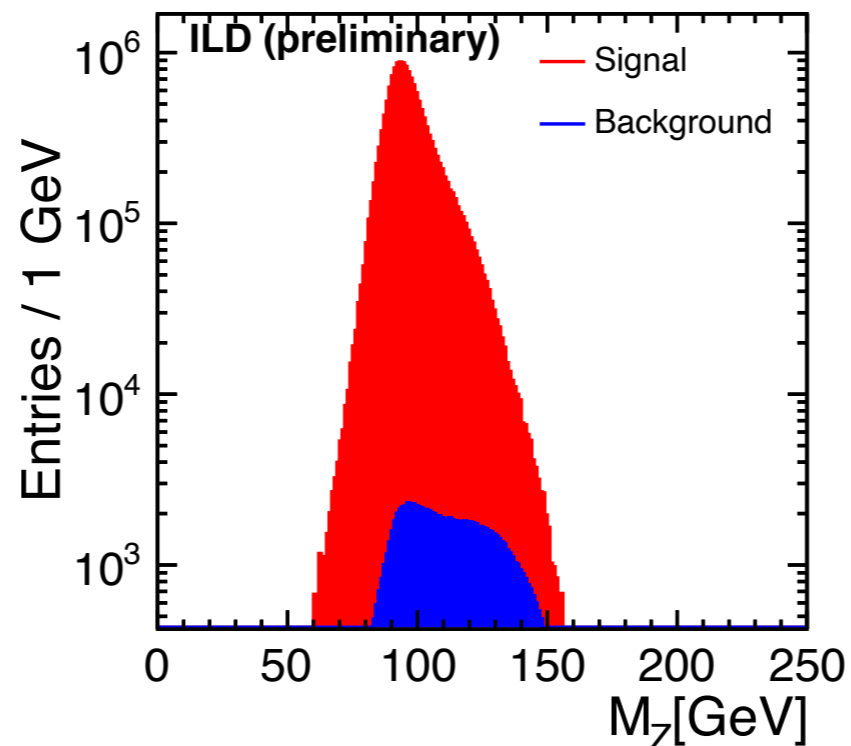
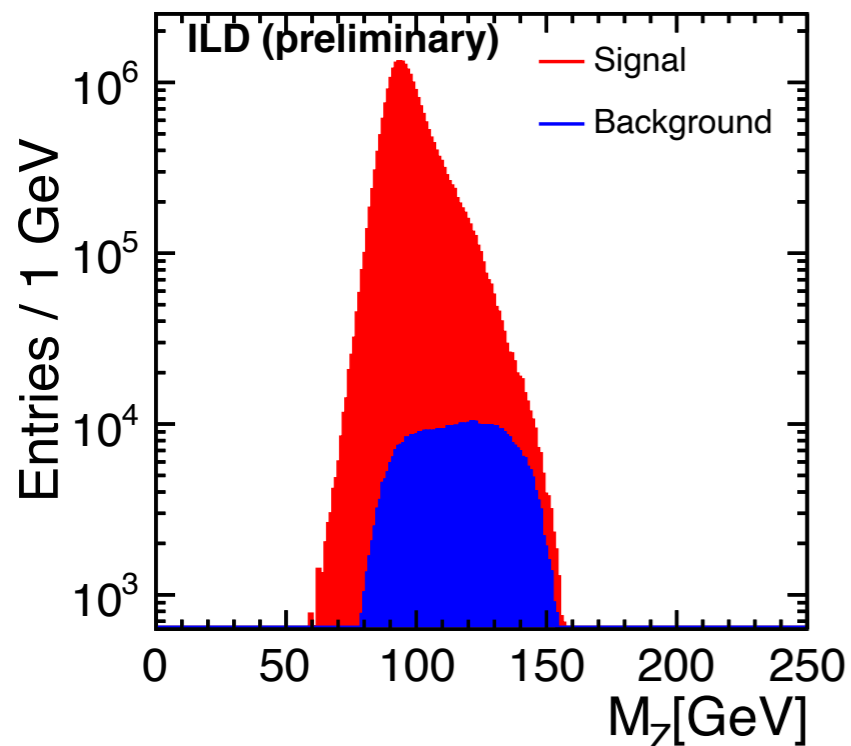
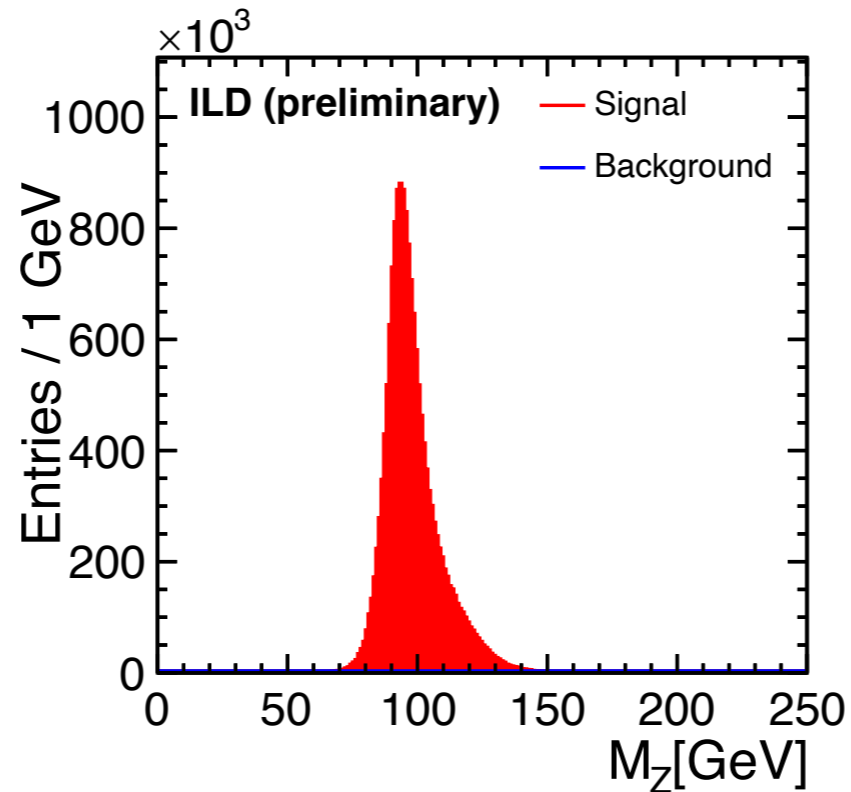
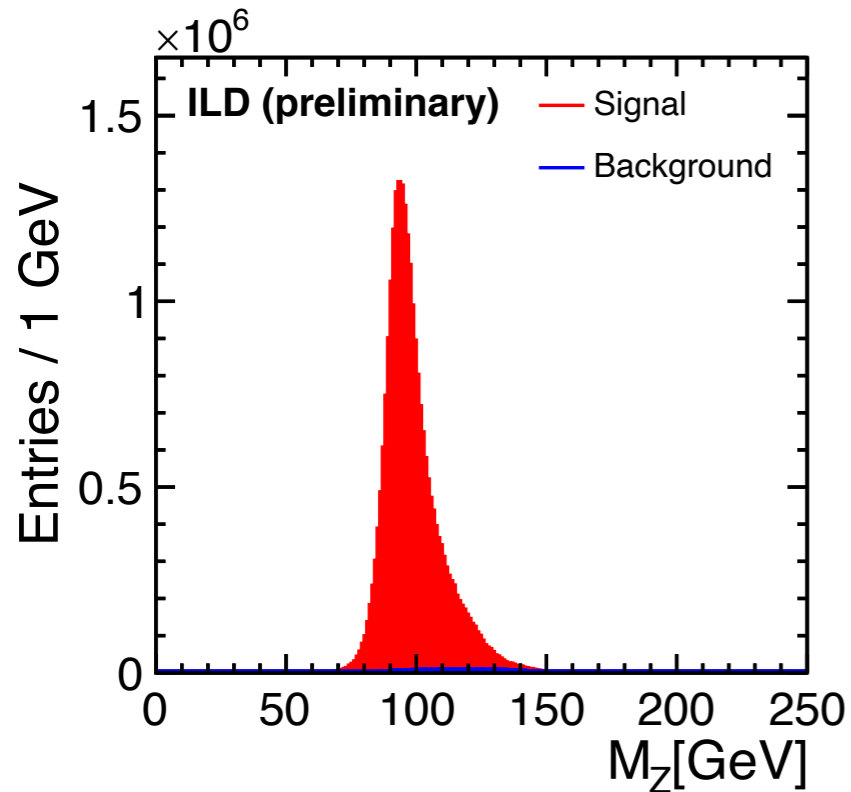
Efficiency = 0.75141 ±0.00026

B/S= 0.00655 for (+0.8, -0.3)

Final Event Sample

$$(P_{e^-}, P_{e^+}) = (-0.8, +0.3)$$

$$(P_{e^-}, P_{e^+}) = (+0.8, -0.3)$$



B/S is indeed very good.

Estimation of A_{LR} precision (1)¹⁵

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}, \quad \text{L/R : 100\% polarization}$$

$$A_{LRobs} \equiv \frac{\sigma_{-+} - \sigma_{+-}}{\sigma_{-+} + \sigma_{+-}} \quad \text{-/+ : Polarization at ILC}$$

$$\sigma_{-+} = \frac{1}{4}(1 + |P_-|)(1 + |P_+|)\sigma_L + \frac{1}{4}(1 - |P_-|)(1 - |P_+|)\sigma_R$$

$$\sigma_{+-} = \frac{1}{4}(1 - |P_-|)(1 - |P_+|)\sigma_L + \frac{1}{4}(1 + |P_-|)(1 + |P_+|)\sigma_R$$

$$A_{LR} = A_{LRobs} \frac{1 + |P_-||P_+|}{|P_-| + |P_+|} = A_{LRobs} \times f$$

The error of the A_{LR} can be expressed as

$$\left(\frac{\Delta A_{LR}}{A_{LR}}\right)^2 = \left(\frac{\Delta A_{LRobs}}{A_{LRobs}}\right)^2 + \left(\frac{\Delta f}{f}\right)^2$$

Estimation of A_{LR} precision (2)¹⁶

Assume $\Delta|P_-|$ and $\Delta|P_+|$ are independent, then

$$\left(\frac{\Delta f}{f}\right)^2 = \left(\frac{|P_-|(1+|P_+|)(1-|P_+|)}{(|P_-|+|P_+|)(1+|P_-||P_+|)}\right)^2 \left(\frac{\Delta|P_-|}{|P_-|}\right)^2 + \left(\frac{|P_+|(1+|P_-|)(1-|P_-|)}{(|P_-|+|P_+|)(1+|P_-||P_+|)}\right)^2 \left(\frac{\Delta|P_+|}{|P_+|}\right)^2$$

As for the error of A_{LRobs} , defining

N: number of events

η : selection efficiency

L: integrated luminosity

$$N_{-+} = \eta_{-+} L_{-+} \sigma_{-+} \quad \alpha \equiv L_{-+} \eta_{-+}$$

$$N_{+-} = \eta_{+-} L_{+-} \sigma_{+-}, \quad \beta \equiv L_{+-} \eta_{+-},$$

$$A_{LRobs} = \frac{\frac{N_{-+}}{\alpha} - \frac{N_{+-}}{\beta}}{\frac{N_{-+}}{\alpha} + \frac{N_{+-}}{\beta}},$$

$$\sim 6.81 \times 10^{-8} \times \sqrt{\langle \Delta\alpha \Delta\beta \rangle}$$

Correlated parts of the error of α and β cancel in A_{LRobs} .

-> $\Delta\alpha$ and $\Delta\beta$ below only refer to uncorrelated parts.

$$\left(\frac{\Delta A_{LRobs}}{A_{LRobs}}\right)^2 = \left(\frac{2 \left(\frac{N_{-+}}{\alpha}\right) \left(\frac{N_{+-}}{\beta}\right)}{\left(\frac{N_{-+}}{\alpha} - \frac{N_{+-}}{\beta}\right) \left(\frac{N_{-+}}{\alpha} + \frac{N_{+-}}{\beta}\right)}\right)^2 \left(\left(\frac{\Delta\alpha}{\alpha}\right)^2 + \left(\frac{\Delta\beta}{\beta}\right)^2 + \left(\frac{\Delta N_{-+}}{N_{-+}}\right)^2 + \left(\frac{\Delta N_{+-}}{N_{+-}}\right)^2 \right)$$

Results

If errors of η , L , and polarization are negligible,

$$A_{LR} = 0.22815 \pm 0.00017$$

If we add polarization error $\Delta f/f = 0.001$,

$$\text{Absolute error of } A_{LR} = 0.00021$$

If $\Delta\alpha/\alpha = \Delta\beta/\beta$ (uncorrelated) = **0.00006 (i.e. 0.006%)**,

$$\text{Absolute error} = 0.000218 \text{ (cf. Abs. error at SLC} = 0.00219)$$

In order to achieve 10 times better precision than SLC, we need to keep the uncorrelated part of the error on product of efficiency and luminosity below 0.006%.

Conclusion of A_{LR} measurement

- As A_{LR} is useful to constrain SMEFT parameters, it is motivated to improve this observable at the ILC. In order to access how much we can improve the precision, full simulation study including $e^+e^- \rightarrow \gamma Z$ process and various background processes are performed.
- In order to exclude the background processes, cut conditions are considered.
- For the radiative return events with hadronic decay and unseen signal photon case, background-to-signal ratios are 0.0215 and 0.00655 for $(P e^-, P e^+) = (-0.8, +0.3)$ and $(+0.8, -0.3)$ polarization respectively after the event selection.
- The **statistical error of the A_{LR}** is estimated to be 1.7×10^{-4} i.e. **13 times better than the overall error at the SLC** and in order to achieve 10 times better precision than SLC, we need to keep the uncorrelated part of the error on product of efficiency and luminosity below **0.006%**.