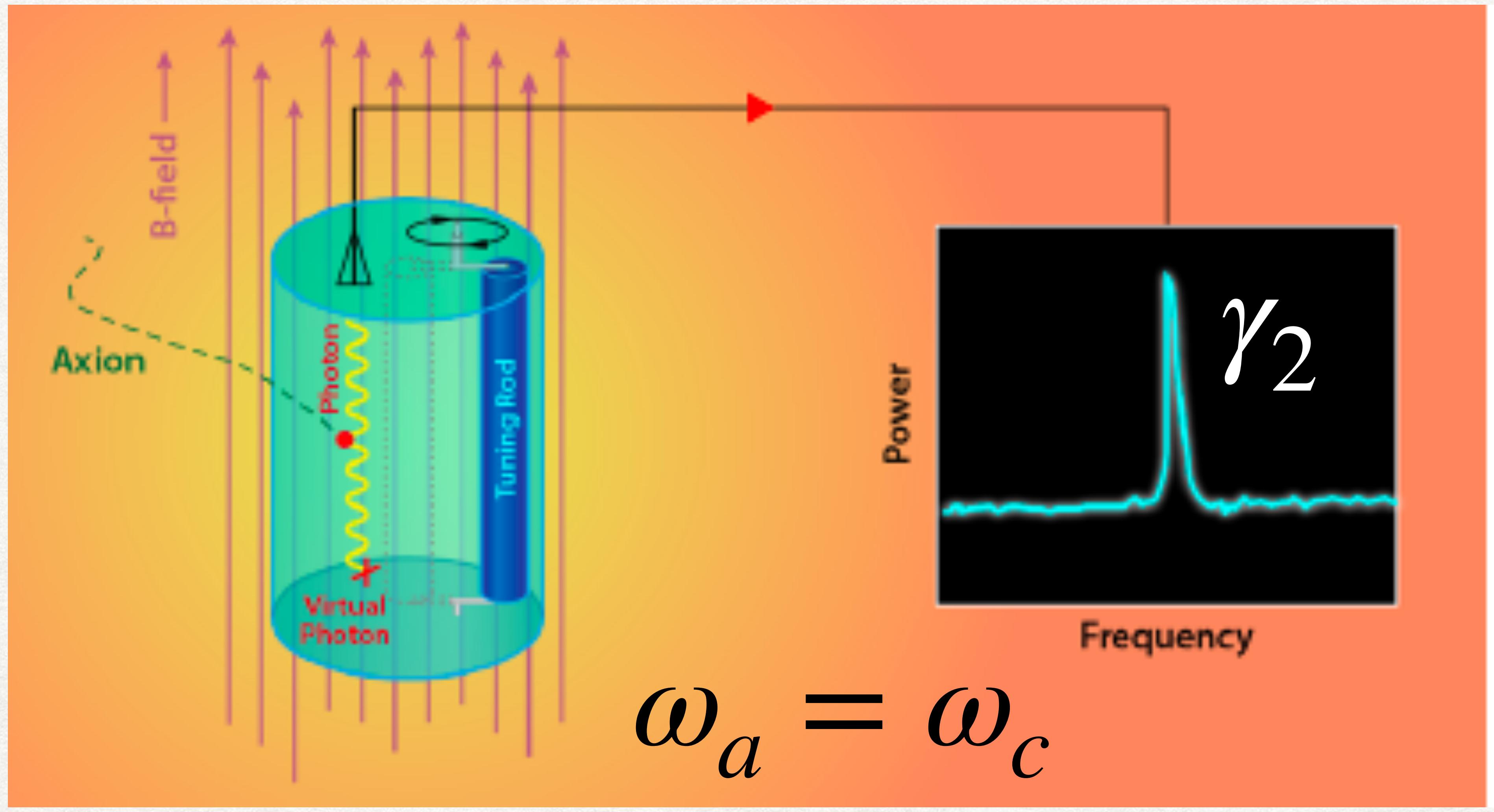
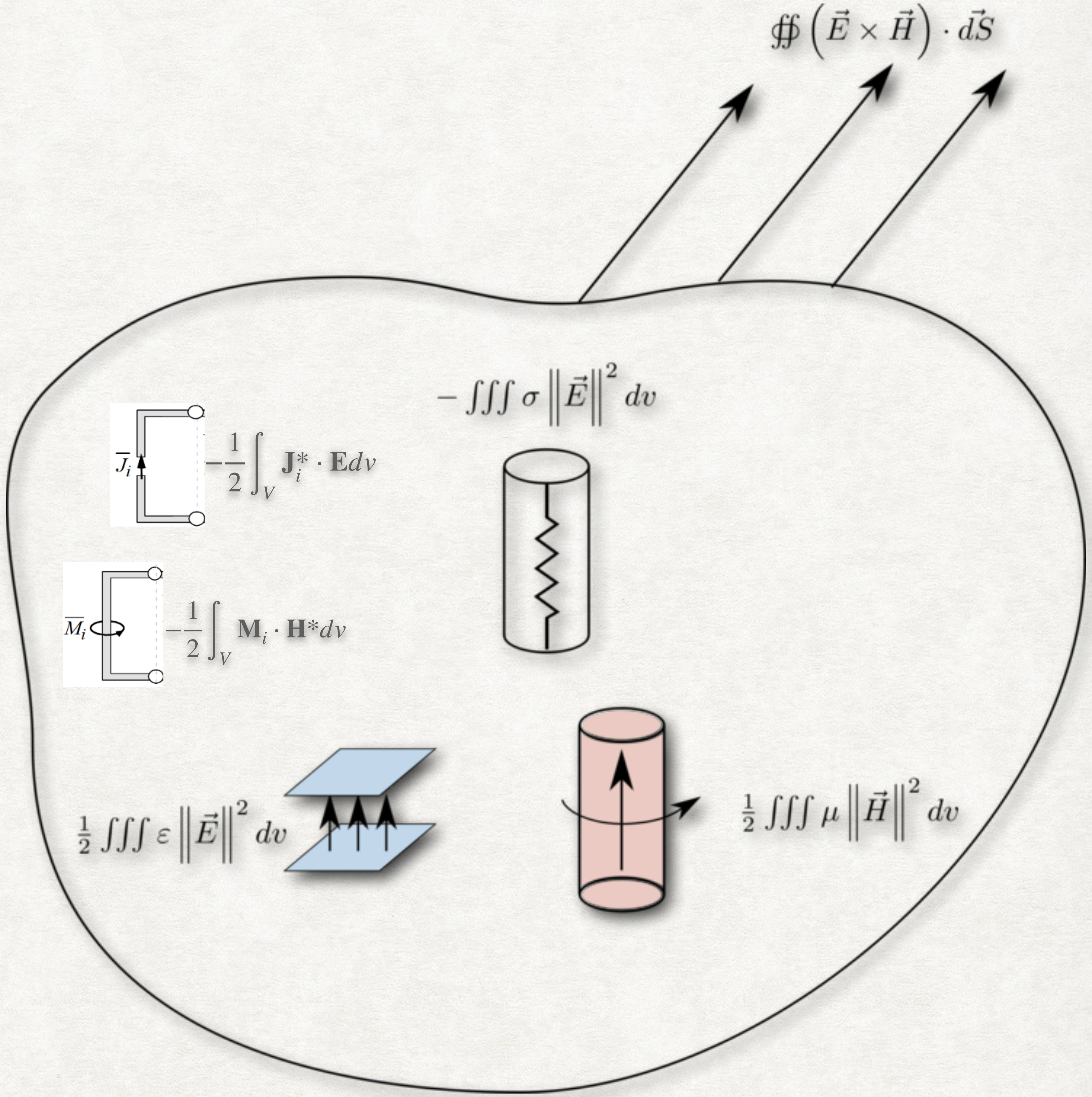
POYNTING VECTOR ANALYSIS OF PHOTONIC CONVERSION OF THE DARK MATTER AXION MIXING WITH A BACKGROUND DC MAGNETIC FIELD





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High Energy Physics – Phenomenology

[Submitted on 9 Sep 2021]

Abraham and Minkowski Poynting vector controversy in axion modified electrodynamics

Michael E Tobar, Ben T McAllister, Maxim Goryachev

The most sensitive haloscopes that search for axion dark matter through the two photon electromagnetic anomaly, convert axions into photons through the mixing of axions with a large DC magnetic field. In this work we apply Poynting theorem to the resulting axion modified electrodynamics and identify two possible Poynting vectors, one similar to the Abraham Poynting vector and the other to the Minkowski Poynting vector in electrodynamics. The latter picks up the extra non-conservative terms while the former does not. To understand the source of energy conversion and power flow in the detection systems, we apply the two Poynting theorems to axion modified electrodynamics, for both the resonant cavity and broadband low-mass axion detectors. We show that both Poynting theorems give the same sensitivity for a resonant cavity axion haloscope, but predict markedly different sensitivity for a low-mass broadband capacitive haloscope. Hence we ask the question, can understanding which one is the correct one for axion dark matter detection, be considered under the framework of the Abraham-Minkowski controversy? In reality, this should be confirmed by experiment when the axion is detected. However, many electrodynamic experiments have ruled in favour of the Minkowski Poynting vector when considering the canonical momentum in dielectric media. In light of this, we show that the axion modified Minkowski Poynting vector should indeed be taken seriously for sensitivity calculation for low-mass axion haloscope detectors in the quasi static limit, and predict orders of magnitude better sensitivity than the Abraham Poynting vector equivalent.

Subjects: **High Energy Physics – Phenomenology (hep-ph)**; Instrumentation and Methods for Astrophysics (astro-ph.IM); General Relativity and Quantum Cosmology (gr-qc); High Energy Physics – Experiment (hep-ex)

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(or arXiv:2109.04056v1 **[hep-ph]** for this version)

arXiv:2109.04056 [hep-ph]

DC Magnetic Haloscopes

- Axions convert into photons in presence of strong magnetic field
- Mass is unknown
- So: narrowband photon signal of an unknown frequency is generated (need to scan frequency)
- Three regimes of haloscope detector

$$\lambda_a = \frac{h}{m_a c}$$

$\lambda_a > d_{exp}$ $\lambda_a \sim d_{exp}$ $\lambda_a < d_{exp}$ ⟷ m_a

$$\omega_a \approx \frac{m_a c^2}{\hbar}$$

- Lumped Element Reactive
- Resonant
- Propagative

$$m_a[eV] \equiv \frac{m_a[kg]c^2}{q_e} \qquad 1eV = 1.8 \times 10^{-36}[kg]$$

Resonant: $\lambda_a \sim d_{exp} \sim 1cm \rightarrow 1m$ $\frac{\omega_a}{2\pi} \sim 300\ MHz \rightarrow 30\ GHz$ $m_a \sim 1\ \mu eV \rightarrow 100\ \mu eV$

DC Magnetic Haloscopes

Low Mass: Lumped Element
Reactive

Middle Mass: Resonant Cavity
Reactive and Dissipative

High Mass: Propagating

$300\ MHz$ — $30\ GHz$ — $\frac{\omega_a}{2\pi}\ Hz$

1.25×10^{-6} — 1.25×10^{-4} — $m_a\ eV$

$\lambda_a > d_{exp}$

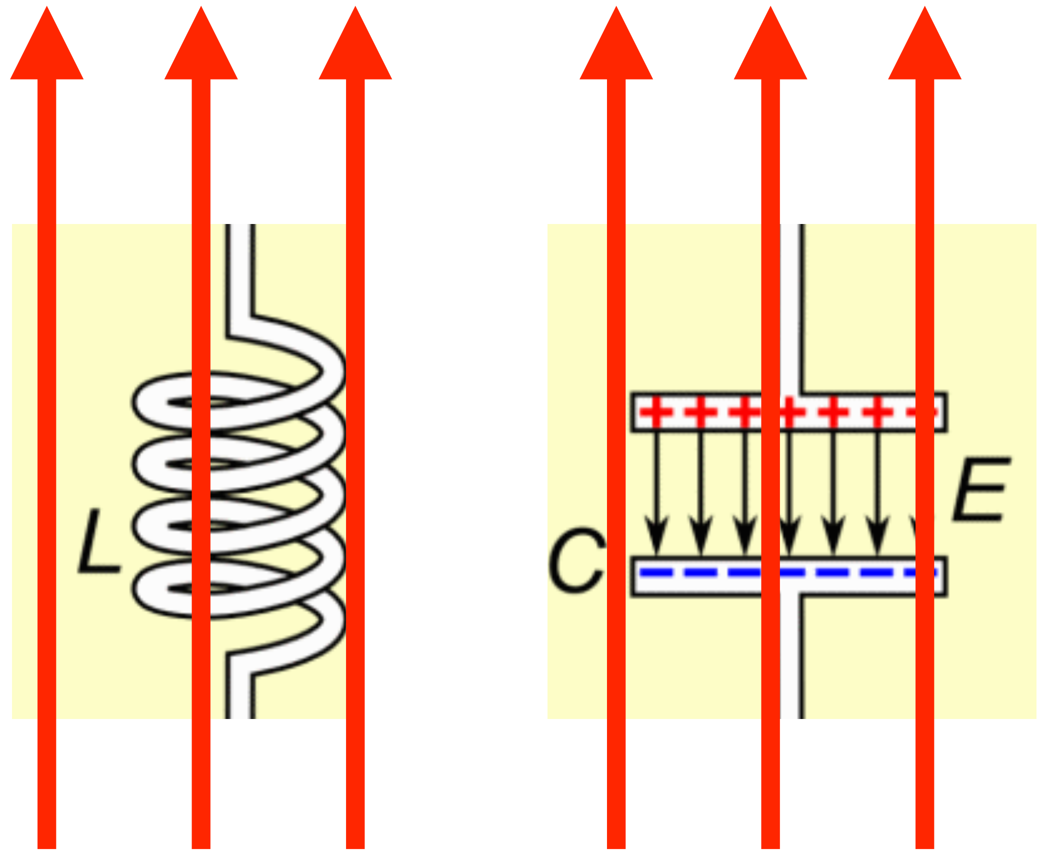
ADMX SLIC
RE-ENTRANT CAVITY
ABRACADABRA
SHAFT
DM RADIO
BEAST

$\lambda_a \sim d_{exp}$

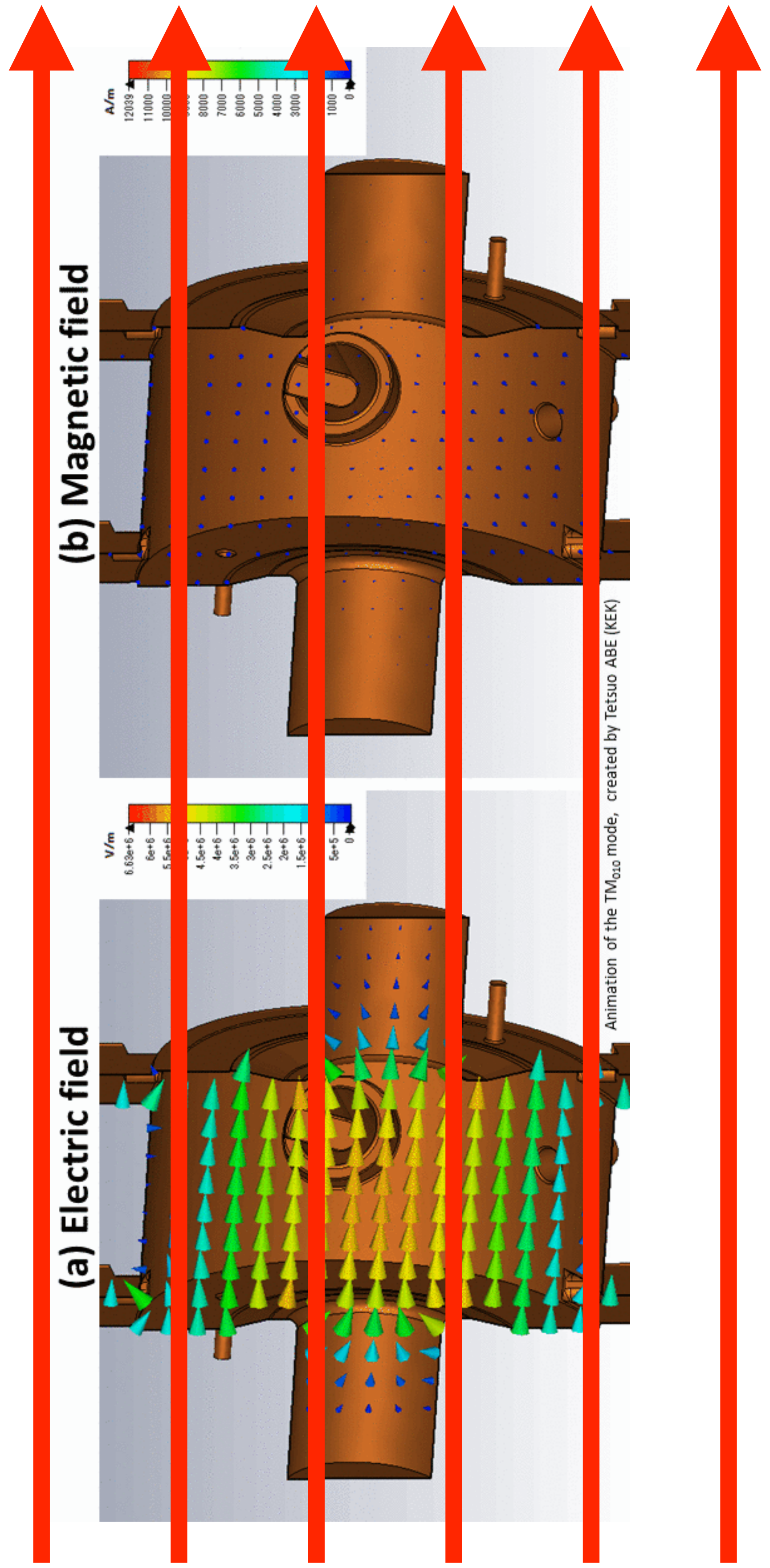
ADMX
CULTASK
ORGAN
QUAX
RADES

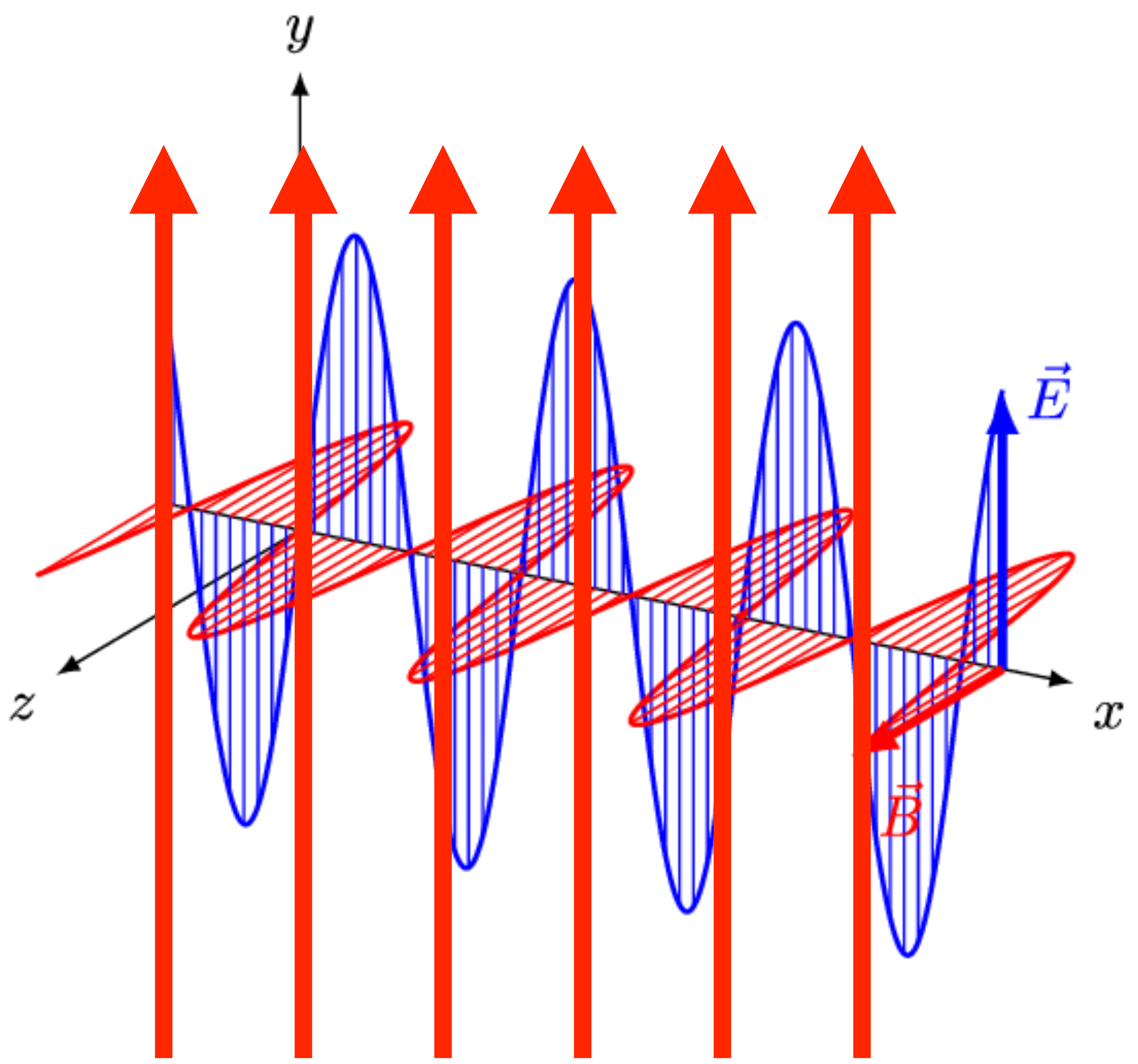
$\lambda_a < d_{exp}$

MADMAX
BREAD



$\overrightarrow{B} = B_{DC}\hat{z}$





REMEMBERING POYNTING THEOREM

- Basic conservation law for electromagnetic energy for AC system
- Describes complex power flow (phasors) in a volume, considering: 1) Sources, 2) Storage, 3) Dissipation, 4) Radiation
- The direction and density of power flow at a point is defined by the instantaneous Poynting vector, $\overrightarrow{S}(t)$ [W/m²]

Instantaneous Poynting vector in vacuum

$$\overrightarrow{S}_1(t) = \frac{1}{\mu_0}\overrightarrow{E}_1(t) \times \overrightarrow{B}_1(t) = \frac{1}{2}\left(\mathbf{E}_1 e^{-j\omega_1 t} + \mathbf{E}_1^* e^{j\omega_1 t}\right) \times \frac{1}{2\mu_0}\left(\mathbf{B}_1 e^{-j\omega_1 t} + \mathbf{B}_1^* e^{j\omega_1 t}\right)$$

$$= \frac{1}{2\mu_0}\mathrm{Re}\left(\mathbf{E}_1 \times \mathbf{B}_1^*\right) + \frac{1}{2\mu_0}\mathrm{Re}\left(\mathbf{E}_1 \times \mathbf{B}_1\ e^{-j2\omega_1 t}\right),$$

$$\langle \overrightarrow{S}_1 \rangle = \frac{1}{T}\int_0^T \overrightarrow{S}_1(t)dt = \frac{1}{T}\int_0^T \left[\frac{1}{2\mu_0}\mathrm{Re}\left(\mathbf{E}_1 \times \mathbf{B}_1^*\right) + \frac{1}{2\mu_0}\mathrm{Re}\left(\mathbf{E}_1 \times \mathbf{B}_1 e^{-2j\omega t}\right)\right] dt = \frac{1}{2\mu_0}\mathrm{Re}\left(\mathbf{E}_1 \times \mathbf{B}_1^*\right)$$

Complex Poynting vector in vacuum

- The corresponding phasor form of the Poynting vector

$$\mathbf{S}_1 = \frac{1}{2\mu_0}\mathbf{E}_1 \times \mathbf{B}_1^* \quad \text{and} \quad \mathbf{S}_1^* = \frac{1}{2\mu_0}\mathbf{E}_1^* \times \mathbf{B}_1,$$

$$\mathrm{Re}\left(\mathbf{S}_1\right) = \frac{1}{2}(\mathbf{S}_1 + \mathbf{S}_1^*) \quad \text{and} \quad j\,\mathrm{Im}\left(\mathbf{S}_1\right) = \frac{1}{2}(\mathbf{S}_1 - \mathbf{S}_1^*).$$

Time Average Power

Reactive Power

Combing the Poynting vector with Maxwell's Equations -> Leads to Poynting Theorem

1) Instantaneous Poynting Theorem
2) Complex Poynting Theorem

CONSIDERATION OF POYNTING VECTOR IN AXION ELECTRODYNAMICS: THE ABRAHAM-MINKOWSKI CONTROVERSY

- Poynting vector in Electrodynamics -> Over a century of Controversy, chose $\mathbf{S}_{\mathrm{M}} = \frac{1}{\epsilon_0 \mu_0}(\mathbf{D} \times \mathbf{B})$ or $\mathbf{S}_{\mathrm{A}} = (\mathbf{E} \times \mathbf{H})$ in matter ?

- Pfeifer et. al., Momentum of an electromagnetic wave in dielectric media, Reviews of Modern Physics 79(4), 1197-1216 (2007). -> Addresses the Abraham-Minkowski controversy, conclude: both valid depends on system.

- Kinsler et al., Four Poynting theorems, Eur. J. Phys. 30 (2009) 983–993. Enables interpretation of four Poynting vectors and interaction with the medium -> choosing the best Poynting vector depend on the medium and experimental set up.

- DJ Griffiths, Resource Letter EM-1: Electromagnetic Momentum, Am. J. Phys. 80, 7 (2012) -> Abraham–Minkowski controversy regarding the field momentum in polarizable and magnetizable media: Correct one depends on the detailed nature of the material.

VII. MOMENTUM OF PHOTONS

$$\mathbf{p}_{\text{total}} = \mathbf{p}_{\text{kinetic}} + \int \mathbf{g}_A \, d^3\mathbf{r} = \mathbf{p}_{\text{canonical}} + \int \mathbf{g}_M \, d^3\mathbf{r}.$$

$$\begin{cases} p_{\mathrm{A}} = \frac{1}{n}\left(\frac{h\nu}{c}\right) & \text{(Abraham)} \\ p_{\mathrm{M}} = n\left(\frac{h\nu}{c}\right) & \text{(Minkowski)} \end{cases}$$

$$\Delta\theta' = \begin{cases} n\,\Delta\theta & \text{(Abraham)} \\ \frac{1}{n}\,\Delta\theta & \text{(Minkowski)} \end{cases}$$

Size of the central maximum in single-slit diffraction

Measured by Jones et al, when media does not move

Four Poynting theorems

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(Dated: November 9, 2016)

The Poynting vector is an invaluable tool for analysing electromagnetic problems. However, even a rigorous stress-energy tensor approach can still leave us with the question: is it best defined as $\vec{E}\times\vec{H}$ or as $\vec{D}\times\vec{B}$? Typical electromagnetic treatments provide yet another perspective: they regard $\vec{E}\times\vec{B}$ as the appropriate definition, because $\vec{E}$ and $\vec{B}$ are taken to be the fundamental electromagnetic fields. The astute reader will even notice the fourth possible combination of fields: i.e. $\vec{D}\times\vec{H}$. Faced with this diverse selection, we have decided to treat each possible flux vector on its merits, deriving its associated energy continuity equation but applying minimal restrictions to the allowed host media. We then discuss each form, and how it represents the response of the medium. Finally, we derive a propagation equation for each flux vector using a directional fields approach; a useful result which enables further interpretation of each flux and its interaction with the medium.

Published in Eur. J. Phys. **30**, *983 (2009).*[1]
This arXiv version has updates not present in the published version.

arXiv:0908.1721v4

II. MAXWELL'S EQUATIONS

Maxwell's equations for the electric field $\vec{E}$ and magnetic field $\vec{B}$ in a medium are

$$\nabla\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho_b + \frac{1}{\varepsilon_0}\rho_f \quad = \frac{1}{\varepsilon_0}\rho \tag{1}$$

$$\nabla\cdot\vec{B} = 0 \tag{2}$$

$$\nabla\times\vec{E} = -\partial_t\vec{B} \tag{3}$$

$$\nabla\times\vec{B} = \mu_0\vec{J}_b + \mu_0\vec{J}_f + \mu_0\varepsilon_0\partial_t\vec{E}, \tag{4}$$

where $(\rho_b, \vec{J}_b)$ and $(\rho_f, \vec{J}_f)$ are respectively the bound and free (charge, current) densities. As an alternative, we can define an electric polarization P and magnetization M, and

$$\vec{J}_b = \vec{J}_P + \vec{J}_M \quad = \partial_t\vec{P} + \nabla\times\vec{M} \tag{5}$$

$$\rho_b = -\nabla\cdot\vec{P} \tag{6}$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} \tag{7}$$

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}. \tag{8}$$

These allow us to rewrite Maxwell's equations as

$$\nabla\cdot\vec{D} = \rho_f \tag{9}$$

$$\nabla\cdot\vec{B} = 0 \tag{10}$$

$$\nabla\times\vec{E} = -\mu_0\partial_t\left(\vec{H}+\vec{M}\right) \tag{11}$$

$$\nabla\times\vec{H} = \vec{J}_f + \partial_t\vec{D} \quad = \vec{J}_f + \partial_t\left(\varepsilon_0\vec{E}+\vec{P}\right). \tag{12}$$

We can even rewrite eqn. (11) in the unconventional form

$$\nabla\times\vec{D} = -\varepsilon_0\mu_0\partial_t\left(\vec{H}+\vec{M}\right) + \nabla\times\vec{P} \tag{13}$$

$$= -\varepsilon_0\mu_0\partial_t\vec{H} - \varepsilon_0\mu_0\vec{K}_b, \tag{14}$$

where we have defined

$$\vec{K}_b = \vec{K}_P + \vec{K}_M \quad = -\frac{1}{\varepsilon_0\mu_0}\nabla\times\vec{P} + \partial_t\vec{M}, \tag{15}$$

$$\sigma_b = -\nabla\cdot\vec{M}. \tag{16}$$

This $\vec{K}_b$ appears in the same place as a monopole current would if such were allowed; σ_b is the *bound* magnetic pole density. Note that $\vec{K}_b$ and σ_b are merely a way of representing the (local) material response; we are not claiming that some process actually generates true magnetic monopoles inside the material [14][2]. Strictly speaking, this is also true of the bound electric charge and its currents – they are a mechanism used solely to represent the behaviour of the medium.

Further, and just as for the ficticious bound electric charge density, the ficticious bound monopole density necessarily integrates to zero over all space. Thus the material response could, in principle, be re-represented as magnetic dipoles instead of monopoles.

GENERAL FORM OF MAXWELL'S EQUATIONS IN MATTER

Bound Electric and/or Magnetic Current Models?

1) Modified Ampere's Law (mmf generator)

$$\nabla \times B = \mu_0(\vec{J}_f + \vec{J}_b) \;; \quad \vec{J}_b = \vec{J}_P + \vec{J}_M = \partial_t \vec{P} + \nabla \times \vec{M}$$

$$\varrho_P = -\nabla \cdot \vec{P} \quad \leftrightarrow \quad \nabla \cdot \vec{J}_P = -\partial_t \rho_b$$

Dielectric term

Polarization Current

Permanent Magnet term

2) Modified Faraday's Law (emf generator)

$$\nabla \times \vec{D} = -\epsilon_0\mu_0\partial_t(\vec{H} + \vec{M}) + \nabla \times \vec{P} = -\epsilon_0\mu_0\partial_t\vec{H} - \epsilon_0 \vec{J}_{mb}$$

$$\vec{J}_{mb} = \vec{J}_{mM} + \vec{J}_{mP} = \mu_0\partial_t\vec{M} - \frac{1}{\epsilon_0}\nabla \times \vec{P}$$

$$\varrho_M = -\mu_0\nabla \cdot \vec{M}$$

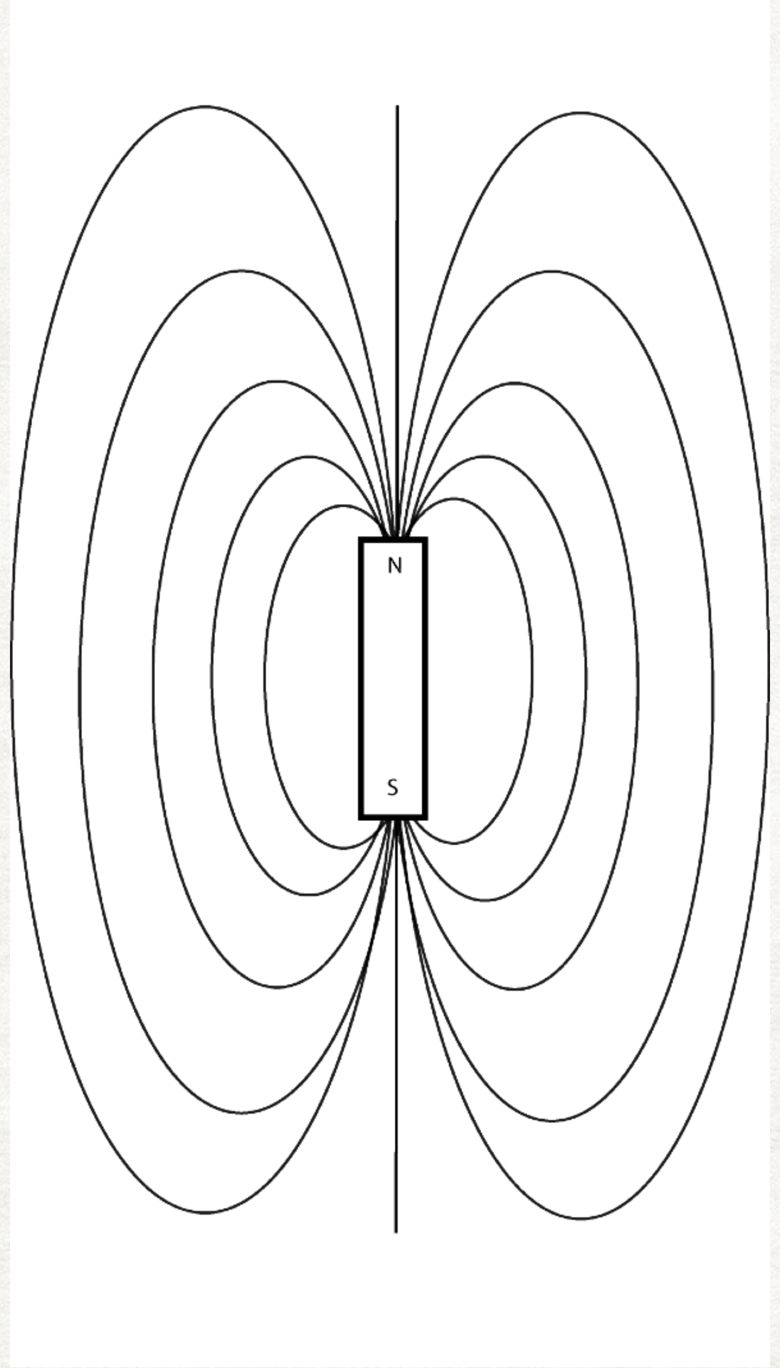
$$\nabla \cdot \vec{J}_{mM} = -\partial_t \varrho_M$$

Magnetisation Magnetic Current

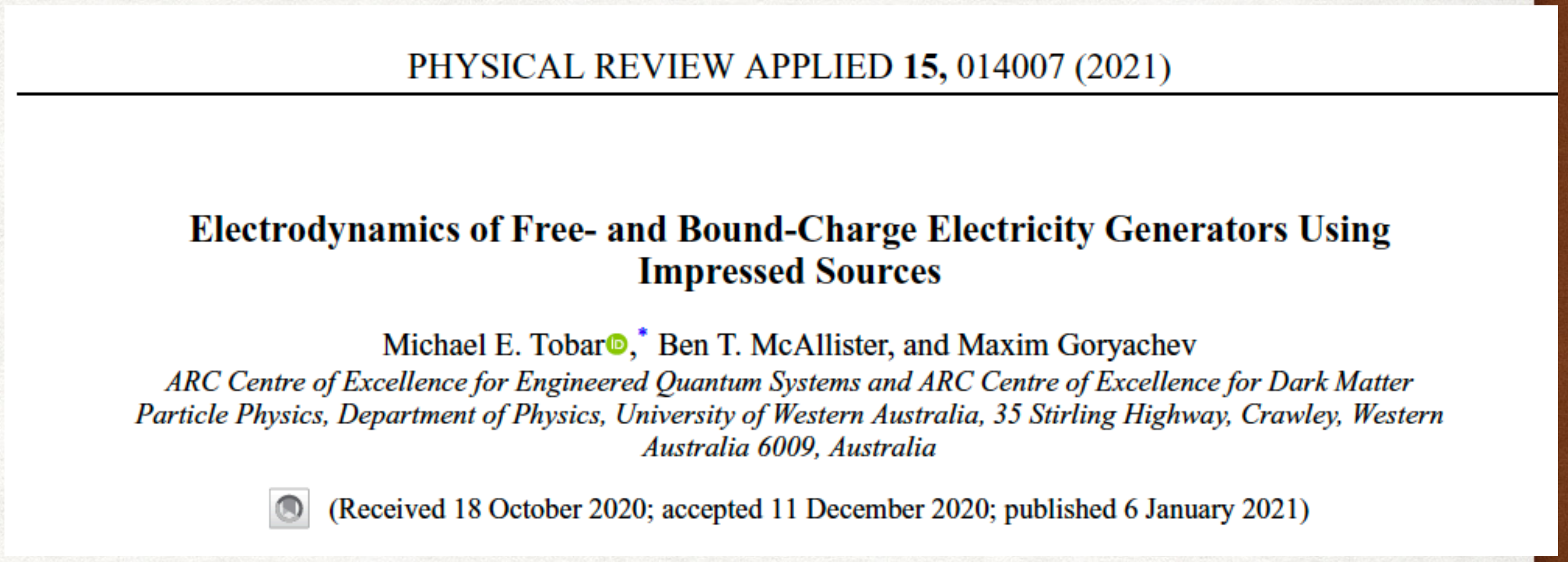
Permanent Electret term (Non-conservative, Metastable)

Permanent Magnet

$$\rightarrow \nabla \times \vec{M} \neq 0$$



Model as fictitious bound monopole currents or fictitious bound "Ampèrian" currents


PHYSICAL REVIEW APPLIED **15**, 014007 (2021)

Electrodynamics of Free- and Bound-Charge Electricity Generators Using Impressed Sources

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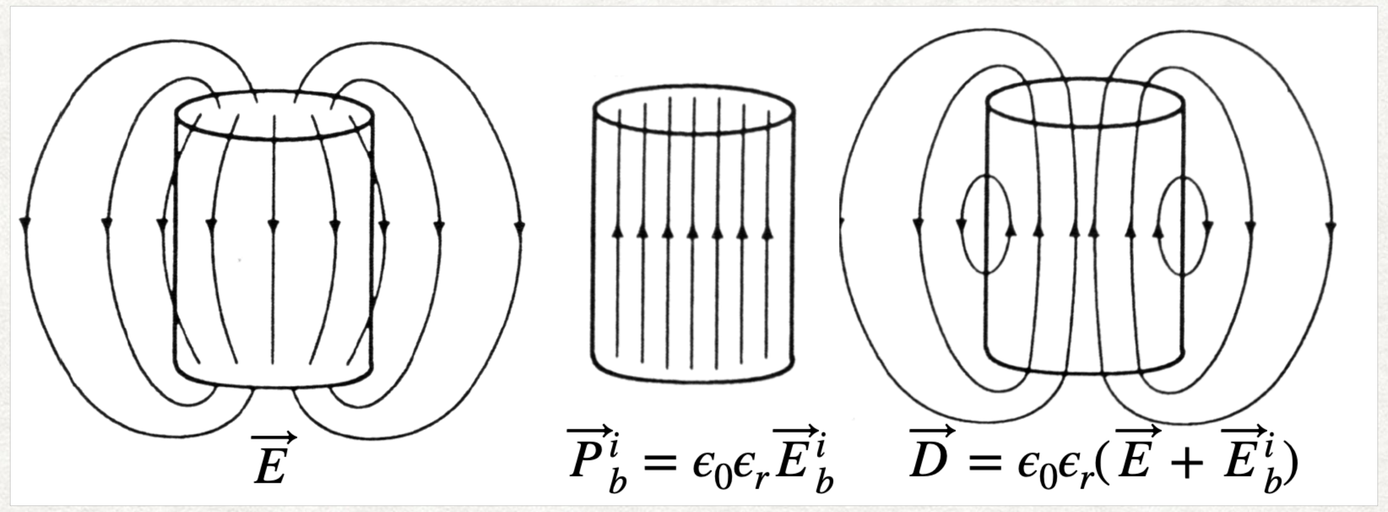
Bound Charge Voltage Source

Electret $\rightarrow \nabla \times \vec{P} \neq 0$

Non-conservative



Polarization can also be defined in free charge voltage source

$$\nabla \times \vec{P} \neq 0$$

$$\epsilon_0\nabla \times \vec{E}_i$$

Engineers call impressed field

Could label as Fictitious Electric field

Electromotive force

From Wikipedia, the free encyclopedia

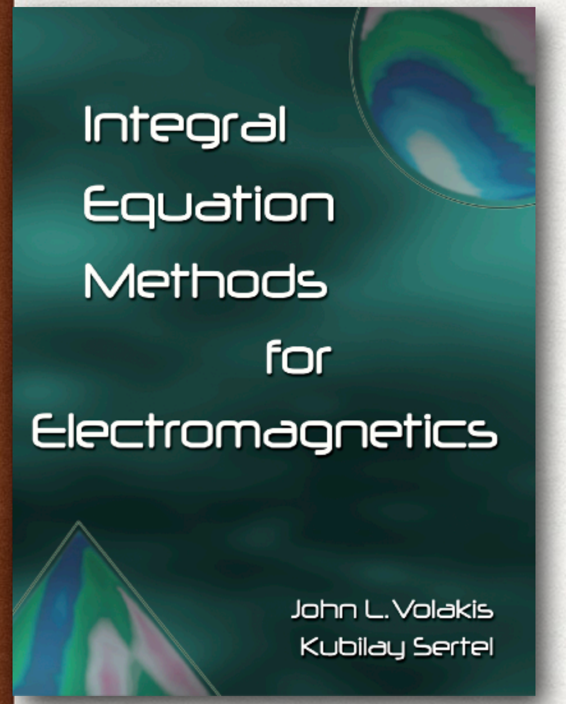
Not to be confused with Electromagnetic field.

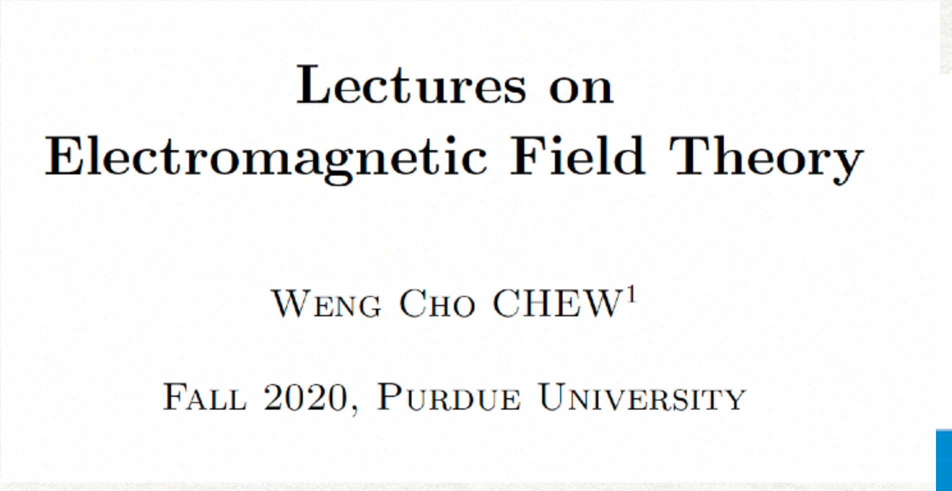
In electromagnetism and electronics, **electromotive force** (**emf**, denoted $\mathcal{E}$ and measured in volts)[1] is the electrical action produced by a non-electrical source.[2] Devices (known as *transducers*) provide an emf[3] by converting other forms of energy into electrical energy,[3] such as batteries (which convert chemical energy) or generators (which convert mechanical energy).[2] Sometimes an analogy to water pressure is used to describe electromotive force.[4] (The word "force" in this case is not used to mean forces of interaction between bodies).

Fictitious Force

- EMF per unit length [V/m], is like a Fictitious Electric field
- Does not conform to Maxwell's equations
- Outside Maxwell's equations

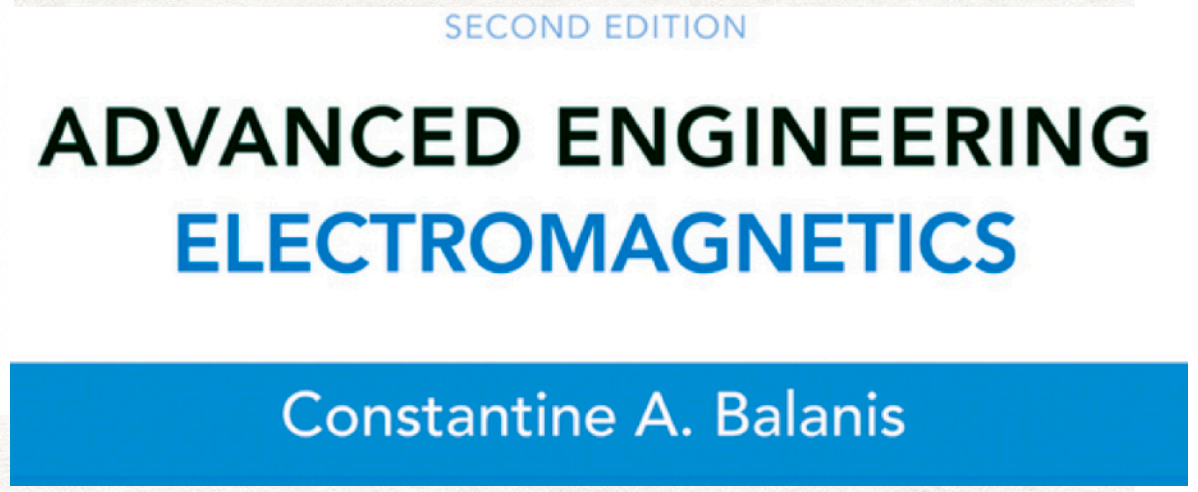
NON CONSERVATIVE MAXWELL'S EQUATIONS




Lectures on Electromagnetic Field Theory

Weng Cho CHEW[1]

Fall 2020, Purdue University



Model of Current and Voltage Source (Impressed)

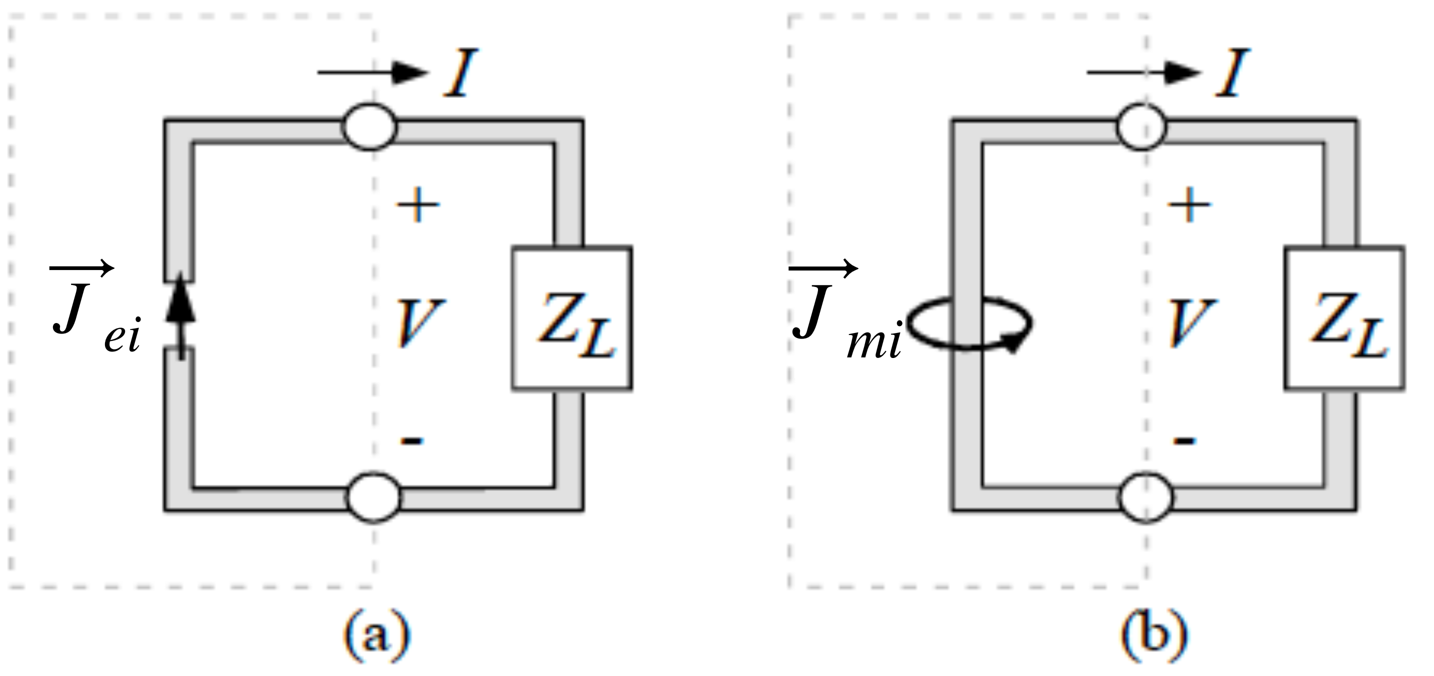
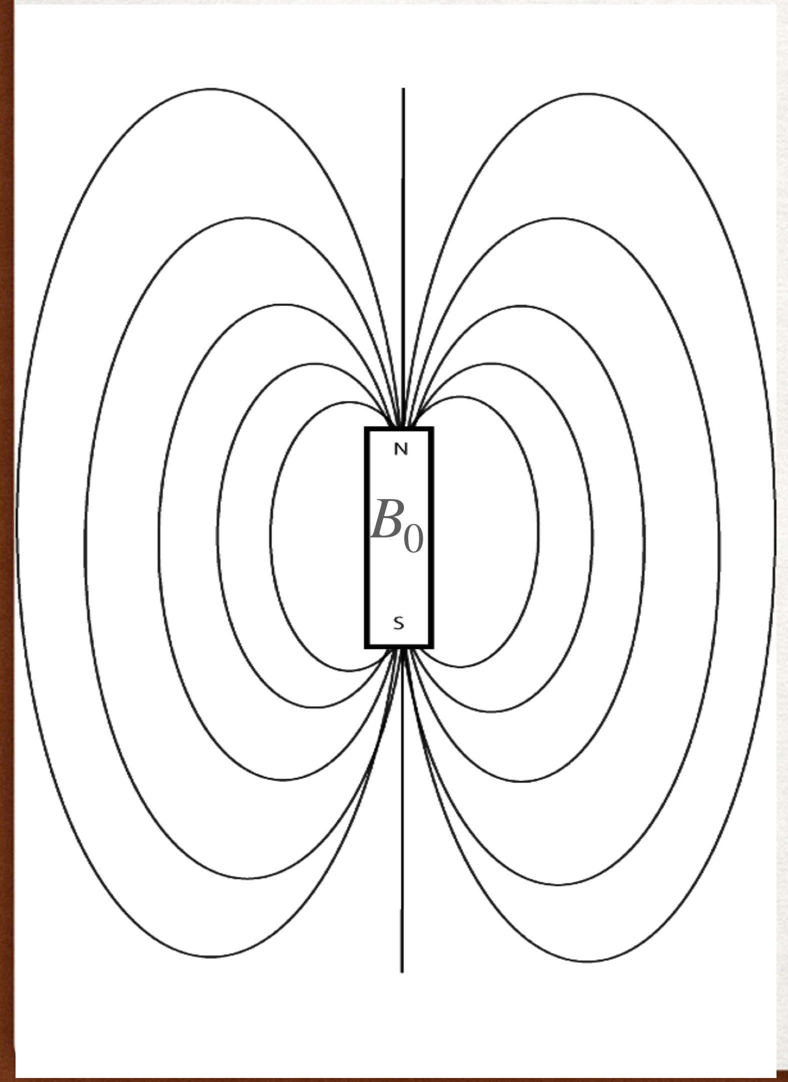


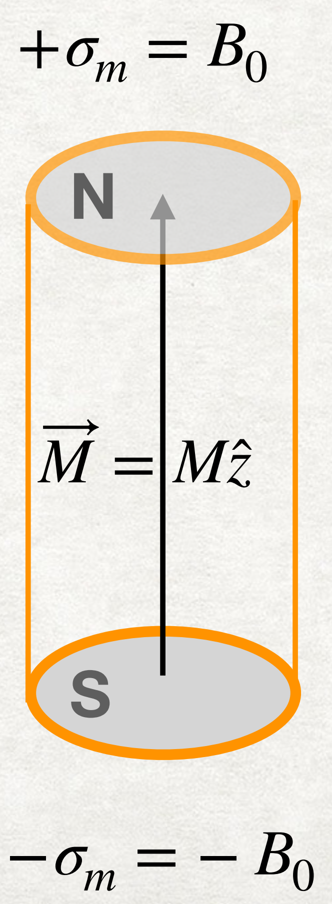
Figure 1.5 Electromagnetic representation of independent circuit sources. (a) Current generator (impressed electric current filament); (b) Voltage generator (impressed magnetic current loop).

- Engineering Systems are in general "non-conservative"
- Engineers keep Maxwell's equations general, with both fictitious magnetic and electric sources
- Magnetic monopoles do not exist, but magnetic dipoles do!
- Magnetic charge occurs in pairs, does not contradict no monopoles

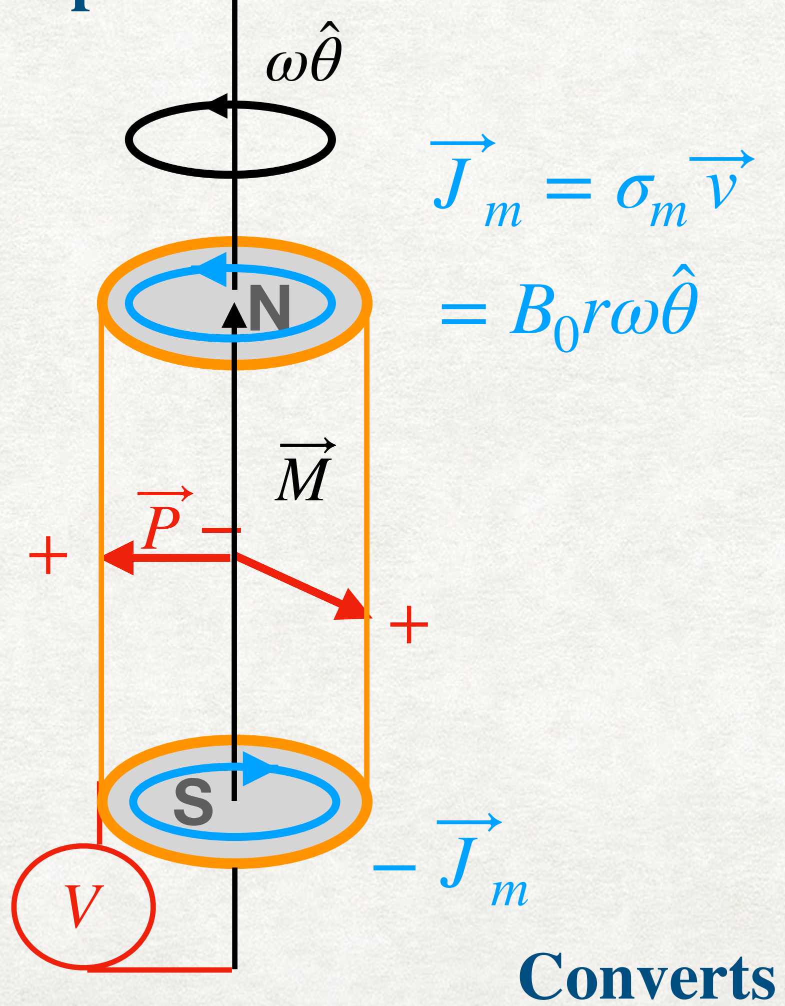
ρ_m: magnetic pole distribution integrates to 0



Surface Magnetic Charge



Spinning magnet of radius R Unipolar Generator



$V = B_0 R\omega$

Converts mechanical motion to EM energy

Fictitious electric field: External Lorentz force/unit charge

Phasor Form for AC Sources

$$\mathbf{J}_e = \mathbf{J}_{ei} + \mathbf{J}_{ec} = \mathbf{J}_{ei} + \sigma_e \mathbf{E}$$
$$\mathbf{J}_m = \mathbf{J}_{mi} + \mathbf{J}_{mc} = \mathbf{J}_{ei} + \sigma_m \mathbf{H}$$

- Subscript *i* -> Impressed (or excitation Currents}
- Subscript c -> Loss term (conductivity)

$$\nabla \times \mathbf{H} = \mathbf{J}_{ei} + j\omega\epsilon_o\tilde{\epsilon}_r \mathbf{E}$$
$$\nabla \times \mathbf{E} = -\mathbf{J}_{mi} - j\omega\mu_o\tilde{\mu}_r \mathbf{H}$$

COMPLEX POYNTING THEOREM: CIRCUITS/ANTENNAS

SECOND EDITION

ADVANCED ENGINEERING ELECTROMAGNETICS

Constantine A. Balanis

Even though magnetic sources do not exist, they can be engineered

Model of Current and Voltage Source (Impressed)

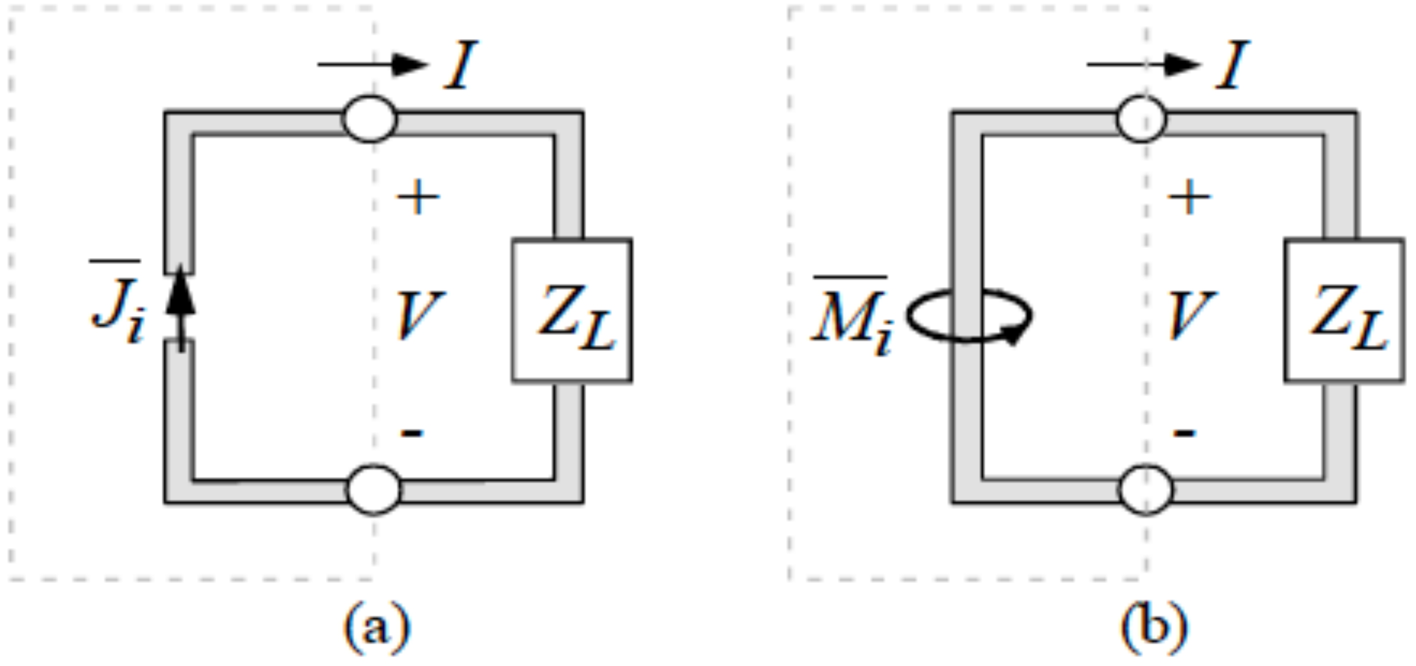


Figure 1.5 Electromagnetic representation of independent circuit sources. (a) Current generator (impressed electric current filament); (b) Voltage generator (impressed magnetic current loop).

Here Balanis uses $\vec{M}$ as magnetic current

$$-\iiint_V \nabla\cdot\left(\tfrac{1}{2}\mathbf{E}\times\mathbf{H}^*\right)dv = -\oint_S \left(\tfrac{1}{2}\mathbf{E}\times\mathbf{H}^*\right)\cdot d\mathbf{s}$$
$$= \frac{1}{2}\iiint_V (\mathbf{H}^*\cdot\mathbf{M}_i + \mathbf{E}\cdot\mathbf{J}_i^*)\,dv$$
$$+\frac{1}{2}\iiint_V \sigma|\mathbf{E}|^2\,dv + j2\omega\iiint_V \left(\tfrac{1}{4}\mu|\mathbf{H}|^2 - \tfrac{1}{4}\varepsilon|\mathbf{E}|^2\right)dv$$

or

$$-\frac{1}{2}\iiint_V (\mathbf{H}^*\cdot\mathbf{M}_i + \mathbf{E}\cdot\mathbf{J}_i^*)\,dv = \oint_S \left(\tfrac{1}{2}\mathbf{E}\times\mathbf{H}^*\right)\cdot d\mathbf{s} + \tfrac{1}{2}\iiint_V \sigma|\mathbf{E}|^2\,dv + j2\omega\iiint_V \left(\tfrac{1}{4}\mu|\mathbf{H}|^2 - \tfrac{1}{4}\varepsilon|\mathbf{E}|^2\right)dv \quad (1\text{-}76)$$

which can be written as

$$P_s = P_e + P_d + j2\omega(\overline{W}_m - \overline{W}_e) \quad (1\text{-}76a)$$

where

$$P_s = -\frac{1}{2}\iiint_V (\mathbf{H}^*\cdot\mathbf{M}_i + \mathbf{E}\cdot\mathbf{J}_i^*)\,dv = \text{supplied complex power (W)} \quad (1\text{-}76b)$$

$$P_e = \oint_S \left(\frac{1}{2}\mathbf{E}\times\mathbf{H}^*\right)\cdot d\mathbf{s} = \text{exiting complex power (W)} \quad (1\text{-}76c)$$

$$P_d = \frac{1}{2}\iiint_V \sigma|\mathbf{E}|^2\,dv = \text{dissipated real power (W)} \quad (1\text{-}76d)$$

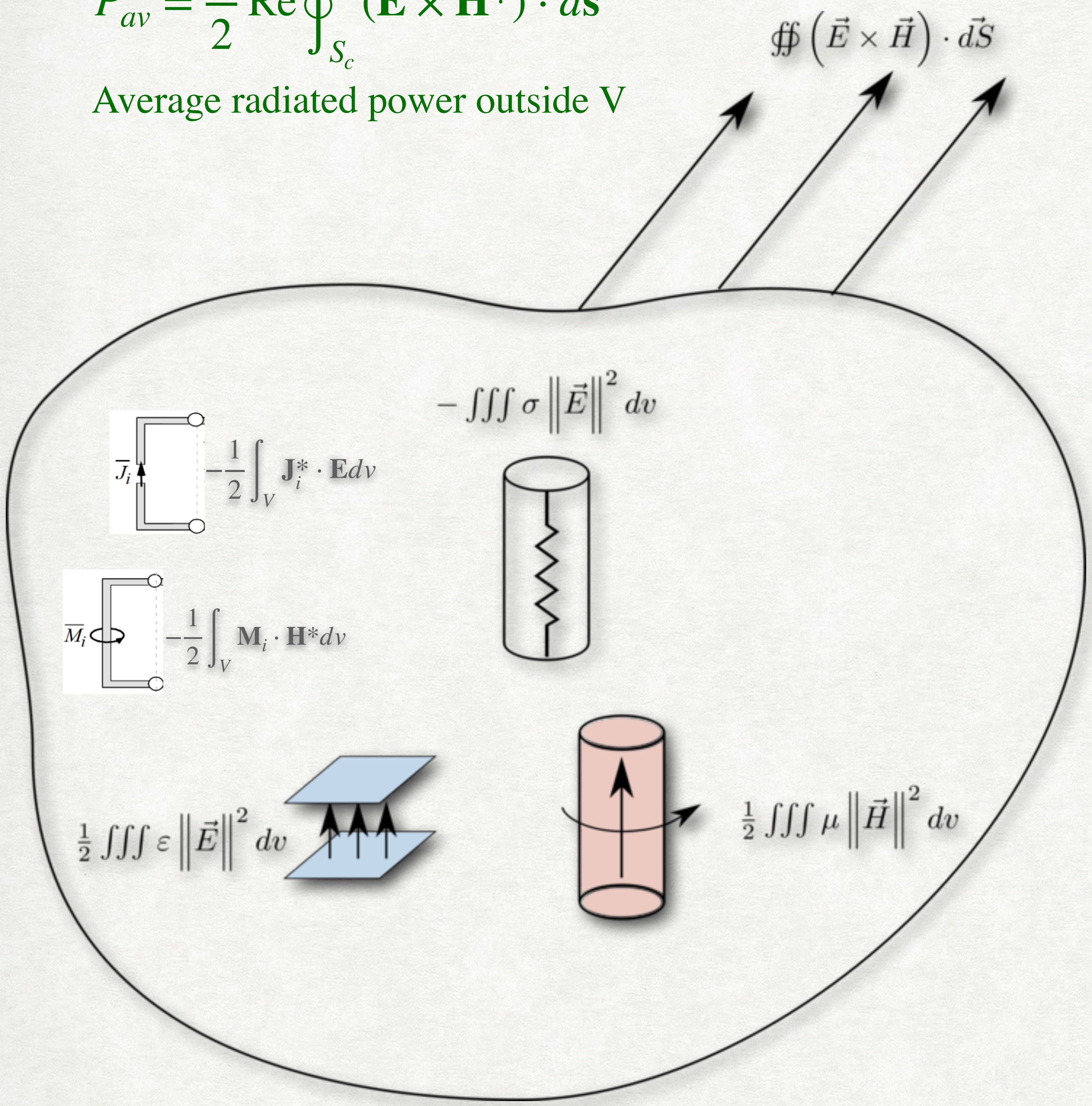
$$\overline{W}_m = \iiint_V \tfrac{1}{4}\mu|\mathbf{H}|^2\,dv = \text{time-average magnetic energy (J)} \quad (1\text{-}76e)$$

$$\overline{W}_e = \iiint_V \tfrac{1}{4}\varepsilon|\mathbf{E}|^2\,dv = \text{time-average electric energy (J)} \quad (1\text{-}76f)$$

COMPLEX POYNTING THEOREM: CIRCUITS/ANTENNAS

$$P_{av} = \frac{1}{2}\mathrm{Re}\oint_{S_c} (\mathbf{E}\times\mathbf{H}^*)\cdot ds$$

Average radiated power outside V



Maxwell's Equations Phasor Form

$$\mathbf{E}\cdot\nabla\times\mathbf{H}^* = \mathbf{J}_i^*\cdot\mathbf{E} - j\omega\epsilon^*\mathbf{E}^*\cdot\mathbf{E} = \mathbf{J}_i^*\cdot\mathbf{E} - j\omega\epsilon^*|\mathbf{E}|^2$$

$$\mathbf{H}^*\cdot\nabla\times\mathbf{E} = -\mathbf{M}_i\cdot\mathbf{H}^* - j\omega\mu\mathbf{H}\cdot\mathbf{H}^* = -\mathbf{M}^i\cdot\mathbf{H}^* - j\omega\mu|\mathbf{H}|^2$$

Vector Identity

$$\nabla\cdot(\mathbf{E}\times\mathbf{H}^*) = \mathbf{H}^*\cdot\nabla\times\mathbf{E} - \mathbf{E}\cdot\nabla\times\mathbf{H}^*$$

$$\nabla\cdot(\mathbf{E}\times\mathbf{H}^*) = j\omega\epsilon^*|\mathbf{E}|^2 - j\omega\mu|\mathbf{H}|^2 - \mathbf{J}_i^*\cdot\mathbf{E} - \mathbf{M}_i\cdot\mathbf{H}^*$$

Integrate over volume, apply Divergence theorem

$$\frac{1}{2}\oint_{S_c}(\mathbf{E}\times\mathbf{H}^*)\cdot ds = \frac{1}{2}\int_V\left[j\omega\epsilon^*|\mathbf{E}|^2 + j\omega\mu|\mathbf{H}|^2 - \mathbf{J}_i^*\cdot\mathbf{E} - \mathbf{M}_i\cdot\mathbf{H}^*\right]dv$$

Real part is average power: Imaginary part is complex power

$$\frac{1}{2}\mathrm{Re}\oint_{S_c}(\mathbf{E}\times\mathbf{H}^*)\cdot ds = P_{ei} + P_{mi} - P_d$$

$P_d = \frac{1}{2}\int_V \sigma|\mathbf{E}|^2 dv =$ average power dissipated in V

$P_{ei} = -\frac{1}{2}\int_V \mathrm{Re}\left(\mathbf{J}_i^*\cdot\mathbf{E}\right)dv =$ average outgoing power due to the source $\mathbf{J}_i^*$

$P_{mi} = -\frac{1}{2}\int_V \mathrm{Re}\left(\mathbf{M}_i\cdot\mathbf{H}^*\right)dv =$ average outgoing power due to the source $\mathbf{M}_i$

complex power

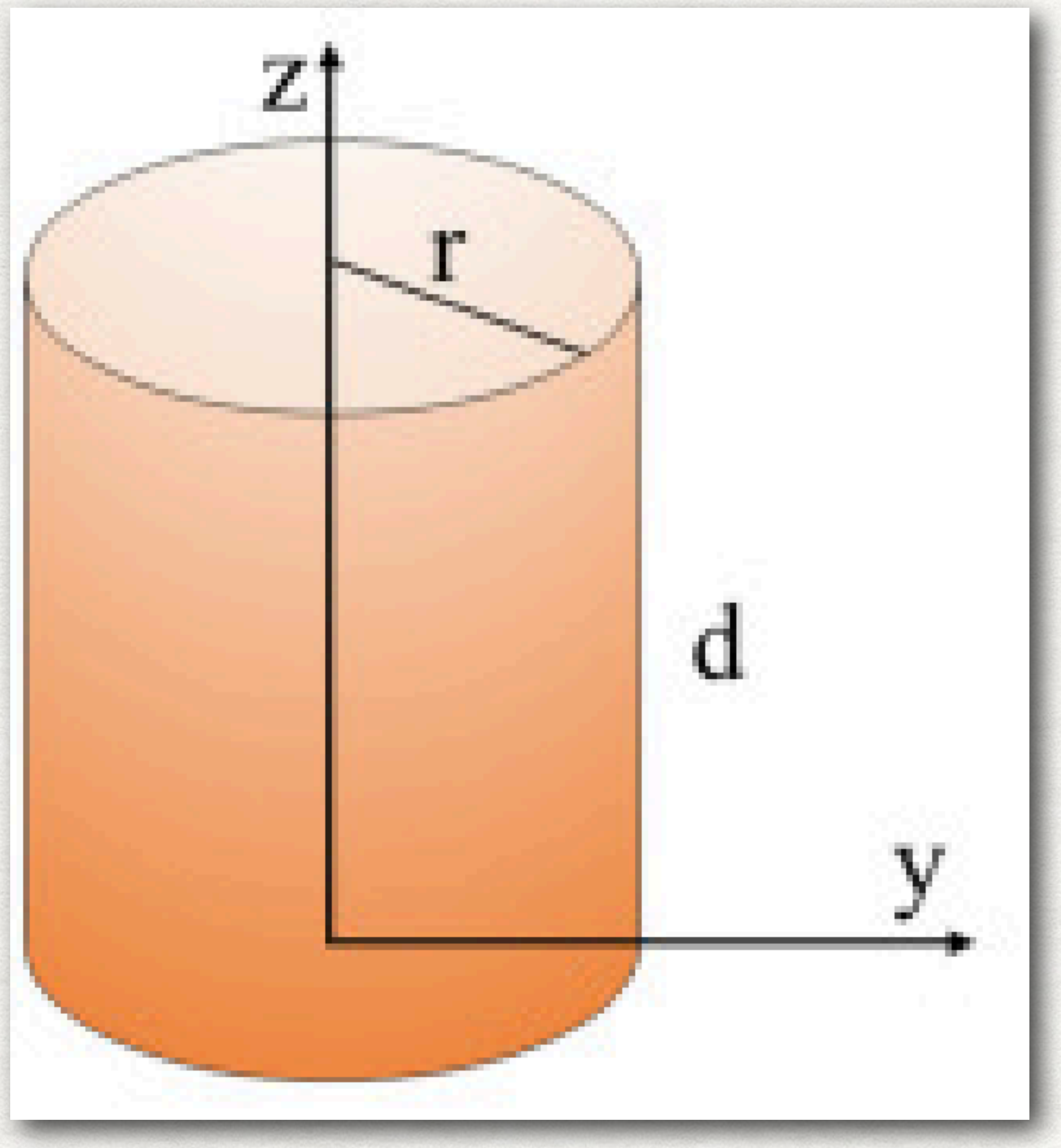
$$\frac{1}{2}Im\oint_{S_c}(\mathbf{E}\times\mathbf{H}^*)\cdot ds = 2\omega\left[W_e - W_m\right] - \frac{1}{2}Im\int\left[\mathbf{J}_i^*\cdot\mathbf{E} + \mathbf{M}_i\cdot\mathbf{H}^*\right]dv$$

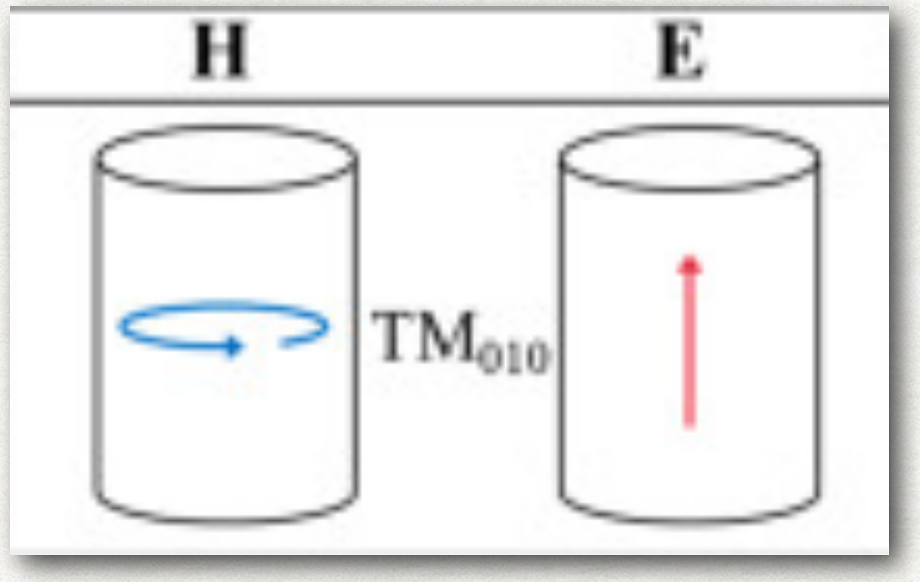
Here Balanis uses $\overrightarrow{M}$ as magnetic current

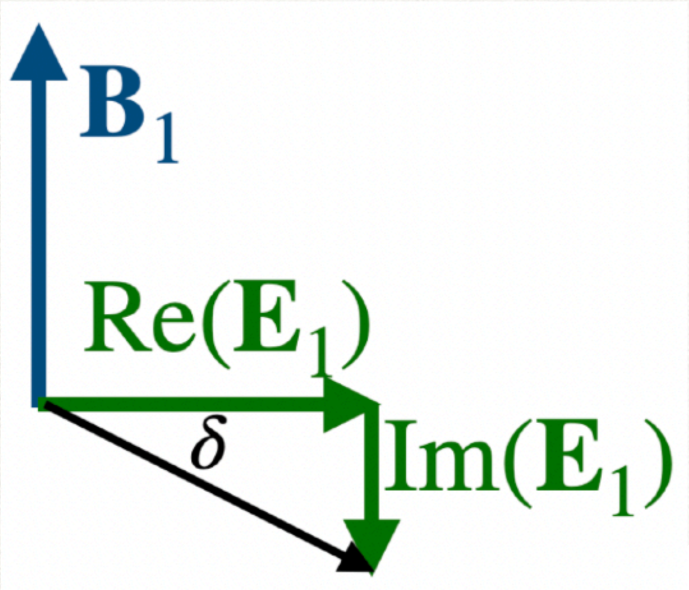
$W_e = \frac{1}{4}\int_V \epsilon_o\epsilon_r|\mathbf{E}|^2 dv =$ average electric energy in V

$W_m = \frac{1}{4}\int_V \mu_o\mu_r|\mathbf{H}|^2 dv =$ average magnetic energy in V

IMAGINARY POYNTING VECTOR INSIDE CAVITY



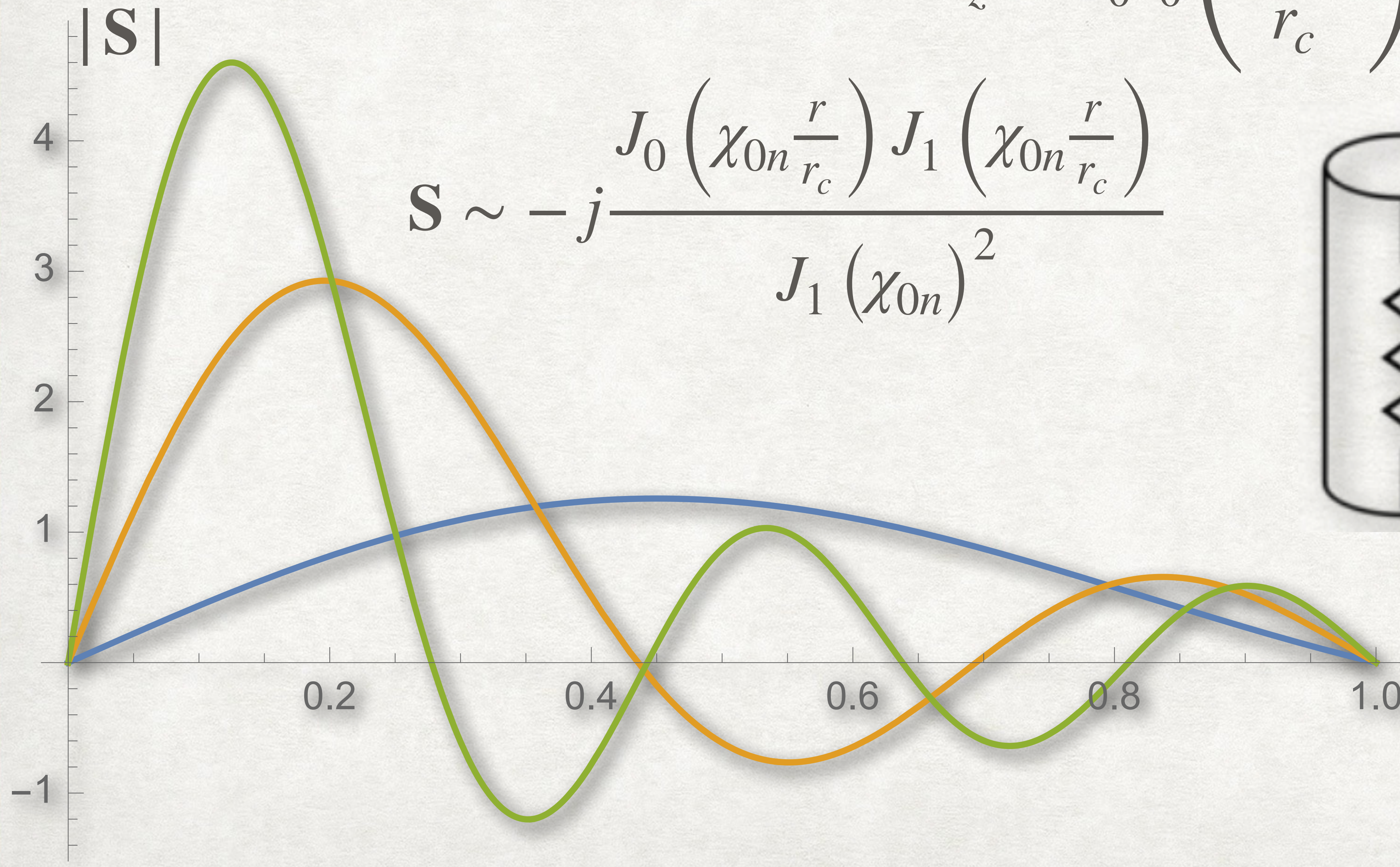




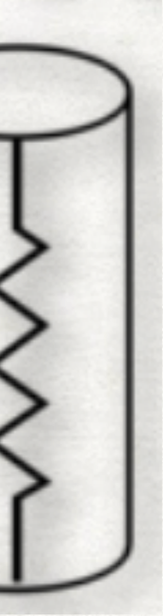
TM_{0n0}

$$\tilde{H}_\phi = -j\tilde{E}_0(\omega\epsilon)\frac{r_c}{\chi_{0n}}J_0'\left(\frac{\chi_{0n}}{r_c}r\right)$$

$$\tilde{E}_z = \tilde{E}_0 J_0\left(\frac{\chi_{0n}}{r_c}r\right)$$

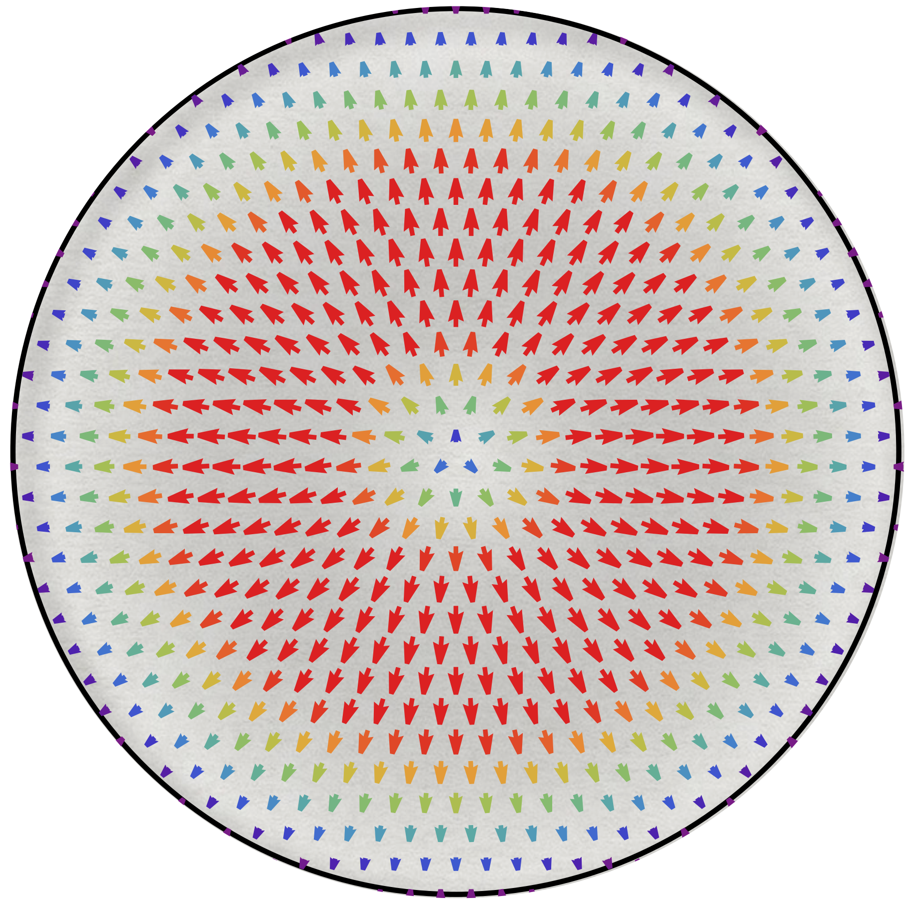


$$\mathbf{S} \sim -j\frac{J_0\left(\chi_{0n}\frac{r}{r_c}\right)J_1\left(\chi_{0n}\frac{r}{r_c}\right)}{J_1\left(\chi_{0n}\right)^2}$$

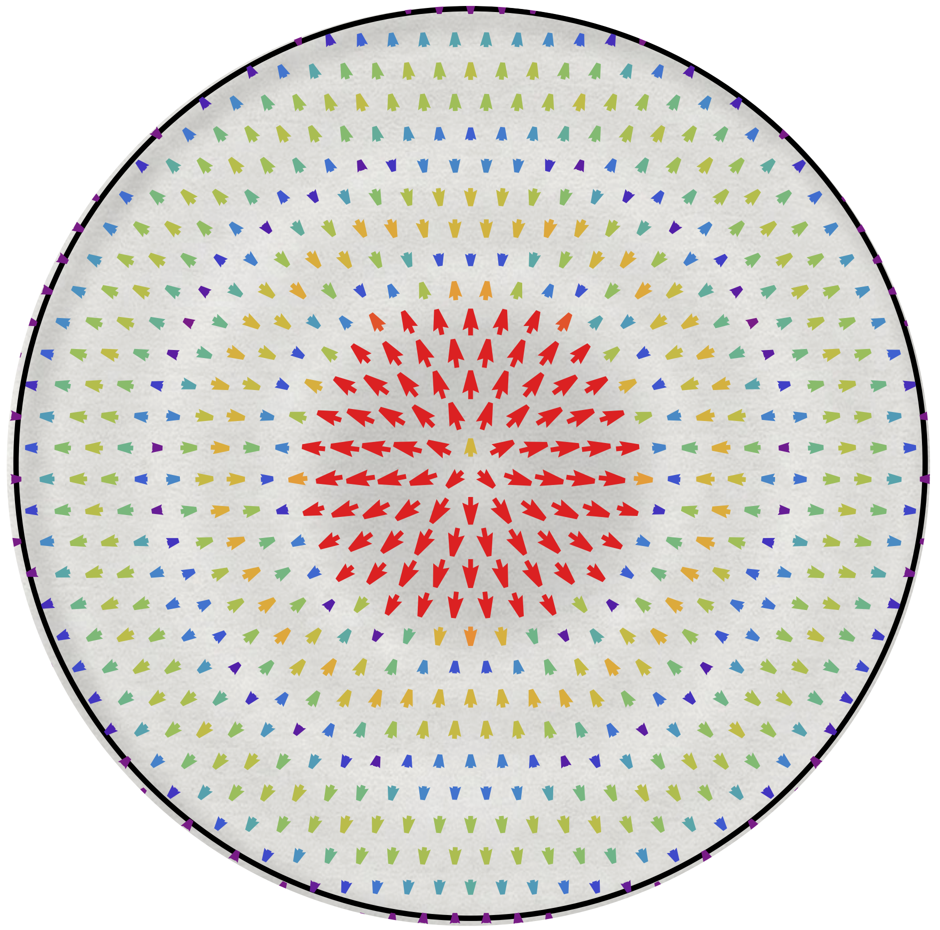


However Dissipation Gives real part of Poynting vector

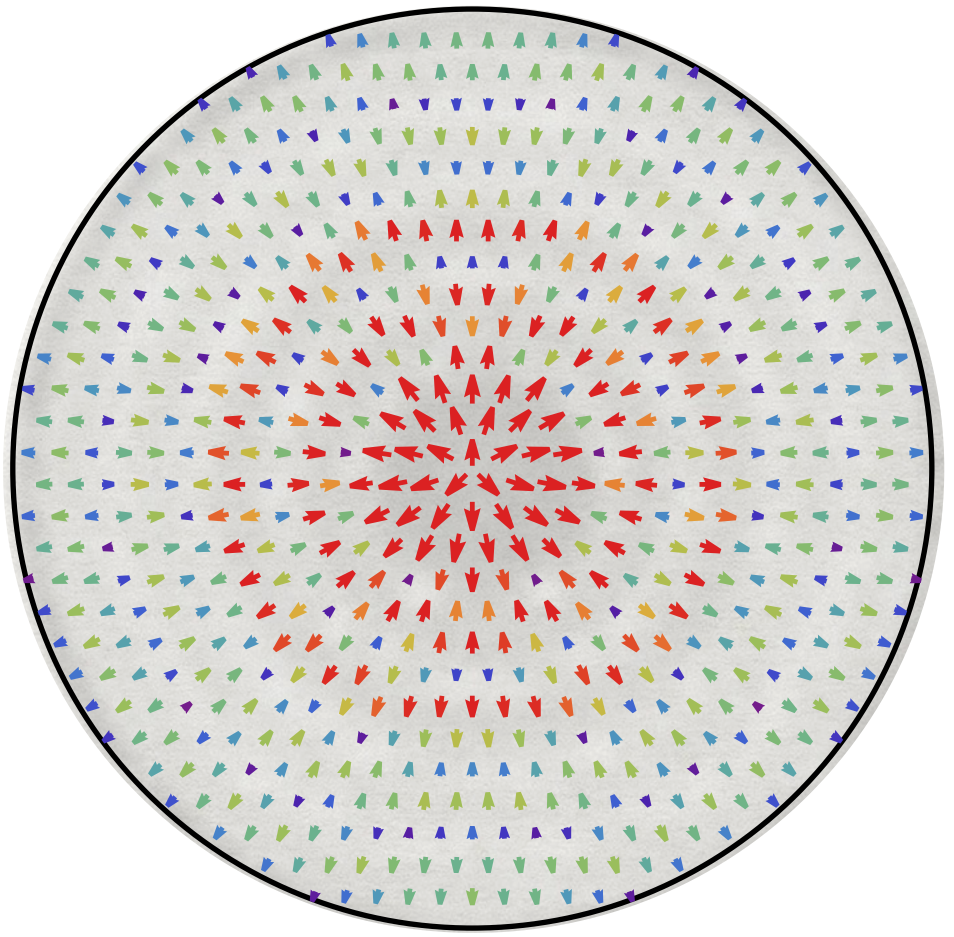
$\vec{S} = \vec{E} \times \vec{H}$



TM_{010}



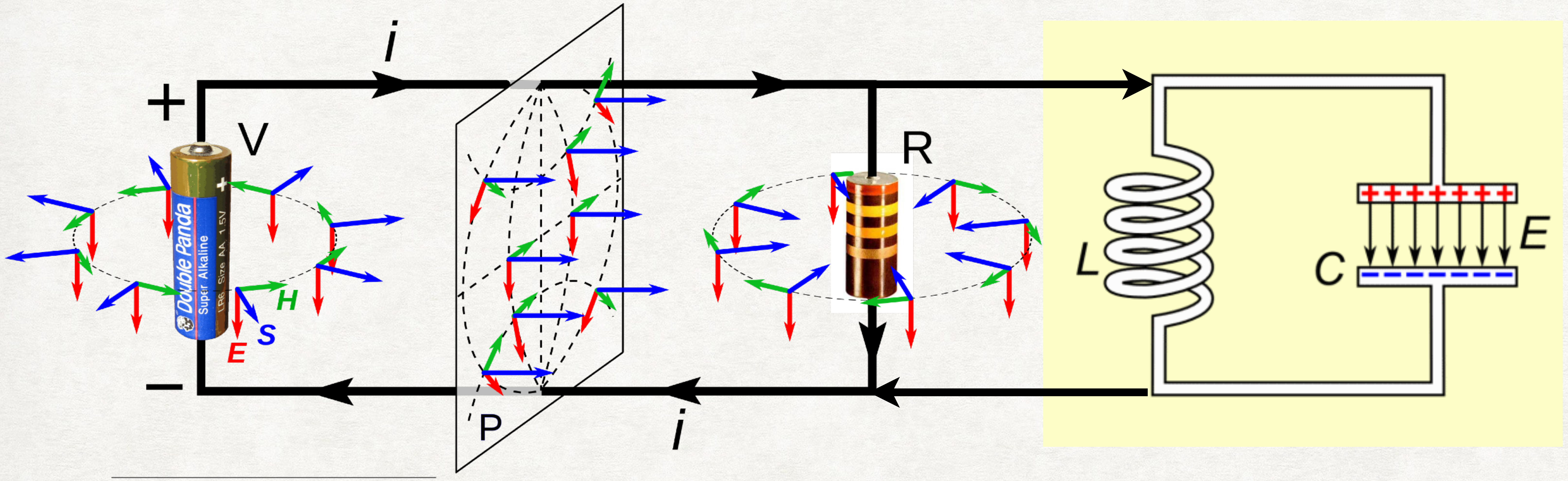
TM_{020}

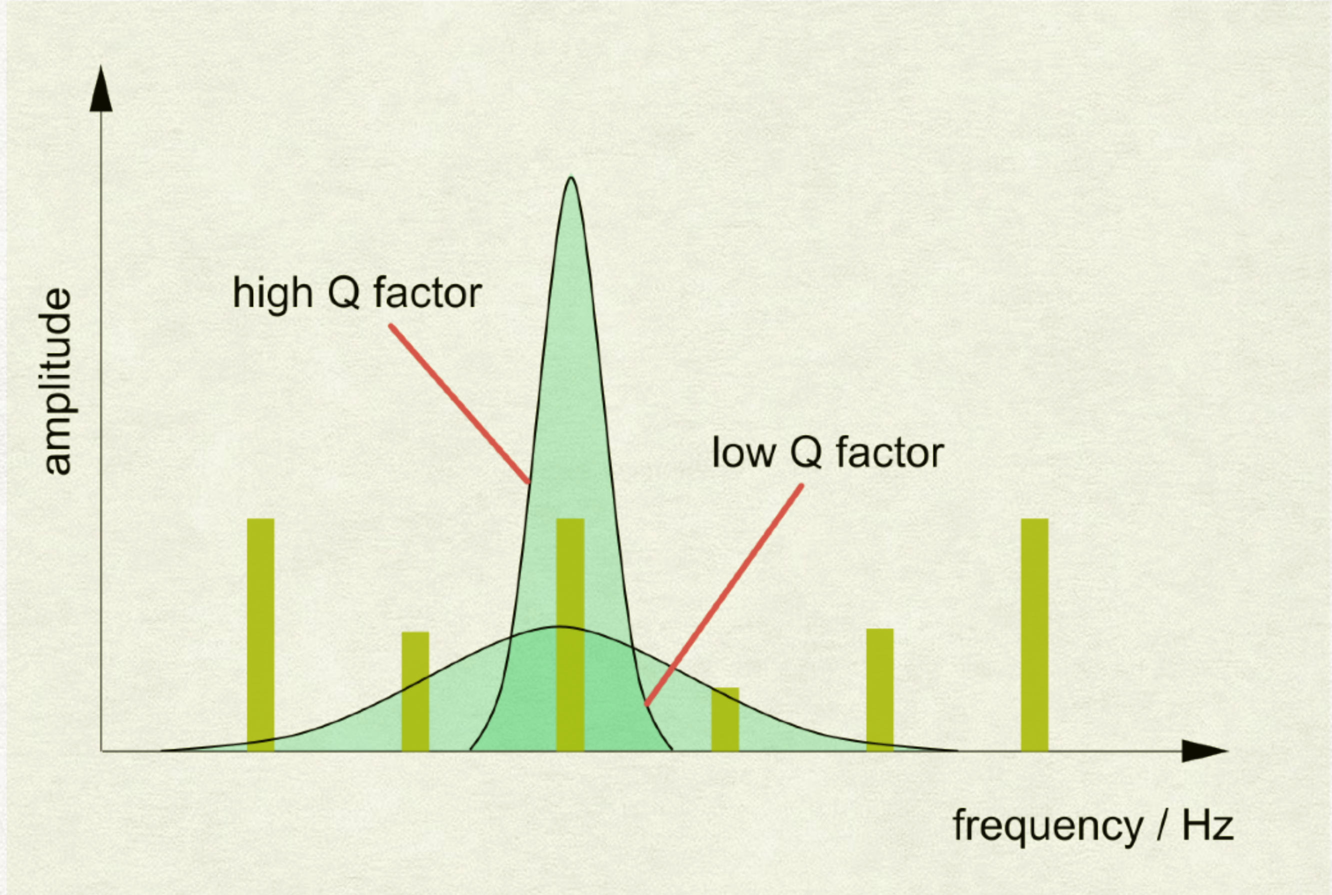


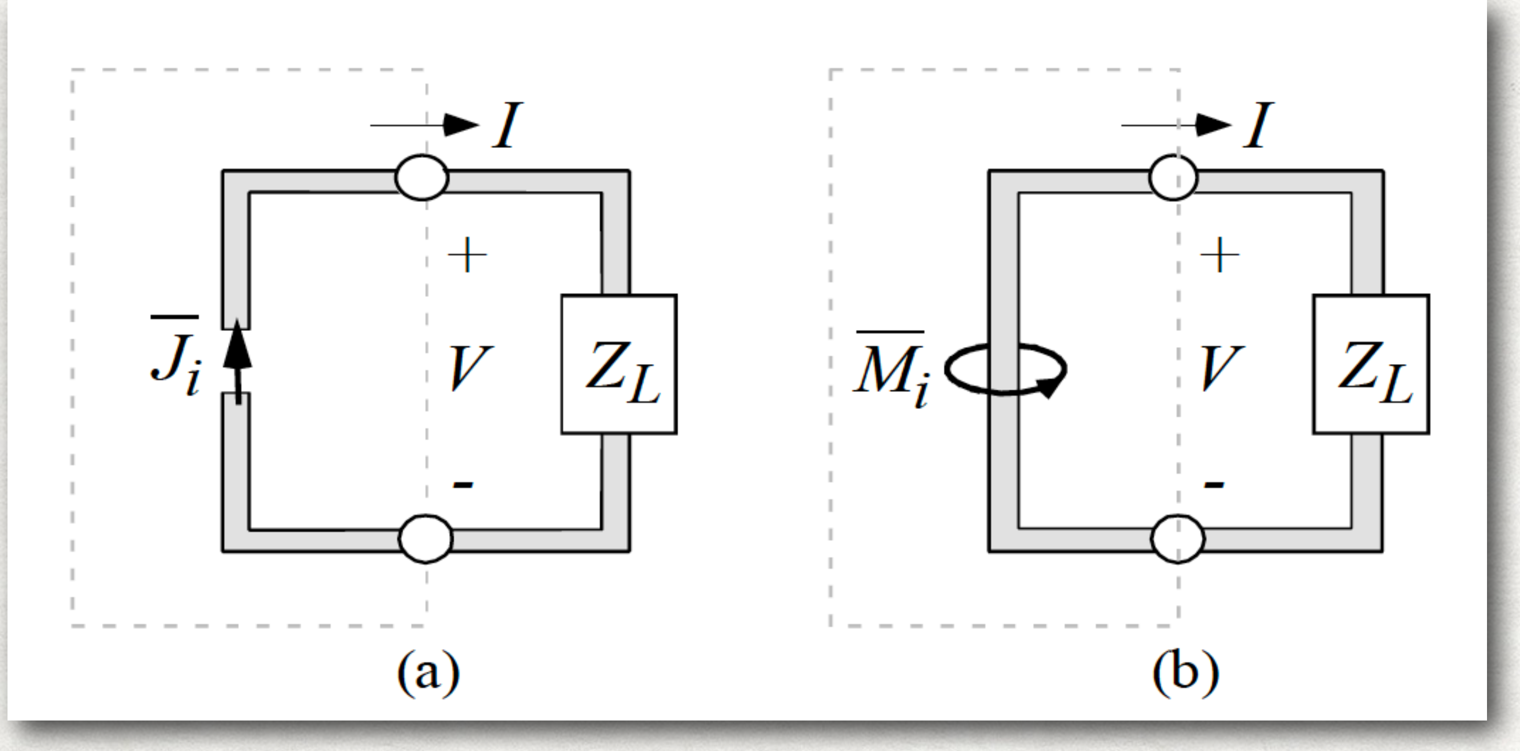
TM_{030}

At resonance Poynting vector is REAL
Small dissipation: Power builds up
Enhanced power per cycle with large Q

EQUIVALENT CIRCUIT AND POYNTING VECTOR







$$P_{in} = \frac{1}{2} V I^* = \frac{1}{2} I I^* \left(R + \jmath\omega L + \frac{1}{\jmath\omega C} \right)$$

$$P_{in} = P_{loss} + 2\jmath\omega \left(U_m - U_e \right)$$

$$U_m = \tfrac{1}{4} L I I^* \text{ and } U_e = \tfrac{1}{4} C V V^*$$

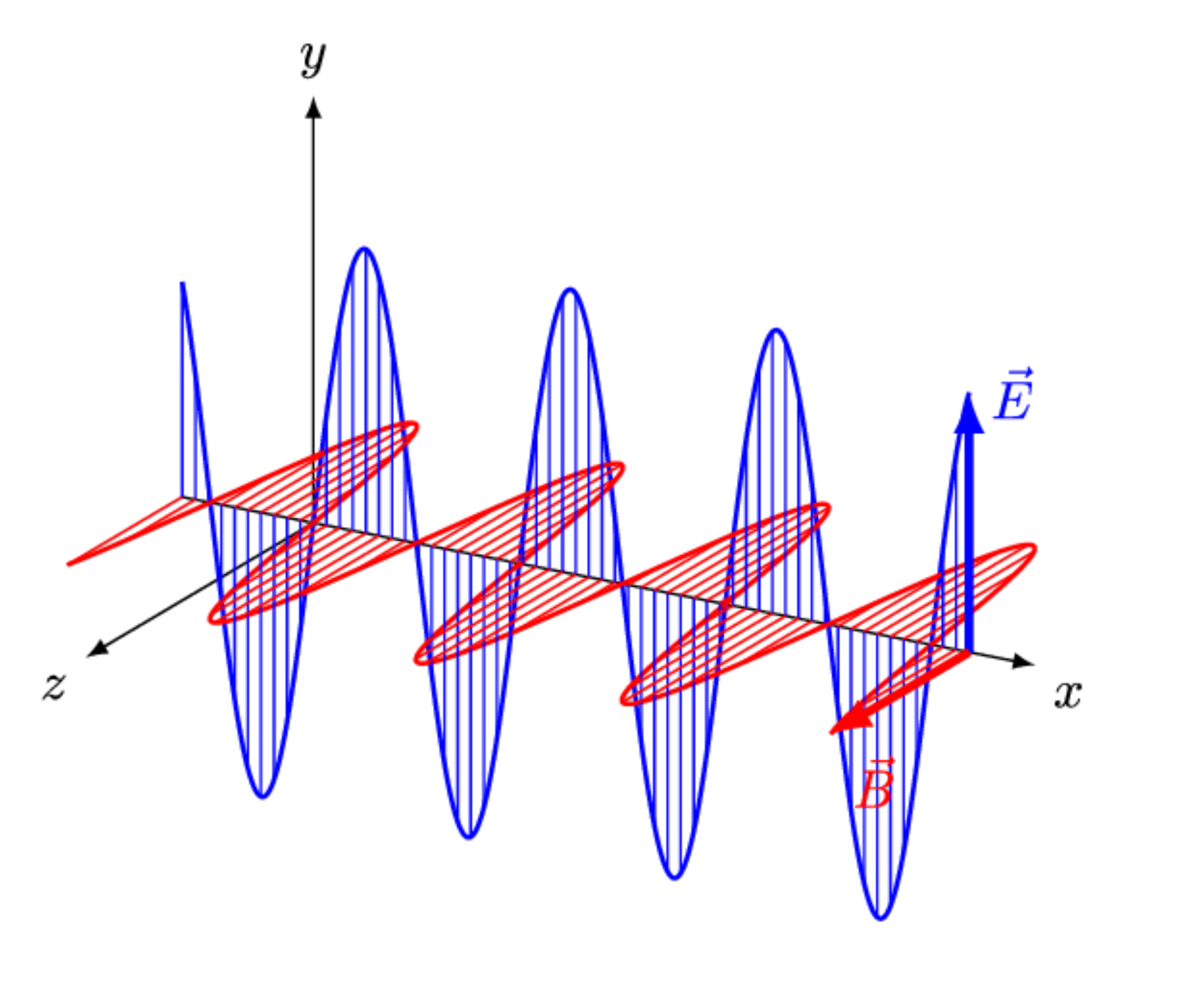
$$P_{loss} = \tfrac{1}{2} R I I^*$$

$$U_m = \frac{1}{4} \mathrm{Re} \iiint \overline{H}^* \cdot \overline{B}\, dV \qquad U_e = \frac{1}{4} \mathrm{Re} \iiint \overline{E} \cdot \overline{D}^* dV$$

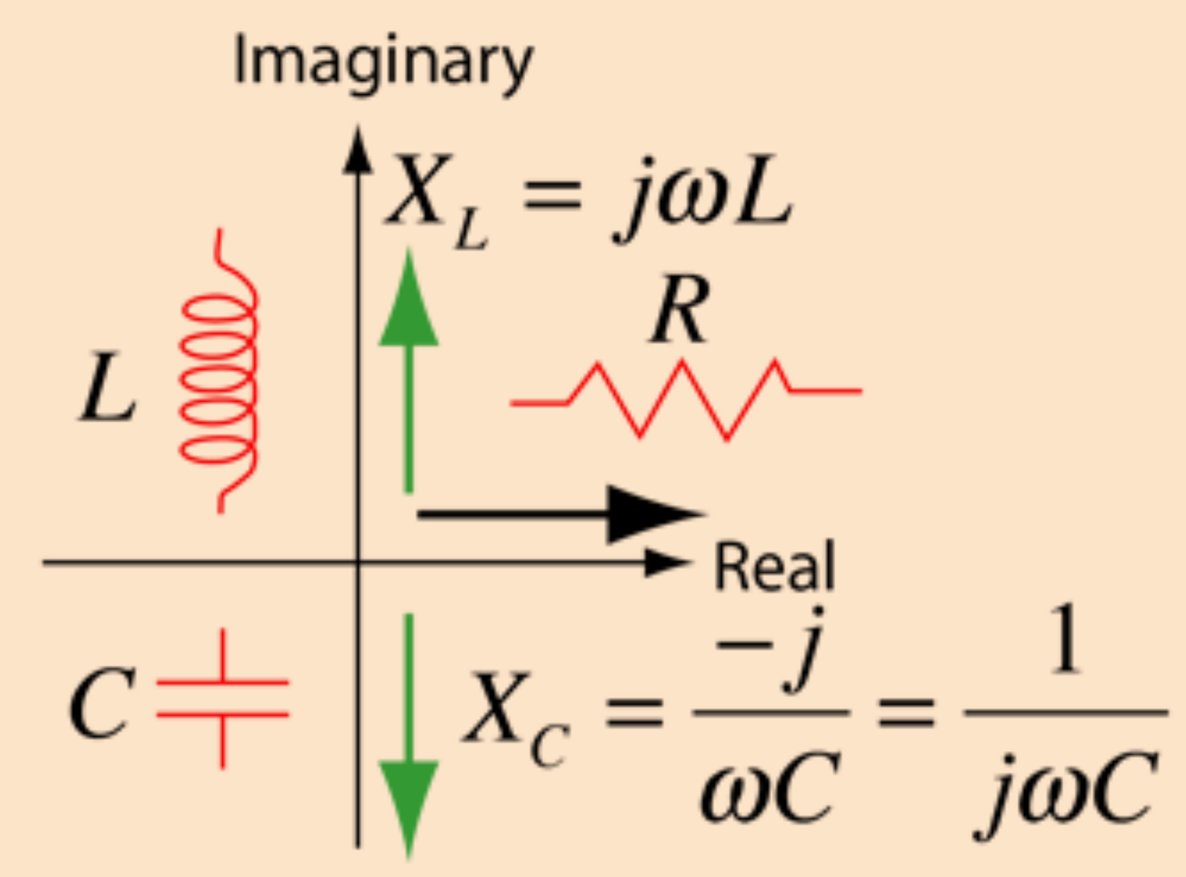
Here Balanis uses $\overrightarrow{M}$ as magnetic current

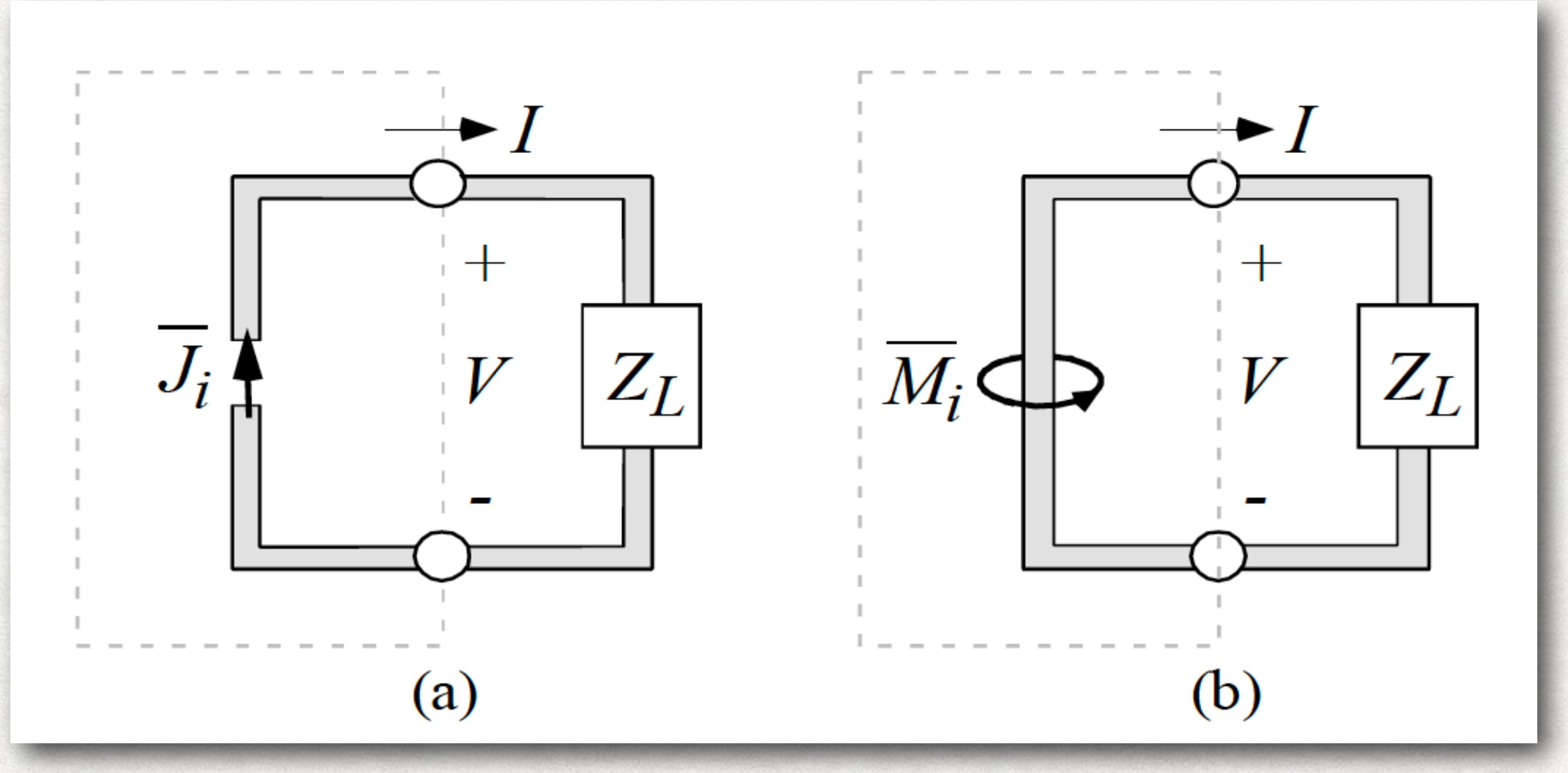
- Reactive Power Oscillates between Electric and Magnetic field in the cavity as Stored Energy
- Source does not need to provide reactive power on resonance
- Steady state: Source power balanced by dissipative power in resonator
- High-Q, low-loss per cycle, power in resonator builds up (circulating power put into narrow frequency bandwidth)

PROPAGATING POYNTING VECTOR IS REAL



INDUCTOR OR CAPACITOR HAS AN IMAGINARY POYNTING VECTOR





Axion-Photon Coupling

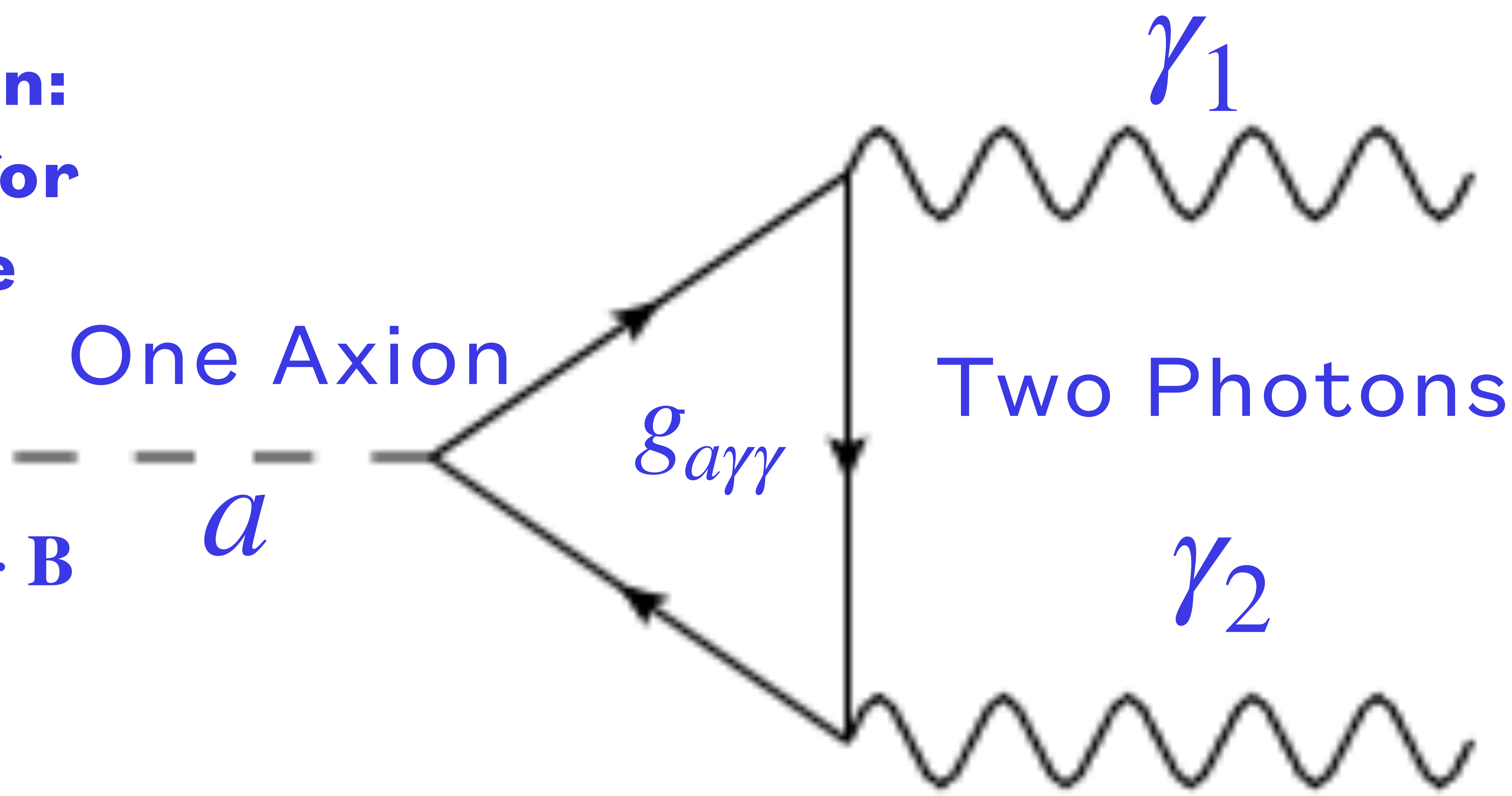
- Axion is predicted to couple to photons, coupling parameter, $g_{a\gamma\gamma}$
- Two-photon transition, interaction Hamiltonian density
$\mathscr{H} = \mathscr{H}_{\text{EM}} + \mathscr{H}_a + \mathscr{H}_{int} \; : \;\; \mathscr{H}_{int} = \varepsilon_0 c g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B}$

Equation of Motion:
Maxwell's Equations

$$\nabla \times \vec{B}_1 = \mu_0 \epsilon_0 \partial_t \vec{E}_1 + \mu_0 \vec{J}_{e1}$$
$$\nabla \times \vec{E}_1 = -\partial_t \vec{B}_1$$
$$\nabla \cdot \vec{B}_1 = 0$$
$$\nabla \cdot \vec{E}_1 = \epsilon_0^{-1} \rho_{e_1}$$

Axion Equation of Motion:
Klein-Gordon equation for massive spin 0 particle

$$\left(\Box + \frac{m_a^2 c^2}{\hbar^2} \right) a = -g_{a\gamma\gamma} \varepsilon_0 c \mathbf{E} \cdot \mathbf{B}$$
$$\Box = c^{-2} \partial_t \partial_t - \nabla^2$$



$$\nabla \times \vec{B}_2 = \mu_0 \epsilon_0 \partial_t \vec{E}_2 + \mu_0 \vec{J}_{e2}$$
$$\nabla \times \vec{E}_2 = -\partial_t \vec{B}_2$$
$$\nabla \cdot \vec{B}_2 = 0$$
$$\nabla \cdot \vec{E}_2 = \epsilon_0^{-1} \rho_{e_2}$$

Axion Coupling to two Photonic Degree of Freedoms Modifies Electrodynamics

Haloscopes

- Axions convert into photons in presence of strong background electromagnetic field

1) Background field (subscript zero)

2) Created Photon Field (subscript 1)

Axion Equation of Motion:

Klein–Gordon equation for massive spin 0 particle

$$a(t) = \frac{1}{2}\left(\tilde{a}e^{-j\omega_a t} + \tilde{a}^* e^{j\omega_a t}\right) = \mathrm{Re}\left(\tilde{a}e^{-j\omega_a t}\right)$$

Modified Axion Electrodynamics

(Represents two photons)

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} + c g_{a\gamma\gamma} \vec{B} \, . \nabla a$$

$$\nabla \times \vec{B} - \frac{1}{c^2}\partial_t \vec{E} = \mu_0 \vec{J}_e - g_{a\gamma\gamma}\epsilon_0 c \left(\vec{B}\partial_t a + \nabla a \times \vec{E}\right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

$$\epsilon_0 \nabla \cdot \vec{E}_1 = \rho_{e1} + \rho_{ab}$$

$$\frac{1}{\mu_0}\nabla \times \vec{B}_1 - \epsilon_0 \partial_t \vec{E}_1 = \vec{J}_{e1} + \vec{J}_{ab} + \vec{J}_{ae}$$

$$\rho_{ab} = g_{a\gamma\gamma}\epsilon_0 c \nabla \cdot \left(a(t)\vec{B}_0(\vec{r}, t)\right)$$

$$\vec{J}_{ab} = - g_{a\gamma\gamma}\epsilon_0 c \partial_t \left(a(t)\vec{B}_0(\vec{r}, t)\right)$$

$$\vec{J}_{ae} = - g_{a\gamma\gamma}\epsilon_0 c \nabla \times \left(a(t)\vec{E}_0(\vec{r}, t)\right)$$

$$\nabla \cdot \vec{J}_{ab} = - \partial_t \rho_{ab}$$

Source Terms generate Photons->
From background fields mixing with axion

Photonic Haloscope Equations in terms of Auxiliary Fields

Modified Axion Electrodynamics

(Represents two photons)

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} + c g_{a\gamma\gamma} \vec{B} . \nabla a$$

$$\nabla \times \vec{B} - \frac{1}{c^2}\partial_t \vec{E} =$$

$$\mu_0 \vec{J}_e - g_{a\gamma\gamma}\epsilon_0 c \left(\vec{B}\partial_t a + \nabla a \times \vec{E} \right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

Applied Background Field

$$\nabla \times \vec{B}_0 = \mu_0\epsilon_0\partial_t \vec{E}_0 + \mu_0 \vec{J}_{e_0}$$

$$\nabla \times \vec{E}_0 = -\partial_t \vec{B}_0$$

$$\nabla \cdot \vec{B}_0 = 0$$

$$\nabla \cdot \vec{E}_0 = \epsilon_0^{-1}\rho_{e_0}$$

Measure Created Photon

$$\nabla \cdot \left(\vec{E}_1(\vec{r},t) - g_{a\gamma\gamma} a(t) c \vec{B}_0(\vec{r},t) \right) = \frac{\rho_{e_1}}{\epsilon_0}$$

$$\nabla \times \left(c\vec{B}_1(\vec{r},t) + g_{a\gamma\gamma} a(t) \vec{E}_0(\vec{r},t) \right)$$

$$-\frac{1}{c}\partial_t \left(\vec{E}_1(\vec{r},t) - g_{a\gamma\gamma} a(\vec{r},t) c \vec{B}_0(\vec{r},t) \right) = c\mu_0 \vec{J}_{e_1}$$

$$\nabla \cdot c\vec{B}_1(\vec{r},t) = 0$$

$$\nabla \times \vec{E}_1(\vec{r},t) + \frac{1}{c}\partial_t c \vec{B}_1(\vec{r},t) = 0.$$

$$\nabla \cdot \vec{D}_1 = \rho_{e_1}$$

$$\nabla \times \vec{H}_1 - \partial_t \vec{D}_1 = \vec{J}_{e_1}$$

$$\nabla \cdot \vec{B}_1(\vec{r},t) = 0$$

$$\nabla \times \vec{E}_1(\vec{r},t) + \partial_t \vec{B}_1(\vec{r},t) = 0,$$

Constitutive Relations(Include Matter)

$$\vec{H}_1(\vec{r},t) = \frac{\vec{B}_1}{\mu_0} - \vec{M}_1 - \vec{M}_{a1};$$

$$\vec{D}_1(\vec{r},t) = \epsilon_0 \vec{E}_1 + \vec{P}_1 + \vec{P}_{a1}$$

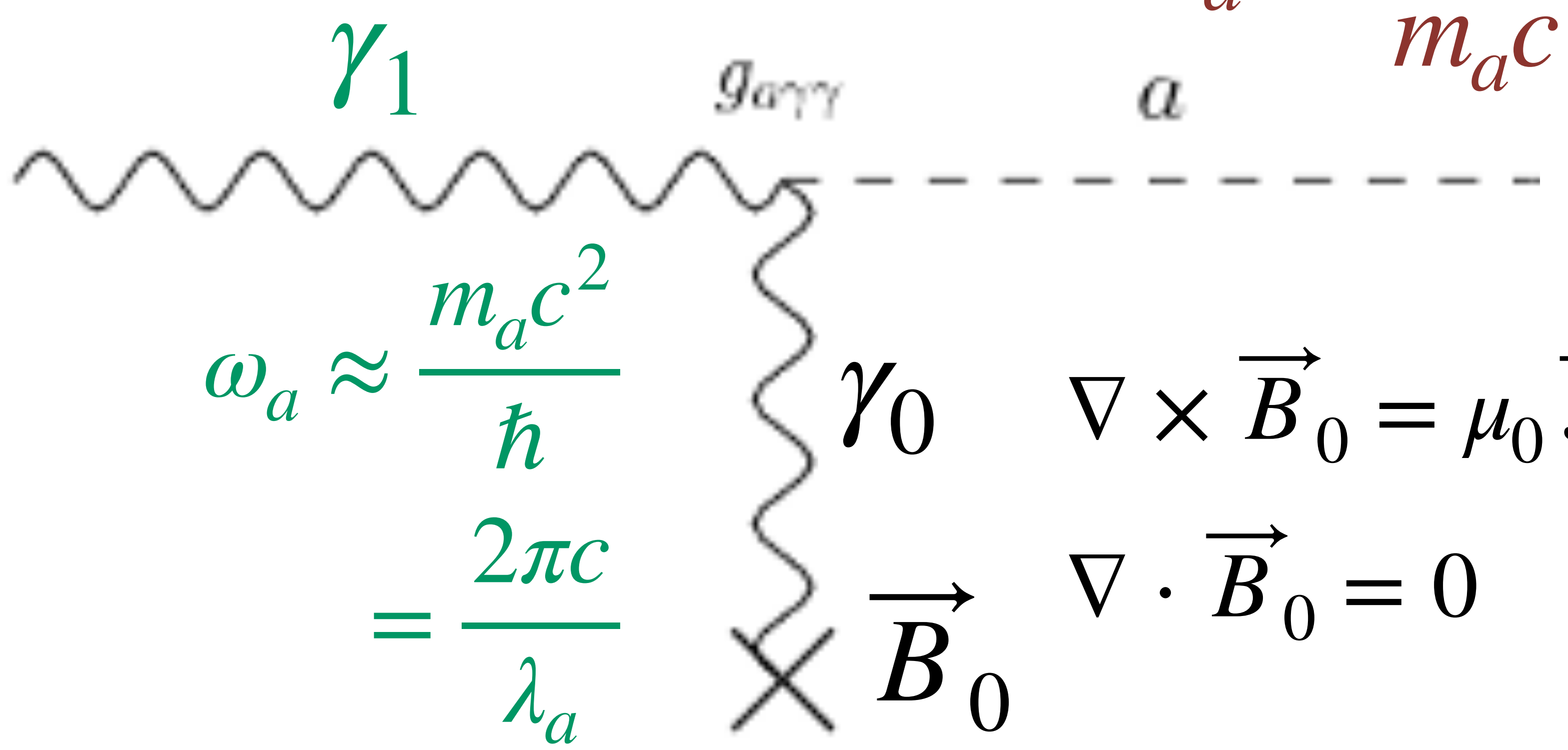
$\vec{M}_{a1} = -g_{a\gamma\gamma} a(t) c \epsilon_0 \vec{E}_0(\vec{r},t)$ and $\frac{1}{\epsilon_0}\vec{P}_{a1} = -g_{a\gamma\gamma} a(t) c \vec{B}_0(\vec{r},t)$

$$\nabla \times \vec{D}_1(\vec{r},t) = -\partial_t \vec{B}_1(\vec{r},t) + \nabla \times (\vec{P}_1 + \vec{P}_{a1})$$

$$\nabla \times \vec{P}_{a1} \neq 0 = -g_{a\gamma\gamma} a(t) c \nabla \times \vec{B}_0(\vec{r},t) \quad (\nabla a = 0)$$

$$\vec{J}_{ab}(\vec{r},t) = \frac{\partial \vec{P}_{a1}(\vec{r},t)}{\partial t}$$

$$\lambda_a = \frac{h}{m_a c}$$



$$\omega_a \approx \frac{m_a c^2}{\hbar} = \frac{2\pi c}{\lambda_a}$$

$$\nabla \times \vec{B}_0 = \mu_0 \vec{J}_{e_0}$$

$$\nabla \cdot \vec{B}_0 = 0$$

Poynting vector analysis of photonic conversion of the dark matter axion mixing with a background DC magnetic Field

- Apply Poynting theorem to axion modified electrodynamics
- Two possible Poynting vectors, analogous to the Abraham Poynting vector and Minkowski Poynting vector
- Minkowski picks up the extra non-conservative terms
- The non-conservative terms -> categorised as “curl forces” -> non-conservative and non-dissipative forces
- Fictitious Electric Field: Outside of the conservative Maxwell’s Equations.
- Both give the same sensitivity for a resonant cavity axion haloscope, but predict markedly different sensitivity for low-mass broad band reactive haloscopes.

Curl Force

PRL 103, 108101 (2009) PHYSICAL REVIEW LETTERS week ending 4 SEPTEMBER 2009

Direct Measurement of the Nonconservative Force Field Generated by Optical Tweezers

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(Received 15 March 2009; published 1 September 2009)

$$\mathbf{F} = -\underbrace{\nabla V}_{\text{conservative}} + \underbrace{\nabla\times\mathbf{A}}_{\text{non-conservative}} \equiv \underbrace{\mathbf{F}_{\text{grad}}}_{\text{conservative}} + \underbrace{\mathbf{F}_{\text{curl}}}_{\text{non-conservative}}, \quad (1)$$

where the nonconservative force $\mathbf{F}_{\text{curl}}$ satisfies

$$\nabla\times\mathbf{F} = \nabla\times\mathbf{F}_{\text{curl}} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A} \neq 0.$$

Classical and quantum complex Hamiltonian curl forces

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Received 23 June 2020, revised 29 July 2020
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Published 10 September 2020

CrossMark

Abstract

Quantized Hamiltonian Curl Forces and Squeezed Light

P. Strange

(Dated: July 3, 2018)

In this paper we discuss quantum curl forces. We present both the classical and quantum theory of linear curl forces. The quantum theory is shown to reproduce the classical theory precisely if appropriate combinations of eigenfunctions are chosen. A series of examples are used to illustrate the theory and to demonstrate its limitations. Furthermore we are able to point out an analogy between the quantum theory of curl forces and some of the squeezed light states of quantum optics.

IOP FTC
IOP Publishing Journal of Physics A: Mathematical and Theoretical
J. Phys. A: Math. Theor. 46 (2013) 422001 (9pp) doi:10.1088/1751-8113/46/42/422001

FAST TRACK COMMUNICATION

Physical curl forces: dipole dynamics near optical vortices

KEY ISSUES REVIEW

Non-conservative optical forces

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Physical curl forces: dipole dynamics near optical vortices

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The European Physical Journal D
Regular Article

Curl forces and their role in optics and ion trapping

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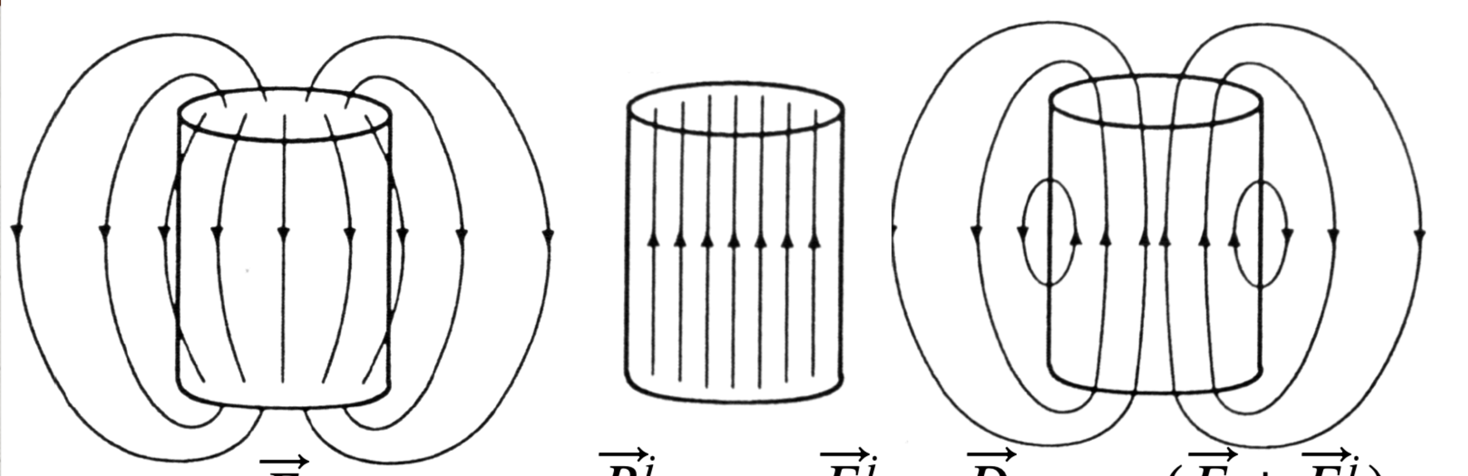
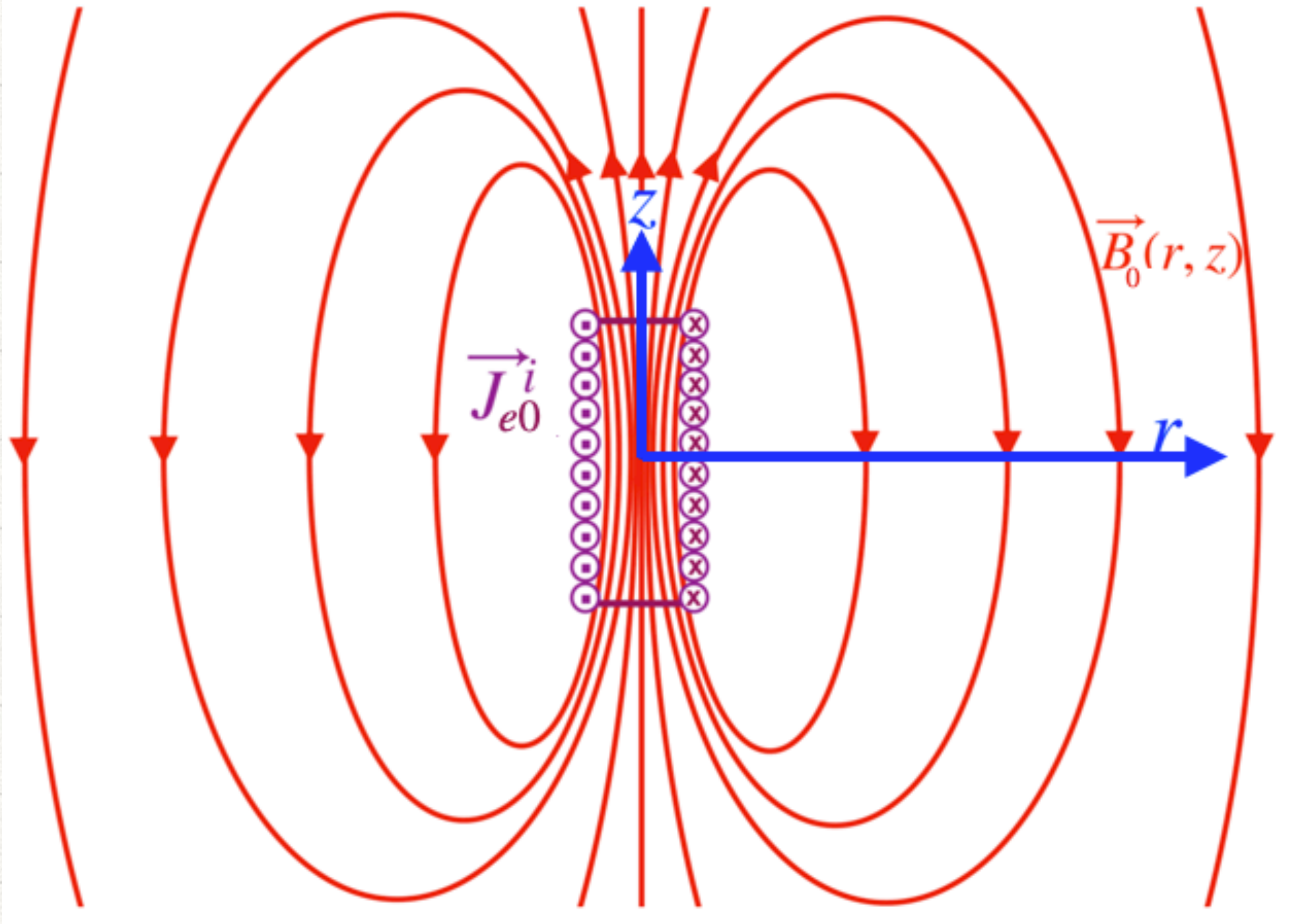
Electret-> Curl Force
Electric Vector Potential

$$\nabla\times\vec{D} = -\epsilon_0\mu_0\partial_t(\vec{B}) + \nabla\times\vec{P}$$



Curl Force in Axions Electrodynamics



$$\vec{B}_0(\vec{r}) = \frac{\mu_0}{4\pi}\int_\Omega \frac{\nabla\times\vec{J}^i_{DC}(\vec{r}')}{|\vec{r}-\vec{r}'|}d^3\vec{r}'$$

$$\vec{E}_{a1}(\vec{r},t) = -g_{a\gamma\gamma}a(t)c\vec{B}_0(\vec{r})$$

$$= -\frac{1}{4\pi}\int_\Omega \frac{\nabla\times\vec{J}^i_{ma}(\vec{r}',t)}{|\vec{r}-\vec{r}'|}d^3\vec{r}'$$

Fictitious Electric field

$$\vec{J}_{m1a} = g_{a\gamma\gamma}a(t)\mu_0 c\vec{J}_{e0}$$

Axion Modified Faraday's Law

$$\frac{1}{\epsilon_0}\nabla\times\vec{D}_1 = \nabla\times\vec{E}_1 - g_{a\gamma\gamma}c\nabla\times(a\vec{B}_0) = -(\partial_t\vec{B}_1 + g_{a\gamma\gamma}a(t)c\mu_0\vec{J}_{e0})$$

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Broadband electrical action sensing techniques with conducting wires for low-mass dark matter axion detection

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Modified axion electrodynamics as impressed electromagnetic sources through oscillating background polarization and magnetization

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Magnetic Vector Potential generates a Conservative Force -> Not a non-conservative Curl force

AXION ELECTRODYNAMICS IN HARMONIC PHASOR FORM

Cavity Electric Field

$$\vec{E}_1(\vec{r},t) = \frac{1}{2}\left(\mathbf{E}_1(\vec{r})e^{-j\omega_1 t} + \mathbf{E}_1^*(\vec{r})e^{j\omega_1 t}\right) = \mathrm{Re}\left[\mathbf{E}_1(\mathbf{r})e^{-j\omega_1 t}\right]$$

Axion Scalar Field

$$a(t) = \frac{1}{2}\left(\tilde{a}e^{-j\omega_a t} + \tilde{a}^* e^{j\omega_a t}\right) = \mathrm{Re}\left(\tilde{a}e^{-j\omega_a t}\right)$$

Axion Phasor

$$\tilde{A} = \tilde{a}e^{-j\omega_a t} \qquad \tilde{A}^* = \tilde{a}^* e^{j\omega_a t}$$

Ampere's law in time dependent form

$$\frac{1}{\mu_0}\nabla \times \vec{B}_1(\vec{r},t)) = \vec{J}_{e_1} + \partial_t\left(\epsilon_0 \vec{E}_1(\vec{r},t) - g_{a\gamma\gamma}a(\vec{r},t)\epsilon_0 c\vec{B}_0(\vec{r},t)\right)$$

Alternative Faraday's law in phasor form (Minkowski)

$$\frac{1}{\epsilon_0}\nabla \times \tilde{\mathbf{D}}_1 = j\omega_1\tilde{\mathbf{B}}_1 - g_{a\gamma\gamma}c\mu_0\tilde{A}\vec{J}_{e0}$$

$$\frac{1}{\epsilon_0}\nabla \times \tilde{\mathbf{D}}_1^* = -j\omega_1\tilde{\mathbf{B}}_1^* - g_{a\gamma\gamma}c\mu_0\tilde{A}^*\vec{J}_{e0}$$

Cavity Electric Field Phasor

$$\tilde{\mathbf{E}}_1(\vec{r},t) = \mathbf{E}_1(\vec{r})e^{-j\omega_1 t} \qquad \tilde{\mathbf{E}}_1^*(\vec{r},t) = \mathbf{E}_1^*(\vec{r})e^{j\omega_1 t}$$

Ampere's law in phasor form

$$\frac{1}{\mu_0}\nabla \times \tilde{\mathbf{B}}_1 = \tilde{\mathbf{J}}_{e1} - j\omega_1\epsilon_0\tilde{\mathbf{E}}_1 + j\omega_a g_{a\gamma\gamma}\epsilon_0 c\tilde{A}\vec{B}_0$$

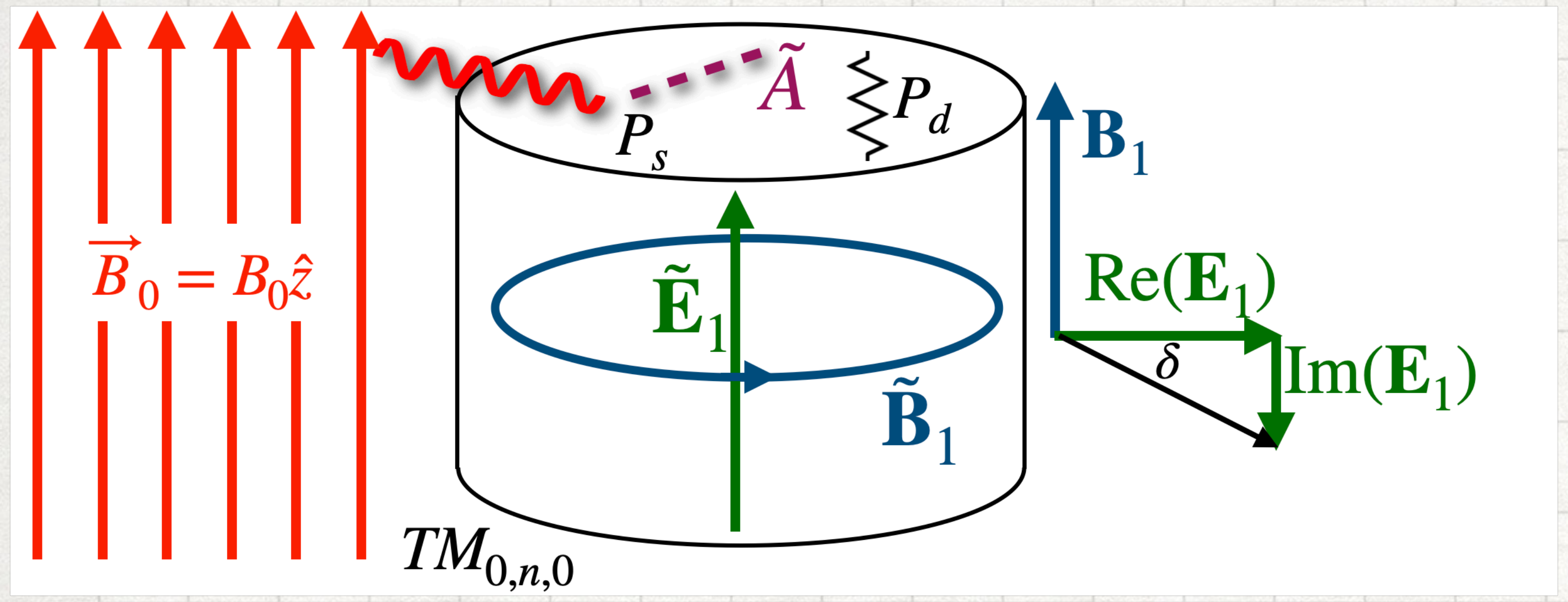
$$\frac{1}{\mu_0}\nabla \times \tilde{\mathbf{B}}_1^* = \tilde{\mathbf{J}}_{e1}^* + j\omega_1\epsilon_0\tilde{\mathbf{E}}_1^* - j\omega_a g_{a\gamma\gamma}\epsilon_0 c\tilde{A}^*\vec{B}_0,$$

Faraday's law in phasor form (Abraham)

$$\nabla \times \tilde{\mathbf{E}}_1 = j\omega_1 \times \tilde{\mathbf{B}}_1$$

$$\nabla \times \tilde{\mathbf{E}}_1^* = -j\omega_1 \times \tilde{\mathbf{B}}_1^*,$$

COMPLEX ABRAHAM POYNTING VECTOR FOR A DC AXION HALOSCOPE



$$\mathbf{S} = \frac{1}{2\mu_0}\mathbf{E}_1 \times \mathbf{B}_1^* \text{ and } \mathbf{S}^* = \frac{1}{2\mu_0}\mathbf{E}_1^* \times \mathbf{B}_1$$

$$\nabla \cdot \mathbf{S} = \frac{1}{2\mu_0}\nabla \cdot (\mathbf{E}_1 \times \mathbf{B}_1^*) = \frac{1}{2\mu_0}\mathbf{B}_1^* \cdot (\nabla \times \mathbf{E}_1) - \frac{1}{2\mu_0}\mathbf{E}_1 \cdot (\nabla \times \mathbf{B}_1^*)$$

$$\nabla \cdot \mathbf{S}^* = \frac{1}{2\mu_0}\nabla \cdot (\mathbf{E}_1^* \times \mathbf{B}_1) = \frac{1}{2\mu_0}\mathbf{B}_1 \cdot (\nabla \times \mathbf{E}_1^*) - \frac{1}{2\mu_0}\mathbf{E}_1^* \cdot (\nabla \times \mathbf{B}_1)$$

Can show

Real part of Poynting Theorem

$$\oint \mathrm{Re}\,(\mathbf{S}) \cdot \hat{n}ds = \int \Big(\frac{j\omega_a}{4}\epsilon_0 g_{a\gamma\gamma} c \vec{B}_0 \cdot (\tilde{a}^*\mathbf{E}_1 - \tilde{a}\mathbf{E}_1^*)) - \frac{1}{4}(\mathbf{E}_1 \cdot \mathbf{J}_{e1}^* + \mathbf{E}_1^* \cdot \mathbf{J}_{e1})\Big)\, d\tau$$

Can show

Reactive part of Poynting Theorem

$$\oint j\,\mathrm{Im}\,(\mathbf{S}) \cdot \hat{n}ds = \int \Big(\frac{j\omega_1}{2}(\frac{1}{\mu_0}\mathbf{B}_1^* \cdot \mathbf{B}_1 - \epsilon_0 \mathbf{E}_1 \cdot \mathbf{E}_1^*) + \frac{j\omega_a}{4}\epsilon_0 g_{a\gamma\gamma} c \vec{B}_0 \cdot (\tilde{a}^*\mathbf{E}_1 + \tilde{a}\mathbf{E}_1^*) - \frac{1}{4}(\mathbf{E}_1 \cdot \mathbf{J}_{e1}^* - \mathbf{E}_1^* \cdot \mathbf{J}_{e1}))\Big)\, d\tau$$

= 0 on resonance

$$\oint \langle \mathbf{S} \rangle \cdot dS = -\frac{1}{4}\int (\mathbf{E}_1 \cdot \mathbf{J}_{e1}^* + \mathbf{E}_1^* \cdot \mathbf{J}_{e1})\, d\tau + \frac{j\omega_a \epsilon_0 g_{a\gamma\gamma}}{4}\int c\vec{B}_0 \cdot (A^*\mathbf{E}_1 - A\mathbf{E}_1^*)\, d\tau$$

Cavity Power dissipation

= 0 for closed system

Axion power input

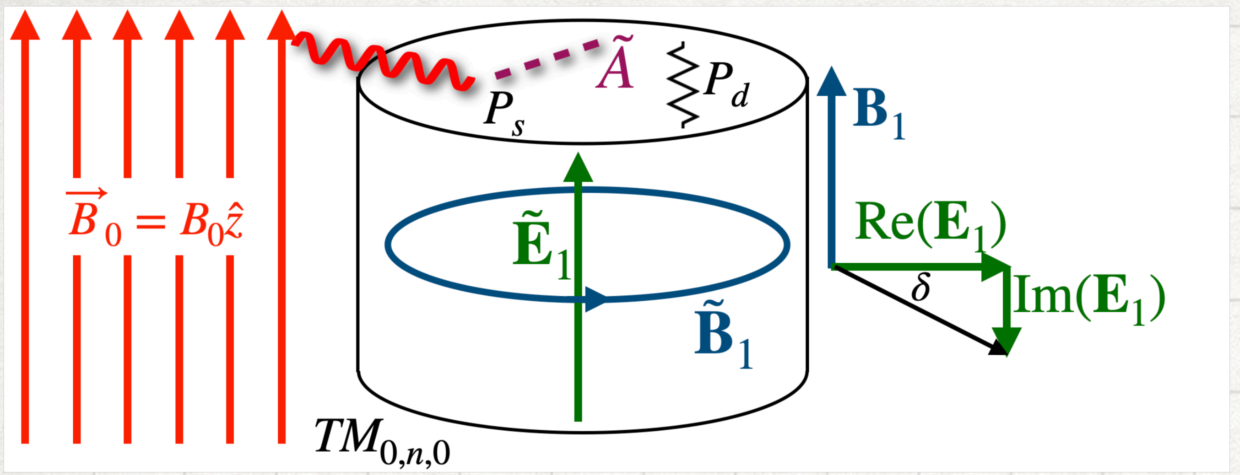
$$P_d = \frac{\omega_a \epsilon_0}{2Q}\int \mathbf{E}_1 \cdot \mathbf{E}_1^*\, d\tau = \frac{\omega_a U}{Q}$$

$$C_1 = \frac{\left(\int \vec{B}_0 \cdot \mathrm{Re}(\mathbf{E}_1)\, dV\right)^2}{B_0^2 V_1 \int \mathbf{E}_1 \cdot \mathbf{E}_1^*\, dV}$$

$$P_1 = \omega_a Q U_1 = g_{a\gamma\gamma}^2 \langle a_0 \rangle^2 \omega_a Q_1 \epsilon_0 c^2 B_0^2 V_1 C_1$$
$$= g_{a\gamma\gamma}^2 \rho_a Q_1 \epsilon_0 c^5 B_0^2 V_1 C_1 \frac{1}{\omega_a},$$

known power output for a Sikivie Haloscope

COMPLEX MINKOWSKI POYNTING VECTOR FOR A DC AXION HALOSCOPE



Real part of Poynting Theorem

$$\oint \mathrm{Re}\left(\mathbf{S}_{DB}\right)\cdot\hat{n}ds = \int\Big(\frac{j(\omega_1-\omega_a)}{4}\epsilon_0 g_{a\gamma\gamma} c\vec{B}_0\cdot(\tilde{a}\mathbf{E}_1^* - \tilde{a}^*\mathbf{E}_1) + \frac{1}{4}g_{a\gamma\gamma}c\vec{B}_0\cdot(\tilde{a}\mathbf{J}_{e_1}^* + \tilde{a}^*\mathbf{J}_{e_1}) - \frac{1}{4}g_{a\gamma\gamma}\vec{J}_{e_0}\cdot(\tilde{a}^*c\mathbf{B}_1 + \tilde{a}c\mathbf{B}_1^*) - \frac{1}{4}(\mathbf{E}_1\cdot\mathbf{J}_{e_1}^* + \mathbf{E}_1^*\cdot\mathbf{J}_{e_1})\Big)\,dV,$$

Reactive part of Poynting Theorem

$$\oint j\,\mathrm{Im}\left(\mathbf{S}_{DB}\right)\cdot\hat{n}ds = \int\Big(\frac{j\omega_1}{2}\big(\frac{1}{\mu_0}\mathbf{B}_1^*\cdot\mathbf{B}_1 - \epsilon_0\mathbf{E}_1\cdot\mathbf{E}_1^*\big) + \frac{j(\omega_1+\omega_a)\epsilon_0 g_{a\gamma\gamma}}{4}c\vec{B}_0\cdot(\tilde{a}\mathbf{E}_1^* + \tilde{a}^*\mathbf{E}_1) + \frac{1}{4}g_{a\gamma\gamma}c\vec{B}_0\cdot(\tilde{a}\mathbf{J}_{e_1}^* - \tilde{a}^*\mathbf{J}_{e_1}) + \frac{1}{4}g_{a\gamma\gamma}\vec{J}_{e_0}\cdot(\tilde{a}^*c\mathbf{B}_1 - \tilde{a}c\mathbf{B}_1^*) - \frac{1}{4}(\mathbf{E}_1\cdot\mathbf{J}_{e_1}^* - \mathbf{E}_1^*\cdot\mathbf{J}_{e_1})\Big)\,dV$$

= 0 on resonance

Axion frequency tuned to resonance $\omega_a = \omega_1$: If Halocope is inside the magnet $\frac{1}{4}g_{a\gamma\gamma}\vec{J}_{e0}\cdot(Ac\mathbf{B}_1^* + A^*c\mathbf{B}_1) = 0$

$$\oint\langle\mathbf{S}\rangle\cdot\hat{n}ds = -\frac{1}{4}\int(\mathbf{E}_1\cdot\mathbf{J}_{e1}^* + \mathbf{E}_1^*\cdot\mathbf{J}_{e1})\,d\tau + \frac{1}{4}\int c\vec{B}_0\cdot(g_{a\gamma\gamma}A\mathbf{J}_{e1}^* + g_{a\gamma\gamma}A^*\mathbf{J}_{e1})\,d\tau$$

Cavity Power dissipation

Axion power input

$$P_d = \frac{1}{4}\int(\mathbf{E}_1\cdot\mathbf{J}_{e1}^* + \mathbf{E}_1^*\cdot\mathbf{J}_{e1}) \qquad P_s = \frac{1}{4}\int c\vec{B}_0\cdot(g_{a\gamma\gamma}A\mathbf{J}_{e1}^* + g_{a\gamma\gamma}A^*\mathbf{J}_{e1})\,d\tau$$

= 0 for closed system

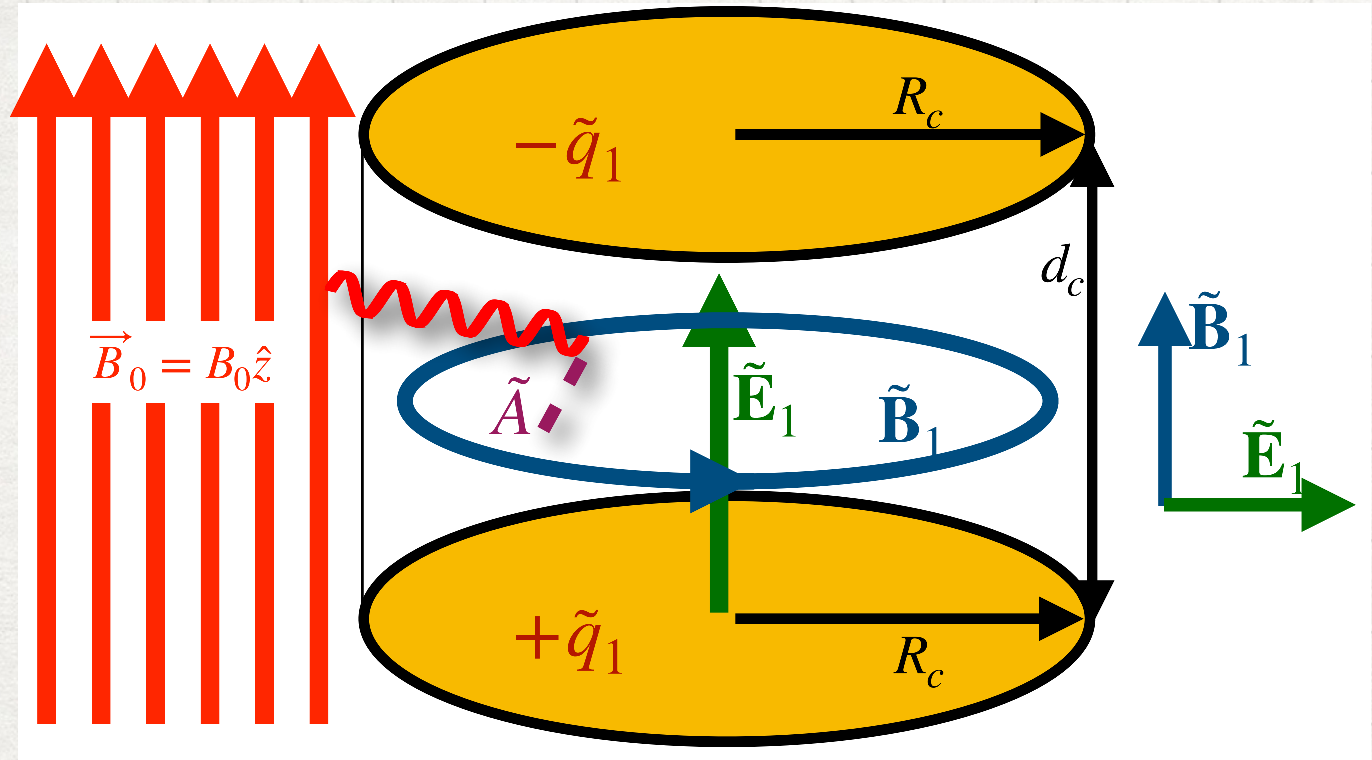
$$P_d = \frac{\omega_a\epsilon_0}{2Q}\int\mathbf{E}_1\cdot\mathbf{E}_1^*\,d\tau = \frac{\omega_a U}{Q}$$

$$C_1 = \frac{\left(\int\vec{B}_0\cdot\mathrm{Re}(\mathbf{E}_1)\,dV\right)^2}{B_0^2V_1\int\mathbf{E}_1\cdot\mathbf{E}_1^*\,dV}$$

$$P_1 = \omega_a Q U_1 = g_{a\gamma\gamma}^2\langle a_0\rangle^2\omega_a Q_1\epsilon_0 c^2 B_0^2 V_1 C_1 = g_{a\gamma\gamma}^2\rho_a Q_1\epsilon_0 c^5 B_0^2 V_1 C_1\frac{1}{\omega_a},$$

known power output for a Sikivie Haloscope

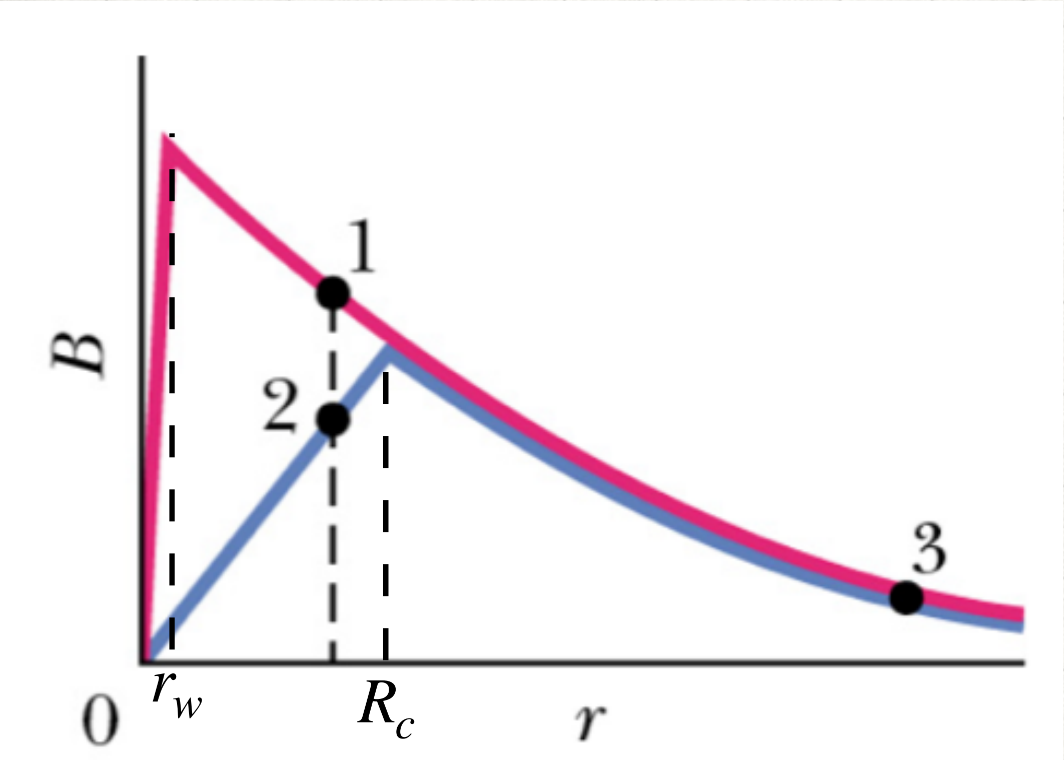
Capacitor under DC Magnetic Field: Quasi-static limit

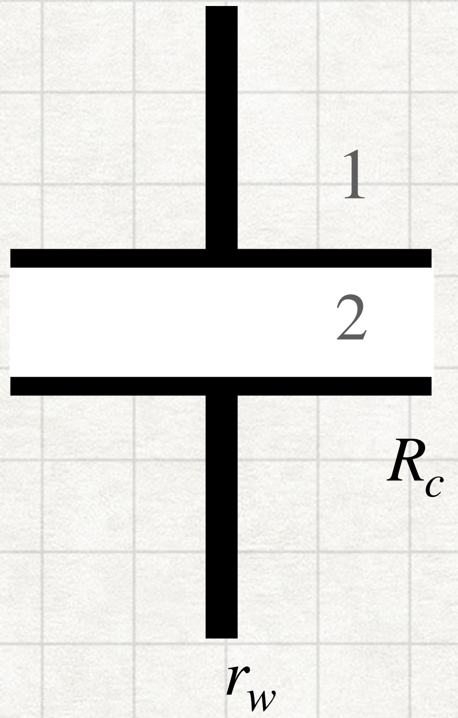


To First order: Real part of Poynting Theorem $= 0$: Reactive part of Poynting Theorem $\neq 0$

$$\oint j\,\mathrm{Im}\left(\mathbf{S}_{EH}\right)\cdot\hat{n}ds = j\omega_a\int\left(\frac{1}{2\mu_0}\mathbf{B}_1^*\cdot\mathbf{B}_1 - \frac{\epsilon_0}{2}\mathbf{E}_1^*\cdot\mathbf{E}_1 + \frac{\epsilon_0}{2}g_{a\gamma\gamma}a_0c\vec{B}_0\cdot Re(\mathbf{E}_1)\right)dV$$

$$\oint j\,\mathrm{Im}\left(\mathbf{S}_{DB}\right)\cdot\hat{n}ds = j\omega_a\int\left(\frac{1}{2\mu_0}\mathbf{B}_1^*\cdot\mathbf{B}_1 - \frac{\epsilon_0}{2}\mathbf{E}_1^*\cdot\mathbf{E}_1 + \epsilon_0 g_{a\gamma\gamma}a_0c\vec{B}_0\cdot Re(\mathbf{E}_1)\right)dV$$





3

First order: Ignore fringing

$$\mathbf{E}_1 = \frac{\tilde{q}_1}{\pi R_c^2\epsilon_0}\hat{z} \qquad \mathbf{B}_1 = -j\omega_a\mu_0\tilde{q}_1\frac{r}{\pi R_c^2}\hat{\theta} \qquad \frac{U_m}{U_e} = \frac{\int_{V_c}\mathbf{B}_1\cdot\mathbf{B}_1^*dV}{\epsilon_0\mu_0\int_{V_c}\mathbf{E}_1\cdot\mathbf{E}_1^*dV} = \frac{R_c^2\omega_a^2}{8c^2} = \frac{\pi^2R_c^2}{2\lambda_a^2}$$

Sensitivity assuming the Modified Minkowski Poynting Vector

$$\nabla\times\vec{D}_1 = \epsilon_0\nabla\times\vec{E}_1 - g_{a\gamma\gamma}\epsilon_0c\nabla\times(a\vec{B}_0) = -\epsilon_0(\partial_t\vec{B}_1 + g_{a\gamma\gamma}a(t)\mu_0c\vec{J}_{e0})$$

$$jP_a = \oint j\,\mathrm{Im}\left(\mathbf{S}_{DB}\right)\cdot\hat{n}ds = \frac{j\omega_a g_{a\gamma\gamma}a_0\epsilon_0c}{2}\int\left(\vec{B}_0\cdot\mathrm{Re}(\mathbf{E}_1)\right)dV$$

$$U_c = g_{a\gamma\gamma}^2\langle a_0\rangle^2\epsilon_0c^2B_0^2V_1$$

$$\mathscr{V}_{rms} = g_{a\gamma\gamma}\langle a_0\rangle cB_0d_c = g_{a\gamma\gamma}d_c\frac{c}{\omega_a}B_0\sqrt{\rho_ac^3}$$

Sensitivity assuming the Modified Abraham Poynting Vector

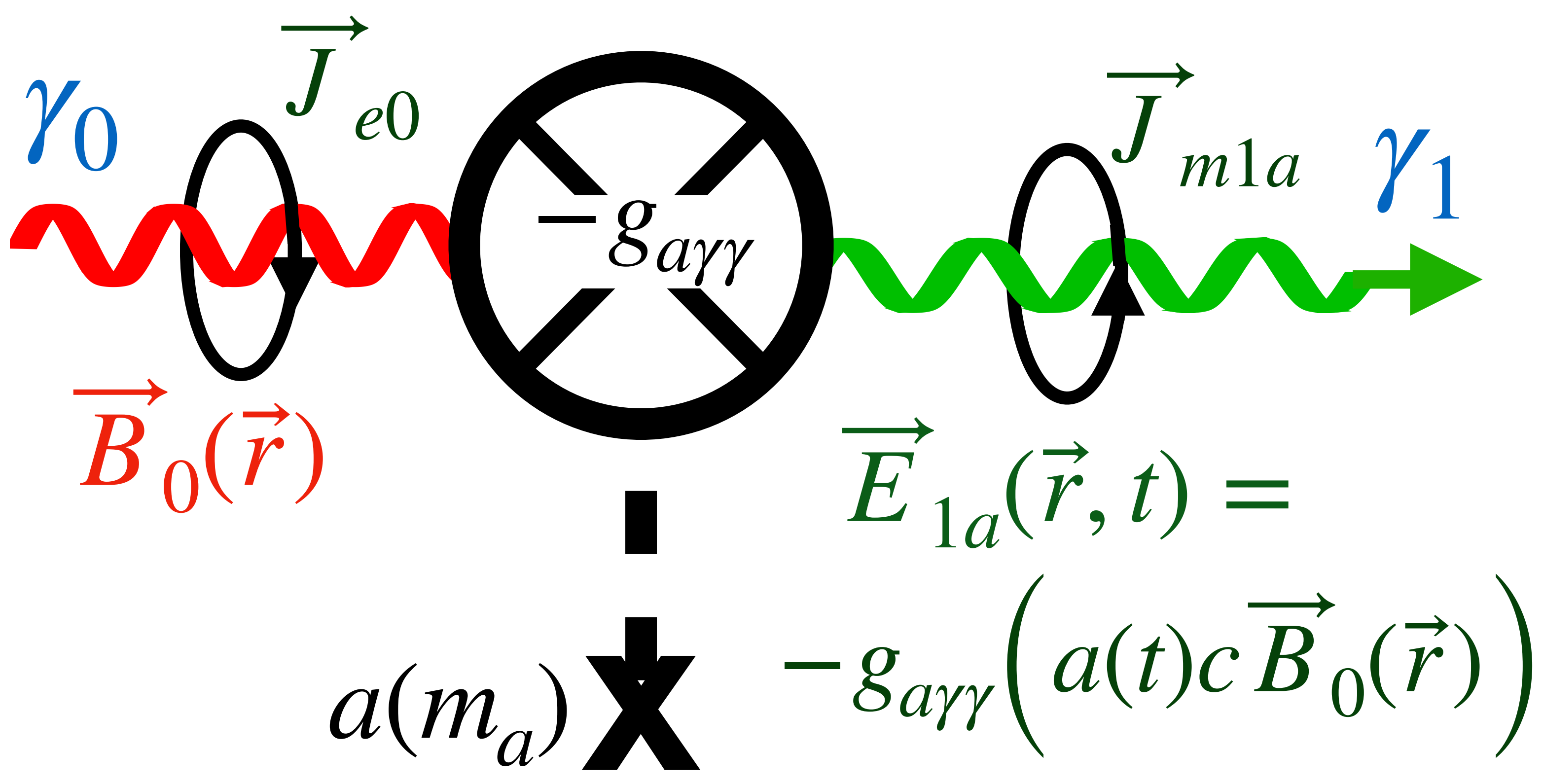
$$\nabla\times\vec{E}_1 = -\partial_t\vec{B}_1$$

$$jP_a = \oint j\,\mathrm{Im}\left(\mathbf{S}_{EH}\right)\cdot\hat{n}ds = \frac{j\omega_a g_{a\gamma\gamma}a_0\epsilon_0c}{2}\int\left(\vec{B}_0\cdot\mathrm{Re}(\mathbf{E}_1)\right)\frac{\pi^2r^2}{\lambda_a^2}dV$$

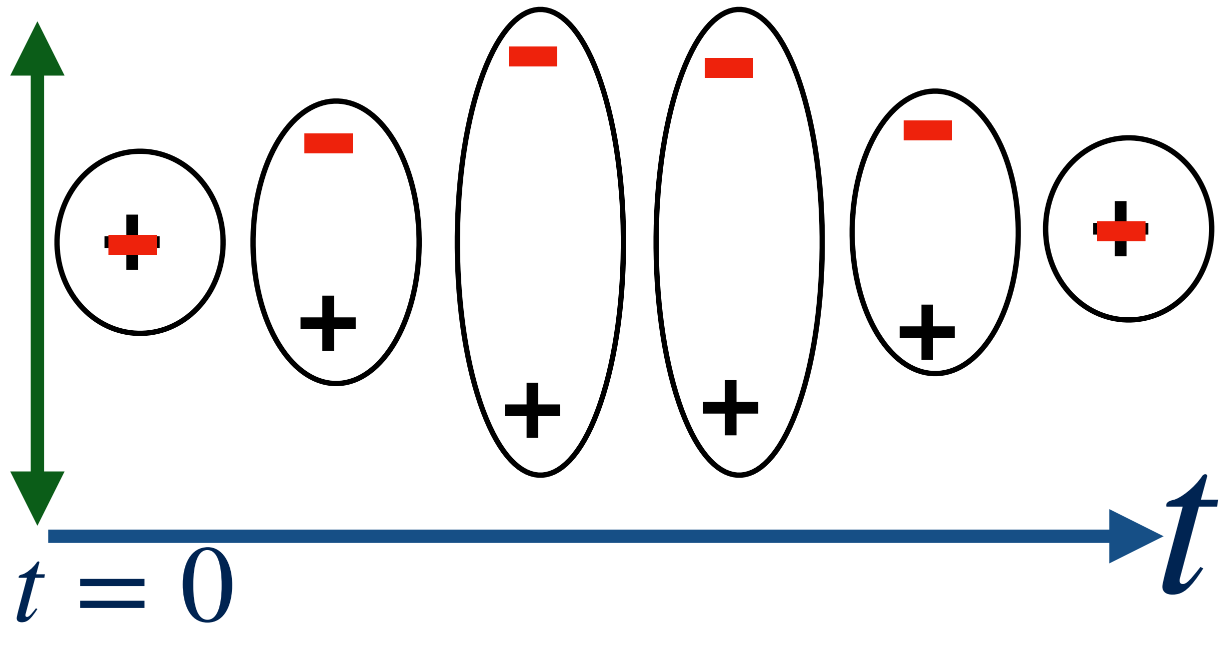
$P_a = \omega_aU_c$, where $U_c = g_{a\gamma\gamma}^2\langle a_0\rangle^2\epsilon_0c^2B_0^2V_1\left(\frac{\pi^2Rc^2}{2\lambda_a^2}\right)^2$ $\qquad U_c = \frac{1}{2}\tilde{\mathscr{V}}\tilde{\mathscr{V}}^*C_a \qquad \left(C_a = \frac{\pi R_c^2\epsilon_0}{d_c}\right)$

$$\mathscr{V}_{rms} = g_{a\gamma\gamma}\langle a_0\rangle cB_0d_c\left(\frac{\pi R_c}{\sqrt{2}\lambda_a}\right)^2 = g_{a\gamma\gamma}d_c\frac{c}{\omega_a}B_0\sqrt{\rho_ac^3}\left(\frac{\pi R_c}{\sqrt{2}\lambda_a}\right)^2$$

Electric Action of the Axion, $a(t)$, mixing with $\vec{B}_0$ -> Creates Electromagnetic Energy at ω_a



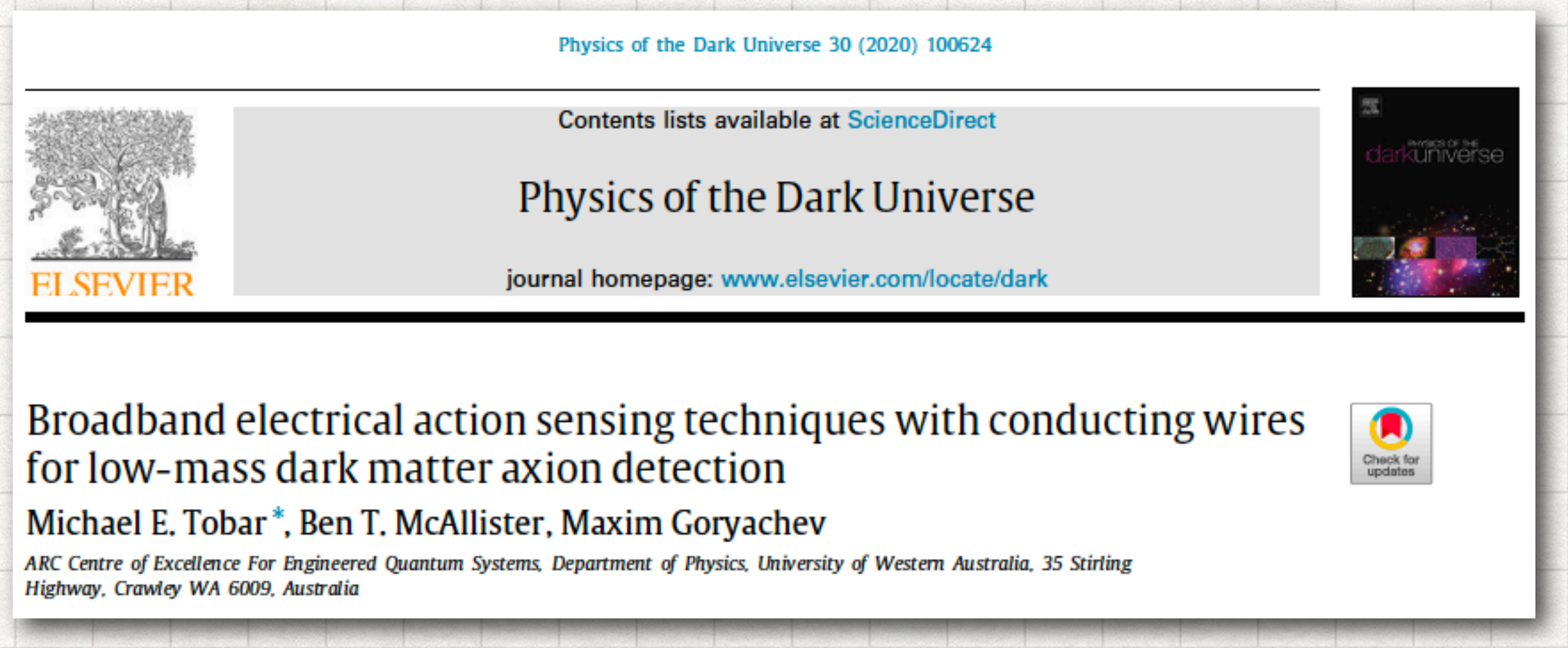
$a(t) = a_0 \sin(\omega_a t)$



$$\vec{J}_{ab}(\vec{r},t) = \epsilon_0 \frac{\partial \vec{E}_{1a}(\vec{r},t)}{\partial t}$$

$\vec{E}_{1a}$

A fictitious Electric Field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Broadband electrical action sensing techniques with conducting wires for low-mass dark matter axion detection

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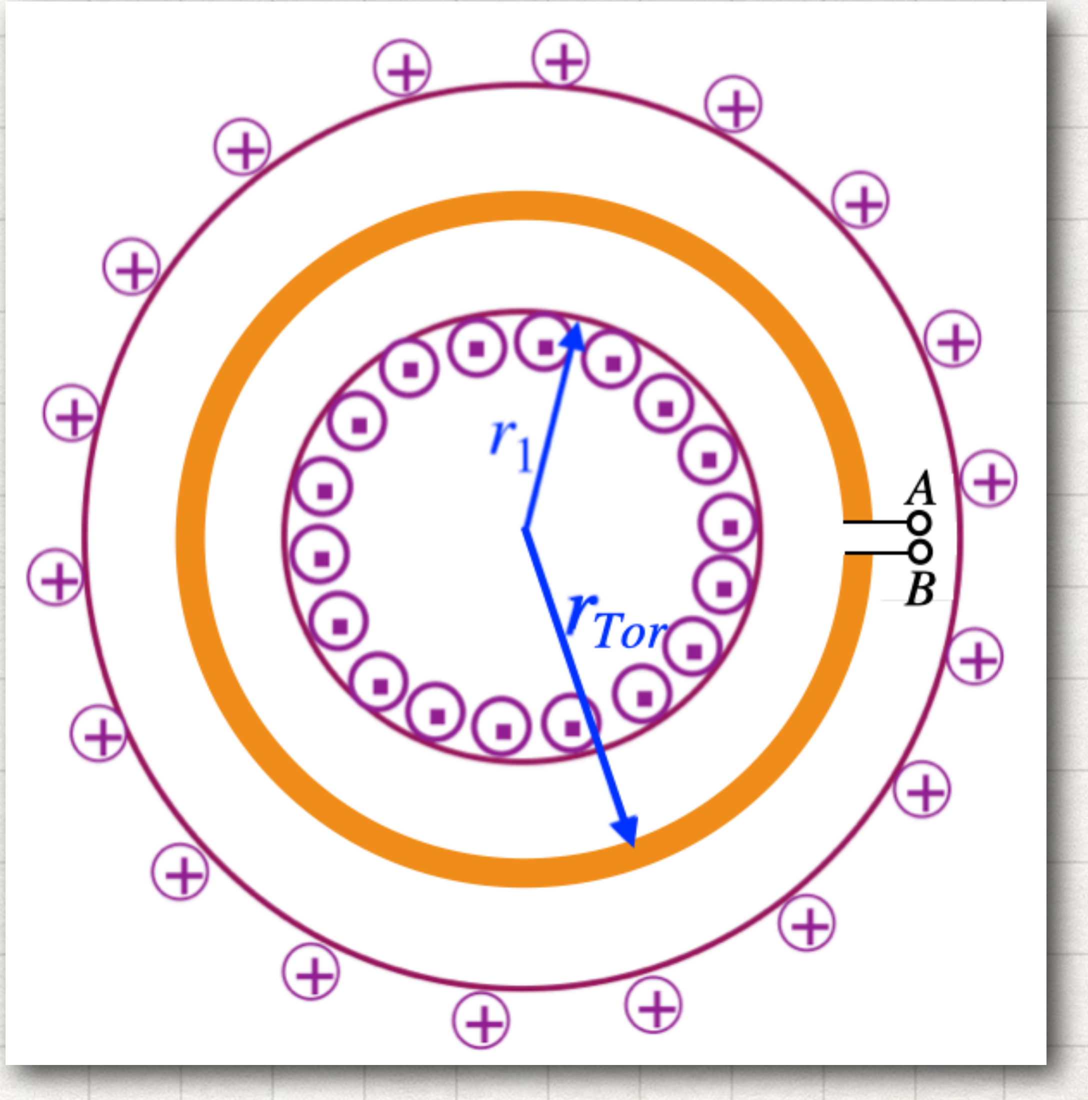

$$\oint \langle \vec{S} \rangle \cdot dS = -\frac{1}{4}\int (\mathbf{E}_1 \cdot \mathbf{J}_{e1}^* + \mathbf{E}_1^* \cdot \mathbf{J}_{e1})\, d\tau + \frac{1}{4}\int c\vec{B}_0 \cdot (g_{a\gamma\gamma} A \mathbf{J}_{e1}^* + g_{a\gamma\gamma} A^* \mathbf{J}_{e1})\, d\tau - \int \frac{1}{4} g_{a\gamma\gamma} \vec{J}_{e0} \cdot (A c \mathbf{B}_1^* + A^* c \mathbf{B}_1)\, d\tau$$