Short-range wake fields in plasma acceleration

Sergei Nagaitsev (Fermilab/UChicago)
In collaboration with Valeri Lebedev and Alexey Burov (Fermilab)

Nov 18, 2021

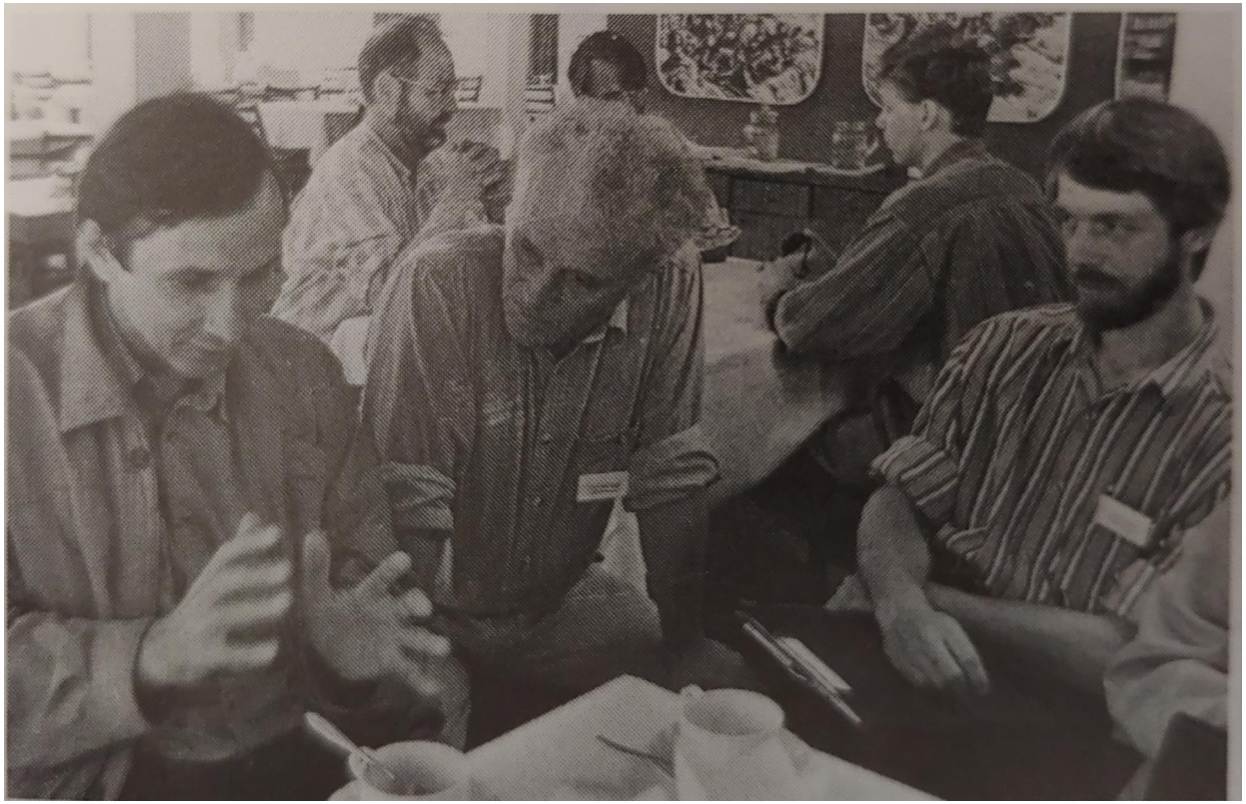
Novosibirsk University

- Valeri: 1973 – 1978
- Me: 1982 – 1989

We share a common advisor: I.N. Meshkov

Dubna, 2010

We reconnected 1996



- The 1996 Beam Cooling workshop, organized by I. Meshkov
  - 30th Anniversary of Electron Cooling

IBS and Coulomb scattering

ELSEVIER

Nuclear Instruments and Methods in Physics Research A 391 (1997) 176–187

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

Single and multiple intrabeam scattering in a laser cooled beam

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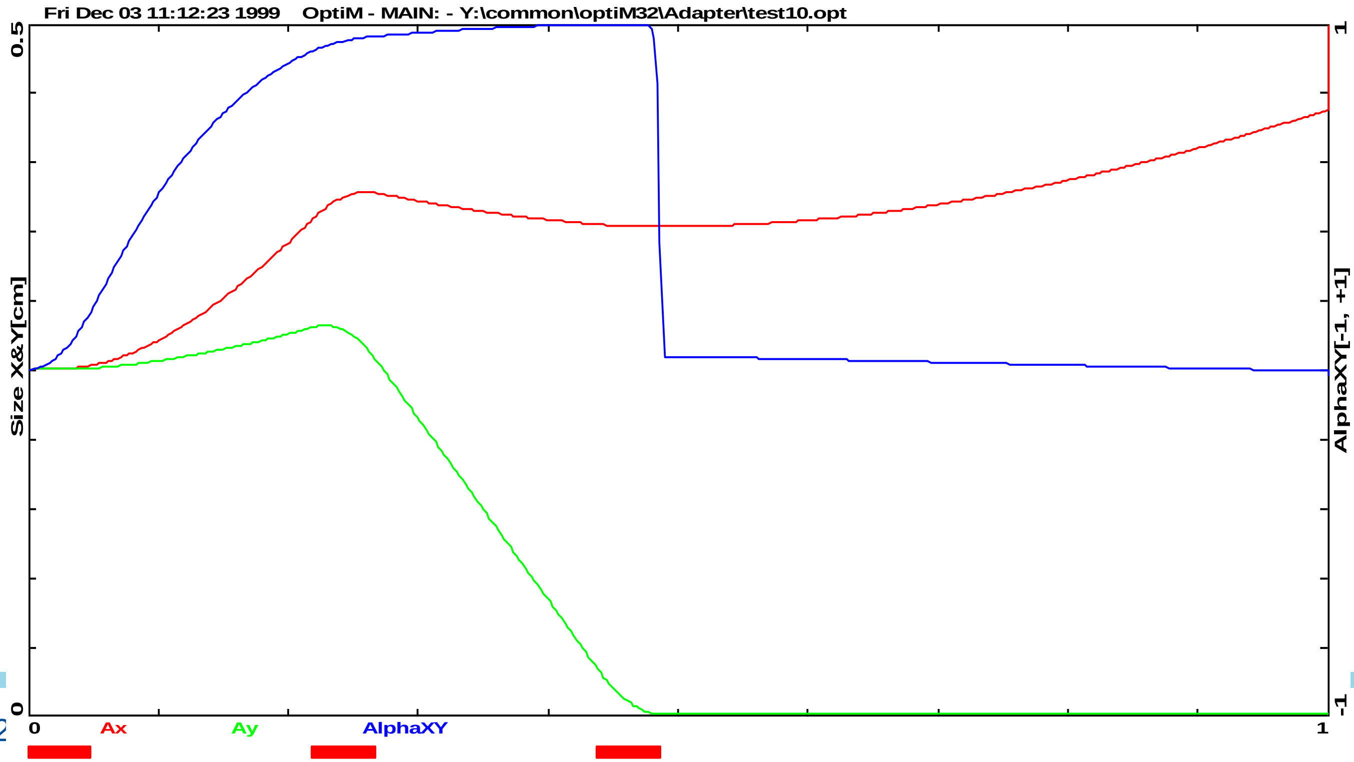
Abstract

The dependence of the laser cooling force on velocity is strongly non-linear and consists of two narrow peaks associated with co- and counter-propagating laser beams. Under these conditions a single intrabeam scattering event can knock a particle out of the cooling range. This creates large non-Gaussian tails in the distribution function for longitudinal velocity. A theoretical model describing single and multiple intrabeam scattering is considered. A detailed analysis of the scattering and its comparison with experimental data for the ASTRID storage ring are performed.

- This is where I’ve learned that the Landau kinetic equation and the Bjorken-Mtingwa IBS approach give the same results for the rms values

OptiM

- In 1999 I became the first US customer (external to JLab) of the OptiM modeling software
- It was immediately put to use in the Fermilab electron cooling project and also in the flat-beam experiments at the A0 photoinjector (a.k.a the Derbenev's adaptor)



Fermilab

Our first collaborative work:

Proceedings of EPAC 2002, Paris, France

PARTICLE DIFFUSION DUE TO COULOMB SCATTERING

V. Lebedev and S. Nagaitsev, FNAL, Batavia, IL 60510, USA

Abstract

Conventionally, the multiple and single particle scattering in a storage ring are considered to be independent. Such an approach is simple and often yields sufficiently accurate results. Nevertheless, there is a class of problems where such an approach is not adequate and the single and multiple scattering need to be considered together. This can be achieved by solving an integro-differential equation for the particle distribution function, which correctly treats particle Coulomb scattering in the presence of betatron motion. A derivation of the equation is presented in the article. A numerical solution for one practical case is also considered.

The solutions of Eq. (2) are commonly used to describe an emittance growth in particle accelerators due to various random diffusion processes. This equation describes well the core of the beam distribution, but completely fails to describe its tail [2]. Far away tails can be sufficiently well estimated using a single scattering approximation, but in many applications a prediction of tails behavior in vicinity of the core is required. It is possible to computer-model the distribution function by Monte-Carlo methods. However, we found it beneficial to advance the analytical treatment of the Coulomb scattering process to a point, where, for a given residual gas pressure, the corrected distribution function can be obtained with the help of a simple computer code. Similar approach was used in Ref

- Still work in progress (see Valeri’s seminar 11/17/20)

Most recent works on the IBS

arXiv:1812.09275: "Multiple intrabeam scattering in X-Y coupled focusing systems"
Valeri Lebedev, Sergei Nagaitsev

Intrabeam Scattering and Stripping

V. Lebedev and S. Nagaitsev, Fermilab

Coulomb scattering of charged particles results in an exchange of energy between different degrees of freedom. The total cross section of two-particle scattering in vacuum diverges; however, it remains finite for collisions in plasma (or beam) due to field screening by other particles [1]. Usually two scattering regimes are considered: single scattering, when a rare single collision makes a large change of particle momentum (the Touschek effect), and multiple scattering, when multiple frequent collisions cause a diffusion. The former phenomenon is usually responsible for creation of distribution tails and beam loss in electron machines, while the latter for changes in the distribution core. Although such approach is useful in many applications, there are cases when it fails to deliver an accurate result.

Multiple Scattering in Single Component Plasma

The temperature exchange in plasma is driven by an

in Ref. [4]. The function $\Psi(x,y,z)$ is chosen so that it depends on the ratios of its variables but not on r. It is symmetric relative to the variables y and z, and is normalized so that $\Psi(0,1,1)=1$. The energy conservation yields: $\Psi(1,0,1) = \Psi(1,1,0) = -1/2$ and $\Psi(x,y,z)+\Psi(y,z,x)+\Psi(z,x,y)=0$. The thermal equilibrium corresponds to $\Psi(1,1,1)=0$. The function $\Psi(0,y,z)$ can be approximated with ~0.5% accuracy by:

$$\Psi(0,y,z) \simeq 1+\frac{\sqrt{2}}{\pi}\ln\left(\frac{y^2+z^2}{2yz}\right)-0.055\left(\frac{y^2-z^2}{y^2+z^2}\right)^2 . \quad (5)$$

The asymptotics are:

$$\Psi(x,y,z)=\begin{cases}\frac{1}{\sqrt{2}\pi}\left(\ln\left(\frac{8r^2}{x^2+y^2}\right)-\frac{3z}{r}\right), & x,y \ll z,\\ \frac{\sqrt{2}}{\pi}\left(\ln\left(\frac{y^2+z^2}{8r^2}\right)+\frac{3x}{r}\right), & y,z \ll x.\end{cases} \quad (6)$$

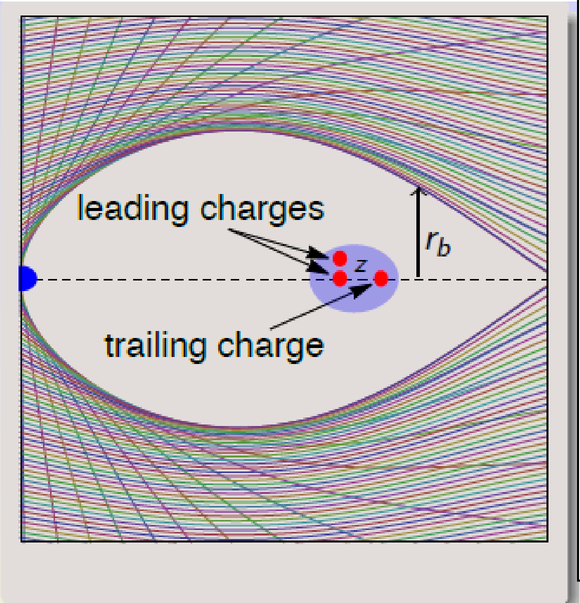
About this talk (our 2014-18 collaboration)

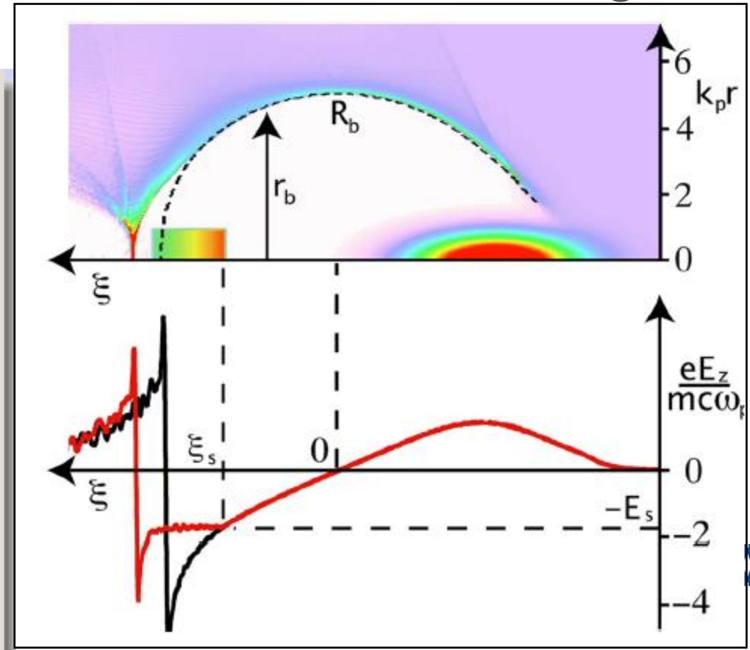
- Some of the results presented in this talk are based on our publications:
  - "Luminosity limitations of linear colliders, based on plasma acceleration", Reviews of Accelerator Science and Technology, Vol. 9 (2016)
  - "Efficiency versus instability in plasma accelerators", Phys. Rev. Accel. Beams 20, 121301 (2017)
  - "Beam Breakup Mitigation by Ion Mobility in Plasma Acceleration", arXiv:1808.03860

Plasma short-range wakefields

- The terminology of wakefields in plasma can be confusing. The original meaning of the wake in plasma is the field generated by the drive bunch, which accelerates the trailing bunch. (The driver could be particle beam or laser)

- In this presentation, by wakefields I mean the fields (longitudinal and transverse) with which the trailing bunch acts on itself.





Fermilab

Transverse beam break-up (BBU) instability

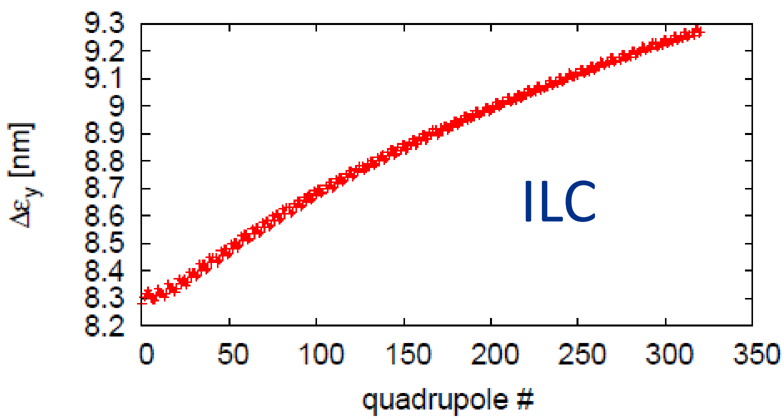
- Transverse wakes act as deflecting force on bunch tail
  - beam position jitter is exponentially amplified

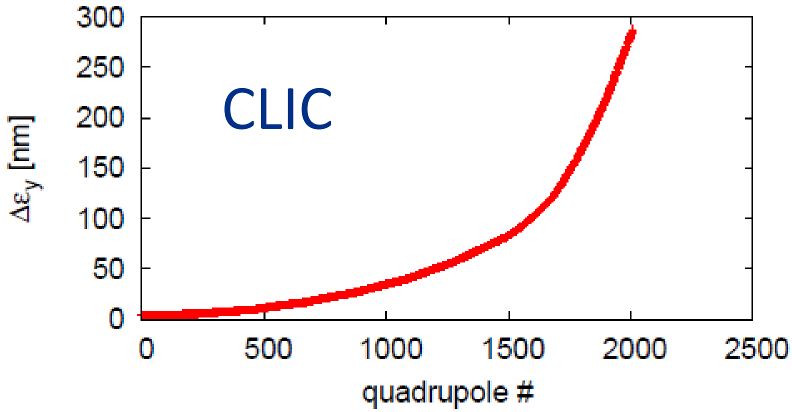
Beam Stability

- Transverse stability of a beam with initial offset of σ_y
  - no energy spread assumed in the beam
  - emittance with respect to the beam axis is shown

$\Rightarrow$ acceptable for ILC (top)

$\Rightarrow$ would be intolerable for CLIC (bottom)





D. Schulte, 6th Linear Collider School 2011, Main Linac Basics 69

Short-range transverse wake (for solid walls)

$$W_\perp(z) = \frac{8z}{a^4}$$

$a \approx 35$ mm (ILC)

$a \approx 3.5$ mm (CLIC)

What about plasma?

$a \sim 0.1$ mm (PWFA)

BBU illustration

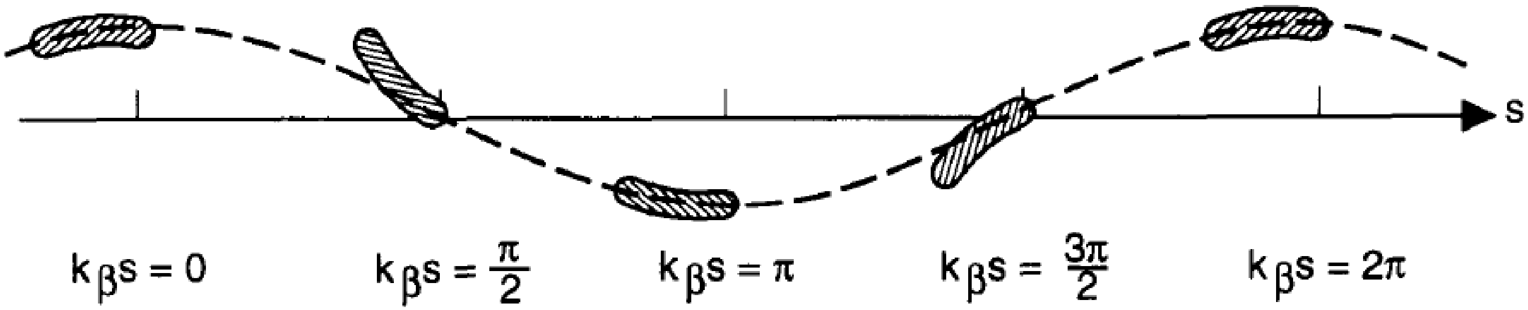


Figure 3.3. Sequence of snapshots of a beam undergoing dipole beam breakup instability in a linac. Values of $k_\beta s$ indicated are modulo 2π. The dashed curves indicate the trajectory of the bunch head.

- See A. Chao, “Physics of collective beam instabilities in high energy accelerators (Wiley, 1993).”
- The growth rate is determined by the ratio of defocusing (wake) force to focusing force (the wake parameter):

$$\eta_t = -\frac{F_t}{F_r} = \frac{e^2 r}{F_r(r)} \int_0^L \frac{dN}{d\xi} W_\perp(\xi) d\xi$$

The BBU instability development

$$\frac{d^2X(\mu,\xi)}{d\mu^2}+X(\mu,\xi)=\frac{2\eta_t}{L_t^{\,2}}\int_0^{\xi}X(\mu,\xi')(\xi-\xi')d\xi'.$$

$$X=\frac{x}{\sqrt{\beta}}\sqrt{\frac{p}{p_0}};\quad \beta=k_p^{-1}\sqrt{2\gamma}$$

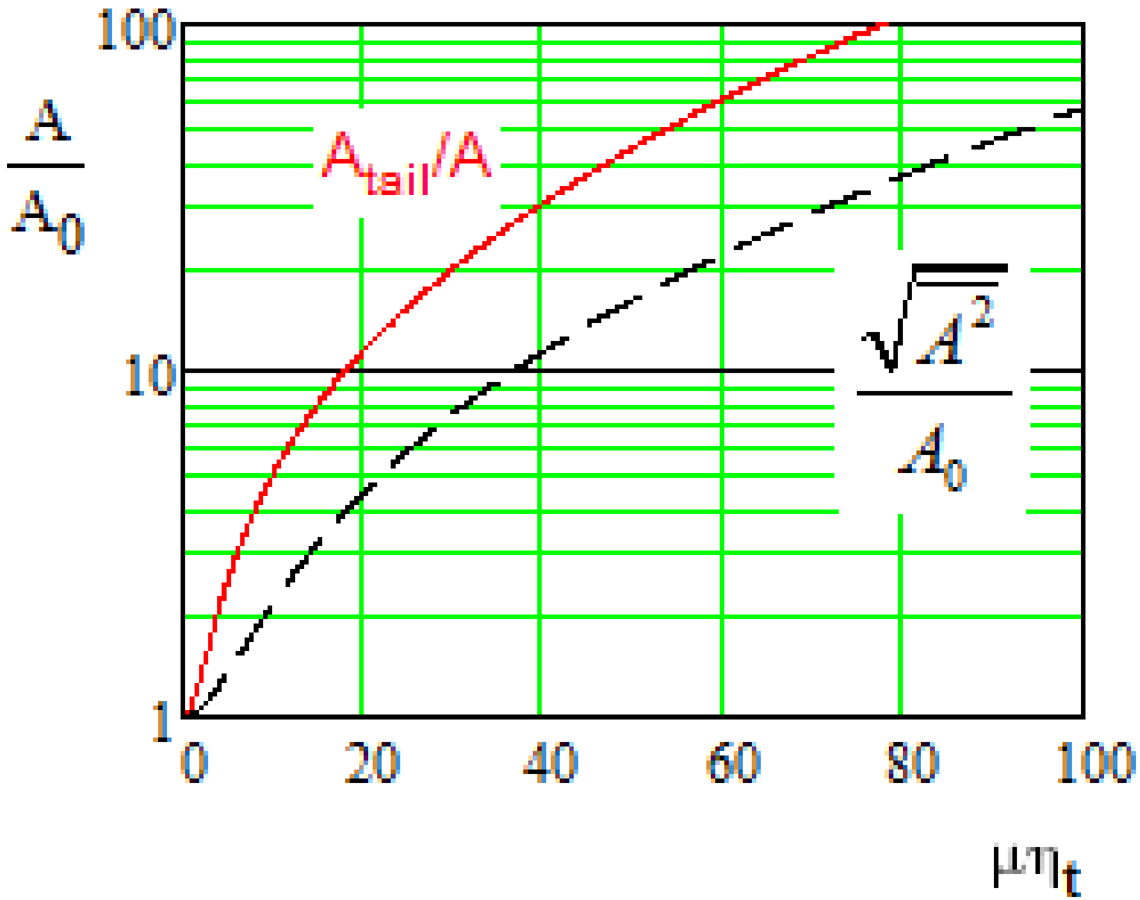
$$d\mu=k_p ds/\sqrt{2\gamma}$$

μ -- betatron phase advance

- For $\eta_t \ll 1$ and $\Delta p/p=0$ it was solved in:
  - C. B. Schroeder, D. H. Whittum, and J. S. Wurtele, "Multimode Analysis of the Hollow Plasma Channel Wakefield Accelerator", Phys. Rev. Lett. **82**, n.6, 1999, pp. 1177-1180.
- Approximate solutions (it's a very good fit, <10% deviation):

$$\frac{A}{A_0}=\exp\left(\frac{(\mu\eta_t)^2}{10+1.4(\mu\eta_t)^{1.57}}\right);\quad \begin{matrix}\mu\eta_t\le 100\\ \eta_t\le 0.1\end{matrix}$$

$$\frac{\sqrt{\overline{A^2}}}{A_0}=\exp\left(\frac{(\mu\eta_t)^2}{60+2.2(\mu\eta_t)^{1.57}}\right);\quad \begin{matrix}\mu\eta_t\le 100\\ \eta_t\le 0.1\end{matrix}$$



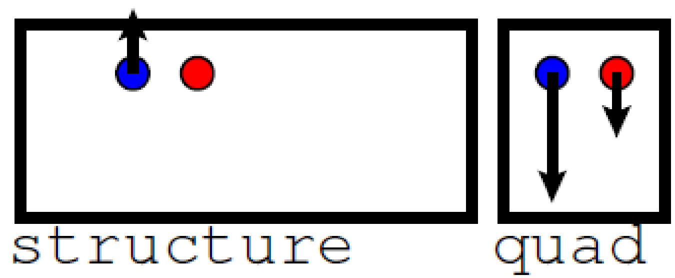
- Note that *A* is a normalized particle amplitude. For a constant plasma density and without instability *A* would stay constant, while the initial physical amplitude *x* should decrease as $1/\gamma^{\frac{1}{4}}$

Beam breakup in various collider concepts

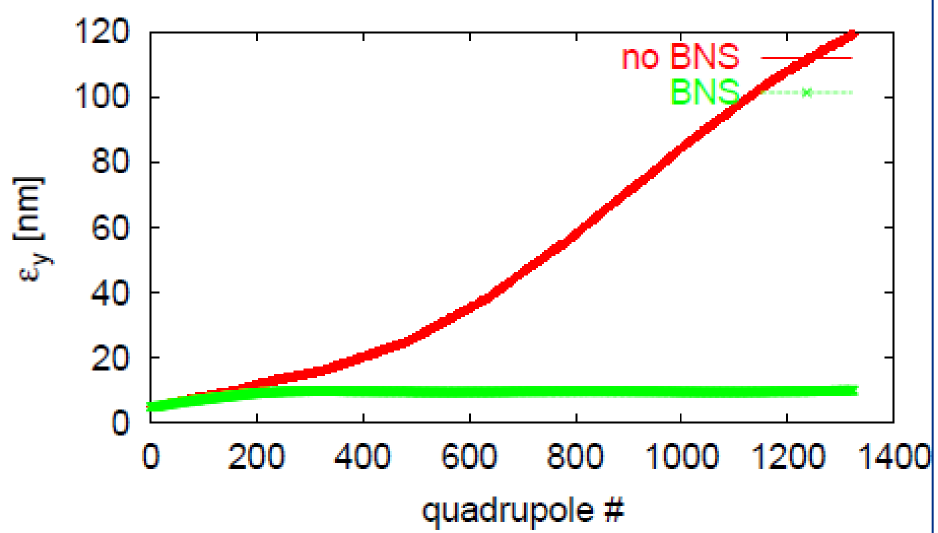
- ILC
  - Not important; bunch rf phase is selected to compensate for long wake and to minimize the momentum spread
- CLIC
  - Important; bunch rf phase is selected to introduce an energy chirp along the bunch for BNS damping (~0.5% rms). May need to be de-chirped after acceleration to meet final-focus energy acceptance requirements
- PWFA – subject of our study
  - Critical;

CLIC strategy: BNS damping + < μm alignment of cavities

Achieving Beam Stability

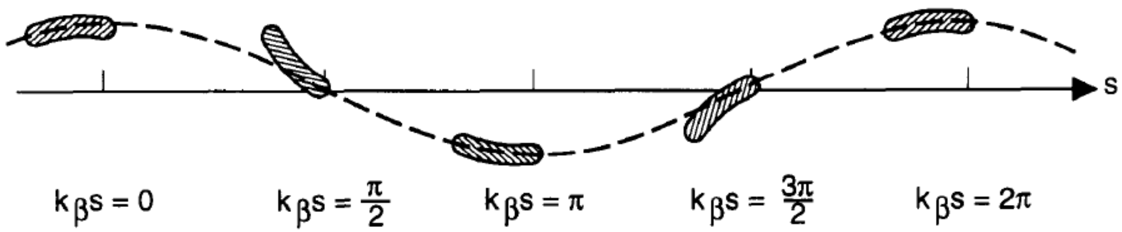


- Transverse wakes act as defocusing force on tail
  - ⇒ beam jitter is exponentially amplified
- BNS (Balakin, Novokhatsky, and Smirnov) damping prevents this growth
  - manipulate RF phases to have energy spread
  - take spread out at end



D. Schulte, 6th Linear Collider School 2011, Main Linac Basics 70

Strategy was also used at the SLC…



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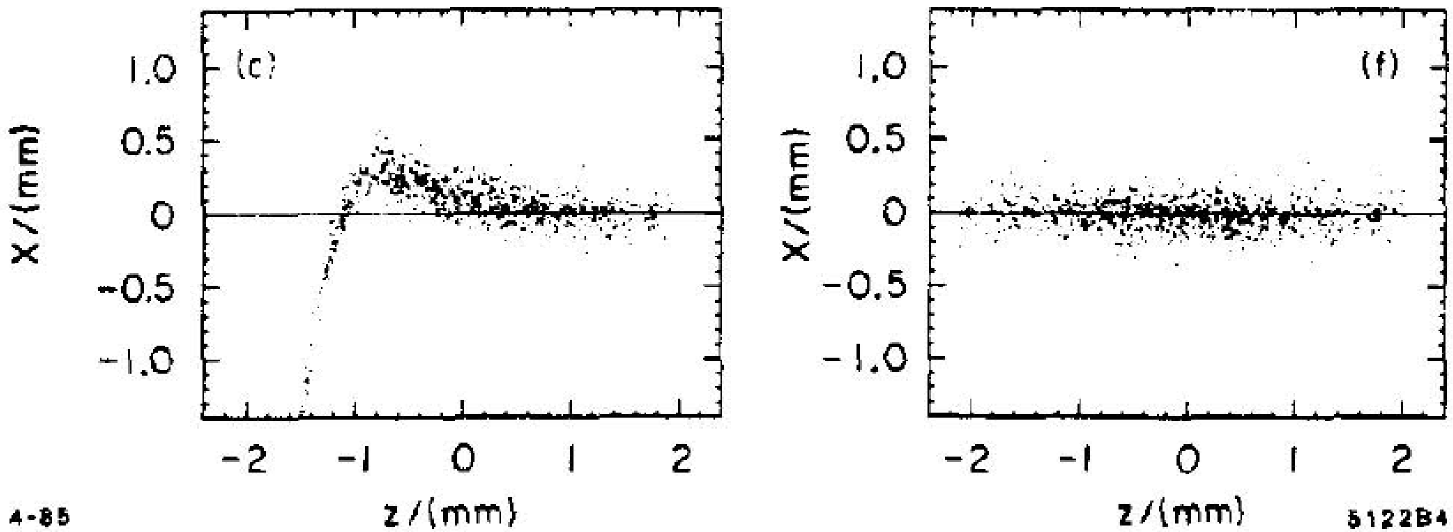


Figure 34: Multiparticle simulation of a particle bunch passing through the SLAC linac without (left) and with BNS damping (right) [36].

BNS damping: what is it?

- Assume a constant long. density of trailing bunch. Chromatic detuning of tail particles allows to keep amplitudes constant

$$\frac{d^2X(\mu,\xi)}{d\mu^2}+\frac{X(\mu,\xi)}{1+\Delta p(\xi)/p}=\frac{2\eta_t}{L_t^{\,2}}\int_0^{\xi}X(\mu,\xi')(\xi-\xi')d\xi'.$$

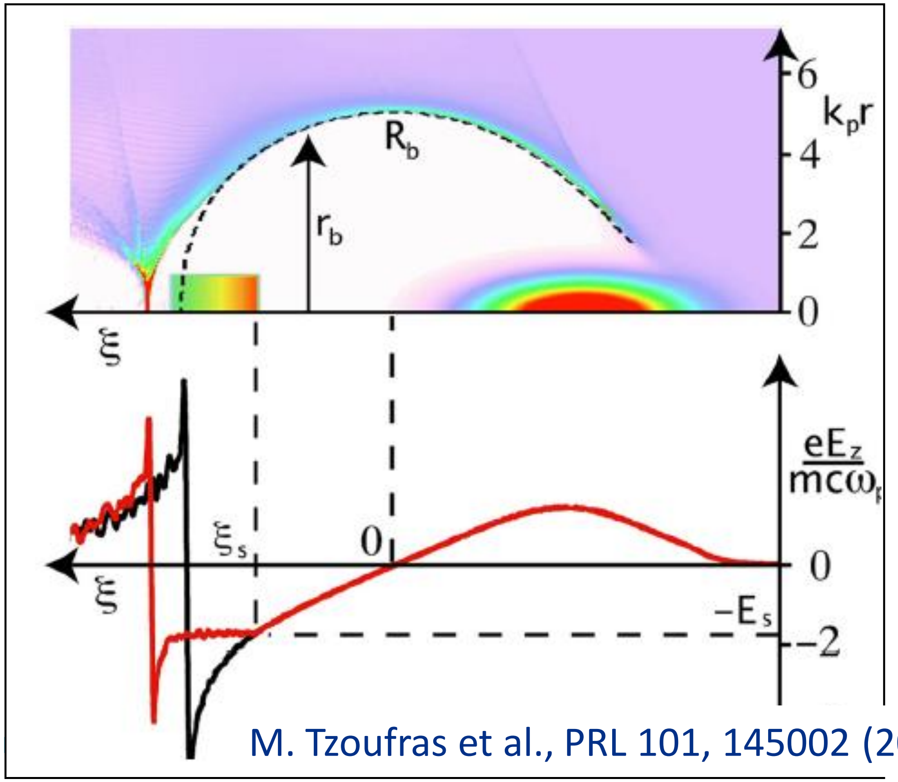
$$\eta_t=-\frac{F_t}{F_r}=\frac{1}{2\pi n_0}\int_0^{L}\frac{dN}{d\xi}W_\perp(\xi)d\xi$$

-- Transverse wake parameter in a PWA blow-out regime

$$X(\xi)=\text{const}\quad\rightarrow\quad\frac{\Delta p(\xi)}{p}=-\eta_t\frac{\xi^2}{L_t^2}$$

Acceleration in a plasma blow-out regime

- The Q-factor is very low (~1) – must accelerate the trailing bunch within the same bubble as the driver!
- Cannot add energy between bunches, thus a single bunch must absorb as much energy as possible from the wake field.



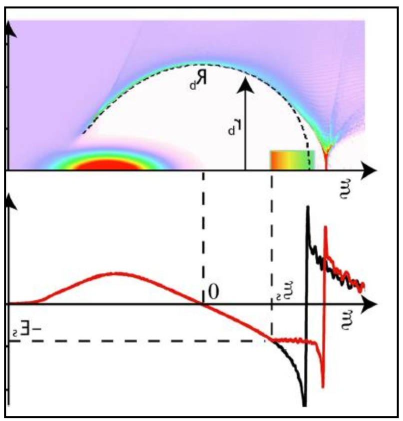
M. Tzoufras et al., PRL 101, 145002 (2008)

To achieve L ~10^{34}, bunches should have ~10^{10} particles (similar to ILC and CLIC). In principle, we can envision a scheme with fewer particles/bunch and a higher rep rate, but the beam loading still needs to be high for efficiency reasons.

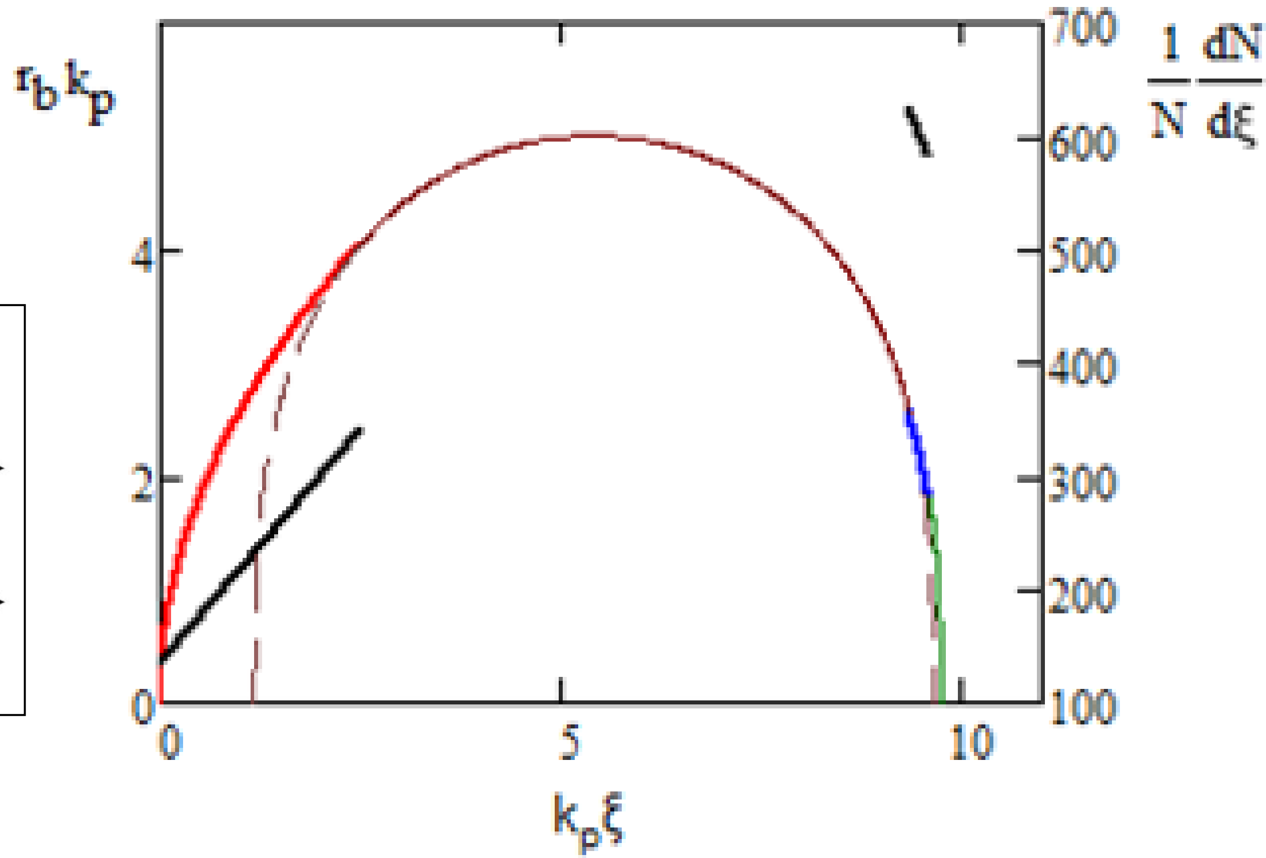
Power transfer from drive to trailing bunches

Trapezoidal charge line density distribution → constant electric field

Example:
50% power transfer efficiency drive to trailing



See M. Tzoufras et al., PRL 101, 145002 (2008)



The power transfer efficiency of 50% and the transformer ratio of 2. For n_0=10^{17} cm^{-3} the drive bunch parameters are chosen to be $R_b k_p$=5, $L_d k_p$=2.5 yielding the decelerating field of E_d = 50 GV/m and N_d=3.55·10^{10}. The trailing bunch parameters are: r_{t2}=0.518R_b, r_{t1}=0.373R_b, E_t = 100 GV/m, N_t=8.86·10^9.

The efficiency-instability relation in a blowout regime

$$\boxed{\eta_t \approx \frac{\eta_P^{\;2}}{4\left(1-\eta_P\right)}, \quad \frac{r_{t2}}{R_b} \le 0.7}$$

- This formula does not include any details of beams and plasma, being amazingly universal!
- Note: this formula is an estimate on a "low side". On a "high side", we estimate it as: $\eta_t \approx \eta_P^{\;2} / \left(4\left(1-\eta_P\right)^2\right)$
- Example: $\eta_P = 50\%$ → $0.125 < \eta_t < 0.25$

  $\eta_P = 25\%$ → $0.021 < \eta_t < 0.028$

See: "Efficiency versus instability in plasma accelerators", PRAB 20, 121301, 2017

Examples (FACET-II)

$$\text{Plasma: } n_0 = 4\times 10^{16}\ \text{cm}^{-3},\ \text{60 cm long channel}$$

- p_i=10 GeV/c for both the drive and the trailing bunches, and the final momentum of trailing bunch p_f=21 GeV/c, N_d=1x10^{10} and N_t=4.3x10^9

$$\eta_P = 50\%,\ \eta_t \approx 0.12,\ \mu\eta_t \approx 11.5 \ \rightarrow\ \frac{A}{A_0} \approx 5.8$$

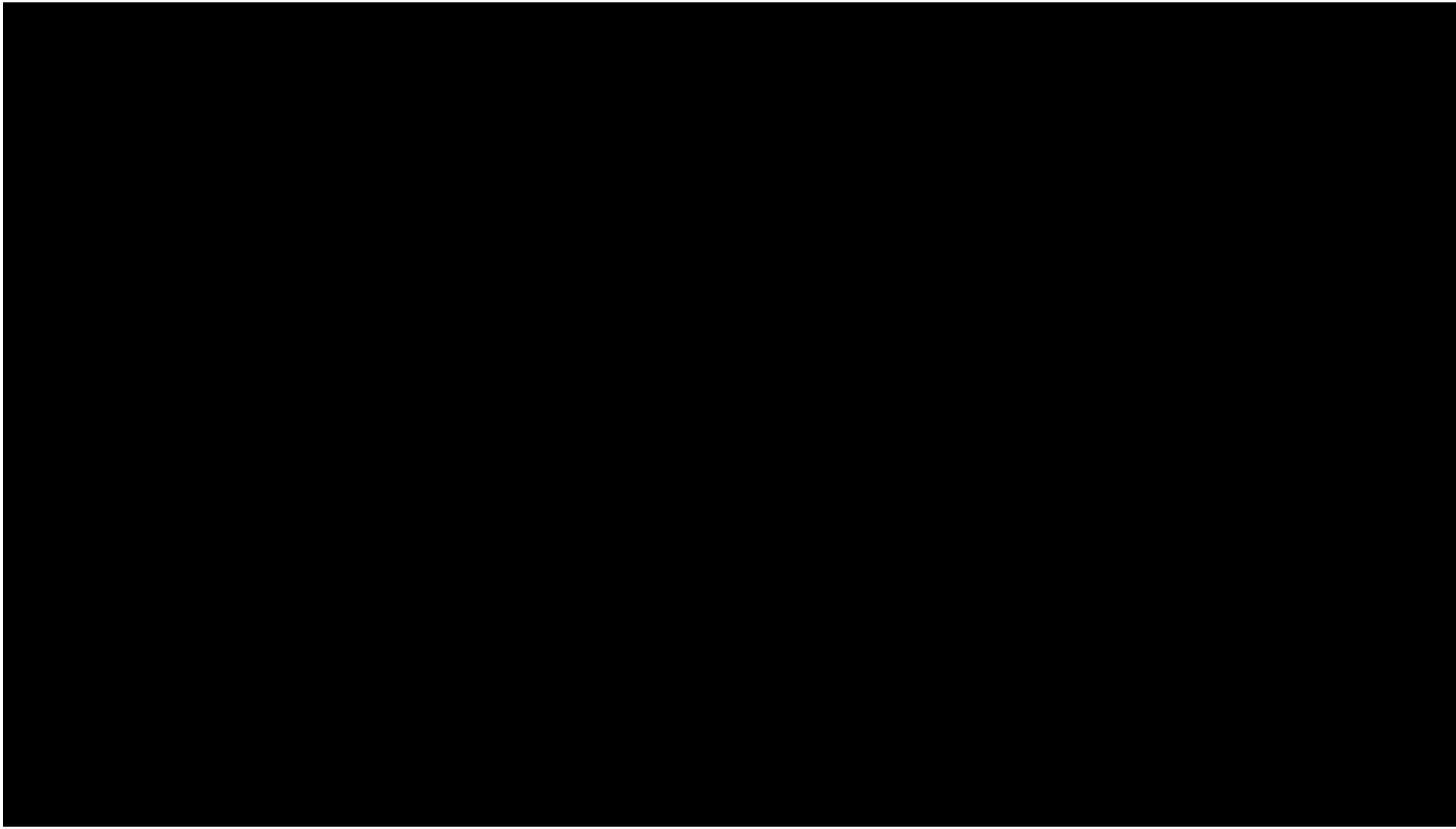
- If one reduces the power efficiency:

$$\eta_P = 25\%,\ \eta_t \approx 0.021,\ \mu\eta_t \approx 2 \ \rightarrow\ \frac{A}{A_0} \approx 1.3$$

- Of course, the final momentum is now p_f=15.5 GeV/c (for the same number of particles)

$$\delta\varepsilon_n = \frac{\delta x^2}{2\beta_i}\gamma_i\left(\frac{\overline{A^2}}{A_0^{\ 2}}\right),\quad \beta_i = \frac{\sqrt{2\gamma_i}}{k_p}$$

Case I: ~50% power efficiency $\eta_P = 50\%,\ \eta_t \approx 0.13$



Courtesy of UCLA

Fermilab

UCLA

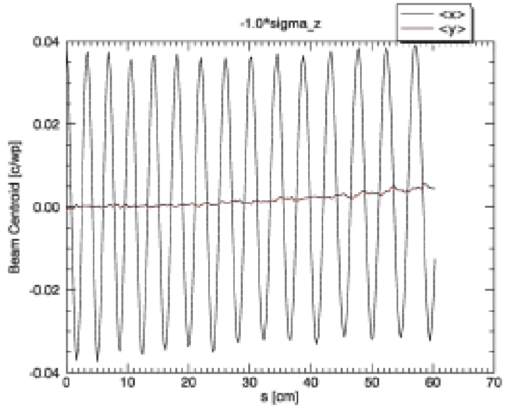
Hosing Study for FACET II: Case I

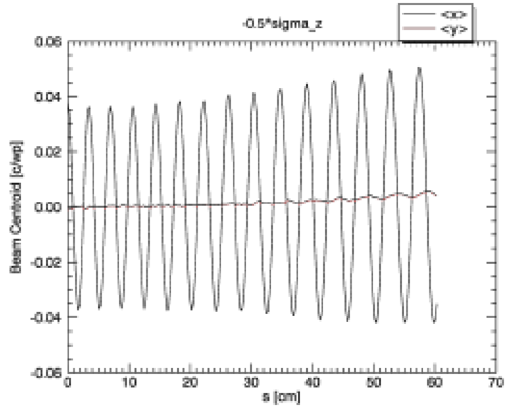
Drive Beam: $E = 10$ GeV, $I_{peak}=15$ kA
$\sigma_r = 3.65\ \mu m$, $\sigma_z = 12.77\ \mu m$,
$N = 1.0 \times 10^{10}$ (1.6 nC), $\varepsilon_N = 10\ \mu m$

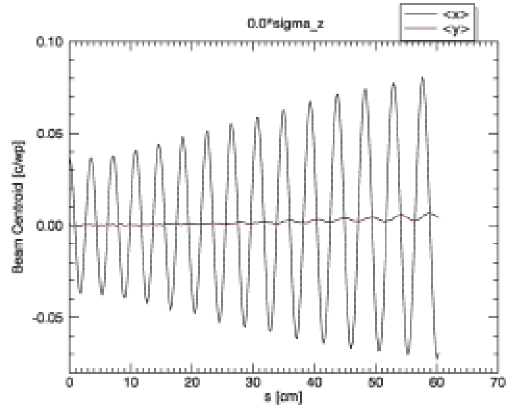
Trailing Beam: $E = 10$ GeV, $I_{peak}=9$ kA
$\sigma_r = 3.65\ \mu m$, $\sigma_z = 6.38\ \mu m$,
$N = 4.33 \times 10^{9}$ (0.69 nC), $\varepsilon_N = 10\ \mu m$
(transversely offset by 1 μm)

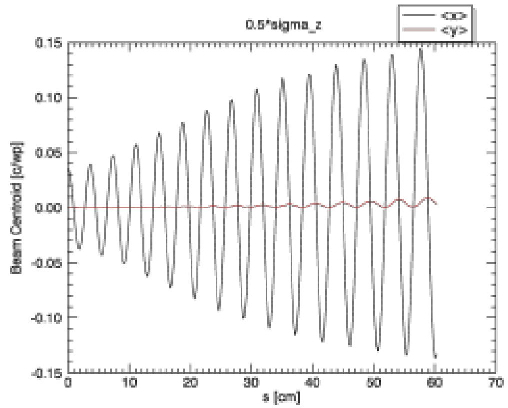
Distance between two bunches: 150 μm
Plasma Density: $4.0 \times 10^{16}\ cm^{-3}$

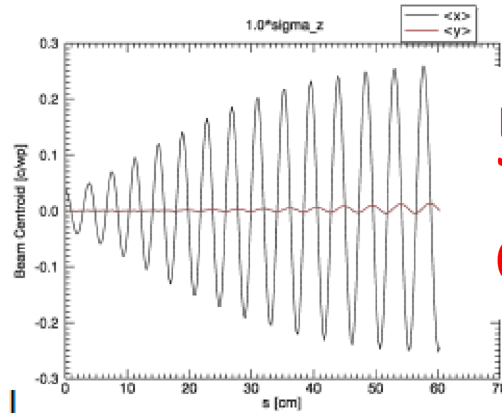
Trailing beam centroid vs s in different slices





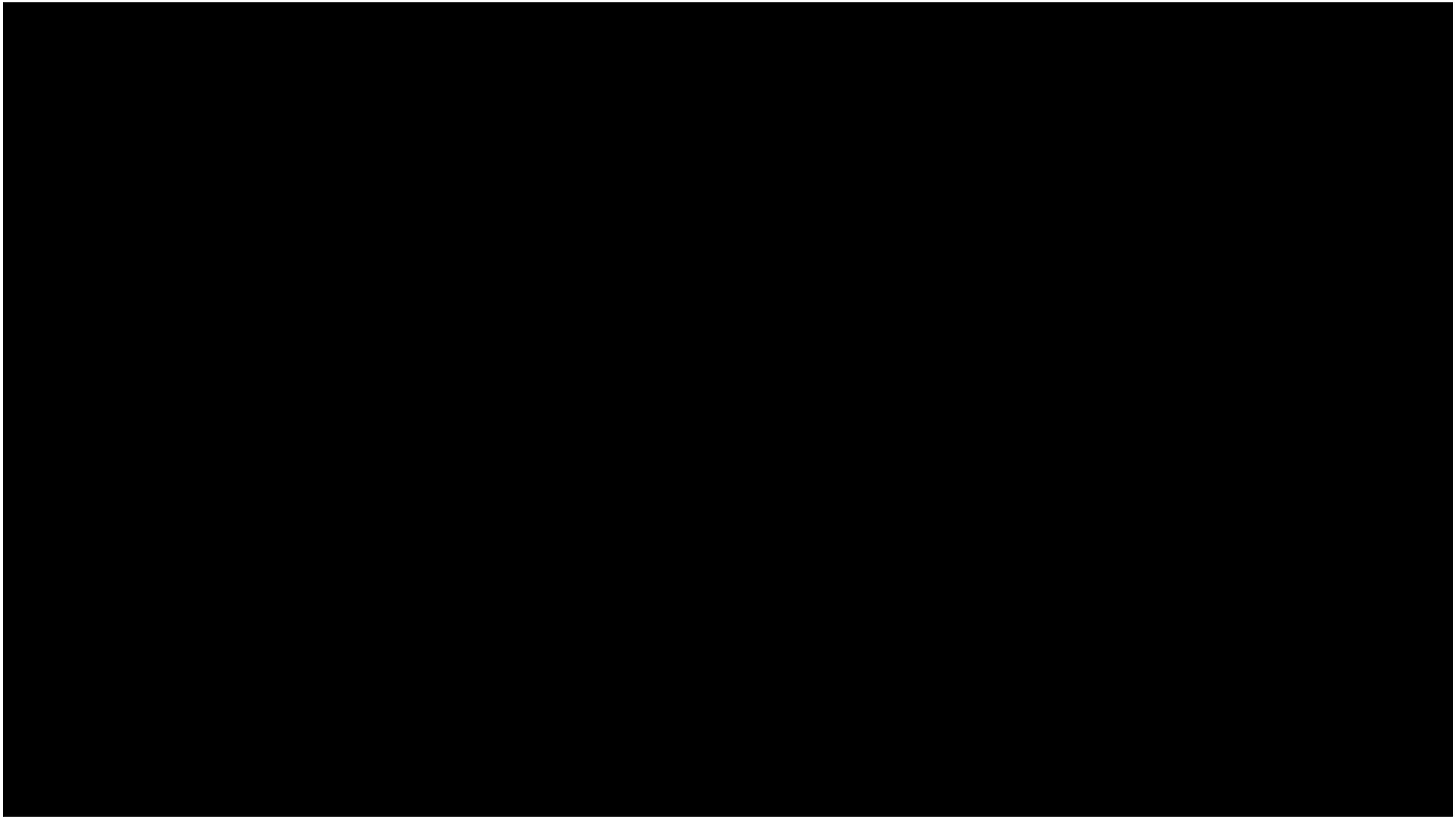






50% power efficiency

Case II: ~25% power efficiency $\eta_P = 25\%,\ \eta_t \approx 0.02$



Courtesy of UCLA

UCLA Hosing Study for FACET II: Case II

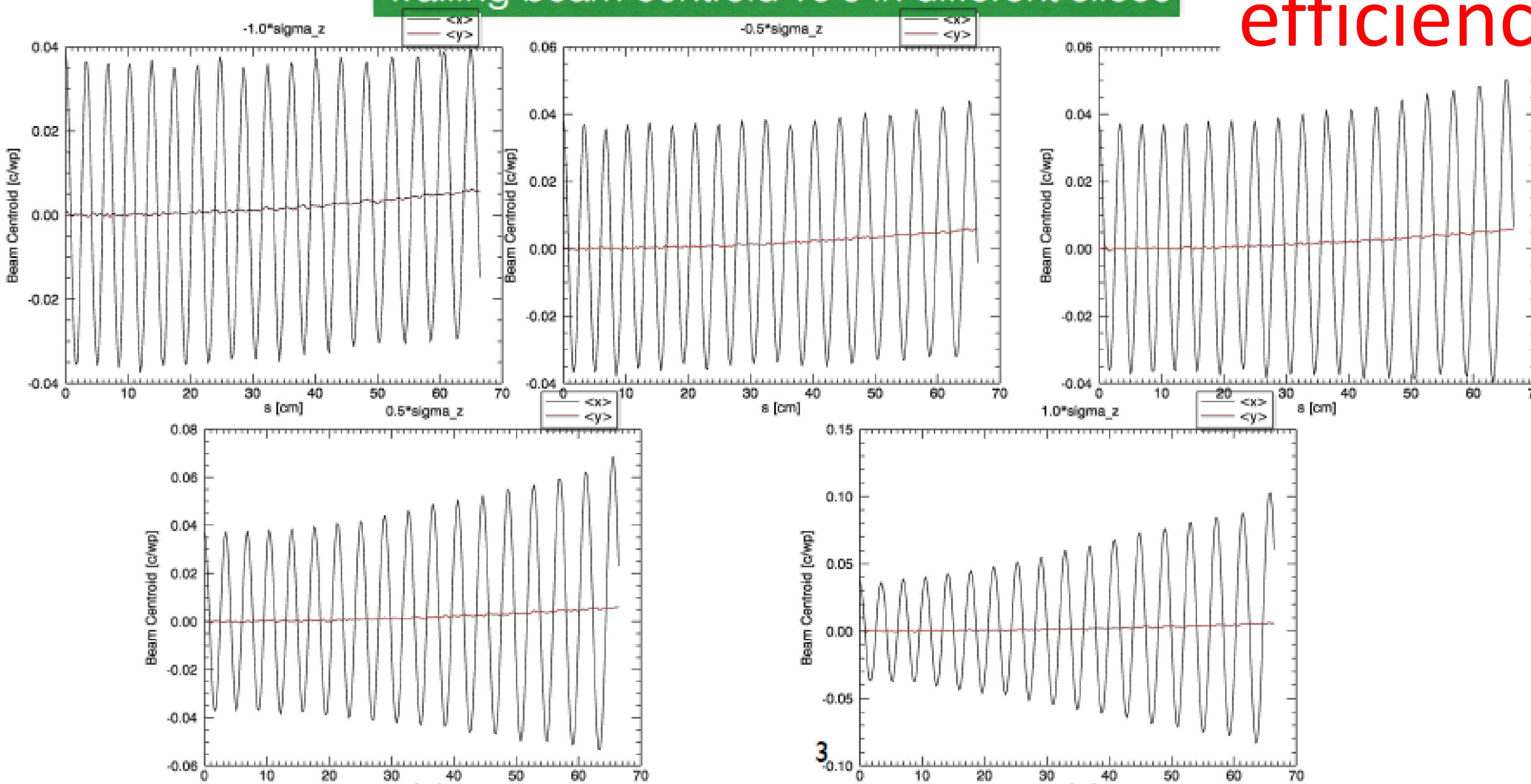
Drive Beam: E = 10 GeV, I_{peak}=15 kA
σ_r = 3.65 μm, σ_z = 12.77 μm ,
N =1.0 x 10^{10} (1.6 nC), ε_N = 10 μm

Trailing Beam: E = 10 GeV, I_{peak}=9 kA
σ_r = 3.65 μm, σ_z = 6.38 μm ,
N =4.33 x 10^9 (0.69 nC), ε_N = 10 μm
(transversely offset by 1 um)

Distance between two bunches: 108 μm
Plasma Density: 4.0 x 10^{16} cm^{-3}

25% power efficiency

Trailing beam centroid vs s in different slices



Mitigation by momentum chirp (classical BNS)

$$\frac{d^2X}{d\mu^2}+\frac{X}{1+\Delta p/p}=\frac{2\eta_t}{L_t^{\,2}}\int_0^{\xi}X(\xi')(\xi-\xi')d\xi'.$$

Goal:

$$X(\xi)=\text{const}\;\rightarrow\;\frac{\Delta p(\xi)}{p}=-\eta_t\frac{\xi^2}{L_t^2}\qquad\boxed{\eta_P=50\%,\;\;\eta_t\approx 0.13}$$

- The maximum allowed momentum spread might be determined by the stage-to-stage transition optics
- If one can tolerate $\frac{\Delta p}{p}\le 1\%$ than $\eta_t\le 0.01$

$$\boxed{\eta_t\approx\frac{\eta_P^{\;2}}{4(1-\eta_P)}}$$

CLIC Design: $\frac{\Delta p}{p}\le 0.5\%$

→ Therefore, the max power efficiency is

$$\boxed{\eta_P\le 18\%}$$

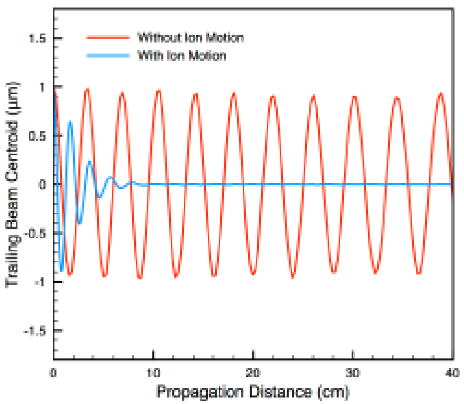
The role of plasma ions

- So far, we considered plasma ions to be stationary (constant transverse focusing).
- In fact, if the bunch density is high enough, the plasma ions are pulled into the electron bunch and create nonlinear focusing.
- Effect was considered first by J. Rosenzweig et al, PRL95, 195002 (2005). Found to be detrimental because of emittance growth.

- However, nonlinear focusing might be helpful to suppress the BBU instability (by allowing some emittance growth)
- Recent simulations performed by Weiming An (UCLA) et al.
  - https://conf.slac.stanford.edu/facet-2-2017/agenda
  - PRL 118, 244801 (2017)

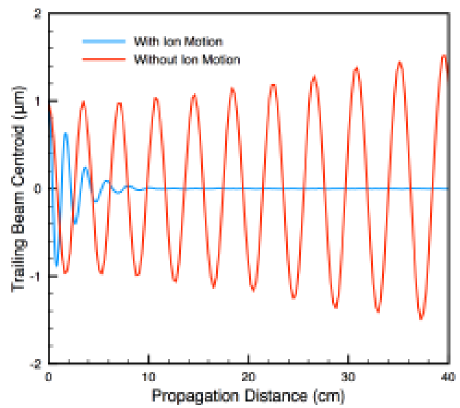
UCLA Eliminate the Hosing Instability

Head



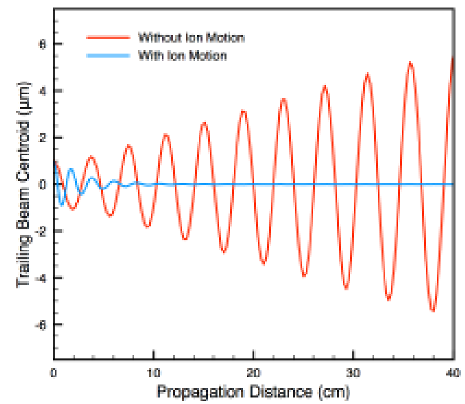
$\xi = -\sigma_z$

Center



$\xi = 0$

Tail



$\xi = \sigma_z$

Drive Beam: E = 10 GeV, I_{peak}=15 kA
σ_r = 0.516 μm, σ_z = 12.77 μm ,
N =1.0 x 10^{10} (1.6 nC), ε_N = 1 μmrad

Trailing Beam: E = 10 GeV, I_{peak}=9 kA
σ_r = 0.516 μm, σ_z = 6.38 μm ,
N =4.33 x 10^9 (0.69 nC), ε_N = 1 μmrad
(transversely offset by 1 μm)

Power efficiency: 50%

Emittance growth: ~a factor of two

See: https://conf.slac.stanford.edu/facet-2-2017/agenda

BNS damping by plasma ions (new idea!)

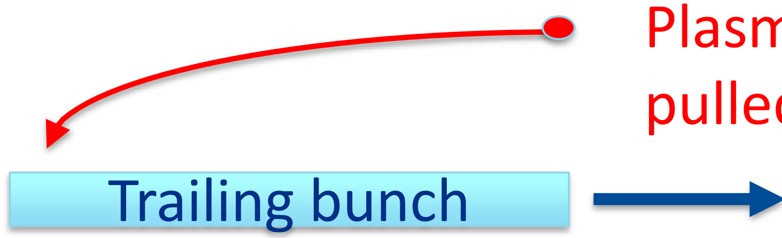
arXiv:1808.03860

$$\frac{d^2X}{d\mu^2}+\left(1+2\frac{\Delta\omega_\perp}{\omega_\perp}\right)X=\frac{2\eta_t}{L_t^{\,2}}\int_0^{\xi}X(\xi')(\xi-\xi')d\xi'.$$

$$2\frac{\Delta\omega_\perp(\xi)}{\omega_\perp}=\eta_t\frac{\xi^2}{L_t^2}$$

-- the betatron frequency increases from bunch head to tail

This focusing variation is normally achieved by an energy chirp, but in PWA, there may be an additional mechanism – plasma ion mobility



Plasma ions are pulled into the beam

Assuming ion density variation is small:

$$\frac{\Delta n_i(\xi)}{n_i}\approx 2\pi n_t r_i \xi^2$$

$$n_t = N/\ \ 4\pi L_t \sigma_\perp^2$$

Plasma ions at FACET-II….

- Since $\frac{\Delta\omega_\perp(\xi)}{\omega_\perp}=\frac{\Delta n_i}{2n_i}$ we would like to have $\boxed{\frac{N r_i L_t}{2\sigma_\perp^2}=\eta_t}$
- For FACET-II parameters: 10 GeV, n_0 = 4x10^{16} cm^{-3}
  N = 10^{10} , L = 5 μm

Goal: $\boxed{\eta_P = 50\%,\ \ \eta_t \approx 0.13}$

For the rms norm emittance 1 μm we should observe BNS damping due to ion moblity (at 50% power efficiency) $\boxed{\frac{N r_i L_t}{2\sigma_\perp^2}=0.13}$

For the rms norm emittance 10 μm we will not observe BNS damping due to ions (at 50% power efficiency)

These examples are based on hydrogen plasma
arXiv:1808.03860 $\boxed{\frac{N r_i L_t}{2\sigma_\perp^2}=0.013}$

Fermilab

Conclusions

- We have found a universal efficiency-instability relation for plasma acceleration. Should allow for tolerance and instability analysis without detailed computer simulations.
  - “Efficiency versus instability in plasma accelerators”, PRAB 20, 121301 (2017)
  - We considered only ideal “trapezoidal” distributions. Real-life distributions may be worse (from the efficiency perspective).
- In a blowout regime, plasma focusing is just strong enough to keep the instability in check for low power efficiencies (<25%)
  - Even for such efficiencies, external focusing and hollow channels are very challenging because of transverse BBU instability.
  - Presents obvious difficulties for positrons
- Classical BNS damping is possible but external optical systems may limit the momentum spread to ~1% max. Thus, the power efficiency (drive to trailing) can not exceed ~18%.
- BNS damping may be based on ion mobility for some range of bunch and plasma parameters. Can be tested at FACET-II.
  - Preparing an experimental proposal