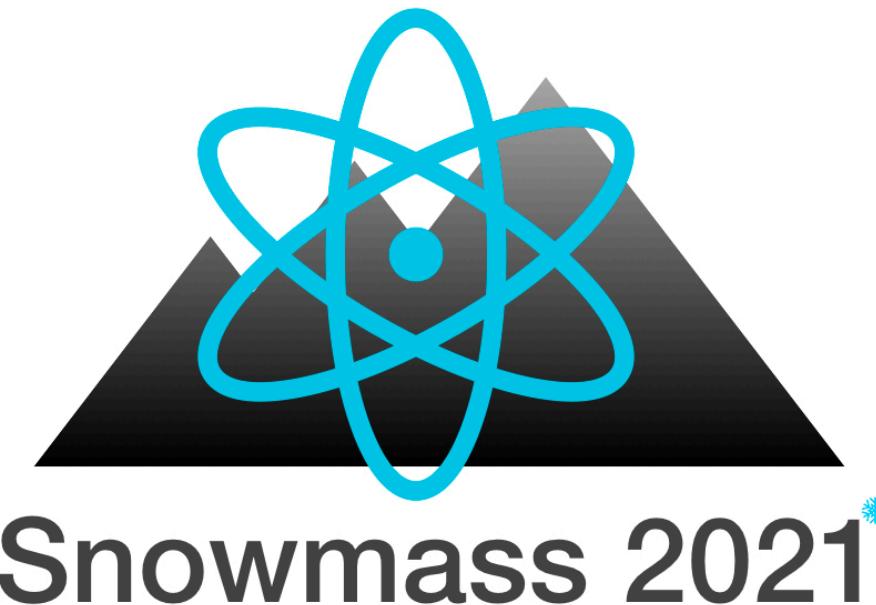


Lattice QCD for RPF: highlights

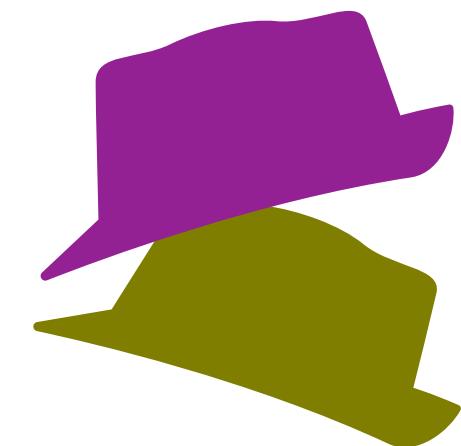


Snowmass Rare and Precision Measurements Frontier
Spring 2022 meeting
University of Cincinnati
16-19 May 2022



Outline

- The role of (lattice) QCD @ RPF
- Introduction to lattice QCD
- Quark flavor physics
 - kaon decay and first-row CKM unitarity
 - $B_{s,d} \rightarrow \mu\mu$
 - Semileptonic B-meson decay form factors
 - inclusive B decay rates
- muon g-2
- Summary
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The role of (lattice) QCD @ RPF

- look for forbidden (in the SM) processes:
 - ★ proton decay
 - ★ Charged Lepton flavor violating processes, for example: $\mu \rightarrow e\gamma, B \rightarrow \tau\mu, \dots$
 - ★ DM
 - ★
- Observation \Rightarrow discovery of new physics
- study rare processes — loop-suppressed in the SM:
 - ★ neutral meson mixing
 - ★ flavor changing neutral current decays
 - ★ MDMs and EDMs
 - ★
- check lepton flavor universality: $\tau/\mu, \mu/e$
- (over)determine SM parameters with high precision
 - ★ test unitarity of CKM matrix (quark-W couplings)

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Desired information on
(short-distance physics) or (SM parameters)
(BSM parameters) (CKM, m_q, α_s, \dots)

is hidden by hadronic/nuclear effects
(nonperturbative QCD).

 \Rightarrow need precise QCD calculations to complement
experimental measurements: Lattice QCD

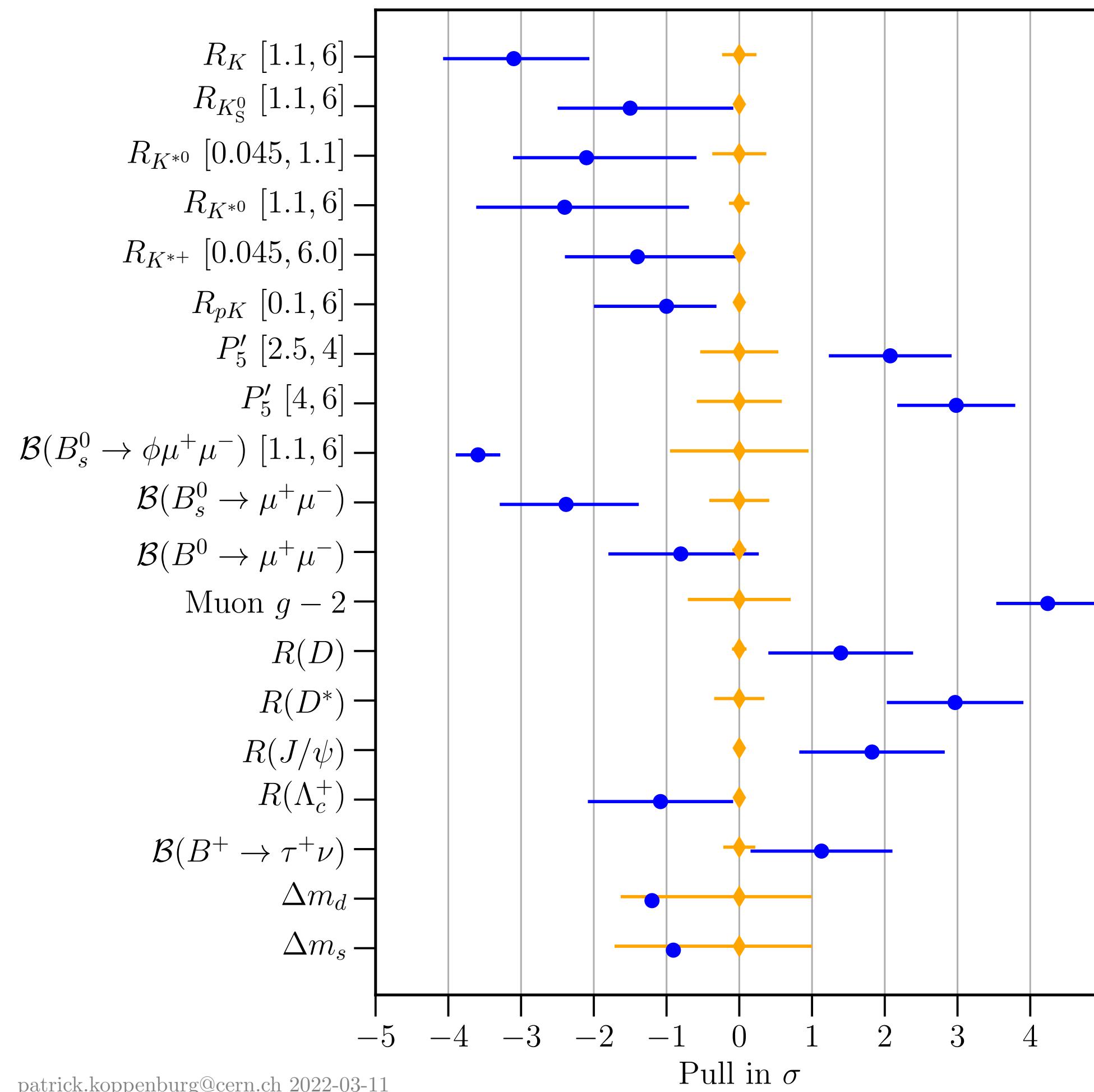
Flavor “Anomalies”

Patrick Koppenburg [http://www.scholarpedia.org/article/Rare_decays_of_b_hadrons,
<https://www.nikhef.nl/~pkoppenb/anomalies.html>]

Cherry-picked selection

theoretical SM
predictions

common source of theory
error: QCD effects

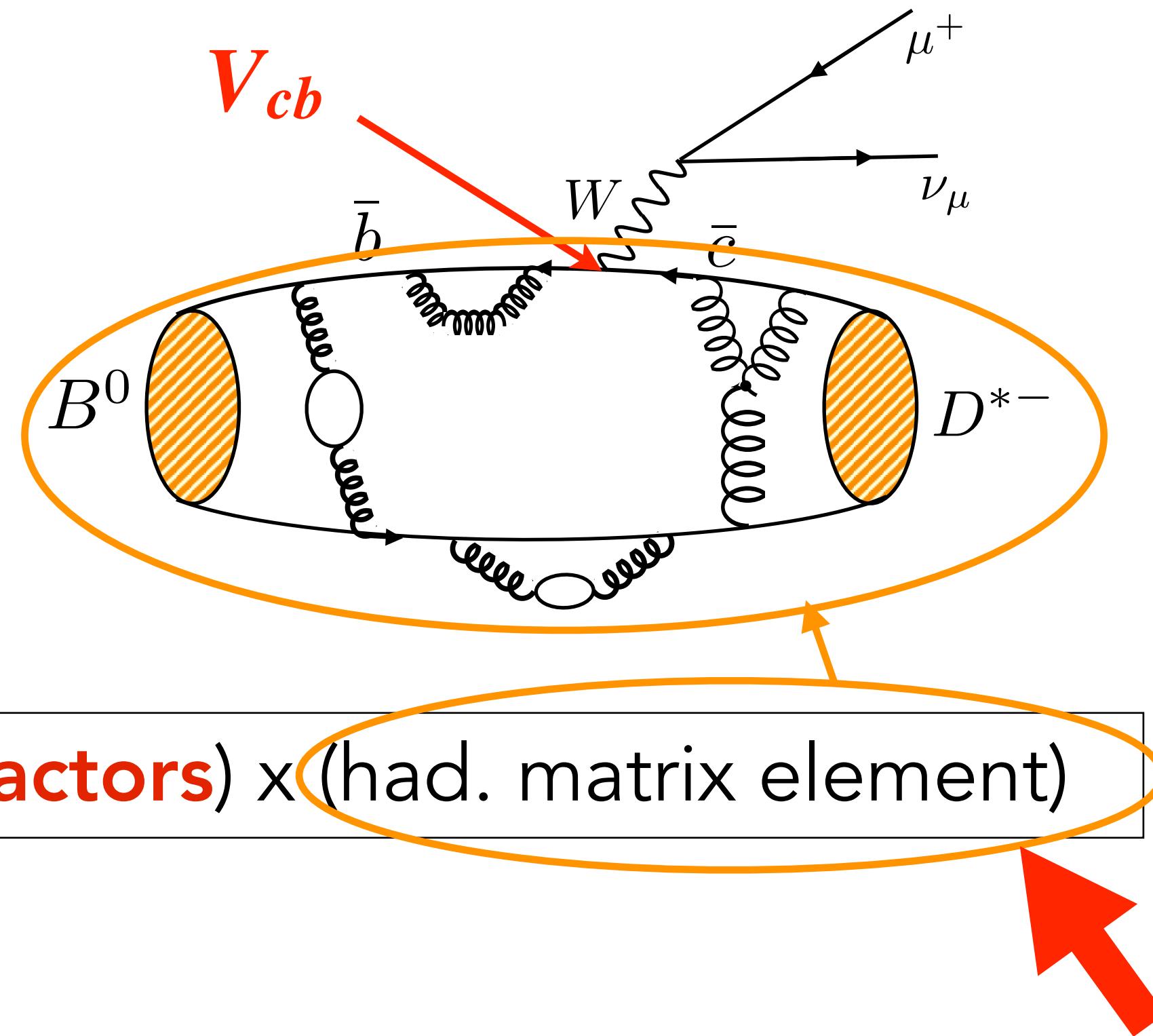


experimental
measurements

The role of lattice QCD in quark flavor physics

example: $B^0 \rightarrow D^{*-} \mu^+ \nu_\mu$

Experiment vs. SM theory:



$$(\text{experiment}) = (\text{known}) \times (\text{CKM factors}) \times (\text{had. matrix element})$$

$$\Gamma(K^+ \rightarrow \ell^+ \nu_\ell(\gamma))$$

$$d\Gamma(B^0 \rightarrow D^{*-} \mu^+ \nu_\mu), \dots$$

$$B(B_s \rightarrow \mu\mu), \dots$$

$$\Delta m_{d(s)} \dots$$

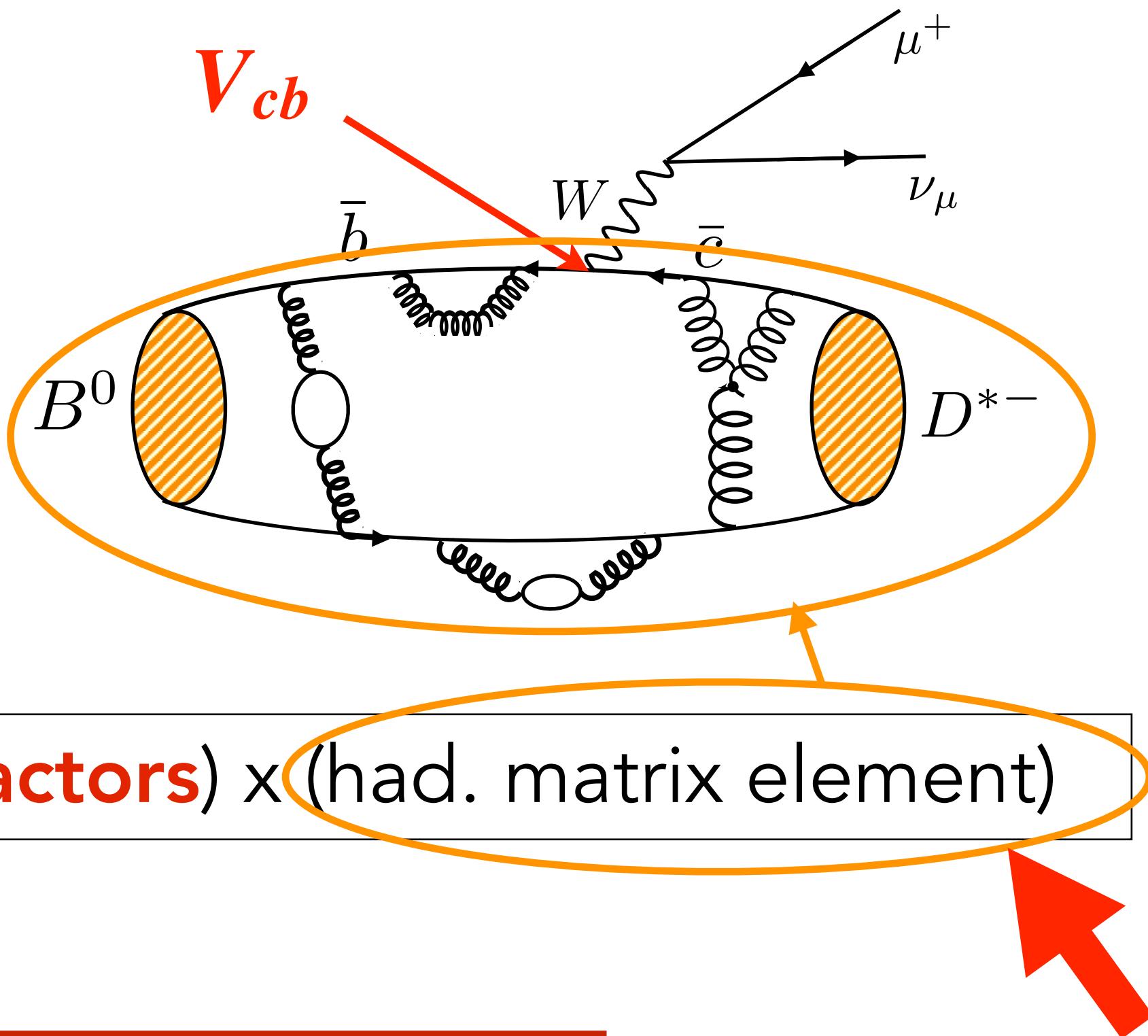
Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

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 $B(B_s \rightarrow \mu\mu), \dots$
 $\Delta m_{d(s)} \dots$

Two main purposes:

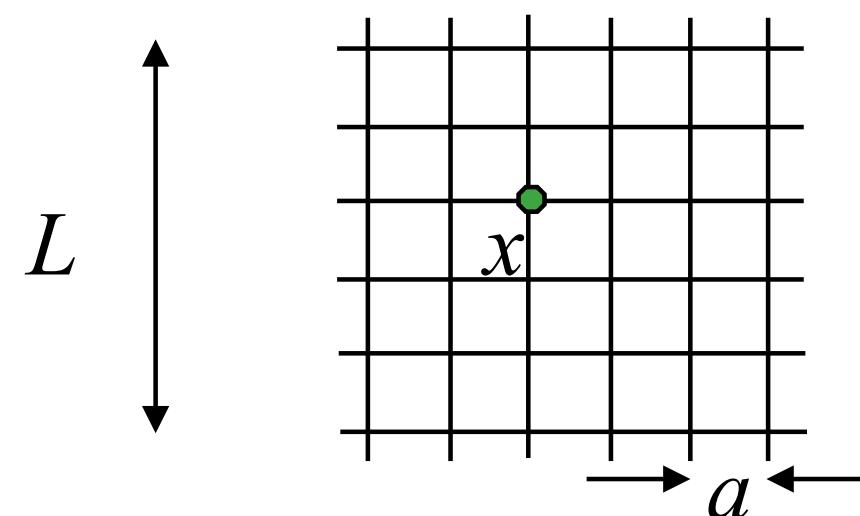
- ◆ combine experimental measurements with LQCD results to determine CKM parameters.
- ◆ confront experimental measurements with SM theory using LQCD inputs.

Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

Lattice QCD Introduction

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f + \frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu}$$

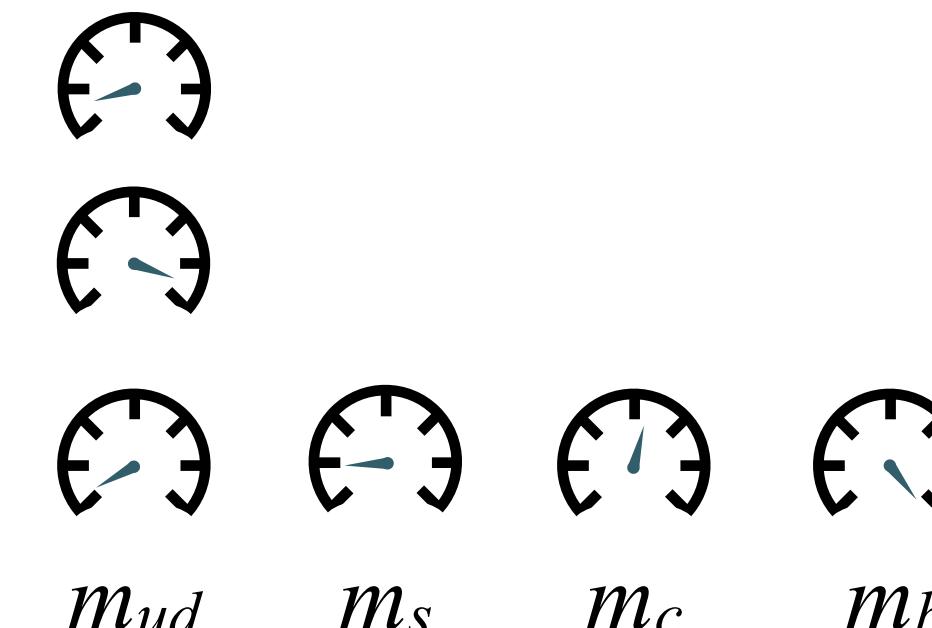


- ◆ discrete Euclidean space-time (spacing a)
derivatives \rightarrow difference operators, etc...
- ◆ finite spatial volume (L)
- ◆ finite time extent (T)

Integrals are evaluated numerically using monte carlo methods.

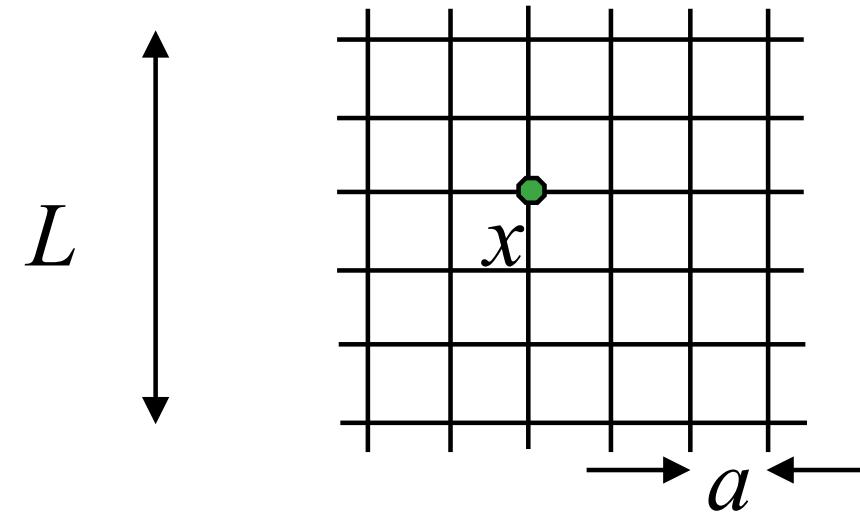
adjustable parameters

- ❖ lattice spacing: $a \rightarrow 0$
- ❖ finite volume, time: $L \rightarrow \infty, T > L$
- ❖ quark masses (m_f):
tune using hadron masses
extrapolations/interpolations $m_f \rightarrow m_{f,\text{phys}}$



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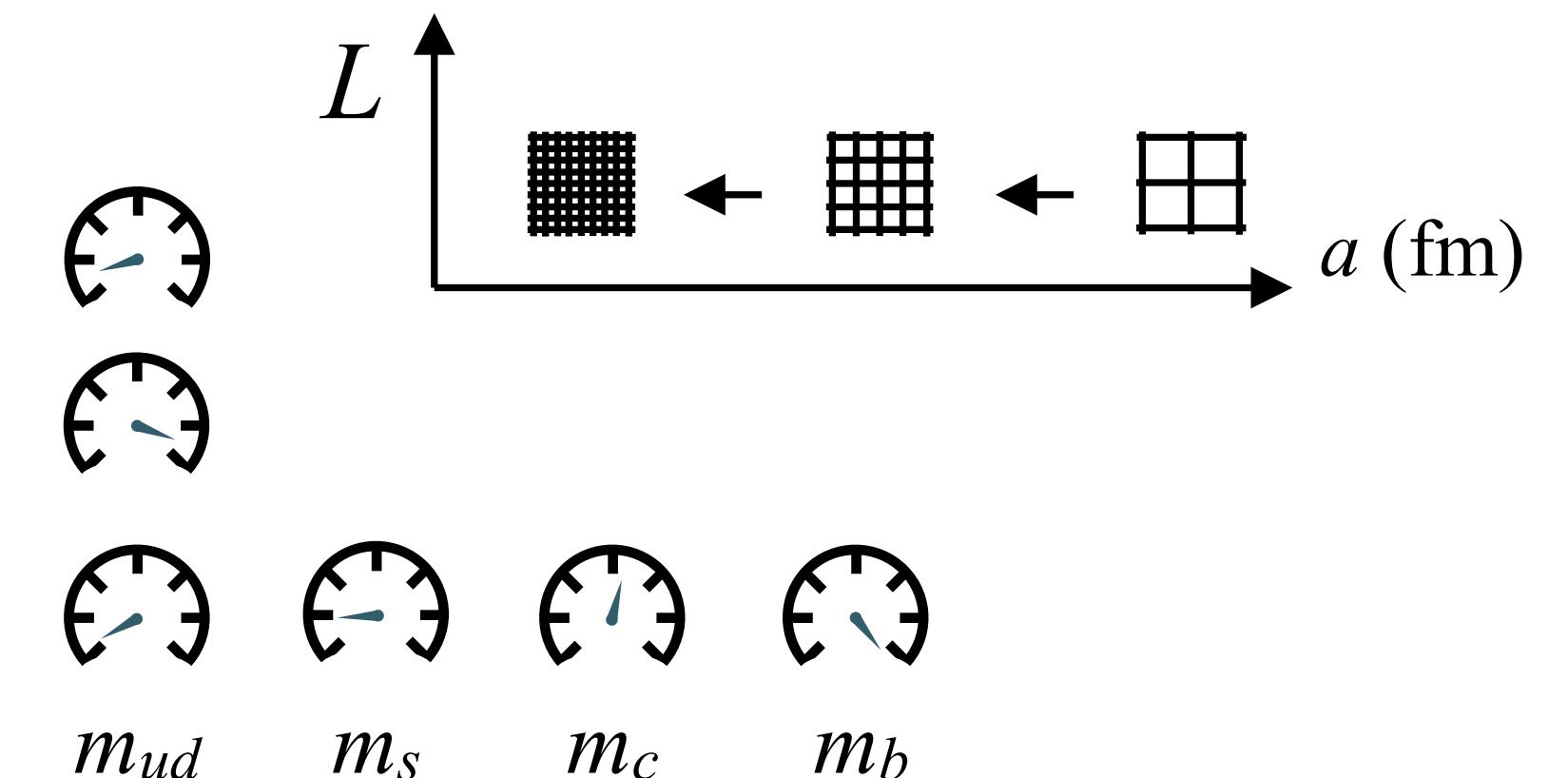


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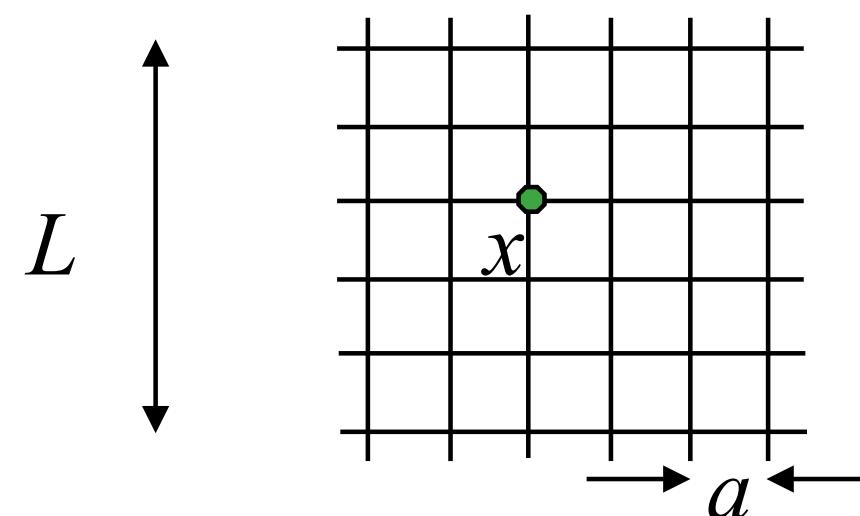
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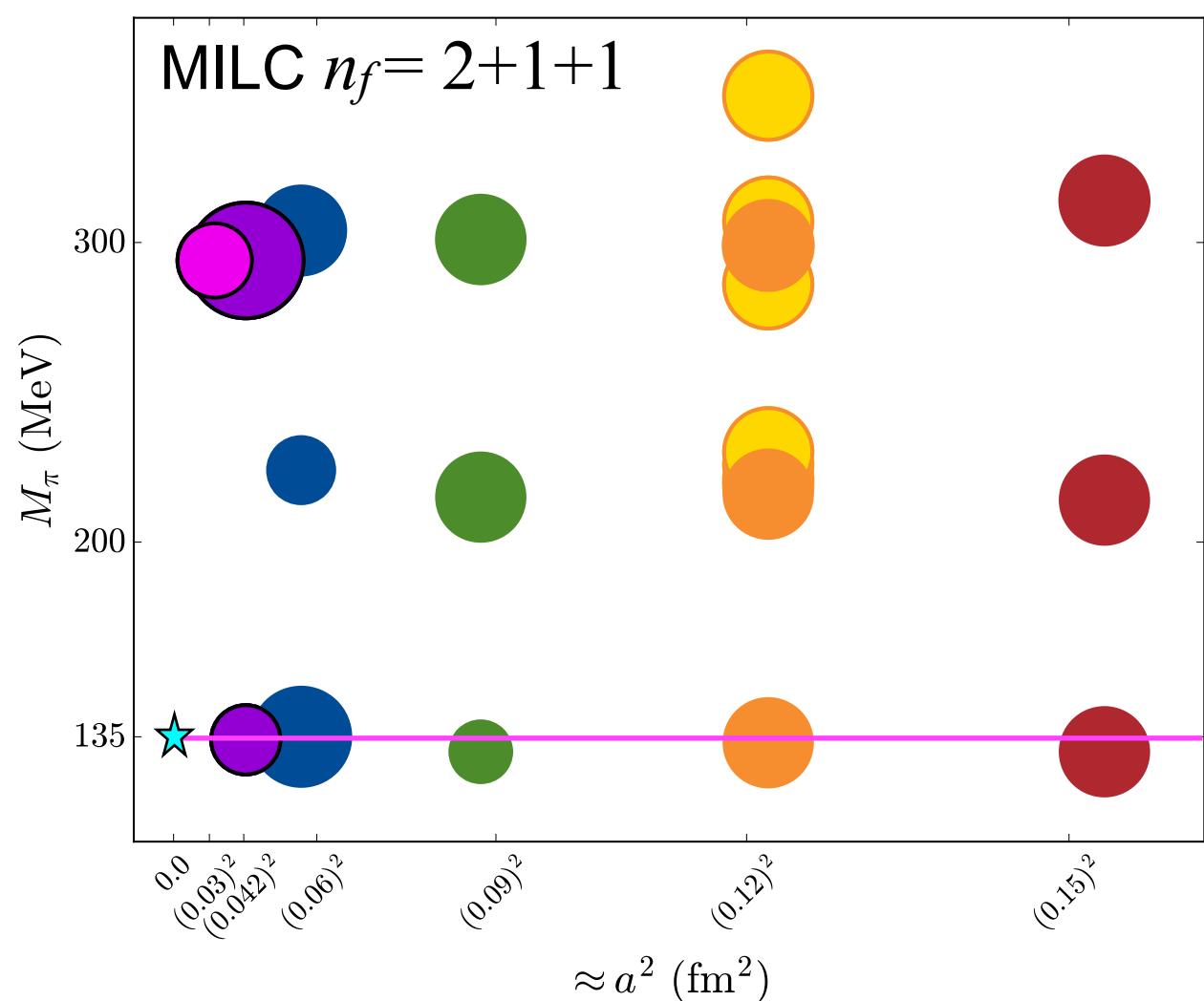
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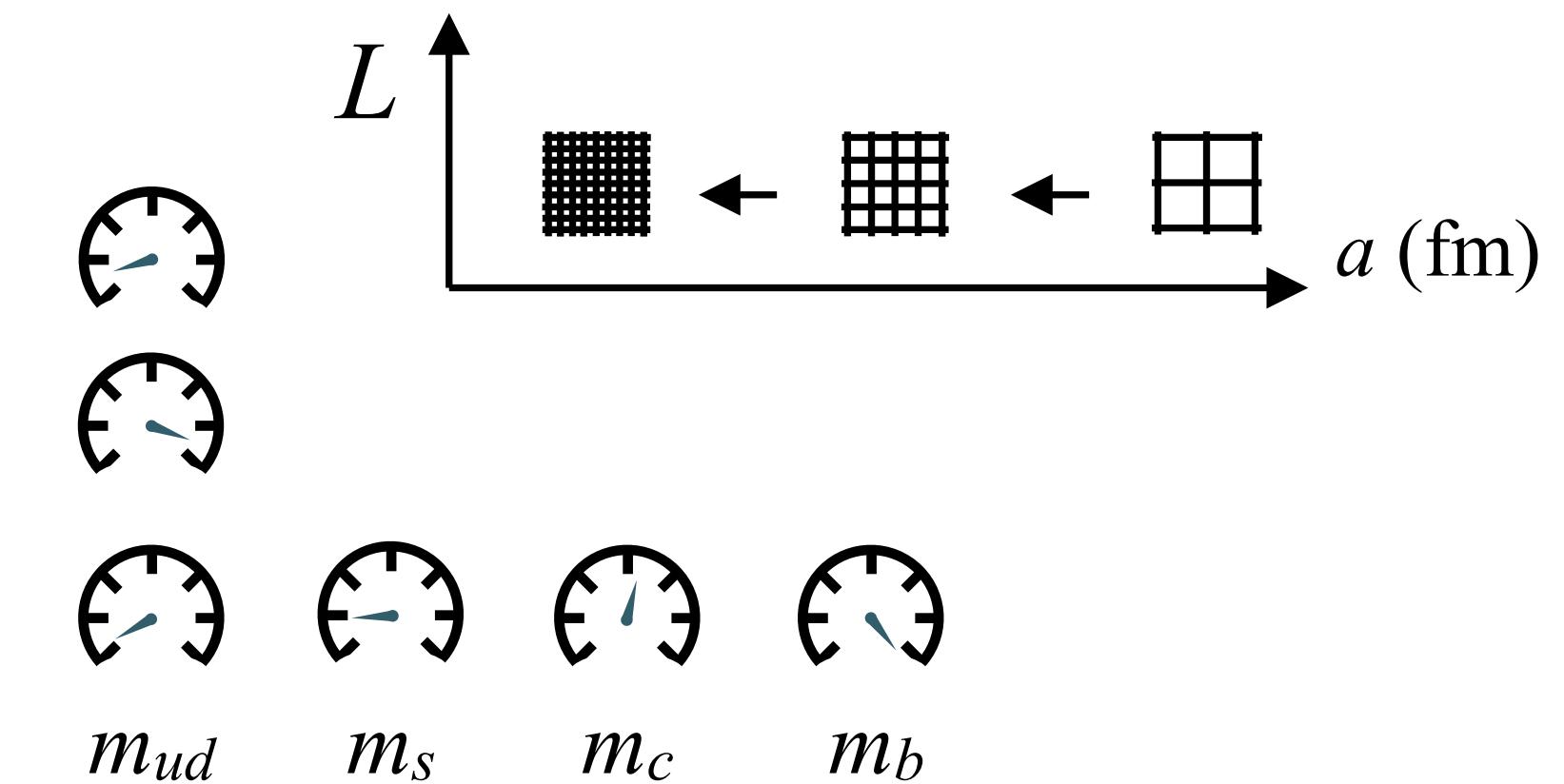
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Leptonic decays of K, π mesons

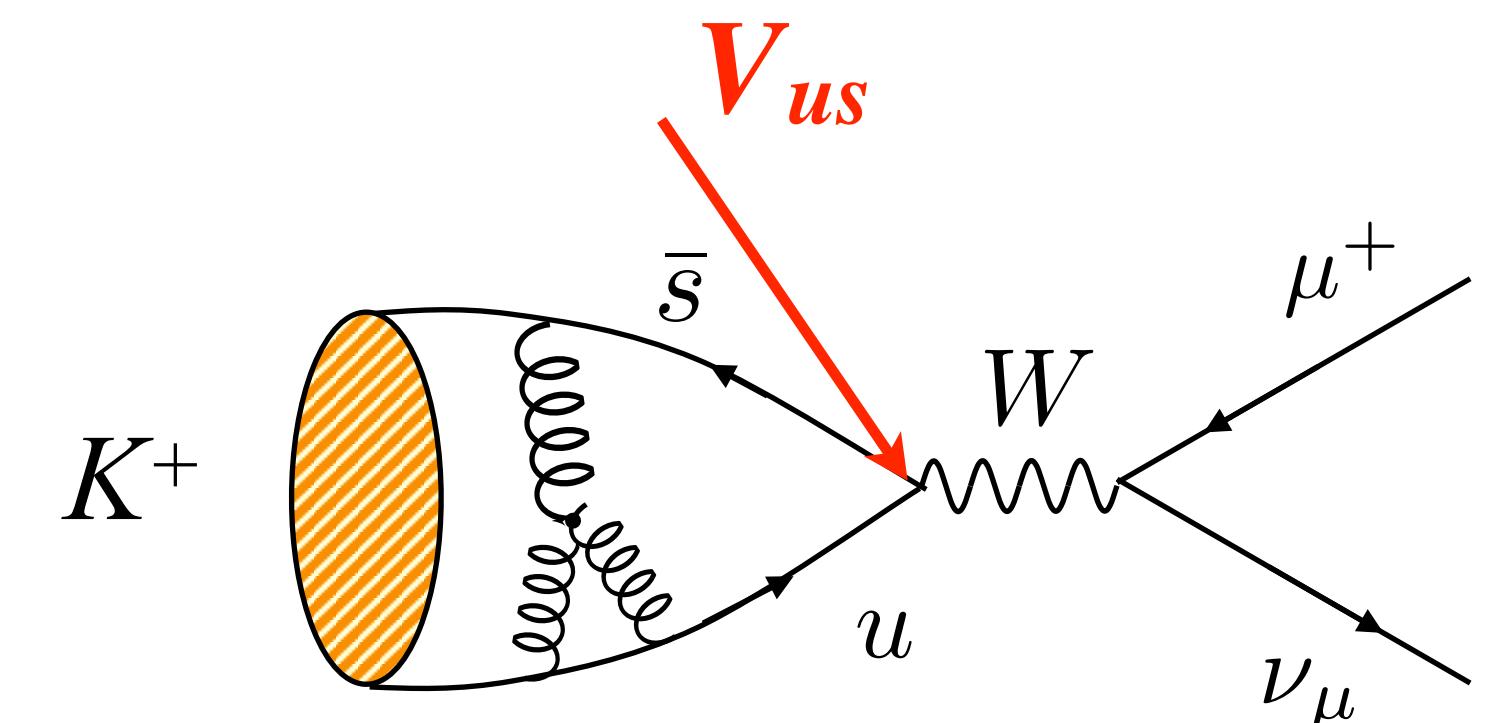
example: $K^+ \rightarrow \mu^+ \nu_\mu$

$$\Gamma(K^+ \rightarrow \ell^+ \nu_\ell(\gamma)) = (\text{known}) \times (1 + \delta_{\text{EM}}^\ell) \times |V_{us}|^2 \times f_{K^+}^2$$

experimental average [PDG]:

$$\frac{\Gamma(K \rightarrow \mu\nu(\gamma))}{\Gamma(\pi \rightarrow \mu\nu(\gamma))} = 1.3367(28)$$

- Needed to relate pure QCD decay constant to experiment
- ChPT + pheno estimate: $-1.12(21)\%$
[Cirigliano et al, arXiv:1107.6001, RMP 2012]
- First LQCD result: $-1.26(14)\%$
[Di Carlo et al, arXiv:1904.08731]
-

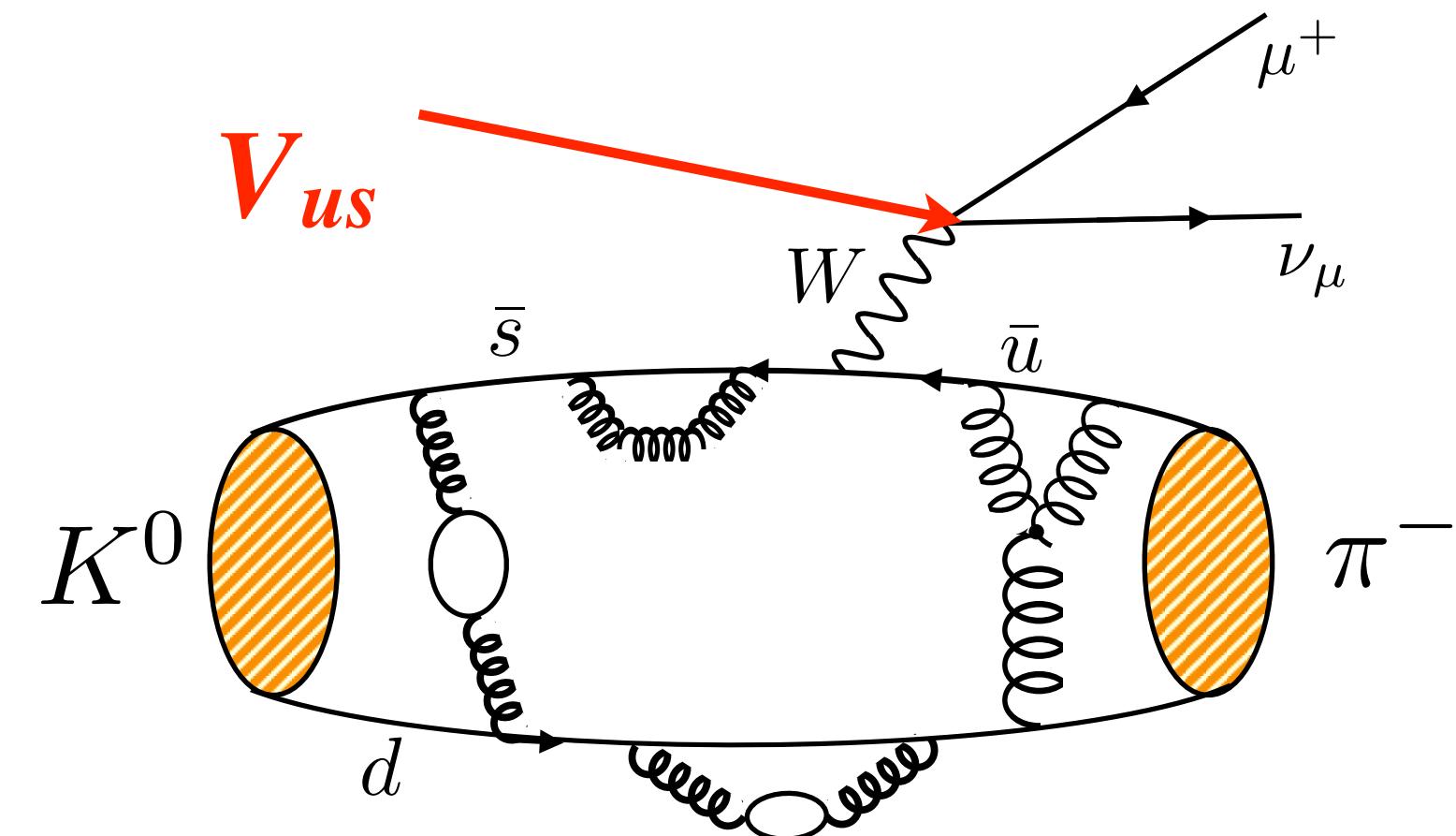


$$\frac{|V_{us}| f_{K^+}}{|V_{ud}| f_{\pi^+}} = 0.27600(37)$$

$$\frac{|V_{us}| f_{K^+}}{|V_{ud}| f_{\pi^+}} = 0.27683(29)_{\text{exp}}(20)_{\text{th}}$$

Semileptonic kaon decay

example: $K^0 \rightarrow \pi^- \ell^+ \nu_\ell$



$$\Gamma_{K\ell 3} = (\text{known}) \times \begin{pmatrix} \text{phase} \\ \text{space} \end{pmatrix} \times (1 + \delta_{\text{EM}}^{K\ell} + \delta_{\text{SU}(2)}^{K\pi}) \times |V_{us}|^2 \times |f_+^{K^0\pi^-}(0)|^2$$

experimental average [PDG]

$$|V_{us}| f_+(0) = 0.21635(39)_{\text{exp}}(3)_{\text{EM}}$$

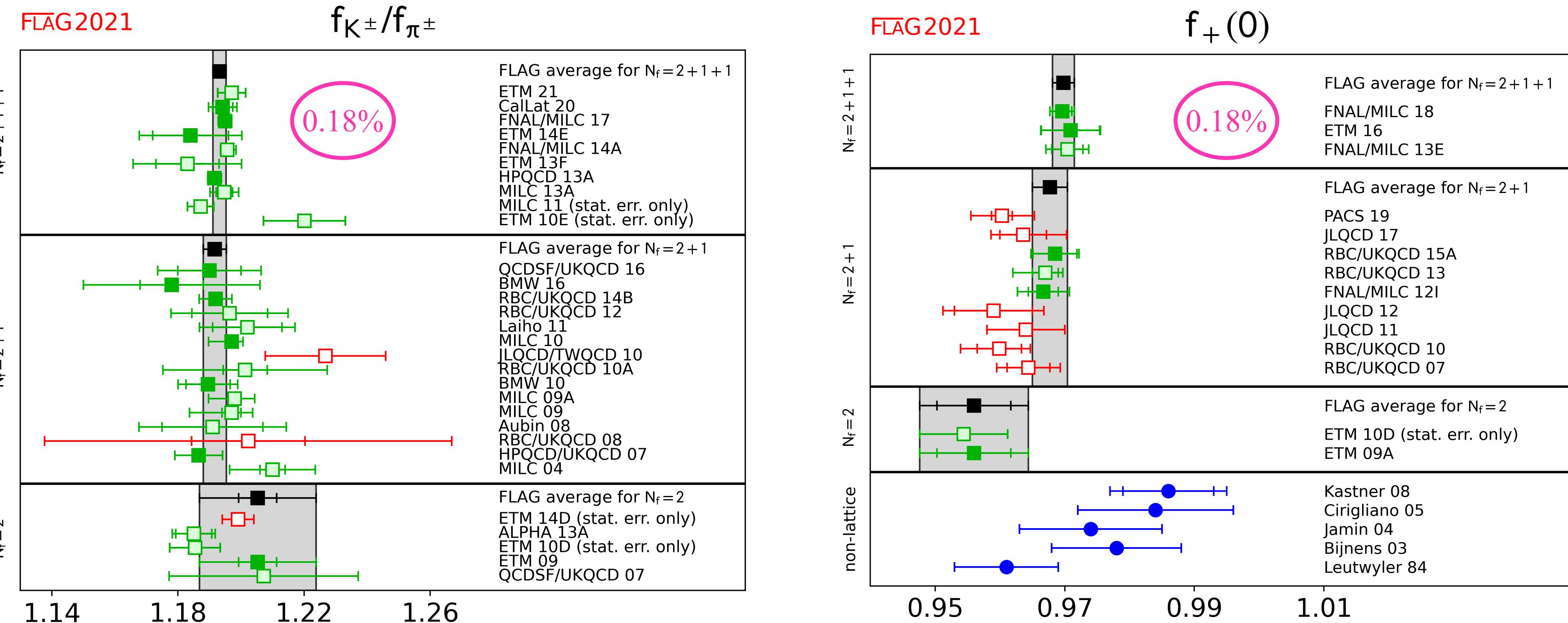
[Ch.Y. Seng et al, arXiv:2107.14708]

Needed to include charged kaon decay in the experimental average.

Needed to relate pure QCD form factor to experiment. Mode dependent.

K/π decay constants and $K_{\ell 3}$ form factor results

S. Aoki et al [FLAG 2021 review, arXiv:2111.09849]

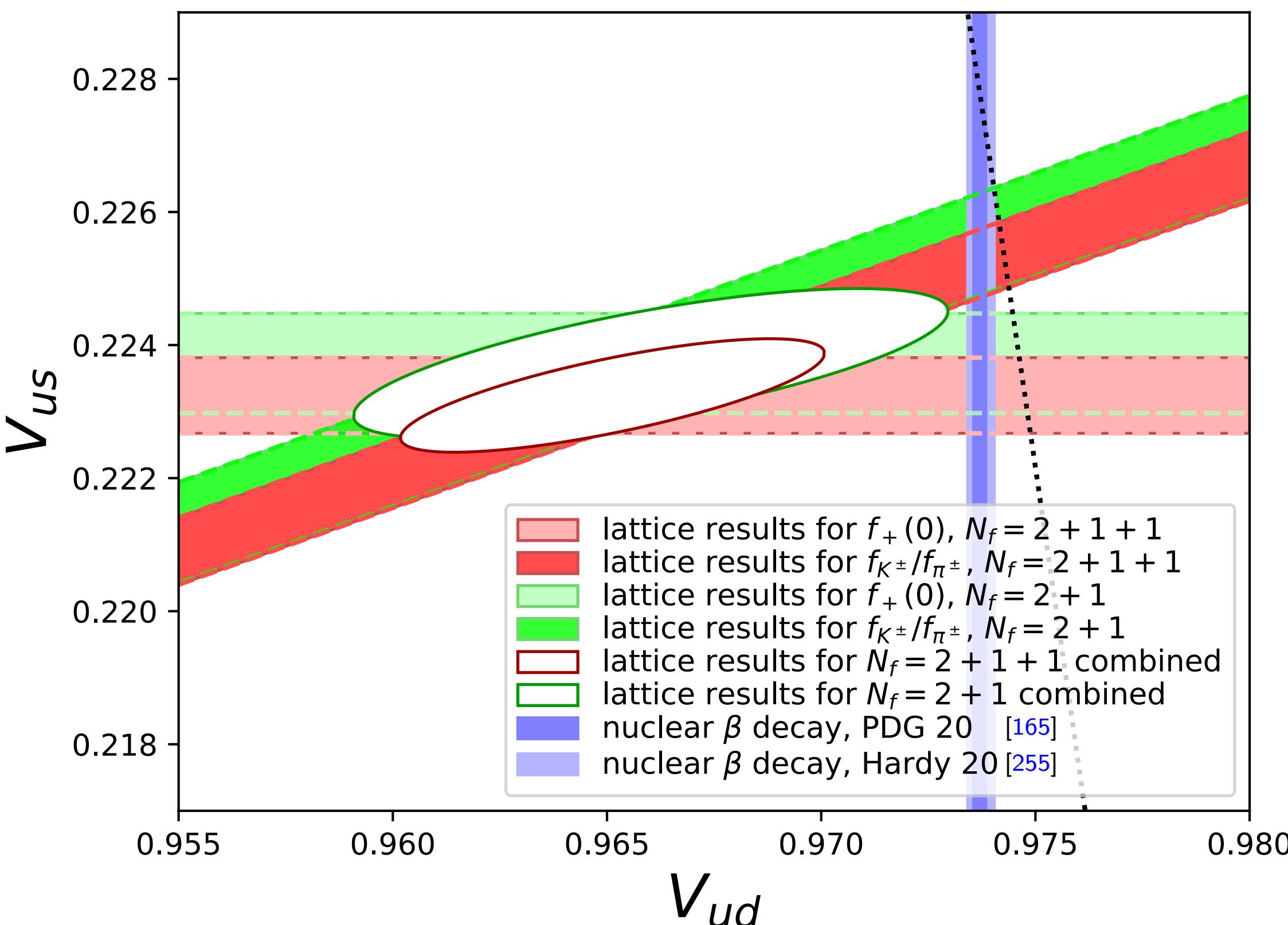


small errors due to **physical light quark masses**,
improved light-quark actions, NPR or no renormalization

First row CKM unitarity

$$\Delta_u \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 \quad V_{ub} \sim 0$$

FLAG2021



$$\Delta_u = -0.0021(2)_{\text{exp},0^+}(2)_{\text{EM}}(5)_{\text{NC}}(2)_{\text{exp},K}(2)_{\text{lat},K}$$

[C.Y. Seng et al, [arXiv:2107.14708](https://arxiv.org/abs/2107.14708)]

3.2 σ tension

$$\Delta_u = -0.00151(39)_{f_+(0)}(36)_{f_K/f_\pi}(36)_{\text{exp}}(27)_{\text{EM}}$$

[FNAL/MILC, [arXiv:1809.02827](https://arxiv.org/abs/1809.02827)]

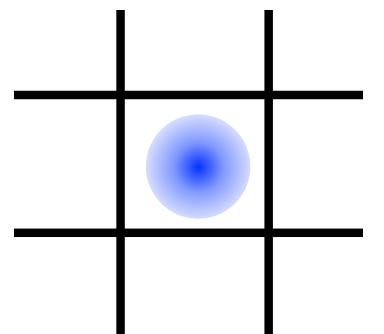
2 σ tension

Structure dependent EM corrections
are now an important source of error

$|V_{ud}| = 0.97373(11)(9)(27)$ from nuclear β -decay
[Hardy & Towner, PRC 102, 045501, 2020]

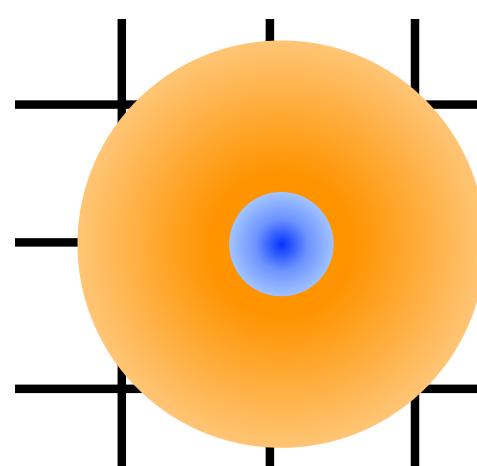
Finding Beauty

b quark



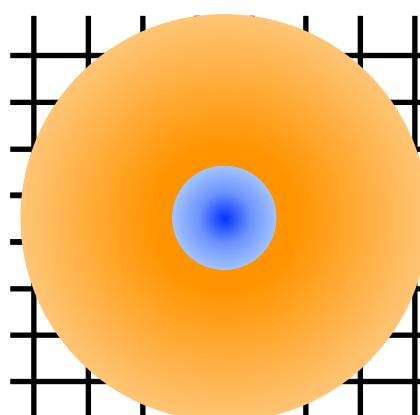
$m_b \gtrsim a^{-1} \gg \Lambda \rightarrow$ leading discretization errors $\sim (am_b)^2$
(using same action as for light quarks)

B meson



use EFT (HQET, NRQCD) $\rightarrow \Lambda/m_b$ expansion

- lattice HQET, NRQCD: use EFT to construct lattice action
complicated continuum limit
nontrivial matching and renormalization \rightarrow (few-5)% errors
- relativistic heavy quark approach (Fermilab)
matching relativistic lattice action via HQET to continuum
nontrivial matching and renormalization \rightarrow (1-3)% errors



$a^{-1} > m_b \gg \Lambda +$ highly improved light quark action

- \rightarrow same action for all quarks
- \rightarrow simple renormalization (Ward identities) \rightarrow < 1% errors

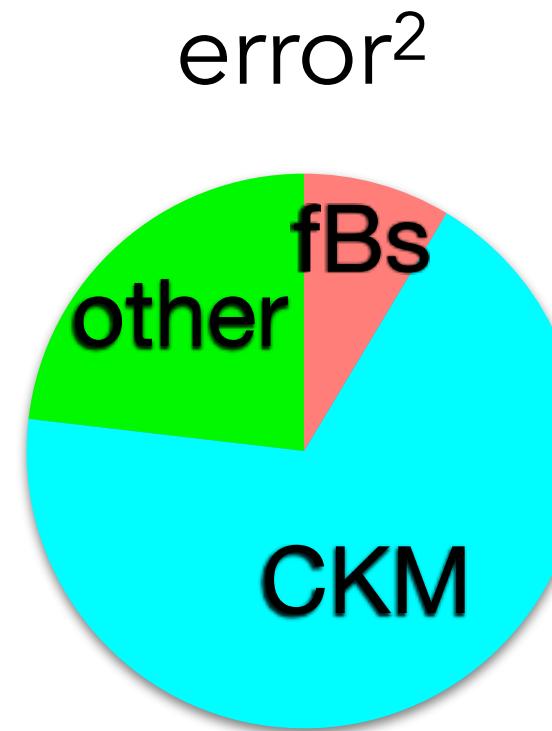
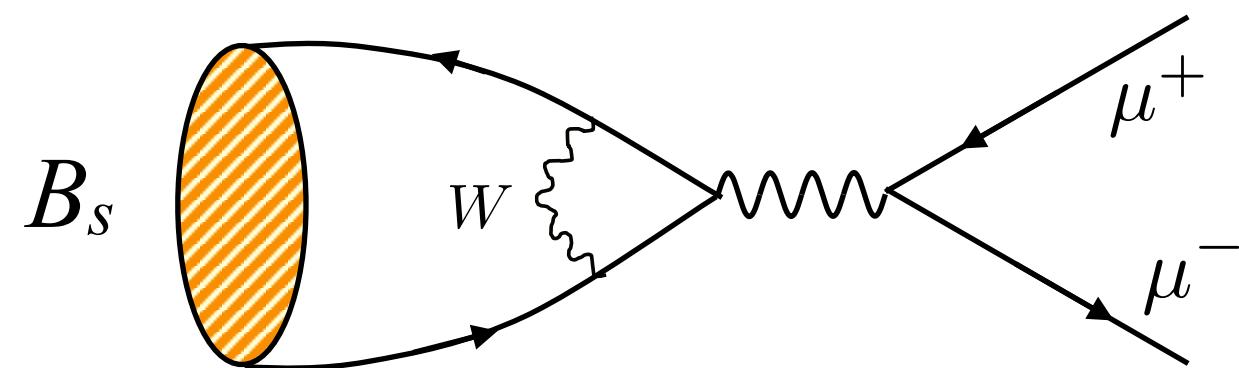




Rare leptonic decay $B_{s,d} \rightarrow \mu\mu$

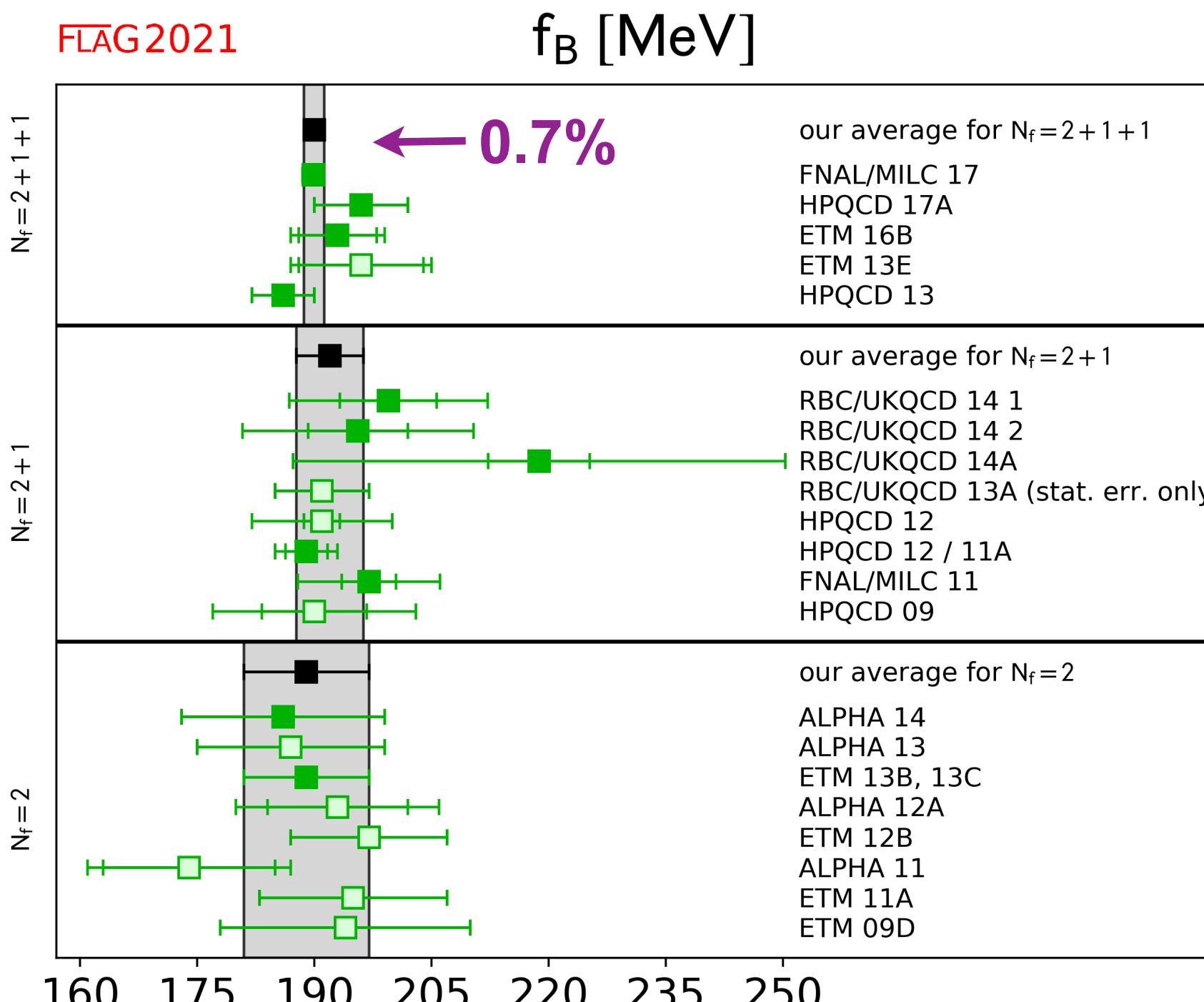
SM prediction for rare leptonic decay rate

$$\overline{\mathcal{B}}(B_s \rightarrow \mu^+ \mu^-) = 3.660(38) \times 10^{-9}$$

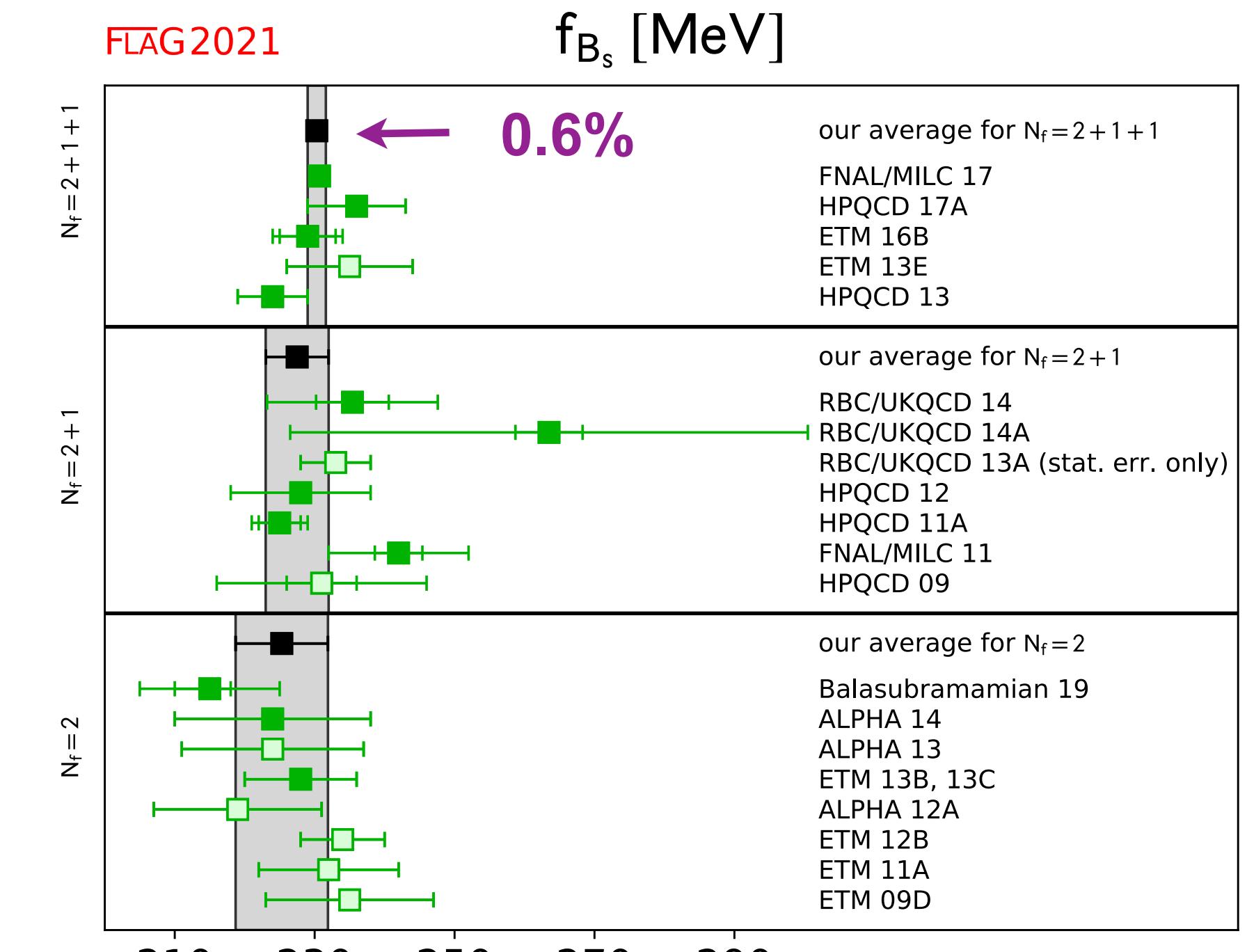


[Beneke et al, arXiv:1908.07011]

- includes structure-dependent QED corrections
- dominant uncertainty due to $|V_{cb}|$
- LQCD decay constant sub dominant source of uncertainty



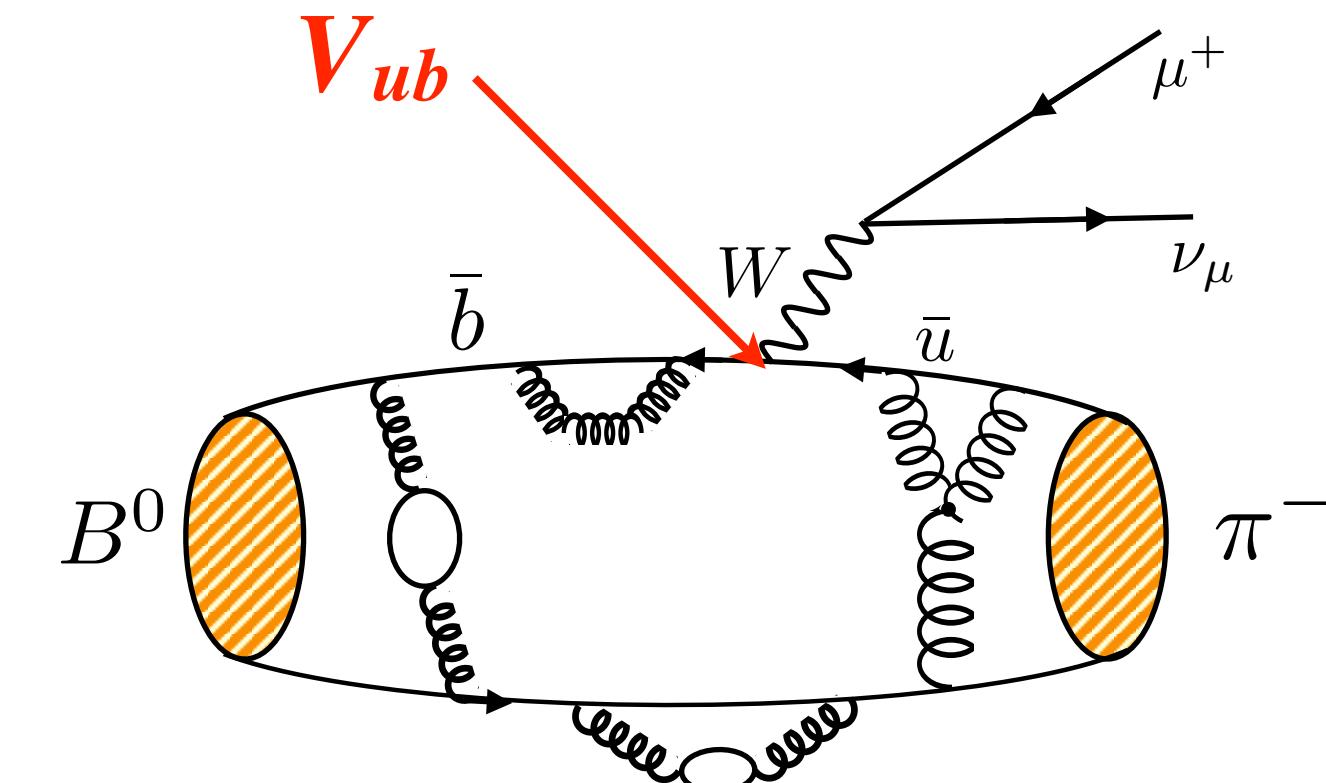
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[FLAG review,
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Semileptonic B-meson decay form factors

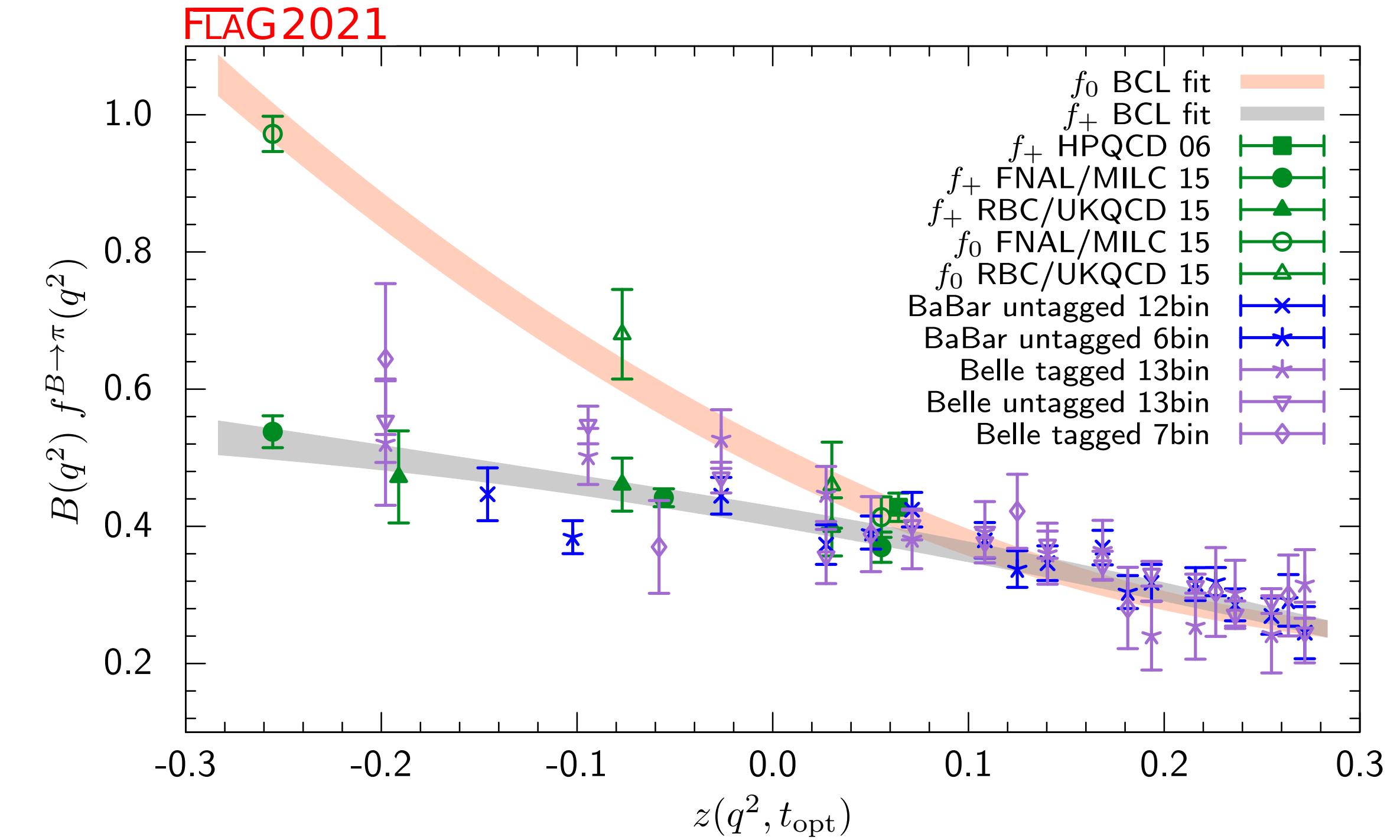
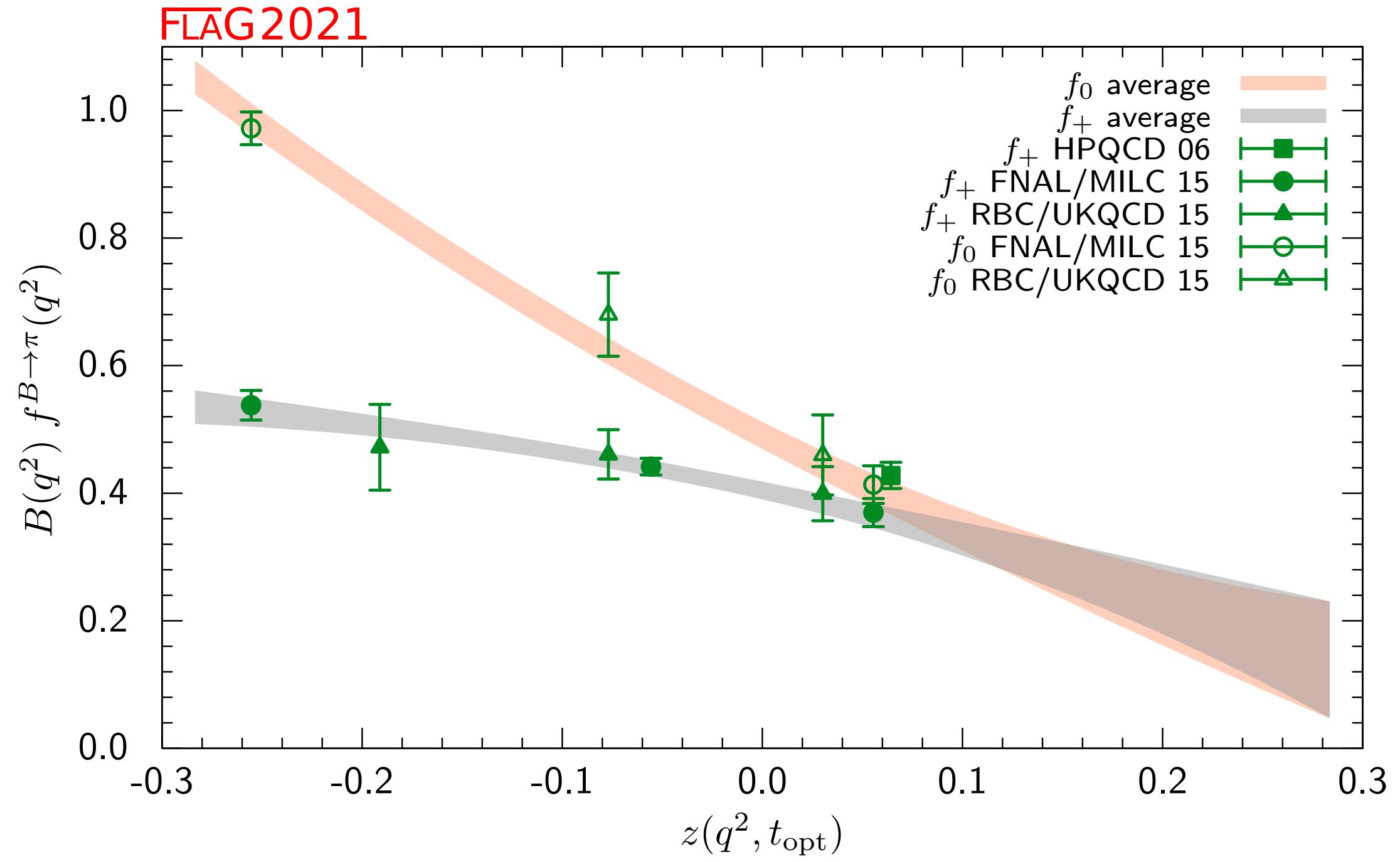
example: $B^0 \rightarrow \pi^- \mu^+ \nu_\mu$

$$\frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2} = (\text{known}) \times |V_{ub}|^2 \times |f_+(q^2)|^2$$



- ★ calculate the form factors in the low recoil (high q^2) range.
- ★ use model-independent parameterization of q^2 dependence.
- ★ calculate the complete set of form factors, $f_+(q^2), f_0(q^2)$ and $f_T(q^2)$.
- ★ for $f_+(q^2)$ compare shape between experiment and lattice.
- ★ lattice results available for $B \rightarrow \pi, B_s \rightarrow K, B \rightarrow K, B \rightarrow D, B_s \rightarrow D_s$ form factors

example: $B \rightarrow \pi$ form factors



- ★ shape of f_+ agrees with experiment
- ★ fit lattice form factors together with experimental data to determine $|V_{ub}|$ and obtain form factors (f_+, f_0) with improved precision
- ★ recent new result by JLQCD in good agreement [B. Colquhoun, et al, arXiv:2203.04938]

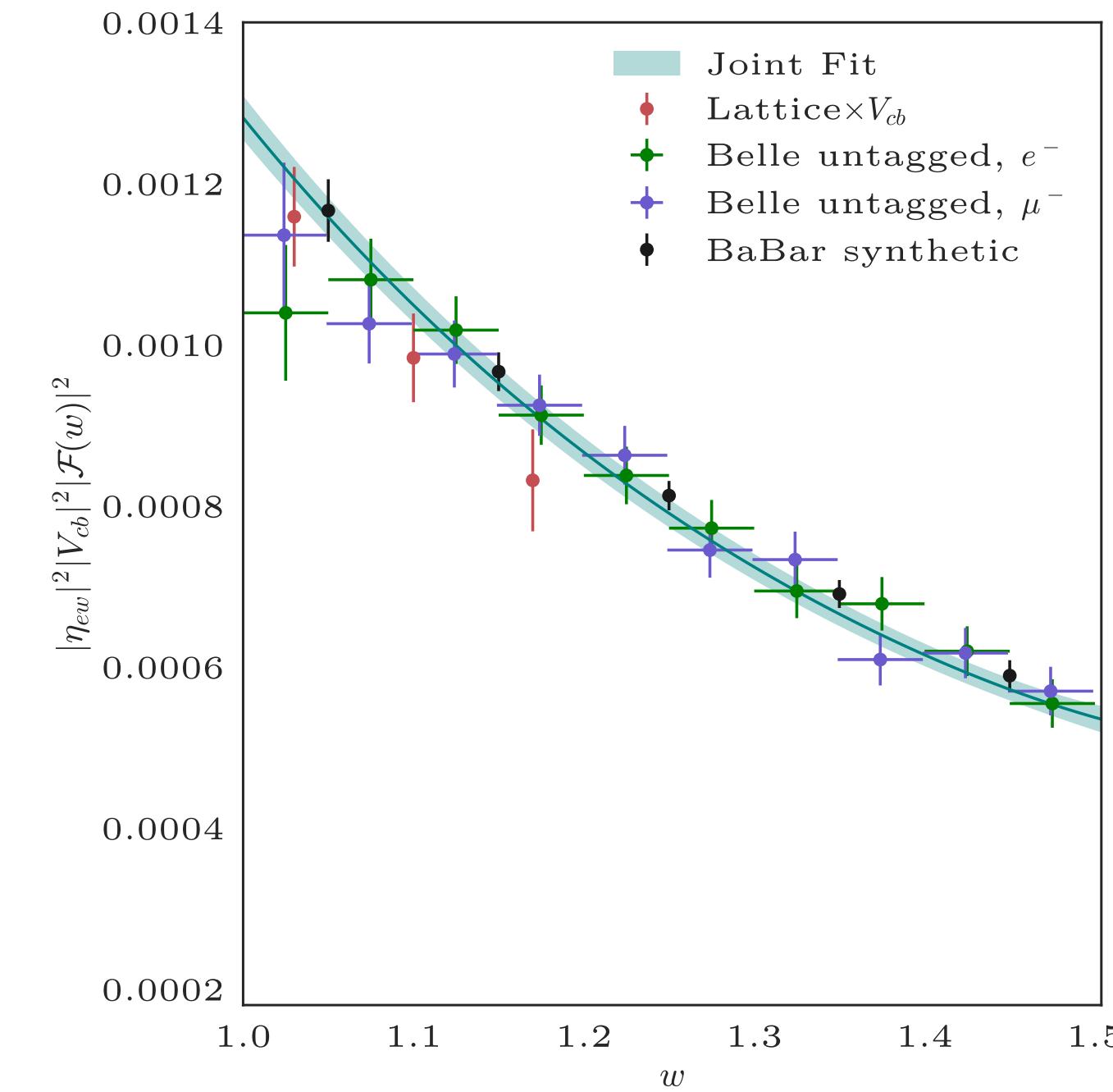
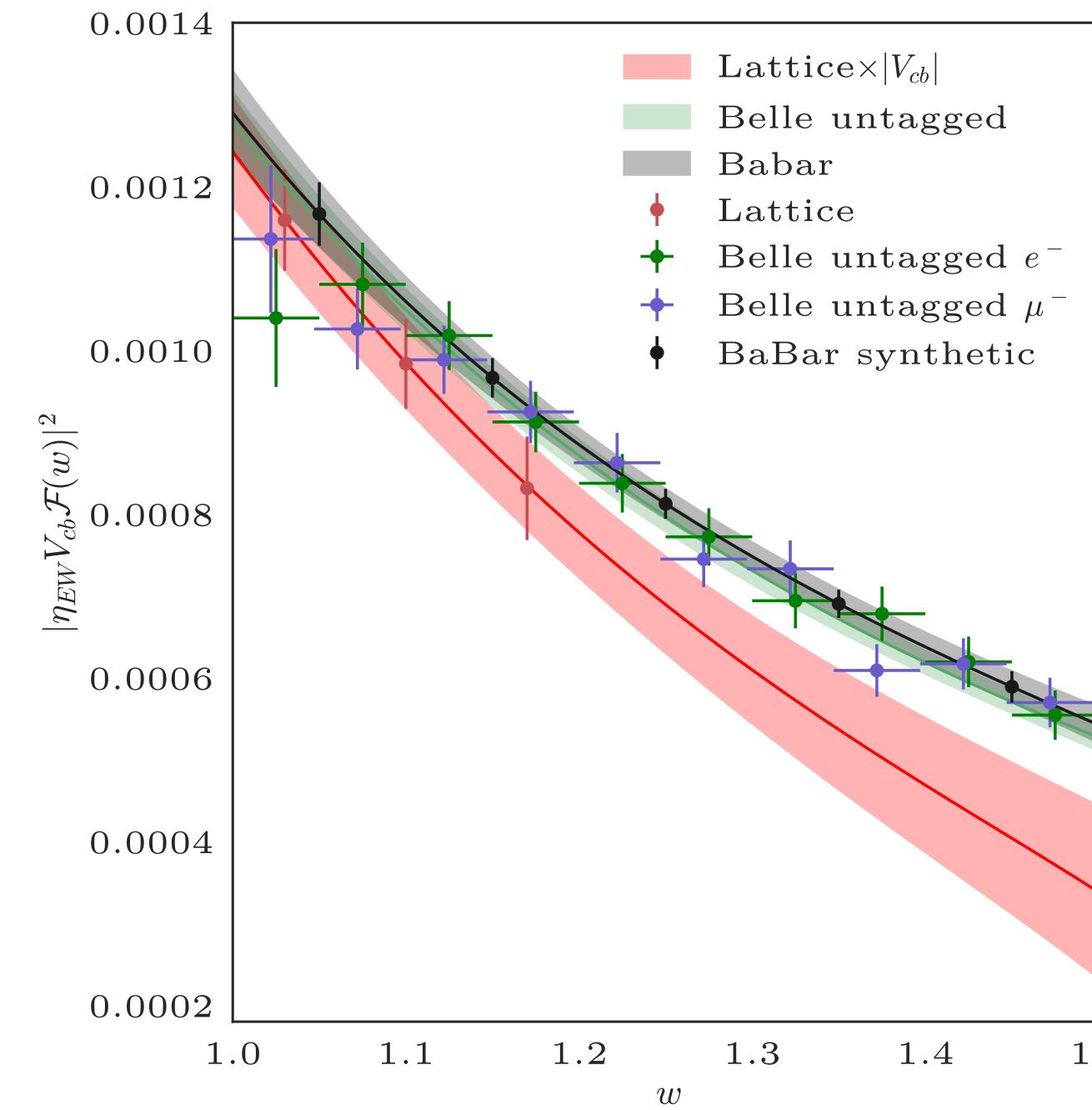
- 📌 LQCD form factors enable determinations of $|V_{ub}|$ from exp. measurements of exclusive (differential) decay rates **with commensurate contributions to total errors from lattice (~3%) and experiment**

Form factors for $B \rightarrow D^* \ell \nu_\ell$ and $|V_{cb}|$

$$\frac{d\Gamma}{dw} = (\text{known}) \times \eta_{\text{EW}}^2 (1 + \delta_{\text{EM}}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} \times \chi(w) |\mathcal{F}(w)|^2$$

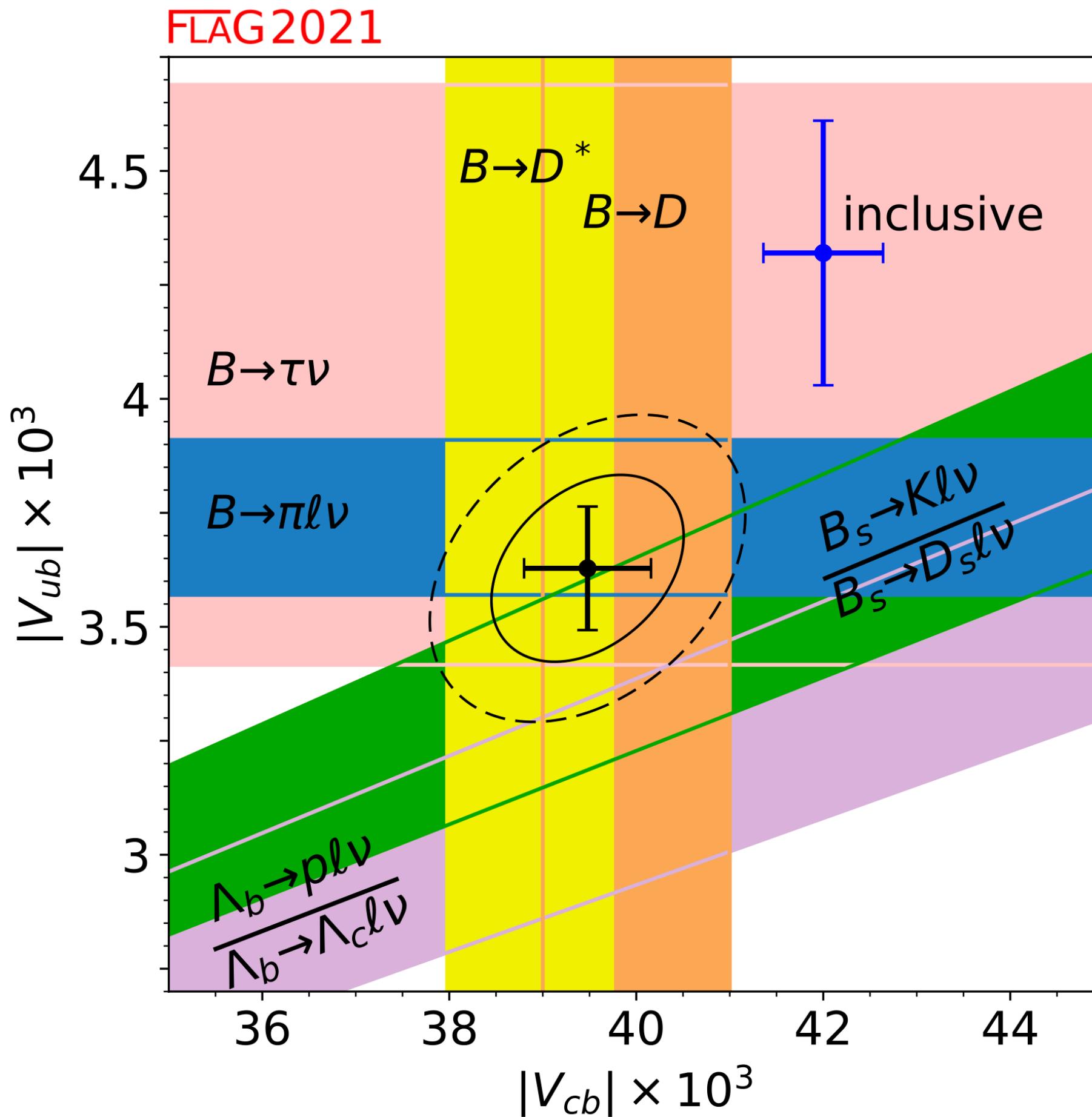
$w = v_B \cdot v_{D^*}$

- ★ First LQCD result for $B \rightarrow D^*$ form factors at non-zero recoil: FNAL/MILC [arXiv:2105.14019]

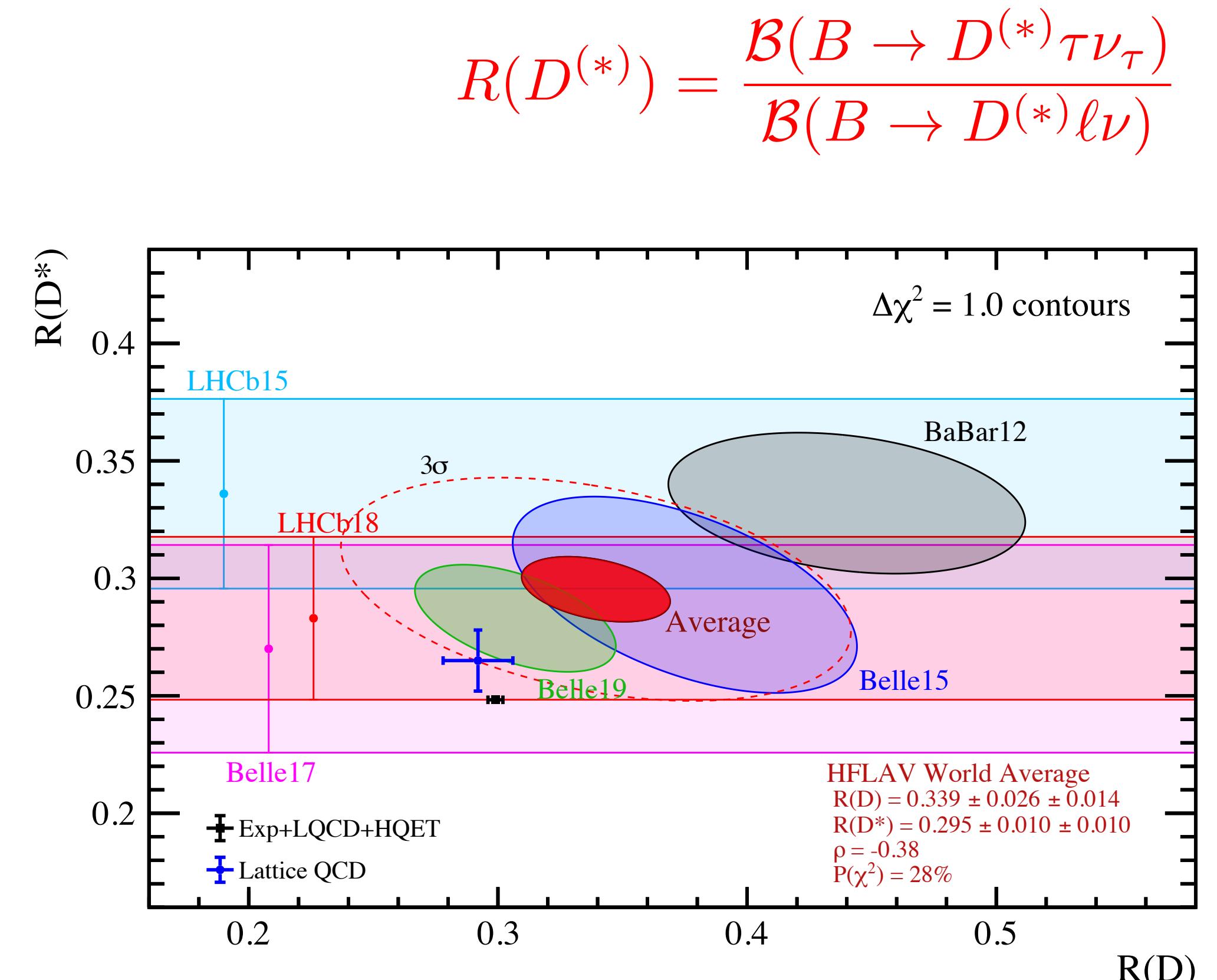


- ★ expect 2nd LQCD result from JLQCD very soon
- ★ also ongoing work by FNAL/MILC, HPQCD, ...

Implications: $|V_{ub}|$, $|V_{cb}|$, LFU ratios



Uncertainties from LQCD form factors are commensurate with experiment.



$\sim 4\sigma$ tension between SM and Exp.

Snowmass 2013 → present

<https://www.usqcd.org/documents/13flavor.pdf> and [J. Butler et al, arXiv:1311.1076]

Quantity	CKM	Present	2007 forecast	Present	2018	2021 FLAG Average
	element	expt. error	lattice error	lattice error	lattice error	
f_K/f_π	$ V_{us} $	0.2%	0.5%	0.4%	0.15%	0.18 %
$f_+^{K\pi}(0)$	$ V_{us} $	0.2%	–	0.4%	0.2%	0.18 %
f_D	$ V_{cd} $	4.3%	5%	2%	< 1%	0.3 %
f_{D_s}	$ V_{cs} $	2.1%	5%	2%	< 1%	0.2 %
$D \rightarrow \pi \ell \nu$	$ V_{cd} $	2.6%	–	4.4%	2%	4.4 %
$D \rightarrow K \ell \nu$	$ V_{cs} $	1.1%	–	2.5%	1%	0.6 %
$B \rightarrow D^* \ell \nu$	$ V_{cb} $	1.3%	–	1.8%	< 1%	1.7 % [from FNAL/MILC, 2105.14019]
$B \rightarrow \pi \ell \nu$	$ V_{ub} $	4.1%	–	8.7%	2%	~3 %
f_B	$ V_{ub} $	9%	–	2.5%	< 1%	0.7 % (0.6 % for f_{B_s})
ξ	$ V_{ts}/V_{td} $	0.4%	2–4%	4%	< 1%	1.3 %
Δm_s	$ V_{ts} V_{tb} ^2$	0.24%	7–12%	11%	5%	4.5 %
B_K	$\text{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	< 1%	1.3 %

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QED
corrections
important

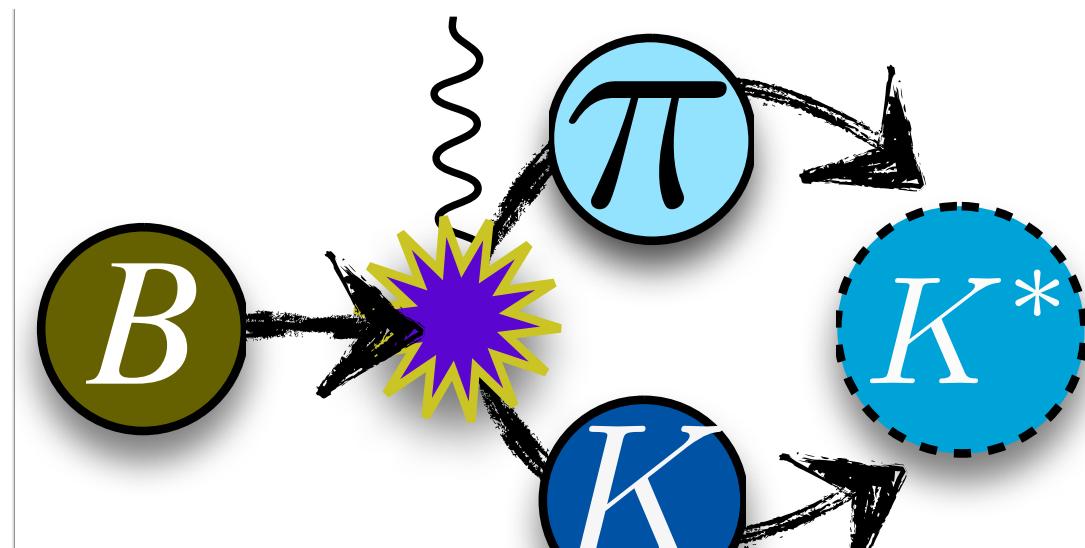
Semileptonic B decays to vector mesons: $B \rightarrow K^* \ell \ell$

existing LQCD results for $B \rightarrow K^*$, $B_s \rightarrow \phi$ form factors assume stable K^* , ϕ (narrow width approximation)
[R. Horgan et al, arXiv:1310.3887, 1310.3722, 1501.00367]

Formalism for multi-channel 1 → 2 transition amplitudes:

[Briceno, Hansen, Walker-Loud, arXiv:1406.5965, PRD 2015; 1502.04314, PRD 2015,...]

weak current



[Figure by R. Briceno]

dedicated effort requires significant computational resources

pilot study [Agadjanov et al, arXiv:1605.03386, NPB 2016]

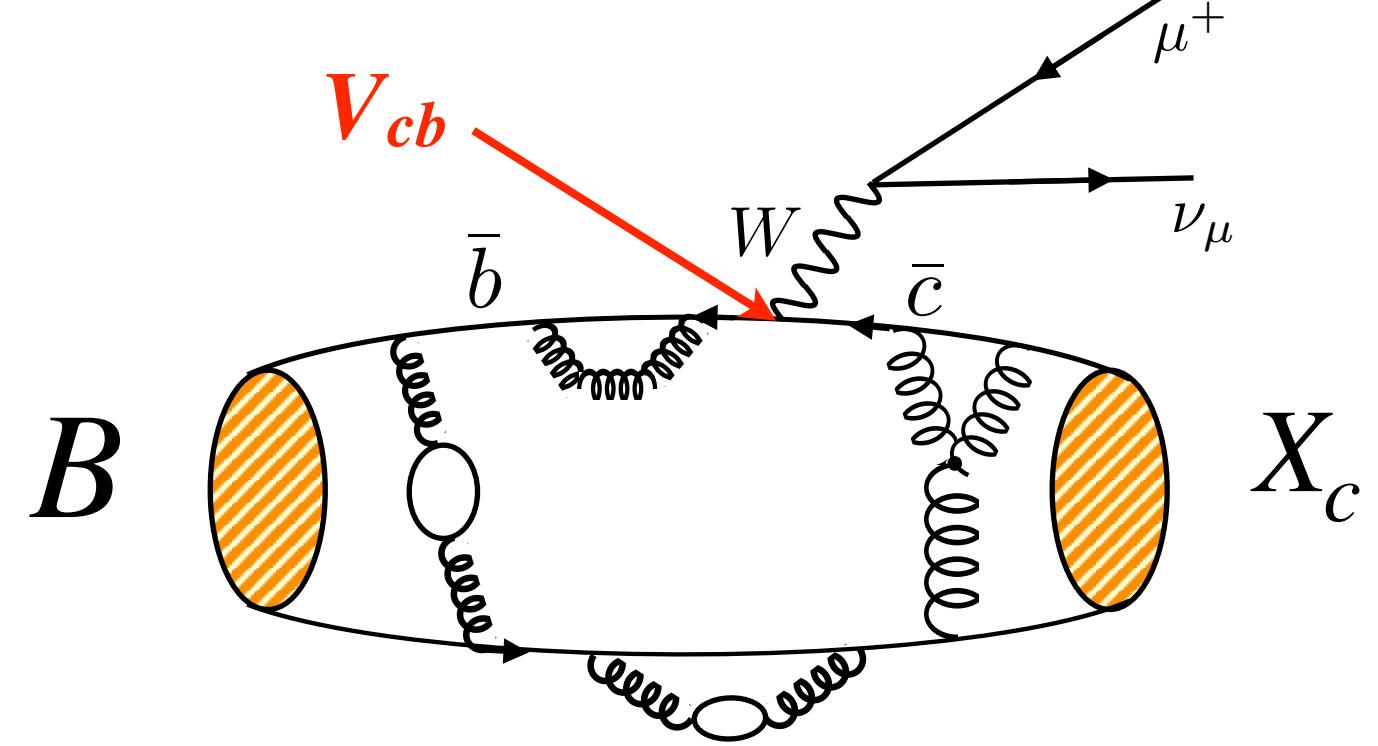
Limitations:

- q^2 reach: small recoil
- invariant mass of two-hadron system: $< 3 m_H$
- recent work to extend formalism to 3 hadrons
[M. Hansen et al, arXiv:2101.10246]

studies of $K\pi$ scattering

[G. Rendon et al, arXiv:1811.10750; D. Wilson et al, arXiv:1904.03188]

Inclusive decay rates with lattice QCD



For example: $B \rightarrow X_c \ell \nu_\ell$

Target: $d\Gamma \sim |V_{cb}|^2 L^{\mu\nu} W_{\mu\nu}$

$$W_{\mu\nu} = \frac{1}{2M_B} \int d^4x e^{-iqx} \langle B | J_\mu^\dagger(x) J_\nu(0) | B \rangle$$

Start with Euclidean four-point function:

$$C_4(q, \tau) = \sum_x e^{iqx} \frac{1}{2M_B} \langle B | J_\mu^\dagger(x) J_\nu(0) | B \rangle$$

Sum over final states:

$$X_c = D, D^*, D\pi, D\pi\pi, D^{**}, \dots$$

Use OPE + pert. QCD to write $d\Gamma$ as a double expansion:

$$d\Gamma \sim \sum_n c_n \frac{\langle O_n \rangle}{m_b^n}$$

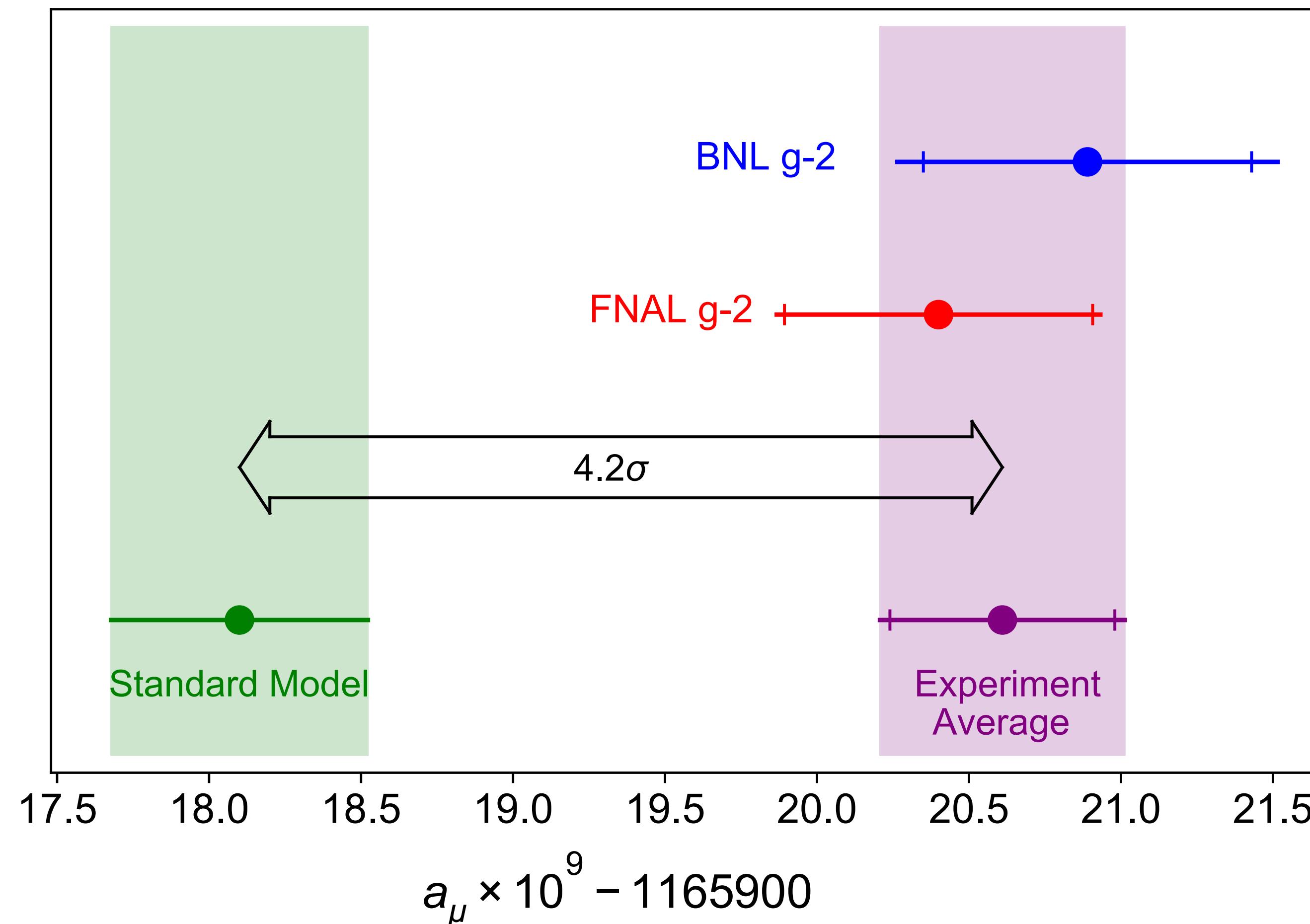
- c_n are calculated in perturbation theory
- $\langle O_n \rangle$ are matrix elements of local operators

- new methods to perform inverse Laplace transform [Liu & Dong (PRL 1994),; Liu (PRD 200); Jian et al (1710.11145); Hansen, Meyer, Robaina (1703.01881, PRD 2017); M. Hansen et al, arXiv:1903.06476; P. Gambino & S. Hashimoto, arXiv:2005.13730; J. Bulava et al, arXiv:2111.12774]
- first application to $B \rightarrow X_c \ell \nu$
good agreement with OPE
[P. Gambino et al, arXiv:2203.11762]

Muon g-2: experiment vs theory

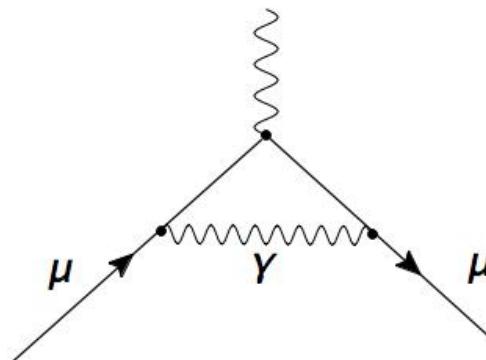
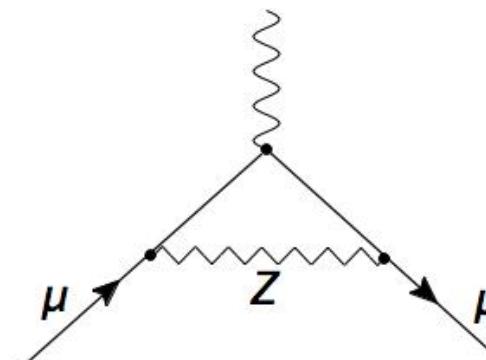
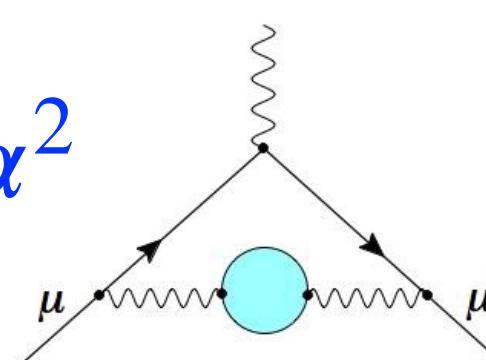
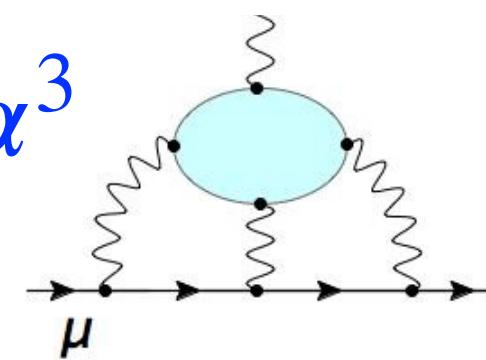
$$a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{Weak}} + a_{\mu}^{\text{HVP}} + a_{\mu}^{\text{HLbL}} = 116591810(43) \times 10^{-11}$$

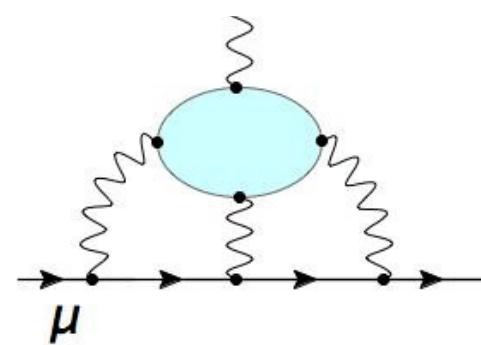
[B. Abi et al (Muon g-2 Collaboration), [Phys. Rev. Lett. 124, 141801 \(2021\)](#)]



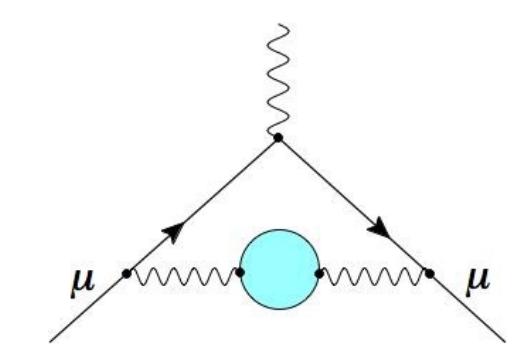
Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

QED		+... (5 loops)	116 584 718.9 (1) $\times 10^{-11}$	0.001 ppm
EW		+... (2 loops)	153.6 (1.0) $\times 10^{-11}$	0.01 ppm
Hadronic corrections				
HVP		+... (NNLO)	6845 (40) $\times 10^{-11}$ [0.6%]	0.34 ppm
HLbL		+... (NLO)	92 (18) $\times 10^{-11}$ [20%]	0.15 ppm



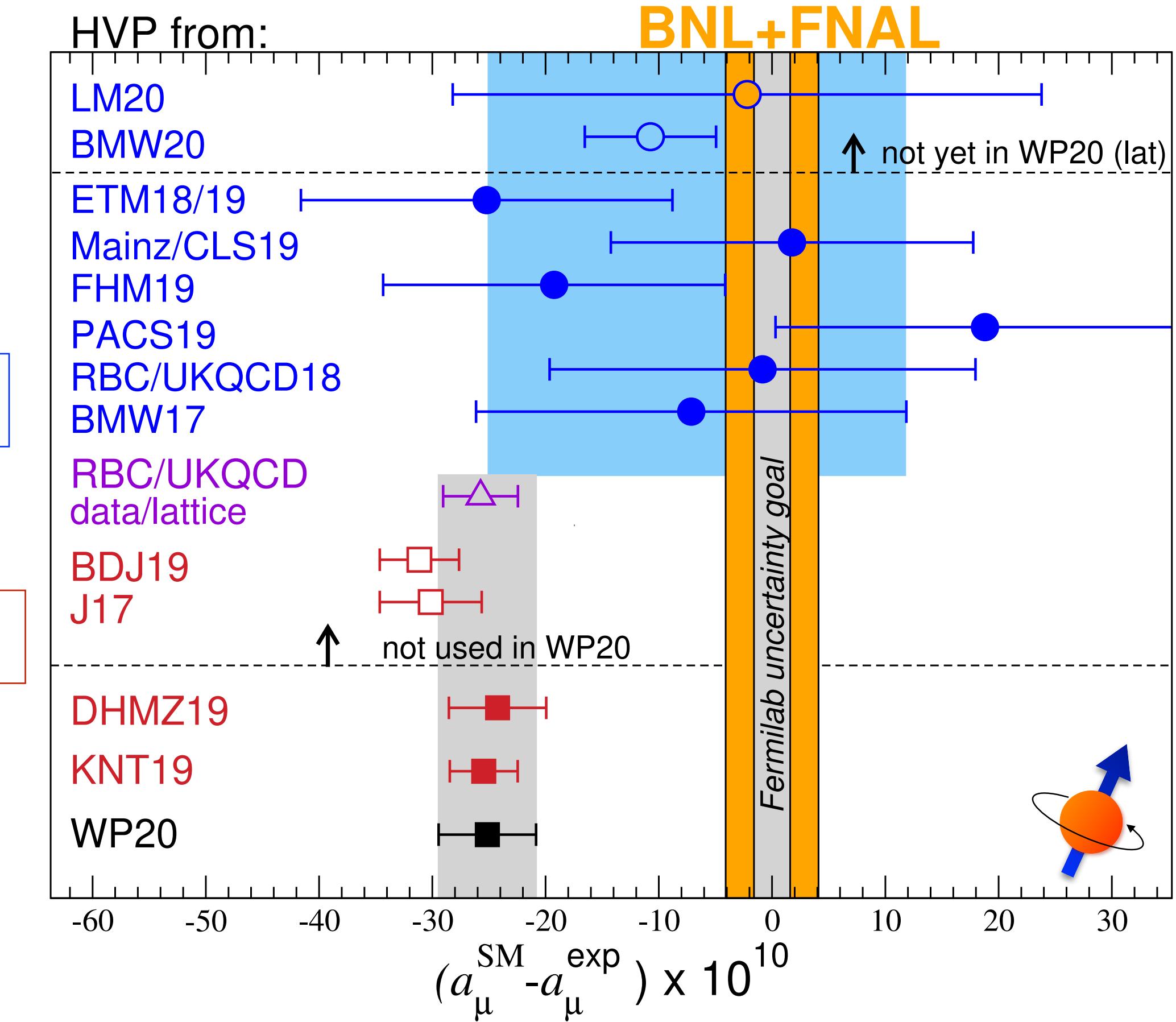
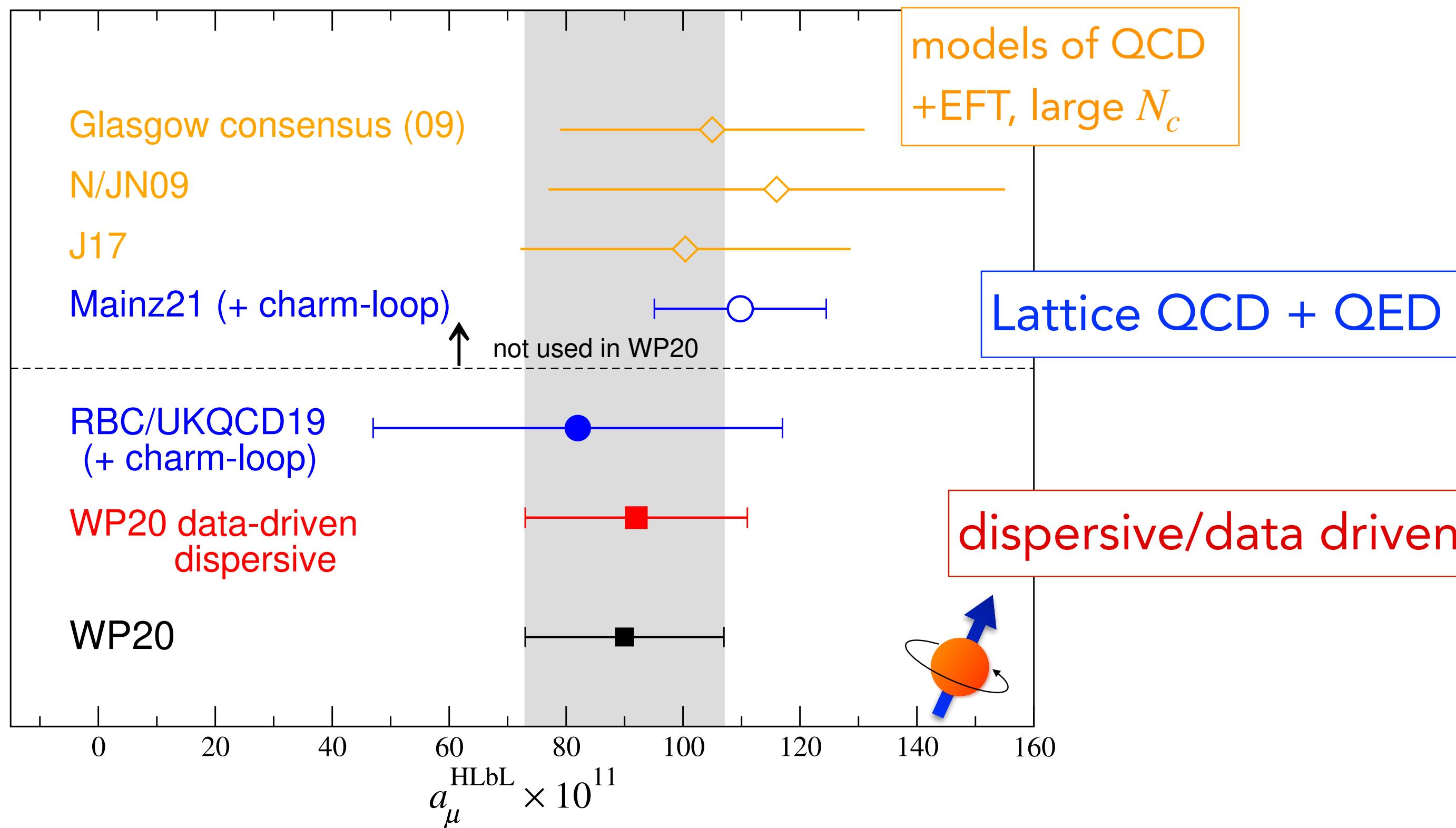
Hadronic Corrections: Comparisons

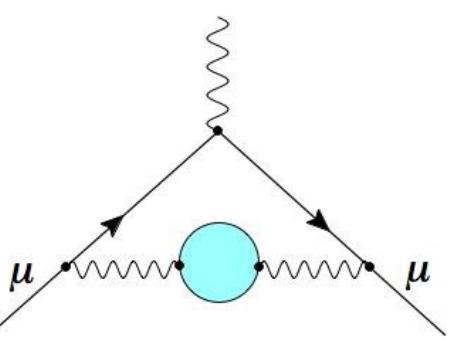


a_μ^{HLbL}

a_μ^{SM}

$$a_\mu^{\text{HVP}} + [a_\mu^{\text{QED}} + a_\mu^{\text{Weak}} + a_\mu^{\text{HLbL}}]$$





HVP: lattice

In 2020 WP:

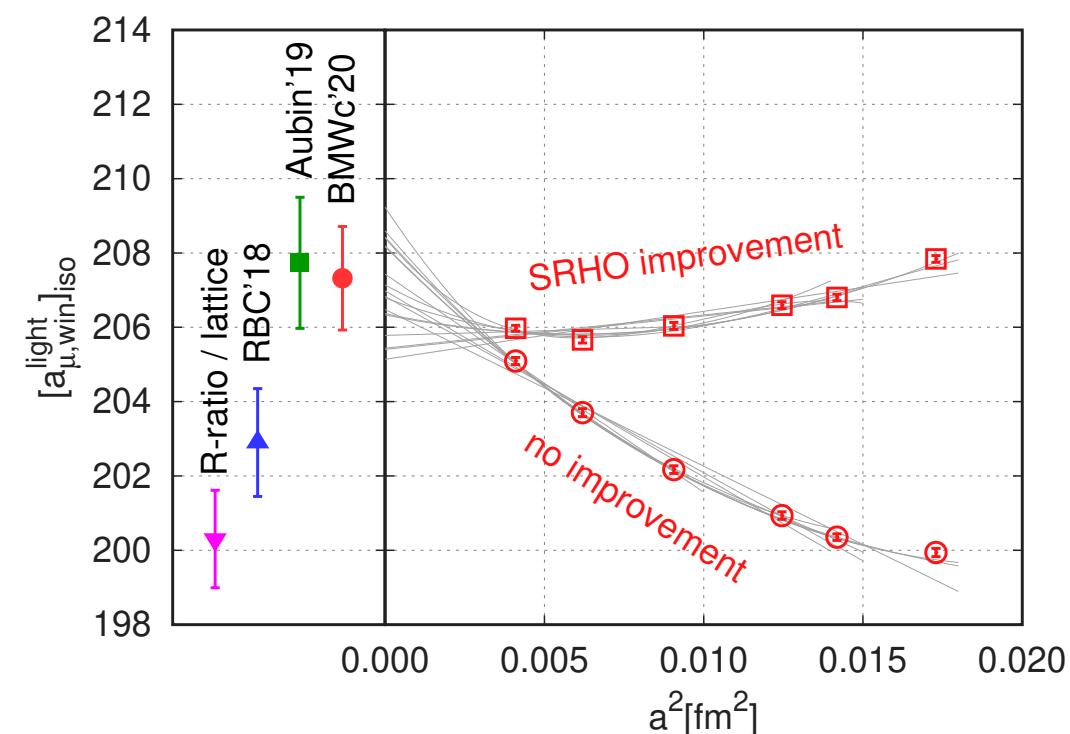
- Lattice HVP average at 2.6 % total uncertainty:

$$a_\mu^{\text{HVP,LO}} = 711.6(18.4) \times 10^{10}$$

- BMW 20 (published in 2021)

first LQCD calculation with sub-percent (0.8 %) error
in tension with data-driven HVP (2.1σ)

- Further tensions for intermediate window:



-3.7σ tension with data-driven evaluation

-2.2σ tension with RBC/UKQCD18

$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt \tilde{w}(t) C(t)$$

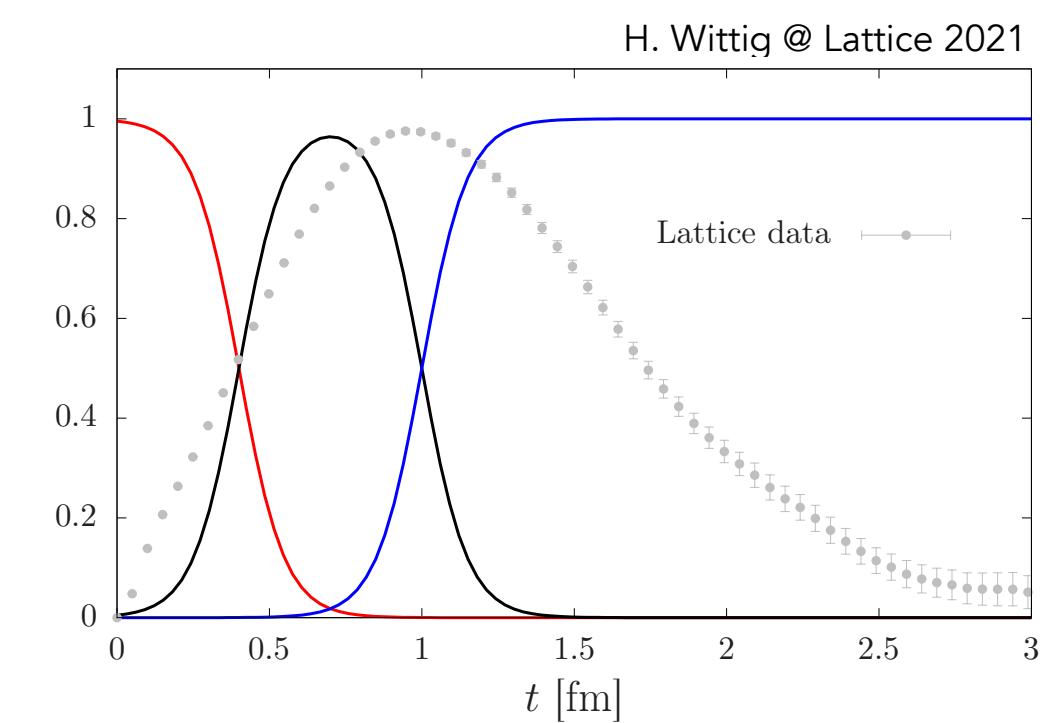
- Use windows in Euclidean time to consider the different time regions separately. [T. Blum et al, arXiv:1801.07224, 2018 PRL]

Short Distance (SD) $t : 0 \rightarrow t_0$

Intermediate (W) $t : t_0 \rightarrow t_1$

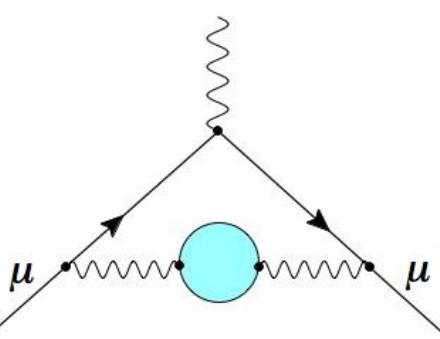
Long Distance (LD) $t : t_1 \rightarrow \infty$

$$t_0 = 0.4 \text{ fm}, t_1 = 1.0 \text{ fm}$$



- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD & using disperse approach:
- Internal cross check:
Compute each window separately (in continuum, infinite volume limits,...) and combine:

$$a_\mu = a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}}$$



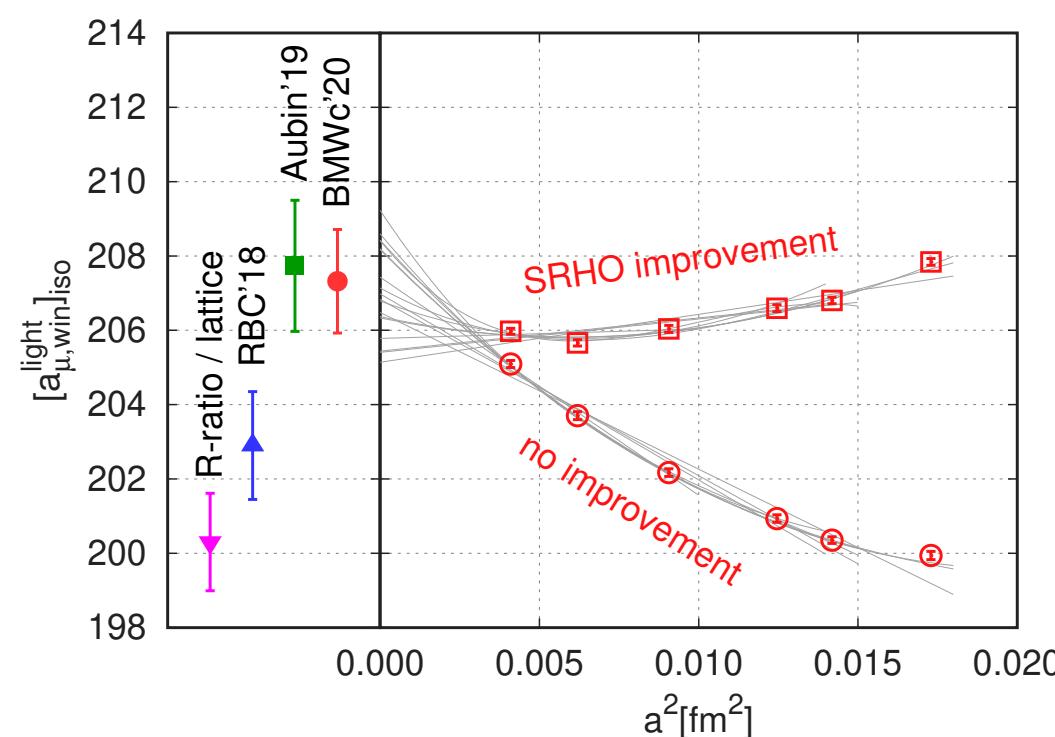
HVP: lattice

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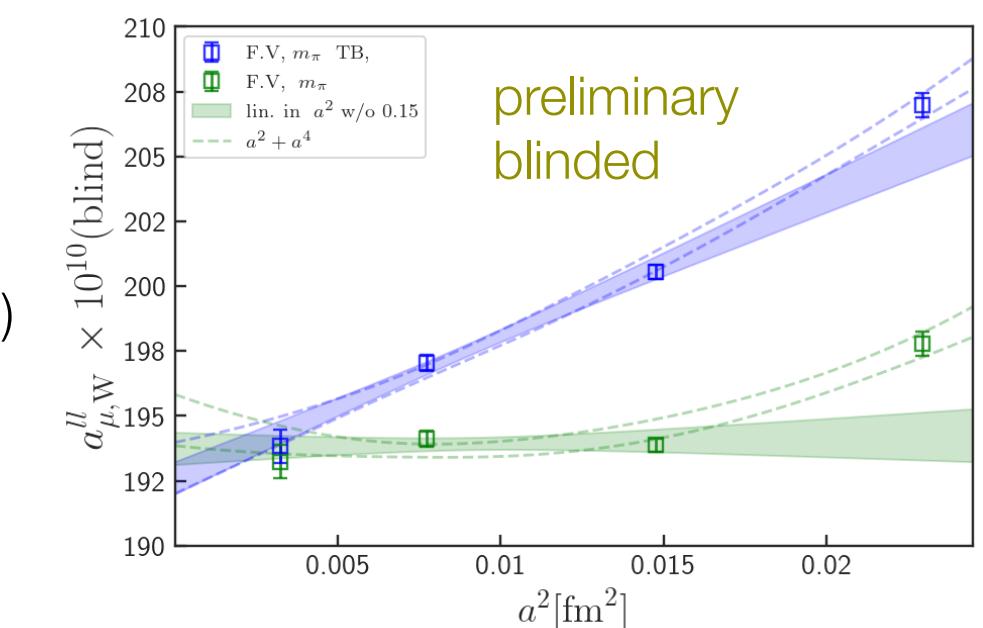


-3.7σ tension with data-driven evaluation
 -2.2σ tension with RBC/UKQCD18

Ongoing work:

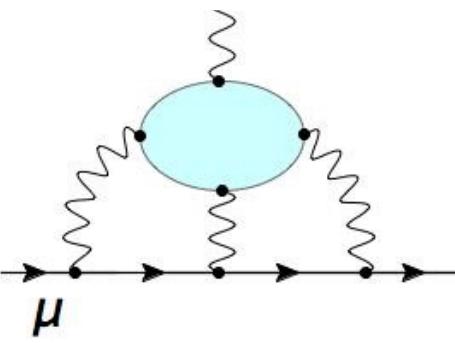
- Expect new results from RBC/UKQCD and FNAL/MILC (and likely other lattice groups) in the coming months:

S. Lahert
(FNAL/MILC)



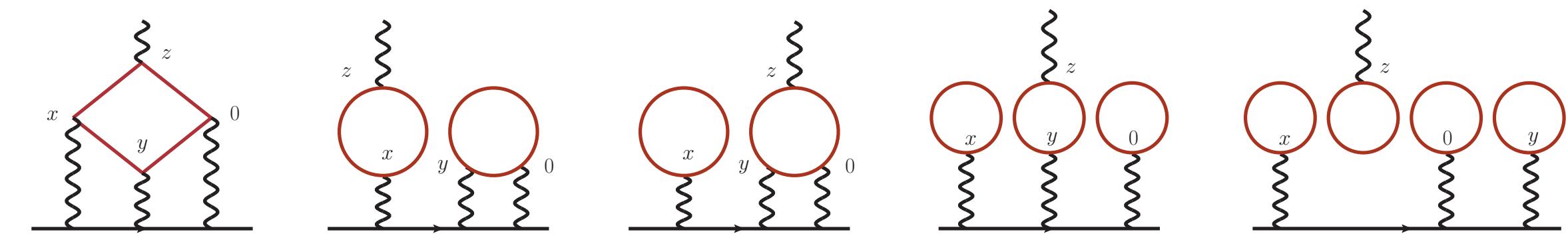
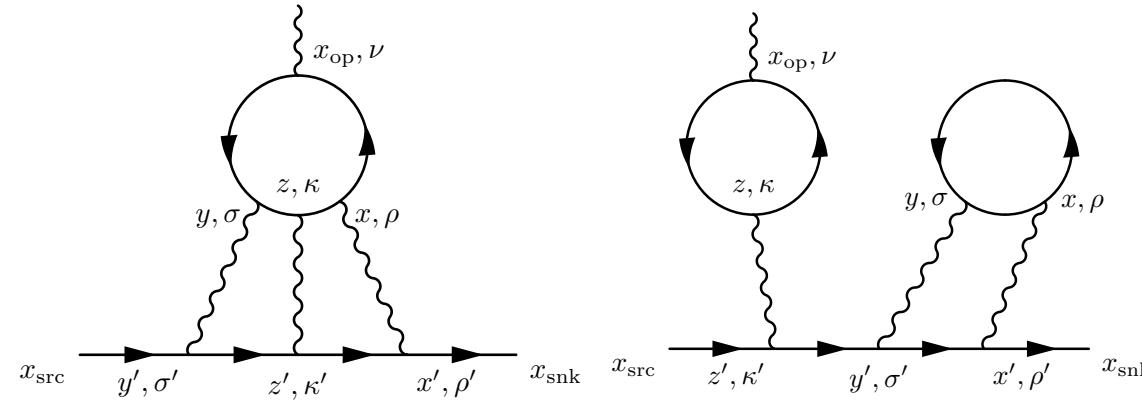
- need blind analyses (already being done in FNAL/MILC and RBC/UKQCD)
- Including $\pi\pi$ states for refined long-distance computation (Mainz, RBC/UKQCD, FNAL/MILC)
- Developing method average for lattice HVP — started at KEK workshop (June 2021), based on detailed comparisons
 - list of sub quantities (and their definitions)
 - common prescription for separating QCD & QED
 - quality criteria for inclusion
- Most groups plan to include smaller lattice spacings to test continuum extrapolations (needs adequate computational resources)

If results are consistent, Lattice HVP (average) with $\lesssim 0.5\%$ errors feasible by 2025



Hadronic Light-by-light

Lattice QCD+QED: Two independent and complete direct calculations of a_μ^{HLbL}



- ◆ RBC/UKQCD

[T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123, 2020 PRL]

- ◆ QCD + QED_L (finite volume)

DWF ensembles at/near phys mass,
 $a \approx 0.08 - 0.2 \text{ fm}$, $L \sim 4.5 - 9.3 \text{ fm}$

- ◆ Mainz group

[E. Chao et al, arXiv:2104.02632]

- ◆ QCD + QED (infinite volume & continuum)

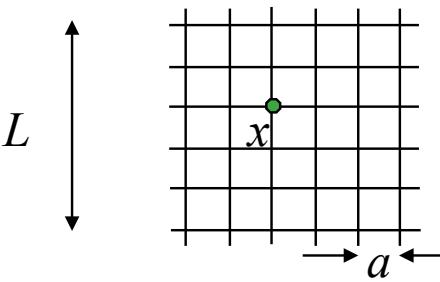
CLS (2+1 Wilson-clover) ensembles

$m_\pi \sim 200 - 430 \text{ MeV}$, $a \approx 0.05 - 0.1 \text{ fm}$, $m_\pi L > 4$

- ◆ Cross checks between RBC/UKQCD & Mainz approaches in White Paper at unphysical pion mass
- ◆ Both groups will continue to improve their calculations, adding more statistics, lattice spacings, physical mass ensemble (Mainz)

Lattice HLbL results with 10 % total uncertainty feasible by ~2025

Summary



The State of the Art

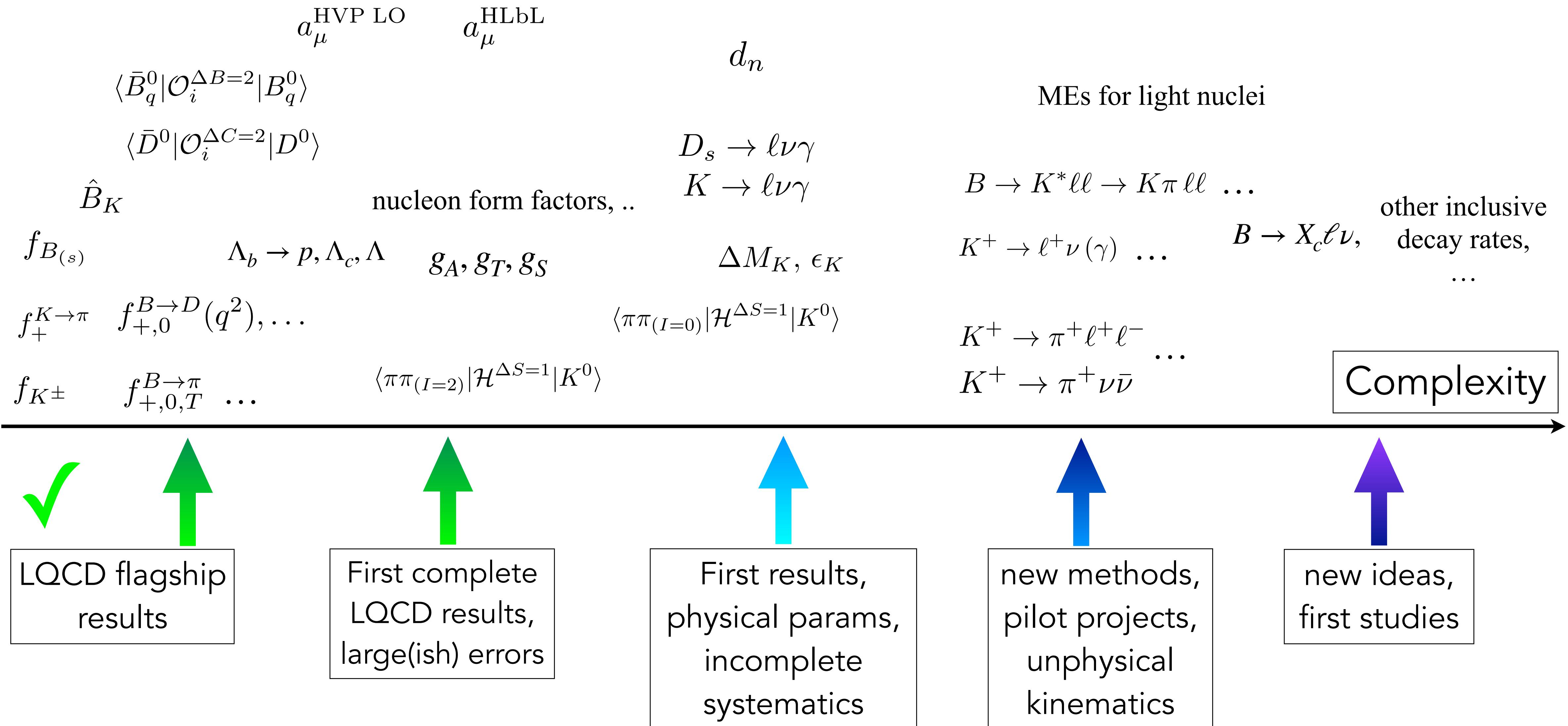
Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,...) in some cases with

- sub percent precision.
- total errors that are commensurate (or smaller) than corresponding experimental uncertainties.

Scope of LQCD calculations is increasing due to continual development of new methods:

- nucleons and other baryons
- nonleptonic decays ($K \rightarrow \pi\pi, \dots$)
- resonances, scattering, long-distance effects, ...
- structure: PDFs, GPDs, ...
- QED effects
- radiative decay rates ...

Summary



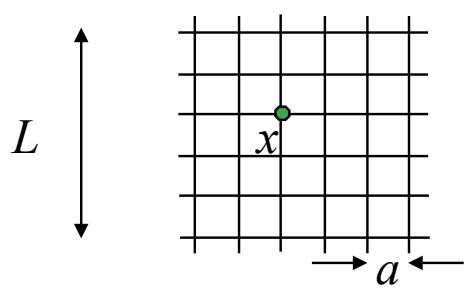
Outlook for 2025+

- ★ Ongoing program to improve precision on “simple quantities”: $\pi, K, D_{(s)}, B_{(s)}$ decay constants, semileptonic form factors (to pseudo scalars), mixing matrix elements ➔ towards the QED wall
 - ➡ talk by Andrew Lytle
 - ★ improved precision for semileptonic $\Lambda_{b,c}$ baryon decay form factors possible (with broader participation)
 - ★ semileptonic form factors for e.g. $B \rightarrow (K^* \rightarrow K\pi) \ell\ell$
 - ➔ treating vector mesons as resonances
 - ★ developing methods to include (radiative) QED effects
 - ➡ talk by Norman Christ
 - ★ studies of rare kaon processes with non-local effects, e.g. $\epsilon'/\epsilon, \Delta m_K, \epsilon, K \rightarrow \pi\ell^+\ell^-$, ...
 - ★ dedicated, continued efforts to improve precision on lattice HVP and HLbL (and related quantities)
 - ➡ talk by Ruth Van de Water
 - ★ nucleon matrix elements (and matrix elements of small nuclei)
 - ★ developing new methods for studies of inclusive B decay rates
 - may be adapted to non-local matrix elements (rare B decays, D mixing, ...)
- ➔ Continued work in Lattice QCD to maximize the discovery potential of experiments within the Rare and Precision Measurements Frontier



Thank you!

Appendix



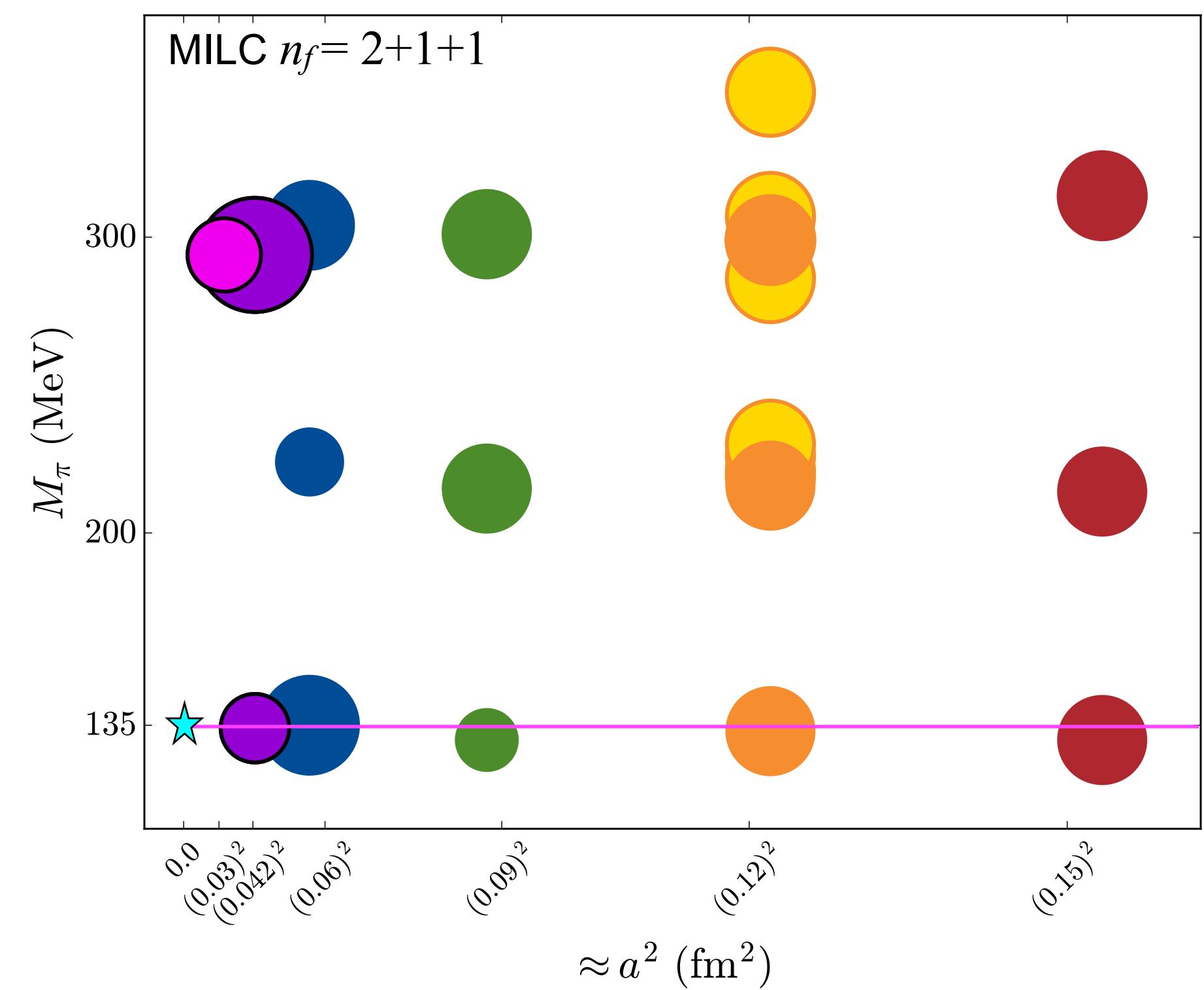
Lattice QCD Introduction

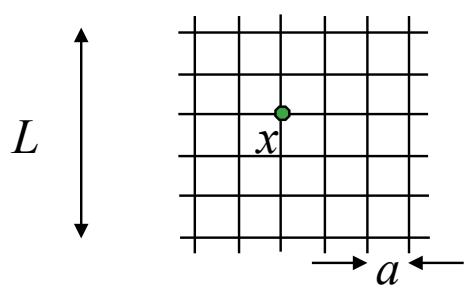
systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on EFT
(Effective Field Theory) descriptions of QCD

→ ab initio

- finite a : Symanzik EFT
- light quark masses: Chiral Perturbation Theory
- heavy quarks: HQET
- finite L : finite volume EFT

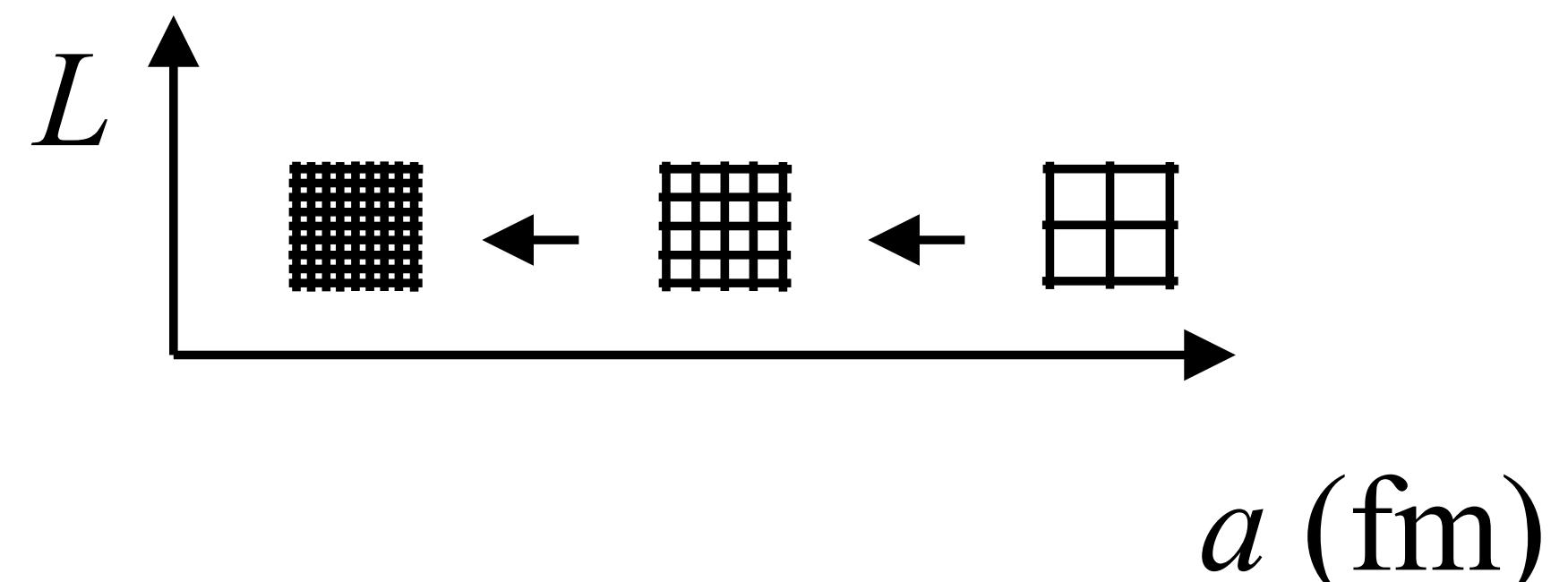


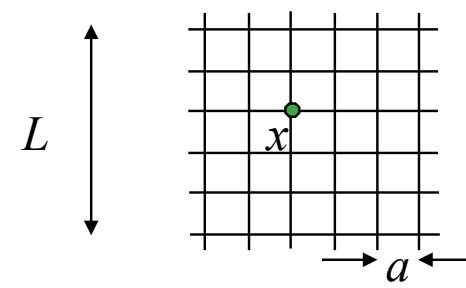


Lattice QCD Introduction

discretization effects — continuum extrapolation

- typical momentum scale of quarks gluons inside hadrons: $\sim \Lambda_{\text{QCD}}$
- make a small to separate the scales: $\Lambda_{\text{QCD}} \ll 1/a$
- Symanzik EFT: $\langle \mathcal{O} \rangle^{\text{lat}} = \langle \mathcal{O} \rangle^{\text{cont}} + O(a\Lambda)^n$, $n \geq 2$
 - provides functional form for extrapolation (depends on the details of the lattice action)
 - can be used to build improved lattice actions
 - can be used to anticipate the size of discretization effects



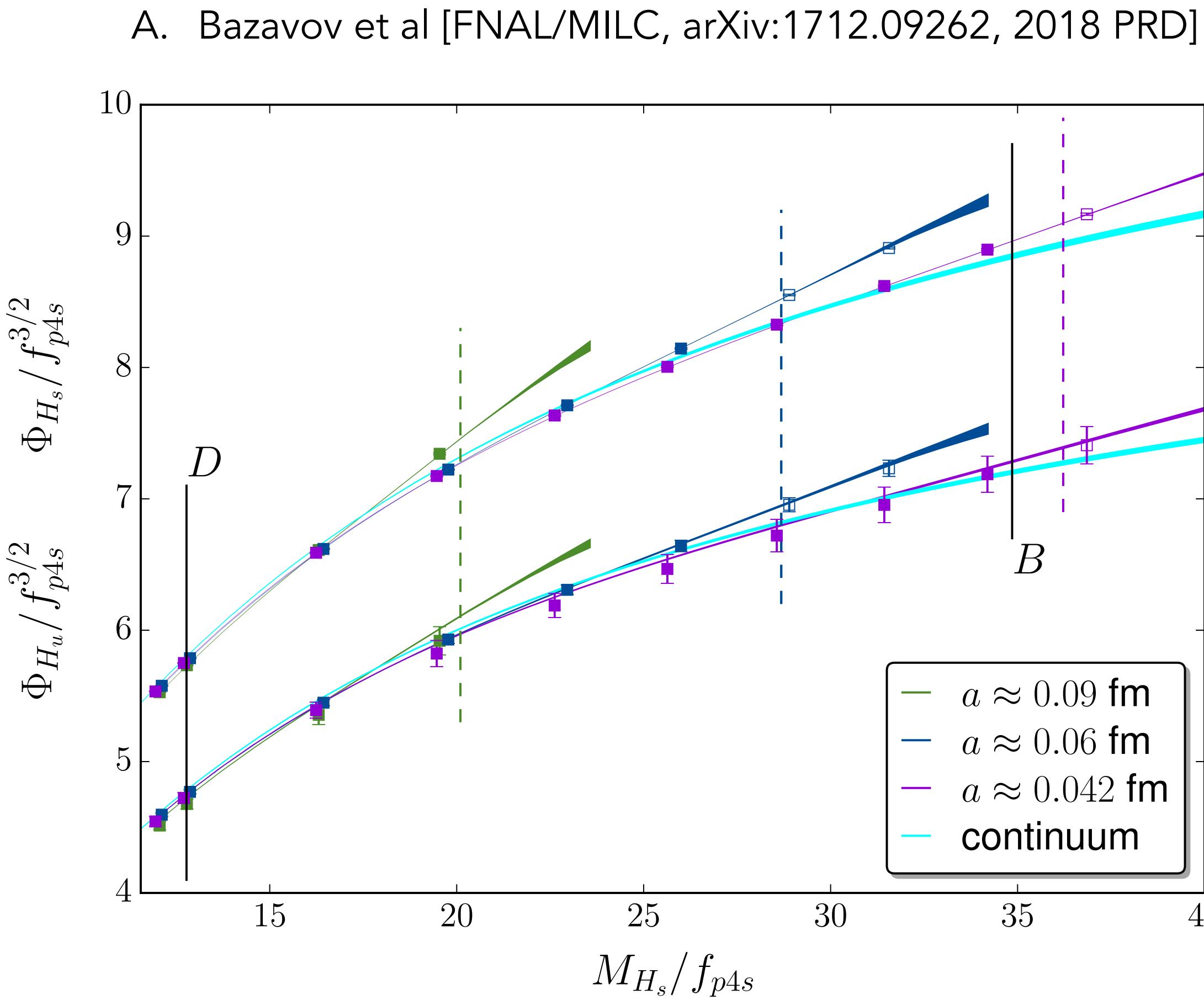
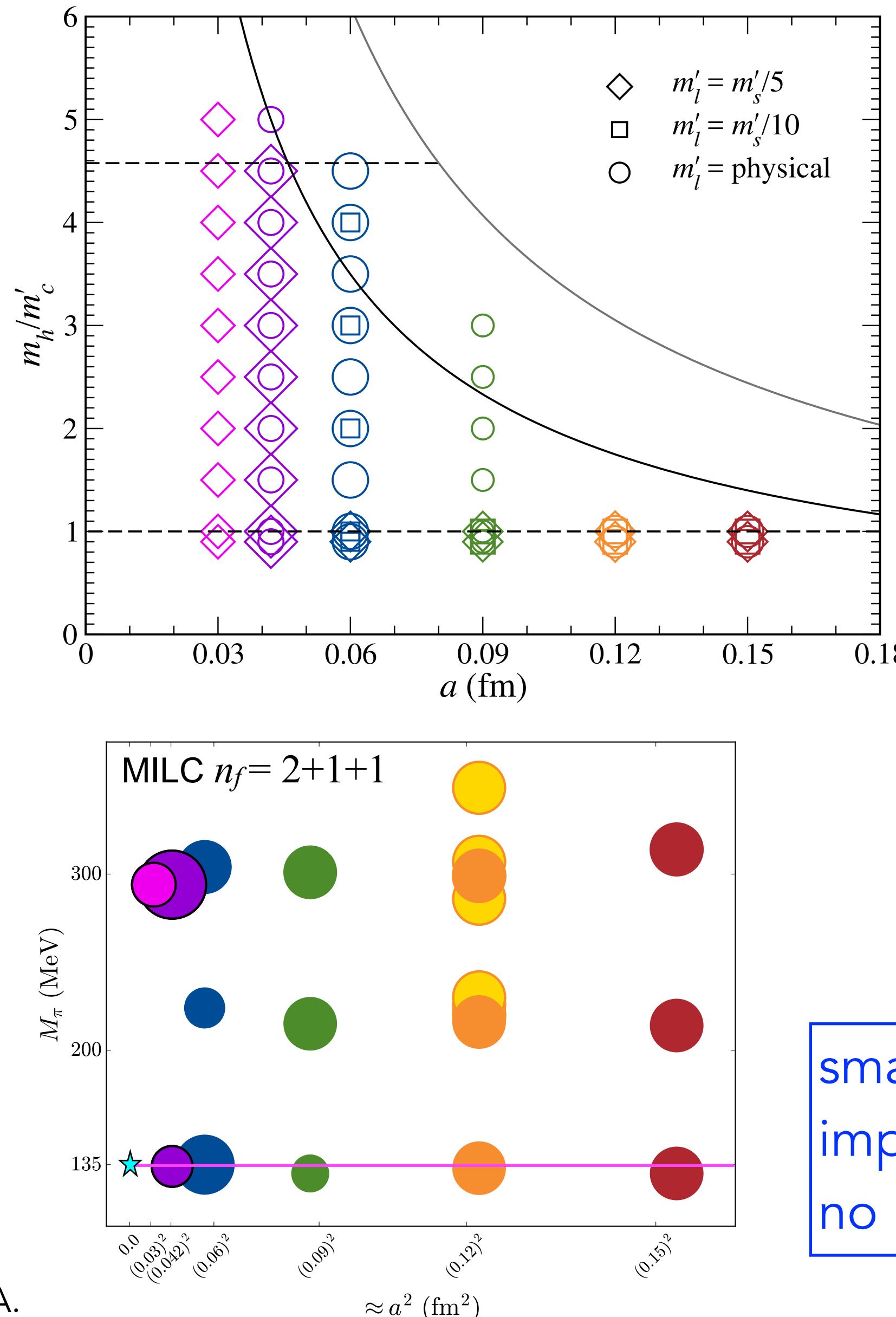


Lattice QCD Introduction: quark discretizations

Fermion doubling problem \Leftrightarrow chiral symmetry

- Staggered quarks (a.k.a Kogut-Susskind)
 - reduce the number of doublers (staggering) but keep some (a.k.a tastes)
 - dominant discretization effects due to taste-breaking effects (can be corrected analytically) $\sim O(a^2)$
 - various improved versions to reduce taste-breaking effects ([HISQ](#),..)
 - computationally inexpensive
- (improved) Wilson quarks
 - no doublers, but chiral symmetry broken explicitly
 - requires improvement to remove $O(a)$ effects (NP improved, twisted mass, ...)
 - moderate computational cost
- Domain wall quarks (live in 5 dimensions)
 - no doublers, chiral symmetry exponentials suppressed
 - small $O(a^2)$ discretization effects
 - high computational cost
- ...

B meson decay constant results



small errors due to **physical light quark masses**
improved quark action with small discretization errors even for heavy quarks
no renormalization (Ward identity)

Connections

$$\sigma(e^+e^- \rightarrow \text{hadrons}) \Leftrightarrow a_\mu^{\text{HVP}} \Leftrightarrow \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \rightarrow \text{hadrons})$, but weighted towards higher energies.

- a shift in a_μ^{HVP} also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: \Rightarrow EW fits
[Passera, et al, 2008, Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]
If the shift in a_μ^{HVP} is in the low-energy region ($\lesssim 1 \text{ GeV}$), the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.

- A shift in a_μ^{HVP} from low ($\lesssim 2 \text{ GeV}$) energies $\Rightarrow \sigma(e^+e^- \rightarrow \pi\pi)$
must satisfy unitarity & analyticity constraints $\Rightarrow F_\pi^V(s)$
can be tested with lattice calculations
[Colangelo, Hoferichter, Stoffer, arXiv:2010.07943]

Connections

$$\sigma(e^+e^- \rightarrow \text{hadrons})$$

$$a_\mu^{\text{HVP}}$$

$$\Delta\alpha_{\text{had}}(M_Z^2)$$

Martin Hoferichter @ Lattice HVP workshop

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[Colangelo, Hoferichter, Stoffer, arXiv:2010.07943]

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) \times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

• Time-like formulation:

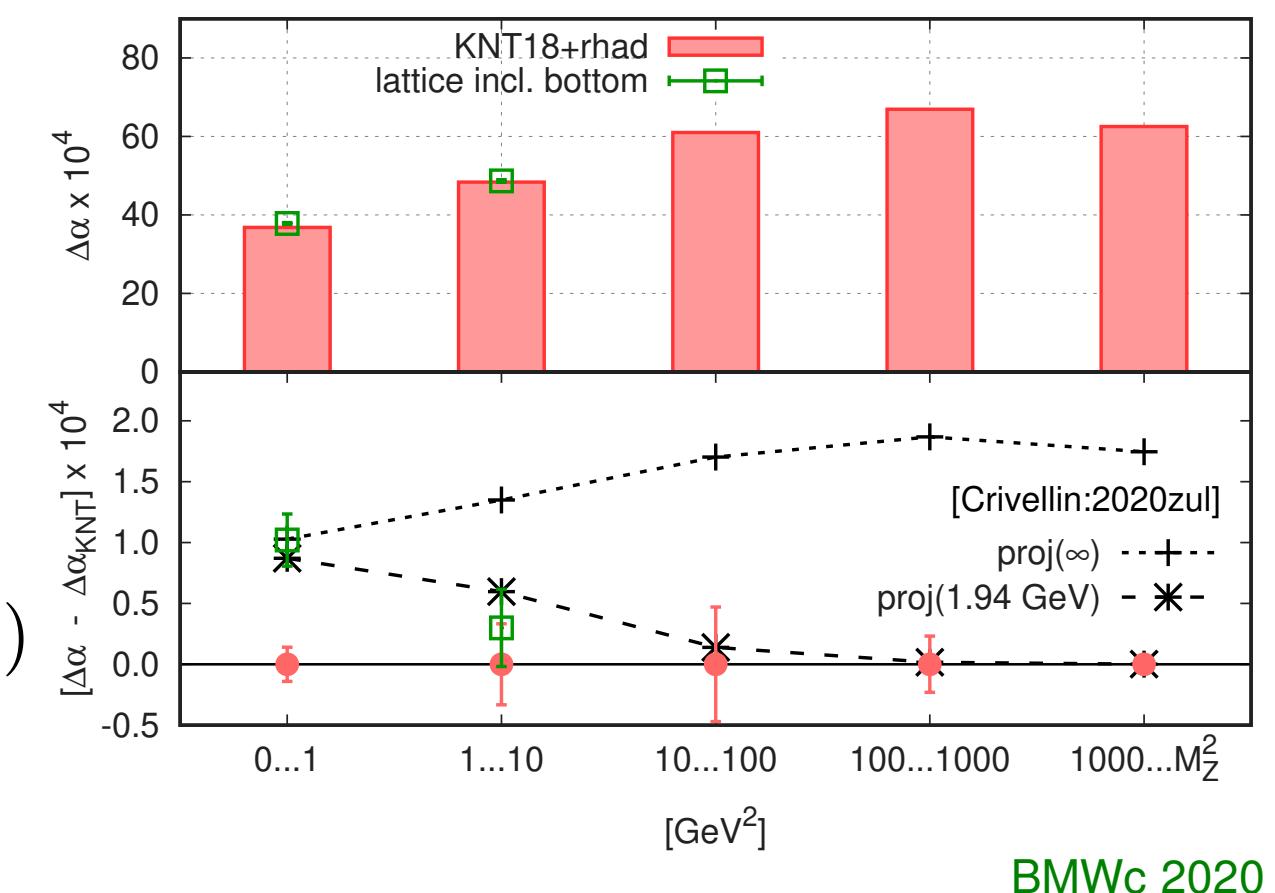
$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

• Space-like formulation:

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{\alpha}{\pi} \hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi} (\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2))$$

• Global EW fit

- Difference between HEPFit and GFitter implementation mainly treatment of M_W
- Pull goes into **opposite direction**



More in talks by M. Passera, B. Malaescu (phenomenology) and K. Miura, T. San José (lattice)

Connections

$$\sigma(e^+e^- \rightarrow \text{hadrons}) \Leftrightarrow a_\mu^{\text{HVP}} \Leftrightarrow \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \rightarrow \text{hadrons})$, but weighted towards higher energies.

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- A shift in a_μ^{HVP} from low ($\lesssim 2 \text{ GeV}$) energies $\Rightarrow \sigma(e^+e^- \rightarrow \pi\pi)$ must satisfy unitarity & analyticity constraints $\Rightarrow F_\pi^V(s)$ can be tested with lattice calculations [Colangelo, Hoferichter, Stoffer, arXiv:2010.07943]

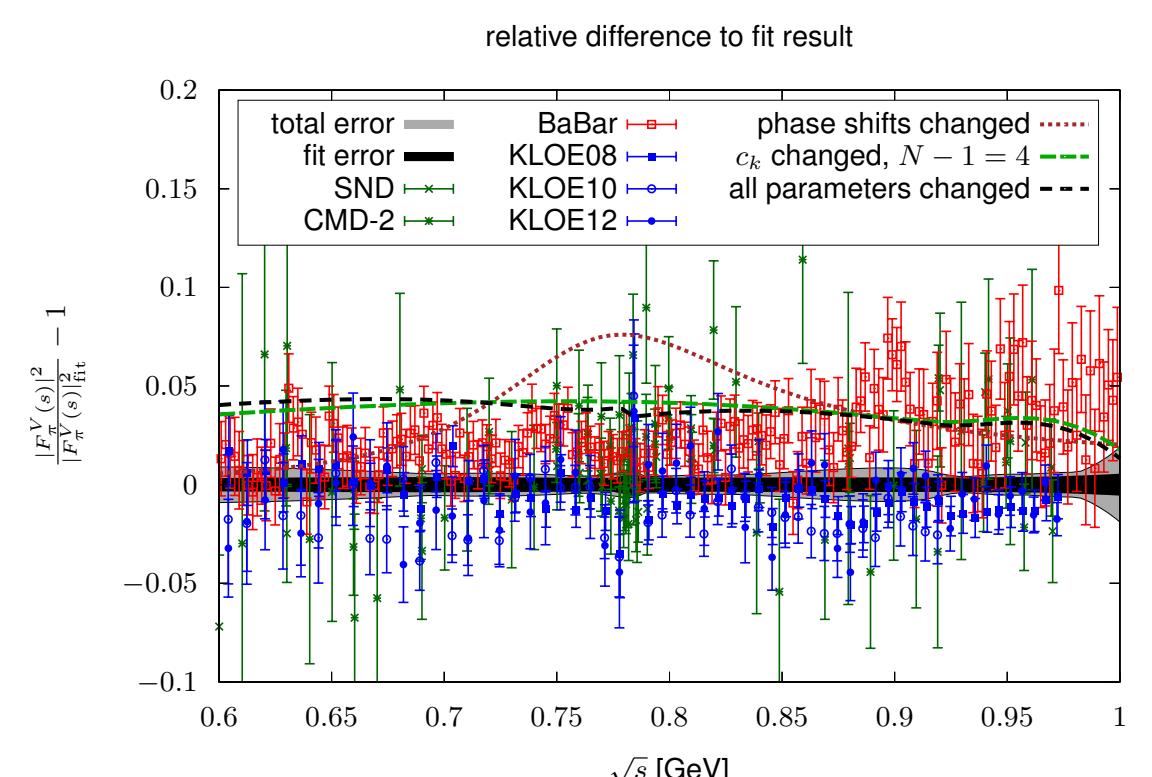
Constraints on the two-pion contribution to HVP

Peter Stoffer @ Lattice HVP workshop

arXiv:2010.07943 [hep-ph]

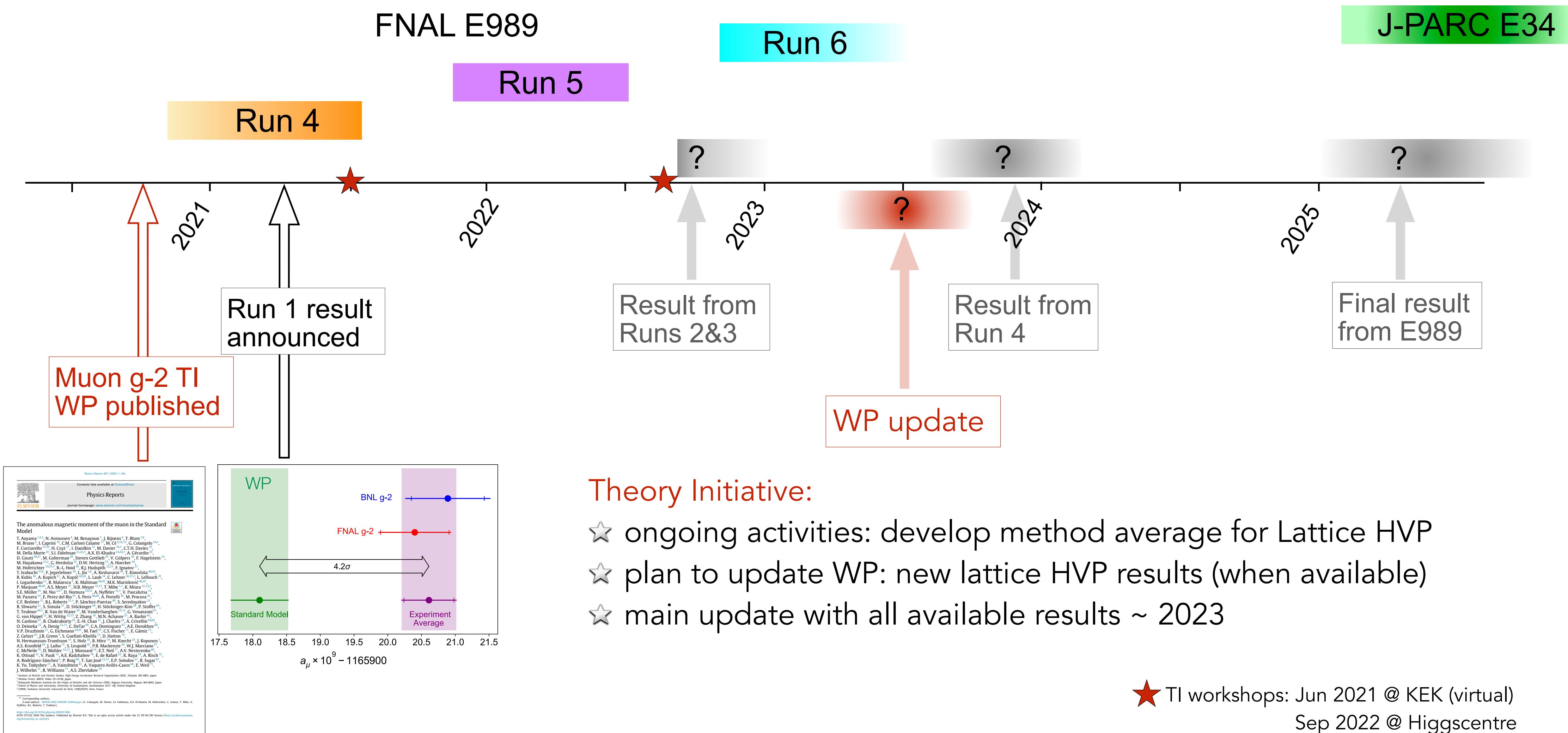
Modifying $a_\mu^{\pi\pi}|_{\leq 1 \text{ GeV}}$

- “low-energy” scenario: local changes in cross section of $\sim 8\%$ **around ρ**
- “high-energy” scenario: impact on **pion charge radius** and space-like VFF \Rightarrow chance for **independent lattice-QCD checks**
- requires **factor ~ 3 improvement** over χQCD result:
 $\langle r_\pi^2 \rangle = 0.433(9)(13) \text{ fm}^2$
 \rightarrow arXiv:2006.05431 [hep-ph]



- Can new physics hide in the low-energy $\sigma(e^+e^- \rightarrow \pi\pi)$ cross section? \Rightarrow No [Luzio, et al, arXiv:2112.08312]

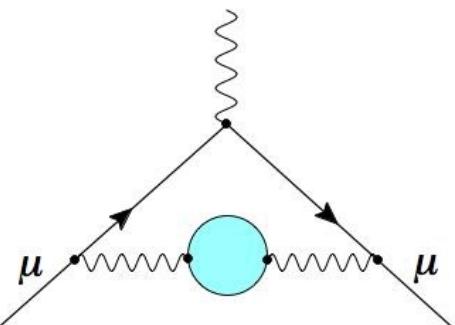
Timeline



Theory Initiative:

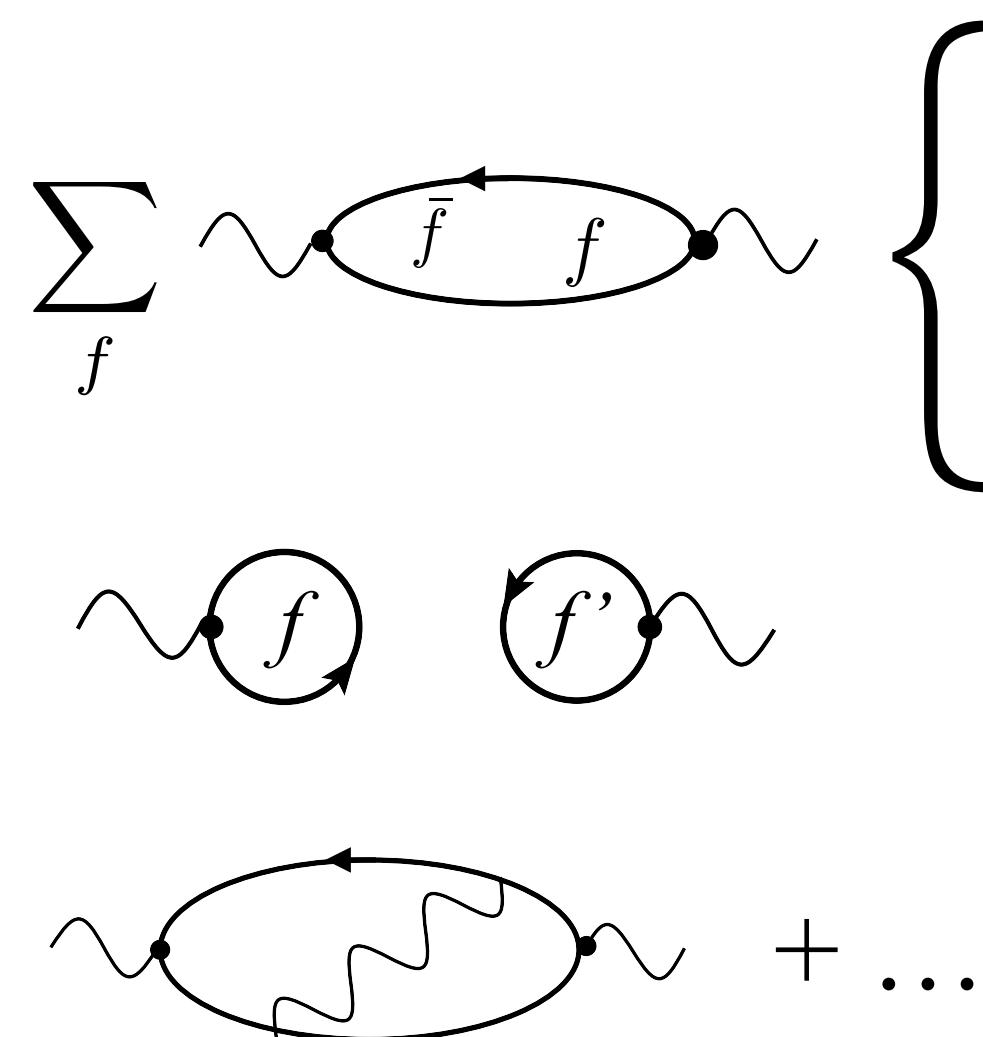
- ★ ongoing activities: develop method average for Lattice HVP
 - ★ plan to update WP: new lattice HVP results (when available)
 - ★ main update with all available results ~ 2023

TI workshops: Jun 2021 @ KEK (virtual) Sep 2022 @ Higgscentre



Lattice HVP: Introduction

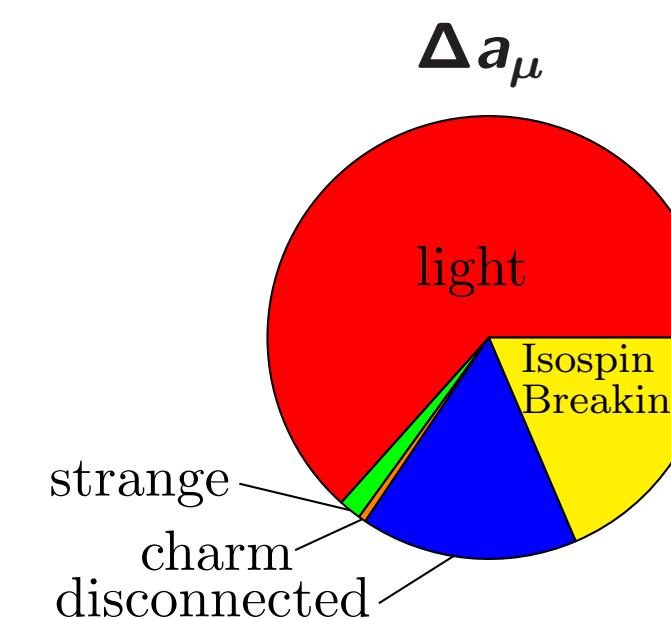
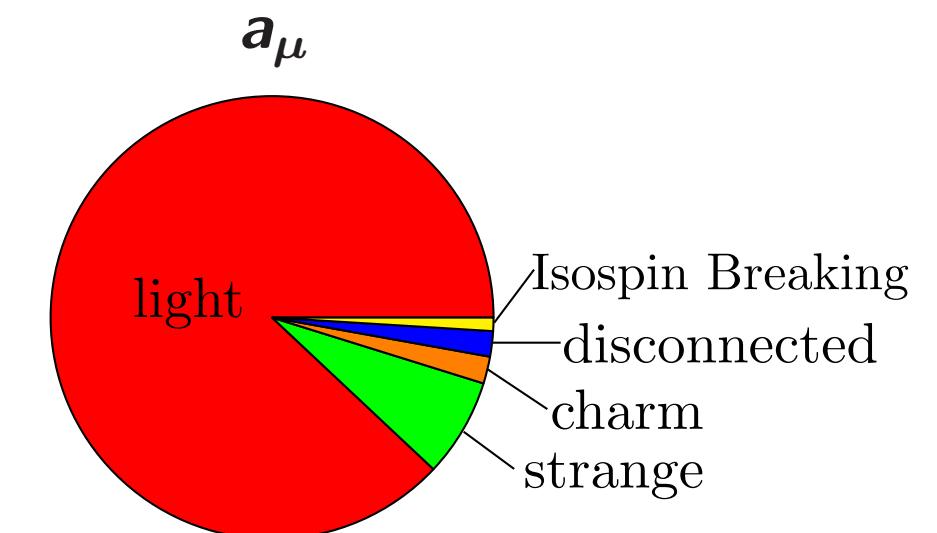
- Target: $\sim 0.2\%$ total error

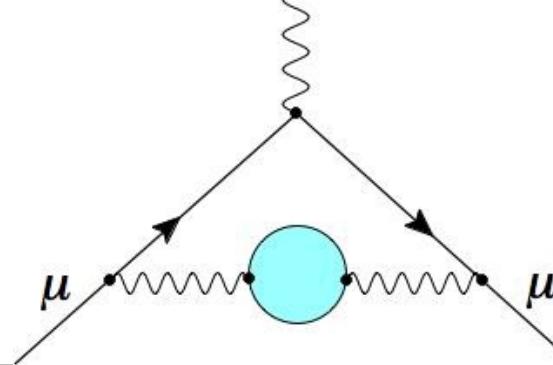


$$a_\mu^{\text{HVP,LO}} = a_\mu^{\text{HVP,LO}}(ud) + a_\mu^{\text{HVP,LO}}(s) + a_\mu^{\text{HVP,LO}}(c) + a_\mu^{\text{HVP,LO}} + \delta a_\mu^{\text{HVP,LO}}$$

- light-quark connected contribution:
 $a_\mu^{\text{HVP,LO}}(ud) \sim 90\%$ of total
- s, c, b -quark contributions
 $a_\mu^{\text{HVP,LO}}(s, c, b) \sim 8\%, 2\%, 0.05\%$ of total
- disconnected contribution:
 $a_{\mu, \text{disc}}^{\text{HVP,LO}} \sim 2\%$ of total
- Isospin breaking (QED + $m_u \neq m_d$) corrections:
 $\delta a_\mu^{\text{HVP,LO}} \sim 1\%$ of total

V. GÜLPERS @ Lattice HVP workshop

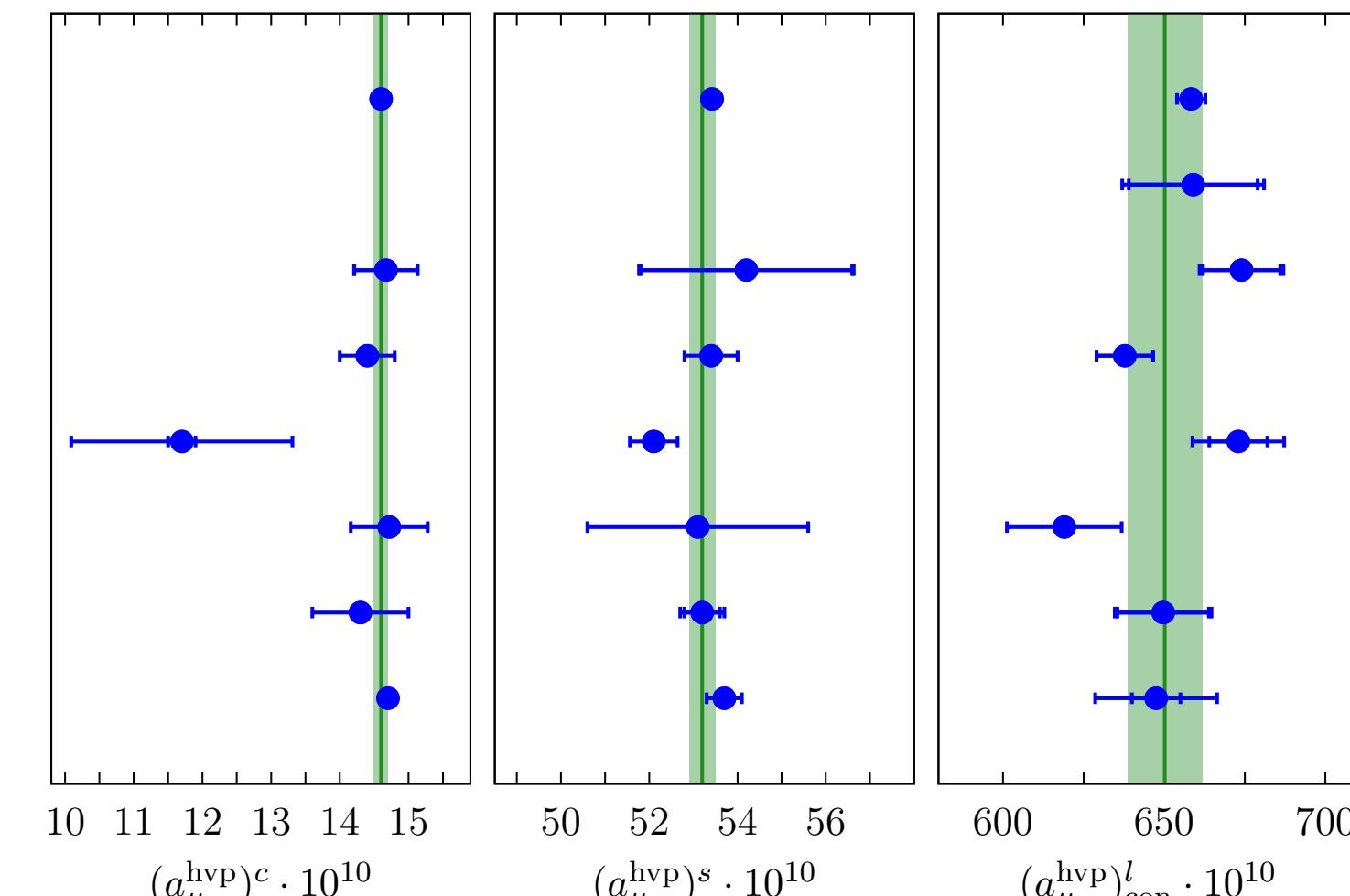




Lattice HVP: comparisons

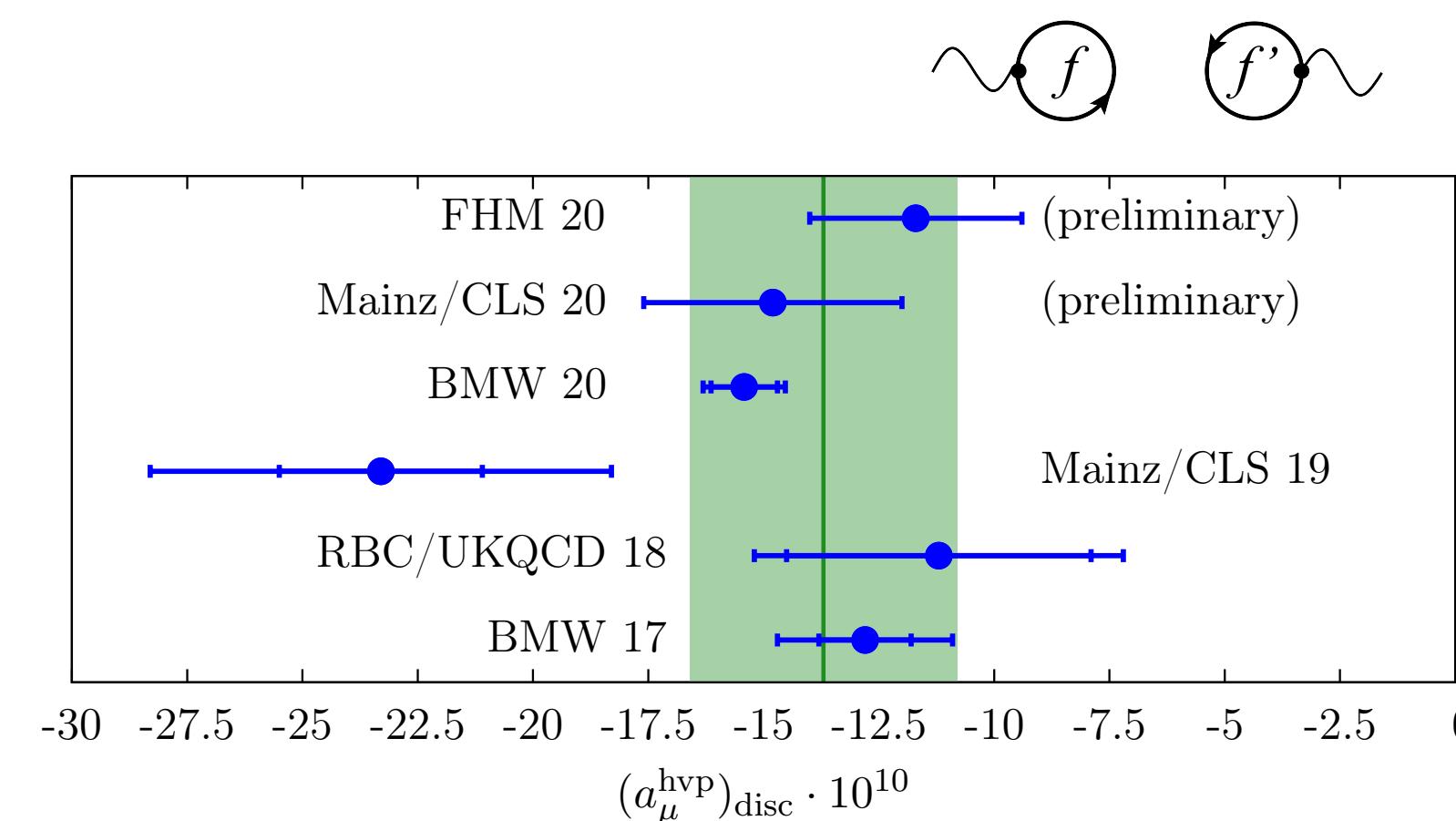
H. Wittig @ Lattice HVP workshop

- Charm, strange contributions already well determined.
- Mild tensions for light contribution



Ongoing efforts by
FNAL-HPQCD-MILC
RBC/UKQCD, Mainz

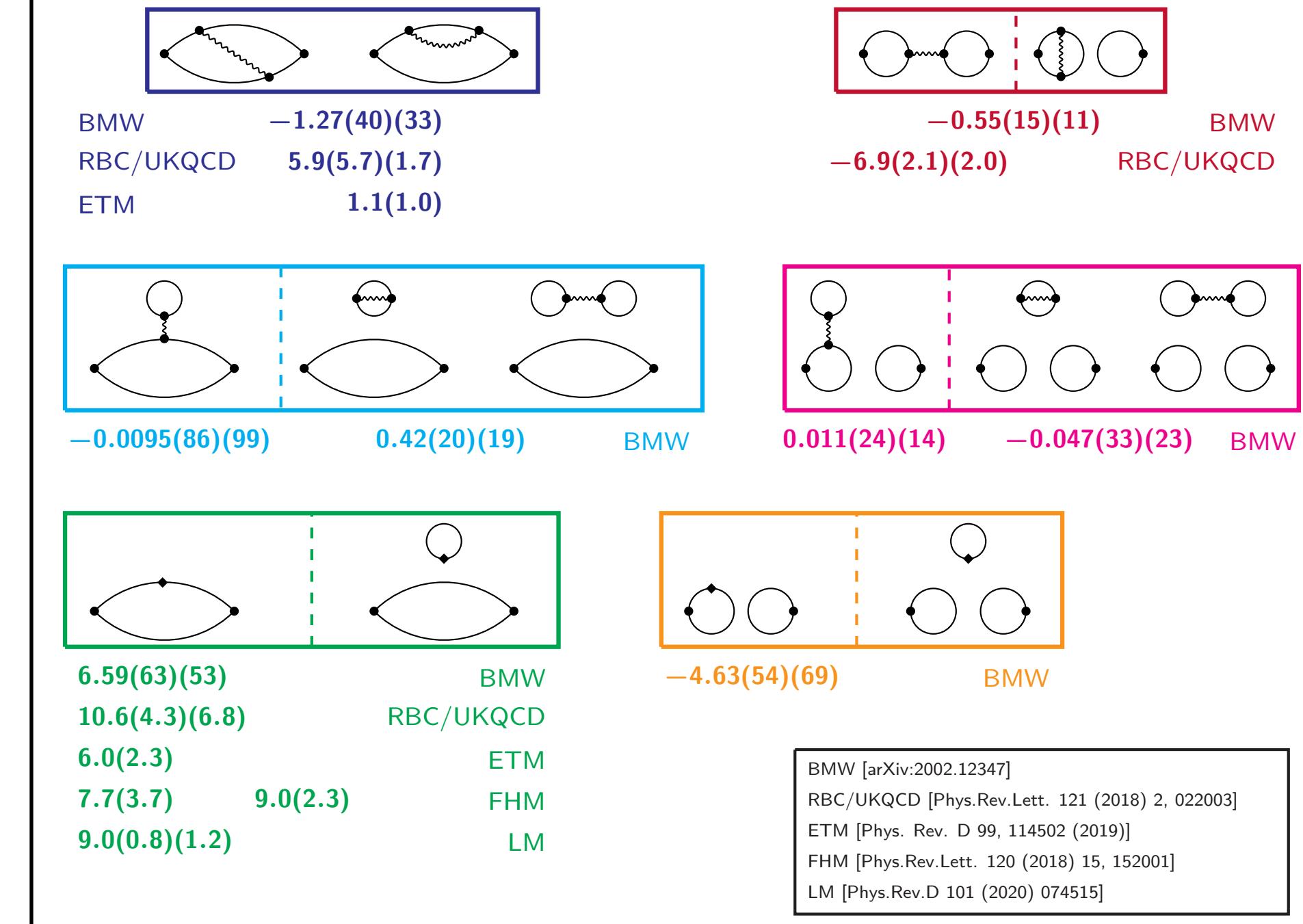
Consistent results with
increasing precision



$$\sum_f \sim \bar{f} f$$

V. Gulpers @ Lattice HVP workshop

Overview of published results - contributions to $a_\mu \times 10^{10}$



- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions:
 $\delta a_\mu^{\text{IB}} \lesssim 1\%$