Discovering new physics in rare kaon decays

Snowmass Rare Processes and Precision Measurements Frontier Spring Meeting

University of Cincinnati

May 16-19, 2022

Snowmass 2021

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RBC and UKQCD Collaborations

RF2 White paper:
Discovering new physics in rare kaon decays
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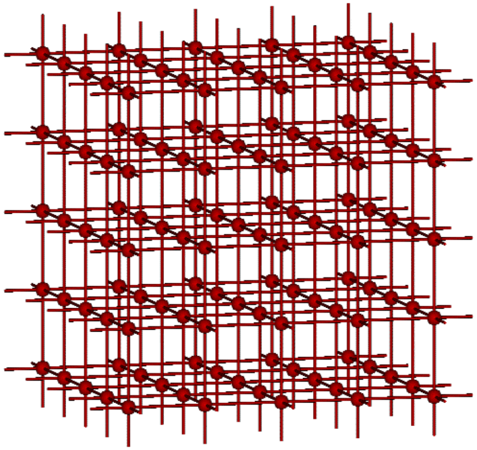
Outline

- Lattice QCD: methods and status
- Sensitive tests of the Standard Model for rare or high precision processes:
  1) $K \rightarrow \pi\pi$ decay and direct CP: ε'
  2) $K_L - K_S$ mass difference
  3) Two photon contribution to the rare kaon decay: $K_L \rightarrow \mu^+\mu^-$
  4) V_{us} from Kμ2 including E&M

State-of-the-art Lattice QCD

Lattice QCD

- Introduce a space-time lattice.
- Evaluate the Euclidean Feynman path integral
  - Study $e^{-H_{QCD}\,t}$
  - Precise non-perturbative formulation
  - Permits numerical evaluation



$$\sum_n \langle n| e^{-H(T-t)} \mathcal{O} e^{-Ht} |n\rangle = \int d[U_\mu(n)] e^{-\mathcal{A}[U]} \det(D+m)\mathcal{O}[U](t)$$

- 96^3 x 192, 1/a=2.8 GeV
  - 5 x 10^9 variables
  - 10^8 x 10^8 determinant

Lattice QCD – 2022

- Physical quark masses (ChPT not needed)
- Chiral quarks (doubling problem solved)
- Large physical volumes: (6 -10 fm)^3
- Small lattice spacing: 1/*a* = 2.77 GeV
  - $(\Lambda_{QCD}\, a)^2$ effects < 1% ☺
  - $(m_{charm}\, a)^2$ effects ~ 20% ☹

$K \to \pi\pi$ Decay

$K \to \pi\,\pi$ and CP violation

- Final $\pi\pi$ states can have I = 0 or 2.

$$\langle \pi\,\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2} \qquad \Delta I = 3/2$$
$$\langle \pi\,\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \qquad \Delta I = 1/2$$

- CP symmetry requires A_0 and A_2 be real.
- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i\,e^{\delta_2-\delta_0}}{\sqrt{2}}\left|\frac{A_2}{A_0}\right|\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}-\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

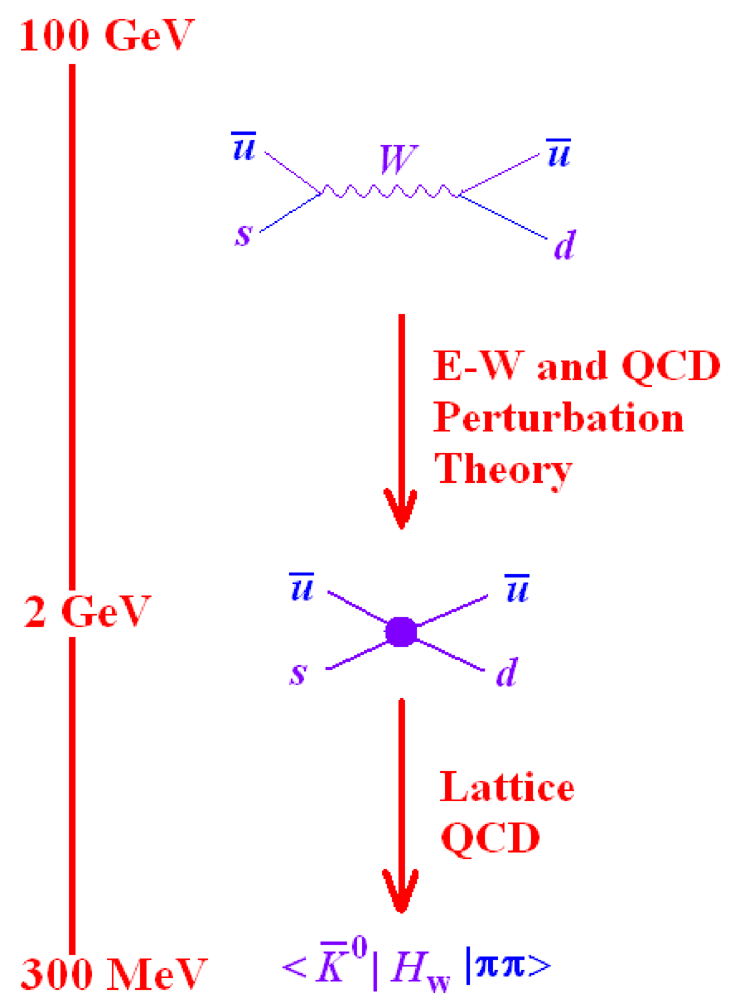
Direct CP violation

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

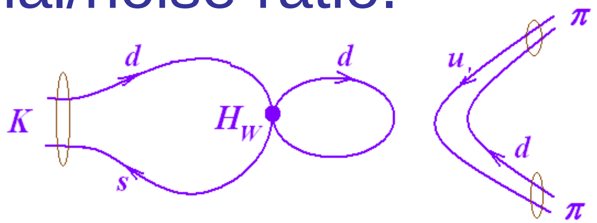
$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i \right\}$$

- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = (1.543 + 0.635i) \times 10^{-3}$
- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



Challenging calculation

- Presence of vacuum implies exponentially falling signal/noise ratio.



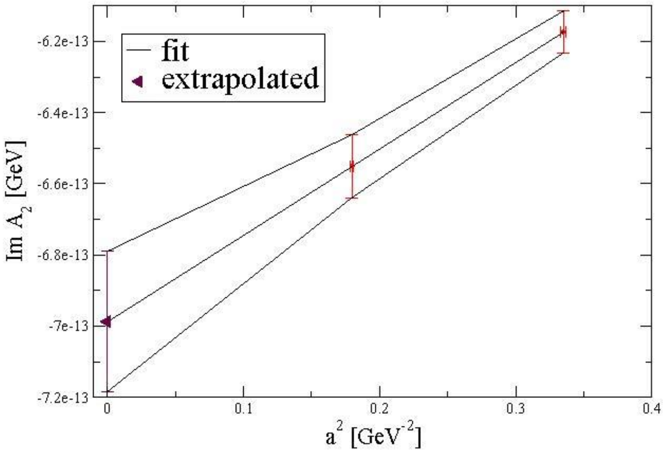
- Two-pion final state was a new challenge.
- We first attempted this calculation in 1997:
- Seven generations of graduate students:
  - Calin Christian (2002)
  - Changhoan Kim (2004)
  - Sam Li (2008)
  - Matthew Lightman (2011)
  - Qi Liu (2012)
  - Daiqian Zhang (2015)
  - Tianle Wang (2021)

Calculation of A_2

ΔI = 3/2 – Continuum Results

(M. Lightman, E. Goode T. Janowski)

- Use two large ensembles to remove a^2 error (m_p=135 MeV, L=5.4 fm)
  - 48^3 x 96, 1/a=1.73 GeV
  - 64^3 x 128, 1/a=2.28 GeV



- Continuum results:
  - Re(A_2) = 1.50(0.04$_{stat}$) (0.14)$_{syst}$×10^{-8} GeV
  - Im(A_2) = - 6.99(0.20)$_{stat}$ (0.84)$_{syst}$×10^{-13} GeV
- Experiment: Re(A_2) = 1.479(4) 10^{-8} GeV
- $E_{\pi\pi}$ → δ_2 = −11.6(2.5)(1.2)^o
- [Phys.Rev. **D91**, 074502 (2015)]

Calculation of A_0 and ε'

Overview of calculation

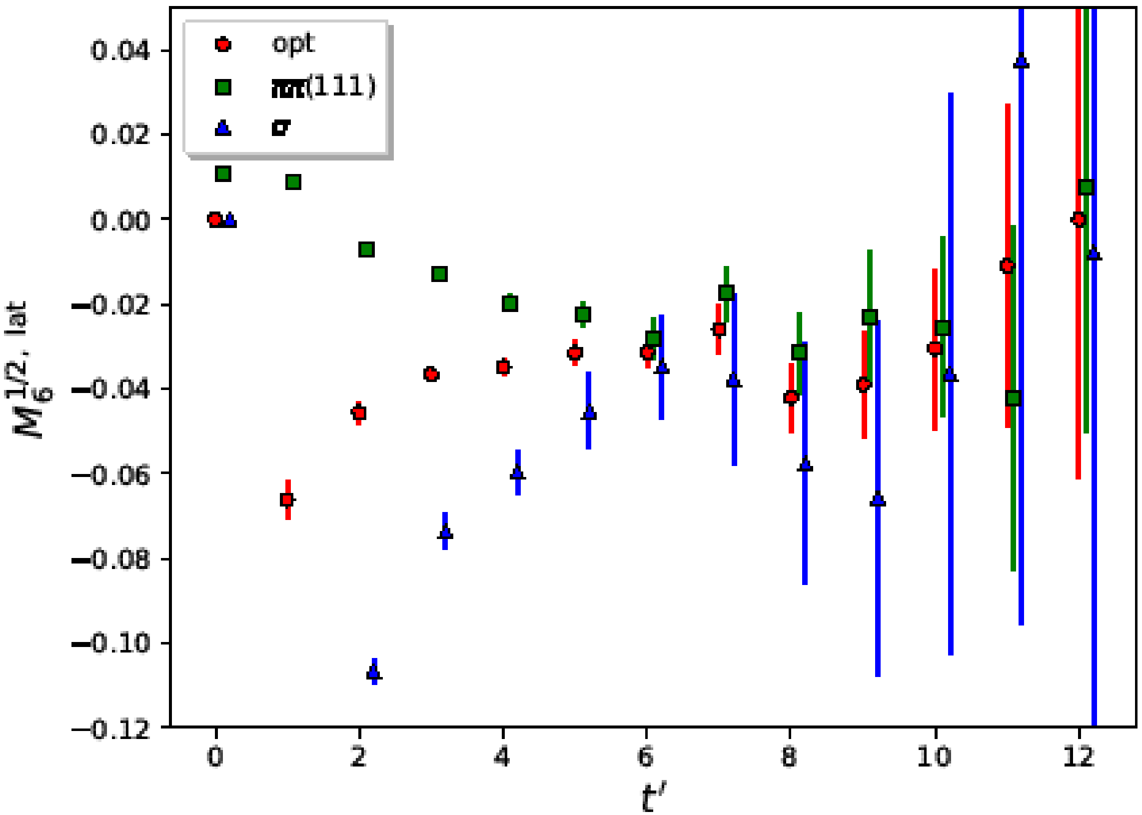
- Use a single lattice spacing $1/a$=1.38 GeV
- 2015 calculation [Phys. Rev. Lett. 115 (2015) 212001]:
  - 216 configurations, single $\pi\pi$ interpolating operator
  - $I = 0$ $\pi\pi$ phase shift: $\delta_0 = 23.8(4.9)(2.2)^o$
  - $\mathrm{Re}(\varepsilon'/\varepsilon) = (1.38 \pm 5.15_{stat} \pm 4.59_{sys}) \times 10^{-4}$
- 2020 calculation [Phys. Rev. D 102 (2020) 054509]:
  - 741 configurations, three $\pi\pi$ interpolating operators
  - ΔI=1/2 rule Re(A0)/Re(A2) = 19.9(2.3)(4.4)

    Expt: = 22.45(6)
  - $I = 0$ $\pi\pi$ phase shift: $\delta_0 = 32.3(1.0)(1.8)^o$
  - $\mathrm{Re}(\varepsilon'/\varepsilon) = (21.7 \pm 2.6_{stat} \pm 6.2_{sys} \pm 5.0_{isospin}) \times 10^{-4}$

    Expt: = 16.6(2.3) x 10^{-4}

Example of $K \to \pi\pi$ data

- Examine dependence on $\pi\pi$ H_W separation
  plot $\langle \pi\pi(t_{\pi\pi})\ H_W\ (t_{op})\ K(t_K)\ \rangle$ versus $t' = t_{\pi\pi}-t_{op}$



$t_{op} - t_K \geq 6$

Accuracy forecast

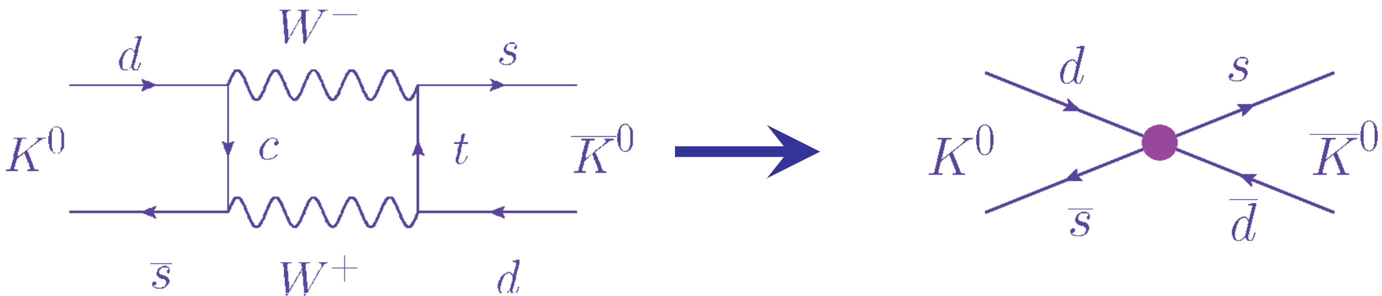
- Continuum limit [12%]:
  - Extend 1/a=1.38 GeV to 1.7 and 2.0 GeV on Perlmutter
- OPE, include active charm [12%]:
  - Raise perturbative scale and remove higher-dim operators
  - Future project on Aurora
- Isospin breaking (E&M + m_u-m_d) [23%]
  - Best estimate from ChPT and large *N*: [V. Cirigliano, et al. JHEP 02 (2020) 032 (1911.01359)]
  - Active target for lattice QCD but some unsolved problems
- Use periodic boundary conditions
  - Independent test of G-parity results
  - Necessary for E&M
- Expect 10% results in the next decade

$K^0 - \bar{K}^0$ mixing
ΔM_K & ε_K

$K^0 - \overline{K^0}$ Mixing

- CP violating: $p \sim m_t$

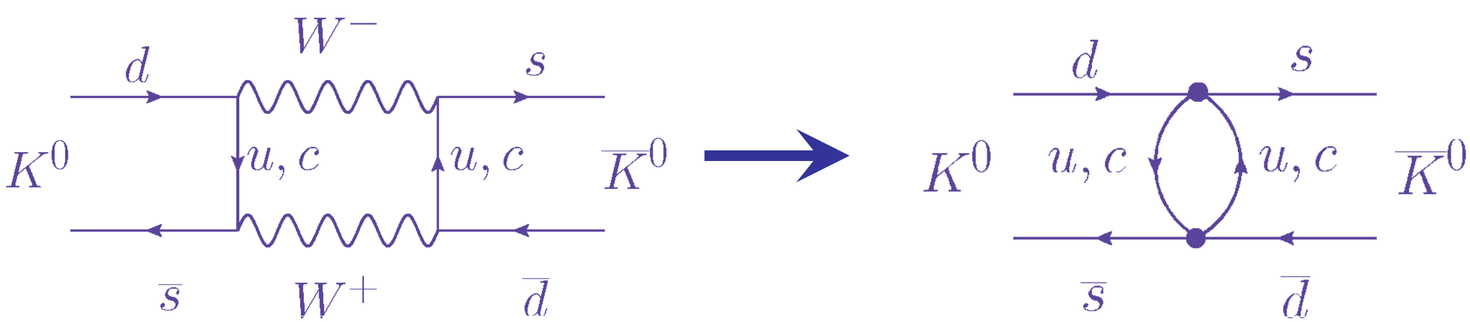
$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im}M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\Gamma_{0\bar{0}}}{\mathrm{Re}M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\Gamma_{0\bar{0}}}\right\} + i\,\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}$$



Compute ~6% LD, reduce uncertainty to 1%

- CP conserving: $p \leq m_c$

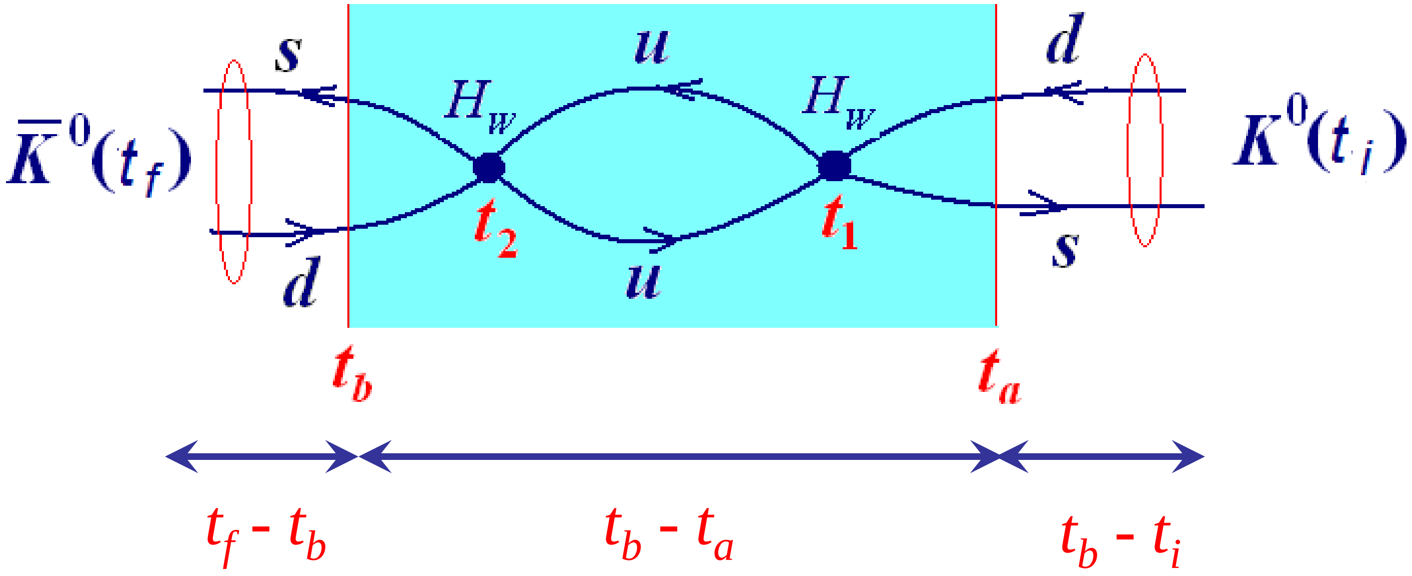
$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\bar{0}}\}$$



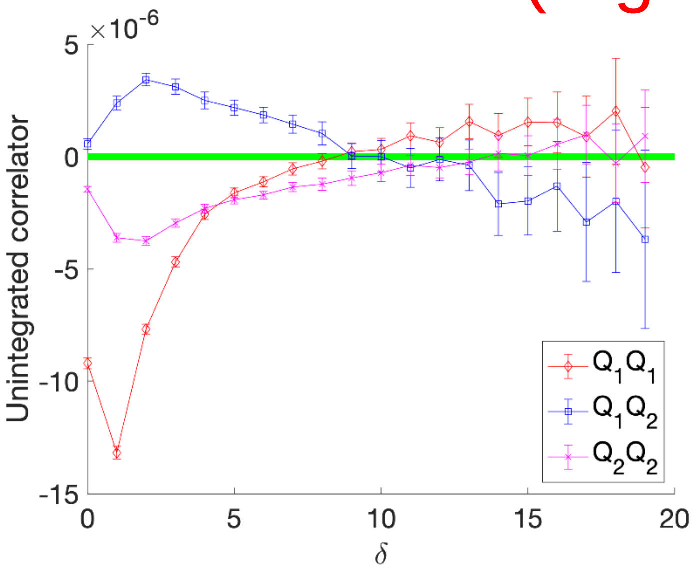
Lattice Version

- Evaluate standard, Euclidean, 2nd order $\overline{K}^0 - K^0$ amplitude:

$$\mathcal{A} = \langle 0|T\left(K^0(t_f)\frac{1}{2}\int_{t_a}^{t_b} dt_2 \int_{t_a}^{t_b} dt_1 H_W(t_2) H_W(t_1) K^{0'}(t_i)\right)|0\rangle$$



ΔM_K Preliminary Results
(Bigeng Wang)



	ΔM_K x 10^{+12} MeV
Types 1-2	6.24(0.24)
Types 3-4	0.33(50)
ΔM_K	5.8(0.6)(2.3)
Expt.	3.483(6)

- $\langle K(t_f)\, H_W\, (t+\delta)\, H_W\, (t)\, K(t_i) \rangle$ as a function of δ.
- $H_W - K$ operator separation ≥ 10.
- Integrate over δ from 0 to 10.

- $m_c^{\overline{MS}}$(2 GeV) ~ 1.2 MeV,
- M_π = 138 MeV
- 64^3x128, $1/a$=2.36 GeV
- 152 configurations
- FV correction ~10%
- a^2 errors: 40%

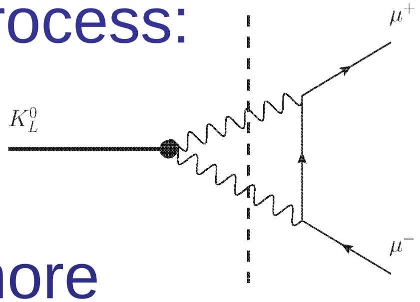
Accuracy forecast

- 10% statistical error better than expected, not difficult to reduce
- 40% a^2 error larger than expected
  - Result of scaling test at larger a and smaller m_c
  - $(m_c a)^2$ *errors ~25%?*
  - $\langle X|O^{(20,1)}|K\rangle$ shows a^2 errors ~20%?
- A continuum limit will require at least a Frontier/Aurora result from: 96^3 x 192, 1/a = 2.77 GeV. Plan a new 2+1+1 flavor series of ensembles.
- Hope for 10% accuracy in 5 years and 5% in 10

$K_L \rightarrow \mu^+\mu^-$

Physics of $K_L \rightarrow \mu^+ \mu^-$

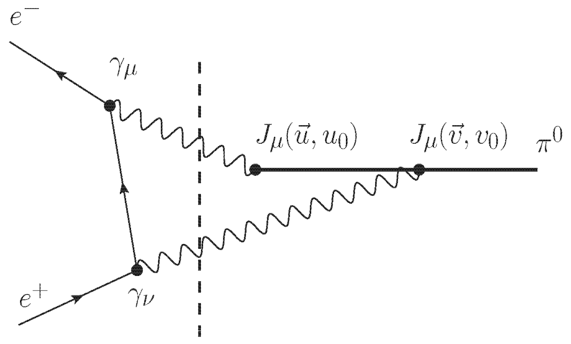
- A second-order weak, "strangeness changing neutral current" – important short distance part.
- $K_L \rightarrow \mu^+ \mu^-$ decay rate is known:
  - $BR(K_L \rightarrow \mu^+ \mu^-) = (6.84 \pm 0.11) \times 10^{-9}$
- Large ``background'' from two-photon process:
  - Third-order electroweak amplitude
  - Optical theorem gives imaginary part.



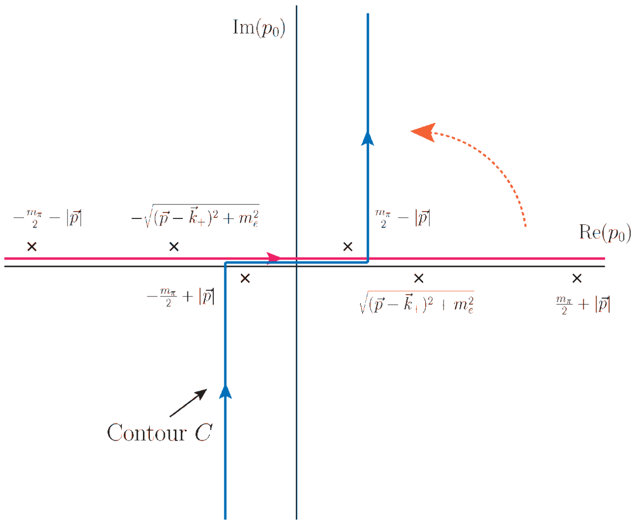
- Lattice calculation of 2γ contribution is more difficult than ΔM_K
  - 5 vertices, 60 time orders
  - many states |n> with $E_n < M_K$
- First try simpler $\pi^0 \rightarrow e^+ e^-$

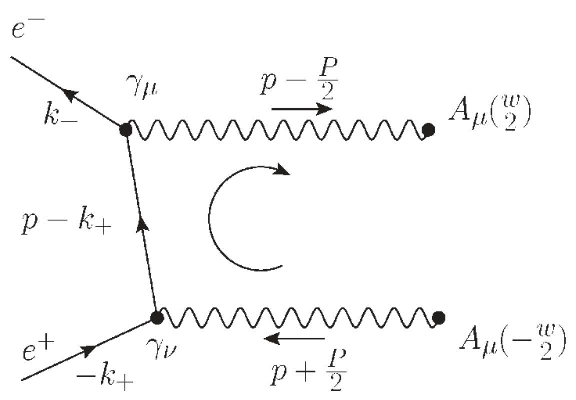
Consider simpler $\pi^0 \rightarrow e^+ e^-$

- Euclidean non-covariant P.T. difficult:
  - 12 time orders,
  - $E_{\gamma\gamma} < M_{\pi 0}$
- Try something different:
  - Evaluate in Minkowski space
  - Wick rotate integral over time argument:



$$\mathcal{A}_{\pi^0 \to e^+ e^-} \to \int d^4 w \, \widetilde{L}(k_-, k_+, w)_{\mu\nu} \langle 0 | T\left\{ J_\mu(\frac{w}{2}) J_\nu(-\frac{w}{2}) \right\} | \pi^0(\vec{P} = 0) \rangle$$

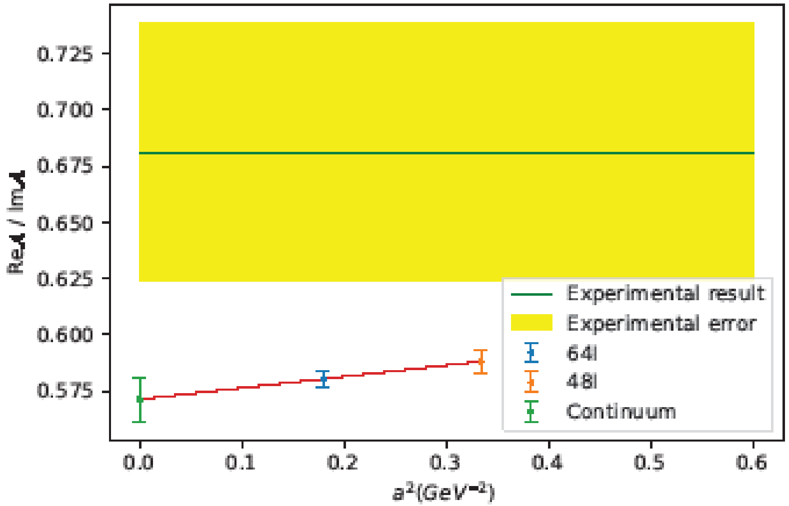




$\pi^0 \rightarrow e^+ e^-$
(Yidi Zhao)

- Continuum limit with disconnected graphs:

$$\mathrm{Re}(A_{\pi\rightarrow e+e-})/\mathrm{Im}(A_{\pi\rightarrow e+e-}) = 0.571(10)_{stat}(4)_{sys}$$



- Next calculate $K \rightarrow \gamma\gamma$
  - Connected part done (1/a=1.0 GeV)
  - Disconnected part needs more statistics
- Finite volume corrections needed for $K_L \rightarrow \mu^+\mu^-$ are not yet known.

Other important decays

- Long distance contribution to $K^{+} \to \pi^{+} \nu \bar{\nu}$
  - 5% correction to well-known SD part
  - Similar to LD contribution to ε_K
- $K \to \pi \ell^{+} \ell^{-}$
  - Long distance dominated
  - Test of lepton universality.

V_{us} and 1st row unitarity

- Kμ2/πμ2: experiment: $V_{us}/V_{ud}\, f_K/f_\pi$ error: 0.15%

  lattice QCD: f_K/f_π error: 0.22%

  - May allow most improvement? Sub 0.1% error
  - E&M corrections from lattice QCD

    [N. Carrasco *et al*. Phys. Rev. D 91 (2015) 074506]
  - New IVR method for E&M (no $1/L^n$ corrections)

    [X. Feng, et al. Lattice PoS LATTICE2019 (2020) 259]

    [Feng and Jin, Phys.Rev.Lett. 128 (2022) 052003]

$m_{\pi+}-m_{\pi 0} = 4.534(42)_{stat}(43)_{sys}$ [1.3% error on E&M]

Outlook

- Lattice QCD has made much of low energy QCD our friend.
- Much previously intractable non-perturbative physics can now be evaluated from first principles:
  - Systematic errors can be estimated and methodically reduced.
  - Total errors on some quantities now at the 1% level and soon the 0.1% (driven by g_μ-2 HVP and HLbL)
- Soon increasing lattice QCD precision will support improving previous experiments and possibly conceiving new ones.