Future of Lattice QCD calculations for Hadron Spectroscopy

White paper *"Hadron Spectroscopy with Lattice QCD"* 2203.03230

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Thanks to:

- expermintal colleagues: for wonerfull data and discoveries
- phenomenology colleagues: for valuable results and insights (many problems are too challenging for LQCD)

- lattice colleagues for collaborative effort in preparing

The white paper of topical group Hadron spectroscopy (RF7), submitted also to Lattice Gauge Theory (TF05)

arxiv:2203.03230
(I'll be refering to references from v2)

23 pages + 211 references

Submitted to the Proceedings of the US Community Study
on the Future of Particle Physics (Snowmass 2021)

Hadron Spectroscopy with Lattice QCD*

John Bulava[1], Raúl Briceño[2,3], William Detmold[4,5], Michael Döring[6,2], Robert G. Edwards[2], Anthony Francis[7,8,9], Francesco Knechtli[10], Randy Lewis[11], Sasa Prelovsek[12,13,†], Sinéad M. Ryan[14], Akaki Rusetsky[15,16], Stephen R. Sharpe[17], Adam Szczepaniak[2,18,19], Christopher E. Thomas[20], Michael L. Wagman[21], Marc Wagner[22,23]

Disclaimers for this talk:

- only **Examples** of available lattice results are given, not complete lists of references
  references are in White paper and even this does not attempt to be review
- current **Status** will be just "rushed over"
- emphasis on what could/needs to be done in the near **Future** of next 5-10 years

Hadron spectroscopy with lattice QCD: in breif

only strong strong decays (no electro-weak decays)

only strong decay threshods matter

How difficult it is to study a certain hadron?

more strong decay channels -> more difficult

"easy / doable": below or near the lowest threshold

"difficult": - above two or three thresholds

- decay to hadrons with spin

T for coupled-channel scattering

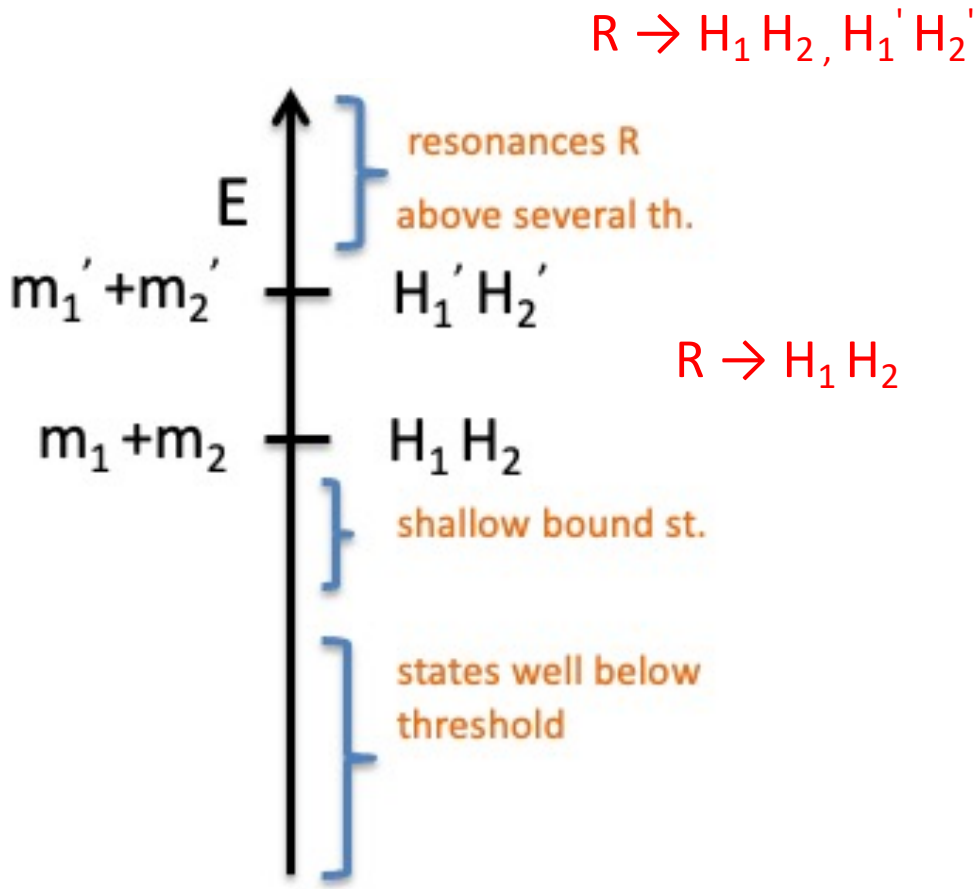
”very chalenging" (for now):

- above more than three thresholds

(if none of them can be "neglected")

- if $H_1 H_2 H_3$ decays are important

in addition to two-hadron decays



QCD: $\mathcal{L}_{QCD} = \frac{1}{4} G_a^{\mu\nu} G_a^{\mu\nu} + \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_a^\mu T^a) q - m_q \bar{q} q$ $\qquad g_s \not\ll 1$ at hadronic energy scale

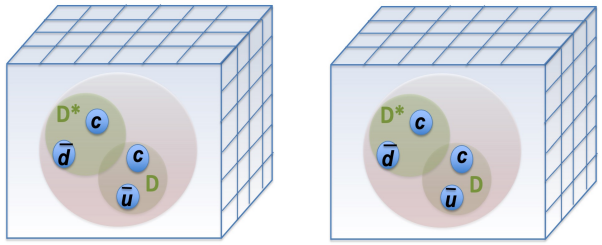
Lattice QCD: nonperturbative approach to QCD

Main quantity extracted: eigen-energy E_n

$\sum_n |n\rangle\langle n|$ $\qquad$ $e^{-iE_n t_M}$ Eucledian time

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^+(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle\, e^{-E_n t} \langle n | \mathcal{O}_j^+ | 0 \rangle \qquad E_n \to m, \Gamma, T(E)$$

t=0, J^P, I $\qquad$ t=0, J^P, I



$$\langle C \rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S^E_{QCD}/\hbar} \qquad S^E_{QCD} = \int d^4 x_E\, \mathcal{L}^E_{QCD}(m_q, g_s)$$

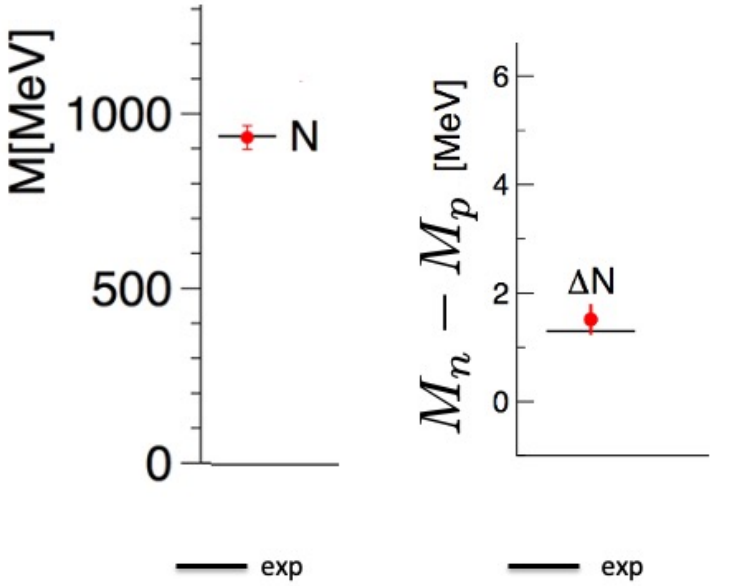
often "non-precision" studies: single a, $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140$ MeV

Strongly stable hadrons well below threshold: *mostly done*

$\pi,\ K,\ D,\ B^{(*)},\ B_c^{(*)},\ p,\ n,\ \Lambda,\ \Lambda_c,\ \Xi_{cc},$ etc. $\qquad m = E\ (\vec{p}=0)$

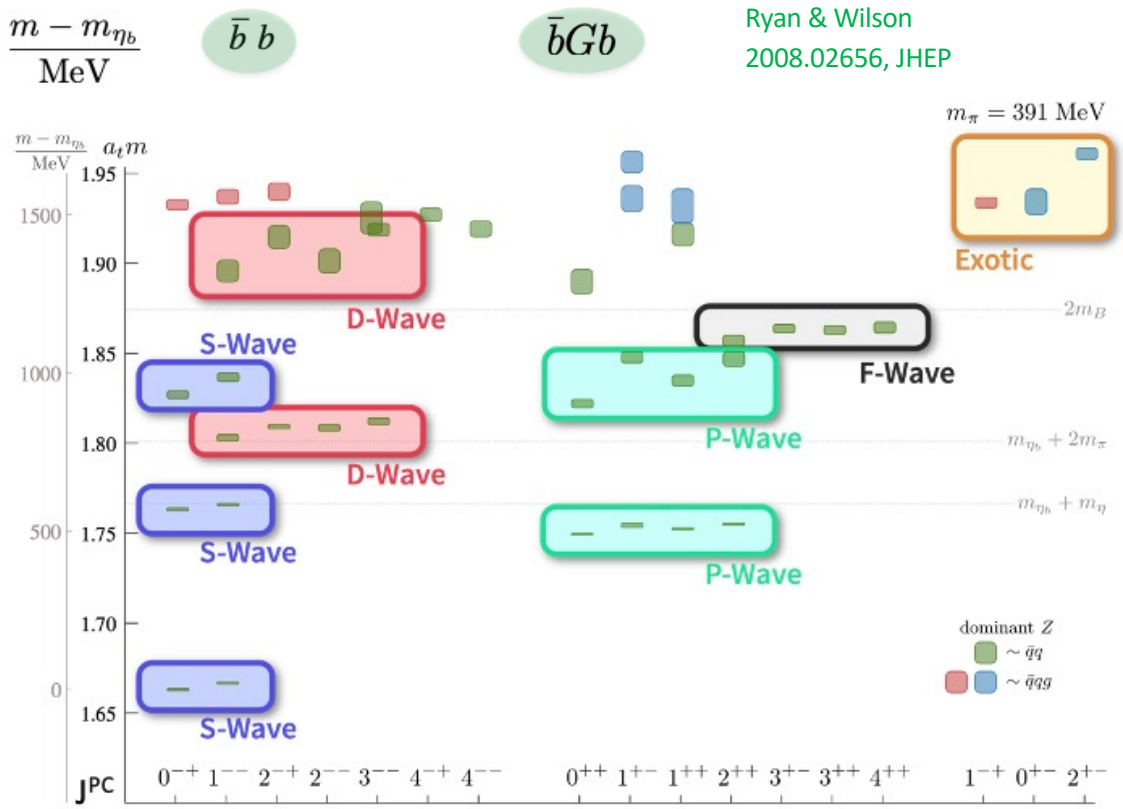
Examples

proton and neutron



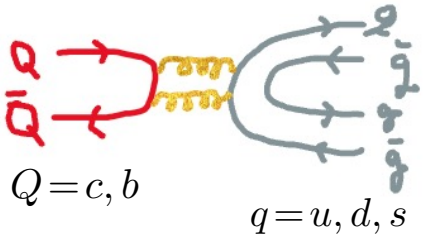
BMW collaboration
Science 322, 2008

BMW collaboration
Science 347,2015



Ryan & Wilson
2008.02656, JHEP

Future:

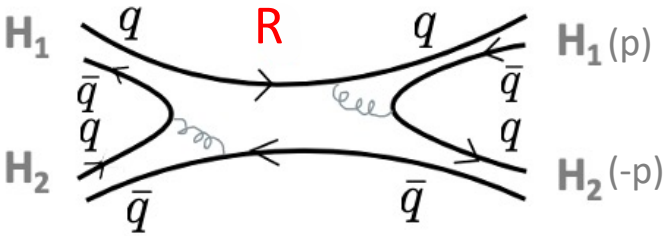


$Q = c, b$ $\qquad q = u, d, s$

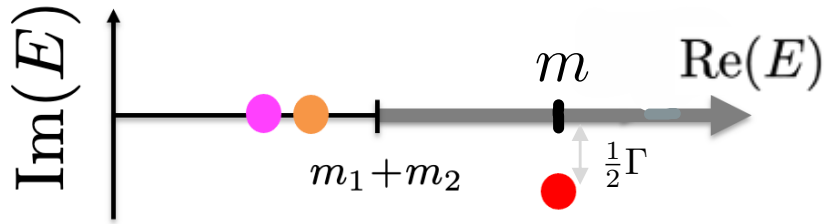
OZI suppressed contributions: very challenging

Isosping breaking effects and decays

Resonances R→ H_1 H_2, bound states near threshold : *significantly explored*



scattering matrix T(E)

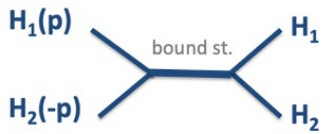


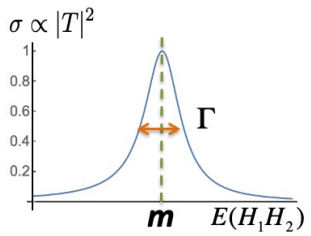
Virtual bound st. Bound st. Resonance

p = - i |p| p = i |p|

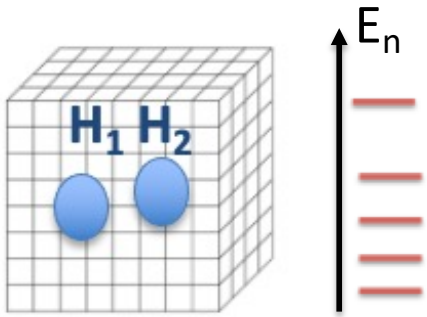
$$T(E) \propto \frac{1}{E^2 - m^2}$$

$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$





Scattering matrix T(E) from lattice QCD



many resonances and bound states extracted in this way by now
(appologies for not covering all)

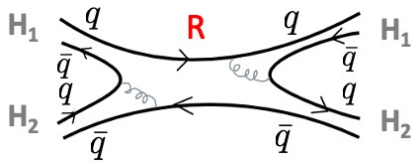
real E for real E for complex E

$$E \rightarrow T(E) \rightarrow T(E^c)$$

analytic relation:
Luscher 1991
generalizations by many authors

analytic contin.
to complex E

Resonances R→ H_1 H_2 : *significantly explored*

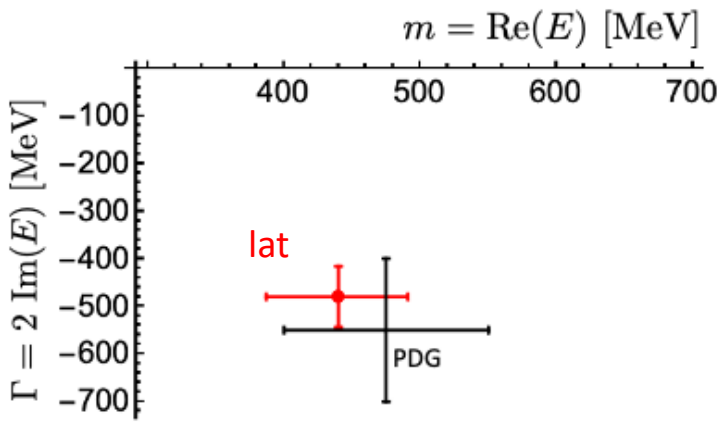


$$\Gamma = g^2 \frac{p^{2l+1}}{E^2}$$

Examples

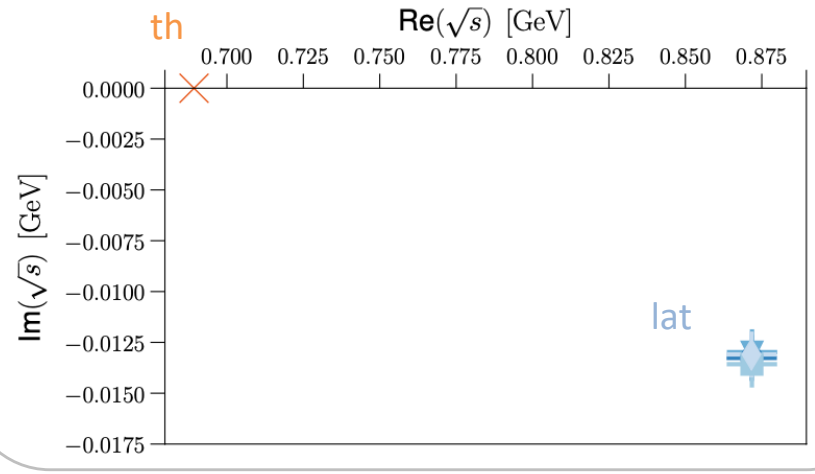
GWU, 1803.02897 $m_\pi = 227,315$ MeV $\to m_\pi^{phy}$ UChPT

$\sigma \to \pi\pi$



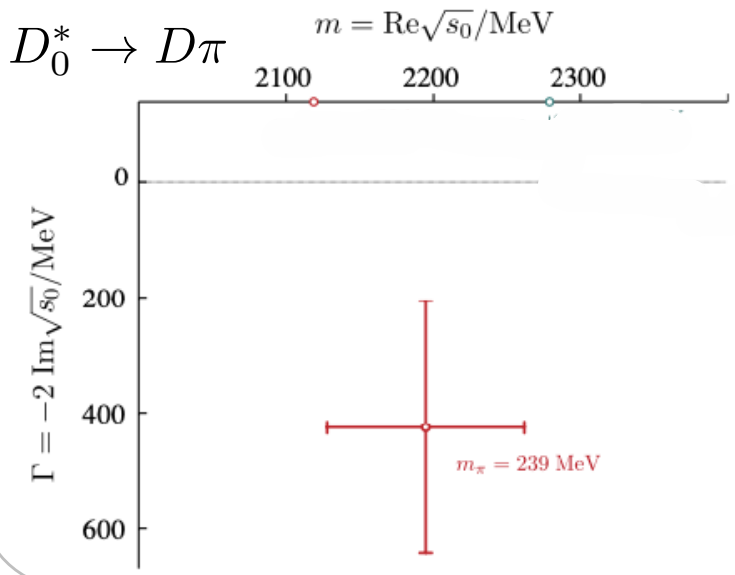
Rendon et al: 2006.14035 $m_\pi \simeq 176$ MeV

$K^* \to K\pi$ coupling g that parametrizes the width agrees with exp.



HadSpec, 2102.04973 JHEP $m_\pi \simeq 239$ MeV

$D_0^* \to D\pi$



PDG: puzzle

$m[D_0^*] \simeq m[D_{s0}^*(2317)]$

lat: OK

$m[D_0^*] < m[D_{s0}^*(2317)]$

Status

$\rho, \sigma \to \pi\pi;\ K^*, \kappa \to K\pi$

$D_0^* \to D\pi$

$\psi(3770), \psi_3(3842) \to D\bar{D}$

$\Delta \to N\pi$

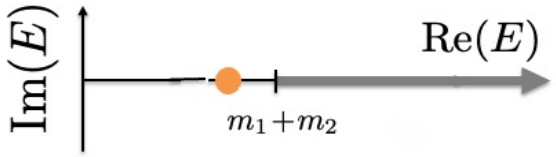
Future

dependence on a, $m_{u,d}$

$B_0^* \to B\pi$

a0 for "all" channels H_1H_2 -> EFT

Bound states closely below th H_1 H_2 : *explored*



	Status	Future
D^*_{s0} below DK	pole found, many studies matured	
B^*_{s0} below BK	pole found, one study	further lat. confirmation, awaits exp. discovery
$X(3872)$ below $D\bar{D}^*$	pole found [SP Leskovec PRL 2013, Padmanath Lang SP 2015]	larger volumes three-hadron decays $X(3872) \to J/\psi\pi\pi$ isospin-breaking effects
deuteron below pn	pole found many studies, mostly near $SU(3)_F$ limit	smaller $m_{u,d}$ distillation method await exp. discovery
H-dibaryon below $\Lambda\Lambda$	pole found, many studies, mostly $SU(3)_F$ limit only one with distillation [Green et al PRL 2021]	
heavy dibaryons	poles found in some systems [Matur et al]	further lat. confirmation, awaits exp. discovery
$bb\bar{b}\bar{b},\ cc\bar{c}\bar{c}$	Q=b: no bound state found [HPQCD, 1710.03236]	$cc\bar{c}\bar{c}$ bound state?

Doubly heavy tetraquarks $QQ'\bar{q}\bar{q}'$

States well below threshold

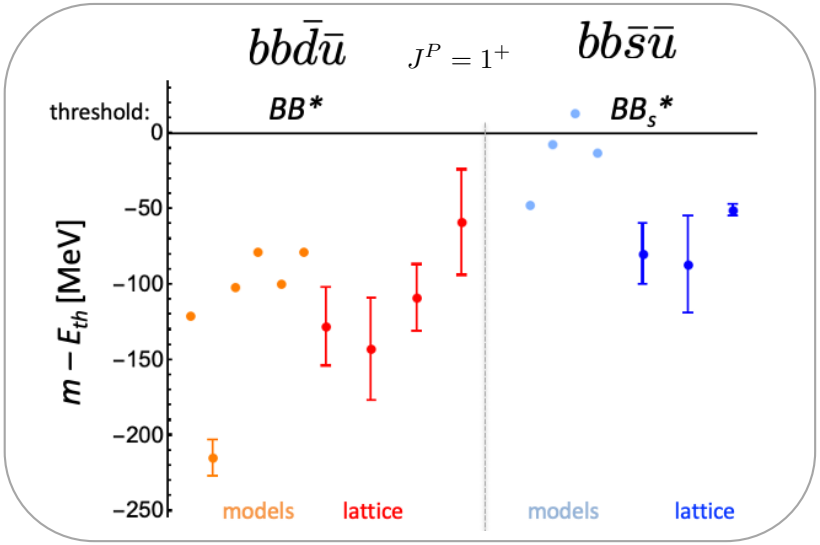
Status

surely bound, matured

explored dependence on m_u, m_b favors $[bb][\overline{ud}]$

[Colquhoun, Hudspith, Maltman @ Lattice 2021]

awaits exp discovery



lattice : from left to right

Leskovec, Meinel, Pflaumer, Wagner (2019) 1904.04197

Junnarkar, Mathur, Padmanth (2018) 1810.12285

Frances, Colquhoun, Hudspith, Maltman (2021) preliminary

Bicudo, Wagner et al. 1612.02758 static potentials

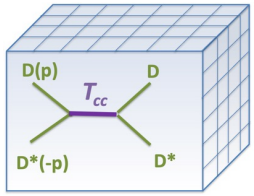
Pflaumer, Leskovec,Meinel,Wagner (2021) 2108.10704

Junnarkar, Mathur, Padmanth (2018) 1810.12285

Frances, Colquhoun, Hudspith, Maltman (2021) preliminary

States near threshold

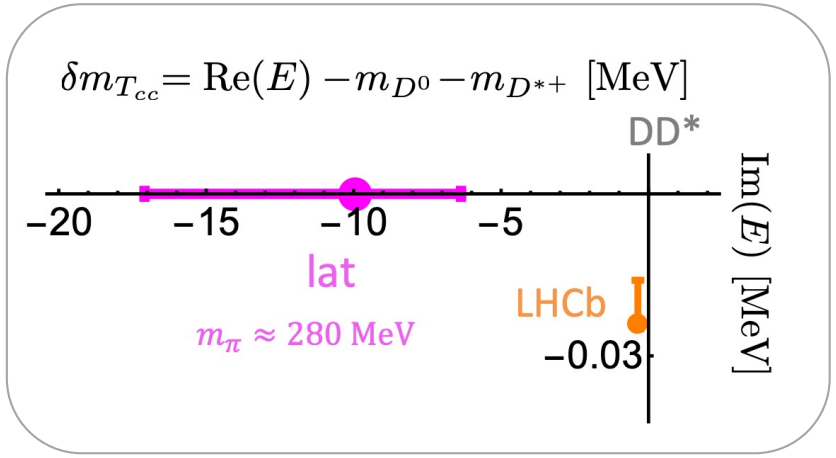
Status



T(E) needs to be extracted and pole established

only one study [Padmanath, SP, 2202.10110]

$cc\bar{d}\bar{u},\ I=0,\ J^P=1^+$ virtual bound state pole found likely related to Tcc



Future

$cc\bar{u}\bar{d},\ cc\bar{u}\bar{s},\ bc\bar{u}\bar{d}$

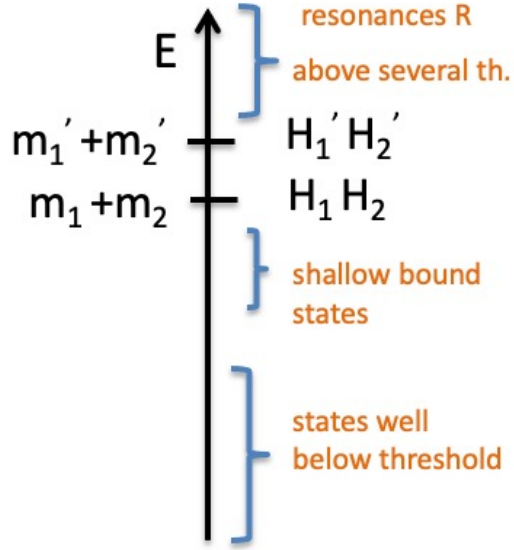
various J^P

dependence on m_Q, $m_{u/d}$

determine T(E)

why Tcc and X(3872) both lie so close to th. ?

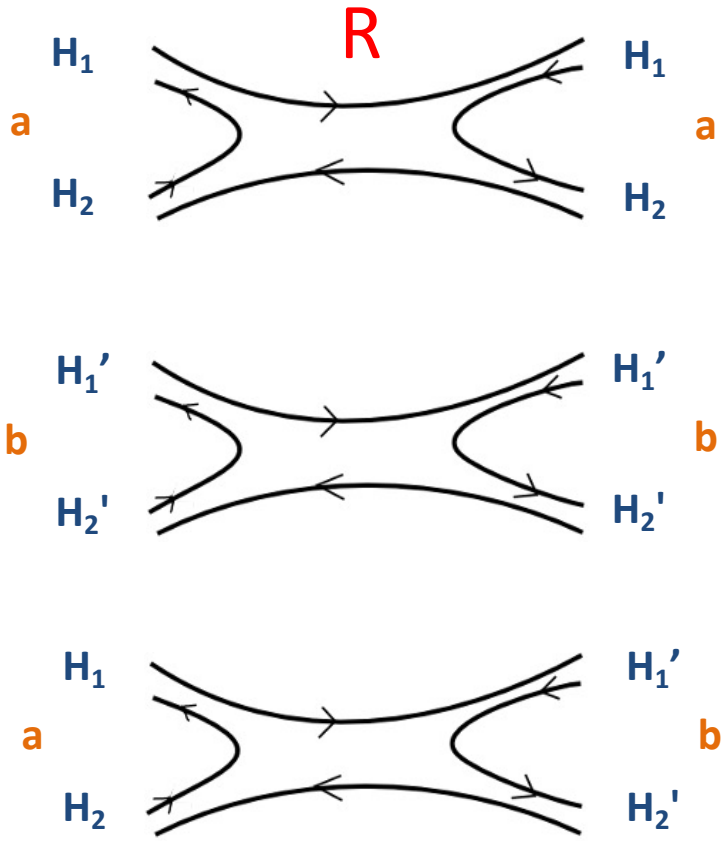
Resonances from coupled-channel scattering:
partly done, challenging



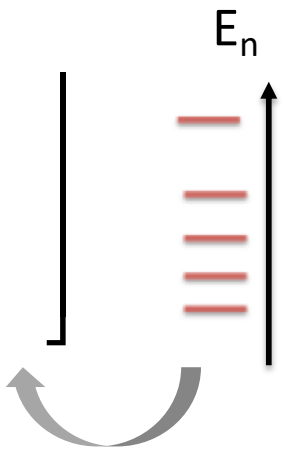
$$R \to H_1 H_2,\ H_1' H_2', \ldots$$

channel a: $H_1 H_2$

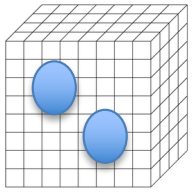
channel b: $H_1' H_2'$



$$T(E) = \begin{array}{c} \begin{array}{cc} a \to a & a \to b \end{array} \\ \begin{bmatrix} T_{aa}(E) & T_{ab}(E) \\ T_{ab}(E) & T_{bb}(E) \end{bmatrix} \\ \begin{array}{cc} b \to a & b \to b \end{array} \end{array}$$



generalized.
Luscher's rel.



Status:

Resonances composed of u,d,s

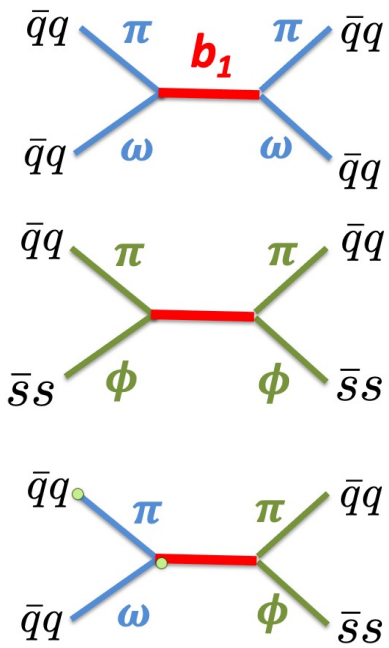
significantly explored by HadSpec. coll.

a0, a1, b1,K0*(1430), ...

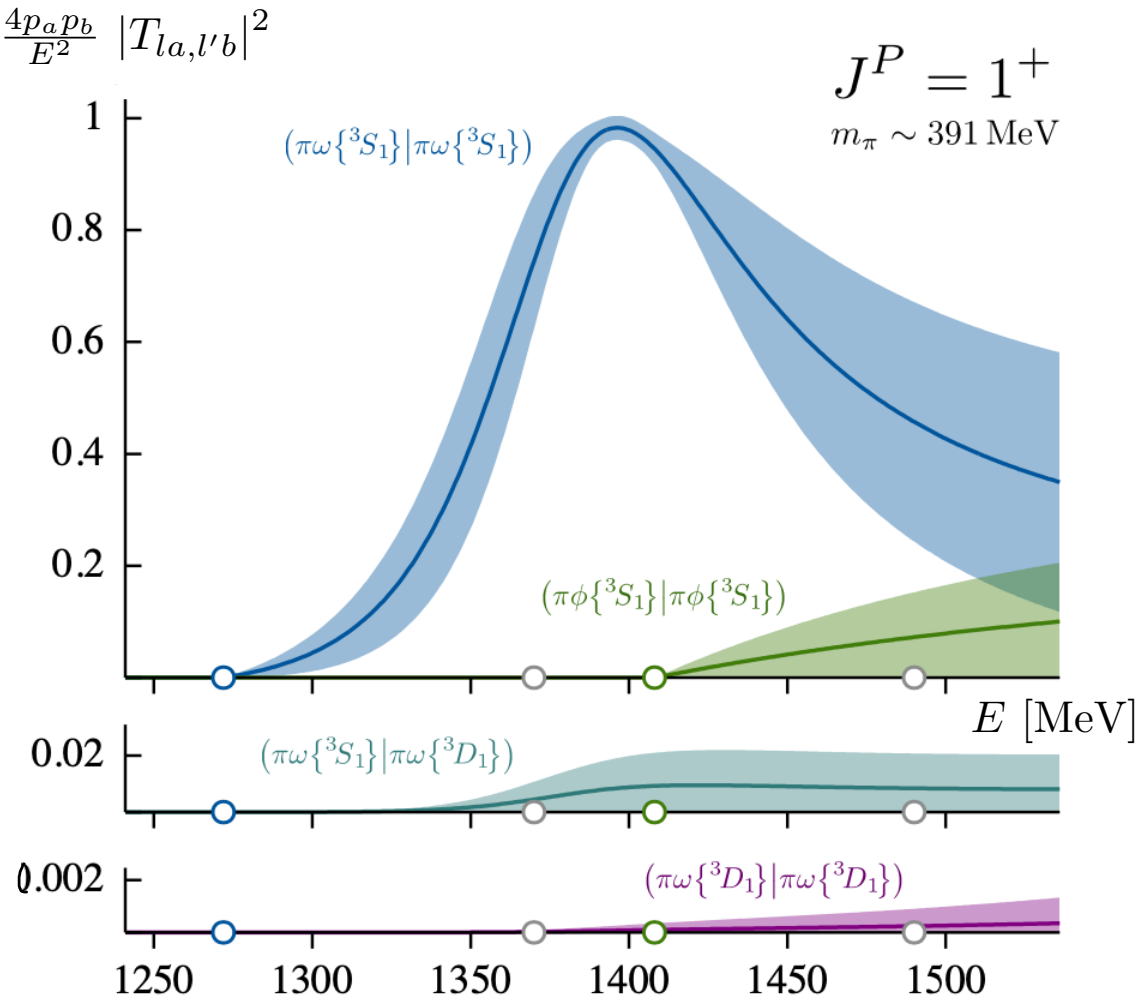
b_1 resonance from lattice

conventional

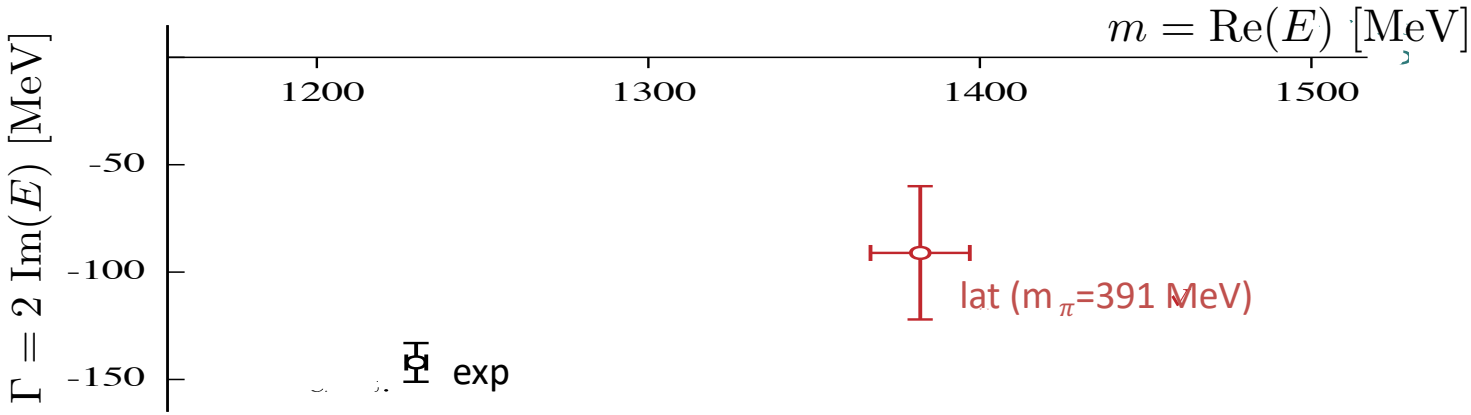
$\bar{d}u$ $J^{PC} = 1^{+-}$



q=u,d



Woss et al, HadSpec, 1904.04136, PRD

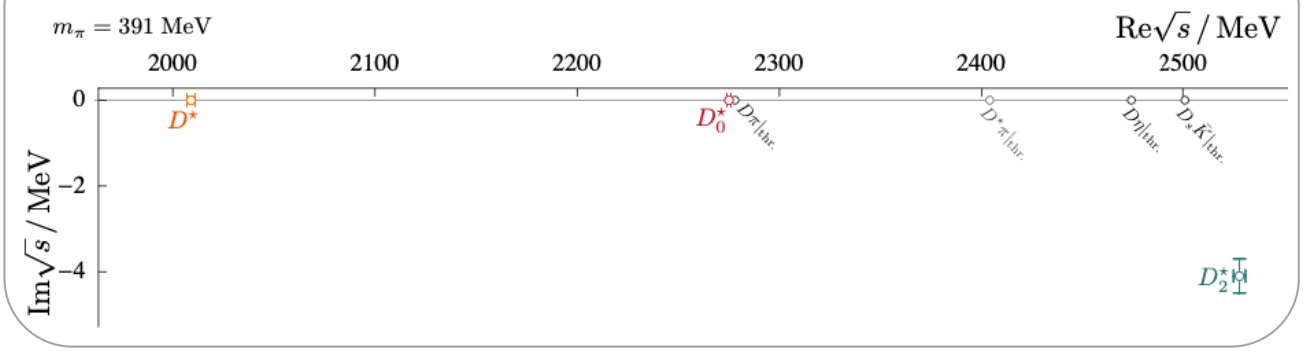


Resonances from coupled-channel scattering: *partly done, challenging*

Resonances containing heavy quarks

Status: only two studies extracted T(E)

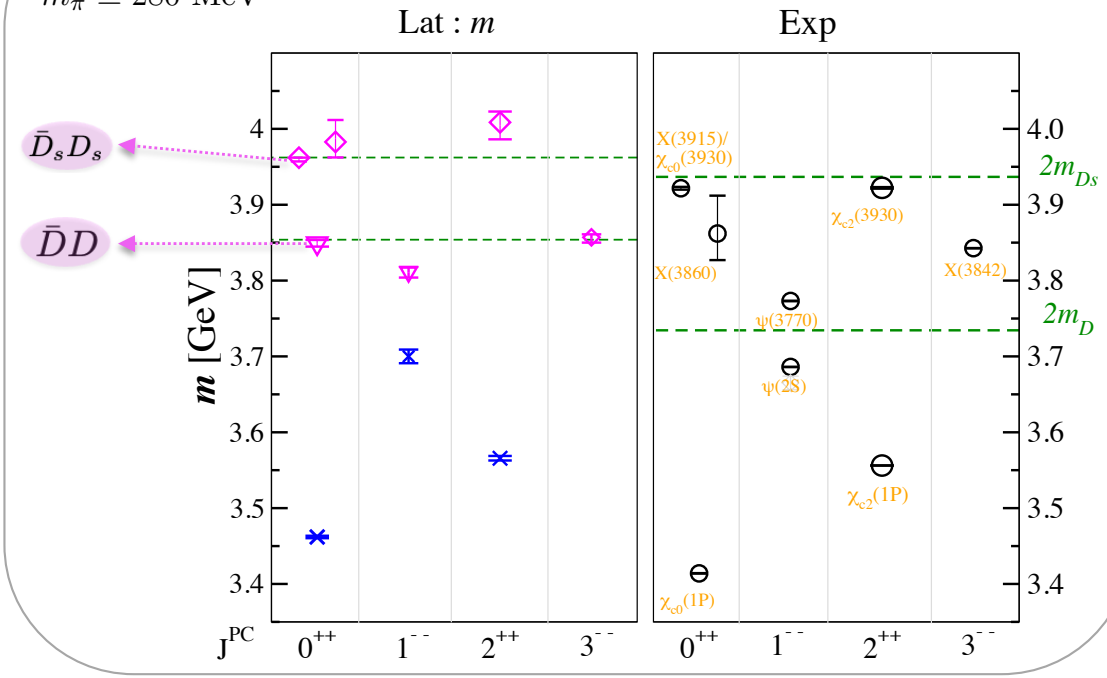
$I = 1: \; D\pi - D\eta - D_s\bar{K}$ HadSpec. 1607.07093, JHEP



$I = 0: \; D\bar{D} - D_s\bar{D}_s$

S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02541 JHEP, 1905.03506 PRD

$m_\pi \simeq 280$ MeV



Several other channels studied,

but T(E) could not be extracted

Hadrons $\bar{Q}Q\bar{q}q$, $\bar{Q}Qqqq$

$\bar{Q}Q\bar{q}q \to \bar{Q}q + \bar{q}Q, \quad \bar{Q}Q + \bar{q}q$

$Z_b = \bar{b}b\bar{d}u$ [Belle 2011] $\quad P_c = \bar{c}cuud$ [LHCb 2015]

$Z_c = \bar{c}c\bar{d}u$ [BessII, Belle, 2013]

$\bar{Q}Qqqq \to \bar{Q}q + Qqq, \quad \bar{Q}Q + qqq$

these decays make studies difficult

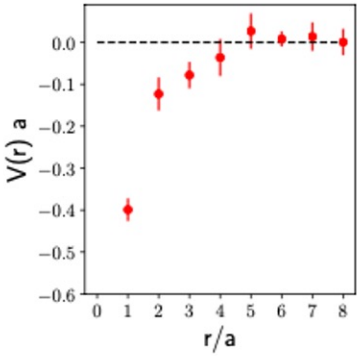
Status: only partial conclusions available

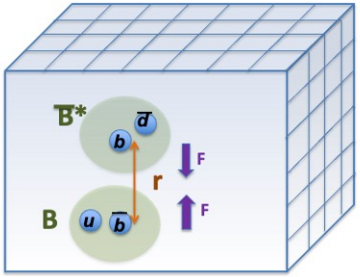
$Z_b = \bar{b}b\bar{d}u$ [Belle 2011]

$\bar{b}b\bar{d}u \to B\bar{B}^*, \ \Upsilon\pi$ static b quarks & Born-Oppenheimer approach

Peter, Wagner, Bicudo

SP, Bahtiyar, Petkovic, Sadl 2019, 2020,





attraction between B and B* likely responsible for Zb

$Z_c = \bar{c}c\bar{d}u$ [BessIII, Belle 2013]

$\bar{c}c\bar{d}u \to D\bar{D}^*, \ J/\psi\pi$ non-static c quarks

several lattice studies found very small interactions in this system

[Leskovec Mohler Lang SP: 1308.2097,1405.7623
HadSpec 1709.01417
Liuming Liu et al. 1907.03371, 1911.08560]

$P_c = \bar{c}cuud$ [LHCb 2015, 2019]

One-channel scattering $J/\psi \ p$ does not lead to observed Pc
-> the channels $(\bar{c}u)(udc)$ must be crucial
[Skerbis SP PRD 2019]

Resonances R from coupled-channel scattering

Future : many many relevant channels, most efforts will likely go into this direction

- more emphasis on R containg Q=c,b (less explored on lat)
- R decaying via 2,3,4 channels
- R decaying to hadrons with spin **1+0, 1+1, ½+0, ½+1 ,...** : more challenging than **0+0**

examples (with heavy quarks)

$Z_c(3900) \simeq \bar{c}c\bar{d}u$	$J/\psi\pi - D\bar{D}^* - \eta_c\rho$	E<=4 GeV, but $Z_c(4430)$ is definitely out of reach!
$\bar{c}c\bar{c}c$:	$\eta_c\eta_c - J/\psi J/\psi$	X(6900)=$\underline{c}c\underline{c}c$ [LHCb] seems to high for near future but theory approaces predict more states near threshold explore at least both threshold regions
$\bar{b}b\bar{b}b$:	$\eta_b\eta_b - \Upsilon\Upsilon$	
D_0^*	$D\pi - D\eta - D_s\bar{K}$	is there one or two poles for scalars ($\underline{3}$ and 6)?
$P_c \simeq \bar{c}cuud$	$J/\psi\ p - \Sigma_c^+\bar{D}^0 - \Sigma_c^+\bar{D}^{0*}$	there are more channels ☹, incorporate at least those three

resonances with light quarks including hybrids

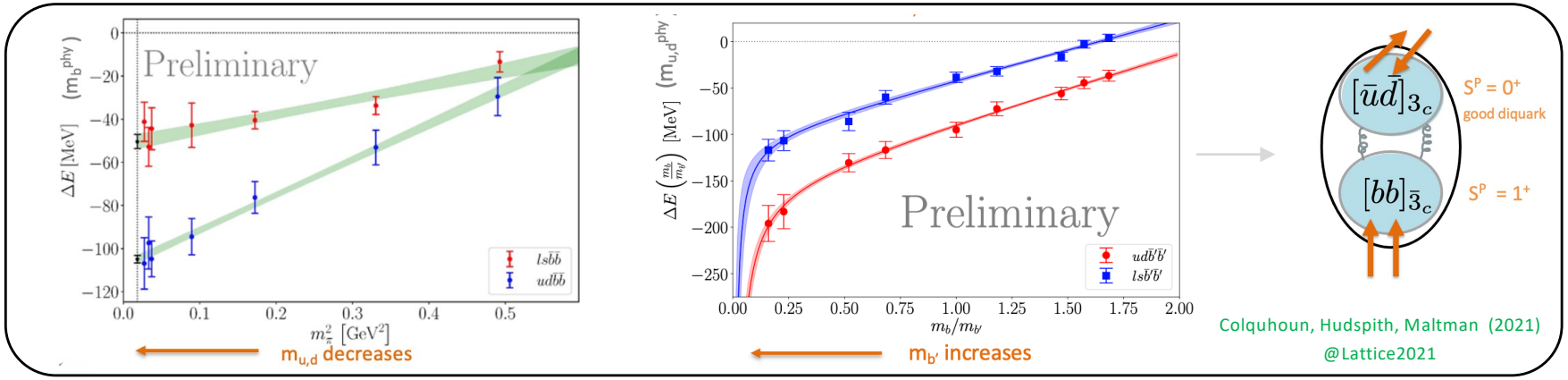
much more has already been done (HadSpec coll)

decrease $m_{u/d}$ (till three-body decays kick in)

Can some reasonable approximations/simplifications be made?

Towards internal structure of hadrons

- Dependence of pole position (m,Γ) on quark masses : can not be explored in exp ...



- generalization of Feynman-Hellmann theorem to distinguish conventional and exotic hadrons [1706.09015]

- hadron structure from various currents K $\langle H|J|H\rangle$ also for H=R [formalism exist: Briceno, Walker-Loud,...]

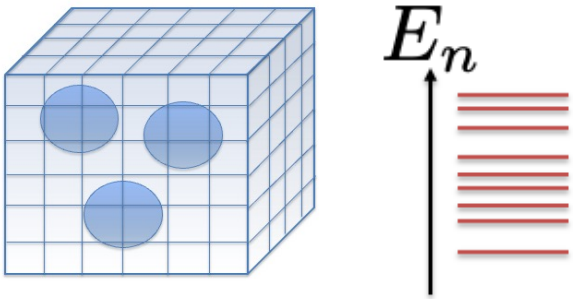
- explore density-density correlations $\langle H|\rho(\vec{x}_1)\rho(\vec{x}_2)|H\rangle$ [for diquarks: Francis et al, 2106.09080 PRL]

- request to pheno community: propose (realistic) lattice observables to learn more about hadron structure

Scattering of three hadrons $H_1 H_2 H_3$, $R \to H_1 H_2 H_3$

Nature $a_1 \to \pi\pi\pi,\ N(1440) \to N\pi\pi,\ X(3872) \to J/\psi\pi\pi,\ \ldots.$

Formalism



3-body force

$$E \to T(E)$$

formalism by 3 groups: agreement
Sharpe, Hansen, Briceno, Lopez...
Doring, Mai,...
Rusetsky, Hammer,...

Status
with actual simulations
impressive progres!!

$\pi^+\pi^+\pi^+,\ K^+K^+K^+,\ \ldots$ [GWU, HadSpec, Hanlon, Horz, NPLQCD, ETMC, . . : 2020 – 2022]

$a_1 \to \pi\pi\pi$ [GWU, 2107.03973, PRL]

Future: generalize the formalism to arbitrary J^P, I, coupled channels,...

improve practical strategies to extract T(E), 3-body force

$R \to H_1 H_2 H_3$

$R \to H_1 H_2 H_3$, $H_1' H_2'$

Electrowek transitions of resonances $\langle H_f | J | H_i \rangle$

Status:

for stable H: vastly explored : topic of RF1 (Weak decays of c and b quarks)

for resonances: only $\langle \rho | J_{em} | \pi \rangle$ $\pi\gamma \to \rho \to \pi\pi$ [HadSpec PRL 2015; Alexandru et al 1807.08357]

formalisms exist [Briceno et al, Walker Loud, Rusestsky,...]

Future: taking into account strong decays of resonances

EM: $N\gamma \to \Delta \to N\pi$ and $K\gamma \to K^* \to K\pi$

weak: $D \to \rho l \bar{\nu}$ and $B \to \rho l \bar{\nu}$ $\rho \to \pi\pi$

$B \to D^* l \bar{\nu}$ $D^* \to D\pi$

$B \to K^* l^+ l^-$ $K^* \to K\pi$

Studies closer to physical world:

- dont require just another "input parameter" (e.g. $m_u, m_d, \ldots$) and more computer resources
- more decay channels
- conceptual improvements, ideas, significant human power

Recommendations

To realize impact of lattice QCD on hadron spectroscopy:

- sustained investment in human capital
- continued access to computer resources

HEP community should work with DOE and NSF to increase funding for the HEP SciDAC programs and other cyber-related initiatives, and to foster partnerships with ASCR and other agencies to strengthen the impact of these programs

- continued support to USQCD lattice QCD collaboration

USQCD : umbrella collaboration, which encompasses nearly all researchers in lattice QCD in the US

SciDAC: program that address the math and computational challenges for predictive modeling

ACSR: Computing office

backup

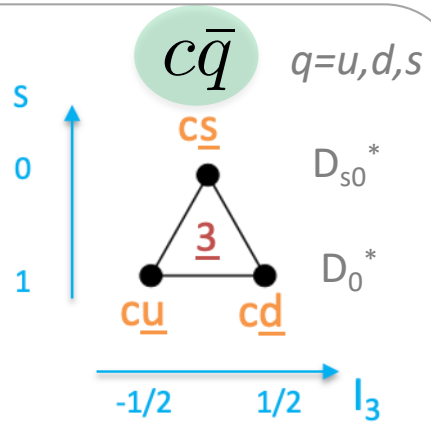
Recommendation concerning USQCD:

Over the last two decades or so, the DOE has provided extensive support to the USQCD lattice-QCD collaboration. This umbrella collaboration, which encompasses nearly all researchers in lattice QCD in the US, has coordinated a highly successful program of precision calculations using lattice methods, including a significant effort in hadronic spectroscopy. Much of the work in lattice QCD described above would not have been possible without this support. As lattice methods are applied to increasingly complex systems, it is essential that computational resources continue to be made available. We recommend the continuation of such support.

Scalar heavy-light mesons

$J^P = 0^+$

Conventional quark model



new paradigm supported by:

- lattice
- effective models ChPT+HQET
- reanalysis of exp data
- states circled by blue seem to feature in the spectrum

Scattering on the lattice

S=1 Mohler et al, 1308.3175, PRL
Lang et al, 1403.8103, PRD
RQCD, 1706.01247, PRD
HadSpec 2008.06432, JHEP

S=0 Lang et al. 1208.4059, PRD (see backup)
HadSpec, 1607.07093, JHEP
HadSpec 2102.04973, JHEP

S=-1 HadSpec, 2008.06432, JHEP

New paradigm

Du et al, 1712.07957, PRD
Albaladejo et al, 1610.06727, PLB
Lutz et al (2003), 0307133, PLB

$c\bar{q}$ + $c\bar{q}\ q\bar{q}$ q=u,d,s n=u,d

$\underline{3} \otimes 8 = \underline{3} \oplus 6 \oplus 15$ $SU(3)_F$

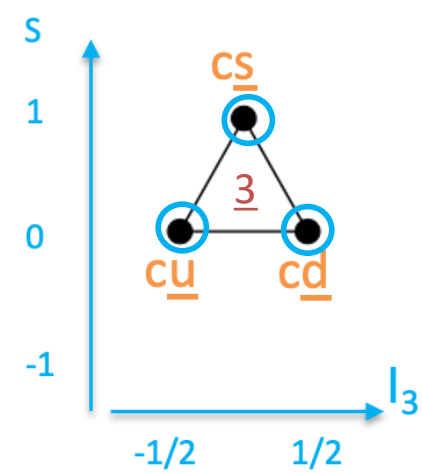
Beveren, Rupp; Dmitrasinovic

2.3 GeV

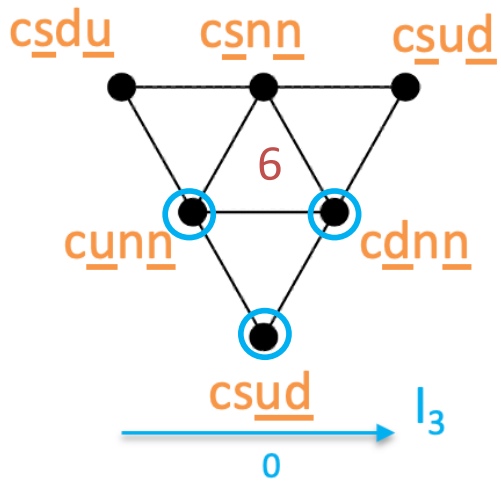
lat: 2.1-2.2 GeV (pole)

PDG: 2.3 GeV (BW)

(see backup)



mix

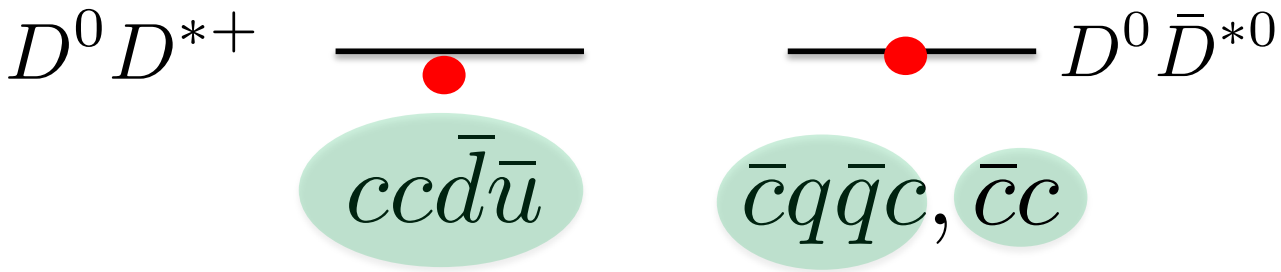


no state (mix with repulsive 15)

2.4-2.5 GeV
reanalysis of lat 1607.07093 by Albaladejo 1610.06727

virtual bound state
HadSpec 2008.06432
partner of X(2900) [LHCb 2009.00025] ?

A puzzle comparing T_{cc} and $X(3872)$



Why both reside within 1 MeV of threshold in exp ? There are many differences ...

Similarities:

- $J^P=1^+$, I=0
- in molecular picture: attraction via light vector exchange [e.g. Guo et al. 2101.01021, 2108.02673]

Differences:

- in molecular picture: attraction from one-pion exchange for X(3872) [Tornquist 1994]
  slight attraction from one-pion exchange for Tcc [eg. Du, Guo, Hanhart, 2110.13765]
- presence of Fock component $\underline{c}c$ for X(3872) [e.g. Padmanath, Lang, SP 1503.03257, PRD]
- presence of Fock component $[cc][\underline{ud}]$ for Tcc

Bound and virtual bound state: simplest example

scattering in square-well potential in QM

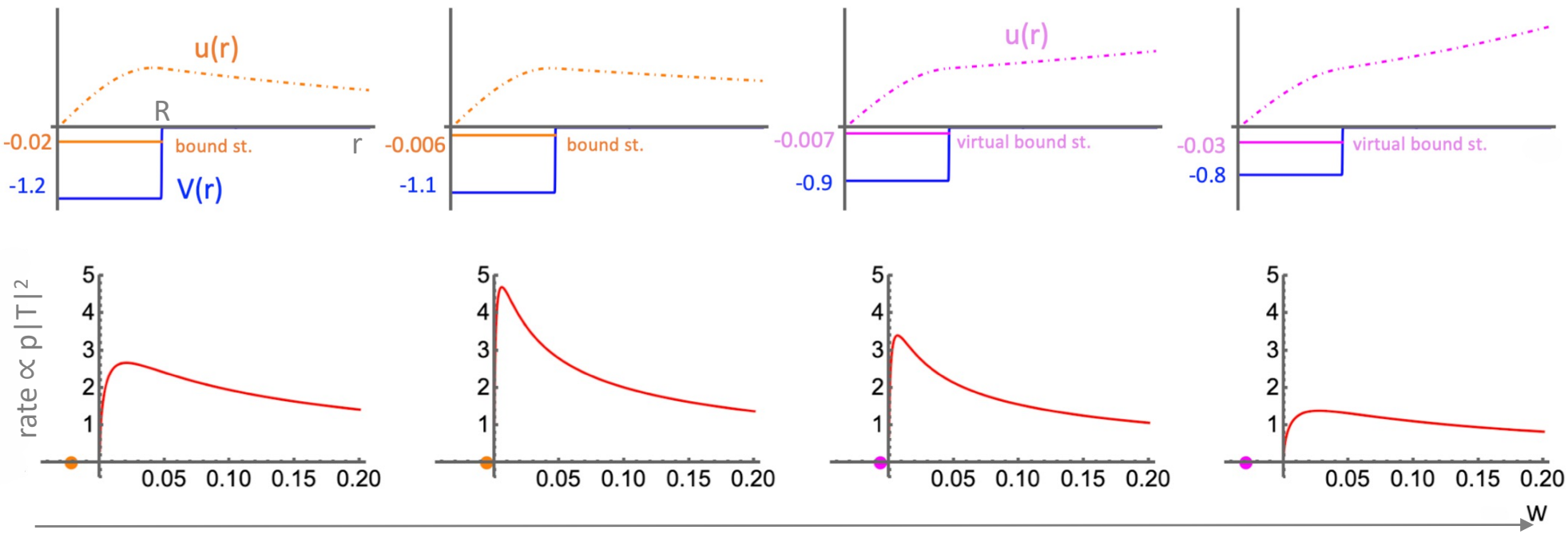
partial wave l=0

$$\delta = \arctan[\tan(qR)\tfrac{p}{q}] - pR$$

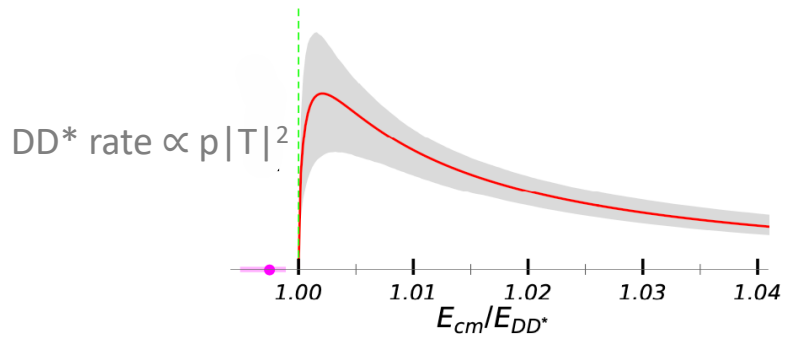
$u(r) = A\sin(qr)$ $\quad u(r) = B\sin(pr+\delta)$

p=i|p| $\quad e^{ipr} = e^{-|p|r}$

p=-i|p| $\quad e^{ipr} = e^{|p|r}$



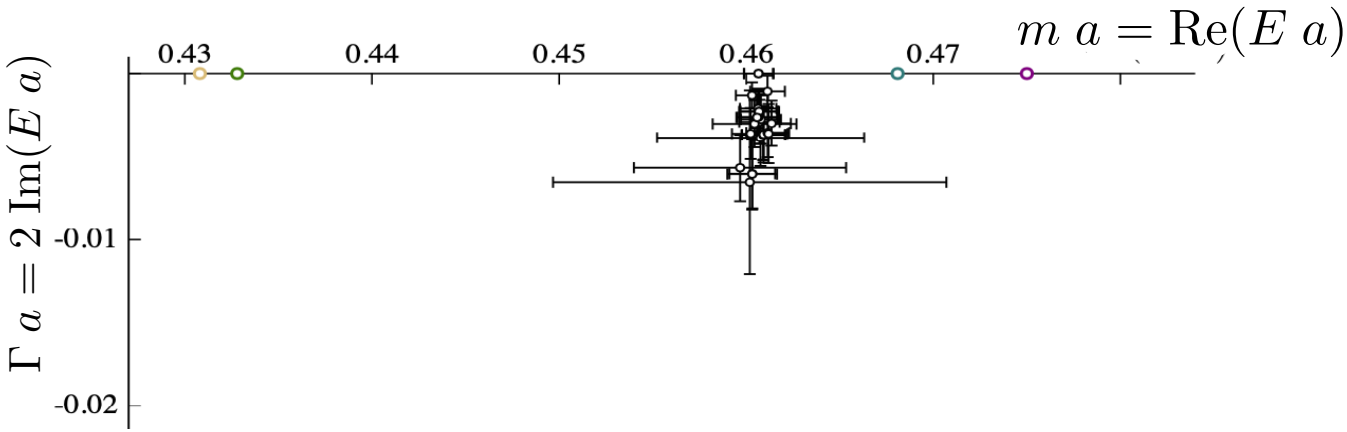
increasing $m_{u/d}$, decreasing attraction

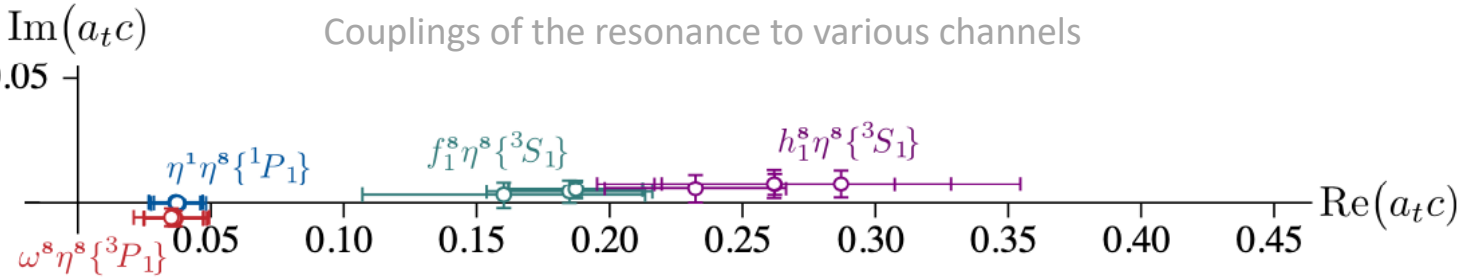


for T_{cc} extracted on lattice

light hybrid meson π_1 from lattice

$\bar{d}Gu$ $J^{PC} = 1^{-+}$





Couplings of the resonance to various channels

$$T_{ij} \sim \frac{c_i\ c_j}{E_p^2 - E^2}$$

Woss et al. (HadSpec)
2009.10034

$m_u = m_d = m_s$, $m_\pi \approx 700\ MeV$

pheno analysis

physical world

$\rho\ \pi$ $\eta'\ \pi$ $f_1\ \pi$ $b_1\ \pi$

dominant coupling

resemblence to experimental $\pi_1(1564)$: COMPASS+JPAC Rodas 1810.04171 [PRL]

$\pi_1(1564)$ in COMPASS+JPAC replaces two older resonances $\pi_1(1400)$ and $\pi_1(1600)$

P_c pentaquarks

$\bar{c}\,c\,u\,u\,d$

$$P_c = \text{uud}\bar{\text{c}}\text{c} \rightarrow (\text{uud})\,(\bar{\text{c}}\text{c}):\ \text{p}\ \text{J}/\Psi,\ldots$$

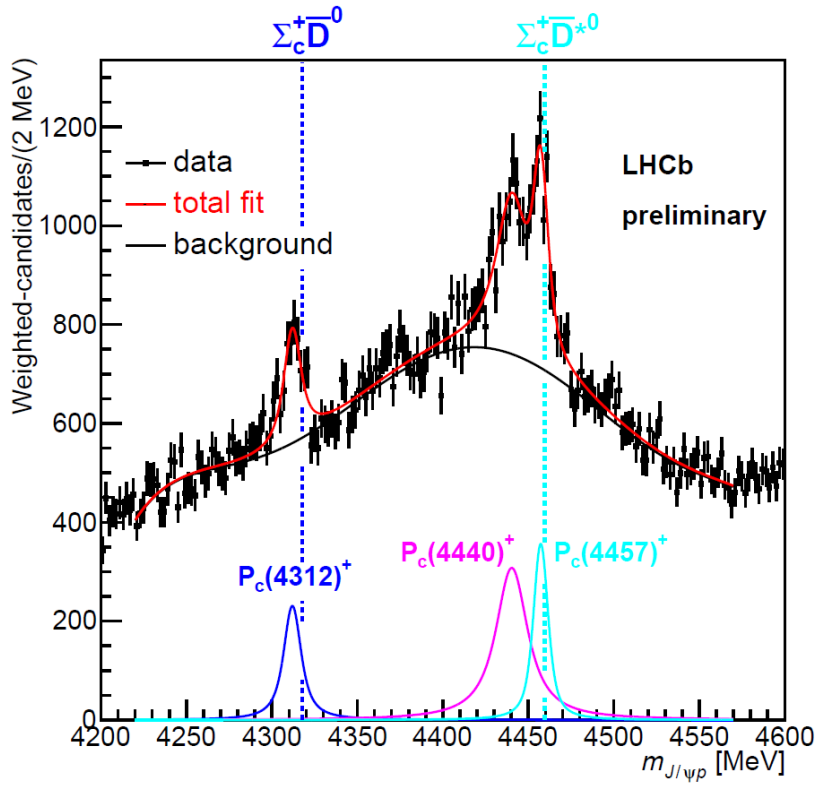
$$\rightarrow (\text{udc})\,(\bar{\text{c}}\text{u}):\ \Sigma_c^+\ \bar{D}^0,\ldots$$

[LHCb 2019, 1904.03947, PRL]

J^P not determined from exp.

Expected J^P for molecule in s-wave:

$$\Sigma_c(\tfrac{1}{2}^+)\bar{D}(0^-) \rightarrow J^P = \tfrac{1}{2}^- \qquad \Sigma_c(\tfrac{1}{2}^+)\bar{D}^*(1^-) \rightarrow J^P = \tfrac{1}{2}^-, \tfrac{3}{2}^-$$



Indications that **Σ_c^+ D$^{(*)}$ molecular** component is important:

- **experiment** finds them slightly below those thresholds
- supported by **phenomenological models** with ρ/ω exchange

  predicted 2010-2012 [Wu, Molina, Oset, Zou, 1007.0573, PRL; Wu et al., 1202.1036. PRC, Yang et al, 1105.2901, Wang et al, 1101.0453, PRC]

- **Lattice QCD** addressed simplified question:

Do Pc resonances appear in one-channel

$$p\ \ J/\psi \rightarrow P_c \rightarrow p\ \ J/\psi$$

scattering if it is decoupled from other channels ?

Answer: No [Skerbis, S. P., 1811.02285, PRD 2019]

T(E)≈0 within large errors, small interaction, no resonance

This indicates that coupling of p J/ψ channel with other two-hadron channels is likely responsible for Pc in experiment (in line with LHCb result)