

Charged Lepton Flavor Violation: Theory

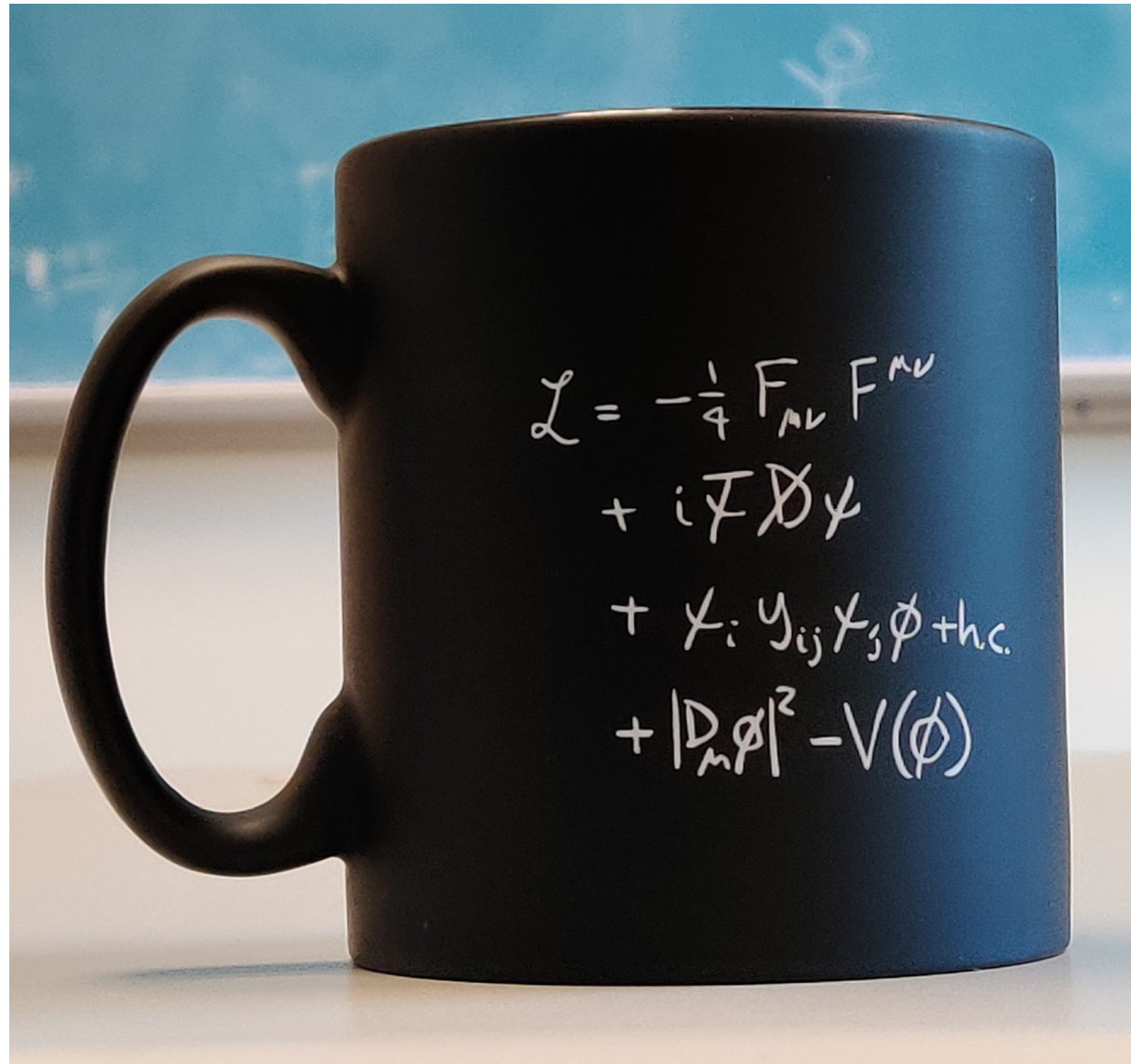
Julian Heeck

Snowmass Rare and Precision Frontier Spring Meeting
Cincinnati, Ohio


















05/17/2022



The Standard Model



Elementary particles

BARYONS/ QUARKS	mass → $\approx 2.3 \text{ MeV}/c^2$ charge → $2/3$ spin → $1/2$  up	mass → $\approx 1.275 \text{ GeV}/c^2$ charge → $2/3$ spin → $1/2$  charm	mass → $\approx 173.07 \text{ GeV}/c^2$ charge → $2/3$ spin → $1/2$  top	mass → 0 charge → 0 spin → 1  gluon	mass → $\approx 126 \text{ GeV}/c^2$ charge → 0 spin → 0  Higgs boson	SCALARS
	mass → $\approx 4.8 \text{ MeV}/c^2$ charge → $-1/3$ spin → $1/2$  down	mass → $\approx 95 \text{ MeV}/c^2$ charge → $-1/3$ spin → $1/2$  strange	mass → $\approx 4.18 \text{ GeV}/c^2$ charge → $-1/3$ spin → $1/2$  bottom	mass → 0 charge → 0 spin → 1  photon		
	mass → $0.511 \text{ MeV}/c^2$ charge → -1 spin → $1/2$  electron	mass → $105.7 \text{ MeV}/c^2$ charge → -1 spin → $1/2$  muon	mass → $1.777 \text{ GeV}/c^2$ charge → -1 spin → $1/2$  tau	mass → $91.2 \text{ GeV}/c^2$ charge → 0 spin → 1  Z boson		
	mass → $< 2.2 \text{ eV}/c^2$ charge → 0 spin → $1/2$  electron neutrino	mass → $< 0.17 \text{ MeV}/c^2$ charge → 0 spin → $1/2$  muon neutrino	mass → $< 15.5 \text{ MeV}/c^2$ charge → 0 spin → $1/2$  tau neutrino	mass → $80.4 \text{ GeV}/c^2$ charge → ± 1 spin → 1  W boson		
LEPTONS				GAUGE BOSONS		

[wikipedia]

Symmetries of the Standard Model

- Rephasing lepton and quark fields:

$$\begin{aligned} &U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \\ &= \\ &\cancel{U(1)_{B+L}} \times U(1)_{B-L} \times U(1)_{L_\mu-L_\tau} \times U(1)_{L_\mu+L_\tau-2L_e} . \end{aligned}$$

- Broken** non-perturbatively, but unobservable. [*t* Hooft, PRL '76]
- True accidental global symmetry:

$$U(1)_{B-L} \times U(1)_{L_\mu-L_\tau} \times U(1)_{L_\mu+L_\tau-2L_e} .$$

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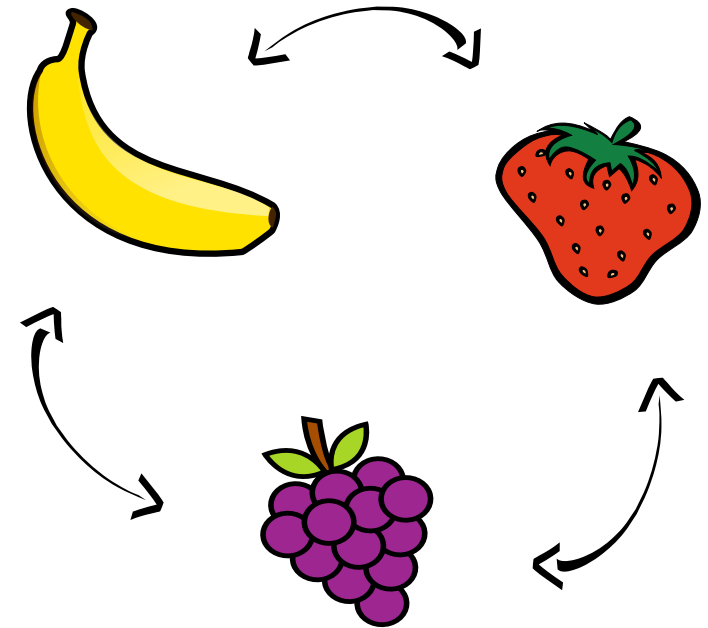
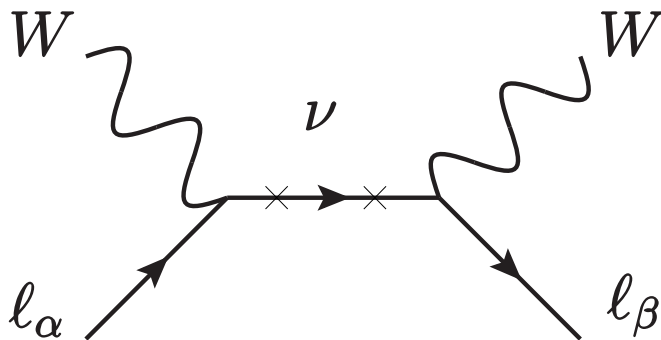
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Lepton flavor conservation!

Neutrino oscillations = flavor violation

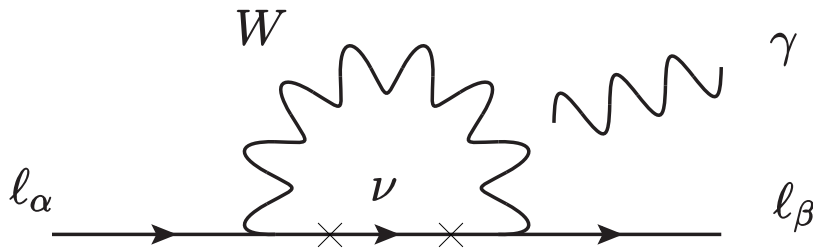
- Observations of $\nu_\alpha \rightarrow \nu_\beta$ prove that $M_\nu \neq 0$ and $U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$ is **broken**!
- Amplitudes for **charged lepton flavor violation** are suppressed:



$$\mathcal{A}(\ell_\alpha^- \rightarrow \ell_\beta^-) \propto \frac{(M_\nu M_\nu^\dagger)_{\alpha\beta}}{M_W^2} < 10^{-24}.$$

Neutrino mass \Rightarrow charged LFV?

- SM + **Dirac neutrinos**: $\mathcal{L} = \mathcal{L}_{\text{SM}} - \underbrace{(y\bar{L}H\nu_R + \text{h.c.})}_{m_\nu} + i\bar{\nu}_R \not{\partial} \nu_R$



$$m_\nu = y\langle H \rangle$$

$$= U \text{diag}(m_1, m_2, m_3) V_R$$

$$\stackrel{!}{\lesssim} \text{eV}$$

- All CLFV is GIM suppressed:

$$\frac{\Gamma(\ell_\alpha \rightarrow \ell_\beta \gamma)}{\Gamma(\ell_\alpha \rightarrow \ell_\beta \nu_\alpha \bar{\nu}_\beta)} \simeq \frac{3\alpha_{\text{EM}}}{32\pi} \left| \sum_{j=2,3} U_{\alpha j} \frac{\Delta m_{j1}^2}{M_W^2} U_{j\beta}^\dagger \right|^2 < 5 \times 10^{-53}.$$

[1977: Petcov; Bilenky, Petcov, Pontecorvo; Marciano, Sanda; Lee, Pakvasa, Shrock, Sugawara]

Seesaw mass \Rightarrow charged LFV?

- SM + **seesaw neutrinos**: $\mathcal{L} = \mathcal{L}_{\text{SM}} + i\bar{N}_R \not{\partial} N_R - \left(\frac{1}{2} M_R \bar{N}_R^c N_R + \underbrace{y \bar{L} H N_R}_{m_D \bar{\nu}_L N_R} + \text{h.c.} \right)$
- Violates $\Delta L = 2$. For large M_R :

$$M_N \simeq M_R, \quad M_\nu \simeq -m_D M_R^{-1} m_D^T = U^* \text{diag}(m_1, m_2, m_3) U^\dagger.$$

- Majorana** neutrinos!

- LFV:
$$\frac{\Gamma(\ell_\alpha \rightarrow \ell_\beta \gamma)}{\Gamma(\ell_\alpha \rightarrow \ell_\beta \nu_\alpha \bar{\nu}_\beta)} \simeq \frac{3\alpha_{\text{EM}}}{8\pi} \underbrace{|(m_D M_R^{-2} m_D^\dagger)_{\alpha\beta}|^2}_{\mathcal{O}(M_\nu^4/m_D^4)}.$$

[Cheng & Li '80]

Not true with fine-tuning or structure in m_D .

Seesaw parameters

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + i\bar{N}_R \not{\partial} N_R - \left(\frac{1}{2} M_R \bar{N}_R^c N_R + m_D \bar{\nu}_L N_R + \text{h.c.} \right)$$

$$\Rightarrow M_\nu \simeq -m_D M_R^{-1} m_D^T \quad \& \quad \text{BR}(\ell_\alpha \rightarrow \ell_\beta \gamma) \propto |(m_D M_R^{-2} m_D^\dagger)_{\alpha\beta}|^2.$$

- One to one correspondence

$$\{m_D, M_R\} \leftrightarrow \{M_\nu, m_D M_R^{-2} m_D^\dagger\}.$$

[Broncano, Gavela, Jenkins,
hep-ph/0210271]

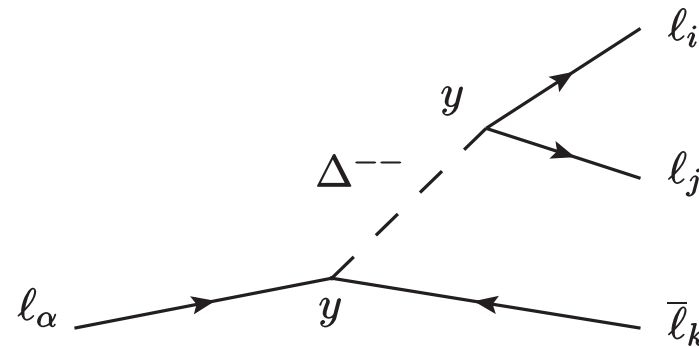
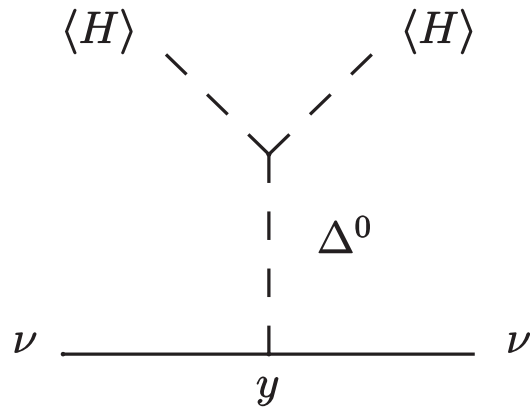
- Or: unique d=6 operator $(y M_R^{-2} y^\dagger)(\bar{L}H)(i\not{\partial})(H^\dagger L)$.
- Gives LFV and non-unitary PMNS.

LFV complementary to M_ν !

Scalar-triplet seesaw

[Konetschny & Kummer '77; Magg & Wetterich, '80; Schechter & Valle '80; Cheng & Li, '80; Mohapatra & Senjanovic, '81]

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + |D_\alpha \Delta|^2 - (y \bar{\ell}^c \Delta L + \mu H \Delta H + \text{h.c.})$$



$$\Rightarrow (M_\nu)_{\alpha\beta} \simeq y_{\alpha\beta} \frac{2\mu v^2}{M_\Delta^2} \quad \& \quad \text{BR}(\ell_\alpha \rightarrow \ell_i \ell_j \bar{\ell}_k) \propto |y_{\alpha k}|^2 |y_{ij}|^2 / M_\Delta^4.$$

[Pich, Santamaria, Bernabeu, '84; Abada++, 0707.4058]

Prediction of LFV *ratios* via M_ν !

CDF's W-mass first hint for this triplet with O(100 GeV) mass? [Heeck, 2204.10274]

Neutrino mass \nRightarrow charged LFV!

- Neutrino-mass induced charged LFV is **unobservable**.

Observation of CLFV \rightarrow beyond SM *and* beyond M_ν !

- $M_\nu \Leftrightarrow$ LFV connection possible but not necessary.
- Can ignore M_ν in CLFV studies!
- How do I pick/organize CLFV channels?
 $\ell \rightarrow \ell' \gamma, \ell \rightarrow \ell' \ell'' \ell''' , \mu \rightarrow e \text{ conv.}, h \rightarrow \ell \ell', \text{ had} \rightarrow \ell \ell', \dots$
- Heavy new physics: **SMEFT**!

SMEFT CLFV

- 864 CLFV operators at d=6:

$$\frac{C_{ijnm}}{\Lambda^2} \ell_i^c \ell_j \ell_n^c \ell_m, \frac{C_{ijnm}}{\Lambda^2} \ell_i^c \ell_j d_n^c d_m, \frac{C_{ij}}{\Lambda^2} \ell_i^c \sigma_{\alpha\beta} \ell_j F^{\alpha\beta}, \dots$$

[Weinberg '79; Buchmüller & Wyler, '86; Grzadkowski++, '10; Fonseca, '17]

- Model-dependent coefficients; can get testable rates:

$$\ell \rightarrow \ell' \gamma, \ell \rightarrow \ell' \ell'' \ell''', \mu \rightarrow e \text{ conv.}, h \rightarrow \ell \ell', \text{had} \rightarrow \ell \ell', \dots$$

Flavor violating decays

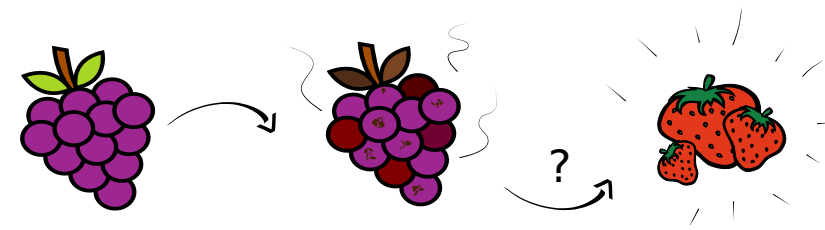


- Prime example: $\mu \rightarrow e \gamma$ @ MEG.
- Observation = new particles (beyond SM and M_ν).
- $\mu \rightarrow e$ conversion @ Mu2e can probe scales up to 10^7 GeV.

See talk by
Sophie Middleton.

LFV	process	current	future	exp
$ \Delta L_\mu = 1$ $ \Delta L_e = 1$	$\mu \rightarrow e \gamma$	4.2×10^{-13}	6×10^{-14}	MEG-II
	$\mu \rightarrow e \bar{e} e$	1.0×10^{-12}	10^{-16}	Mu3e
	$\mu \rightarrow e$ conv.	$\mathcal{O}(10^{-12})$	10^{-16}	Mu2e, COMET
	$h \rightarrow e \bar{\mu}$	6.1×10^{-5}	10^{-5}	LHC
	$Z \rightarrow e \bar{\mu}$	7.5×10^{-7}	10^{-10}	FCC-ee
	$\text{had} \rightarrow e \bar{\mu}(\text{had})$	4.7×10^{-12}	10^{-12}	NA62

Flavor violating decays



- Produce tauons at B factories (BaBar, Belle, LHCb).
- Observation = **new particles** (beyond SM *and* M_ν).
- $\tau^- \rightarrow e^- e^+ e^-$ @ Belle II will probe scales up to $2 \times 10^4 \text{ GeV}$.

LFV	process	current	future	exp
$ \Delta L_\tau = 1$	$\tau \rightarrow e \gamma$	3.3×10^{-8}	10^{-9}	Belle II
	$\tau \rightarrow e \bar{\ell} \ell$	2.7×10^{-8}	10^{-9}	Belle II
	$\tau \rightarrow e \text{ had}$	$\mathcal{O}(10^{-8})$	10^{-9}	Belle II
$ \Delta L_e = 1$	$h \rightarrow e \bar{\tau}$	4.7×10^{-3}	10^{-4}	LHC
	$Z \rightarrow e \bar{\tau}$	9.8×10^{-6}	10^{-9}	FCC-ee
	$\text{had} \rightarrow e \bar{\tau}(\text{had})$	$\mathcal{O}(10^{-6})$	–	Belle II

See talk by
Swagato Banerjee.

SMEFT CLFV

- 864 CLFV operators at d=6:

$$\frac{C_{ijnm}}{\Lambda^2} \ell_i^c \ell_j \ell_n^c \ell_m, \frac{C_{ijnm}}{\Lambda^2} \ell_i^c \ell_j d_n^c d_m, \frac{C_{ij}}{\Lambda^2} \ell_i^c \sigma_{\alpha\beta} \ell_j F^{\alpha\beta}, \dots$$

[Weinberg '79; Buchmüller & Wyler, '86; Grzadkowski++, '10; Fonseca, '17]

- Model-dependent coefficients; can get testable rates:

$$\ell \rightarrow \ell' \gamma, \ell \rightarrow \ell' \ell'' \ell''', \mu \rightarrow e \text{ conv.}, h \rightarrow \ell \ell', \text{had} \rightarrow \ell \ell', \dots$$

- Not all constrained, e.g. $\Delta L_\tau = 2$ operators.

- CLFV even sensitive to some d=8 operators, e.g. $\frac{\bar{L}_\mu \text{He}_R G G}{\Lambda^4}$.

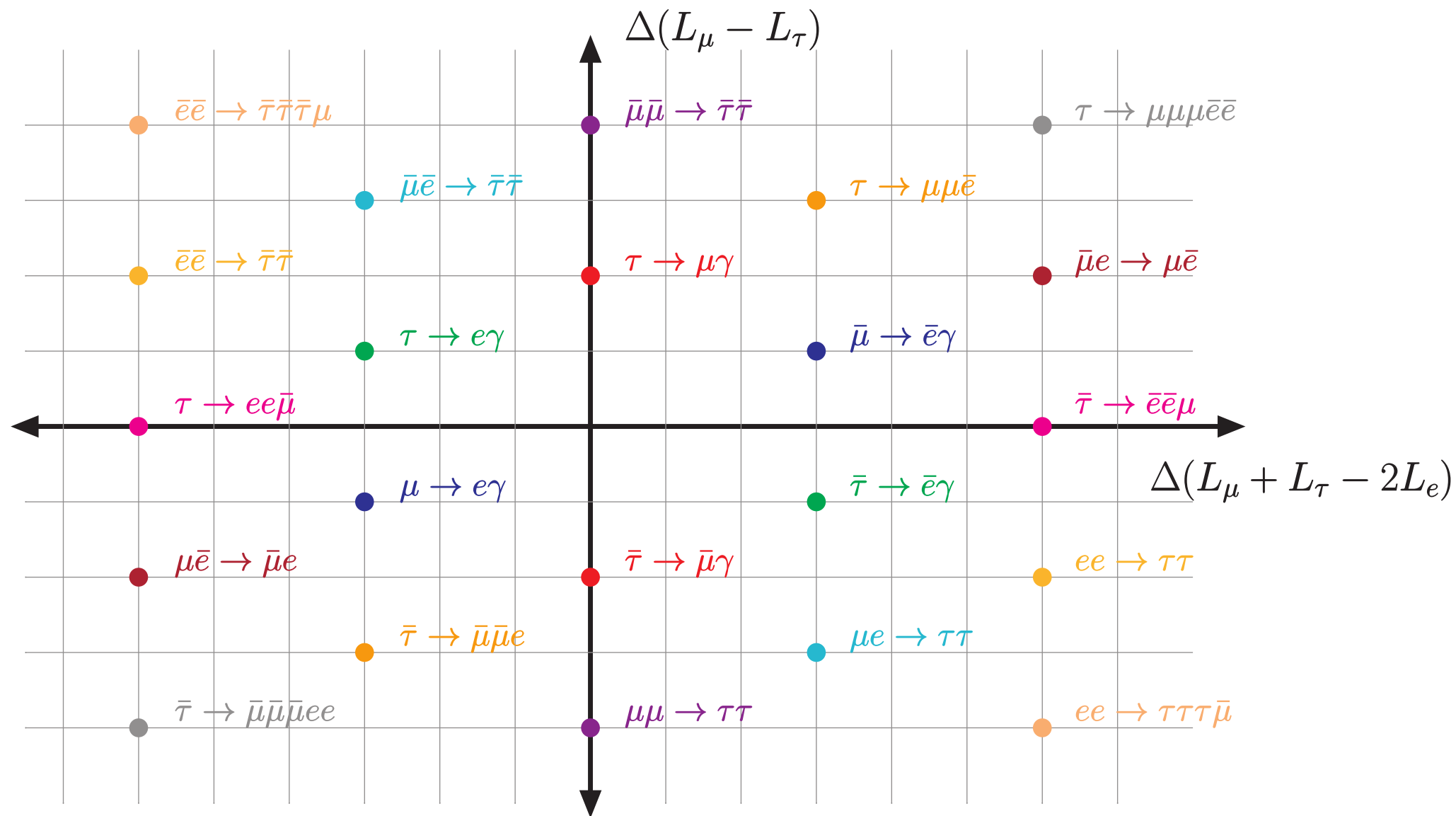
[Davidson, Kuno, Uesaka, Yamanaka, 2007.09612;
Ardu & Davidson, 2103.07212]

- Not clear if / how $U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$ is broken in CLFV.

[Lew & Volkas, 9410277; Heeck, 1610.07623]

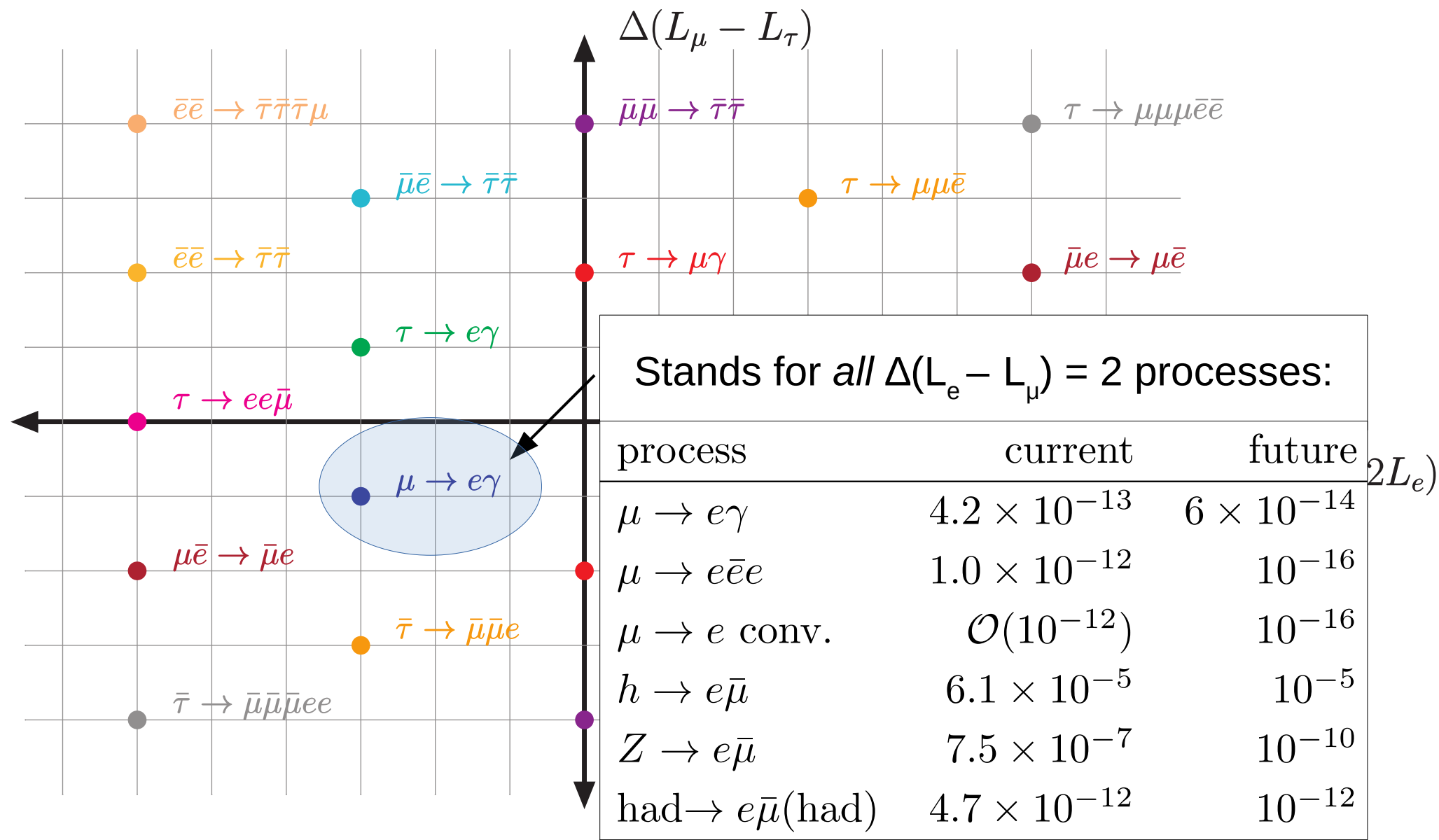
CLFV = breaking of $U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$

[Heeck, 1610.07623]



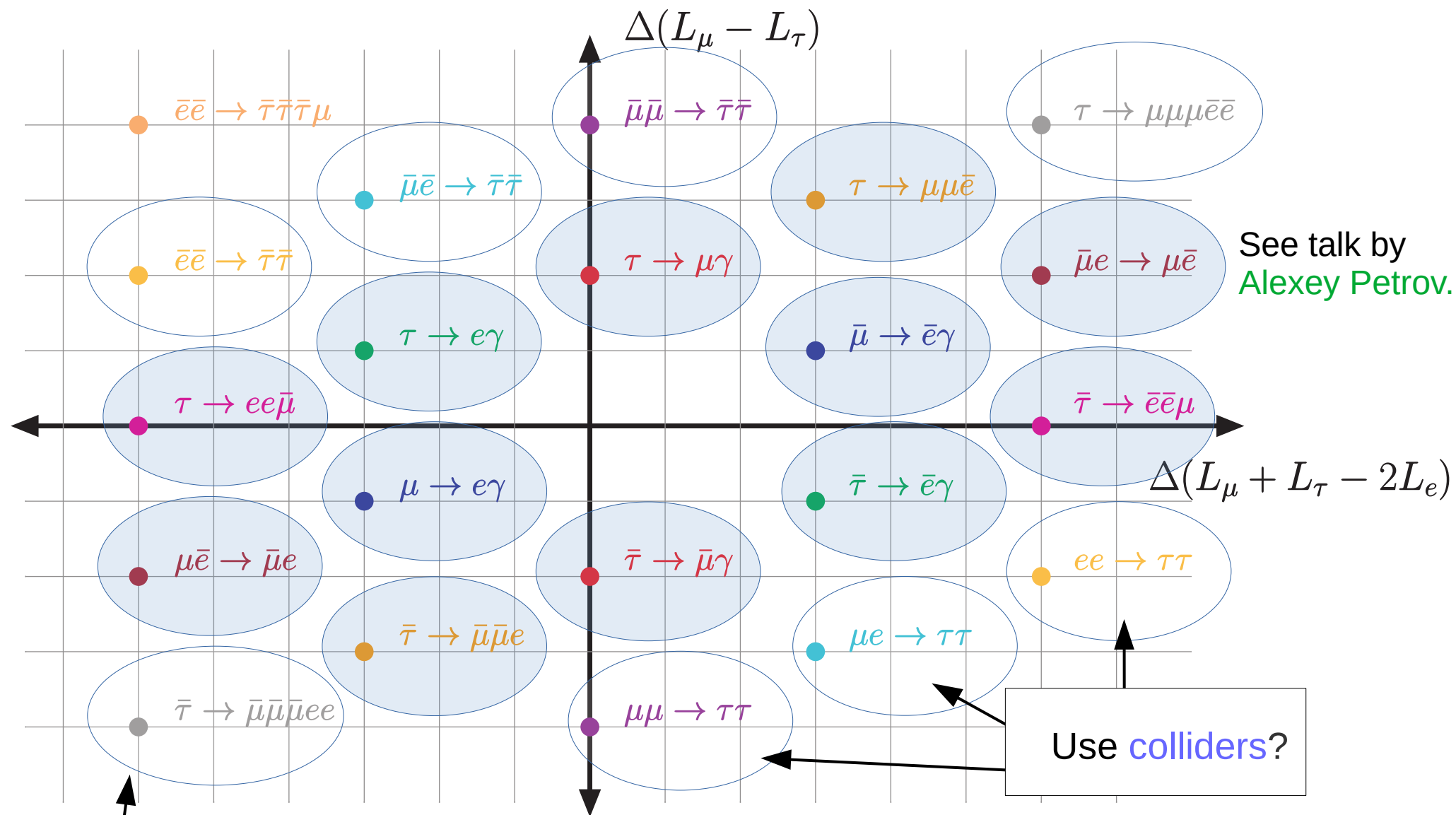
CLFV = breaking of $U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$

[Heeck, 1610.07623]



Currently being probed: Future:

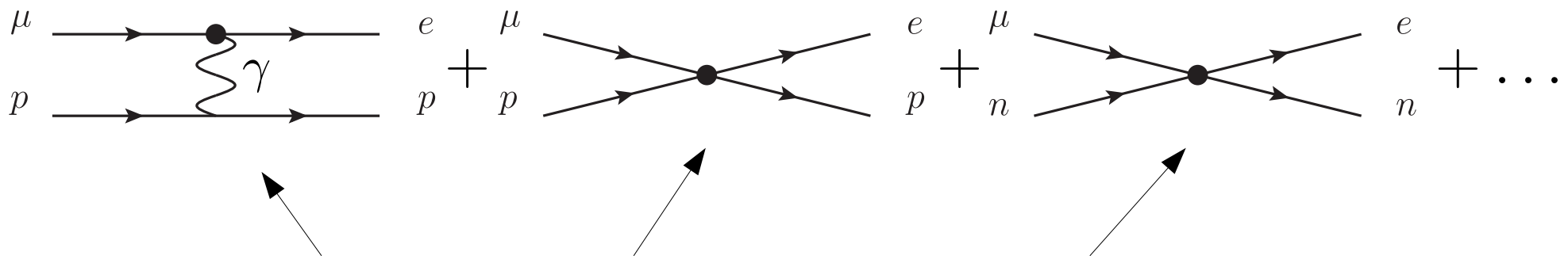
[Heeck, 1610.07623]



[Altmannshofer, Chen, Dev, Soni, PLB '16]

The inverse problem

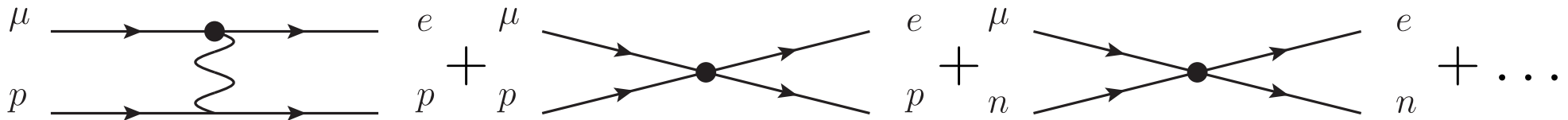
- If we see CLFV, can we pin down the underlying operator?
 - In many cases: **Yes!** (e.g. $\mu \rightarrow e\gamma \leftrightarrow$ dipole)
 - $\mu \rightarrow e$ conversion in nucleus: **No!**



Relative contributions depend on **nucleus**: Z, N, spin!

- **Need to observe $\mu \rightarrow e$ conversion in different nuclei!**
 [Kitano, Koike, Okada, PRD '07; Cirigliano++, PRD '09; Davidson++, '18]

$\mu \rightarrow e$ conversion



- Assuming spin-*independent* conversion:

$$\text{BR}_{\text{SI}} = \frac{32G_F^2}{\Gamma_{\text{capture}}} [|\mathbf{v} \cdot \mathbf{C}_L|^2 + |\mathbf{v} \cdot \mathbf{C}_R|^2]$$

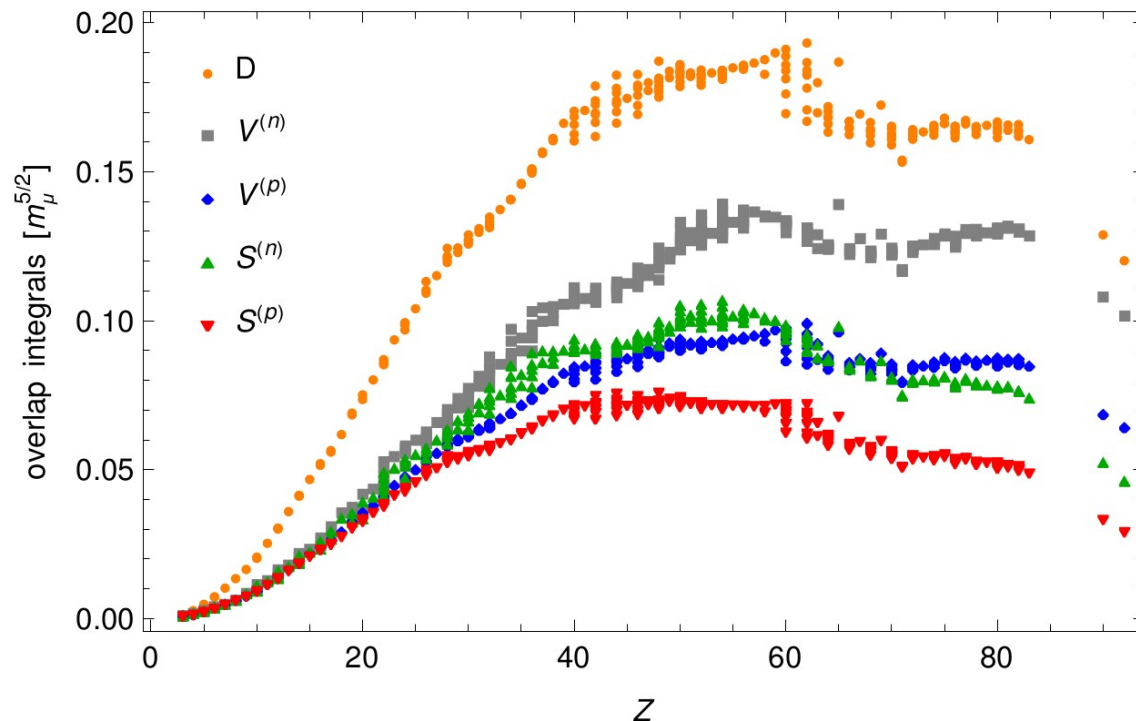
Overlap integrals

Wilson coefficients

$$\mathbf{v} \equiv \left(\frac{D}{4}, V^{(p)}, S^{(p)}, V^{(n)}, S^{(n)} \right)$$

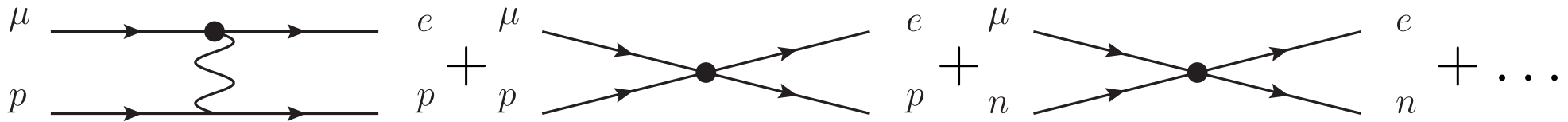
To measure the Wilson coefficients, use nuclei whose \mathbf{v} are maximally misaligned.

[Davidson, Kuno, Yamanaka, PLB '19]

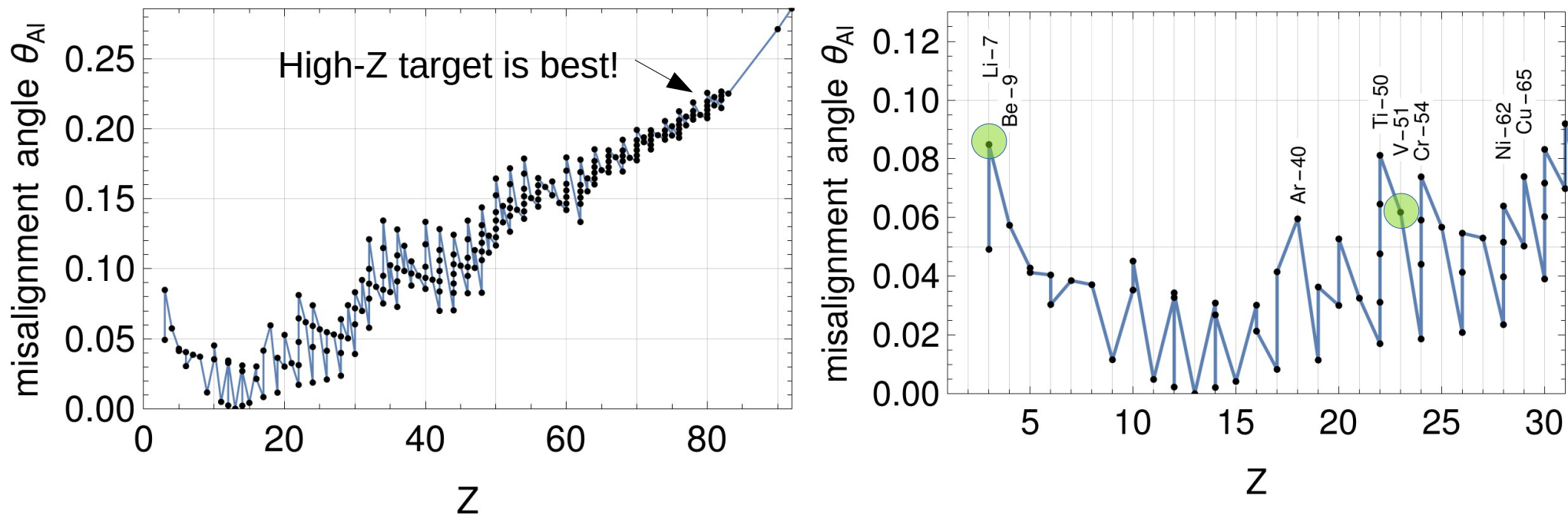


[Kitano, Koike, Okada, PRD '07; Heeck, Szafron, Uesaka, NPB '22]

$\mu \rightarrow e$ conversion



- Misalignment with aluminium (target in COMET & Mu2e):



- At low Z , **Li-7** and **V-51** can distinguish proton/neutron.

[Davidson, Kuno, Yamanaka, PLB '19; Heeck, Szafron, Uesaka, NPB '22]

Probing *light* particles

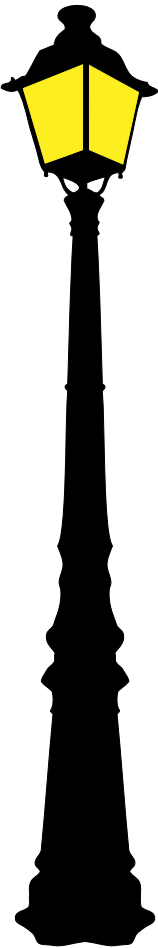
- SMEFT only works for *heavy* new particles!
- *Light* new particles X give new signatures:
- $\mu \rightarrow e X$ or $\tau \rightarrow \ell X$, followed by (displaced) $X \rightarrow \ell^+ \ell^-$, $\gamma\gamma$?
[Heeck & Rodejohann, PLB '18; Cheung++, JHEP '21]
- **Mu3e** and **Belle II** can improve limits, maybe others too?
[i Tormo++, PRD '11; Uesaka, PRD '20; Calibbi, Redigolo, Ziegler, Zupan, JHEP '21]
- Light particles as **mediators** change rate expectations.
- X = axion/ALP/majoron/familon/ Z' , connected to DM?
- Or: SMEFT + X .
[Georgi, Kaplan, Randall, '86; Brivio++, '17; Dror, Lasenby, Pospelov, '17 & '19]

Far from finished!

Summary

- Charged LFV gives info *complementary* to ν oscillations.
- **Generically** predicted by BSM, could be around the corner.
- Difficult to **predict** LFV *rates*, needs
 - Fixed flavor structure (neutrino mass, CKM?)
 - Fixed new physics scale (DM, anomalies?)
- **Light new physics** open new avenues.
- Hope for sign in Mu3e, MEG-II, Belle-II, Mu2e, LHC(b),...

Explore every corner of our lamppost!



Backup

Effective field theory view

- SM symmetry: $G = U(1)_{B-L} \times U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$.
- Effective field theory with **Majorana ν** :

$$L = L_{\text{SM}} + \frac{\overbrace{LLHH}^{M_\nu}}{\Lambda} + \underbrace{\sum_j \frac{\mathcal{O}_j}{\Lambda^2} + \sum_j \frac{\mathcal{O}'_j}{\Lambda^3} + \sum_j \frac{\mathcal{O}''_j}{\Lambda^4} + \dots}_{\text{could conserve } G \text{ or subgroup} \Rightarrow \text{'weird' channels dominate!?!}}$$

\nearrow conserves G
 \nearrow violates G

Upcoming CLFV

84

LORENZO CALIBBI and GIOVANNI SIGNORELLI

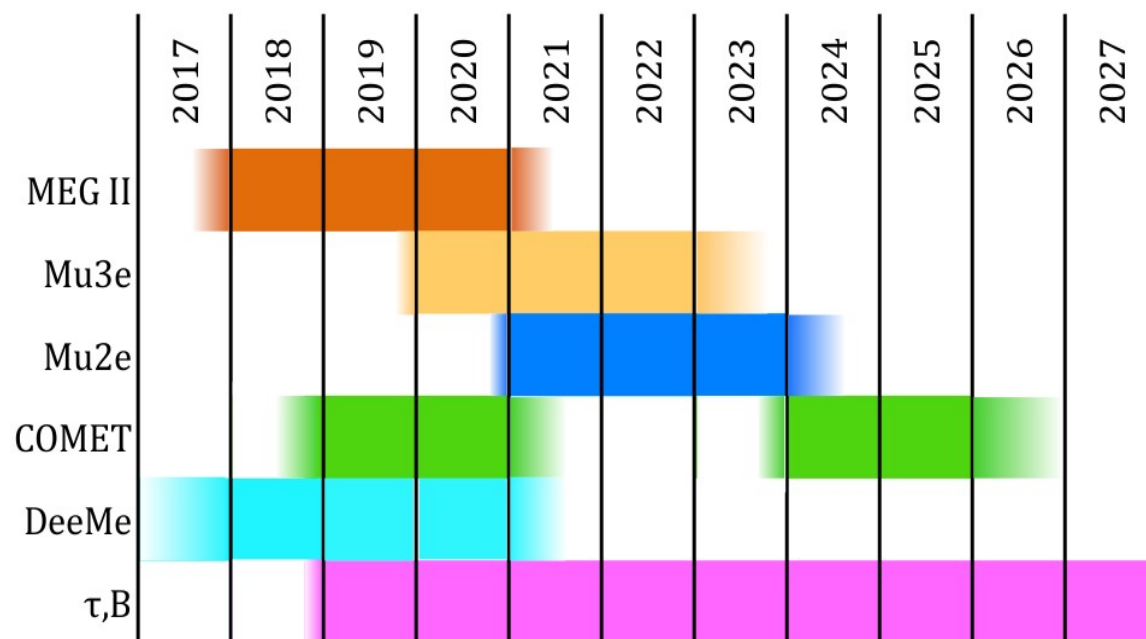
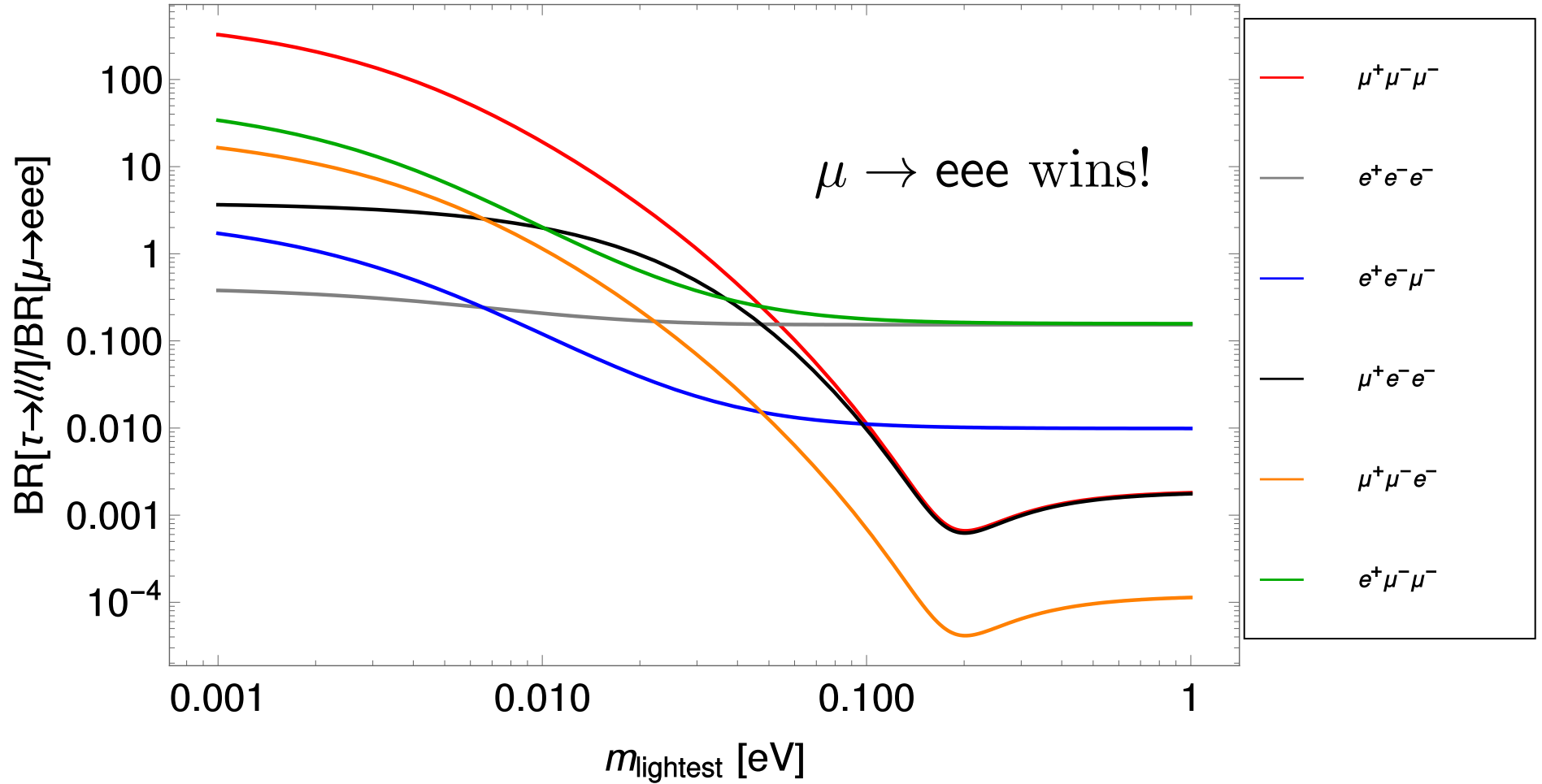


Figure 47. – Projected time lines for different projects searching for CLFV decays. MEG II is expected to start data taking in 2018 after an engineering run in 2017; Mu3e magnet and detectors are expected at the end of 2019; Mu2e foresees three years of data taking starting in 2021; COMET Phase-I is expected to start commissioning and data taking in 2018 for two-three years, followed by a stop to develop and deploy the beamline and detectors for Phase-II; DeeMe is expected to start soon and take data with graphite and silicon carbide targets in sequence; Belle II is schedule to start data taking at end 2018.

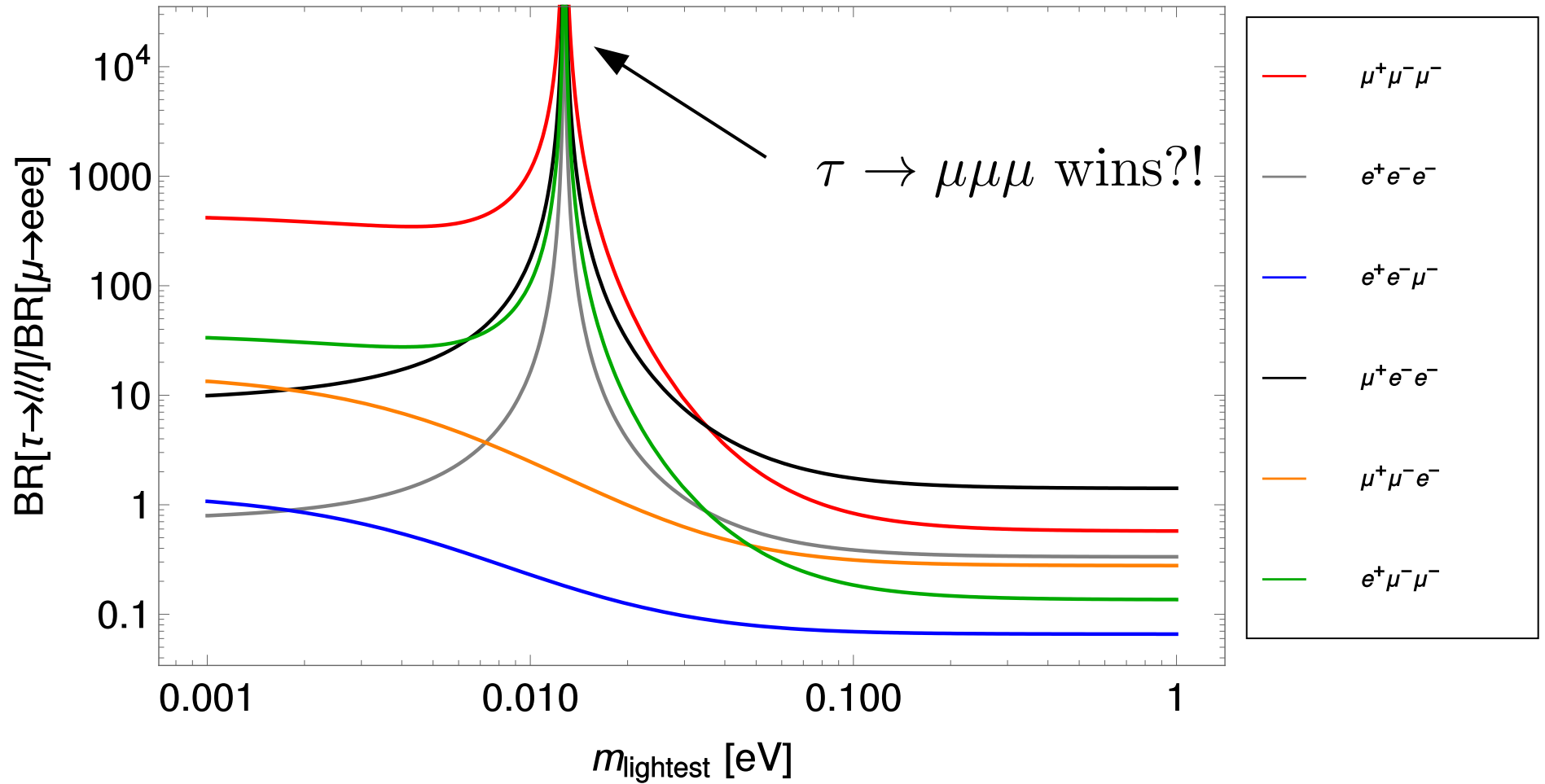
[Calibbi & Signorelli, 1709.00294]

Normal hierarchy, $\alpha=\beta=0$



$$(M_\nu)_{\alpha\beta} \simeq y_{\alpha\beta} \frac{2\mu v^2}{M_\Delta^2} \quad \& \quad \text{BR}(\ell_\alpha \rightarrow \ell_i \ell_j \bar{\ell}_k) \propto |y_{\alpha k}|^2 |y_{ij}|^2 / M_\Delta^4.$$

Normal hierarchy, $\alpha, \beta: (M_\nu)_{e\mu} \sim 0$

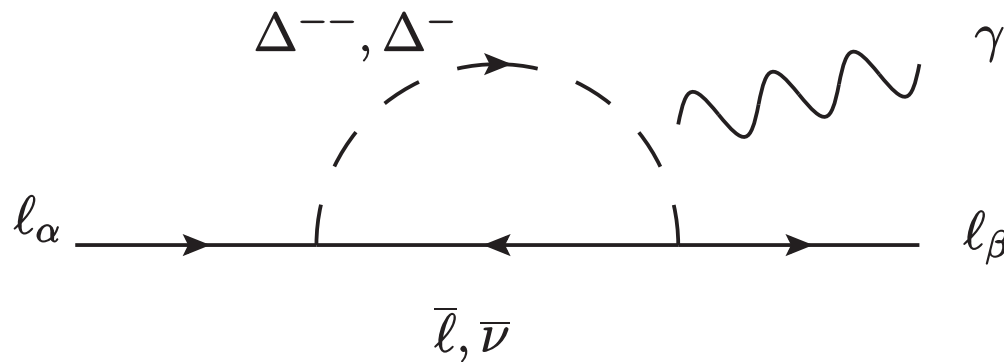


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Scalar-triplet seesaw

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- But at loop level:



$$\text{BR}(\ell_\alpha \rightarrow \ell_\beta \gamma) \propto \frac{|(y^\dagger y)_{\alpha\beta}|^2}{M_\Delta^4}.$$

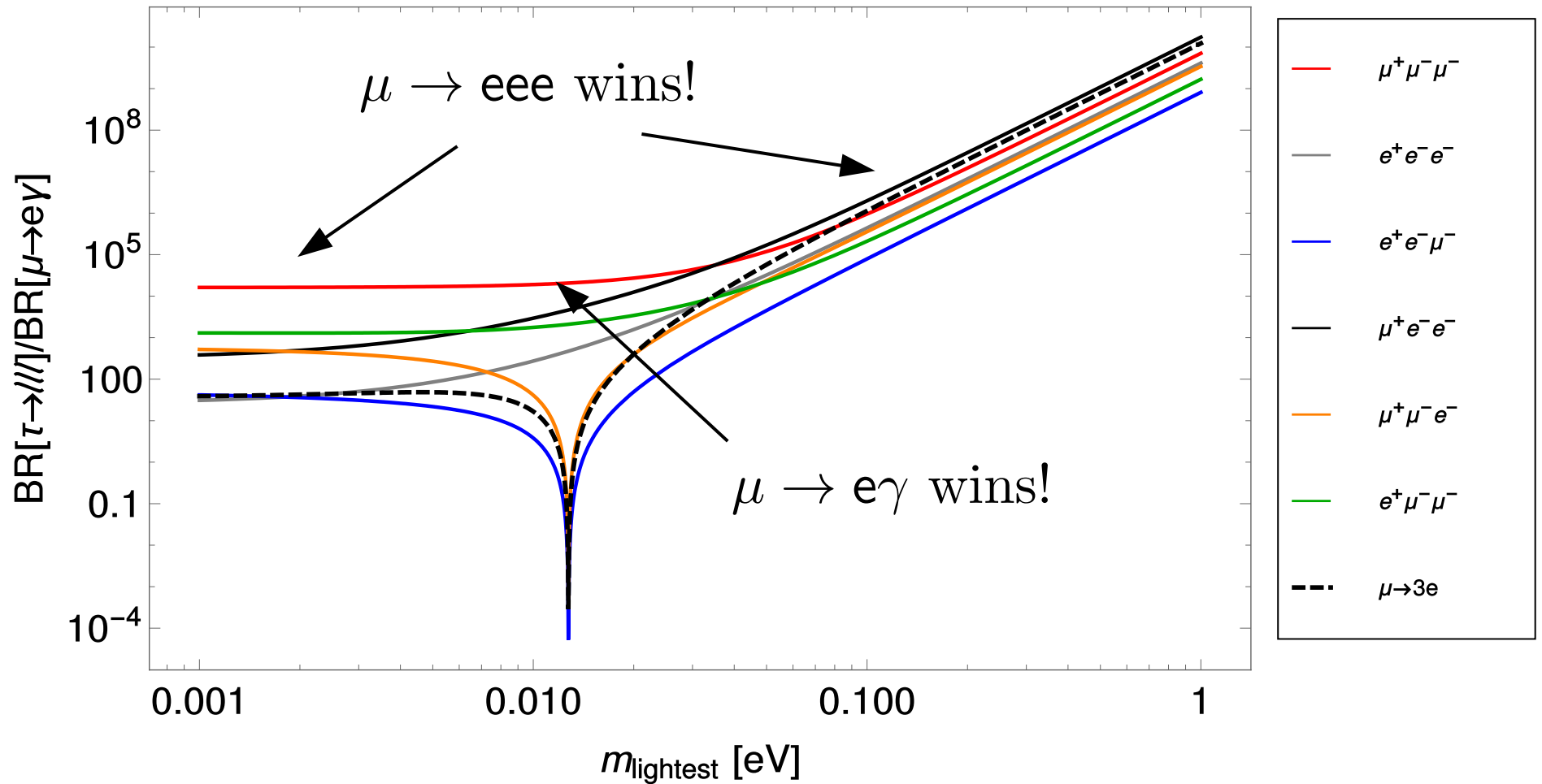
[Pich, Santamaria, Bernabeu, '84]

- $\mu \rightarrow 3e$ could be 0, but $\mu \rightarrow e\gamma$ cannot (since θ_{13}).

[Chakraborty++, 1204.1000]

Prediction of LFV ratios via M_ν !

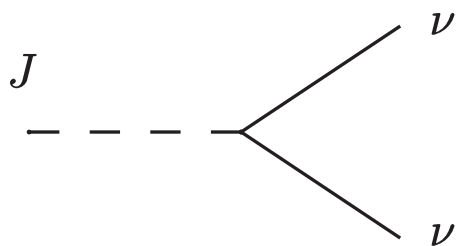
Normal hierarchy, $\alpha, \beta: (M_\nu)_{e\mu} \sim 0$

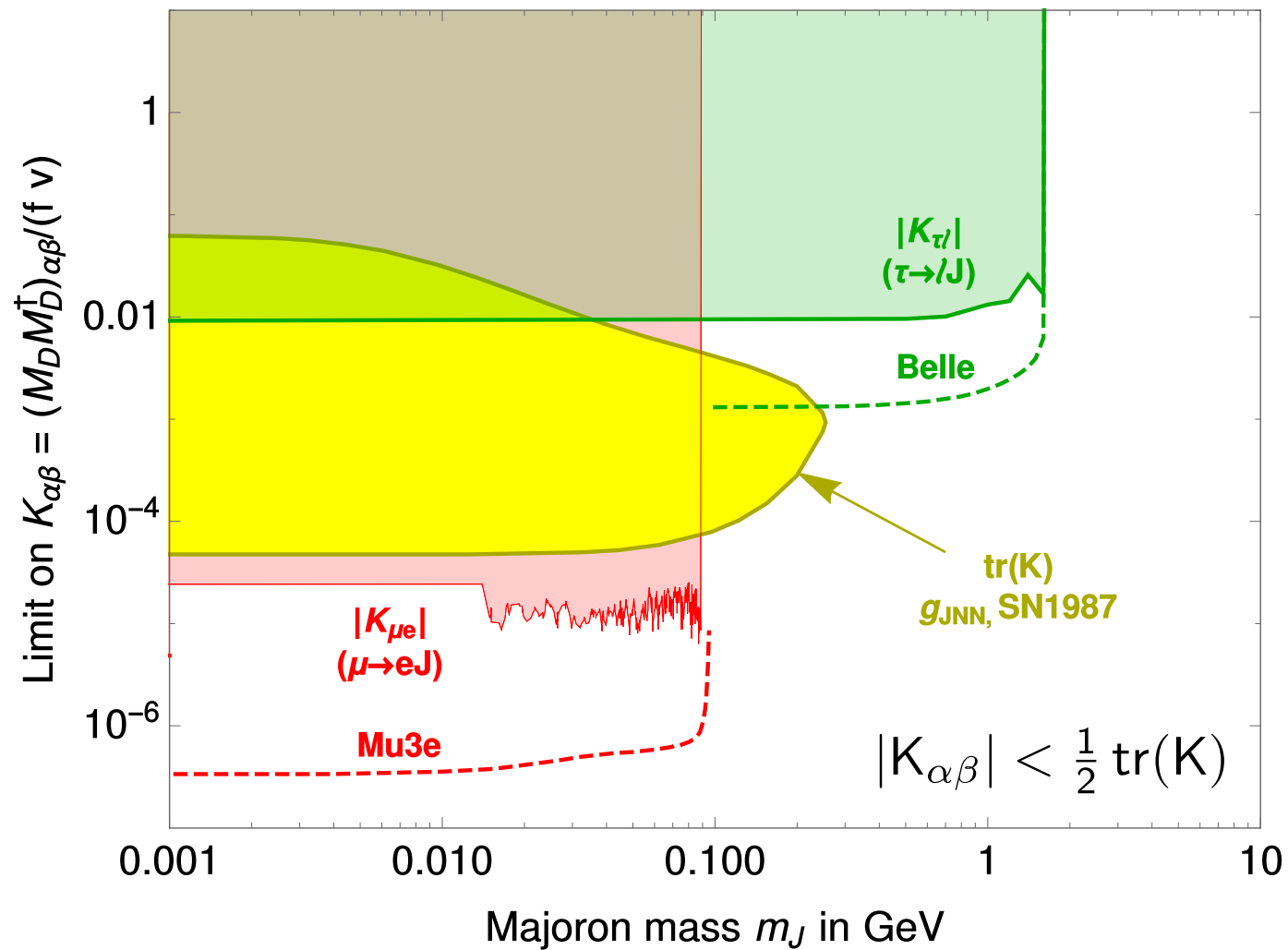
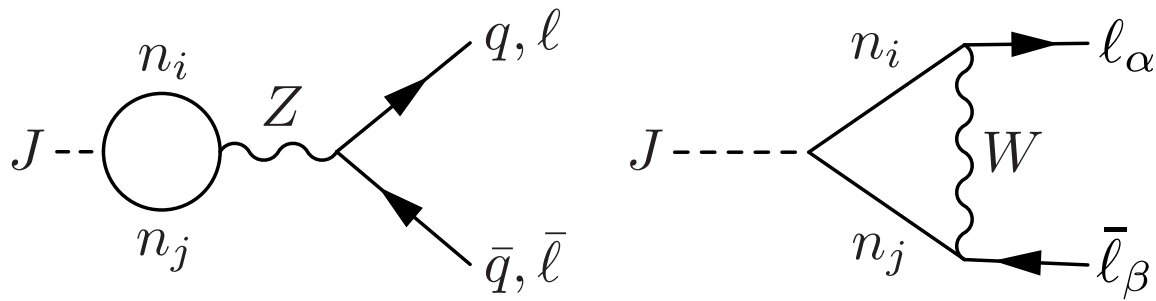


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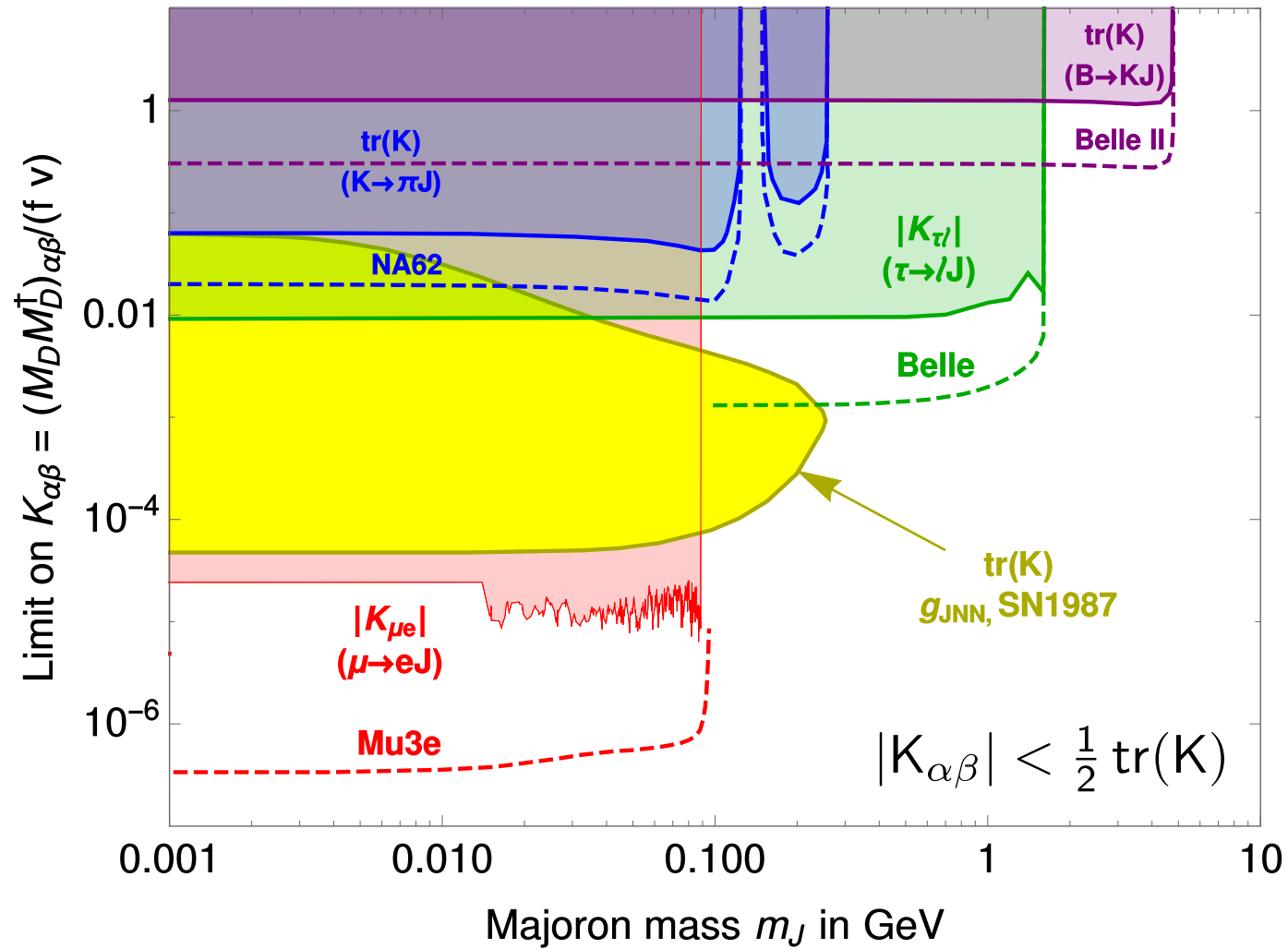
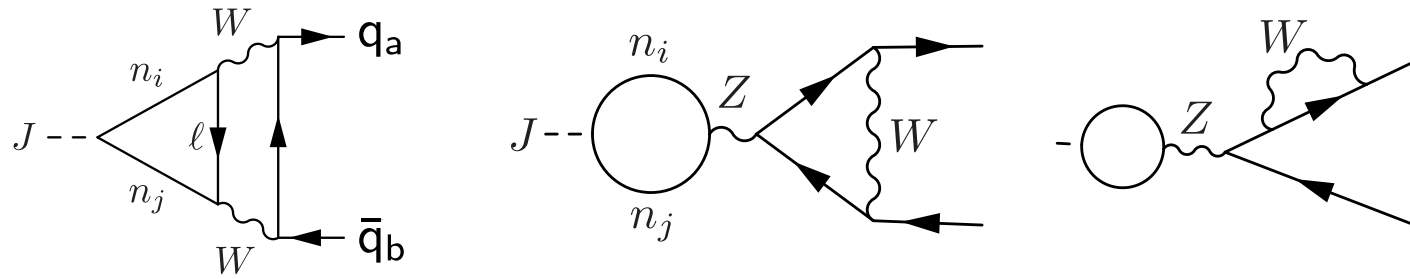
Probing light particles

- Mu3e: $\text{BR}(\mu \rightarrow e X)$ from 10^{-6} to 10^{-8} .
- Belle II: $\text{BR}(\tau \rightarrow \ell X)$ from 10^{-3} to 10^{-5} . [JH, PLB '16]
- Followed by (displaced) $X \rightarrow \ell^+ \ell^-, \gamma\gamma$? [JH, Rodejohann, PLB '18]
- Example: Majoron.
 - Pseudo-Goldstone boson of lepton number.
 - Potential dark matter candidate. [JH, Garcia-Cely, JHEP '17]
 - Tree-level coupling only to neutrinos.





[JH, Garcia-Cely, JHEP '17]



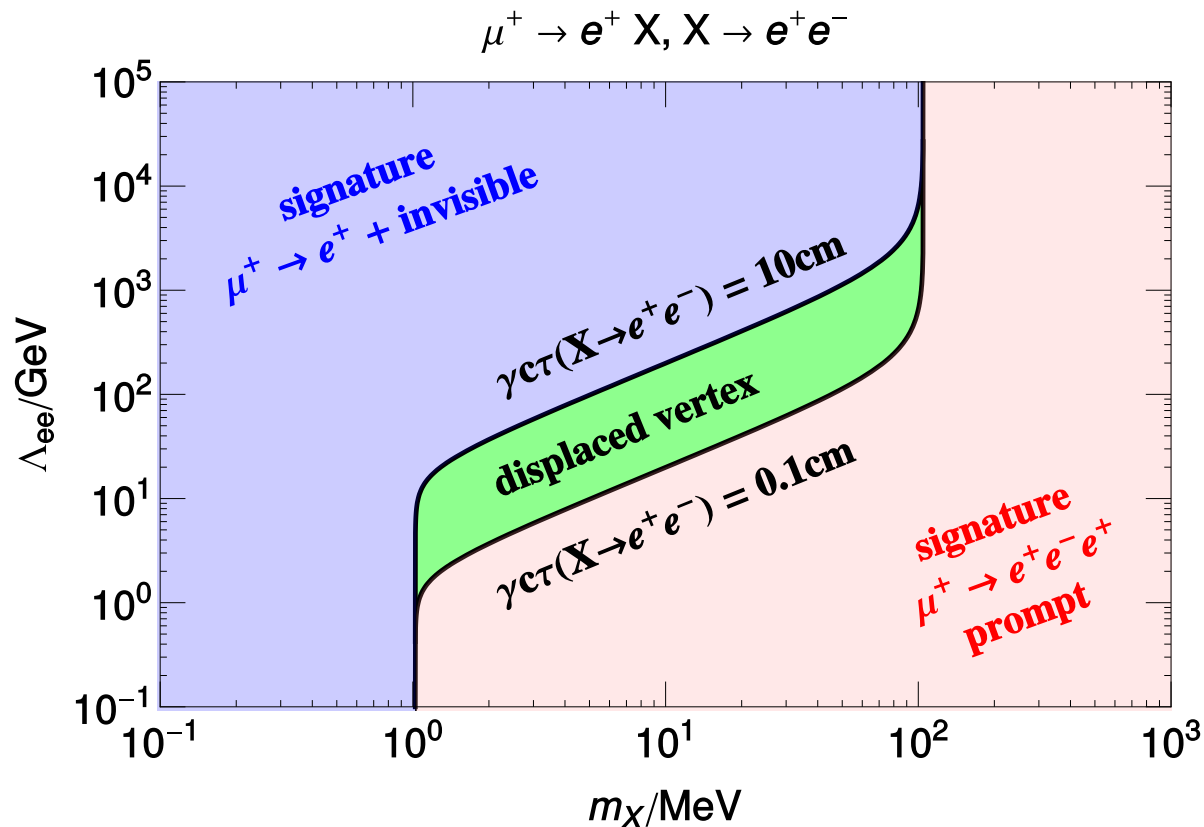
[JH, Patel, PRD '19]

$\mu \rightarrow e X$ with $X \rightarrow$ visible

- Take $X \bar{e} \gamma_5 e$ m_e / Λ_{ee} .
- Decay length determines signature.
- Displaced vertex gives new observable.
[JH, Rodejohann, PLB '18]

- Muon at rest:

$$\gamma c \tau \simeq \frac{\pi m_\mu \Lambda_{ee}^2}{m_e^2 m_X^2} \simeq 2.5 \text{ cm} \left(\frac{\Lambda_{ee}}{100 \text{ GeV}} \right)^2 \left(\frac{10 \text{ MeV}}{m_X} \right)^2.$$

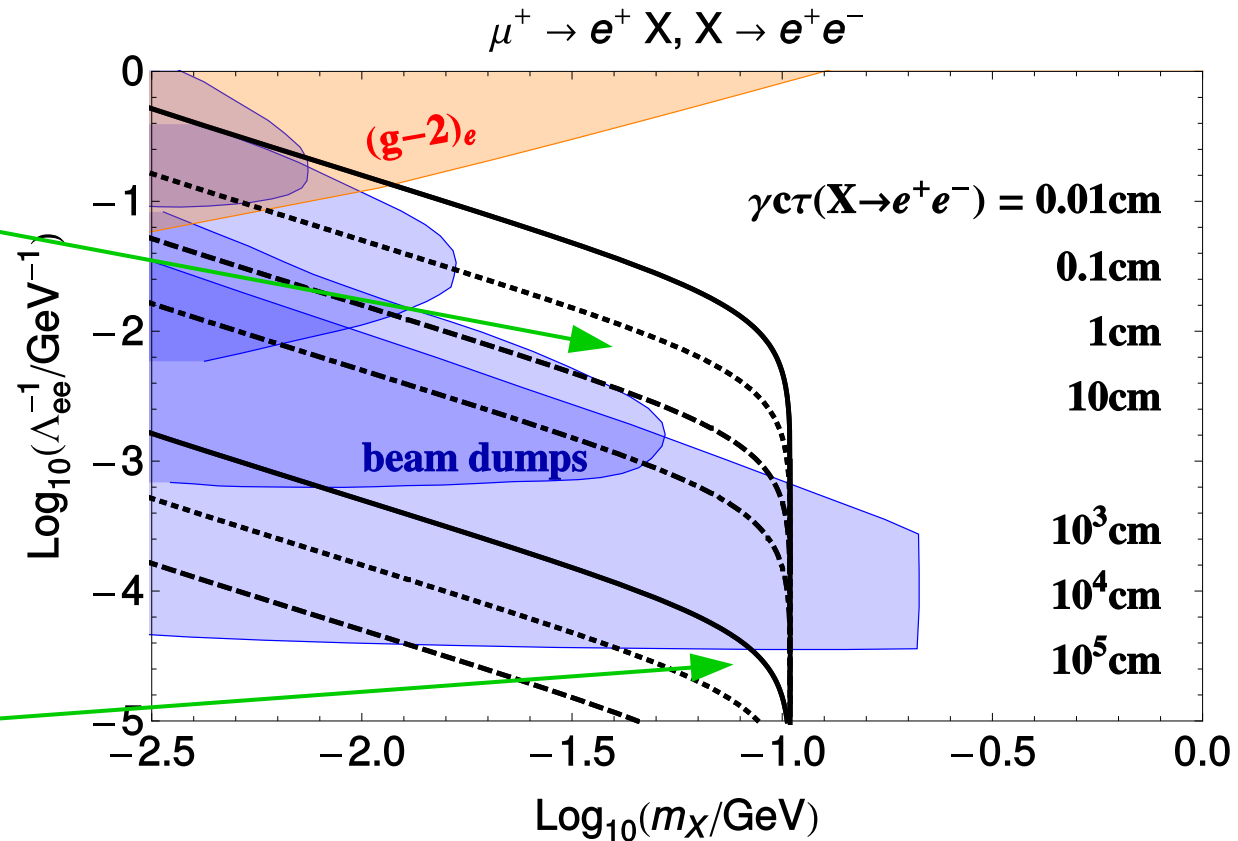


Sub-GeV X with ee coupling allowed?

$\mu \rightarrow e X$ with $X \rightarrow \bar{e}e$

[JH, Rodejohann, PLB '18]

- Decay length typically below cm.
⇒ looks prompt.
- Below beam dump:
 $\Lambda_{ee} > 30$ TeV;
mostly invisible, but some DV!



$$\text{BR}(\mu \rightarrow eX) \text{BR}(X \rightarrow ee) (1 - P(l_{\text{dec}}))$$

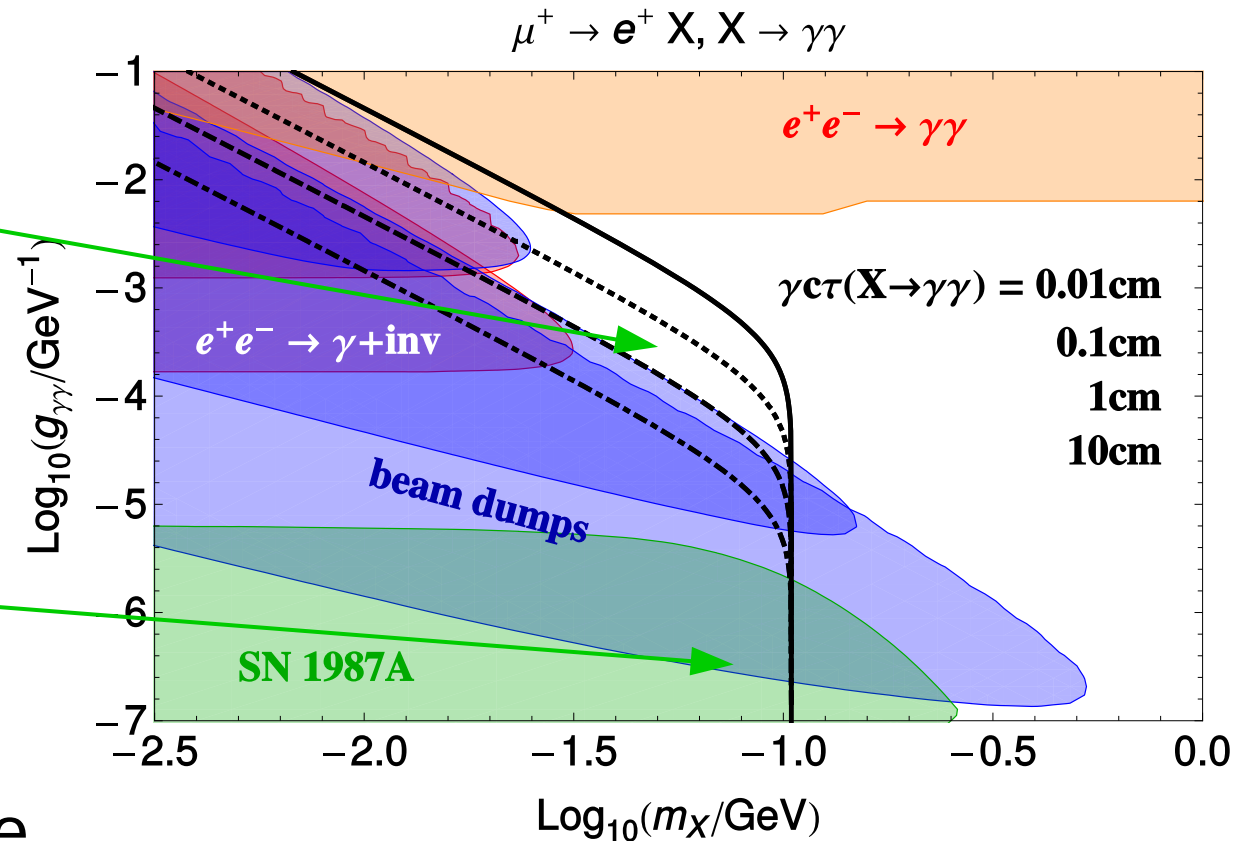
$$\simeq \text{BR}(\mu \rightarrow eX) \frac{l_{\text{dec}}}{\gamma c \tau}.$$

Possible in
Mu3e!

$\mu \rightarrow e X$ with $X \rightarrow \gamma\gamma$

[JH, Rodejohann, PLB '18]

- Decay length always below cm.
⇒ looks prompt.
- Below beam dump:
supernova constraints!
- Prompt channel still interesting, maybe MEG(II) or Mu3e extension?



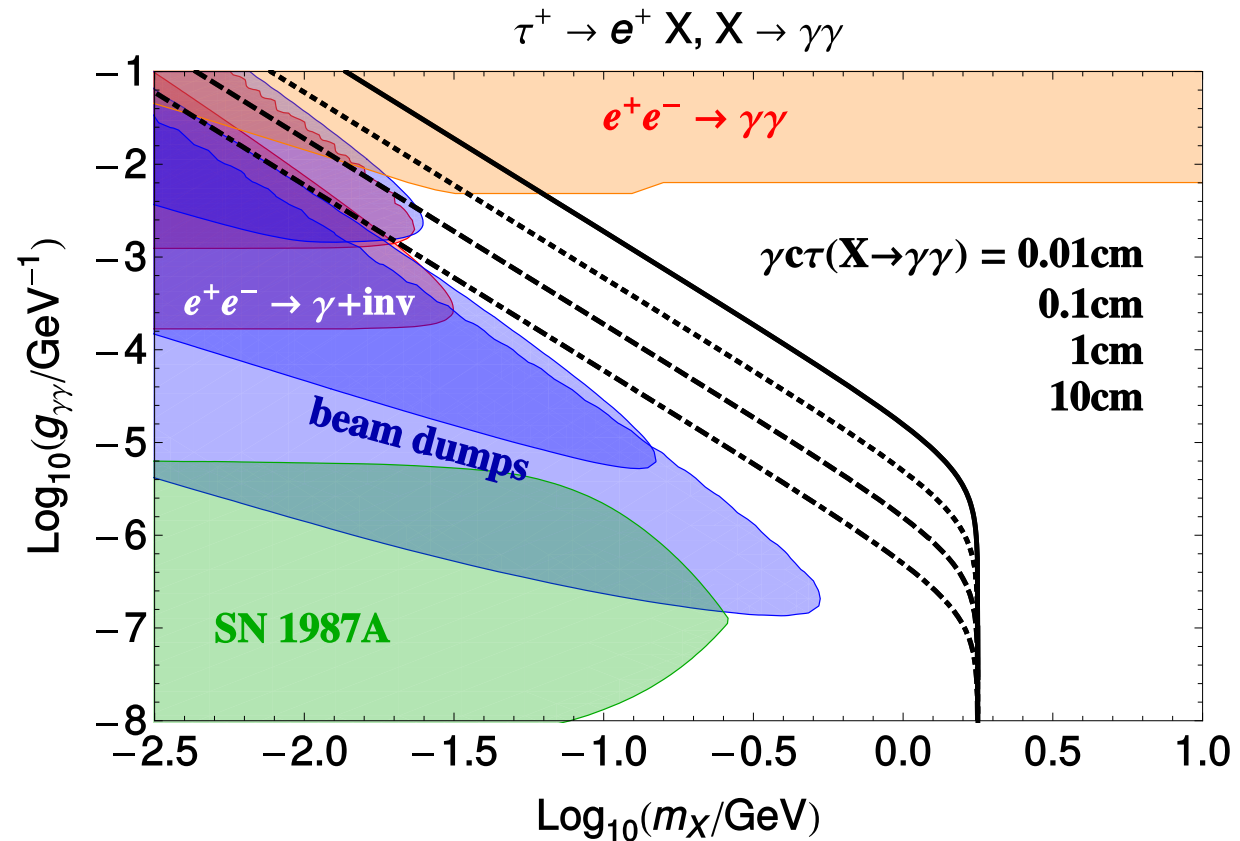
[Limits: Dolan et al, JHEP '17]

Muons difficult, taus easier.

$\tau \rightarrow e X$ with $X \rightarrow$ visible

[JH, Rodejohann, PLB '18]

- Tau at rest, higher X boost.
- Arbitrary decay lengths possible.
- Similar for $X \rightarrow ee, \mu\mu, \mu e$.
- Worthwhile in LHCb and Belle (II).



[Limits: Dolan et al, JHEP '17]

New signatures from light physics!