Simulating 1+1D Z_2 Gauge Theory on NISQ Hardware

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Yang, Liu, Gorshkov, and **TI**, PRL **124**, 207602 (2020)

TI and Schecter, PRB **101**, 024306 (2020)

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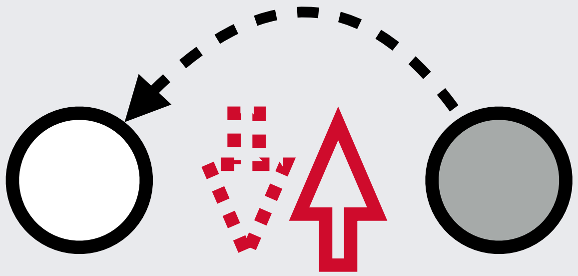
1+1D Z_2 Gauge Theory w/ Fermionic Matter

$$H = \lambda \sum_i \left(c_i^\dagger \, \sigma^x_{i,i+1} \, c_{i+1} + \mathrm{H.c.} \right) + h \sum_i \sigma^z_{i,i+1}$$



Gauge generators

$$G_i = \sigma^z_{i-1,i} (-1)^{c_i^\dagger c_i} \sigma^z_{i,i+1}$$

Locality preserving map

[Borla *et al.*, PRL **124**, 120503 (2020)]

Fermions ⟷ Ising domain walls

Simple spin model (one spin-1/2 per site)

$$H = \frac{\lambda}{2} \sum_i (X_i - Z_{i-1} X_i Z_{i+1}) + h \sum_i Z_i$$

Gauss's law is "baked in!"

Attractive to simulate on NISQ hardware?

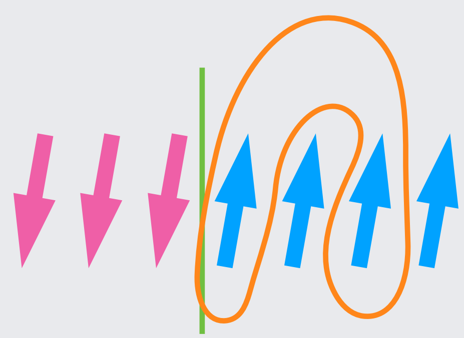
Simulating the Spin Model

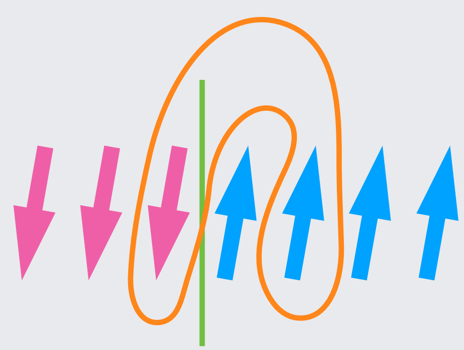
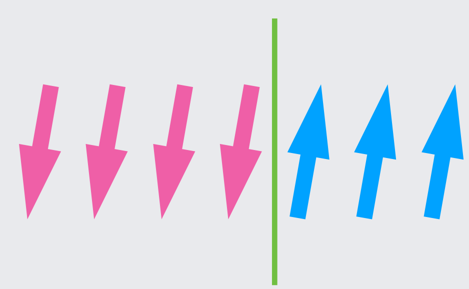
Trotter circuit for this model? $H = \frac{\lambda}{2}\sum_i (X_i - Z_{i-1}X_iZ_{i+1}) + h\sum_i Z_i$

Field term is easy, kinetic term is weird

$$\frac{1}{2}(X_i - Z_{i-1}X_iZ_{i+1}) = X_i\left(\frac{1 - Z_{i-1}Z_{i+1}}{2}\right)$$

Domain-wall hopping

 $\longrightarrow$ 0

 $\longrightarrow$ 

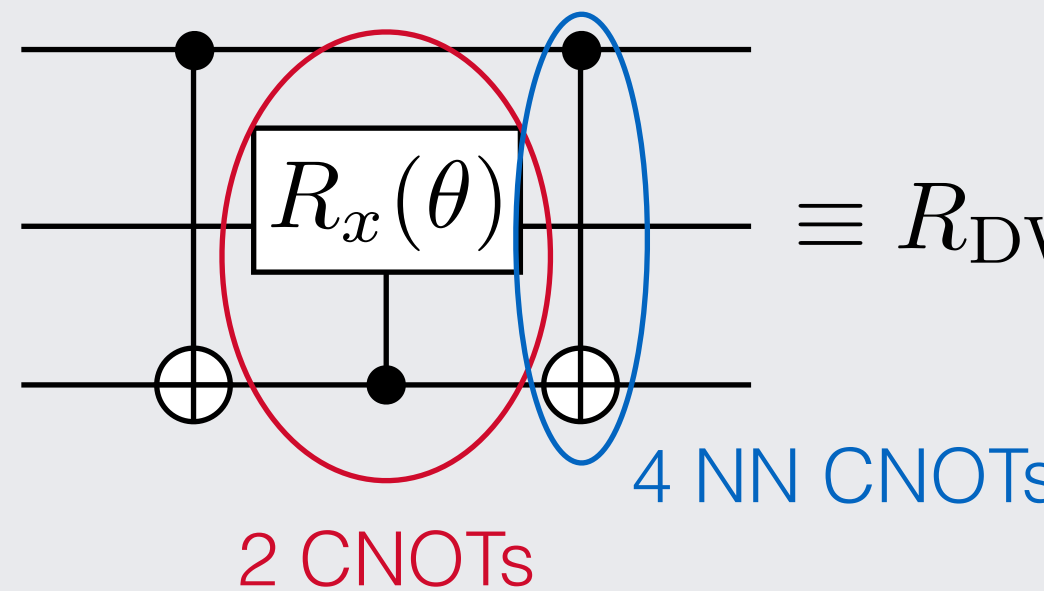
Spin flip conditioned on joint parity of nearest neighbors

Simulating the Spin Model

$$\frac{1}{2}(X_i - Z_{i-1}X_iZ_{i+1}) = X_i\left(\frac{1 - Z_{i-1}Z_{i+1}}{2}\right)$$

Spin flip conditioned on joint parity of nearest neighbors

$$\exp\left[i\frac{\theta}{2}X_i\left(\frac{1 - Z_{i-1}Z_{i+1}}{2}\right)\right] =$$



$$\equiv R_{\mathrm{DW},i}(\theta)$$

4 CNOTs per gate

10 assuming NN connectivity :(

<u>Comparison</u>:

Ising/XY—2 CNOTs

Heisenberg—3 CNOTs

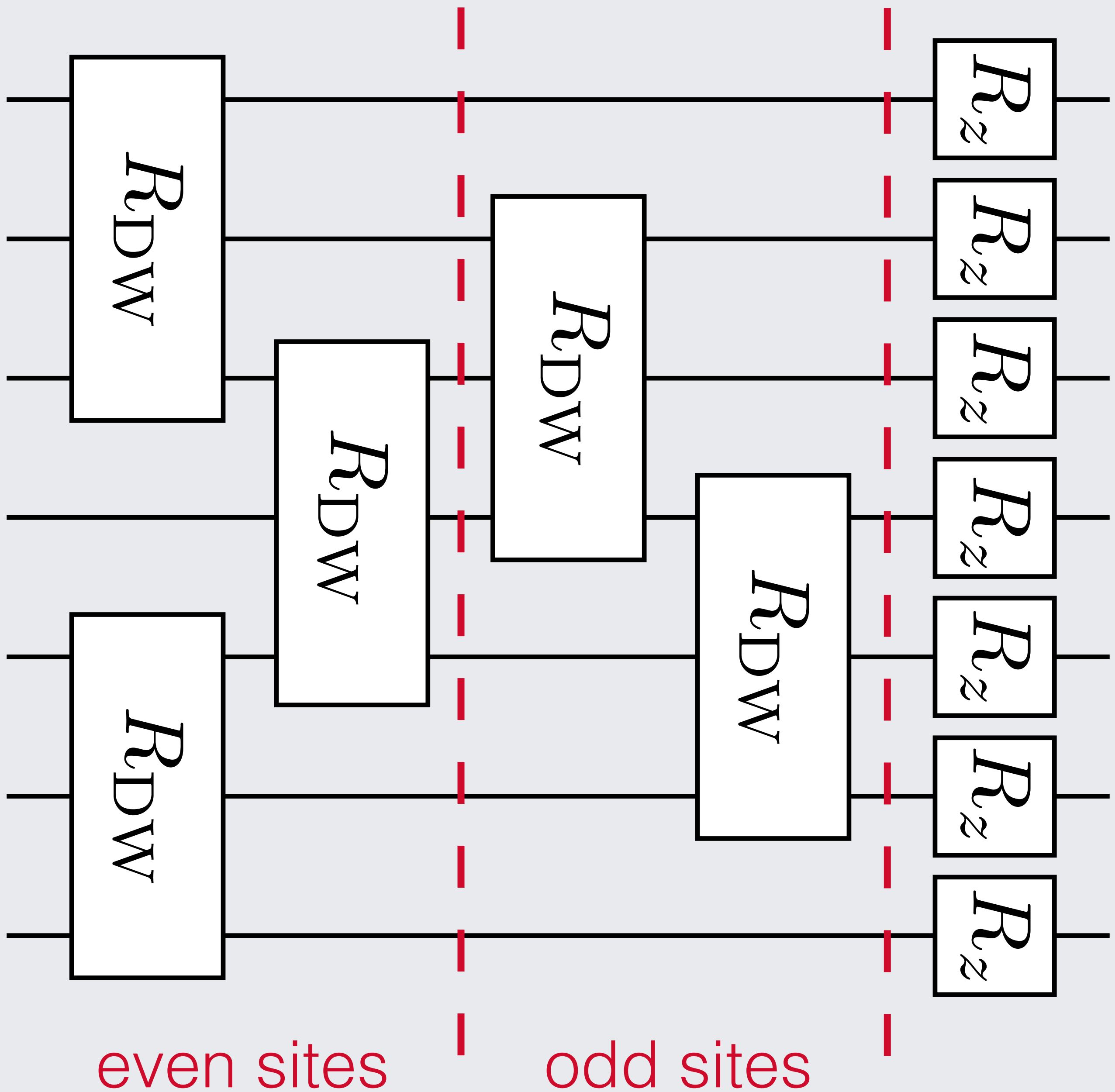
<u>Note</u>:

$$[R_{\mathrm{DW},i}(\theta), R_{\mathrm{DW},i+2}(\phi)] = 0$$

Trotter Circuit

$$H = \frac{\lambda}{2}\sum_i (X_i - Z_{i-1}X_iZ_{i+1}) + h\sum_i Z_i$$

Open boundary conditions (fix boundary spins)



What to Calculate?

confining field
(string tension)

$$H = \frac{\lambda}{2}\sum_i (X_i - Z_{i-1}X_iZ_{i+1}) + h\sum_i Z_i$$

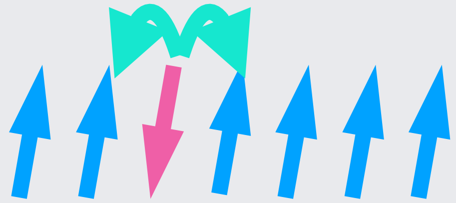
- Domain-wall (fermion) dynamics

$$n_{\mathrm{DW},i} = \frac{1 - Z_iZ_{i+1}}{2}$$

$$\langle\psi_0|n_{\mathrm{DW},i}(t)|\psi_0\rangle \qquad \langle\psi_0|n_{\mathrm{DW},i}(t)\,n_{\mathrm{DW},j}(0)|\psi_0\rangle$$

<u>Confinement (large *h*)</u>:
Freezing of configurations like

vs.



Response functions?

Viscosity?

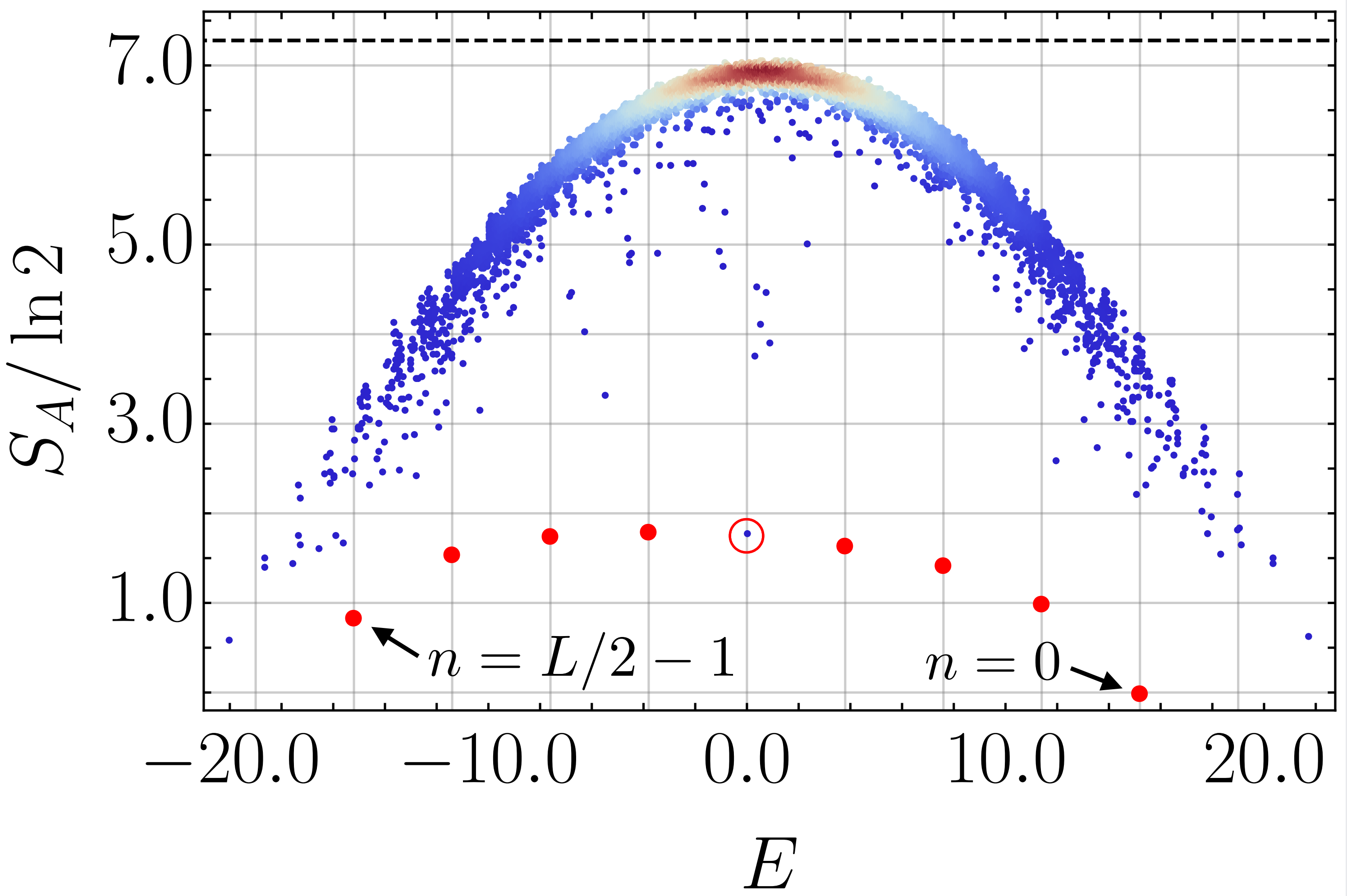
Focus on short-ish times

Yang *et al.*, PRL **124**, 207602 (2020)

What to Calculate?

- Quantum many-body scars (QMBS)

Coherent dynamics from certain special initial states due to a tower of low entanglement eigenstates



$$|\mathcal{S}_n\rangle \propto \left(\sum_i Q_i^\dagger\right)^n |\Omega\rangle$$

Bernien *et al.*, Nature **551**, 579 (2017)

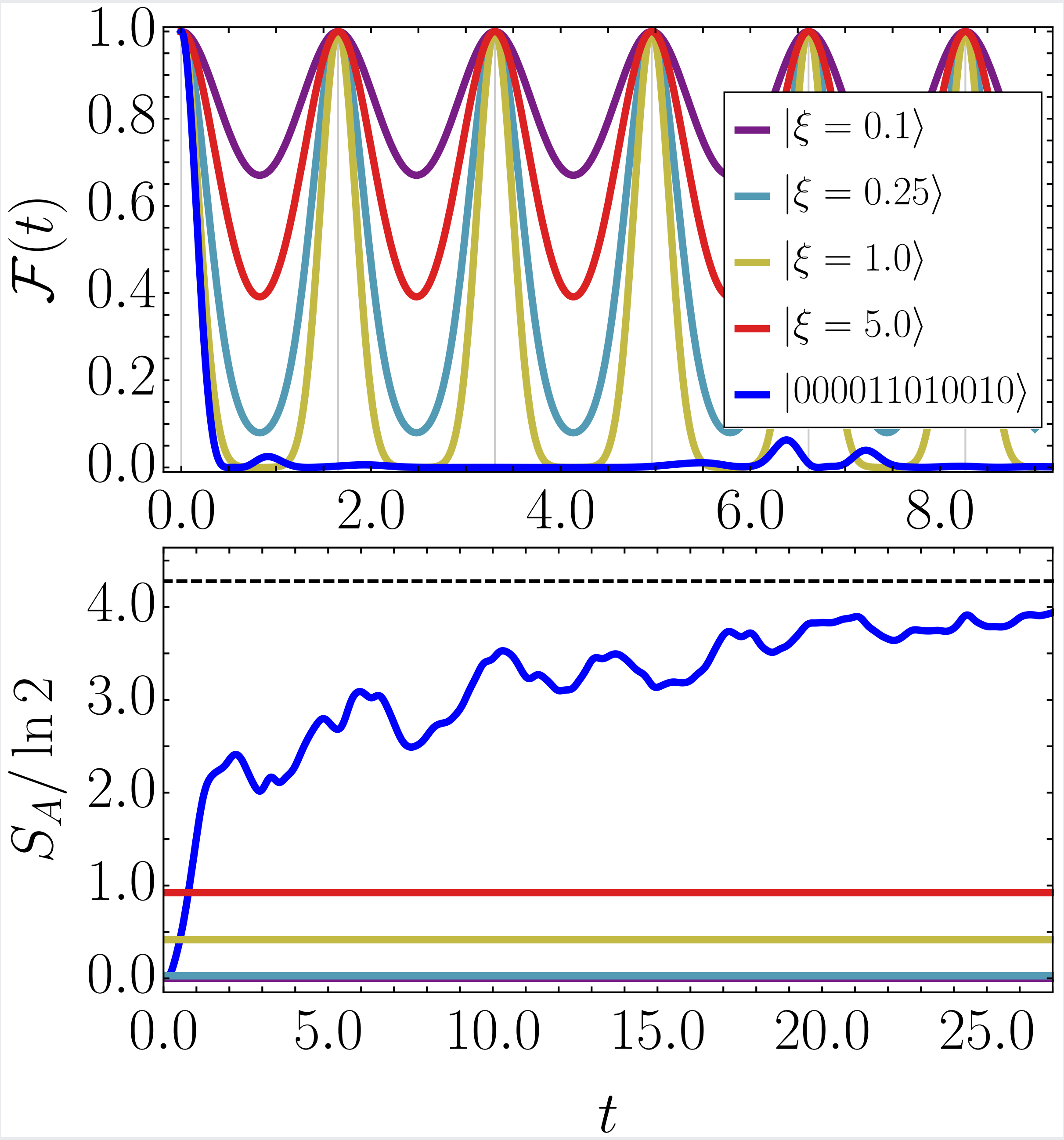
Turner et al., Nat. Phys. **14**, 745 (2018)

TI and Schecter, PRB **101**, 024306 (2020)

What to Calculate?

- Quantum many-body scars (QMBS)

Coherent dynamics from certain special initial states due to a tower of low entanglement eigenstates



$$|\mathcal{S}_n\rangle \propto \left(\sum_i Q_i^\dagger\right)^n |\Omega\rangle$$

$$|\xi\rangle \propto \mathcal{P}\prod_i \left[1 + \xi\,(-1)^i\,\sigma_i^+\right]|\Omega\rangle$$

$$= \sum_n c_n\,|\mathcal{S}_n\rangle$$

Oscillations of the many-body state:

$$\mathcal{F}(t) = |\,\langle\psi(t)|\psi(0)\rangle\,|^2$$

TI and Schecter, PRB **101**, 024306 (2020)

What to Calculate?

- Quantum many-body scars (QMBS)

$$|\xi\rangle \propto \mathcal{P}\prod_i \left[1+\xi\,(-1)^i\,\sigma_i^+\right]|\Omega\rangle$$

Area-law entangled state. How to prepare on NISQ device?

1) $|\xi\rangle$ has a known MPS representation w/ bond-dimension 2.

Translate to a circuit using, e.g., Ran, PRA **101**, 032310 (2020)

2) $|\xi\rangle$ is the GS of a known family of parent Hamiltonians.

<u>Ex ($\xi = 1$)</u>: $$H_0 = -\sum_i \left[(-1)^i P_{i-1}X_iP_{i+1} - P_{i-1}P_{i+1}\right]$$

$$P_i = \frac{1-Z_i}{2}$$

Combine VQE state preparation w/ Trotter?

TI and Schecter, PRB **101**, 024306 (2020)

What to Calculate?

- Quantum many-body scars (QMBS)

  Why study QMBS in this model?

  — Fundamental interest

  Mechanism to avoid thermalization in isolated quantum systems

  — So far, have only been realized in analog quantum simulators (e.g. Rydberg atoms)

  Limited variety of models, limited initial states

  Demonstrate advantage of digital quantum simulation?

Thank you!