MAGIS imager simulations / development

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Parameter space for camera

Assumptions:
- distance to target: 155mm
- number of atoms in cloud: 1E6
- saturated transition
- 4 clouds need to be imaged, each with (sigma) width 1.5mm
- spatial frequency of interest is the fringes (which are periodic every 100 microns in object space)

NB in what follows we include "sampling MTF" in the calculations, even though with accurate diagnostic imaging we will likely be able to unfold this from the final imaging

Procedure

Minimum focal length determined by pixel MTF at fringe spatial frequency
Maximum focal length determined by needing to fit 4 clouds in field of view

Useful figure of merit: ratio of out of focus MTF (edge of cloud) to in focus MTF(centre of cloud), depends on f/# and focal length (via magnification).

For each camera (with figures on QE, full well, noise, pixel size, total size), make maps of MTF and signal to noise ratio in (focal length, f/#) space.

Camera Parameters

Name	Pixel pitch (µm)	Footprint (mm)	Read noise (e-)	Full well (ke-)	QE @ 461nm
Andor Ixon ultra (EMCCD)	13	13.3	0.001 - 6*	80*	0.85
Andor Zyla 5.5 (sCMOS)	6.5	13.3	2.3	30	0.52
Sony IMX541 (CMOS)	2.74	12.3	2.1	9.5	0.62
Gsense 2020 BSI (sCMOS)	6.5	13.3	1.6	55	0.80

* assuming no EM-gain – much lower effective read noise with EM-gain

Equations used #1 -MTF

$$\mathrm{MTF_{pixel}}(s) = \left|\mathrm{sinc}\left(\frac{p\,s\,(D-f)}{f}\right)\right|$$

$$\mathrm{MTF_{detector}}(s) = \underbrace{\left|\mathrm{sinc}\left(\frac{p\,s\,(D-f)}{f}\right)\right|}_{\text{pixel}}\underbrace{\left|\mathrm{sinc}\left(\frac{N_{\text{pixels}}\,p\,s\,(D-f)}{f}\right)\right|}_{\text{format}}\underbrace{\left|\mathrm{sinc}\left(\frac{p\,s\,(D-f)}{f}\right)\right|}_{\text{sampling}}$$

$$\mathrm{MTF_{detector}}(s) \propto \left|\mathrm{sinc}^3\left(\frac{p\,s\,(D-f)}{f}\right)\right|$$

$$\mathrm{MTF_{diffraction}}(s) = \frac{2}{\pi}\left(\Phi - \cos\Phi\sin\Phi\right)$$

$$\Phi = \cos^{-1}\left(\frac{\lambda\,s\,N(D-f)}{f}\right)$$

$$\mathrm{MTF_{defocus}}(s) \approx \int_0^{\sqrt{1-(\lambda N s)^2}} \sin\left(\frac{\pi\,s\,|\Delta\varphi|\,f^2}{N\,(D-\Delta\varphi)(D-f)}\,\sqrt{1-y^2} - \lambda\,N\,s\right) dy$$

p – pixel pitch
s – spatial frequency
D – distance to target
n – f/#
f – focal length

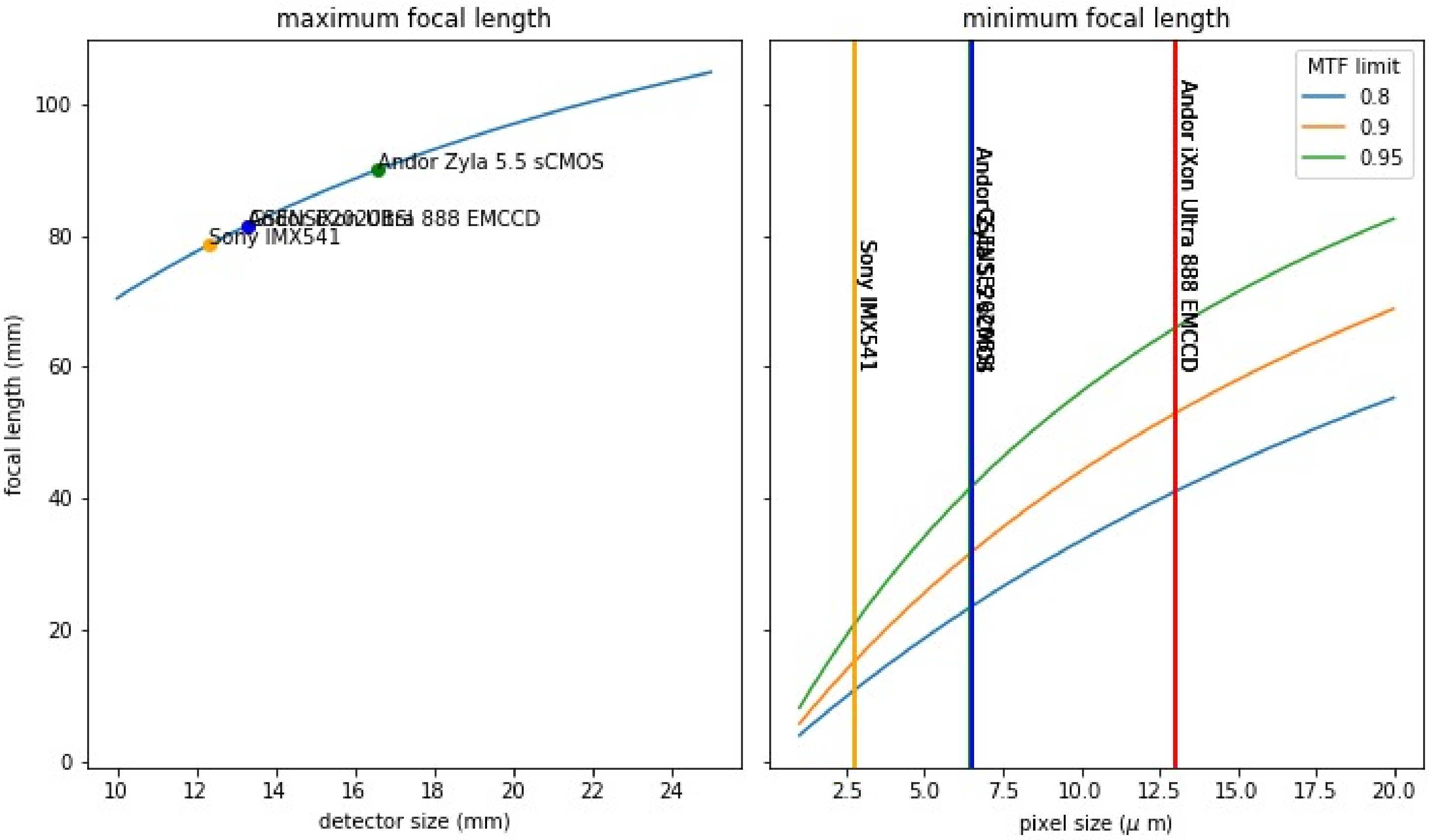
Equations used #2 - SNR

$$I \approx \frac{N_{\text{atoms}}\,\Gamma\pi}{8\sigma\,\sqrt{2\,\pi}\,N^2\left(1+\frac{f}{D-f}\right)^2}$$

$$\frac{d\,\text{SNR}}{dt} = \frac{I\eta p^2}{\sigma_r}$$

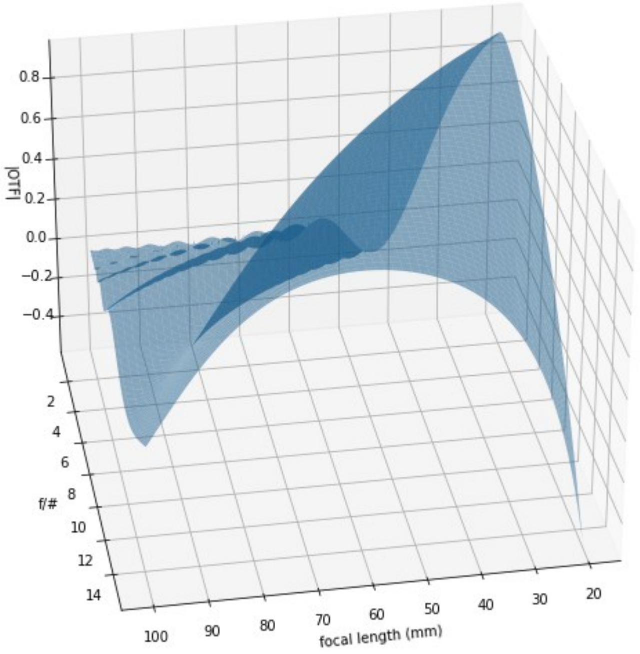
λ – wavelength of illumination
Γ – photon emission rate
σ – cloud width
σ_r – read noise
η – quantum efficiency

Minimum / maximum focal lengths



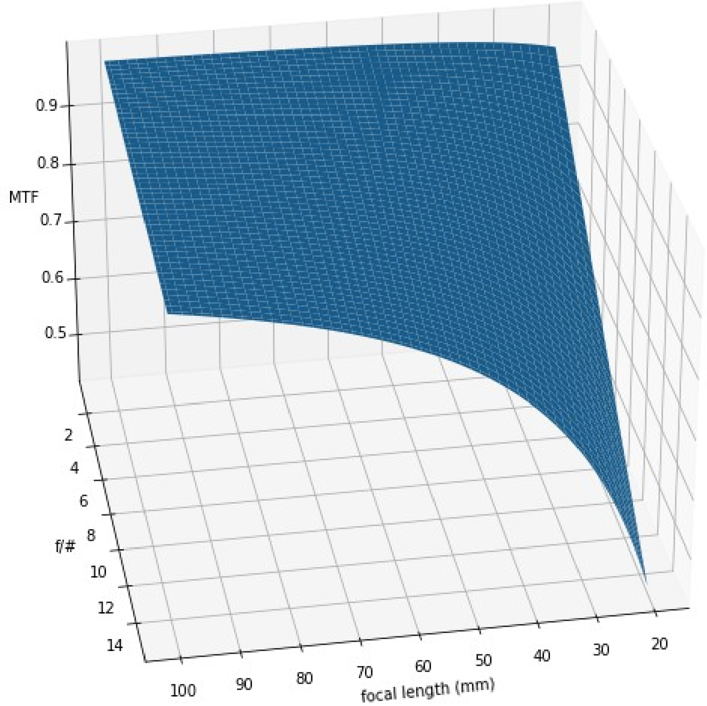
NOTE: Andor Zyla & GSense2020BSI are On top of of each other (they share pixel sizes and detector format)!

Optics MTF calculations

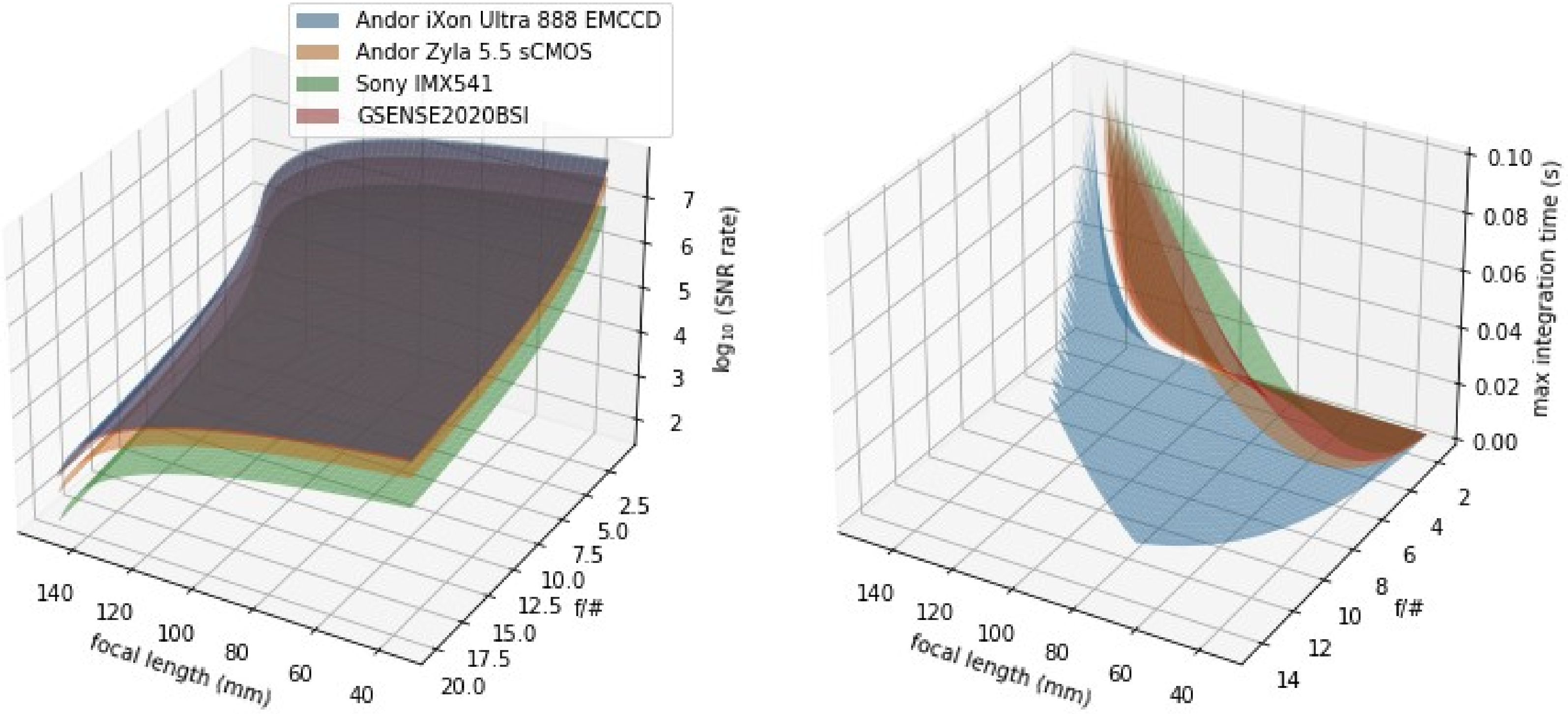


Out of focus (geometric/ CoC ONLY)

In focus (diffraction limited)

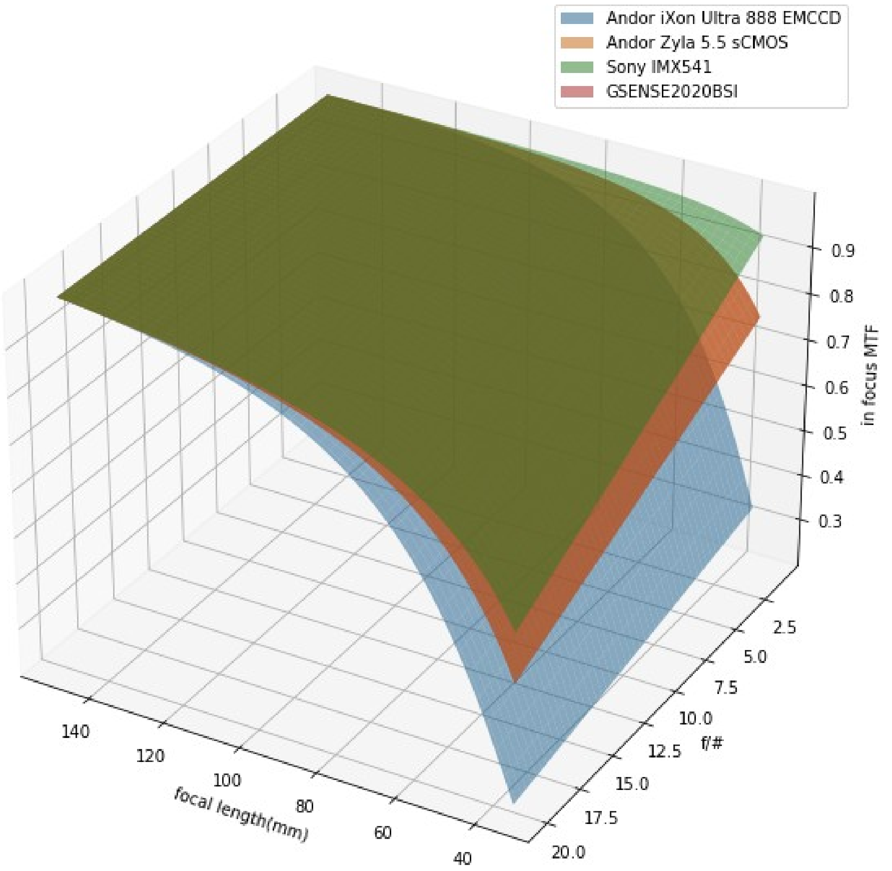


SNR rate of various sensors



Radiative transfer equation used to find rate of signal accumulation at sensor from the **brightest** point of the cloud (which determines maximum integration time)

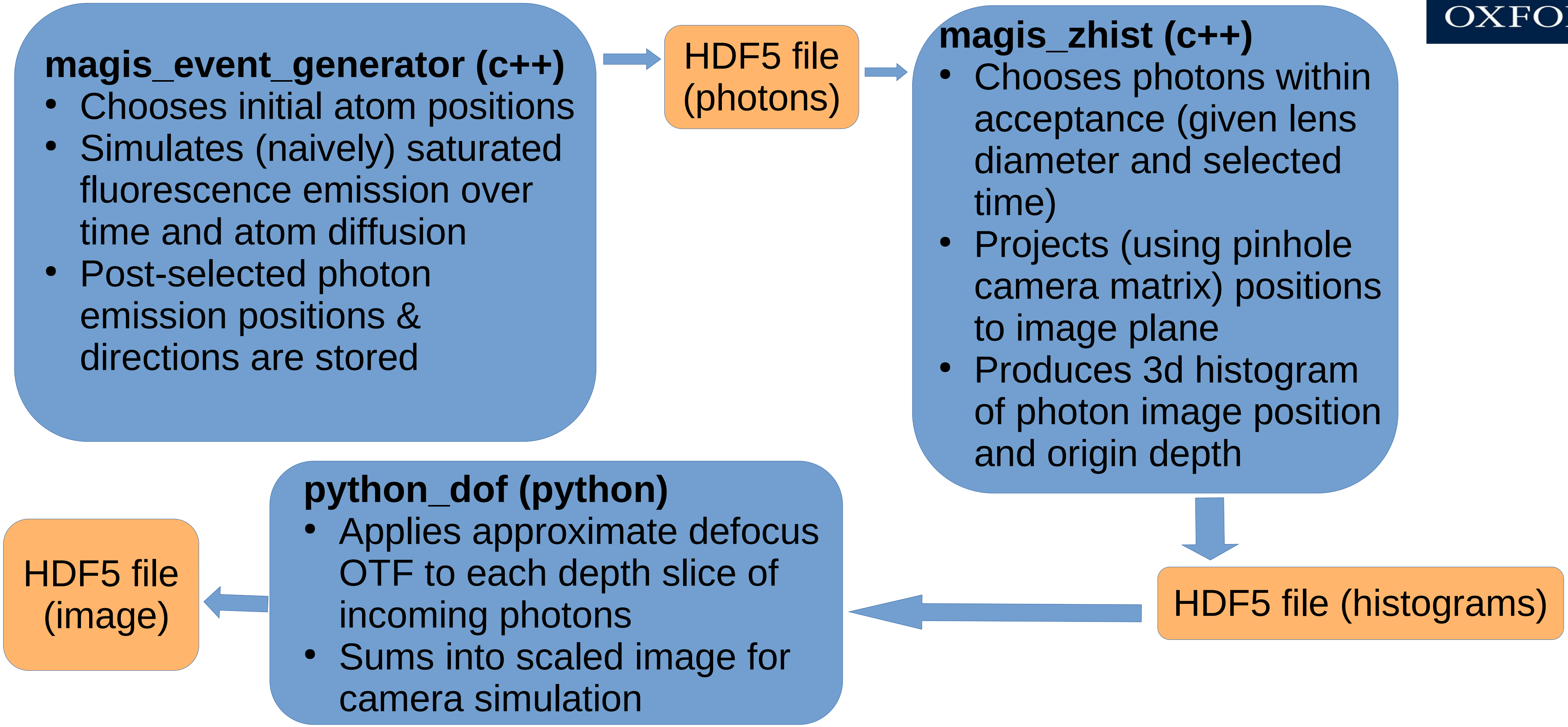
MTF comparison of sensors



Diffraction limited MTF
(including sampling error
And detector size cutoff)

For several sensors.

Overview of Simulation (optics)



magis_event_generator (c++)

- Chooses initial atom positions
- Simulates (naively) saturated fluorescence emission over time and atom diffusion
- Post-selected photon emission positions & directions are stored

HDF5 file (photons)

magis_zhist (c++)

- Chooses photons within acceptance (given lens diameter and selected time)
- Projects (using pinhole camera matrix) positions to image plane
- Produces 3d histogram of photon image position and origin depth

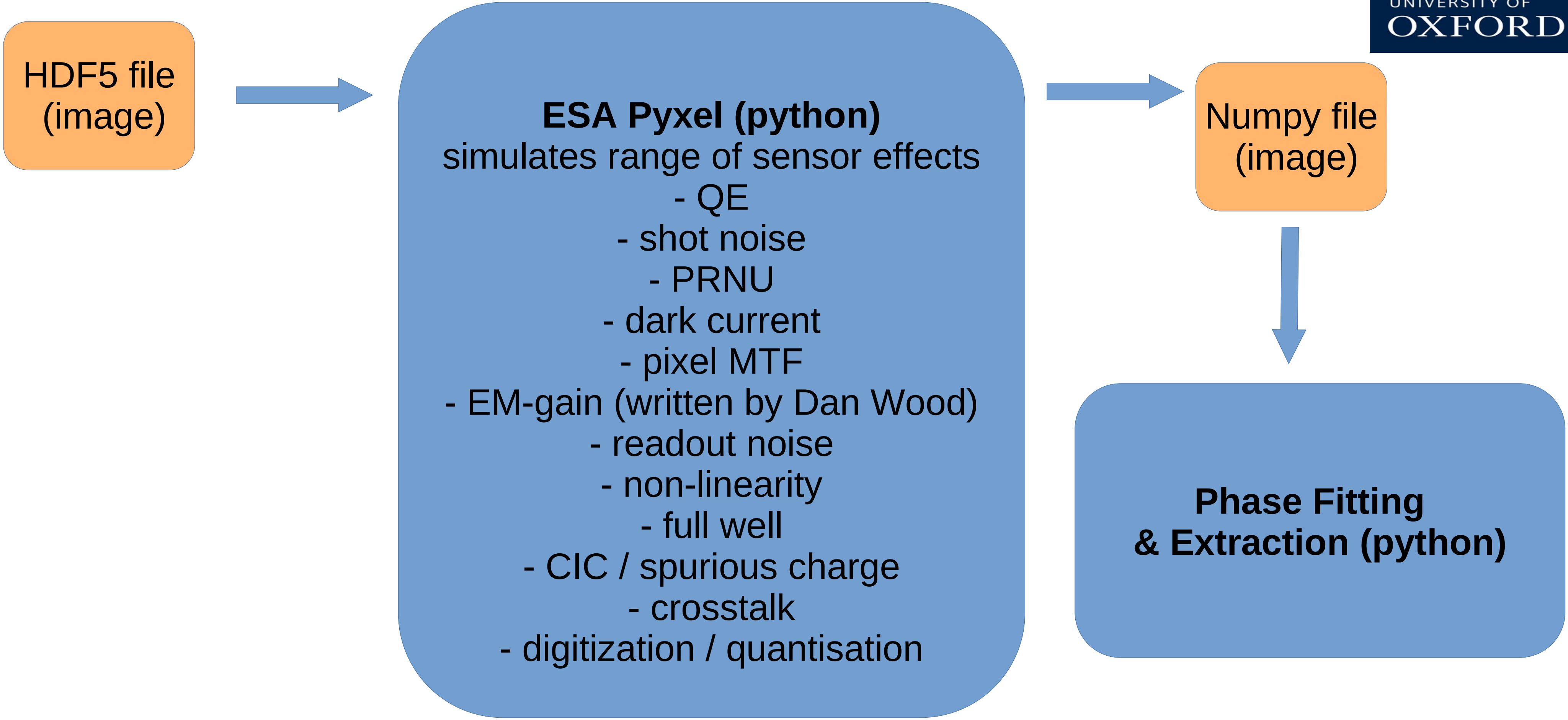
HDF5 file (histograms)

python_dof (python)

- Applies approximate defocus OTF to each depth slice of incoming photons
- Sums into scaled image for camera simulation

HDF5 file (image)

Overview of simulation (detector)



HDF5 file (image)

ESA Pyxel (python)
simulates range of sensor effects
- QE
- shot noise
- PRNU
- dark current
- pixel MTF
- EM-gain (written by Dan Wood)
- readout noise
- non-linearity
- full well
- CIC / spurious charge
- crosstalk
- digitization / quantisation

Numpy file (image)

Phase Fitting & Extraction (python)

Effects included / not included

INCLUDED

- **Depth of Field** - Given lens diameter, distance to object & wavelength, we get reasonably realistic defocussing effects (see later)
- **Perspective –** pinhole camera model gives approximate (but perhaps useful?) perspective distortion
- **Atom Diffusion** – following method used by previously seen MAGIS simulations
- **Intensity** – sampled raytracing should give realistic intensity distributions at image plane
- **Diffraction Effects** – included via OTF (assuming diffraction limited optics for now)

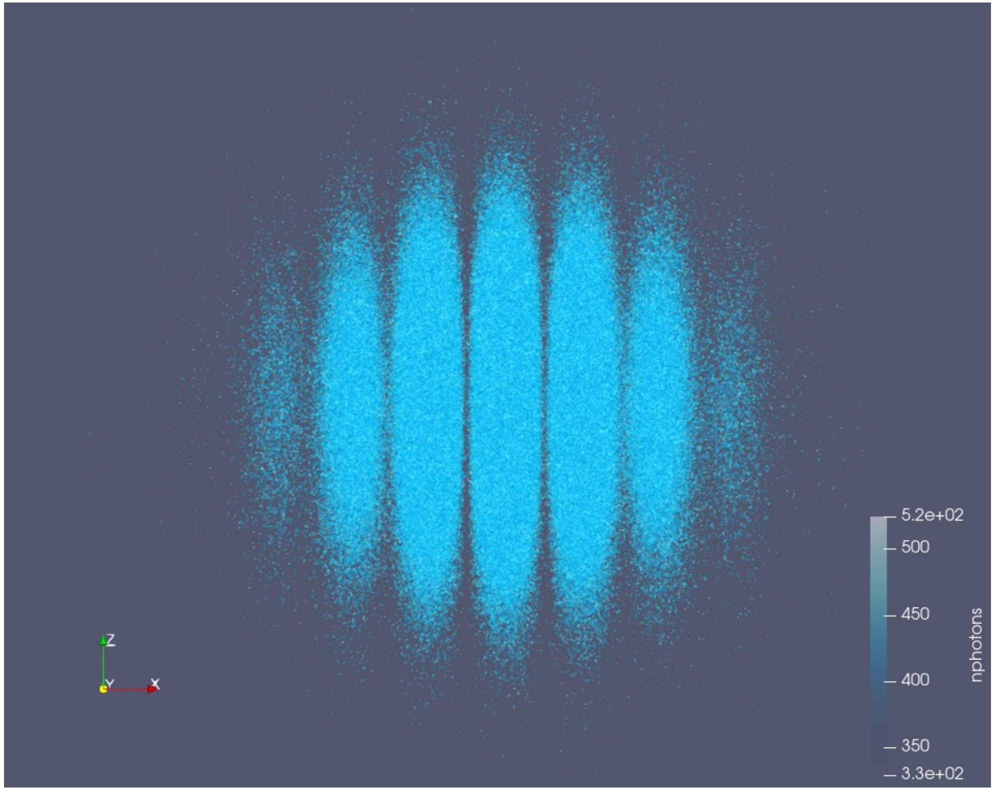
NOT INCLUDED

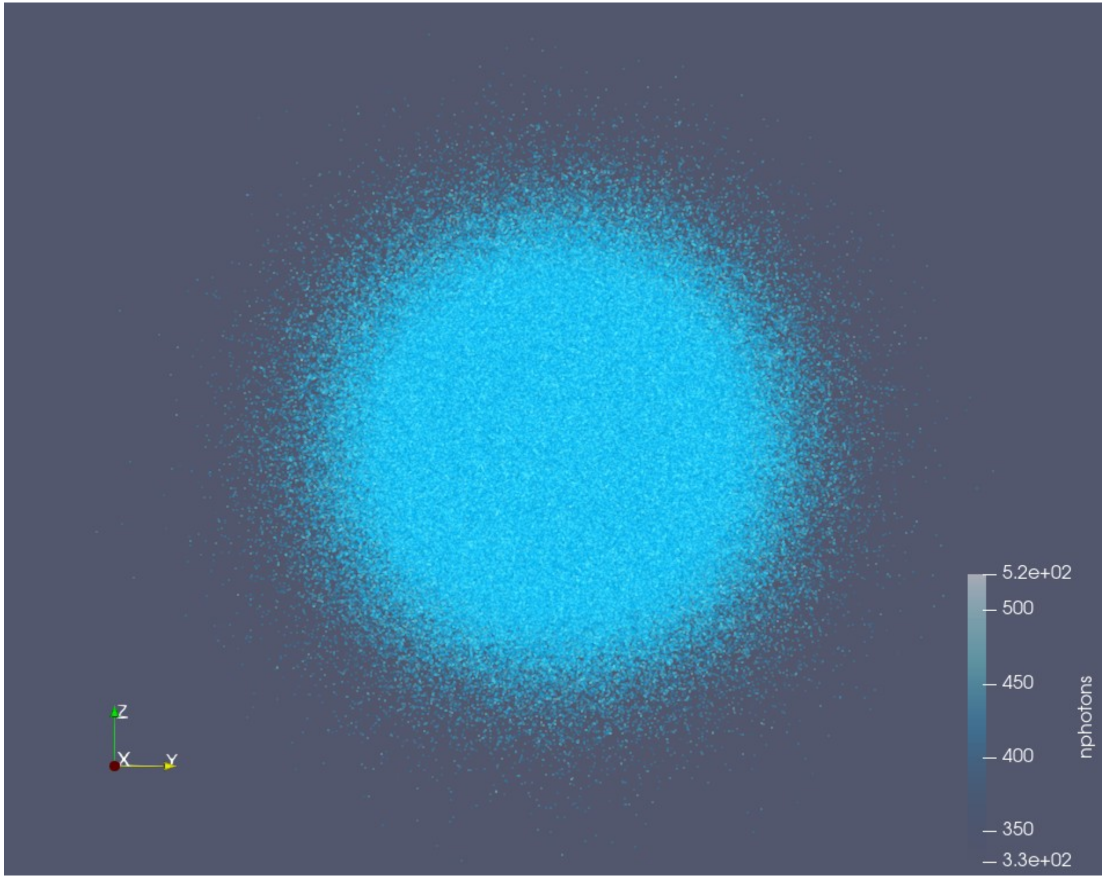
- **Astigmatism, spherical aberrations etc –** can't include since we don't know specifics of optics yet
- **Coma** / **skew ray effects**– naive model of acceptance means we are effectively doing a meridional approximation (probably a **bad** idea! But will be solved by integration with SLAC's differentiable_optics simulations)
- **Chromatic Aberration –** reasons fairly obvious
- **Atom cloud movement** – i.e. no overall residual velocity of the atom cloud (could very easily be included)
- **Effect of glass window** – again could be relatively easily added, effectively more spherical aberration + differential OPL

Implementation Notes

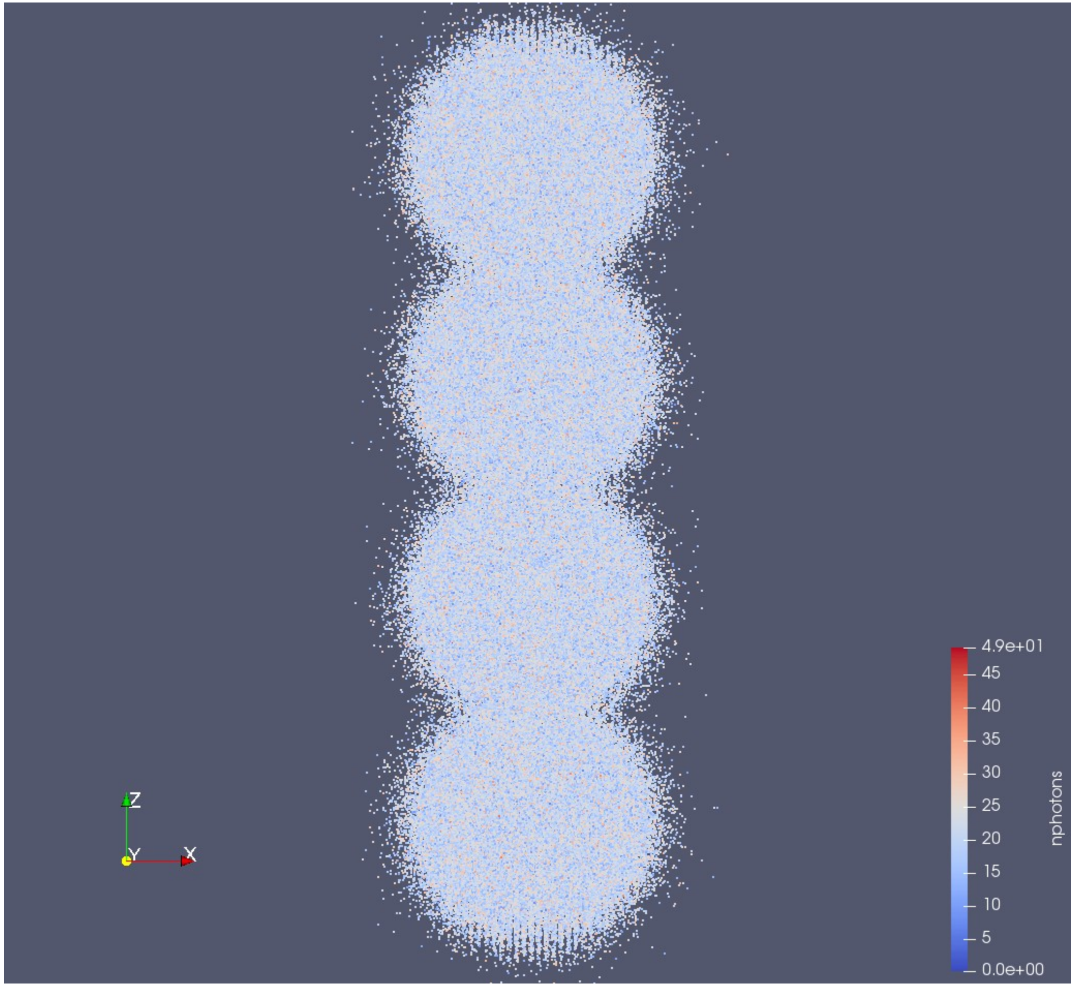
- **magis_event_generator** treats each atom separately, producing poisson distributed emissions, rather than globally timestepping the entire state and choosing atoms to emit from – this is slower, but allows trivial parallelisation, almost completely CPU bound (we currently run on 16 cores at ~98% saturation, can scale higher but limited by our local cluster not properly supporting MPI jobs)
- Due to data volumes, we only keep photons emerging within a certain solid angle (larger than any we would need for optics simulation), and also apply a random selection (typically ~1%) of these photons
- Since we keep all photon direction and position information, we can reconstruct easily imaging scenarios of different integration times from the same photon data. Multiple runs of photon data are needed with different random seeds to reproduce photon shot noise in subsequent images.
- Our camera projection is pinhole (standard in computer graphics), using homogeneous co-ordinates. The ray-tracing is thus entirely geometric, but we apply then a physically based OTF for defocussing intensity calculation (see next slides)

Initial Atom Positions (Single Cloud)





Initial Atom Positions (4 clouds)



- More realistic simulation of actual MAGIS-100 viewport.
- 4 clouds, fringe periodicity 100um

Quick Depth of Field notes

- We are imaging a 3D target, therefore **depth of field** is an important consideration.
- The optics are **not** planned to be telecentric, thus we need to consider also **perspective distortion**
- For object distance u, focal length f, circle of confusion c and f-number N, depth of field usually approximated (from simple geometric optics considerations) as:

$$\mathrm{DOF} \approx \frac{2u^2Nc}{f^2}$$

So:

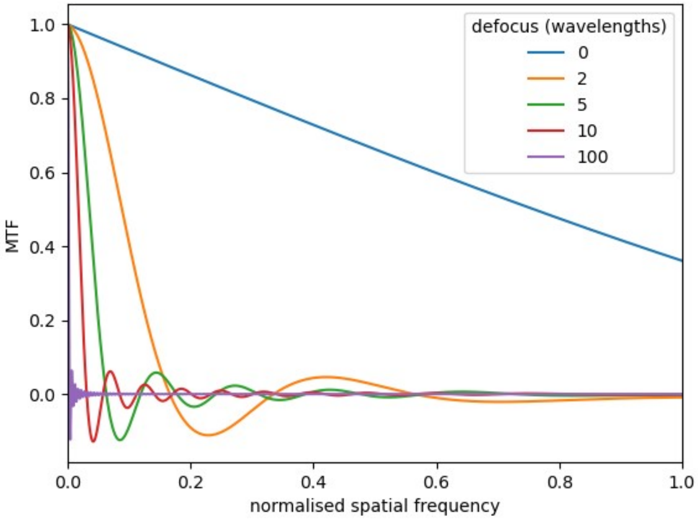
- longer focal lengths have less depth of field (we need somewhere around 100mm -150mm focal length for MAGIS given an iXon camera)
- DoF decreases with wider aperture (i.e. lower f-number). So it may be necessary to sacrifice light gathering capability to be able to image the entire cloud in focus for phase recovery.

For example, given f=100mm, u=140mm, N=2.8 (quite a wide aperture), c=13.3 um (i.e. single pixel iXon circle of confusion), DoF ~ **0.1mm**!! So, given the cloud is several mm across, this is a real concern!

2D defocussing / OTF

$$
\begin{aligned}
I &= \left(\frac{2}{u}\right)^2 \left(U_1^2(u,v) + U_2^2(u,v)\right) I_0 \\
I_0 &= \left(\frac{\pi a^2 \left|A\right|}{\lambda f^2}\right) \\
U_n &= \sum_{s=0}^{\infty} (-1)^s \left(\frac{u}{v}\right)^{n+2s} J_{n+2s}(v)
\end{aligned}
$$

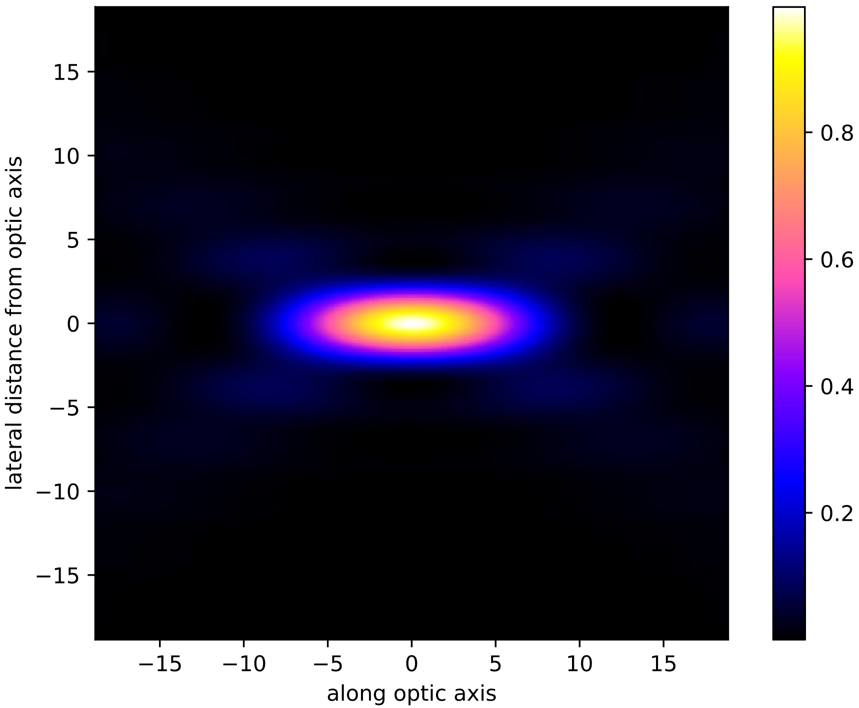
- Calculating 2D defocussed PSF is very intensive (slowly converging series of Bessel functions, many many terms needed at high defocus)
- Going via the OTF (fourier transform of PSF) is faster – there are several published extremely accurate approximations available , and very convenient for simulation
- We calculate an OTF for each defocus depth



$$
\begin{aligned}
T = \frac{4}{\pi a}\cos\left(\frac{as}{2}\right)&\left(\beta J_1(a) + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin 2n\beta}{2n}\left(J_{2n-1}(a) - J_{2n+1}(a)\right)\right) \\
&- \frac{4}{\pi a}\sin\left(\frac{as}{2}\right)\left(\sum_{n=1}^{\infty}(-1)^{n}\frac{\sin(2n+1)\beta}{2n+1}\left(J_{2n}(a) - J_{2(n+1)}(a)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
a &= \frac{4\pi}{\lambda} ws \\
\beta &= \cos^{-1}\left(\frac{s}{2}\right)
\end{aligned}
$$

2D defocussing PSF / OTF



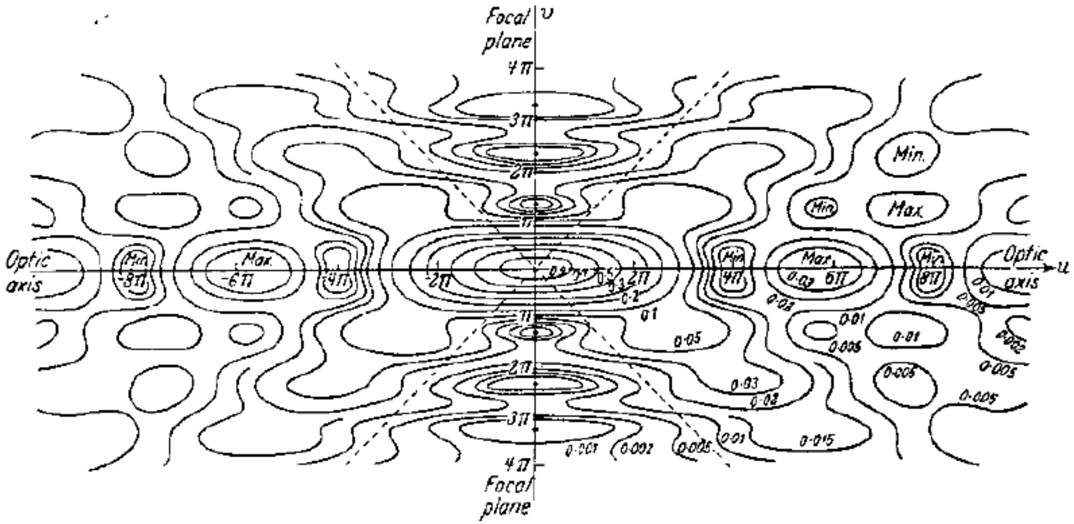


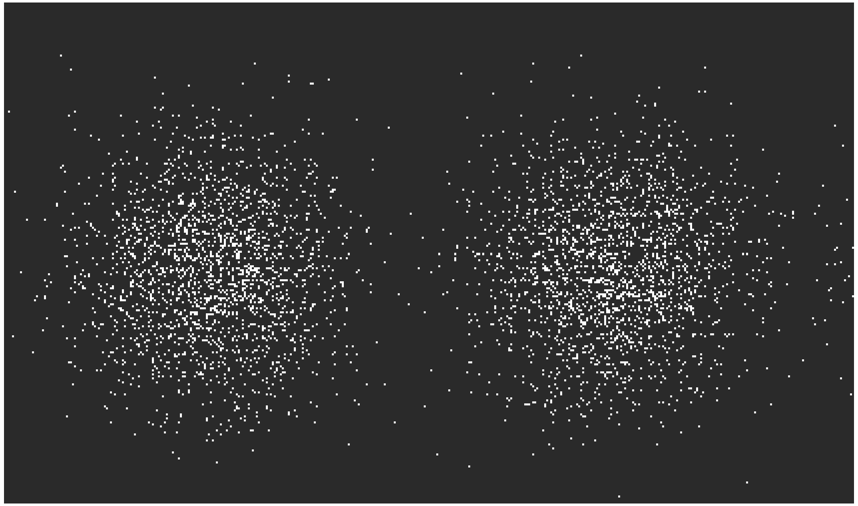
Fig. 8.41. Isophotes [contour lines of the intensity $I(u, v)$] in a meridional plane near focus of a converging spherical wave diffracted at a circular aperture. The intensity is normalized to unity at focus. The dotted lines represent the boundary of the geometrical shadow. When the figure is rotated about the u-axis, the minima on the v-axis generate the Airy dark rings.

(Adapted from E. H. Linfoot and E. Wolf, *Proc. Phys. Soc.*, B, **69** (1956), 823.)

The theory on this is very interesting, but you have to go back to the 1950s for detailed treatments on actually evaluating the 2D Lommel functions (if you want to directly calculate the defocus PSF). Key point: even for very high defocus, geometric optics prediction of intensity is not accurate! (can send papers about this if interested). Modern approach always goes via OTF rather than PSF

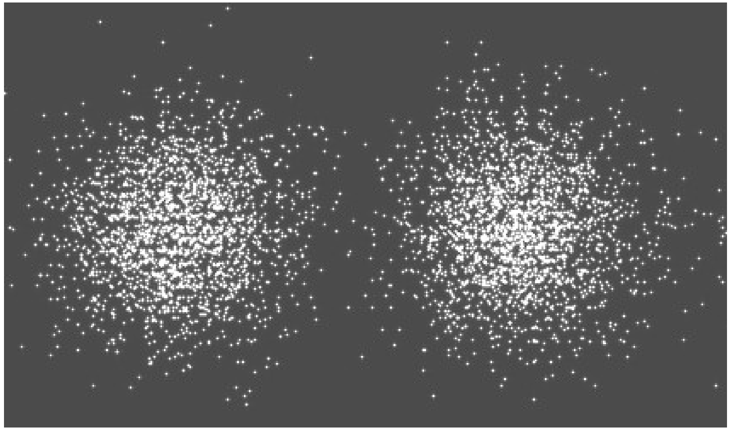
Effect of DoF on image slices

Photons originating from near centre of cloud

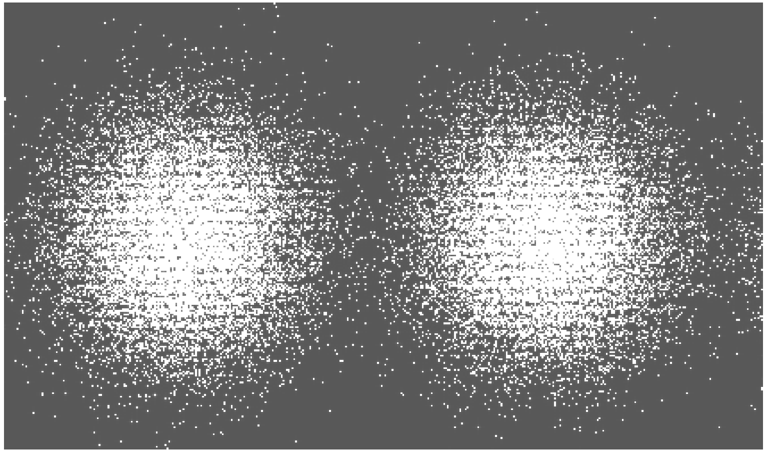


- No DoF effect

DoF with 10mm lens

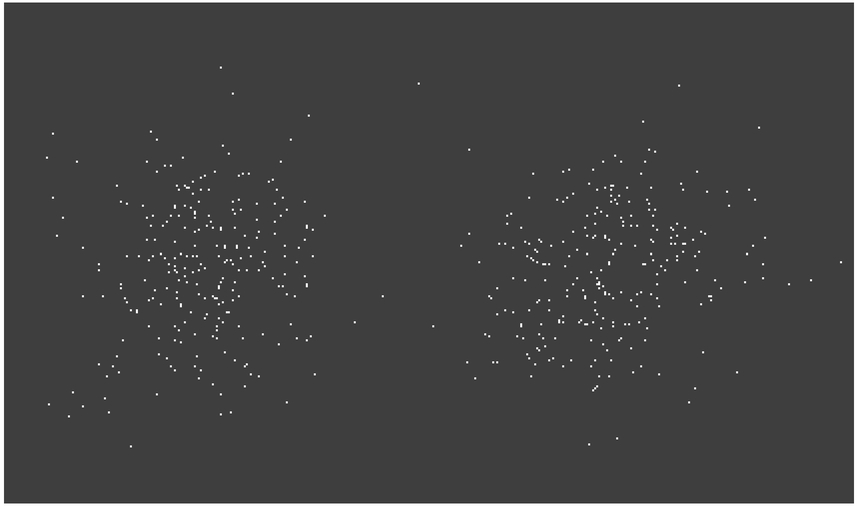


DoF with 30mm lens



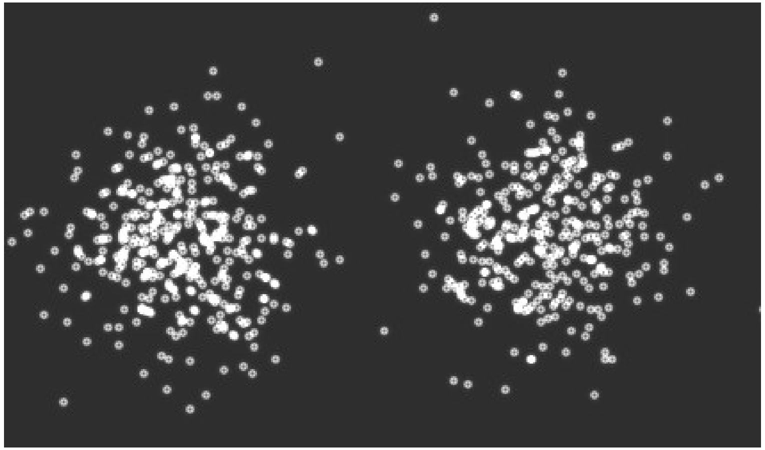
Effect of DoF on image slices

Photons from near edge of cloud

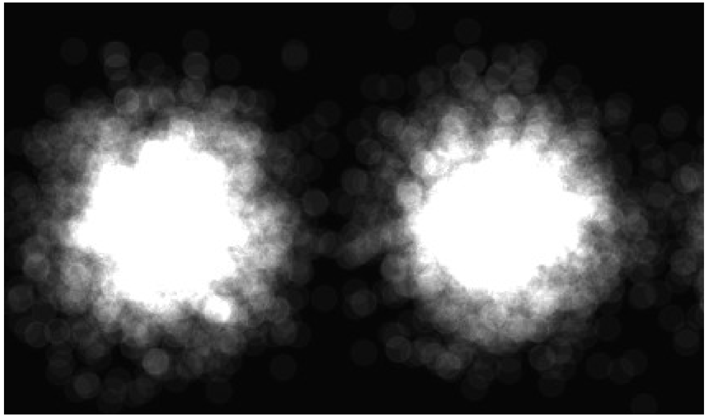


- No DoF effect

DoF with 10mm lens



DoF with 30mm lens



Simulations so far...

- Simulated optics at several f-numbers (corresponding to diameters between 5mm and 70mm).
- Range of integration times used.
- Both Andor iXon (EMCCD, 13um pixels) and Andor Zyla (sCMOS, 6.5um pixels).
- Regardless of details, for all sensible choices, given 1E6 atoms per cloud, we can get good imaging with integration time of <50us.

→

However, our current understanding is that both simulated camera options are physically **too large** to fit within the MAGIS connection node.

Size reductions potentially available through:

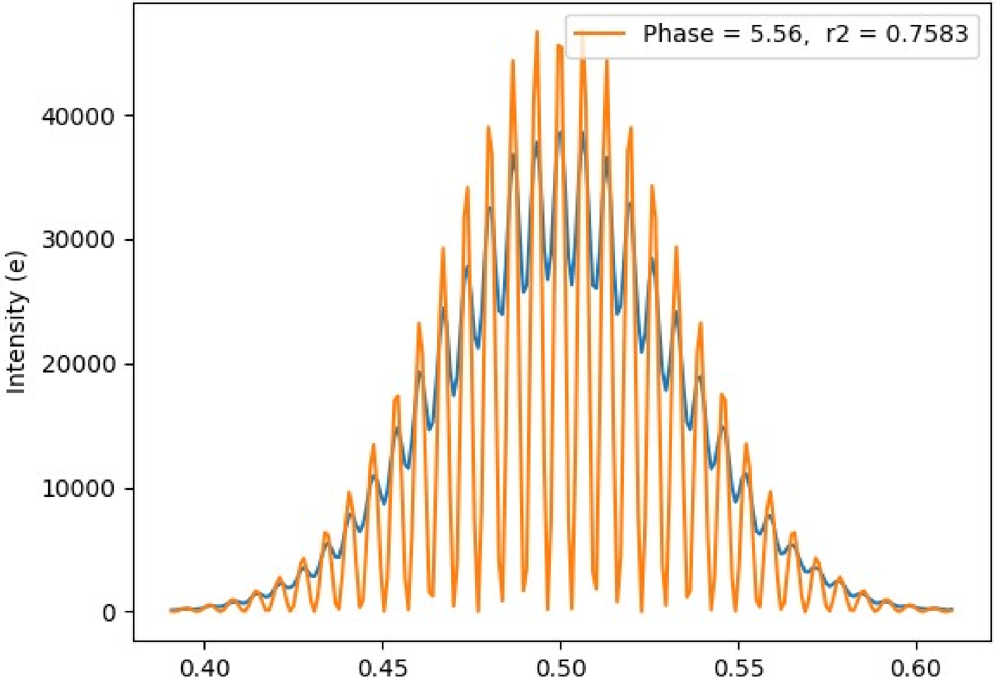
- Use of shorter focal lengths (see following work)
- Removal of cooling mechanisms
- Use of C-mount lens (no adapter)

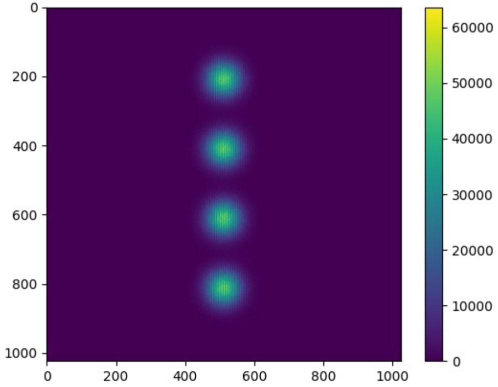
Non-linear fitting

$$s(x) = N \times f_\gamma \times \left[1 + Cos\left(\frac{2\pi}{\lambda}x + \phi\right)\right] \times \frac{1}{\sigma}\, e^{-\frac{1}{2}[\frac{x-\mu}{\sigma}]^2}$$

~20us integration time

mm aperture diameter





Very sensitive to input parameters.

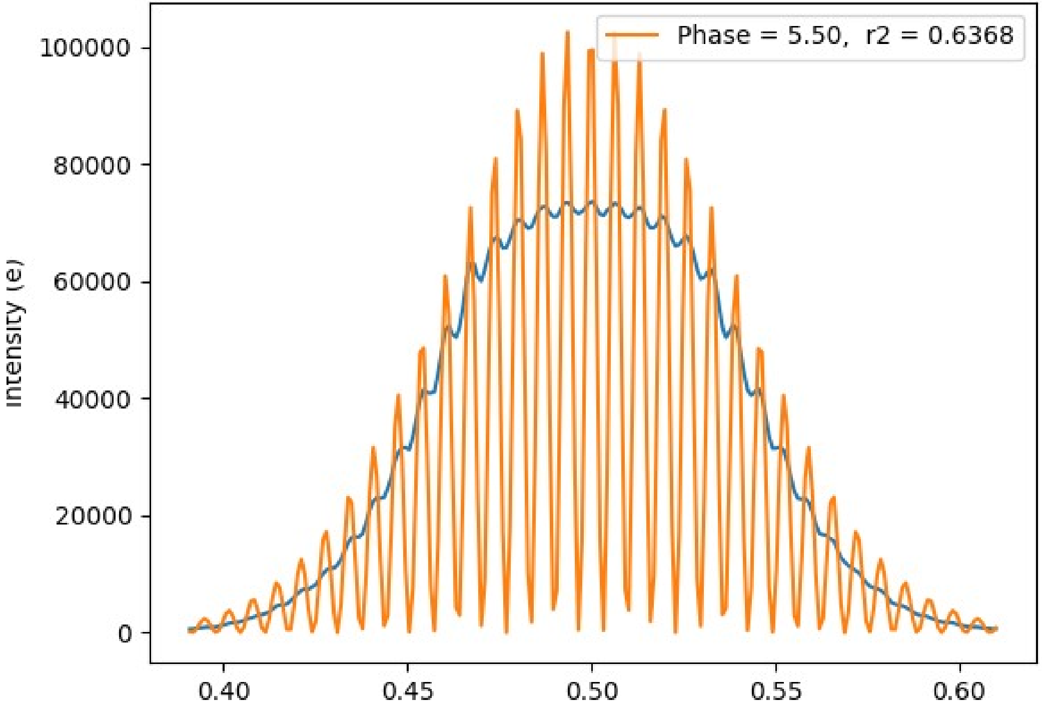
GoF strongly influenced by DoF effects.

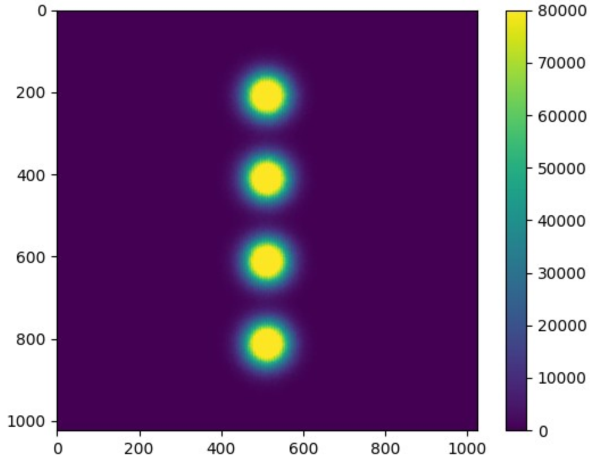
Non-linear fitting

$$s(x) = N \times f_\gamma \times \left[1 + Cos\left(\frac{2\pi}{\lambda}x + \phi\right)\right] \times \frac{1}{\sigma} e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}$$

~50us integration time

30mm aperture diameter

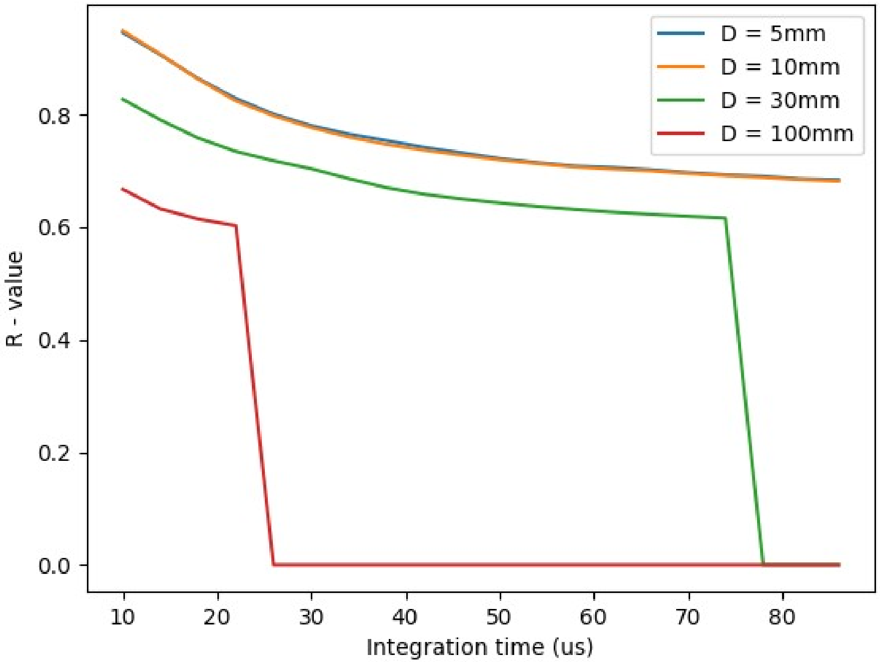


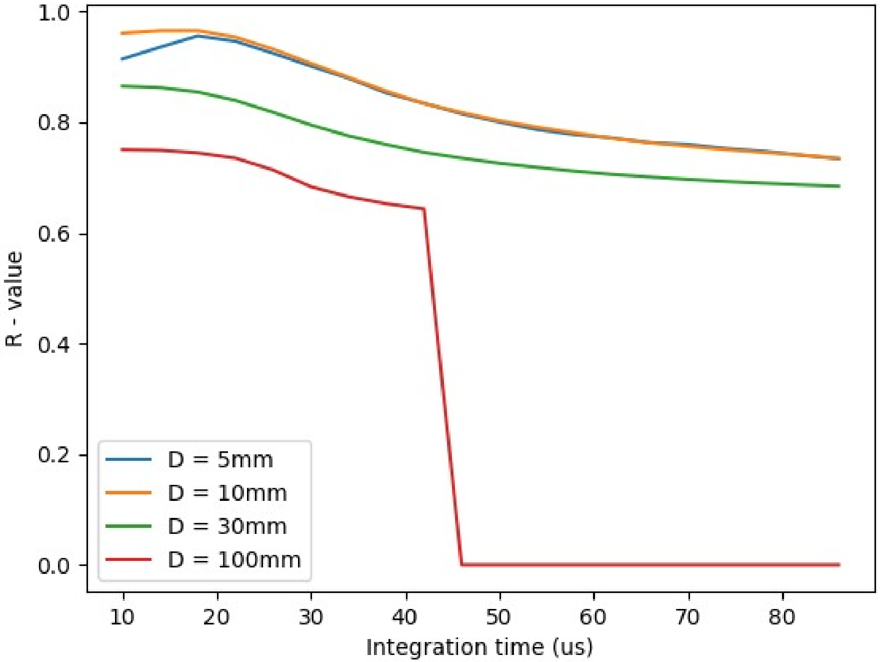


GoF comparisons

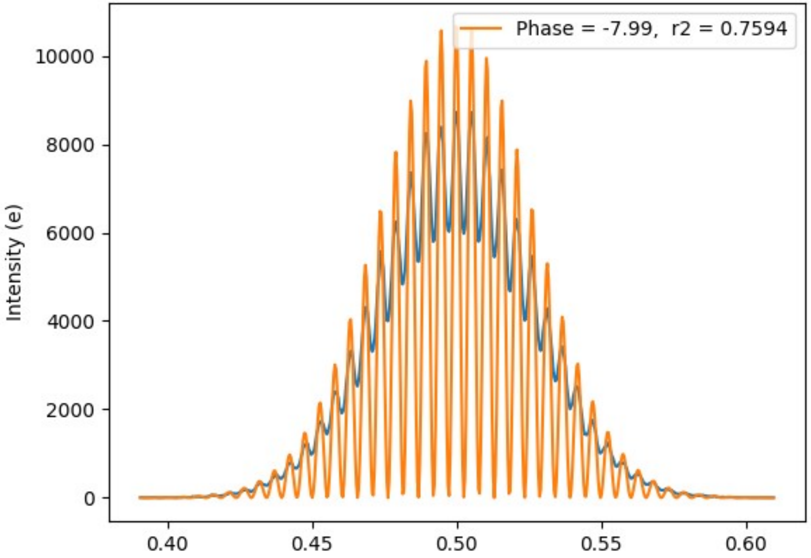
EMCCD

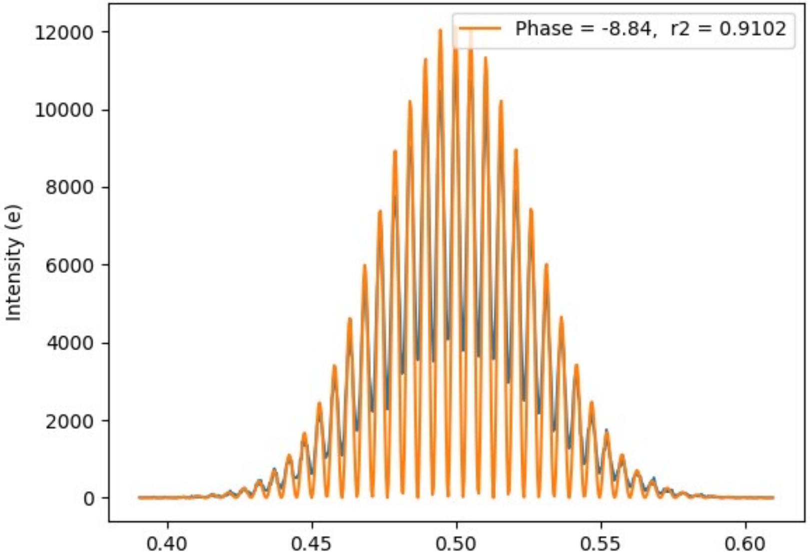
sCMOS

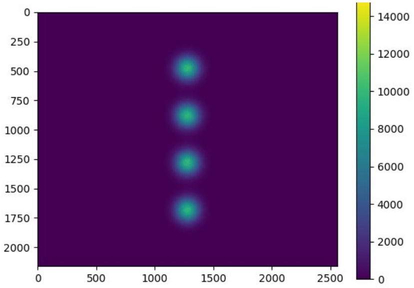




DoF effects


DoF defocussing


No DoF defocussing



sCMOS - ~20us integration time, 30mm aperture diameter

Analysis via 2D FFTs?

$$h(x,z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-\left(x^2+z^2\right)}{2\sigma^2}\right)(1+\cos(kx+\phi))$$

Start from function we are looking at

$$\begin{aligned} f(x) &= 1+\cos(kx+\phi) \\ g(x,z) &= \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-\left(x^2+z^2\right)}{2\sigma^2}\right) \end{aligned}$$

$$\tilde{H}(u,v) = \tilde{F}(u) * \tilde{G}(u,v)$$

Split into gaussian and cos parts, and apply convolution theorem

$$\tilde{F}(u) = \delta(u) + \frac{1}{2}e^{i\phi}\delta(u-k) + \frac{1}{2}e^{-i\phi}\delta(u+k)$$

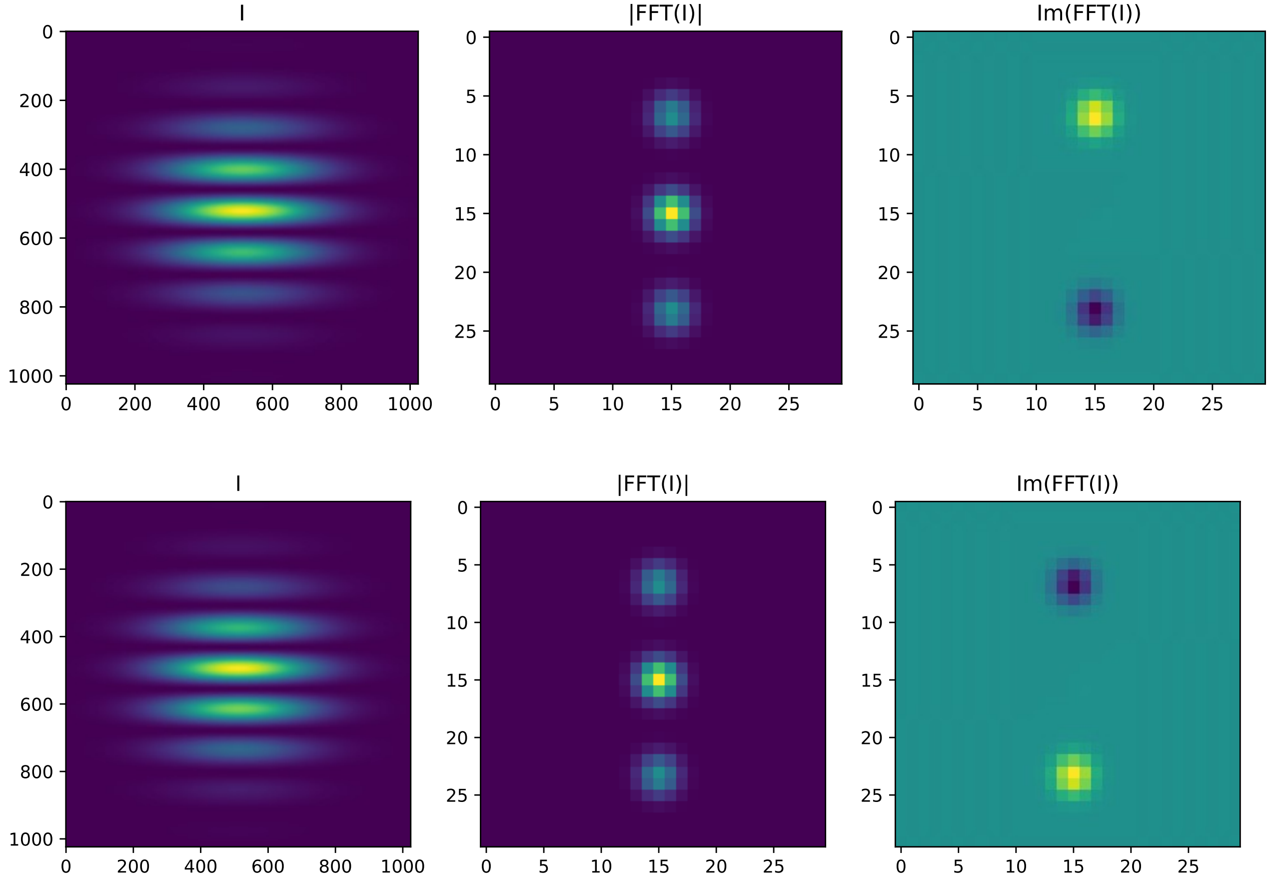
$$\tilde{H}(u,v) = \tilde{G}(u,v) + \frac{1}{2}e^{i\phi}\tilde{G}(u+k,v) + \frac{1}{2}e^{-i\phi}\tilde{G}(u-k,v)$$

Can then write down analytical 2D FT quite easily – three shifted Gaussians and a phase factor

$$\Im\left(\tilde{H}(u,v)\right) = \frac{1}{2}\sin(\phi)\tilde{G}(u+k,v) - \frac{1}{2}\sin(\phi)\tilde{G}(u-k,v)$$

Bottom line – can recover phase information by looking at imaginary part of this

2D FFT analysis demo



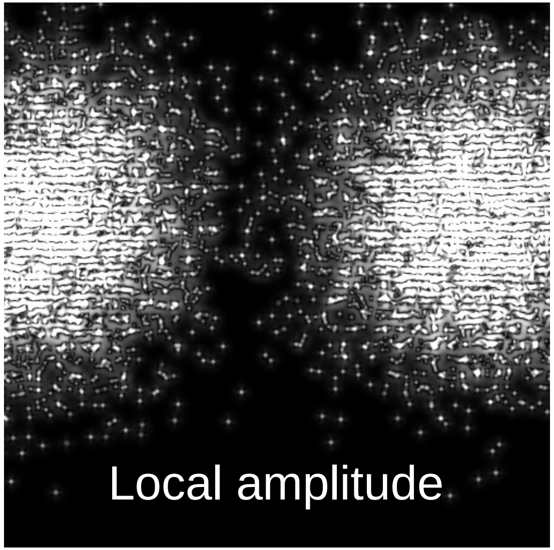
Examples:
-30 degree phase shift (top)

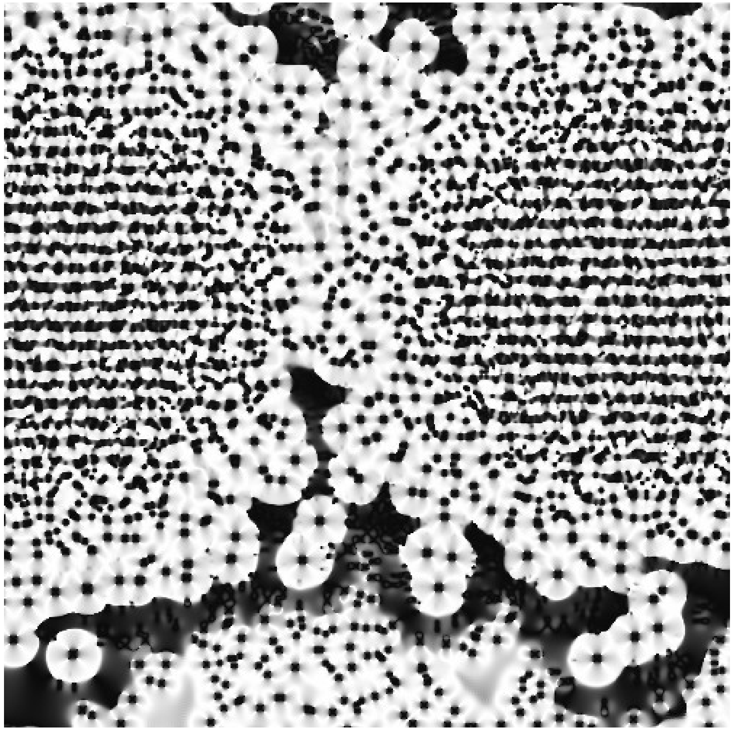
+50 degree phase shift (bottom)

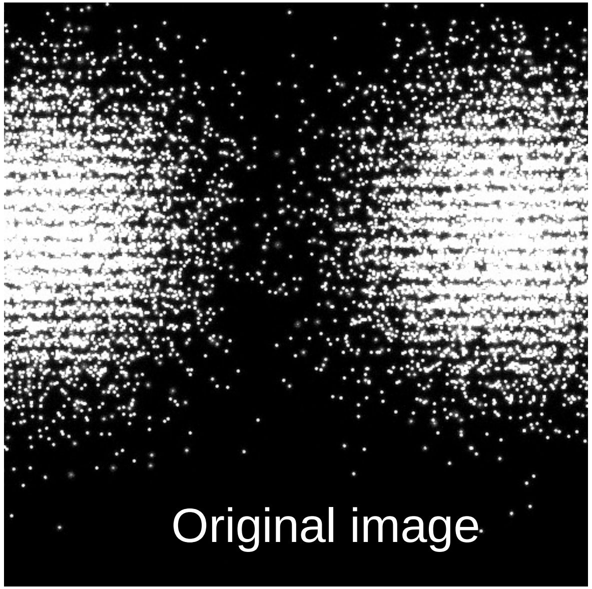
If one were to change frequency or width of gaussian cloud, blobs in fourier space would shift relative to each other (not shown here)

Monogenic Signal Analysis

- Extension to FFT analysis – a 2D generalisation of the Hilbert transform (via the Riesz transform) allows us to obtain local amplitude and phase information
- First apply a log-Gabor filter in Fourier domain (eliminating noise modulation), then extract phase & amplitude (see right).
- Early stages, but promising


Local amplitude

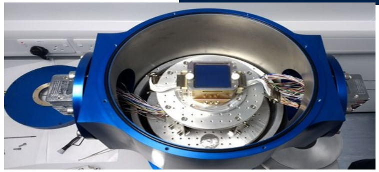

Local phase


Original image

Testing / Commissioning plans

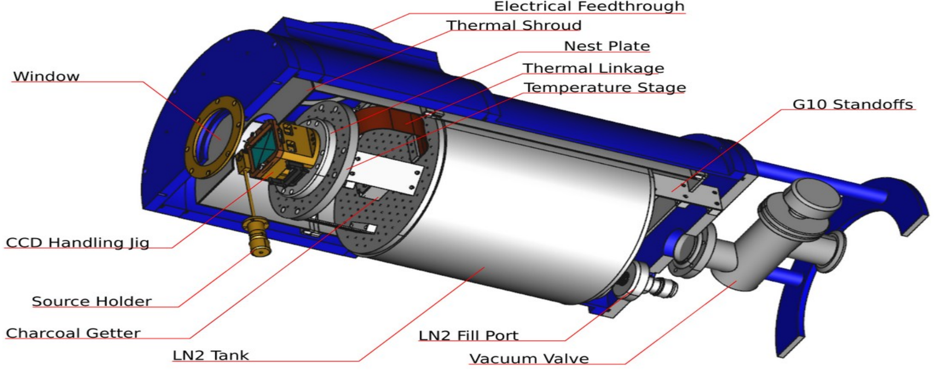
- Measure MTF and aberrations of lens assemblies via shearing interferometer (not as good as Shack-Hartmann, but much cheaper!)
- Check DoF of optics using special slanted Ronchi grating target (from Edmund optics, very expensive!)
- Calibrate MTF (via USAF, slit projections & laser speckle), noise profile, PTC, flat field and QE @ target wavelength of each detector (using LSST test stand light source / radiometry)
- Measure representative MTF and aberration of total system (using combined USAF / Ronchi / sector star target, rear illuminated)
- Ship 6 individually calibrated & tested imaging systems to SLAC - **May 2022 (!!!)**
- Current hardware plan:
  - sCMOS cameras
  - Canon 75mm EF mount macro lens adjustable down to 2.2 f/#.
  - EF lens controller from INSSI systems (for focus & aperture control)
  - Bit of ethernet / USB plumbing and readout software to tie it together!
- Given timescale, everything needs to be quite “plug and play”
- Have already taken some test data on loaned iXon camera

LSST Test System

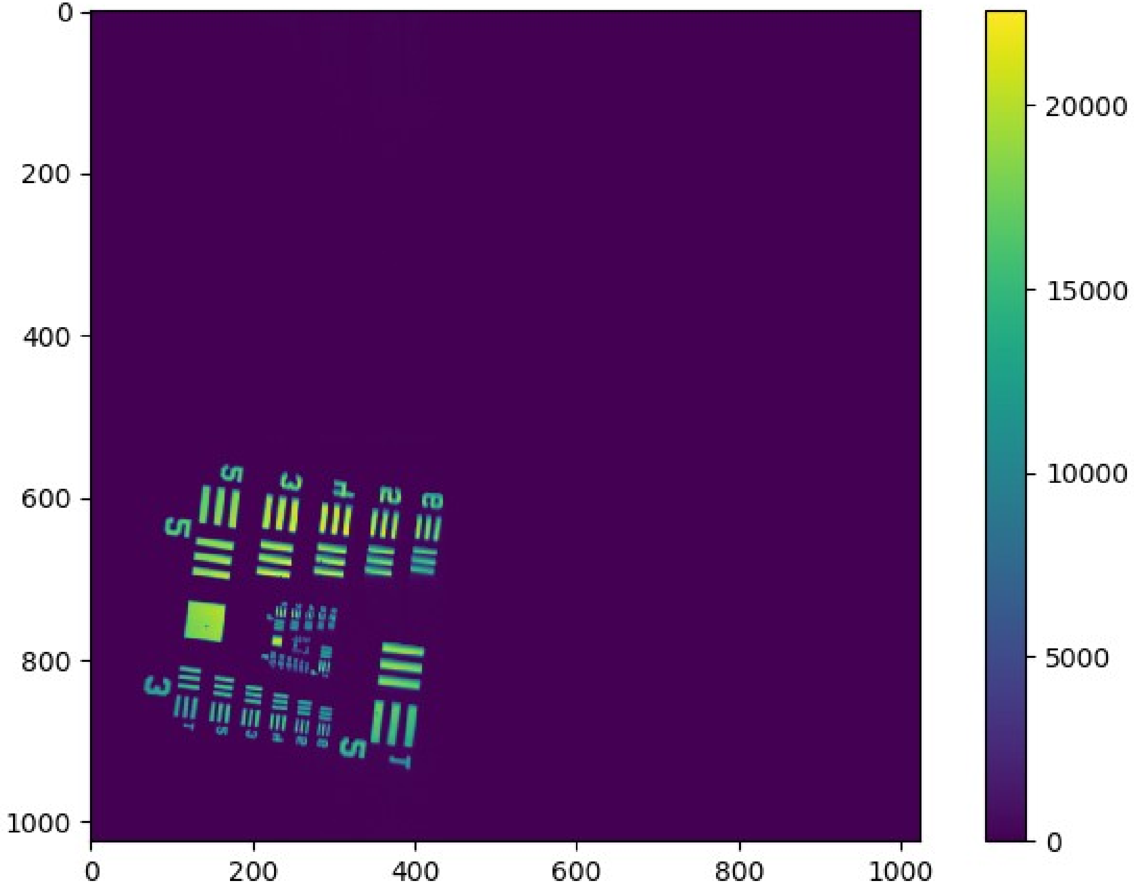


250W QTH light source + monochromator (300nm – 1600nm wavelength)

- Online radiometry and spectrosopy (at integrating sphere)
- Can quickly integrating sphere with projection optics for PSF etc measurements.

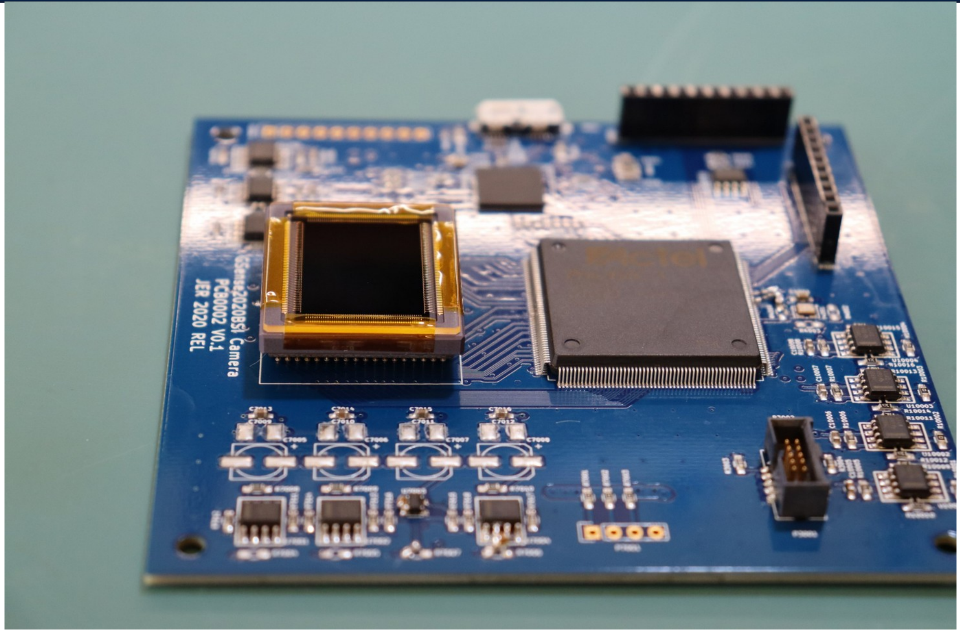


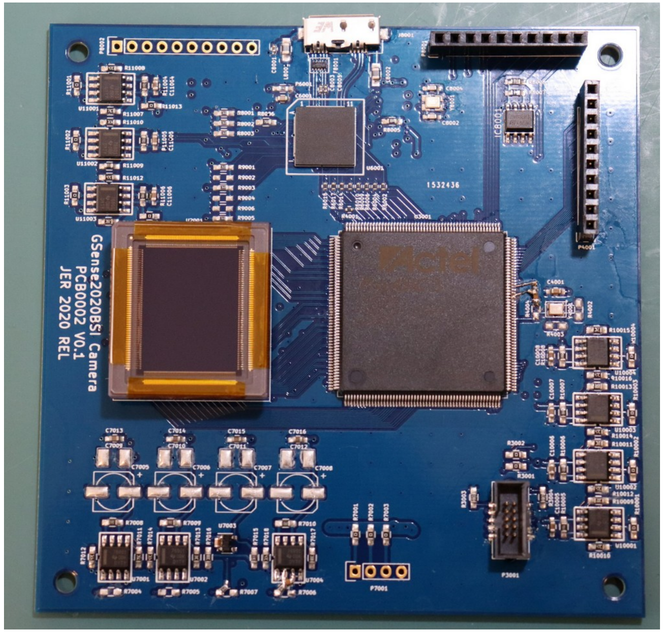
Ixon (EMCCD) test image



USAF test pattern projected onto sensor plane. No lens used.

REL1 Camera Concept





Since the integration times we are using are relatively short (<= 100us), we believe that TE cooling is **not required** on an sCMOS camera to suppress dark current below read noise to maintain effective SNR. The REL1 concept (based on Gsense2020BSI) would allow us to use an sCMOS sensor but fit within the original MAGIS space envelope available. Engineering model to arrive at OPMD in December for initial test / qualification.

Outstanding Questions

Questions to MAGIS simulations people...

- **What focal length do we actually need (i.e. field of view)?**
- **Are we drastically underestimating photon losses for some reason?**
- **What is the residual velocity of the atom clouds if any?**
- **Is there any useful general "image quality" metric we can usefully target rather than doing some physics analysis?**

Questions to MAGIS hardware people...

- **How much space do we actually have for the camera system?**
- **Can we really get away without global shutter (i.e. illumination of target comes from probe beam only)**
- **Is there an expected application for which ultimate read noise (as opposed to signal to noise ratio) is most important, e.g. extremely low number of atoms in a cloud?**
- **What are we expecting to receive as a trigger signal for acquisition (i.e. timings, jitter etc)?**

Thanks

All questions, comments & suggestions very enthusiastically received

Should there be interest in reviewing / using the code let us know!

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Right: the consequences of getting Depth of field wrong!