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A High-Accuracy Technique to Measure the Phase-Space of a Dispersive Particle Beam

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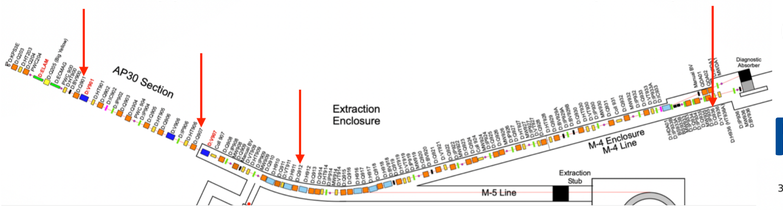
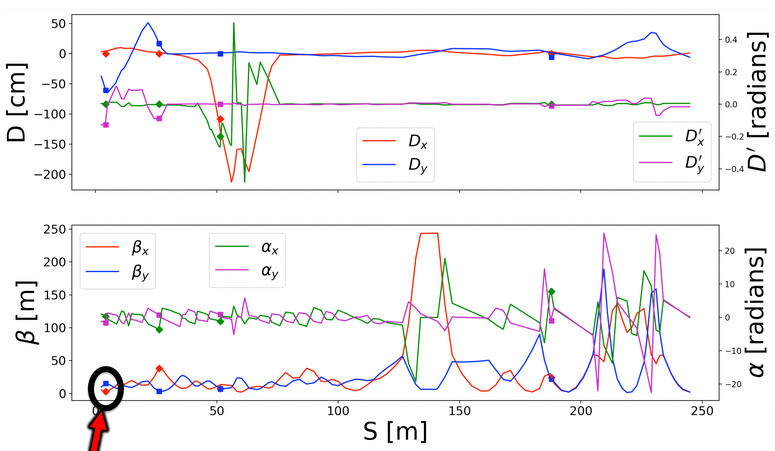
Fermilab Mu2e Experiment:

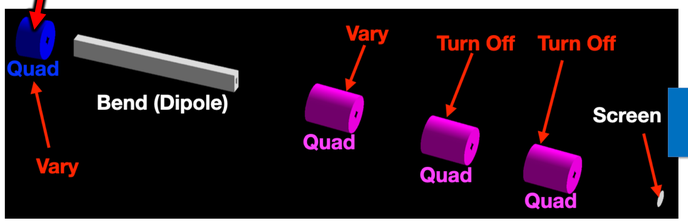
The Fermilab Mu2e experiment aims to study the possibility of charged lepton flavor violation (CLFV) through the conversion of muons to electrons in the field of an aluminum nucleus. This requires careful transport of an 8 GeV proton beam along the Muon Campus lines to a tungsten production target to achieve the statistics required. In accelerator physics, control of the beam dynamics and optimization of injection requires knowledge of the transverse properties: the Twiss parameters $(\epsilon, \beta, \alpha, \gamma)$ and the dispersion parameters (D, D', σ_δ^2). We propose a methodology for reconstructing these parameters with beam profile measurements alone, and we test our approach in simulation for the M4 line (below).



Below: Lattice functions for the entire M4 line. Solid lines are calculated from particle distributions generated by virtual detectors in G4Beamline. Markers show reconstructed values for the 4 locations of interest in our study which have the required optics for our methods. D, D' are assumed to be zero in non-bending plane



Below: G4Beamline visualization of one scan location (Q901).
G4Beamline is a particle tracking simulation program based on Geant4



Methods:

Reconstructing both the betatronic Twiss parameters and the dispersion parameters at a point of interest along the beamline with only profile measurements requires solving a two-part problem:

(1) Need models which minimize the transport error through the lattice. We treat the dispersion function as a first order perturbation to the betatron oscillations and derive the general scan formula:

$$\sigma^2 = m_{11}^2 A_1 + 2m_{11}m_{12}A_2 + m_{12}^2 A_3 + 2m_{11}m_{13}A_4 + 2m_{12}m_{13}A_5 + m_{13}^2 A_6$$

Fit Parameters:

$$A_1 = \epsilon\beta + D^2\sigma_\delta^2 \qquad A_4 = D\sigma_\delta^2$$
$$A_2 = -\epsilon\alpha + DD'\sigma_\delta^2 \qquad A_5 = D'\sigma_\delta^2$$
$$A_3 = \epsilon\gamma + D'^2\sigma_\delta^2 \qquad A_6 = \sigma_\delta^2$$

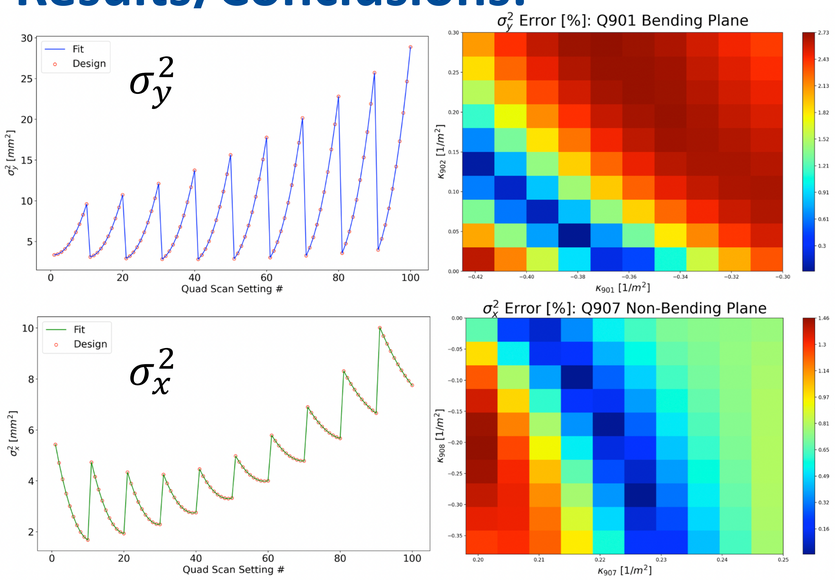
Ratios:

$$R_1 = (D^2\sigma_\delta^2)/A_1$$
$$R_3 = (D^2\sigma_\delta^2)/A_3$$

$$\epsilon = \sqrt{(\epsilon\beta)(\epsilon\gamma) - (\epsilon\alpha)^2}$$

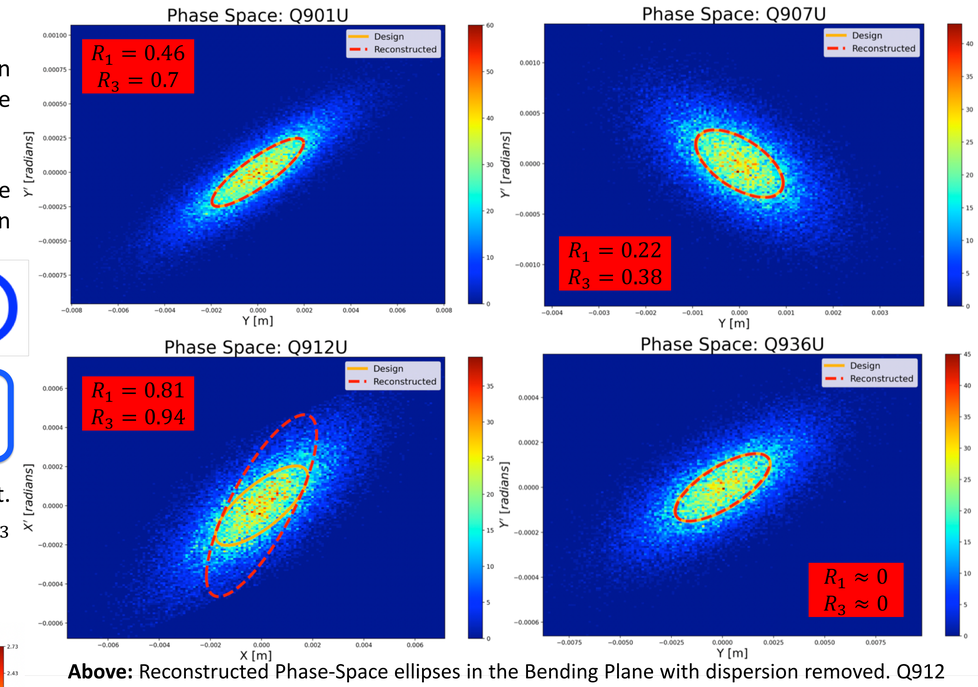
(1) A minimum choice of optics: two quadrupoles and one bending magnet. Varying 2 quads simultaneously allows for sufficient sampling of the m_{13} matrix element required to reconstruct the momentum spread.

Results/Conclusions:

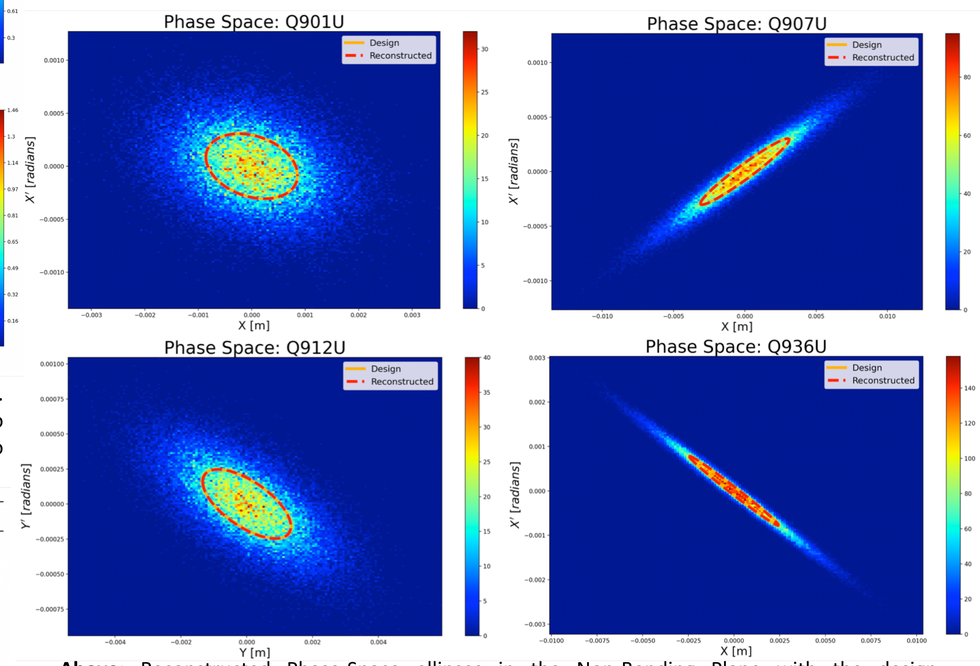


Above: Two example scans for both the Bending-Plane and the Non-Bending Plane. Color maps show the % Error of the model predictions as a function of the two normalized quadrupole gradients. A large variation of beam sizes is necessary to constrain all 7 parameters.

	Parameter	Design	Fit	Error: ±	% Difference
Q901U	$\beta_y[m]$	15.42	15.5	1.29	0.52
	α_y	-1.66	-1.68	0.14	1.58
	$\epsilon_y[m^{-6}]$	0.2497	0.251	0.021	0.52
	$D_y[m]$	-0.614	-0.606	0.0035	1.31
	D'_y	-0.1281	-0.1284	0.00086	0.28
	$\sigma_\delta^2[\times 10^{-6}]$	8.53	8.57	0.049	0.47



Above: Reconstructed Phase-Space ellipses in the Bending Plane with dispersion removed. Q912 Scan illustrates a limitation of the method. If the ratios R_1 and R_3 are very large, then it is difficult to estimate the Twiss parameters. Even when the dispersion parameters are known, the fit would overestimate γ and underestimate β (Q912). A solution is to simply choose a point of interest farther upstream. However, this may lead to larger transport errors in practice. Additionally, we find that including more quadrupoles has limited success in this case.



Above: Reconstructed Phase-Space ellipses in the Non-Bending Plane with the design assumptions of the M4 line. We achieve excellent agreement in all 4 cases.

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