J/Ψ and $\Psi(2s)$ production as a probe of low x evolution - an update

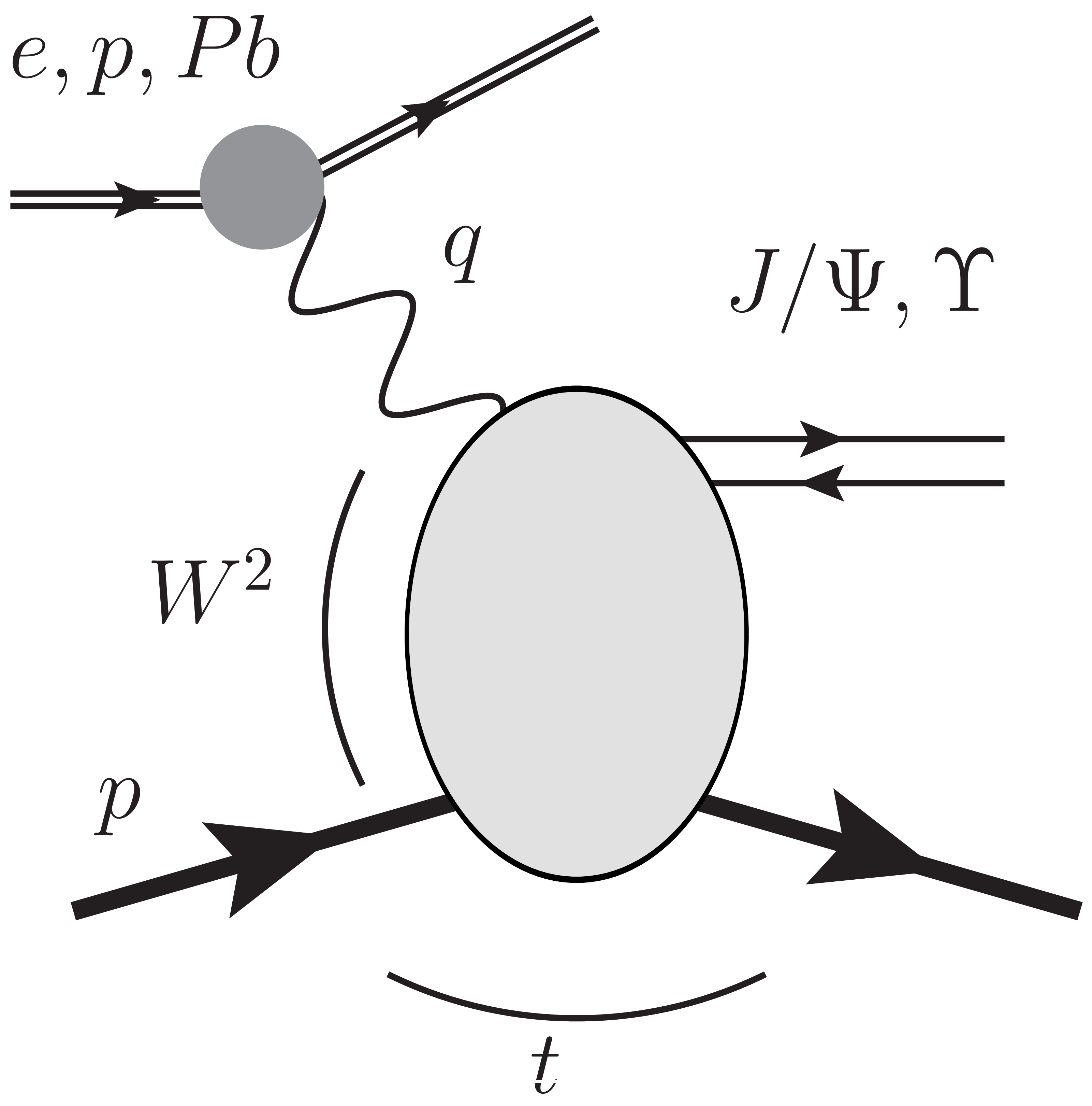
Martin Hentschinski
Universidad de las Americas Puebla
Ex-Hacienda Santa Catarina Martir S/N
San Andrés Cholula
72820 Puebla, Mexico
martin.hentschinski@gmail.com

based on:

- I. Bautista, Fernandez Tellez, MH, PRD 94 (2016) 5, 054002, arXiv:1607.05203
- A. Arroyo Garcia, MH, K.Kutak, PLB 795 (2019) 569-575, arXiv:1904.04394
- MH, E. Padron Molina, *Phys.Rev.D* 103 (2021) 7, 074008 arXiv:2011.02640
- Alcazar Peredo, MH, in preparation

Snowmass 2021 contributions from EF06, 2021, 08 of December 2021, Online

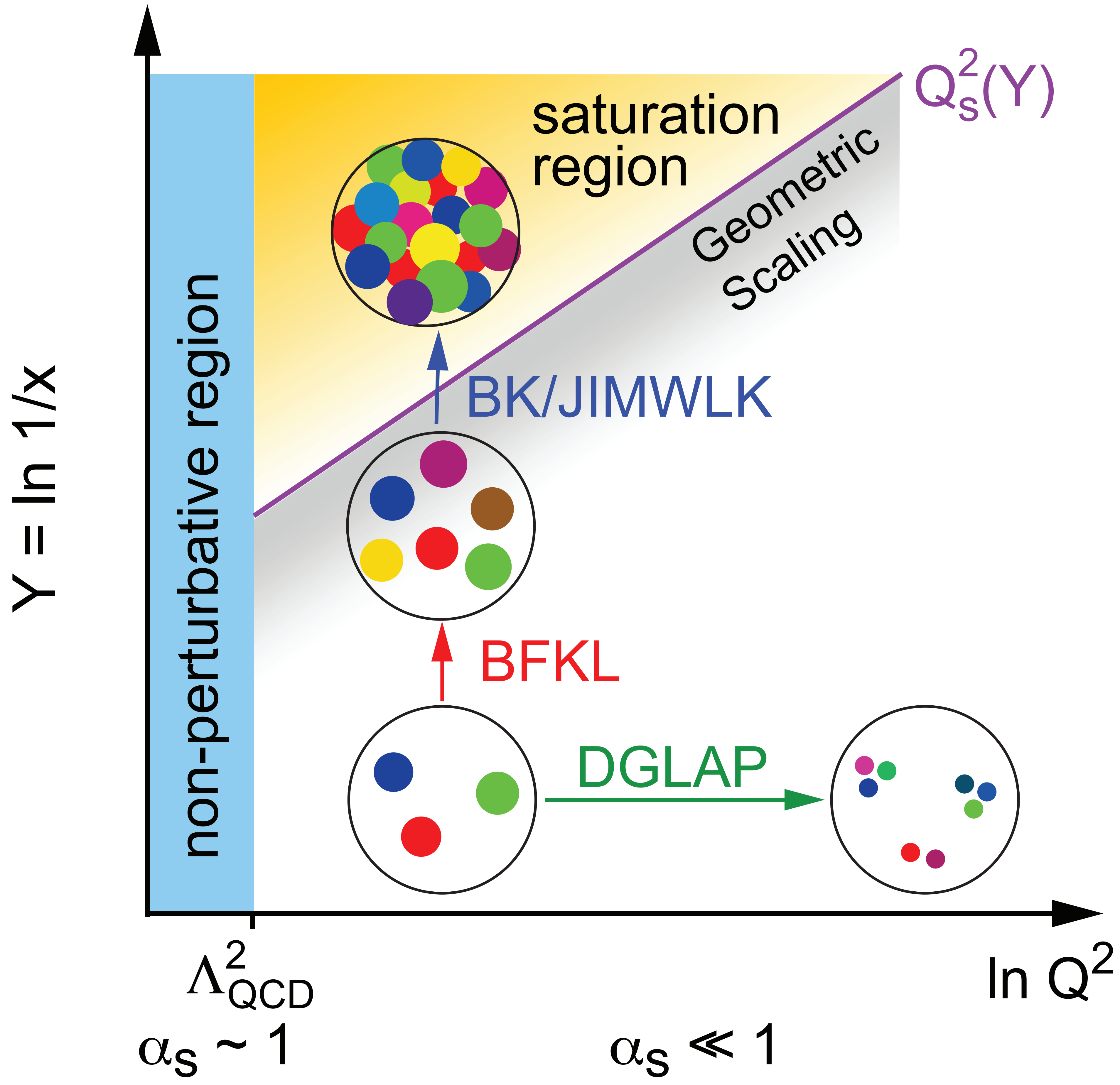
photo induced exclusive photo-production of J/Ψs and $\Psi(2s)$



technical details: see appendix

- hard scale: charm mass (small, but perturbative)
- reach up to x≳.5·10^{-6}
- perturbative cross-check: ϒ (b-mass)
- measured at **LHC** (LHCb, ALICE, CMS) & **HERA** (H1, ZEUS)

Goal: confront linear vs. non-linear



BK evolution for dipole amplitude $N(x,r) \in [0,1]$

[related to gluon distribution]

kernel calculated in pQCD

$$\frac{dN(x,r)}{d\ln\frac{1}{x}} = \int d^2\boldsymbol{r}_1 K(\boldsymbol{r},\boldsymbol{r}_1) \boxed{[N(x,r_1) + N(x,r_2) - N(x,r)} - \boxed{N(x,r_1)N(x,r_2)]}$$

linear BFKL evolution = subset of complete BK

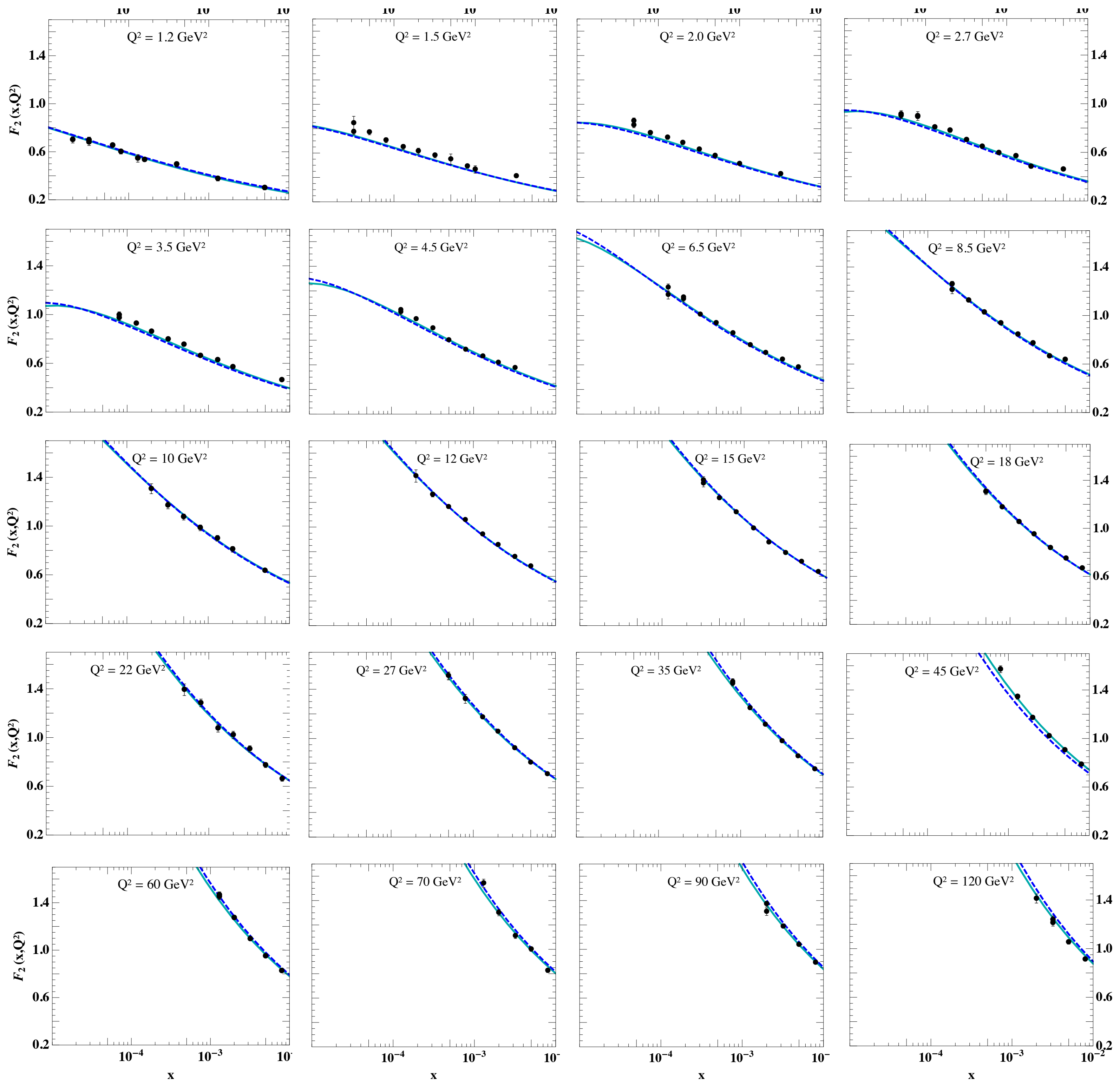
non-linear term relevant for N~1 (=high density)

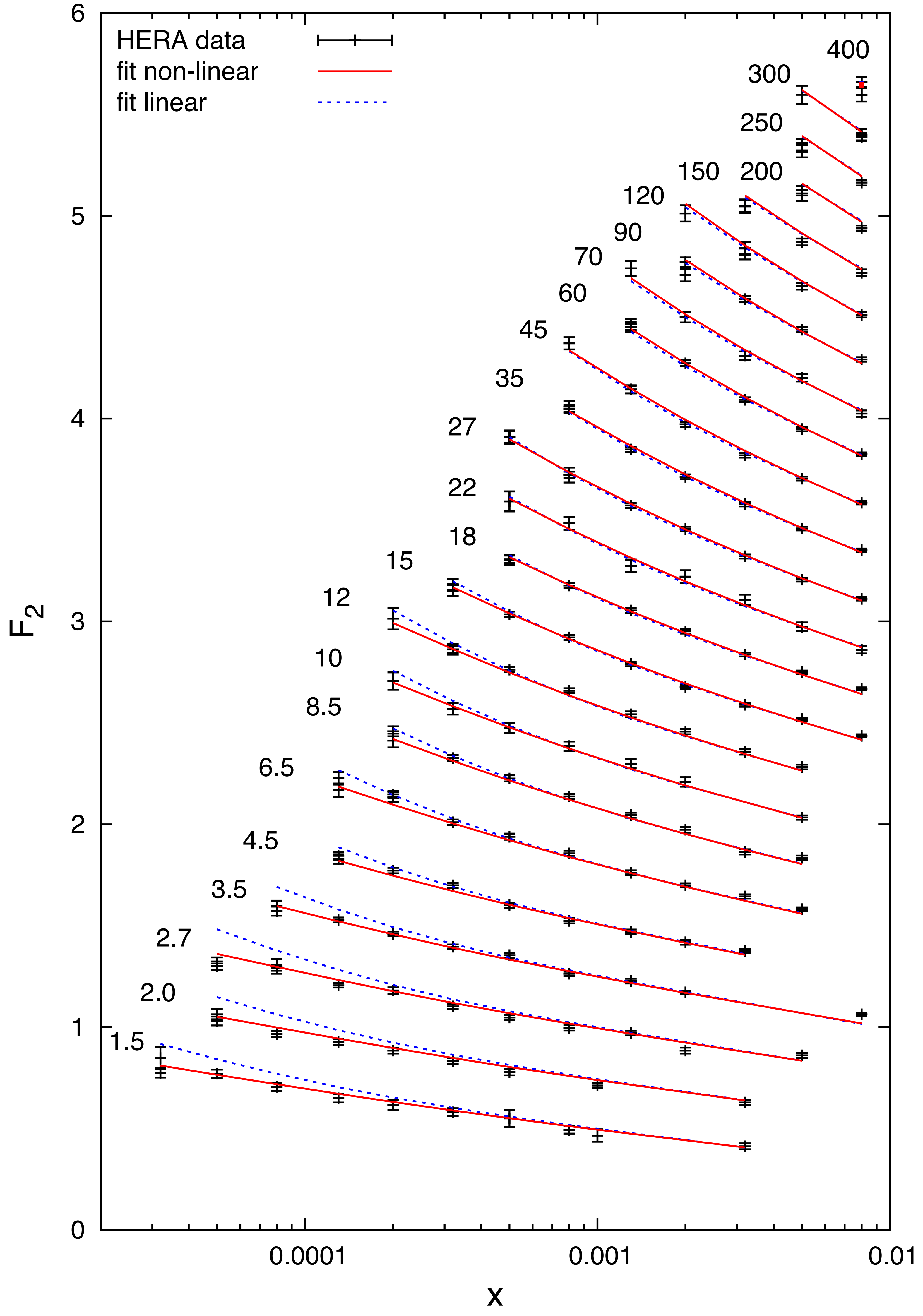
Most recent study

[MH, E. Padron Molina,:2011.02640]

- Use HSS NLO BFKL fit for linear evolution [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]
- Use KS LO BK fit for non-linear evolution [Kutak, Sapeta; 1205.5035]

Both fitted to combined HERA data

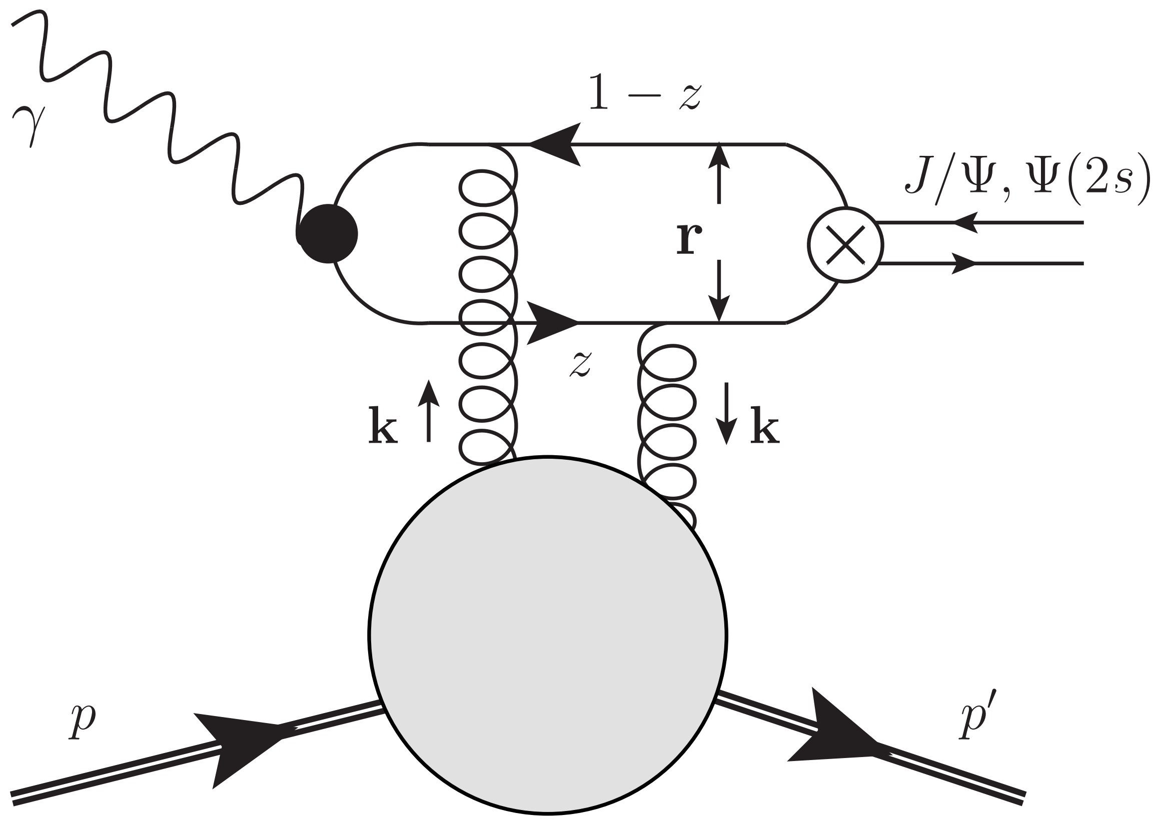




improved transition amplitude $\gamma \to$ VM

includes relativistic spin rotation effects + (more) realistic $c\bar{c}$ potential both for J/Ψ and $\Psi(2s)$

[Hufner, Y. Ivanov, B. Kopeliovich, A. Tarasov; hep-ph/0007111],
[M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; 1812.03001; 1901.02664]



$$\Im\mathrm{m}\mathcal{A}_T(W^2, t=0) = \int d^2\boldsymbol{r}\left[\sigma_{q\bar{q}}\left(\frac{M_V^2}{W^2}, r\right)\Sigma_T^{(1)}(r) + \frac{d\sigma_{q\bar{q}}\left(\frac{M_V^2}{W^2}, r\right)}{dr}\Sigma_T^{(2)}(r)\right]$$

- depends both on dipole cross-section and its derivative
- wave functions have been obtained in [M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; 1812.03001; 1901.02664] through numerical solution to corresponding Schrödinger equation
- transition function factorizes for real photon ($Q = 0$)

$$\Sigma_T^{(i)}(r) = \hat{e}_f\sqrt{\frac{\alpha_{e.m.}N_c}{2\pi^2}}K_0(m_f r)\,\Xi^{(i)}(r), \qquad i = 1, 2$$

$$\Xi^{(1)}(r) = \int_0^1 dz\int\frac{d^2\boldsymbol{p}}{2\pi}e^{i\boldsymbol{p}\cdot\boldsymbol{r}}\frac{m_T^2 + m_T m_L - 2p_T^2 z(1-z)}{m_T + m_L}\Psi_V(z, |\boldsymbol{p}|),$$

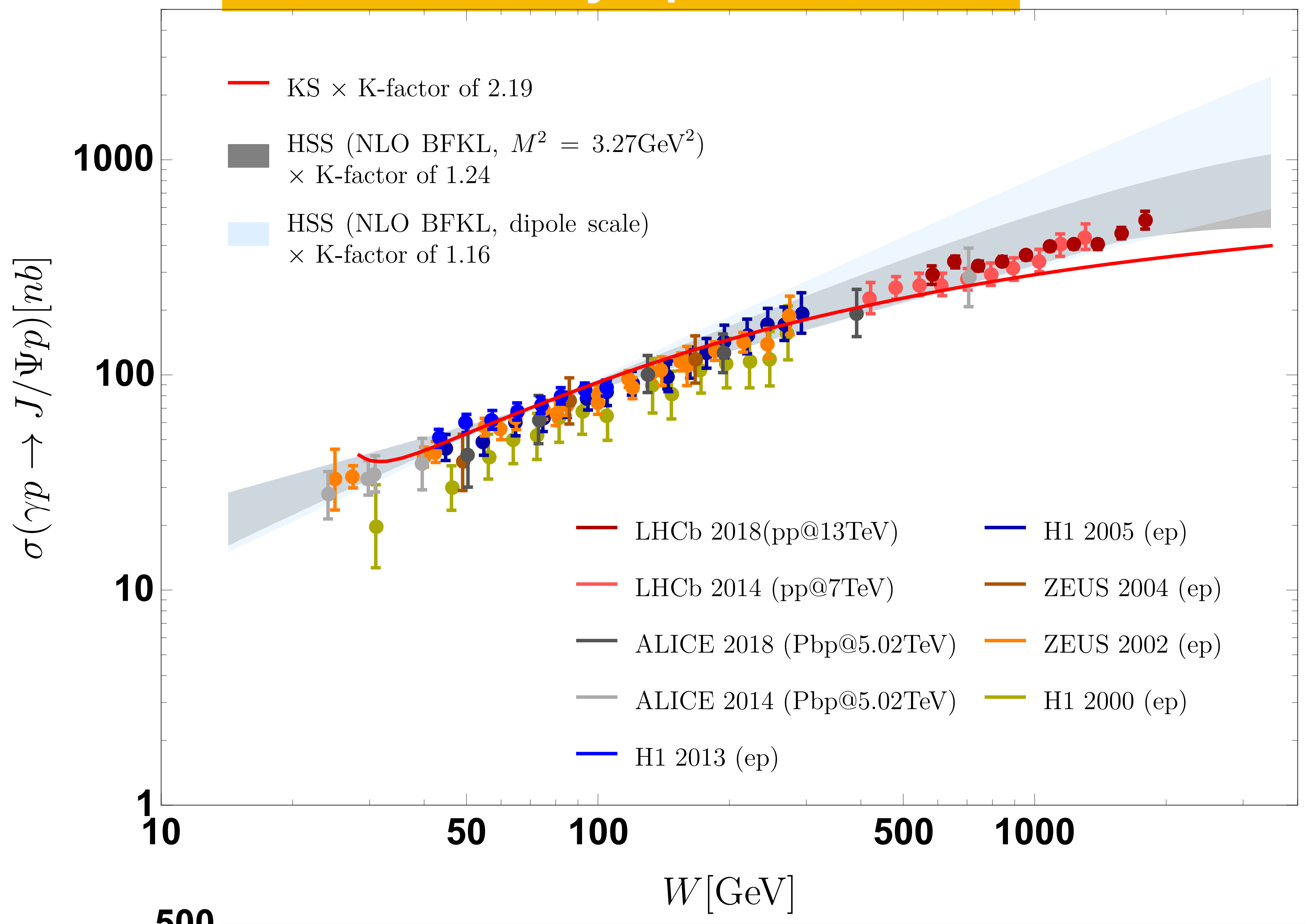
$$\Xi^{(2)}(r) = \int_0^1 dz\int\frac{d^2\boldsymbol{p}}{2\pi}\,e^{i\boldsymbol{p}\cdot\boldsymbol{r}}|\boldsymbol{p}|\frac{m_T^2 + m_T m_L - 2\boldsymbol{p}^2 z(1-z)}{2m_T(m_T + m_L)}\Psi_V(z, |\boldsymbol{p}|),$$

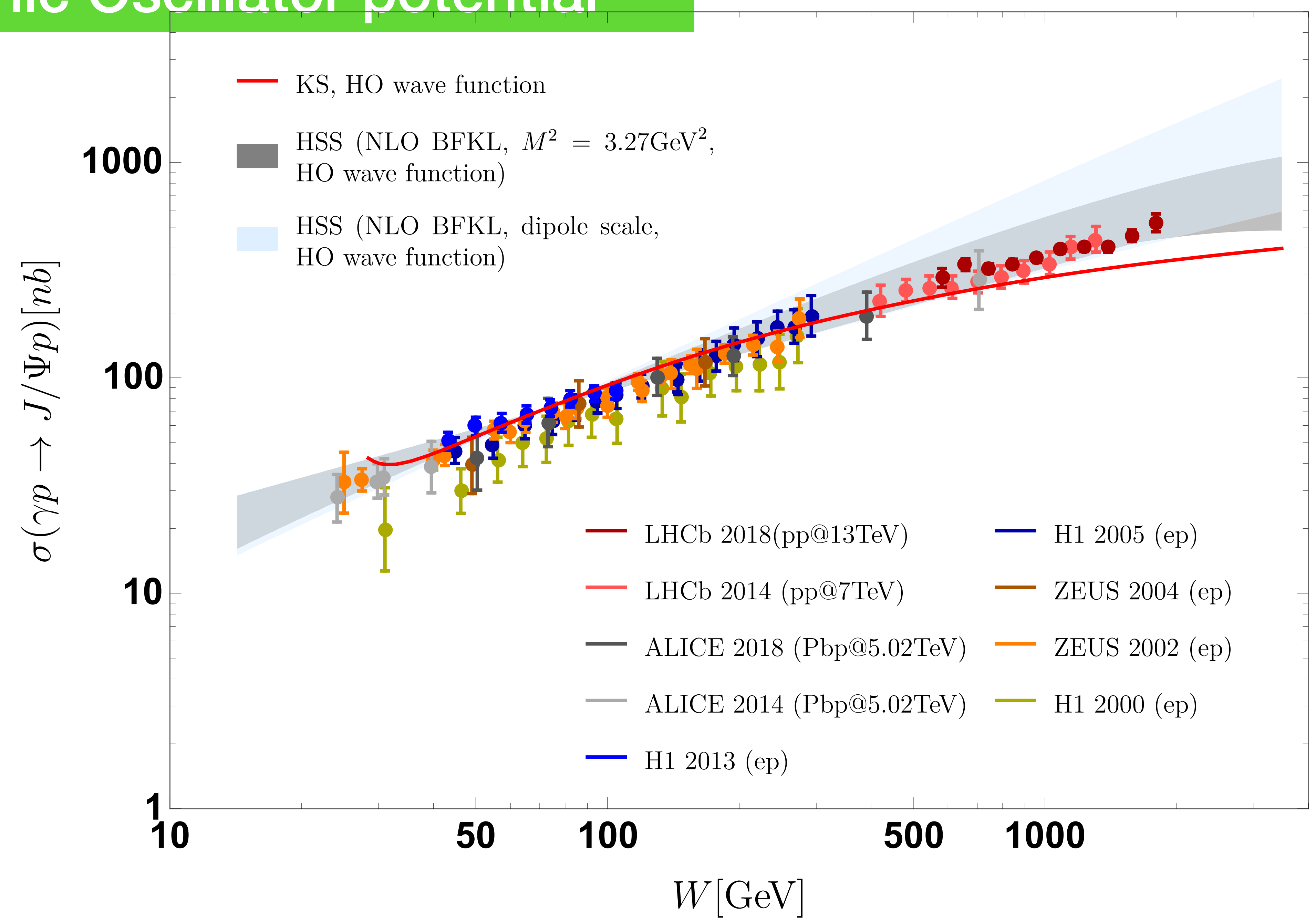
- $\Psi_V(z, \mathbf{p})$ provided as table by authors of [1812.03001; 1901.02664]
- $m_T^2 = m_f^2 + \boldsymbol{p}^2$, $m_L^2 = 4m_f^2 z(1-z)$,

Buchmüller-Tye potential

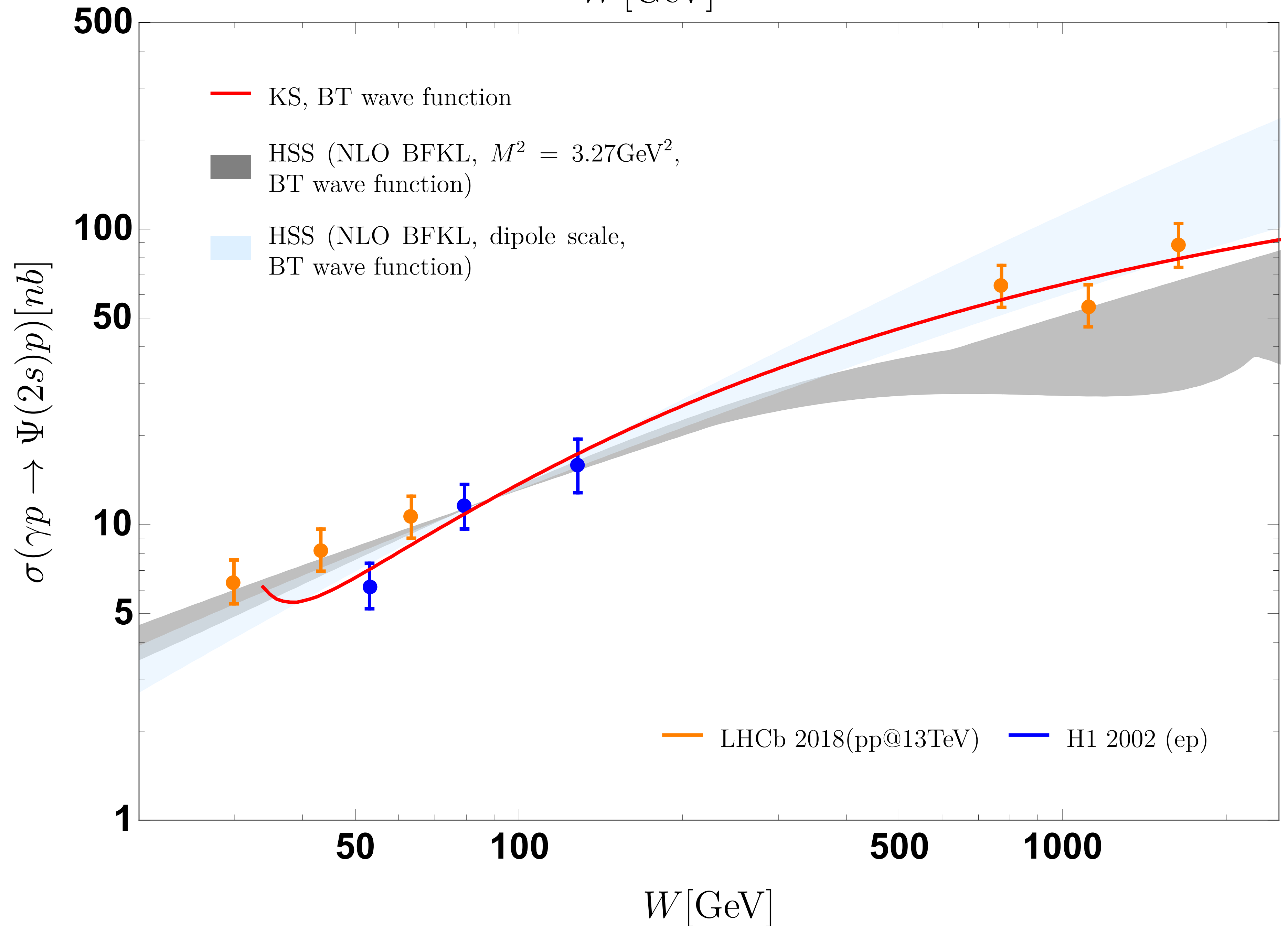
Harmonic Oscillator potential

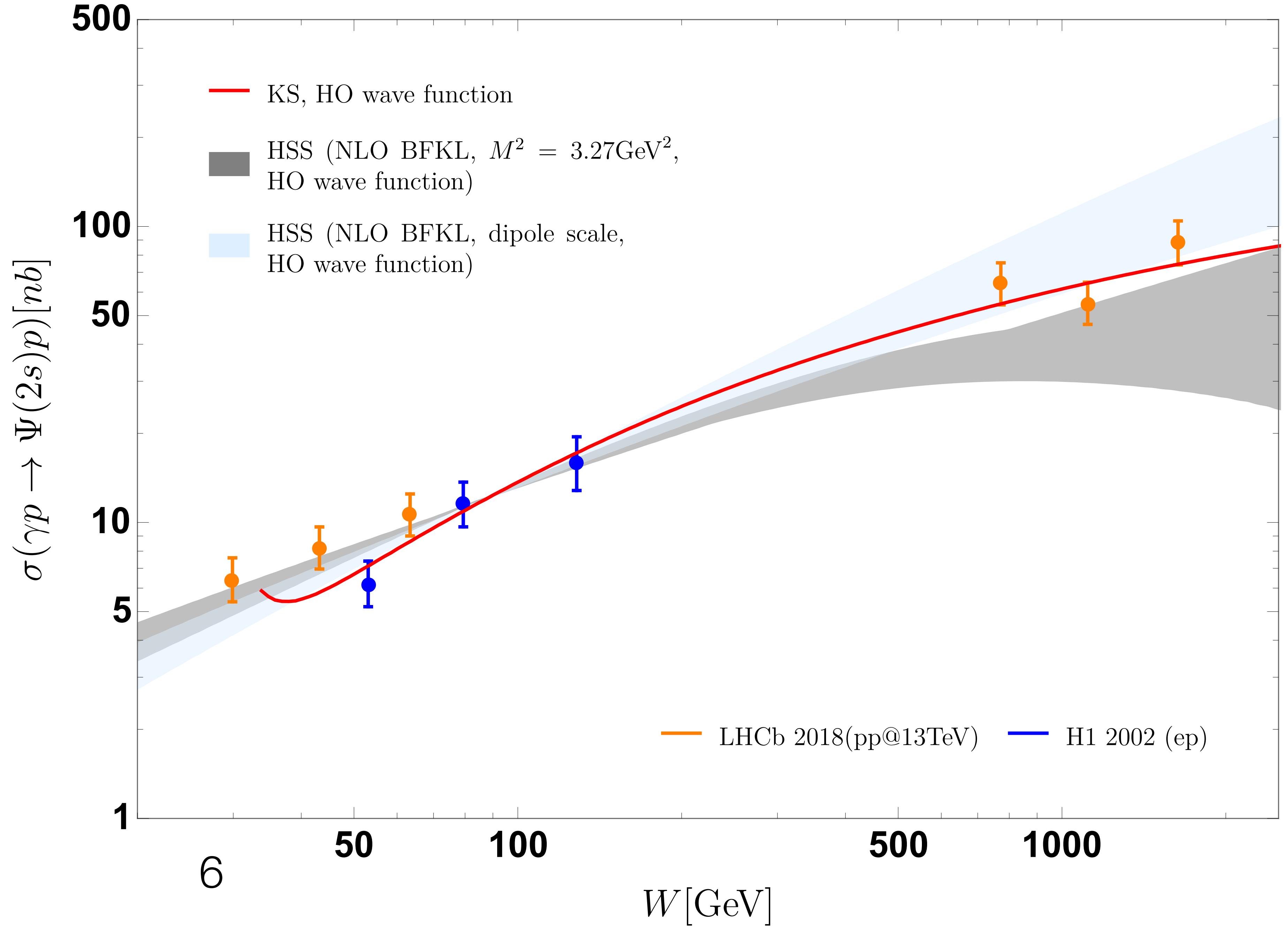
J/Ψ





$\Psi(2s)$





Observations:

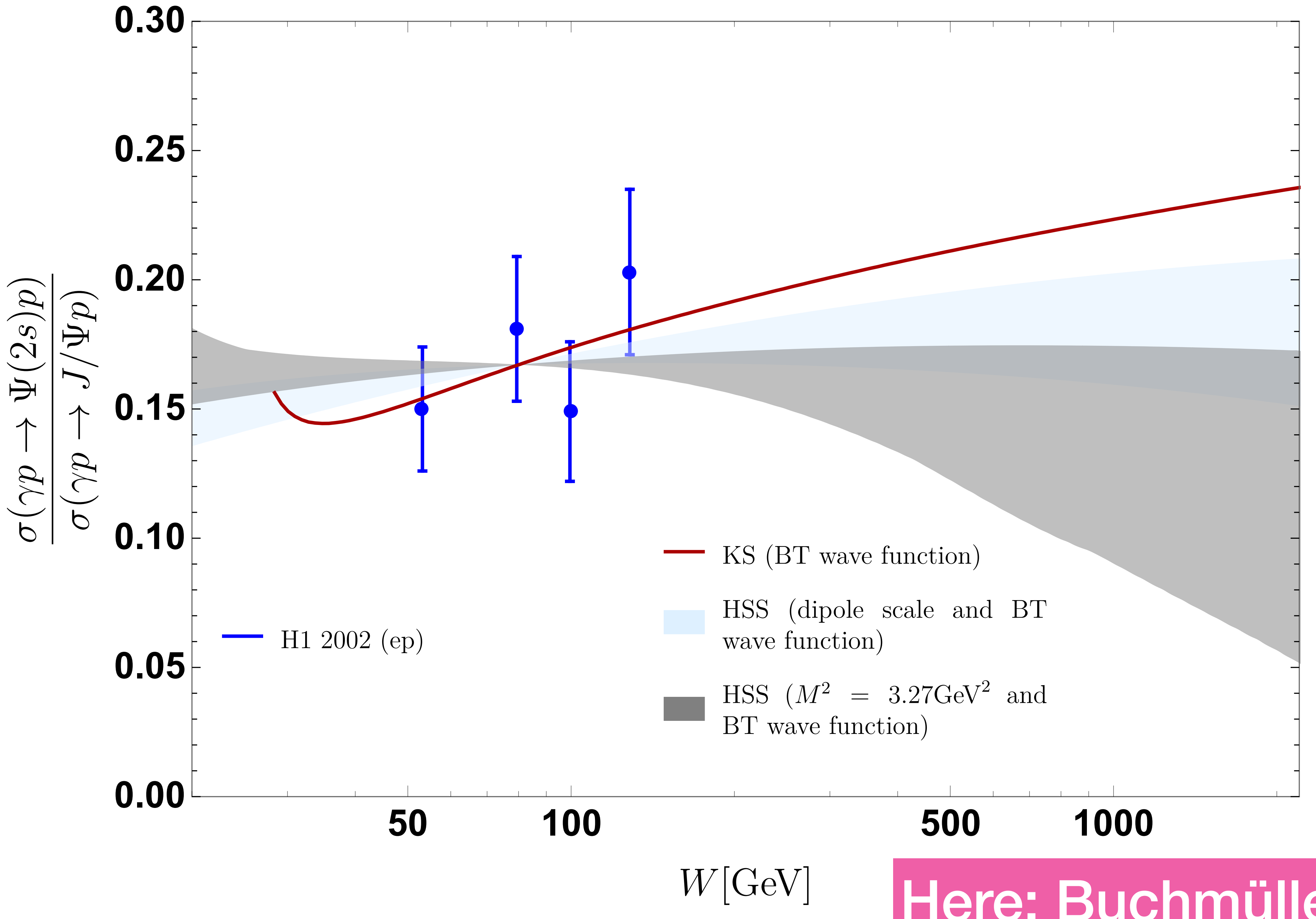
- Fixed scale BFKL (used for the original fit) develops instability
- Can be cured by setting renormalization scale $\mu^2 = \frac{1}{\mathbf{r^2}} + \mu_0^2$

  r : transverse size of dipole

There are difference between BFKL (HSS fit) and BK (KS fit), but they do not really allow to distinguish between both descriptions

- theory uncertainties [expect same size for BK as for BFKL]
- Experimental uncertainties are underestimated [error bars = propagation of the uncertainty of the rapidity distribution]

More interesting: the ratio $\sigma[\Psi(2s)]/\sigma[J/\Psi]$



Here: Buchmüller-Tye, HO very similar

- Rise of the non-linear gluon
- No rise is present for HSS (NLO BFKL) gluon (stabilized version)
- both slope & curvature differ
- general feature of perturbative QCD evolution?

problem: no data at high energies

(J/Ψ and $\Psi(2s)$ LHCb data in different W-bins)

- rise of non-linear gluon also observed in [M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; 1812.03001; 1901.02664] →KST dipole X-section [Kopeliovich, Schäfer, Tarasov, hep-ph/9908245]
- here: confirmed for KS (BK) gluon

Feature of the fits or something more general?

constant ratio → linear
Growing ration → non-linear

The ratio within the GBW model

general feeling: it would be good to understand the observed behavior a bit better
how? use a simple model & see what it tells us

GBW model: [Golec-Biernat, Wusthoff, hep-ph/9807513]

$$\sigma_{q\bar{q}}(x,r) = \sigma_0 \left(1 - \exp(-\frac{r^2 Q_s^2(x)}{4} \right)$$ with saturation scale $Q_s^2(x) = Q_0^2 \left(\frac{x}{x_0} \right)^\lambda$

linearized version: $$\sigma_{q\bar{q}}^{lin.}(x,r) = \sigma_0 \frac{r^2 Q_s^2(x)}{4}$$

use most recent fit [Golec-Biernat, Sapeta, 1711.11360] to combined HERA data with $Q^2 \leq 10\mathrm{GeV}^2$ and $\chi^2/N_{dof} = 352/219 = 1.61$

$\sigma_0[mb]$	λ	$x_0/10^{-4}$
27.43±0.35	0.248±0.002	0.40±0.04

The ratio for the GBW model

work in progress

Recall: $$\Im\mathrm{m}\mathcal{A}_T(W^2,t=0) = \int d^2\boldsymbol{r}\left[\sigma_{q\bar{q}}\left(\frac{M_V^2}{W^2},r\right)\overline{\Sigma}_T^{(1)}(r) + \frac{d\sigma_{q\bar{q}}\left(\frac{M_V^2}{W^2},r\right)}{dr}\overline{\Sigma}_T^{(2)}(r)\right]$$

$\Downarrow$ Linear GWB

$$\Im\mathrm{m}\mathscr{A}^{lin.}(x) \sim Q_s^2(x)\cdot\int dr\ldots$$

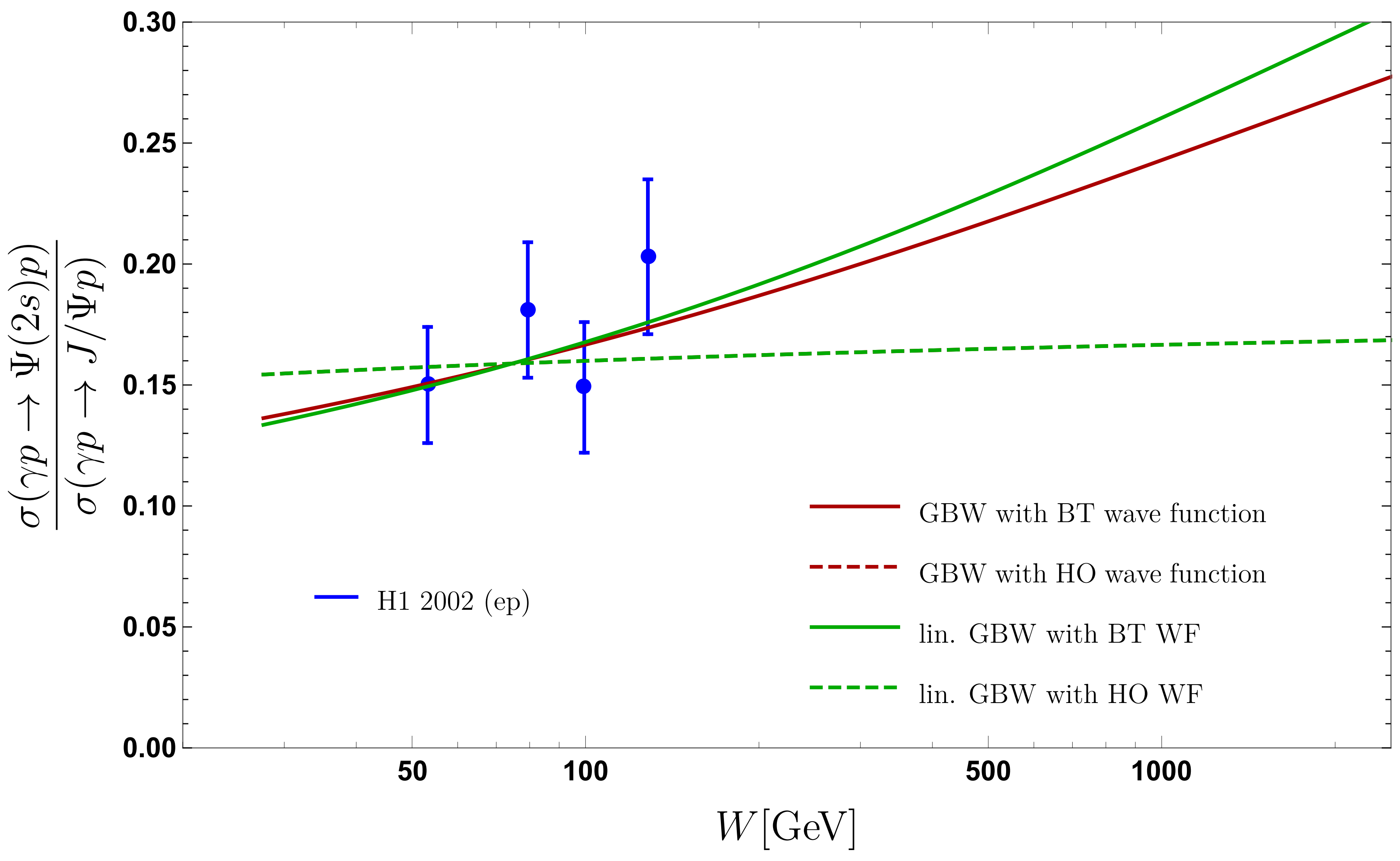
Cross-section:

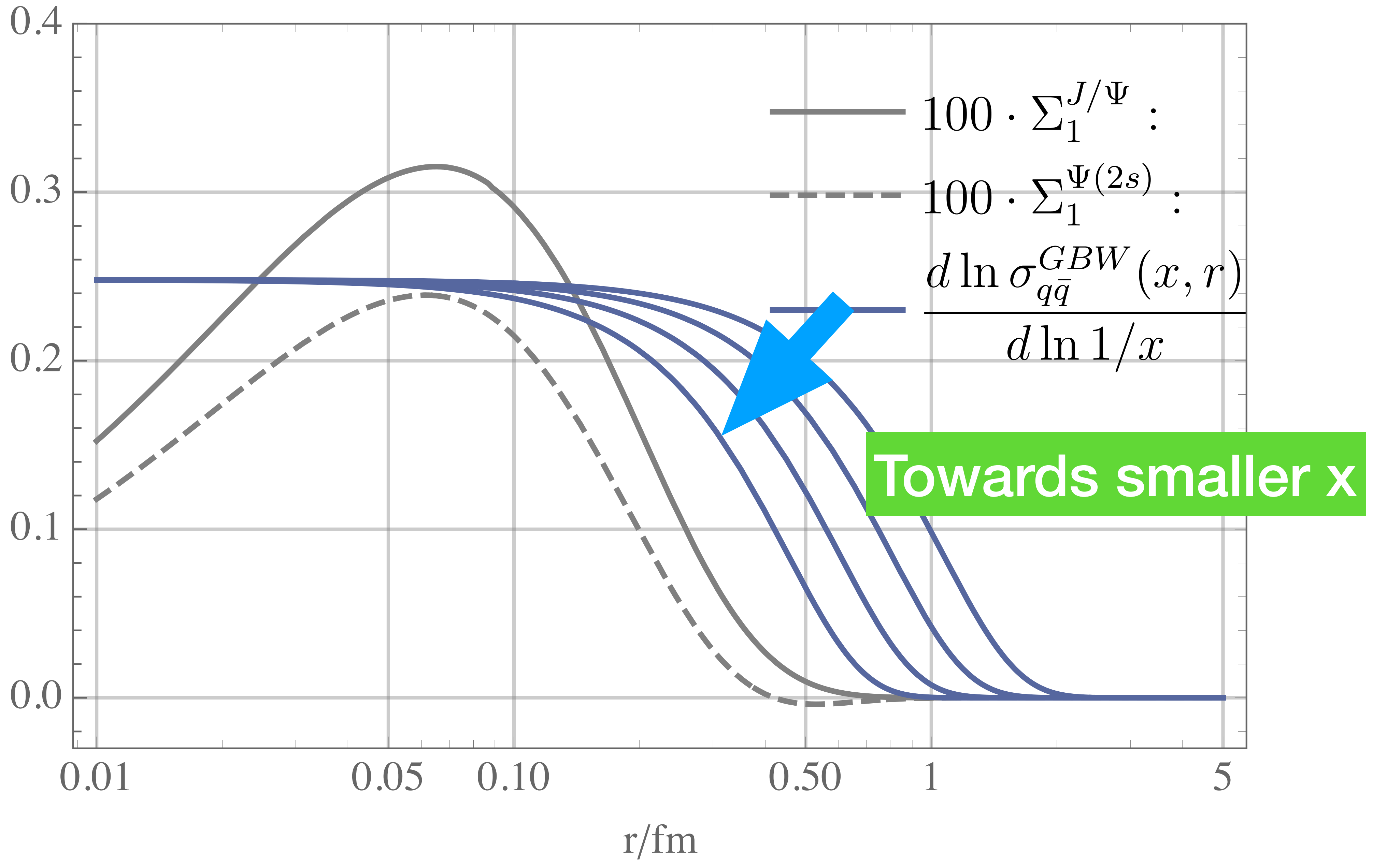
$$\frac{d\sigma}{dt}(\gamma p\to Vp)\Big|_{t=0} = \frac{1}{16\pi}\left|\mathcal{A}^{\gamma p\to Vp}(W^2,t=0)\right|^2$$

$$\sigma^{\gamma p\to Vp}(W^2) = \frac{1}{B_D(W)}\frac{d\sigma}{dt}(\gamma p\to Vp)\Big|_{t=0}$$

- $Q_s(x) = Q_s(M_V^2/W^2)$ cancels for the ratio
- Ratio constant with energy for **linear GBW**

The ratio within the GBW model





r-dependence of the "slope" $\dfrac{d \ln \sigma_{q\bar{q}}}{\ln 1/x}$

- for linear model x-dependence in $Q_s^2(x) = Q_0^2 \left(\dfrac{x}{x_0}\right)^\lambda$ we have $\dfrac{d \ln \sigma_{q\bar{q}}}{\ln 1/x} = \lambda$=const.
- Non-trivial r-dependence for complete GBW model → rise of the ratio

The DGLAP improved saturation model

[Bartels, Golec-Biernat, Kowalski; hep-ph/0203258]

Essentially the GBW model with DGLAP evolution

$$\sigma_{\mathrm{dip}}(r,x) = \sigma_0 \left\{ 1 - \exp\left(-\frac{\pi^2 r^2\, \alpha_s(\mu^2)\, xg(x,\mu^2)}{3\sigma_0} \right) \right\} ,$$

Factorization scale originally: $$\mu^2 = \frac{C}{r^2} + \mu_0^2 \, .$$

Recent fit: $$\mu^2 = \frac{\mu_0^2}{1 - \exp(-\mu_0^2\, r^2/C)}$$

[Golec-Biernat, Sapeta; 1711.11360]

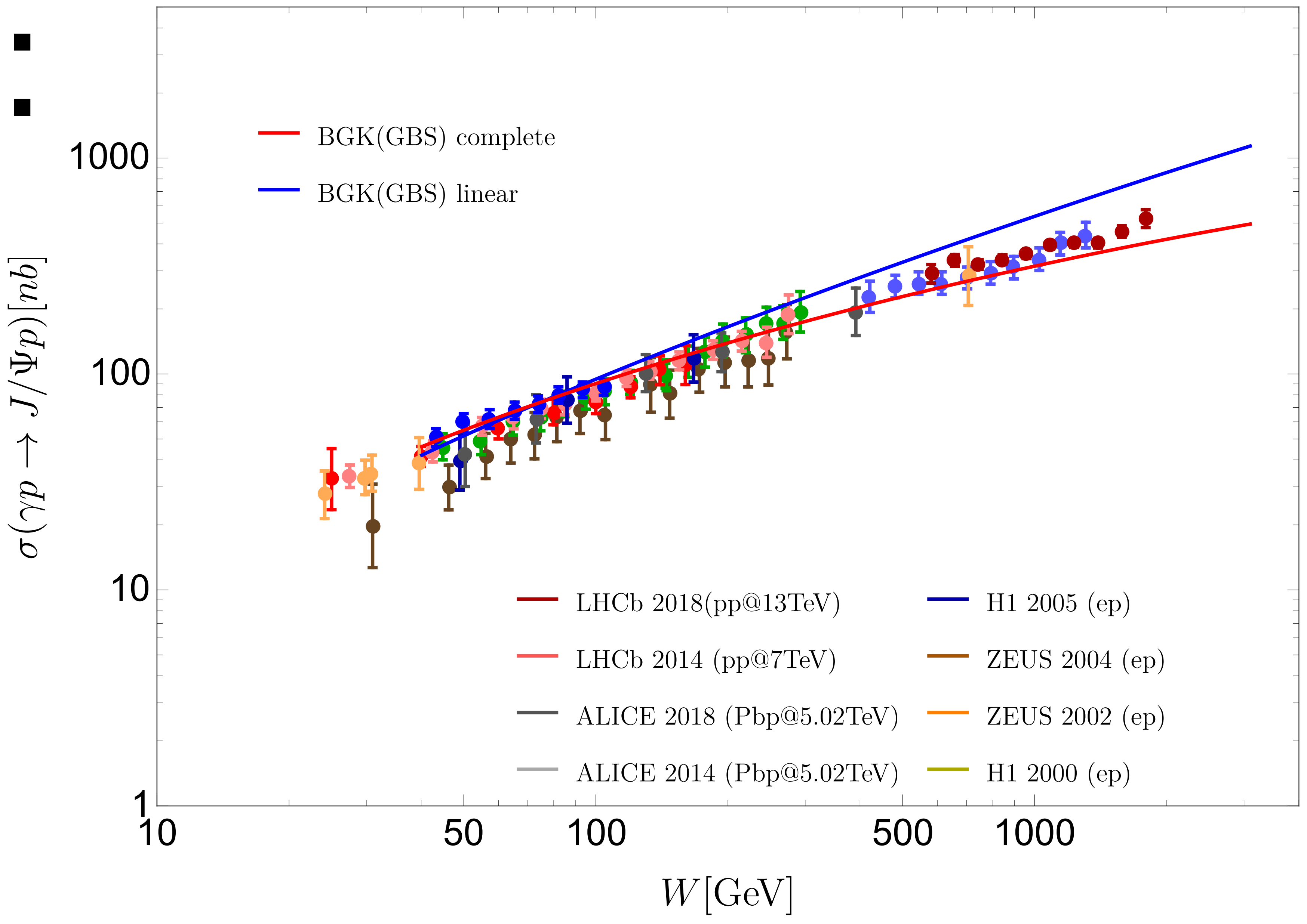
In common:

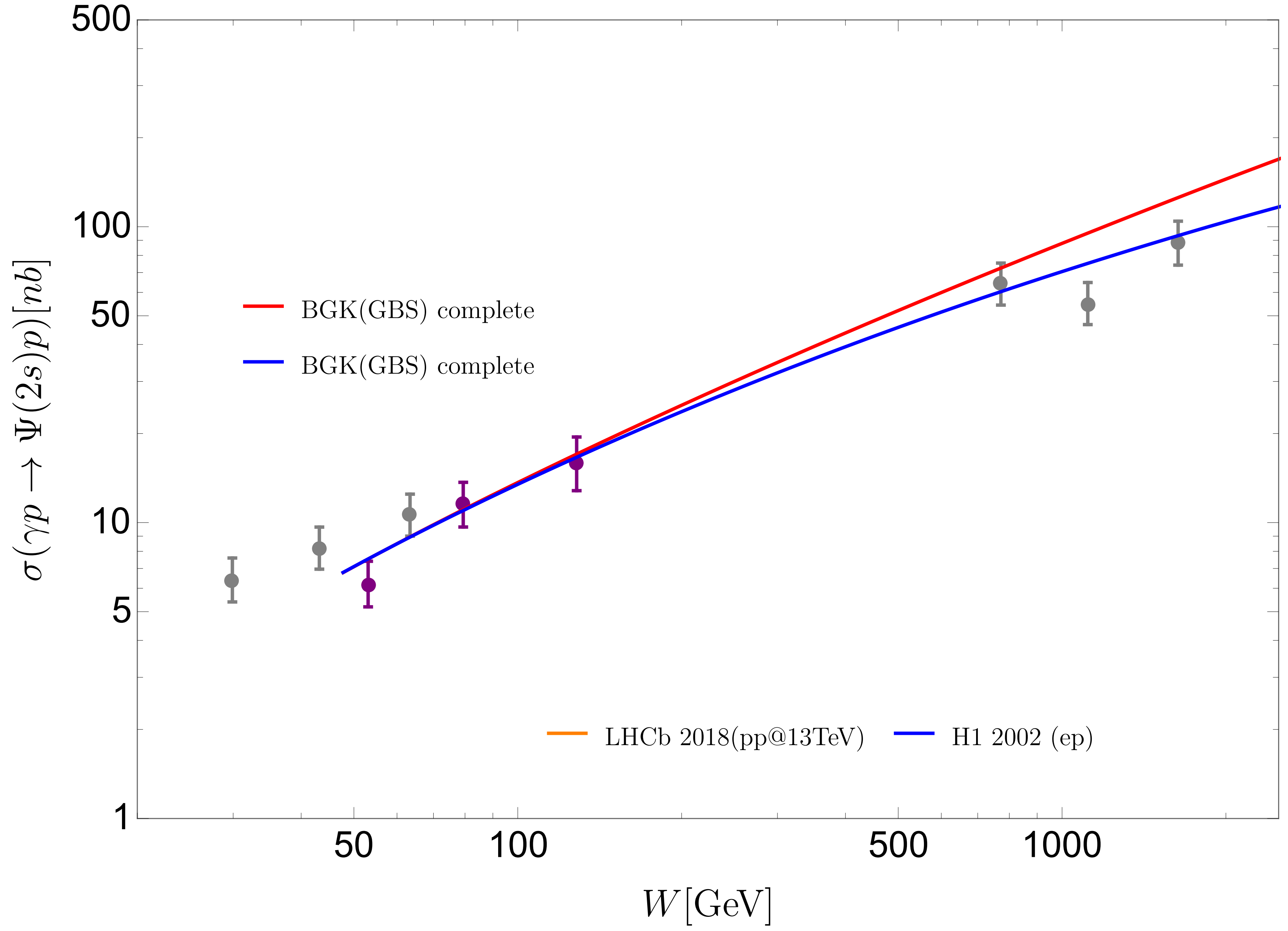
- for large dipole sizes r, $\mu \to \mu_0$
- Otherwise $\sim C/r^2$

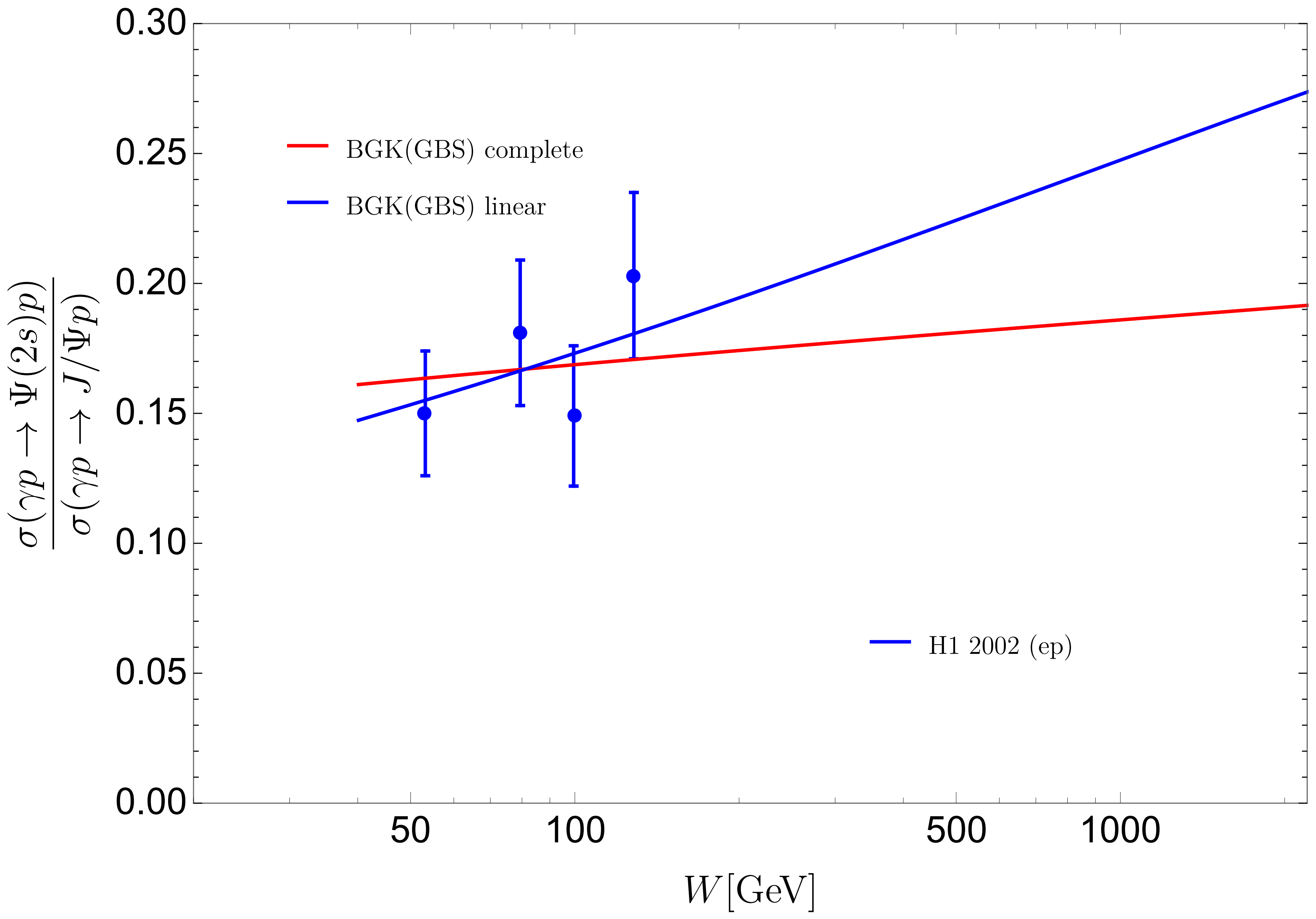
Saturation scale becomes r-dependent → includes correct DGLAP limit for small r

Complementary to BFKL/BK study

Results:

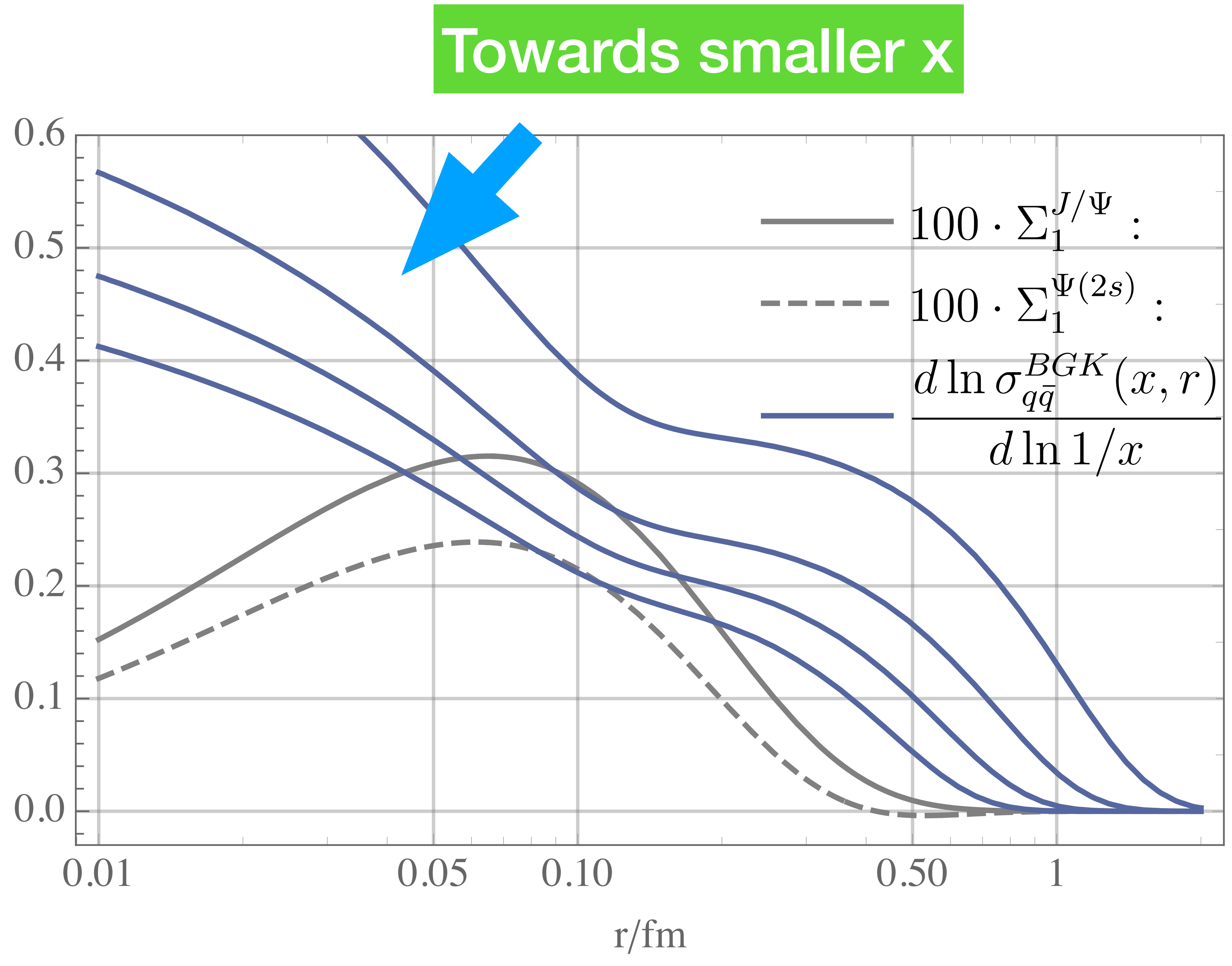




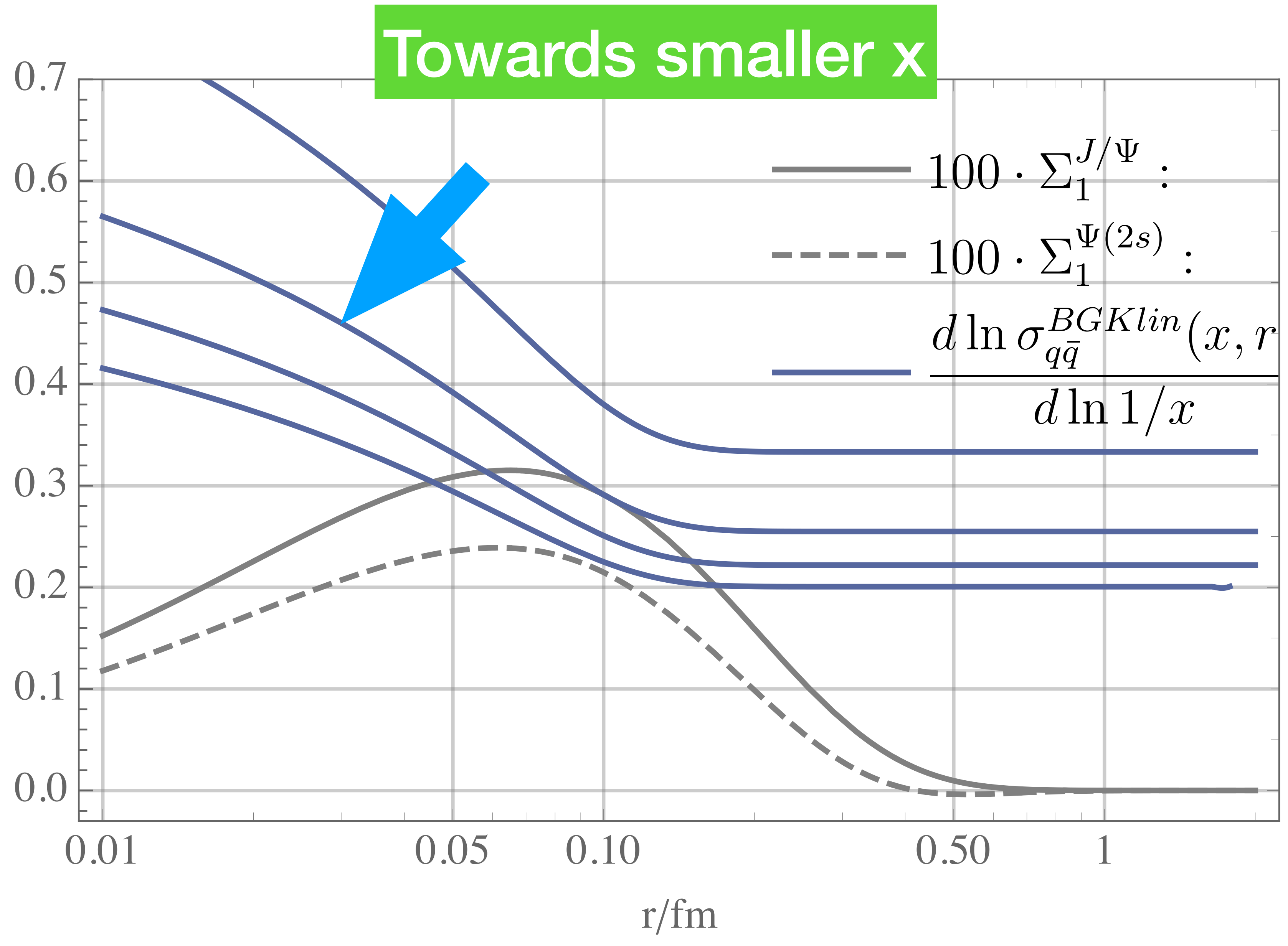


- ratio is not constant (influence of DGLAP evolution), but clear difference between linearized version and complete BGK model
- Challenge: difficult to estimate uncertainties
- It would be good to have data here
  [re-binning of LHCb data would already help a lot]

Discussion & Conclusion



“Slope" for complete BGK



"Slope" for linear BGK

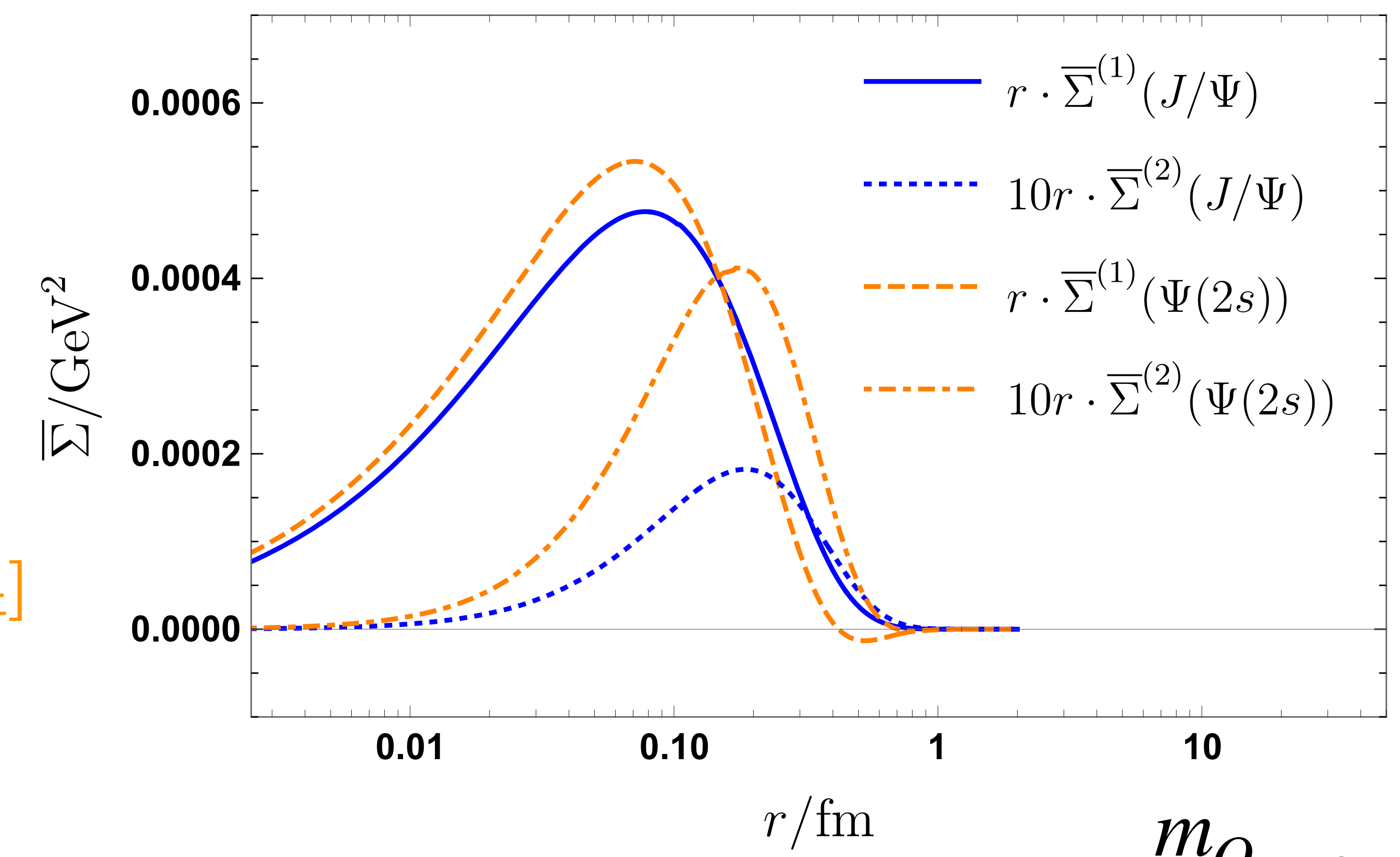
$$\lambda = \frac{d \ln \sigma_{q\bar{q}}}{\ln 1/x}$$

- Difference between J/Ψ and $\Psi(2s)$ at relative large dipole size r
- Full non-linear model: non-trivial x-dependence in this region
- Linear model with factorization scale frozen at large dipole size r, there is not much happening
  → constant ratio
- Trivial for GBW model; also seen for BFKL vs BK (QCD low x evolution)
- Prediction depends on VM wave function, but the tendency should be stable

Appendix

potentials for wave functions:

as implemented in [M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; 1812.03001; 1901.02664]

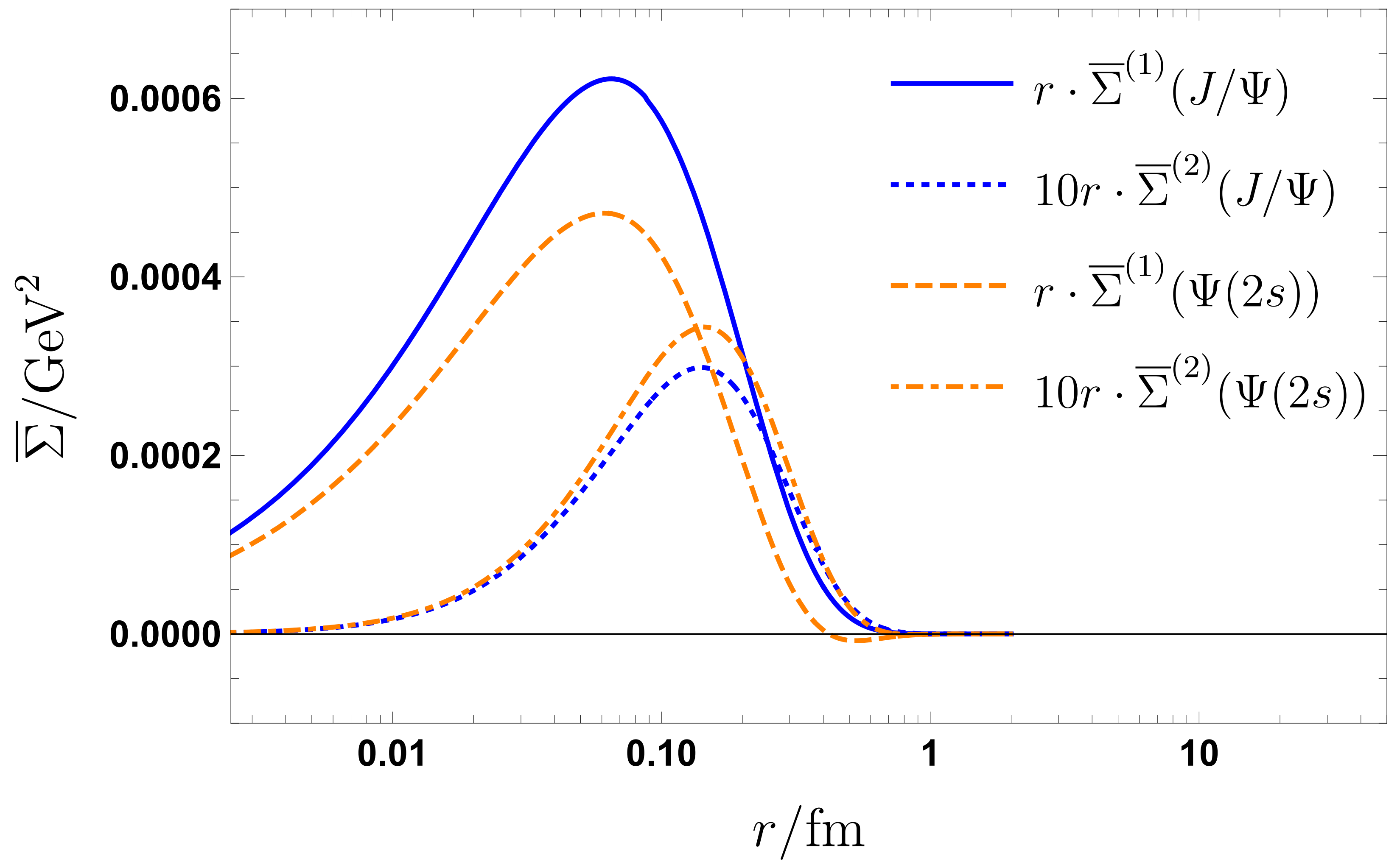


harmonic oscillator (HO): $U(r) = \frac{m_Q}{2}\omega^2 r^2$

$\omega = 0.3$GeV→ Gaussian shape

Note:

- plots show transition function $\gamma \to VM$, not wave function
- $\Psi(2s)$: node structure of wave function absent in transition after integration over photon momentum fraction z
- $\overline{\Sigma}^{(2)}(r)$ enhanced for $\Psi(2s)$, but still considerable smaller



→$\Psi(2s)$ gives access to a (slightly) different region in r than J/Ψ

→ requires separate diffractive slopes $B_D(W)$ as obtained in [M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; 1812.03001; 1901.02664]

Buchmüller-Tye Potential: Coulomb-like behavior at small r and a string-like behavior at large r [Buchmüller, Tye; PRD24, 132 (1981)]

how to compare to experiment?

(sort of standard procedure for comparing inclusive gluon to exclusive data)

a) analytic properties of scattering amplitude → real part

$$\mathcal{A}^{\gamma p \to Vp}(x, t=0) = \left(i + \tan\frac{\lambda(x)\pi}{2}\right) \cdot \Im\mathrm{m}\mathcal{A}^{\gamma p \to Vp}(x, t=0)$$

with intercept

$$\lambda(x) = \frac{d \ln \Im\mathrm{m}\mathcal{A}(x,t)}{d \ln 1/x}$$

b) differential Xsection at t=0:

$$\frac{d\sigma}{dt}(\gamma p \to Vp)\Big|_{t=0} = \frac{1}{16\pi}\left|\mathcal{A}^{\gamma p \to Vp}(W^2, t=0)\right|^2$$

c) from experiment:

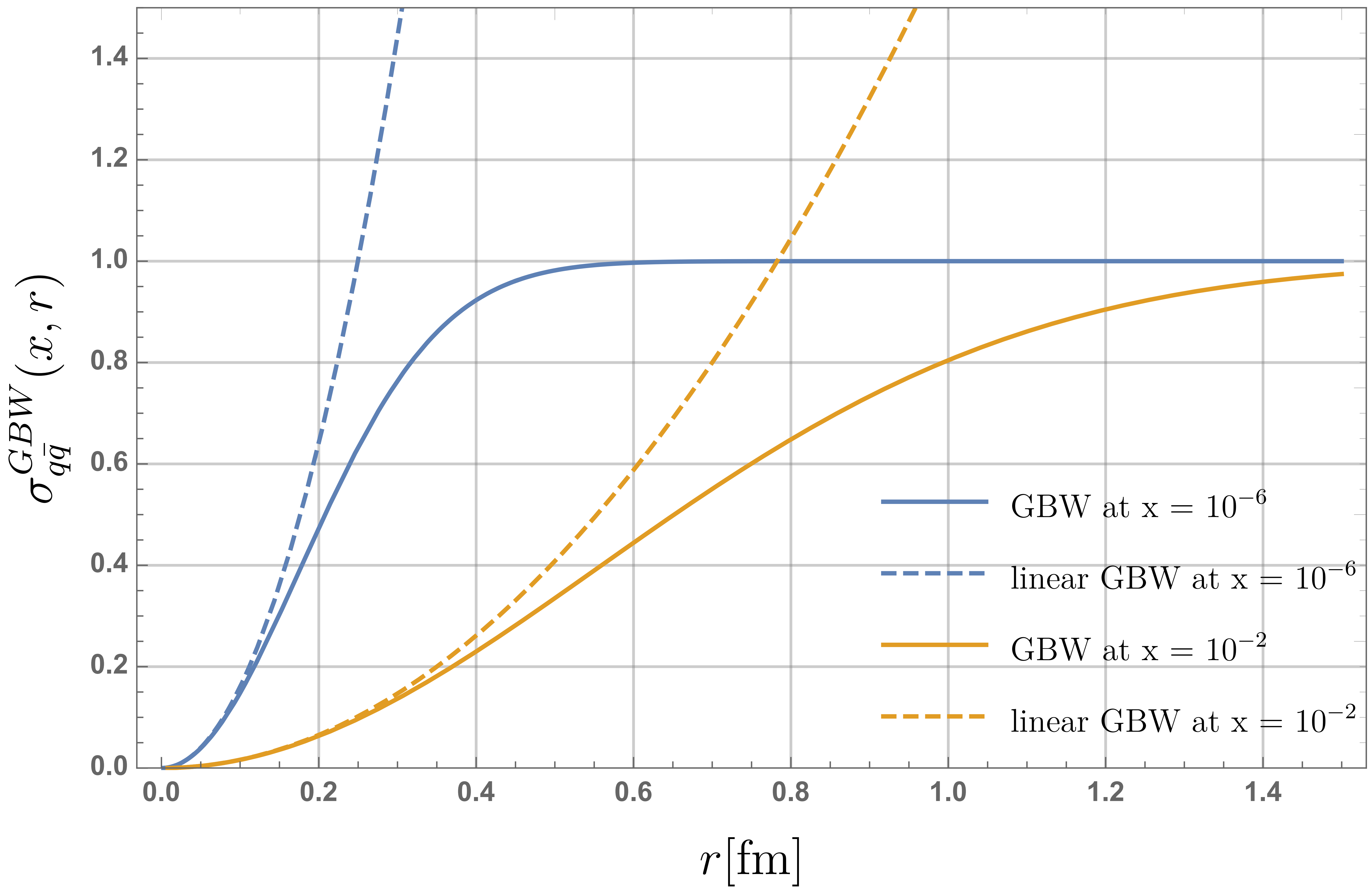
$$\frac{d\sigma}{dt}(\gamma p \to Vp) = e^{-B_D(W)\cdot|t|} \cdot \frac{d\sigma}{dt}(\gamma p \to Vp)\Big|_{t=0}$$

$$\sigma^{\gamma p \to Vp}(W^2) = \frac{1}{B_D(W)}\frac{d\sigma}{dt}(\gamma p \to Vp)\Big|_{t=0}$$

extracted from data

weak energy dependence from slope parameter

$$B_D(W) = \left[b_0 + 4\alpha' \ln\frac{W}{W_0}\right] \mathrm{GeV}^{-2}.$$



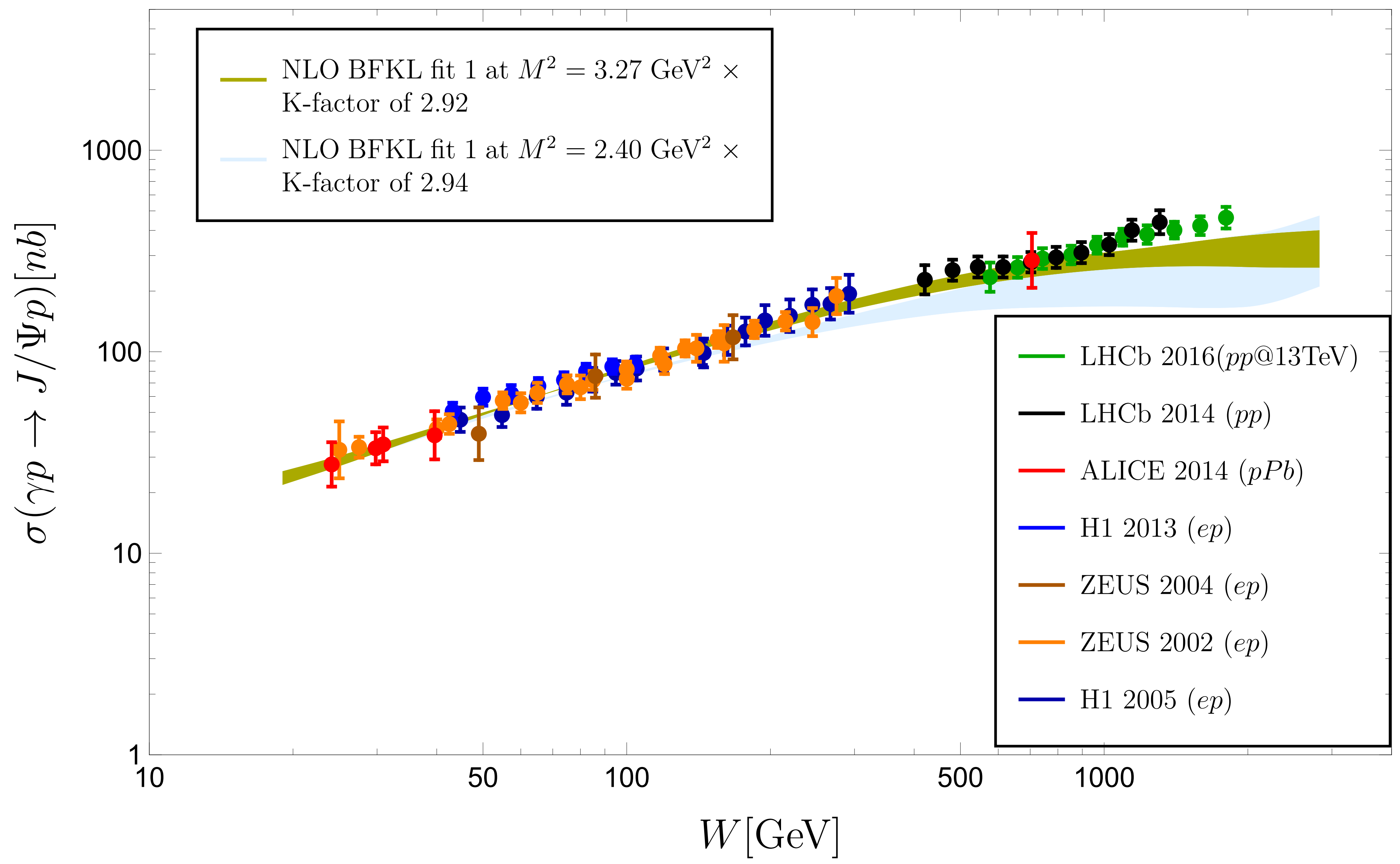
work in progress

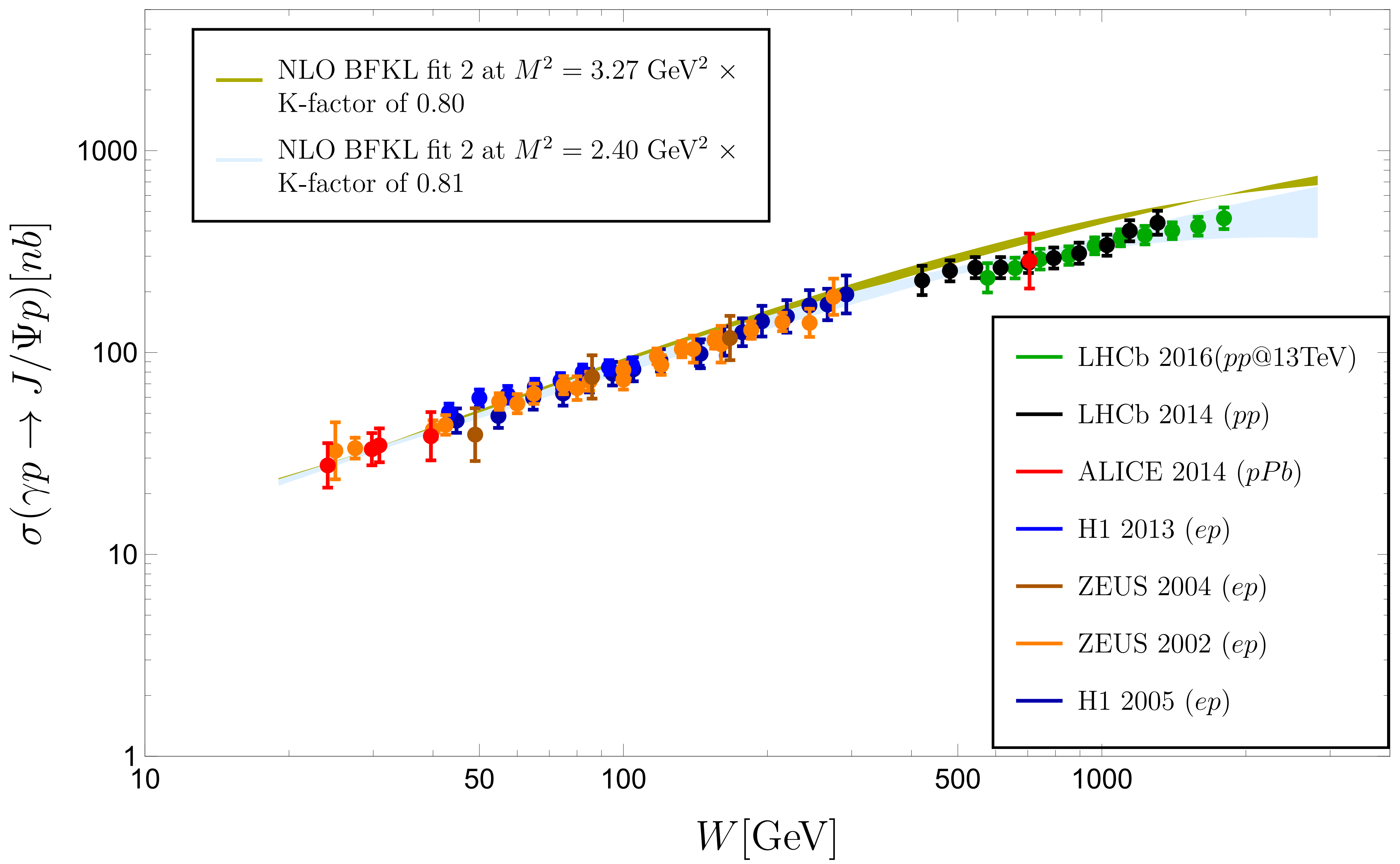
- as expected linear and complete GBW model agree for small dipole sizes
- for large dipole sizes linearized version breaks overshoots complete saturation model

First study (BFKL only, also for Υ)

[Bautista, MH, Fernandez-Tellez;1607.05203]

NLO BFKL describes energy dependence, but …..



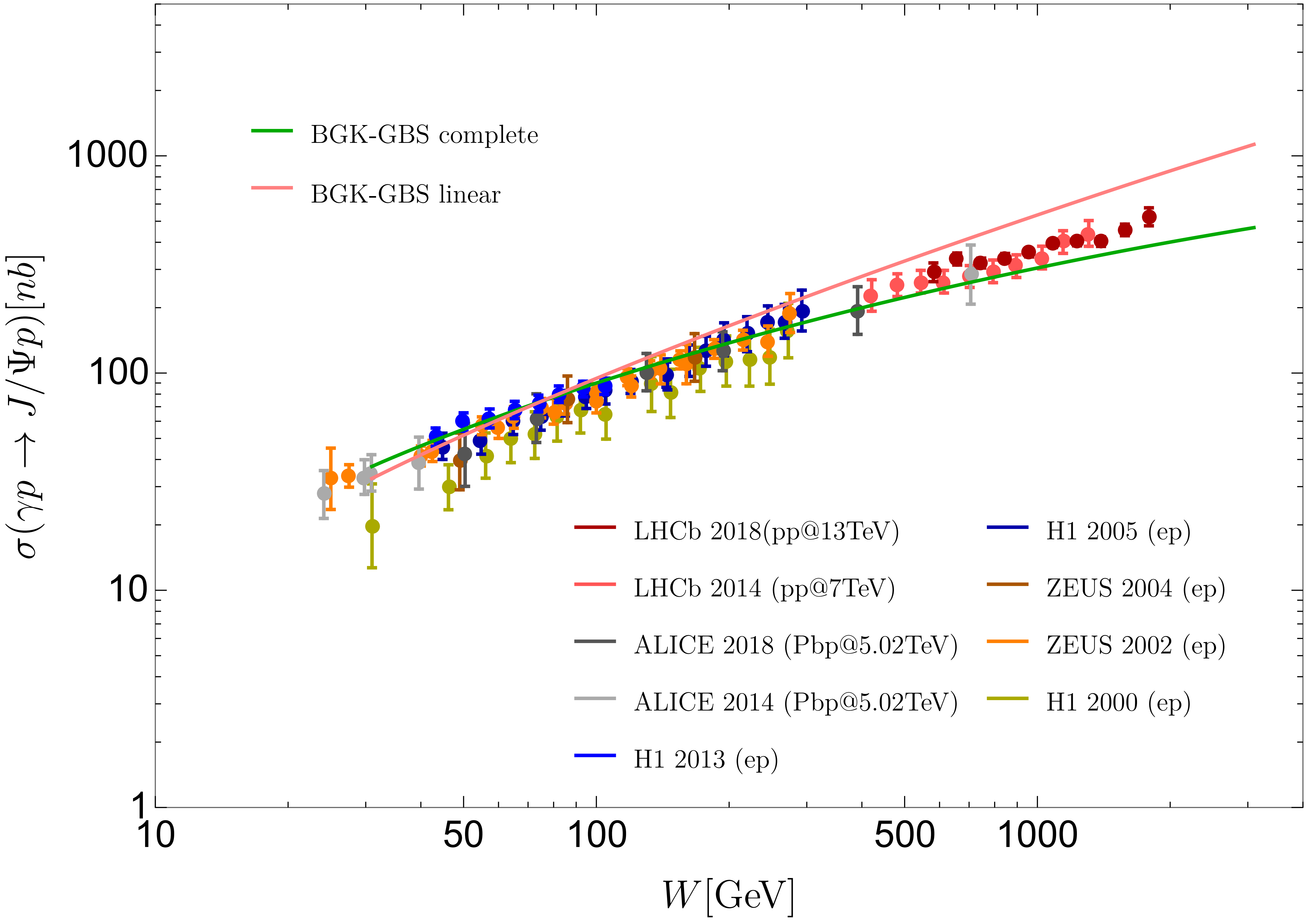


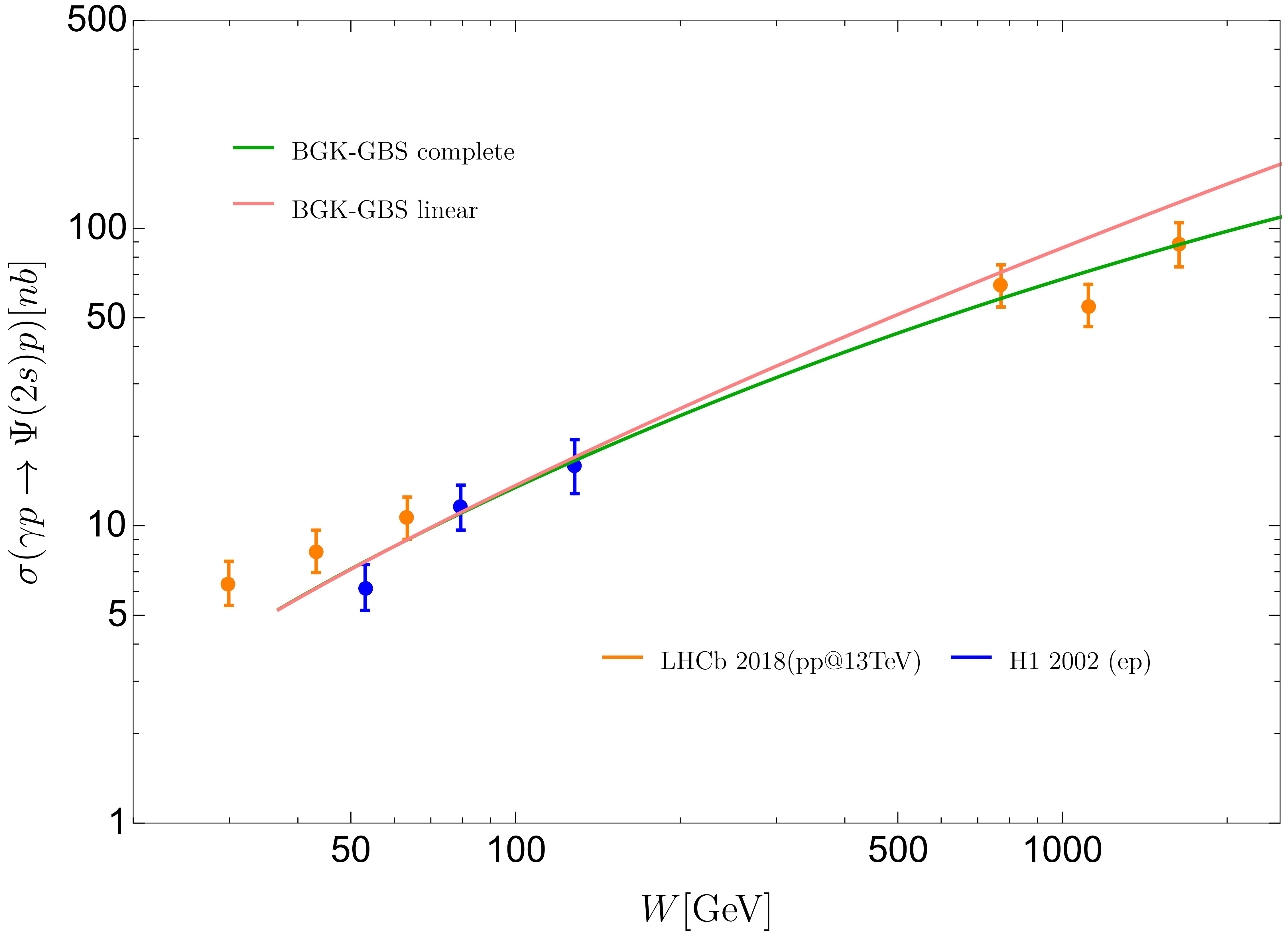
error band: variation of renormalization scale
→ in general pretty small = stability

…but error blows up for highest energies

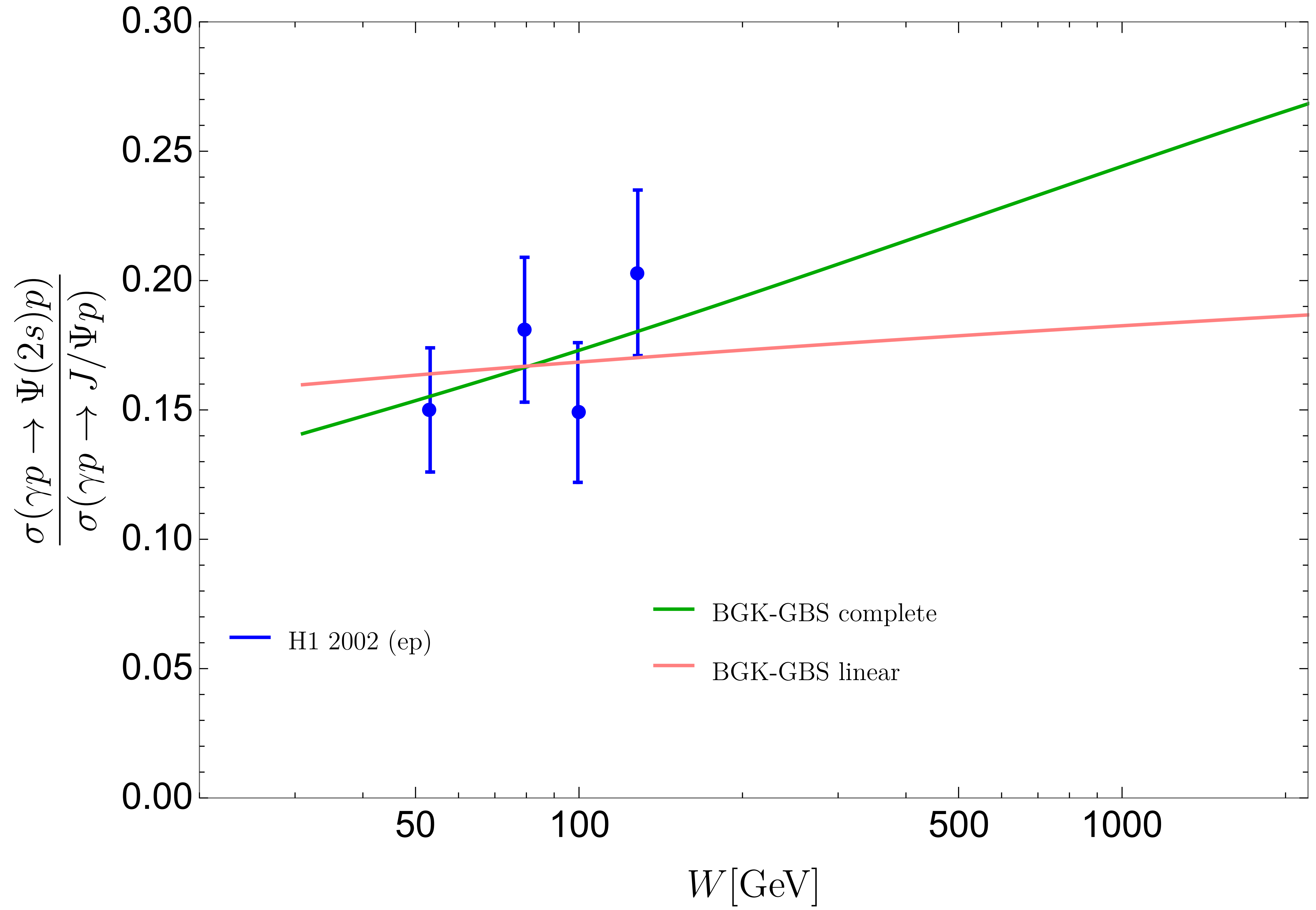
does it mean something?

DGLAP improved saturation model with Gaussian wave function





Ratio



0.30
0.25
0.20
0.15
0.10
0.05
0.00
$\frac{\sigma(\gamma p \to \Psi(2s)p)}{\sigma(\gamma p \to J/\Psi p)}$
50
100
500
1000
$W\,[\mathrm{GeV}]$
H1 2002 (ep)
BGK-GBS complete
BGK-GBS linear