

# Heavy flavor production and hadronic structure at small-x

EF06 WORKING GROUP MEETING - SM21

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Based on

[A.D. Bolognino, F. G. Celiberto, D.Yu. Ivanov, A. Papa, W. Schäfer, A. Szczurek, *Eur. Phys. J. C* 81 (2021) 9 ]

[F. G. Celiberto, M. Fucilla, D. Yu. Ivanov, M. M. A. M., A. Papa [arXiv:2109.11875]],  
to appear in *Phys. Rev. D*

# Introduction

- ➊ Semihard collision processes, featuring the scale hierarchy

$$s \gg Q^2 \gg \Lambda_{\text{QCD}}^2, \quad Q \text{ a hard scale,}$$

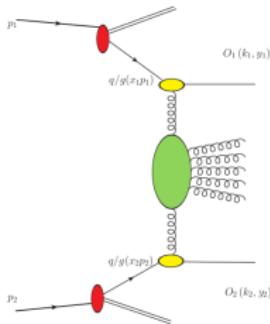
represent a challenge for perturbative QCD,

$$\Rightarrow \alpha_s(Q) \ln s/Q^2 \sim 1 \text{ need to be resummed.}$$

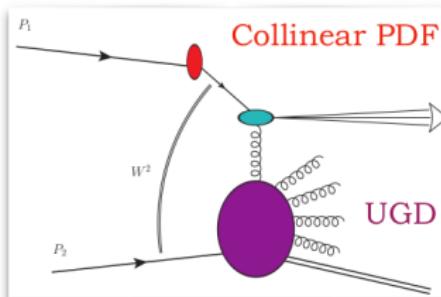
- ➋ The Balitsky-Fadin-Kuraev-Lipatov (**BFKL**) approach provides a general framework for the resummation of high-energy logs.

[V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977)]  
 [Y.Y. Balitskii, L.N. Lipatov (1978)]

- \* forward/backward two-particles emissions:



- \* single forward emissions:

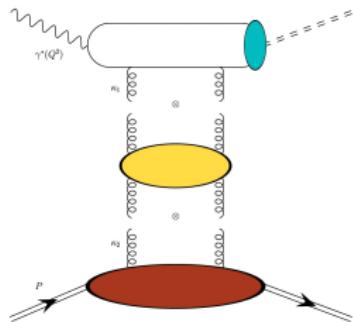


- ▷ test the BFKL dynamics as resummation energy logarithms in the  $t$ -channel.

- ▷ possibility to probe the proton content via (UGD):   
 $\Rightarrow$  hadronic structure at small- $x$ .

# Exclusive production of the $\rho$ meson in ep collisions

the subprocess:  $\gamma^*(\lambda_\gamma)p \rightarrow \rho(\lambda_\rho)p$



in the high-energy regime:

$$s \equiv W^2 \gg Q^2 \gg \Lambda_{\text{QCD}}^2 \implies x \simeq Q^2/W^2 \ll 1$$

▷  $\Phi^{\gamma^* \rightarrow \rho} = [\text{HF}_{\text{LO}}] \otimes [\text{DA}]$

▷  $\mathcal{F}(x, \kappa^2) = [\text{BFKL } \mathbf{G}_\omega] \otimes [\text{proton IF}]$



The forward helicity amplitude:

$$T_{\lambda_\rho \lambda_\gamma}(s, Q^2) = \frac{is}{(2\pi)^2} \int \frac{d^2 \kappa}{(\kappa^2)^2} \Phi^{\gamma^*(\lambda_\gamma) \rightarrow \rho(\lambda_\rho)}(\kappa^2, Q^2) \mathcal{F}(x, \kappa^2)$$

- different  $\kappa$ -dependence of the  $\Phi_{L,T}^{\gamma^* \rightarrow \rho}$   $\implies$  constraint on the  $\kappa$ -dependence of the UGD in the HERA energy range:

$$2.5 \text{ GeV}^2 < Q^2 < 60 \text{ GeV}^2, \quad 35 \text{ GeV} < W < 180 \text{ GeV}$$

# UGD models

- ★ x-independent model (ABIPSW):  $\mathcal{F}(x, \kappa^2) = \frac{A}{(2\pi)^2 M^2} \left[ \frac{\kappa^2}{M^2 + \kappa^2} \right]$

[I. V. Anikin et al. (2011)]

- ★ Gluon mom. derivative:  $\mathcal{F}(x, \kappa^2) = \frac{dxg(x, \kappa^2)}{d \ln \kappa^2}$

- ★ IN:  $\mathcal{F}(x, \kappa^2) = \mathcal{F}_{\text{soft}}^{(B)}(x, \kappa^2) \left[ \frac{\kappa_s^2}{\kappa^2 + \kappa_s^2} \right] + \mathcal{F}_{\text{hard}}(x, \kappa^2) \left[ \frac{\kappa_h^2}{\kappa^2 + \kappa_h^2} \right]$

[I. P. Ivanov and N. N. Nikolaev (2002)]

- ★ HSS model:  $[G_\omega^{(\text{BFKL})}] \otimes [\text{LO proton IF}]$

[M. Hentschinski, A. Sabio Vera, and C. Salas, Phys. Rev. Lett. 110, 041601 (2013)]

- ★ GBW UGD: FT of dipole cross section

[K.J. Golec-Biernat, M. Wüsthoff (1998)]

- ★ WMR model: angular ordering of gluon emissions

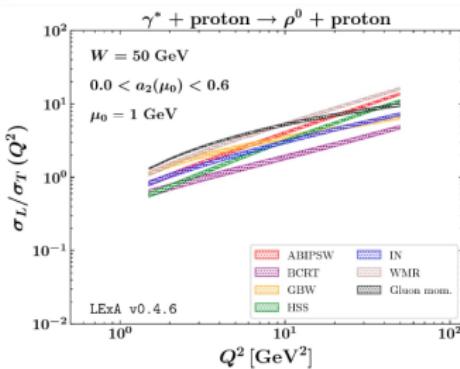
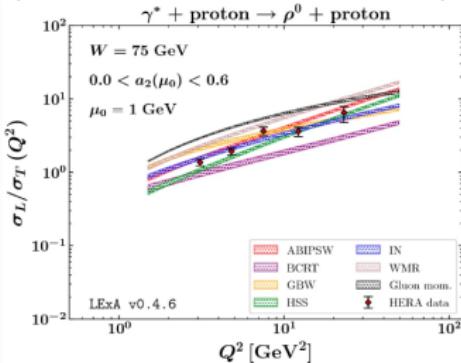
[G. Watt, A.D. Martin, M.G. Ryskin (2003)]

- ★ BCRT distribution:  $\mathcal{F}(x, \kappa^2) = \kappa^2 \int_M^\infty dM_X \rho_X(M_X) \hat{f}_1^G(x, \kappa^2; M_X)$

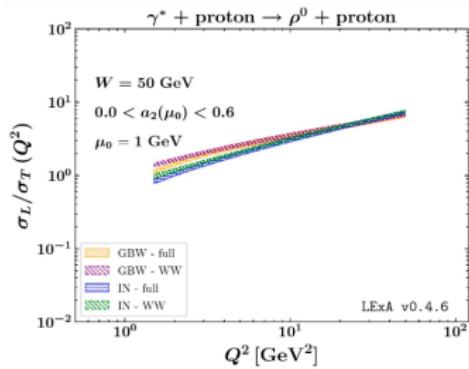
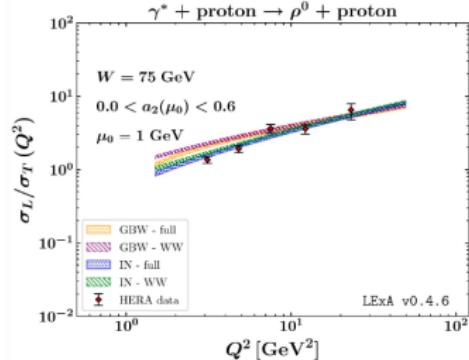
[A. Bacchetta, F. G. Celiberto, M. Radici, and P. Taels (2020,2021)]

# $Q^2$ -dependence of the polarized cross-section ratio: $\sigma_L/\sigma_T$ at $W = 75$ GeV together with the HERA data and at $W = 50$ GeV for EIC

(All the considered UGD models)



(GBW and IN UGD models)

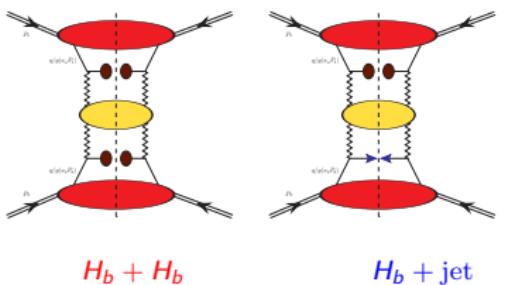


# Inclusive b-hadron production

Processes:

$$\text{proton}(P_1) + \text{proton}(P_2) \rightarrow H_b(\vec{p}_1, y_1) + X + O_2(\vec{p}_2, y_2), \text{ where } O_2 \equiv \{H_b, \text{jet}\}$$

- ▷ Partons **probability density**.
- ▷ Collinear **fragmentation** of the parton  $r$  into a hadron  $H_b$ .



- ▶ project onto the eigenfunctions of the LO BFKL kernel.
- ▶ suitable definition of the **azimuthal coefficients**:

- large transverse masses:  
 $m_{1,2\perp} = \sqrt{|\vec{p}_{1,2}|^2 + m_{1,2}^2} \gg \Lambda_{\text{QCD}}$ .  
 where  $m_{2\perp} = \begin{cases} \sqrt{|\vec{p}_2|^2 + m_{H_b}^2}, & (\text{double } H_b) \\ p_{2\perp}, & (\text{jet}) \end{cases}$
- QCD collinear factorization:
- large rapidity interval  $\Delta y = y_1 - y_2 \Rightarrow \text{BFKL resummation.}$

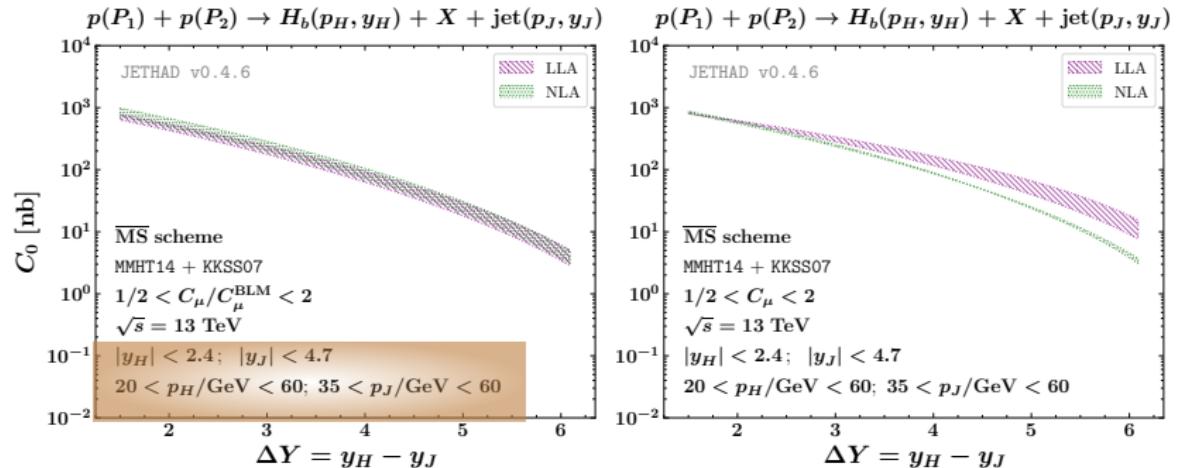
$$\frac{d\sigma}{dy_1 dy_2 d|\vec{p}_1| d|\vec{p}_2| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2} \left[ c_0 + \sum_{n=1}^{\infty} 2 \cos(n\varphi) c_n \right],$$

where  $\varphi = \phi_1 - \phi_2 - \pi$

# Phenomenology: $\phi$ -averaged cross section $\mathcal{C}_0$

for  $H_b + \text{jet}$  channel at natural and BLM-optimized scales  $\sqrt{s} = 13 \text{ TeV}$

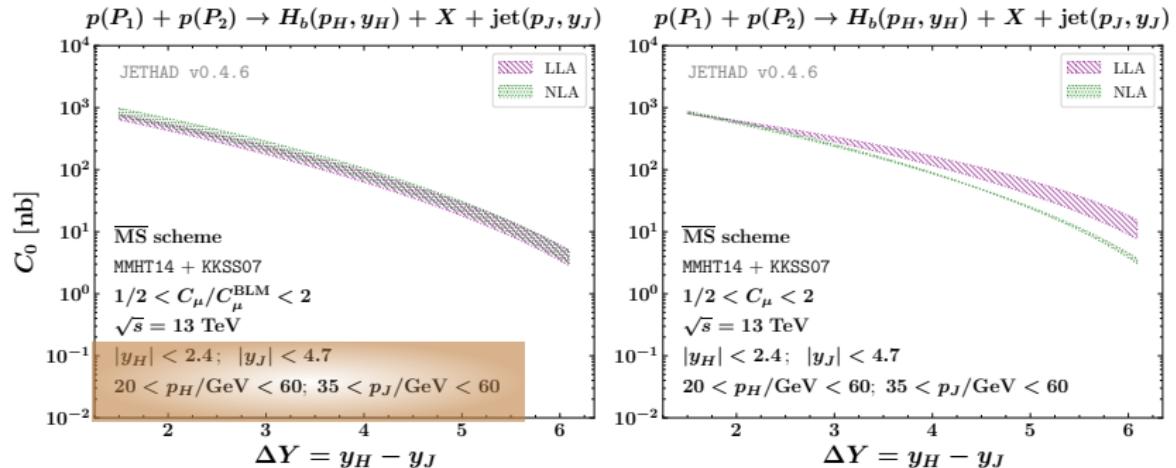
$$\mathcal{C}_0(\Delta Y, s) = \int_{p_1^{\min}}^{p_1^{\max}} d|\vec{p}_1| \int_{p_2^{\min}}^{p_2^{\max}} d|\vec{p}_2| \int_{y_1^{\min}}^{y_1^{\max}} dy_1 \int_{y_2^{\min}}^{y_2^{\max}} dy_2 \delta(y_1 - y_2 - \Delta Y) \mathcal{C}_0,$$



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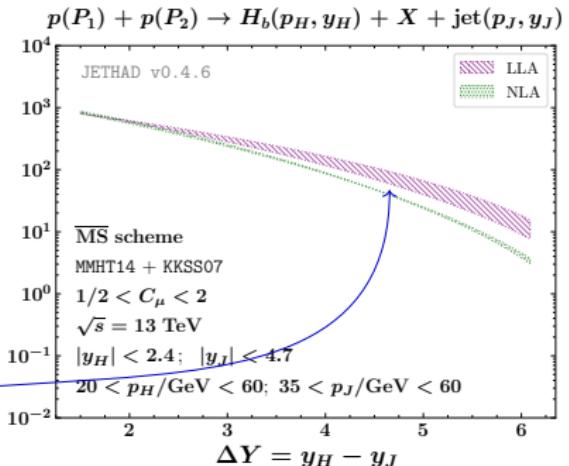
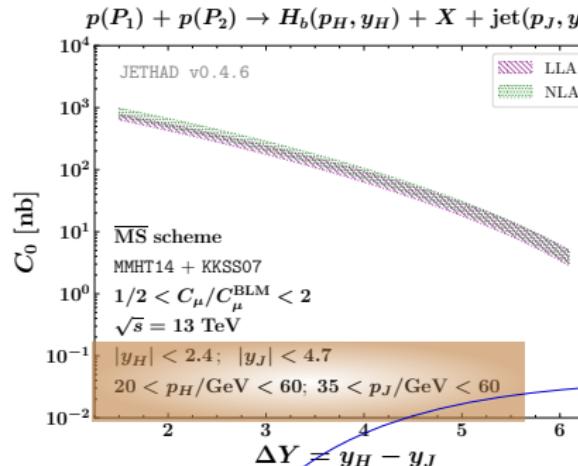
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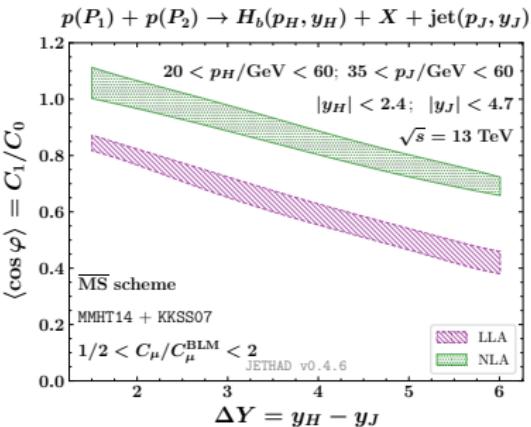
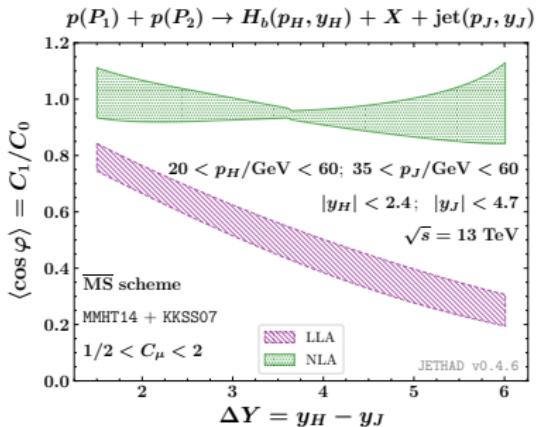
$$\mathcal{C}_0(\Delta Y, s) = \int_{p_1^{\min}}^{p_1^{\max}} d|\vec{p}_1| \int_{p_2^{\min}}^{p_2^{\max}} d|\vec{p}_2| \int_{y_1^{\min}}^{y_1^{\max}} dy_1 \int_{y_2^{\min}}^{y_2^{\max}} dy_2 \delta(y_1 - y_2 - \Delta Y) \mathcal{C}_0,$$



the **decoupling** behavior  $\Rightarrow$  NLA series are very **stable** under **scale variation**.

# Phenomenology: Azimuthal correlations $R_{n0}(\Delta Y, s)$

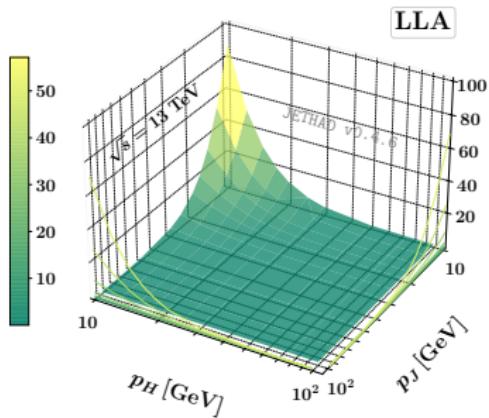
$$R_{10}(\Delta Y, s) = C_1/C_0 \equiv \langle \cos \phi \rangle$$



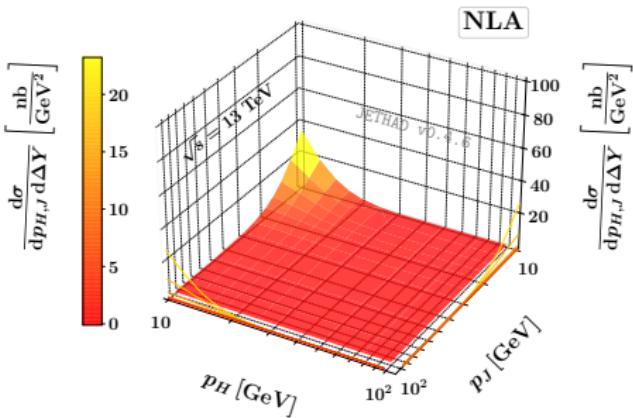
# Phenomenology: double differential $p_T$ -distribution

$$\frac{d\sigma(|\vec{p}_{1,2}|, \Delta Y, s)}{d|\vec{p}_1| d|\vec{p}_2| d\Delta Y} = \int_{y_2^{\min}}^{y_2^{\max}} dy_2 \int_{y_1^{\min}}^{y_1^{\max}} dy_1 \delta(y_1 - y_2 - \Delta Y) \mathcal{C}_0,$$

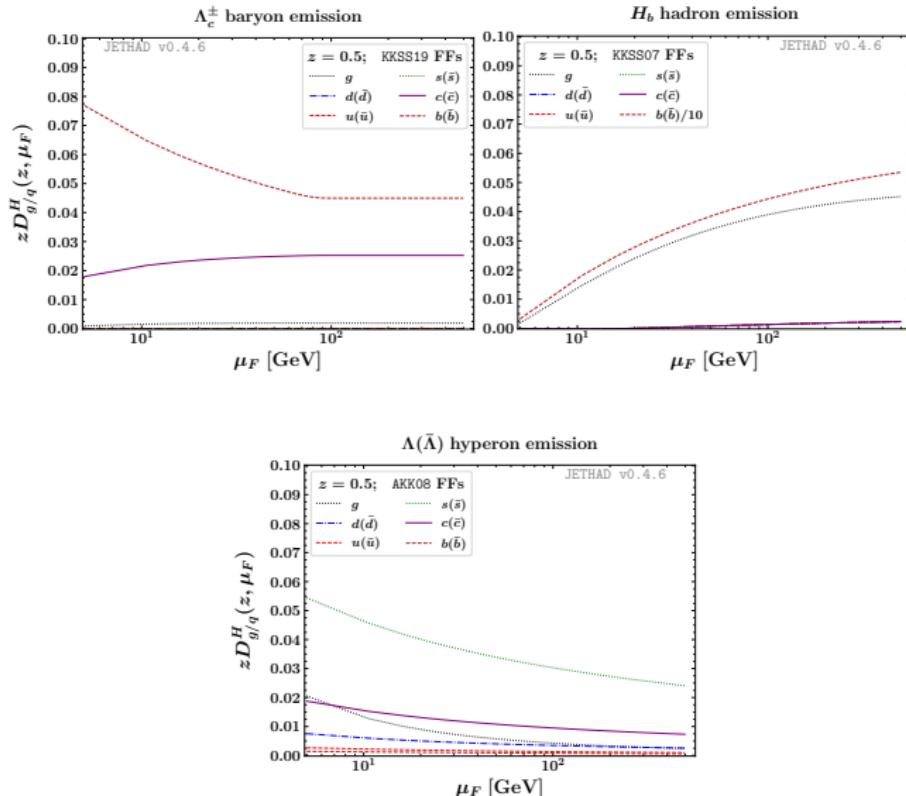
$H_b + \text{jet}$  ( $\Delta Y = 5$ ;  $C_\mu = 1$ )



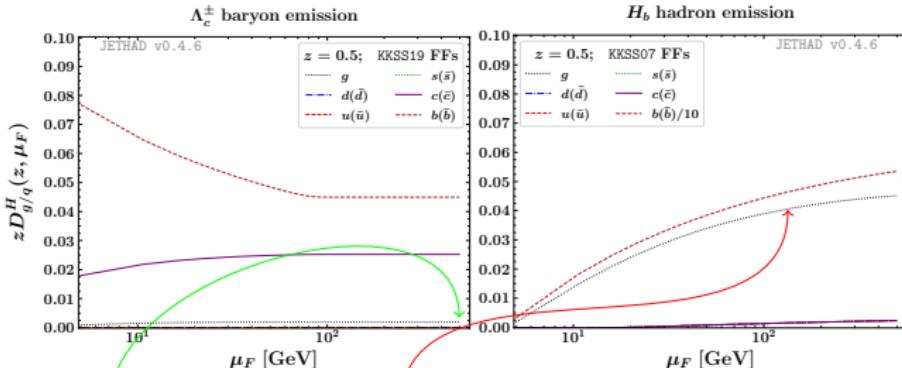
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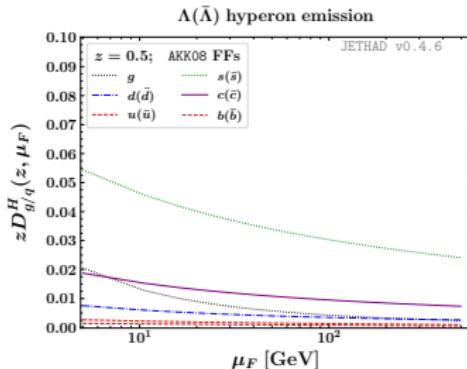
# Stabilizing effects of b-flavor fragmentation



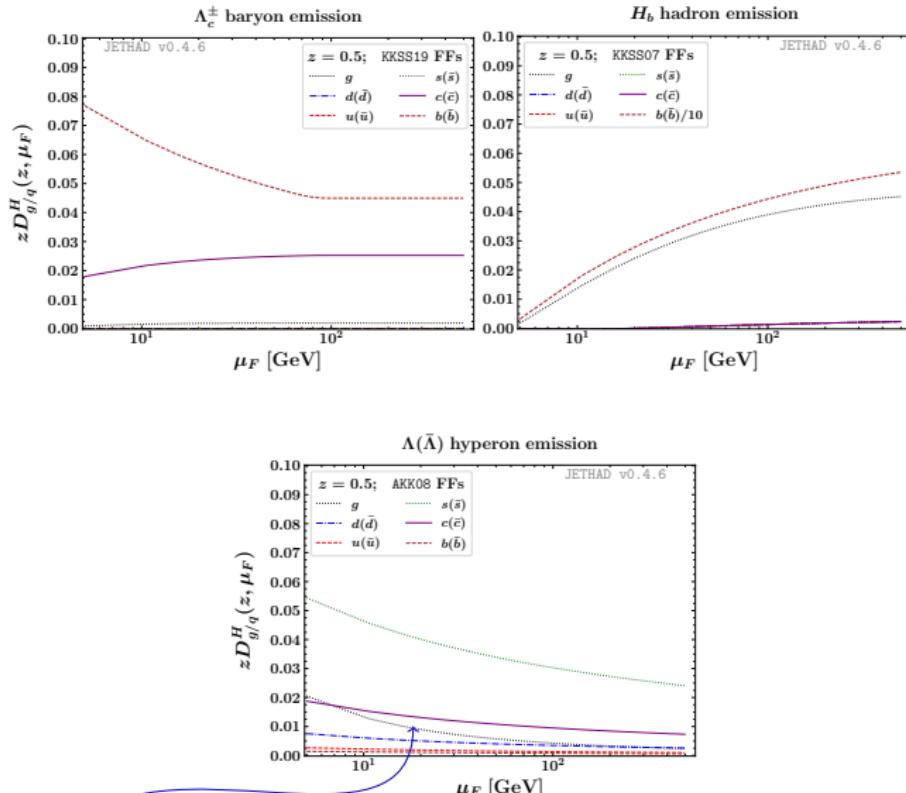
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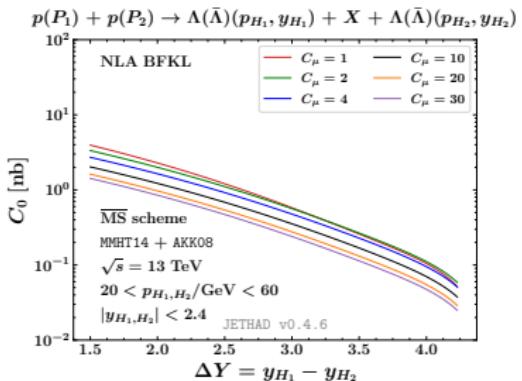
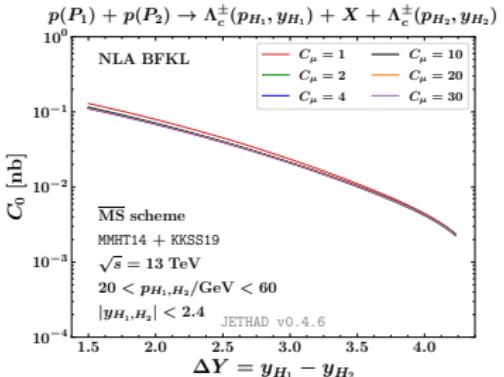
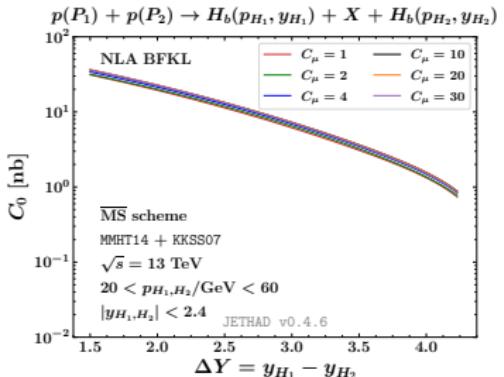
non-decreasing (for KKSS19  $\Lambda_c$ ) and growth (KKSS07) with  $\mu_F$  gluon FF  $\Rightarrow$  (stability).



# Stabilizing effects of b-flavor fragmentation



decreasing  $\mu_F$  behavior of the gluon AKK08  $\implies$  (increased energy-scale sensitivity).



$\Lambda_c$ : gluon FF plays a dominant role.

[F. G. Celiberto, M. Fucilla, D. Yu. Ivanov, and A. Papa (2021)] UNIVERSITÀ DELLA CALABRIA

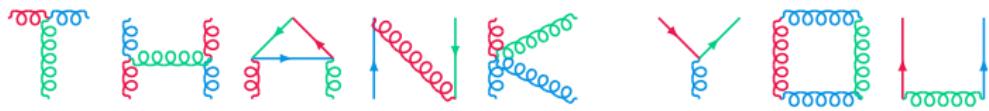


$H_b$ : gluon FF grows with  $\mu_F$  (compensated with the higher  $\mu_R \implies$  stability).



# Summary

- Heavy-flavored emissions of bound states act as fair **stabilizers** of the high-energy series.
- In the  $H_b + \text{jet}$  channel it was possible to study azimuthal moments at **natural** scales.
- The polarization dependence of  $\rho$ -production has been proposed as a sensitive probe of the shape of the investigated UGD.
- The cross section ratio  $\sigma_L/\sigma_T$  indeed appears to have potential to **discriminate** further between UGDs, for the case at hand the **IN** UGD gives the best description of HERA data.



**FOR YOUR ATTENTION!!**

# BACKUP

## JETHAD

*JETHAD, BFKL inspired but for HEP purposes!*

It is a Fortran2008-Python3 hybrid library by Cosenza collaboration

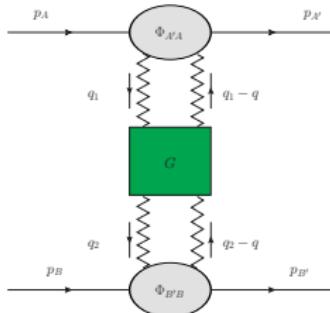
- ▶ Main features:
  1. Modularity
  2. Extensive use of structures and dynamic memory
  3. Smart management of final-state phase-space integration
- ▶ Developed software:
  1. BFKL tools (BFKL kernel and Impact factors)
  2. UGD modular package
- ▶ External interfaces:
  1. LHAPDF and native FF parametrizations
  2. CUBA multi-dim integrators
  3. QUADPACK one-dim integrators
  4. CERNLIB (multi-dim integrators, special functions, MINUIT, etc.)

# The high-energy resummation



## BFKL resummation:

Scattering in  $A + B \rightarrow A' + B'$  Regge kinematical region  $s \rightarrow \infty, t$  fixed.



→ BFKL factorization for  $\text{Im}_s \mathcal{A}$ :

$$\Phi_A(\vec{q}_1, \mathbf{s}_0) \otimes G_\omega(\vec{q}_1, \vec{q}_2) \otimes \Phi_B(-\vec{q}_2, \mathbf{s}_0)$$

Valid both in

- ▷ leading logarithmic approximation (LLA):  $\alpha_s^n (\ln s)^n$ .
- ▷ next-to-leading logarithmic approximation (NLA):  $\alpha_s^{n+1} (\ln s)^n$ .



**Green's function** is process-independent → determined through the **BFKL equation**.

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^2(\vec{q}_1 - \vec{q}_2) + \int d^2 \vec{q} K(\vec{q}_1, \vec{q}) G_\omega(\vec{q}, \vec{q}_1)$$



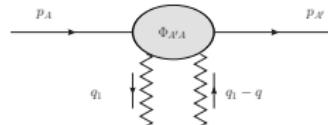
**Impact factors** are process-dependent

→ known in the NLA just for limited cases.

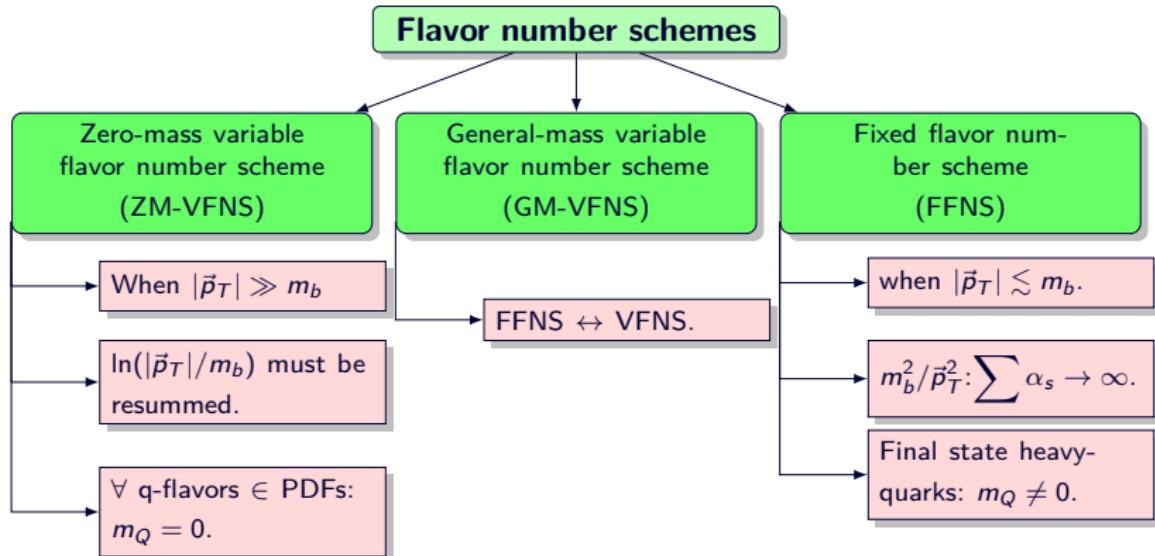
Universal property:  $\Phi_{A'A}|_{q \rightarrow 0}^{q_1 - q \rightarrow 0} \rightarrow 0$ ,

which guarantees the **infra-red finiteness** of the BFKL amplitudes.

[Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]



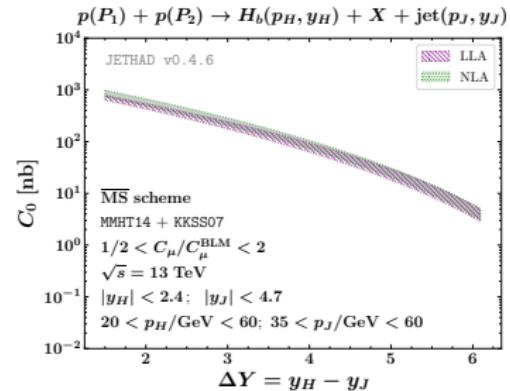
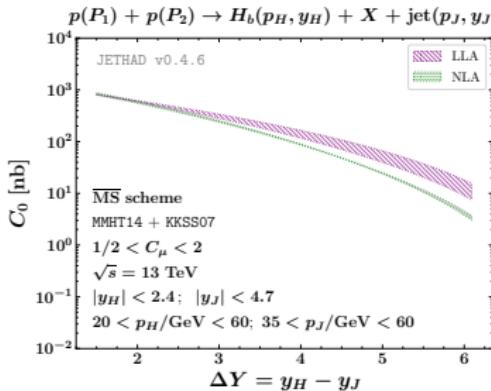
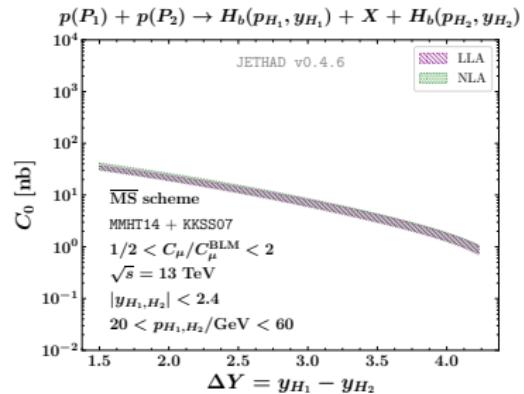
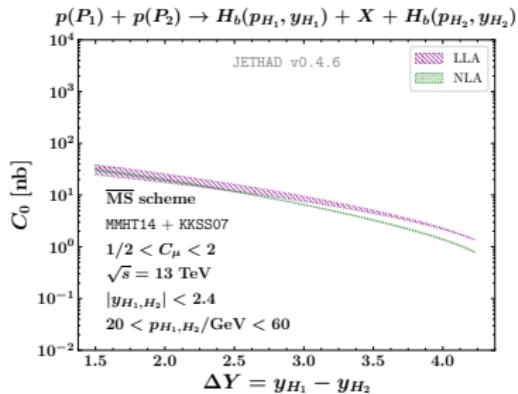
# bottom-flavor phenomenology



- ▷  $m_b$  plays a crucial role.
- ▷ different implementations of this scheme have been proposed so far.
  - [M. Krämer, F. I. Olness, and D. E. Soper (2000)]
  - [S. Forte, E. Laenen, P. Nason, and J. Rojo, (2010)]
  - [J. Blümlein, A. De Freitas, C. Schneider, and K. Schönwald, (2018)]
- ▷ approaching particular kinematic regions → (gap of knowledge).

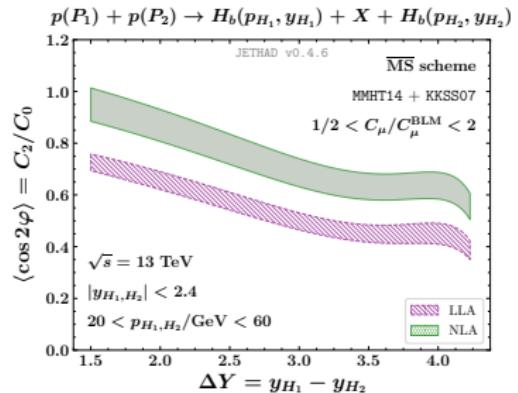
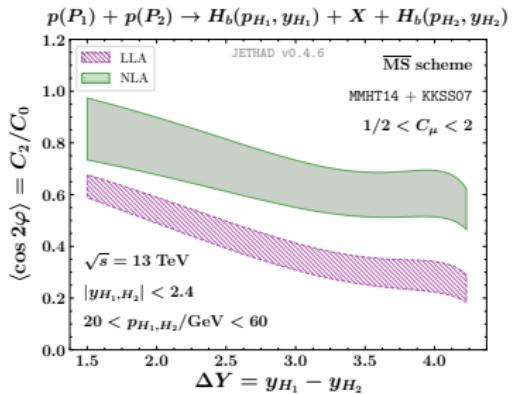
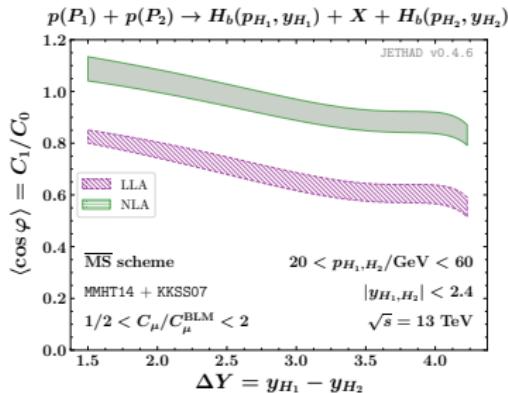
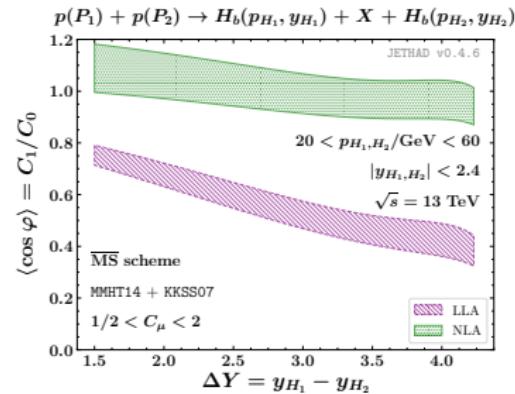
# Results:

$\Delta Y$ -shape of  $C_0$  at natural and BLM-optimized scales for  $\sqrt{s} = 13$  TeV



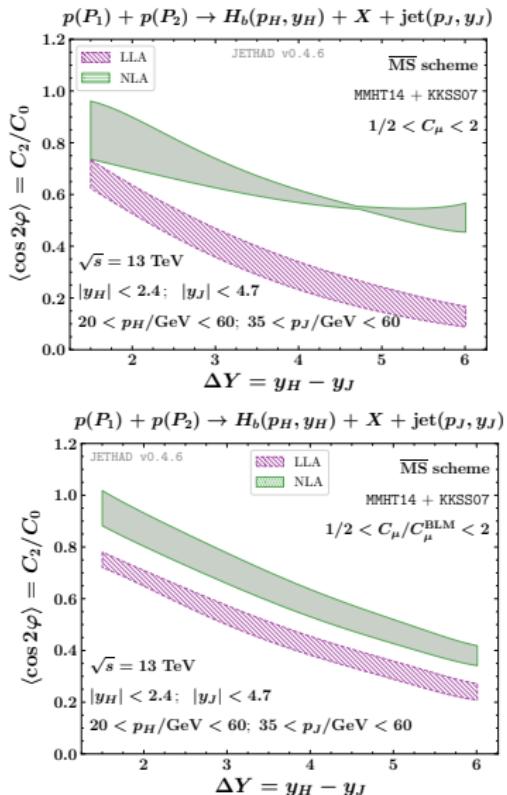
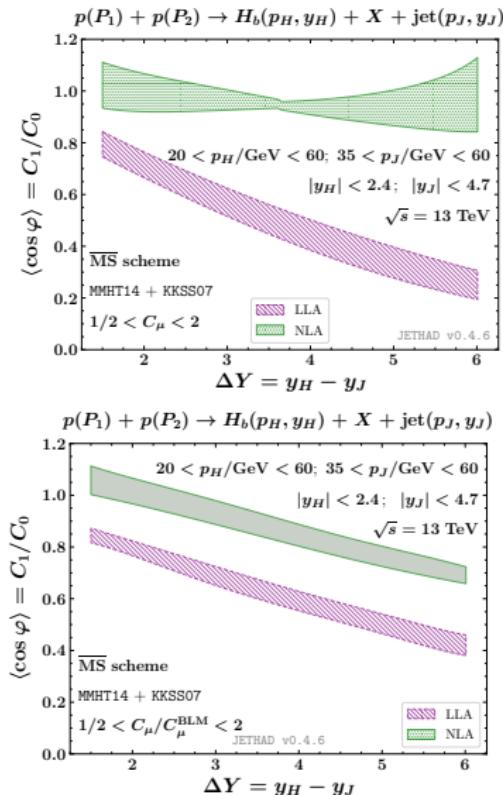
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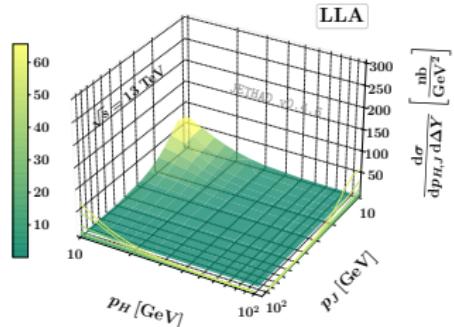
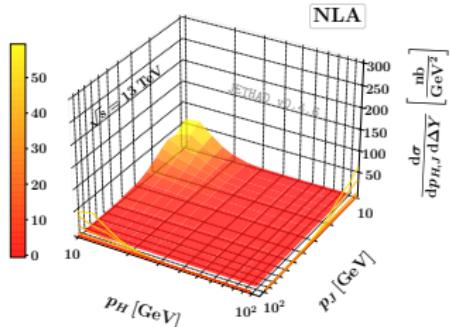
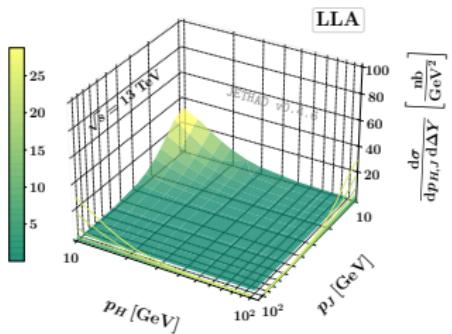
double  $H_b$



# Results:

## $H_b + \text{jet}$



$H_b + \text{jet } (\Delta Y = 3; C_\mu = 1/2)$  $H_b + \text{jet } (\Delta Y = 3; C_\mu = 1/2)$  $H_b + \text{jet } (\Delta Y = 5; C_\mu = 1/2)$  $H_b + \text{jet } (\Delta Y = 5; C_\mu = 1/2)$ 