Forward dijet production at small x within saturation formalism

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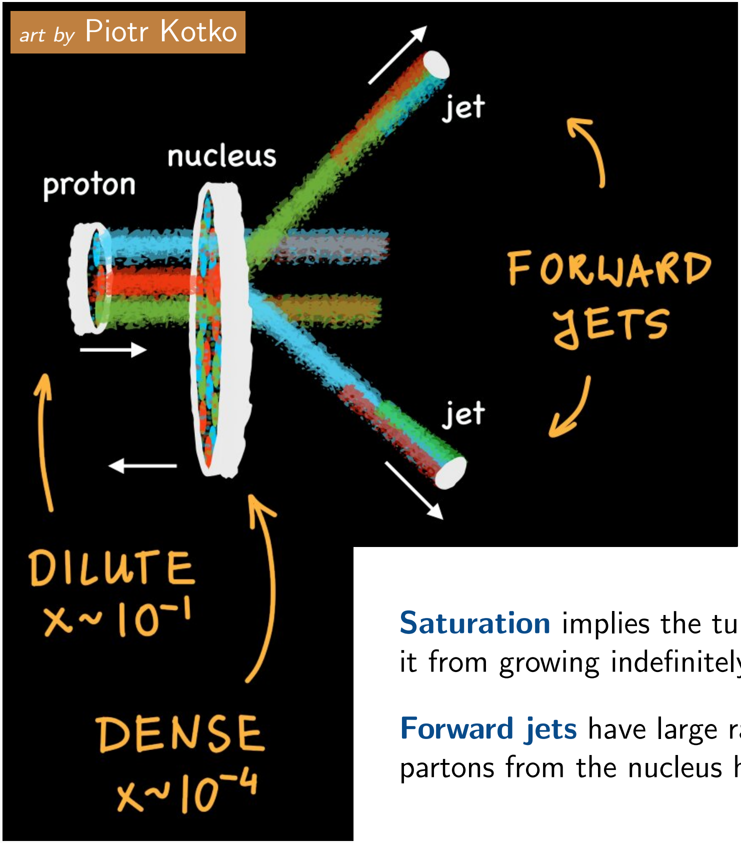
Outline

- small-x Improved TMD (ITMD) factorization
- Sudakov resummation
- results for dijets at hadron colliders and electron-ion colliders

QCD evolution, dilute vs. dense, forward jets



A dilute system carries a few high-x partons contributing to the hard scattering.

A dense system carries many low-x partons.

At high density, gluons are imagined to undergo recombination, and to saturate.

This is modeled with non-linear evolution equations, involving explicit non-vanishing k_T.

Saturation implies the turnover of the gluon density, stopping it from growing indefinitely for small x.

Forward jets have large rapidities, and trigger events in which partons from the nucleus have small x.

ITMD Factorization

For forward dijet production
in dilute-dense hadronic collisions

Generalized TMD factorization (Dominguez, Marquet, Xiao, Yuan 2011)

$$d\sigma_{AB\to X} = \int dk_T^2 \int dx_A \sum_i \int dx_B \sum_b \Phi_{gb}^{(i)}(x_A, k_T, \mu)\, f_{b/B}(x_B, \mu)\, d\hat{\sigma}_{gb\to X}^{(i)}(x_A, x_B, \mu)$$

For $x_A \ll 1$ and $P_T \gg k_T \sim Q_s$ (jets almost back-to-back).
TMD gluon distributions $\Phi_{gb}^{(i)}(x_A, k_T, \mu)$ satisfy non-linear evolution equations.
Partonic cross section $d\hat{\sigma}_{gb}^{(i)}$ is on-shell, but depends on color-structure i.

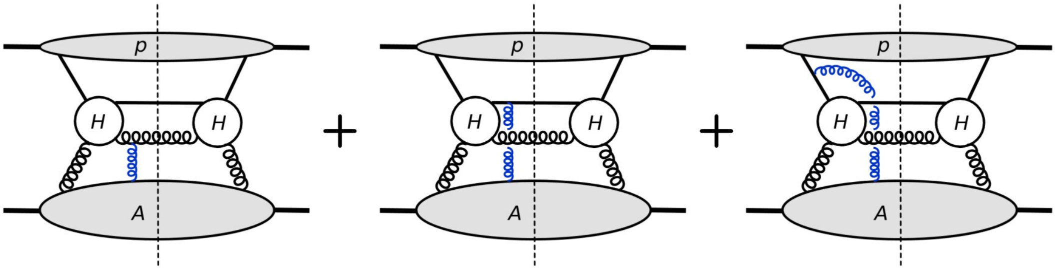
Improved TMD factorization (Kotko, Kutak, Marquet, Petreska, Sapeta, AvH 2015)

$$d\sigma_{AB\to X} = \int dk_T^2 \int dx_A \sum_i \int dx_B \sum_b \Phi_{gb}^{(i)}(x_A, k_T, \mu)\, f_{b/B}(x_B, \mu)\, d\hat{\sigma}_{gb\to X}^{(i)}(x_A, x_B, k_T, \mu)$$

Originally a model interpolating between High Energy Factorization and Generalized TMD factorization: $P_T \gtrsim k_T \gtrsim Q_s$.
Partonic cross section $d\hat{\sigma}_{gb}^{(i)}$ is off-shell and depends on color-structure i.

ITMD formalism is obtained from the CGC formalism, by including so-called kinematic twist corrections (Antinoluk, Boussarie, Kotko 2019).

Definition of gluon TMDs



+ similar diagrams with $2, 3, \ldots$ gluon exchanges

Resummation of gluon exchanges leads to Wilson line $U_\gamma = \mathcal{P}\exp\left\{ -ig \int_\gamma dz \cdot A(z) \right\}$ acting as a gauge link for the gauge invariant definition of a TMD

$$\mathcal{F}_{g/A}(x, k_T) = 2 \int \frac{d^4\xi\, \delta(\xi^+)}{(2\pi)^3\, p_A^+} \exp\left\{ixp_A^+\xi^- - i\vec{k}_T \cdot \vec{\xi}_T\right\} \left\langle A \middle| \mathrm{Tr}\left\{\hat{F}^{i+}(\xi) U_{\gamma(\xi,0)} \hat{F}^{i+}(0)\right\} \middle| A \right\rangle$$

ITMD Factorization

Schematic hybrid (non-ITMD) factorization fomula

$$d\sigma = \sum_{y=g,u,d,\dots} \int dx_1 d^2k_T \int dx_2 \, d\Phi_{g^*y\to n} \, \frac{1}{\mathrm{flux}_{gy}} \, \mathcal{F}_g(x_1, k_T, \mu) \, f_y(x_2, \mu) \sum_{\mathrm{color}} \left|\mathcal{M}^{(\mathrm{color})}_{g^*y\to n}\right|^2$$

ITMD* formula: replace color matrix in color sum in terms of partial amplitudes

$$\mathcal{F}_g \sum_{\mathrm{color}} \left|\mathcal{M}^{(\mathrm{color})}\right|^2 = \mathcal{F}_g \sum_{\sigma\in S_{n+2}} \sum_{\tau\in S_{n+2}} \mathcal{A}^*_\sigma \, \mathcal{C}_{\sigma\tau} \, \mathcal{A}_\tau \qquad , \qquad \mathcal{C}_{\sigma\tau} = N_c^{\lambda(\sigma,\tau)}$$

with "TMD-valued color matrix"

$$(N_c^2 - 1) \sum_{\sigma\in S_{n+2}} \sum_{\tau\in S_{n+2}} \mathcal{A}^*_\sigma \, \tilde{\mathcal{C}}_{\sigma\tau}(x, |k_T|) \, \mathcal{A}_\tau \quad , \quad \tilde{\mathcal{C}}_{\sigma\tau}(x, |k_T|) = N_c^{\bar{\lambda}(\sigma,\tau)} \tilde{\mathcal{F}}_{\sigma\tau}(x, |k_T|)$$

where each function $\tilde{\mathcal{F}}_{\sigma\tau}$ is one of 10 functions

$$\mathcal{F}^{(1)}_{qg} \quad , \quad \mathcal{F}^{(2)}_{qg} \quad , \quad \mathcal{F}^{(3)}_{qg}$$

$$\mathcal{F}^{(1)}_{gg} \quad , \quad \mathcal{F}^{(2)}_{gg} \quad , \quad \mathcal{F}^{(3)}_{gg} \quad , \quad \mathcal{F}^{(4)}_{gg} \quad , \quad \mathcal{F}^{(5)}_{gg} \quad , \quad \mathcal{F}^{(6)}_{gg} \quad , \quad \mathcal{F}^{(7)}_{gg}$$

*This gives the full contribution for 2 jets, and incomplete but manifestly gauge-invariant contribution for more jets.

ITMD* factorization for more than 2 jets

$$\mathcal{F}_{qg}^{(1)}(x,k_T) = \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle \quad , \quad \left\langle \cdots \right\rangle = 2\int \frac{d^4\xi\,\delta(\xi_+)}{(2\pi)^3 P^+}\, e^{ik\cdot\xi} \left\langle P\middle| \cdots \middle|P\right\rangle$$

$$\mathcal{F}_{qg}^{(2)}(x,k_T) = \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(3)}(x,k_T) = \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(1)}(x,k_T) = \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x,k_T) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\right]\mathrm{Tr}\left[\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x,k_T) = \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(4)}(x,k_T) = \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[-]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(5)}(x,k_T) = \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(6)}(x,k_T) = \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(7)}(x,k_T) = \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

ITMD gluons

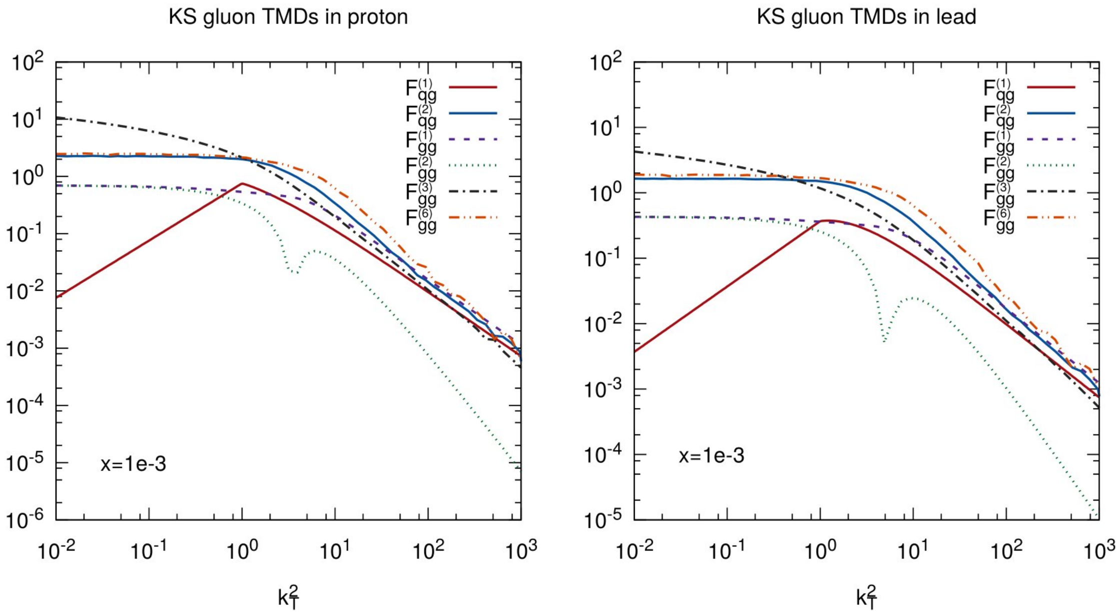
Start with dipole distribution $\mathcal{F}_{qg}^{(1)}(x, k_T) = \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$ evolved via the BK equation formulated in momentum space supplemented with subleading corrections and fitted to F_2 data (Kutak, Sapeta 2012)

All other distribution appearing in dijet production, $\mathcal{F}_{qg}^{(2)}, \mathcal{F}_{gg}^{(1)}, \mathcal{F}_{gg}^{(2)}, \mathcal{F}_{gg}^{(6)}$, in the mean-field approximation (AvH, Marquet, Kotko, Kutak, Sapeta, Petreska 2016).

This is, at leading order in $1/N_c$. In this approximation, the same distributions suffice for trijets.

ITMD gluons

Bury, AvH, Kotko, Kutak 2020

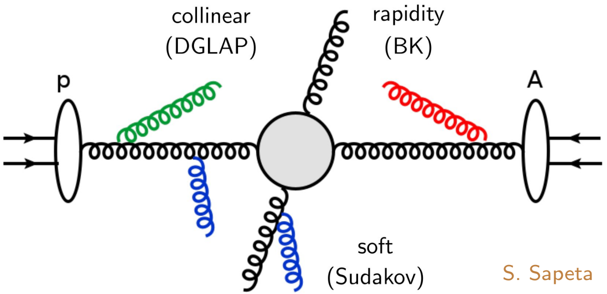


Dependence of $\mathcal{F}_{qg}^{(1)}$ on k_T below 1GeV approximated by power-like fall-off. For higher values of $|k_T|$ it is a solution to the BK equation.

TMDs decrease as $1/|k_T|$ for increasing $|k_T|$, except $\mathcal{F}_{gg}^{(2)}$, which decreases faster (even becomes negative, absolute value shown here).

Sudakov resummation for dijets

Having hard jets in the final state, large logarithms associated with the hard scale have to be resummed. This resummation can be accounted for by inclusion of the Sudakov factor.



Within the small-x saturation formalism, Sudakov effects are most conveniently included in b-space (Mueller, Xiao, Yuan 2013; Staśto, Wei, Xiao, Yuan 2018)

$$\mathcal{F}_{g^*/B}^{ag\to cd}(x, q_T, \mu) = \frac{-N_c S_\perp}{2\pi\alpha_s} \int \frac{b_T db_T}{2\pi} J_0(b_T q_T)\, e^{-S_{\text{Sud}}^{ag\to cd}(\mu, b_T)}\, \nabla_{b_T}^2 S(x, b_T)$$

where $S_\perp$ is the transverse area of the target, and $S(x, b_T)$ the dipole scattering amplitude. This can be translated into a relation for momentum dependent distributions as

$$\mathcal{F}_{g^*/B}^{ag\to cd}(x, k_T, \mu) = \int db_T\, b_T\, J_0(b_T k_T)\, e^{-S_{\text{Sud}}^{ag\to cd}(\mu, b_T)} \int dk_T'\, k_T'\, J_0(b_T k_T')\, \mathcal{F}_{g^*/B}(x, k_T')$$

Sudakov resummation for dijets

The Sudakov receives perturbative and non-perturbative contributions for each cannel

$$S_{Sud}^{ab\to cd}(\mu, b_T) = \sum_{i=a,b,c,d} S_p^i(\mu, b_T) + \sum_{i=a,c,d} S_{np}^i(\mu, b_T)$$

Perturbative part Mueller, Xiao, Yuan 2013; Staśto, Wei, Xiao, Yuan 2018

$$S_p^i(Q, b_T) = \frac{\alpha_s}{2\pi} \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A^i \ln \frac{Q^2}{\mu^2} - B^i \right]$$

$$\{A, B\}^{qg\to qg} = \left\{ 2(C_A + C_F), 3C_F + 2C_A\beta_0 \right\} \ , \ \{A, B\}^{gg\to gg} = \left\{ 4C_A, 6C_A\beta_0 \right\}$$

$$\mu_b = 2e^{-\gamma_E}/b_* \ , \ b_* = b_T/\sqrt{1 + b_T^2/b_{max}^2} \ , \ b_{max} = 0.5\text{GeV}^{-1}$$

Non-perturbative part Sun, Isaacson, Yuan, Yuan 2014; Prokudin, Sun, Yuan 2015

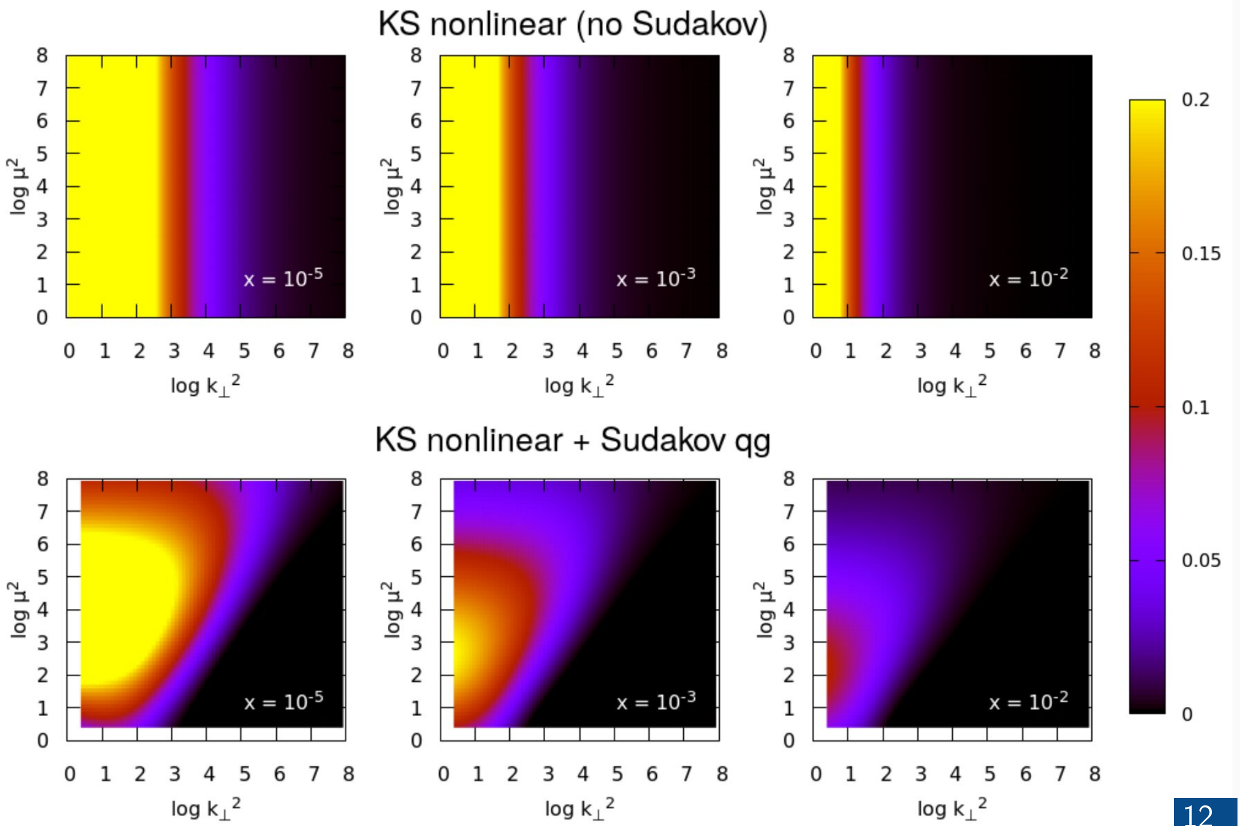
$$S_{np}^i(Q, b_T) = C^i \left[g_1 b_T^2 + g_2 \ln \frac{Q}{Q_0} \ln \frac{b_T}{b_*} \right] , \ C^{qg\to qg} = 1 + \frac{C_A}{2C_F} , \ C^{gg\to gg} = \frac{3C_A}{2C_F}$$

$$g_1 = 0.212 \ , \ g_2 = 0.84 \ , \ Q_0^2 = 2.4\text{GeV}^2$$

Non-perturbative contribution for small-x gluon already in TMD and omitted here.

Effect of Sudakov on TMD

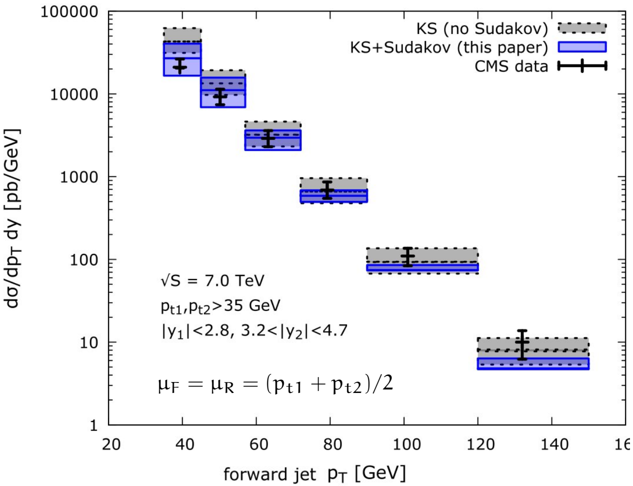
AvH, Kotko, Kutak, Sapeta 2021

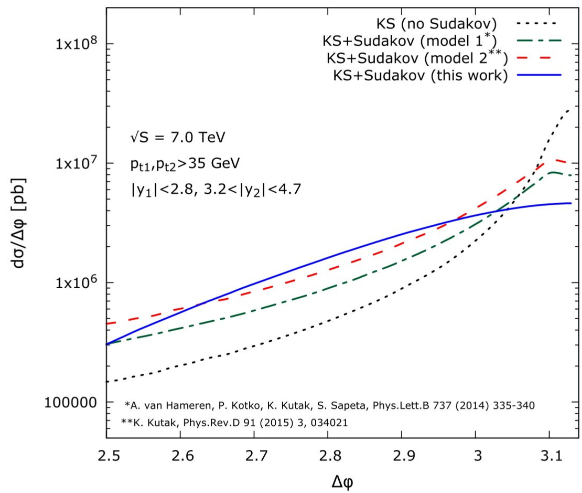


Sudakov resummation for central-forward dijets

AvH, Kotko, Kutak, Sapeta 2021

from pp collisions at $\sqrt{S} = 7\text{TeV}$





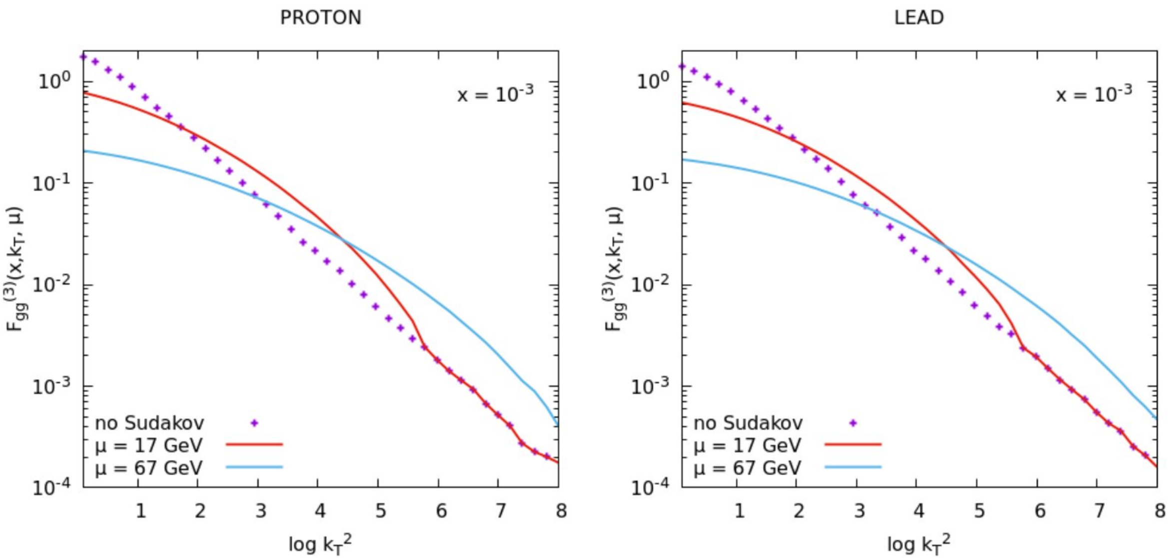
The p_T distribution describes the data reasonably well. It is a bit closer to the data for small values of p_T if the Sudakov factor is included.

$\Delta\varphi$ is the angle between the jets. The Sudakov factor suppresses the peak at $\Delta\varphi = \pi$, and makes the distribution concave.

Calculations performed independently with LxJet (Kotko) and KaTie (AvH).

$\mathcal{F}_{gg}(3)$ with Sudakov

AvH, Kotko, Kutak, Sapeta 2021



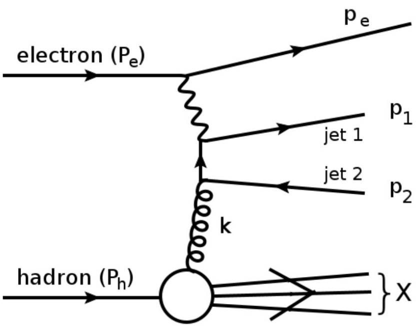
Within the Gaussian approximation, $\mathcal{F}_{gg}^{(3)}$ can be obtained from $\mathcal{F}_{qg}^{(1)}$ via

$$\mathcal{F}_{gg}^{(3)}(x,k_T) = \frac{\pi\alpha_s}{N_c k_T^2 S_\perp}\int_{k_T^2} dr_T^2 \,\ln\frac{r_T^2}{k_T^2}\int\frac{d^2q_T}{q_T^2}\,\mathcal{F}_{qg}^{(1)}(x,q_T)\,\mathcal{F}_{qg}^{(1)}(x,r_T-q_T)$$

where $S_\perp$ is the target's transverse area.

Dijets in DIS

AvH, Kotko, Kutak, Sapeta 2021

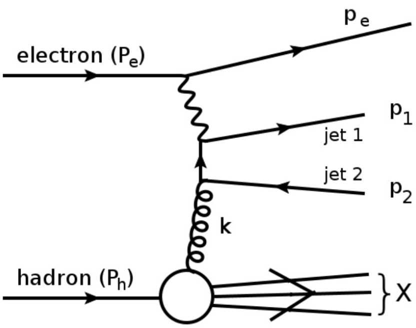


$$d\sigma_{eh\to e'+2j+X} = \int \frac{dx}{x}\frac{d^2k_T}{\pi}\,\mathcal{F}^{(3)}_{gg}(x,k_T,\mu) \times \frac{1}{4xP_e\cdot P_h}\,d\Phi(P_e,k;p_e,p_1,p_2)\,|\overline{M}_{eg^*\to e'+2j}|^2$$

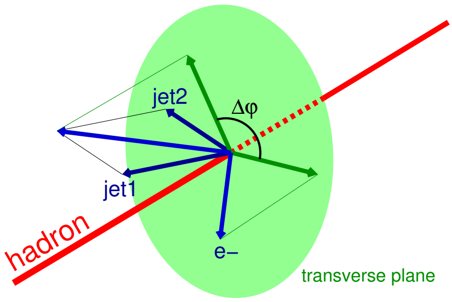
ITMD for DIS only requires $\mathcal{F}^{(3)}_{gg}$,
aka the Weizsäcker-Williams density

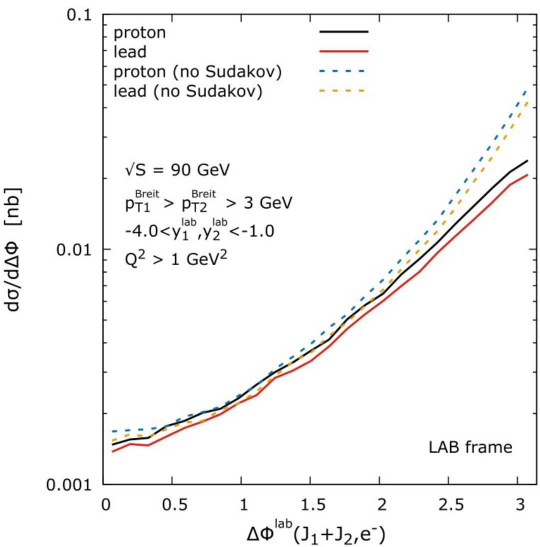
Dijets in DIS

AvH, Kotko, Kutak, Sapeta 2021



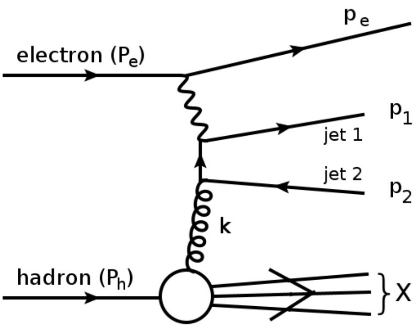
$$
\begin{aligned}
d\sigma_{eh\to e'+2j+X} &= \int \frac{dx}{x}\frac{d^2k_T}{\pi}\,\mathcal{F}^{(3)}_{gg}(x,k_T,\mu) \\
&\times \frac{1}{4xP_e\cdot P_h}\,d\Phi(P_e,k;p_e,p_1,p_2)\,|\overline{M}_{eg^*\to e'+2j}|^2
\end{aligned}
$$





Dijets in DIS

AvH, Kotko, Kutak, Sapeta 2021



$$
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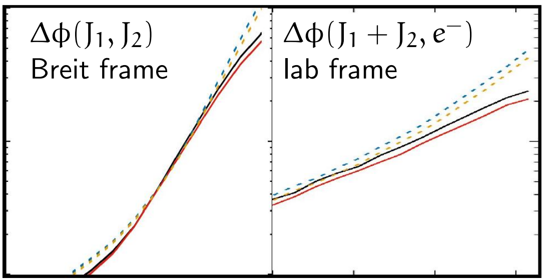
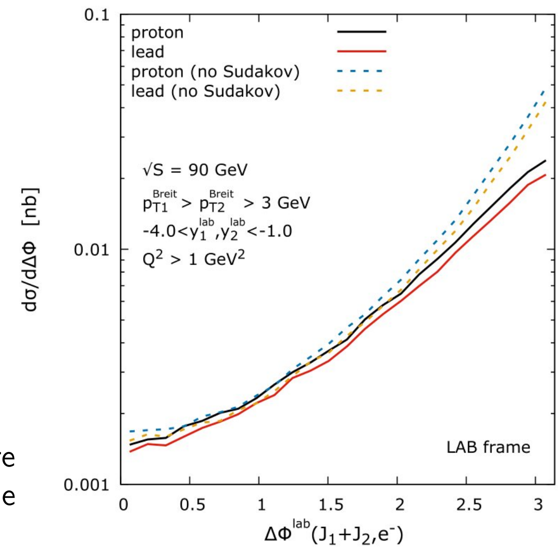
&\quad\times \frac{1}{4xP_e\cdot P_h}\,d\Phi(P_e,k;p_e,p_1,p_2)\,|\overline{M}_{eg^*\to e'+2j}|^2
\end{aligned}
$$





Differences between curves slightly more pronounced for $\Delta\phi(J_1+J_2,e^-)$ in lab frame than for $\Delta\phi(J_1,J_2)$ in Breit frame.

Conclusions and outlook

- small-x Improved TMD factorization allows to consistently include saturation effects in calculations for forward dijets, both at hadron colliders and electron-ion colliders
- in particular decorrelation-type of observables are sensitive to saturation effects
- inclusion of Sudakov resummation also has a sizable effect such observables
- we so far included both effects for DIS
- we plan to include the Sudakov resummation in all TMDs necessary for dijets at hadron colliders within ITMD factorization

Thank you for your attention.

Augmented TMD evolution

Kwieciński, Martin, Staśto 1997
Kwieciński, Kutak 2003

linear BFKL with kinematic constraint

$$\phi(x,k^2) = \phi^{(0)}(x,k^2)$$

$$+\frac{\alpha_s(k^2)N_c}{\pi}\int_x^1\frac{dz}{z}\int_{k_0^2}^{\infty}\frac{dl^2}{l^2}\left\{\frac{l^2\phi(\frac{x}{z},l^2)\theta(\frac{k^2}{z}-l^2)-k^2\phi(\frac{x}{z},k^2)}{|l^2-k^2|}+\frac{k^2\phi(\frac{x}{z},k^2))}{\sqrt{|4l^4+k^4|}}\right\}$$

$$+\frac{\alpha_s(k^2)}{2\pi k^2}\int_x^1 dz\left(P_{gg}(z)-\frac{2N_c}{z}\right)\int_{k_0^2}^{k^2}dl^2\,\phi\left(\frac{x}{z},l^2\right)+\frac{\alpha_s(k^2)}{2\pi}\int_x^1 dz\,P_{gq}(z)\Sigma\left(\frac{x}{z},k^2\right)$$

$$-\frac{2\alpha_s^2(k^2)}{R^2}\left[\left(\int_{k^2}^{\infty}\frac{dl^2}{l^2}\phi(x,l^2)\right)^2+\phi(x,k^2)\int_{k^2}^{\infty}\frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\phi(x,l^2)\right]$$

non-linear term from triple-pomeron vertex, with $R_A = R\,A^{1/3}$

DGLAP corrections

Kutak, Sapeta 2012:

Starting distribution $\phi^{(0)}(x,k^2) = \dfrac{\alpha_s(k^2)}{2\pi k^2}\displaystyle\int_x^1 dz P_{gg}(z)\frac{x}{z}g\left(\frac{x}{z}\right)$, $xg(x) = N(1-x)^{\beta}(1-Dx)$

fitted to combined HERA F_2 data, and with $\phi(x,k^2<1) = k^2\phi(x,1)$.