WORK DONE ON VLENF SENSITIVITY

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This document is intended to serve as a review of the work done to determine the sensitivity of a VLENF using the golden channel. Motivation towards building a VLENF will not be given nor will details of a VLENF be discussed beyond what is pertinent to this study.

Appendices

Appendix A. Polarization

The polarization $\mathcal{P}$ of the beam should be measured since it affects the neutrino fluxes (Eq. **??**) in a way that, in the worst case, could turn off the ν_e flux in the lab frame[1].The muons used to generate the NF's beam are the daughters of pion decays within the storage ring. Given the spin of the pion and that it decays weakly, within the rest-frame of the pion the resulting muon in the suppressed decay $\pi^- \to \mu^- + \bar{\nu}_\mu$ is right-handed with a helicity of essentially unity[2]. Thus each particle within the bunch has some spin $\vec{S}$ which can Thomas precess.

Within the context of accelerator physics, the polarization of a beam is defined as the ensemble average of the spin vectors:

$$\mathcal{P} = \begin{pmatrix} \mathcal{P}_x \\ \mathcal{P}_y \\ \mathcal{P}_z \end{pmatrix} = \langle \vec{S} \rangle = \frac{1}{N} \sum_i^N \vec{S}_i. \tag{1}$$

The projection of the polarization vector on the beam axis $\mathcal{P} \cdot \hat{z} = \mathcal{P}_z$ is called the *longitudinal polarization* in the language of accelerator physics. Some of this polarisation can be lost due to, for example, sign flips when the muons interacts with matter, but this is a negligible effect for the accelerator in question.

The initial polarization in the storage ring can be determined using similar methods that have been used in [2] for neutrino factory studies. However, in the case of a VLENF life is easier since polarization isn't lost due to cooling and other interactions with matter. The muon helicity in the lab frame for a single pion decaying is:

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[1] If the polarization $\mathcal{P}$ is unity then a μ-decay's ν_e flux vanishes (See Eq. **??**) since the ν_es are at rest in the lab-frame. For μ^- flip the polarization sign. This has been explored extensively in [1].

[2] There is helicity suppression here since V-A theory wants the muon to be left-handed in the massless limit, which is why electrons are suppressed at the 10^{-4} level. Note that helicity h satisfies the relation $h \sim \sigma \cdot p$. Helicity transforms like angular momentum $r \times p$ with respect to CPT – so r flips – and a μ^+ is left-handed with helicity -1.

$$h = \frac{EE^* - \gamma_\pi m^2}{pp^*} \tag{2}$$

where m, p, and E are the muon's mass, momentum, and energy, respectively. The starred frame is the pion rest frame and the unstarred frame is the lab frame. For a more comprehensive treatment, see [3]. We can thus determine the beam polarization if we know the kinematics of the pion decay.

Until we have a full simulation of the decay kinematics within the storage ring, we can make some approximations to help determine h. The VLENF storage ring lattice has been designed to accept muons that decay backwards in the pion's rest frame such that muons and pions can be differentiated in energy. The muon distribution in the pion rest frame is isotropic. Writing a toy MC and only taking muons that decay backwards, we can compute the helicity in the lab frame of each of these muons using Eq. 2, thus can compute the polarization of the beam. We find that the lab longitudinal polarization is -48%.

The polarization can then be evolved through the lattice since we are only interested in the beam after it has made a complete turn. The spin precession frequency for a polarized muon is:

$$\nu = \frac{g_\mu - 2}{2}\frac{E_{\text{beam}}}{m_\mu} \tag{3}$$

$$= \frac{E_{\text{beam}}\ (\text{GeV})}{90.6233} \tag{4}$$

where $g-2$ is well measured and predicted (See [1] and references therein). For the VLENF, we are far away from any spin resonance. Without an energy spread, the muons would Thomas precess each time they pass around the ring. Once we introduce an energy spread, the spins will precess at different frequencies thus decohere. The polarization is the spin expectation value so will thus average out to zero and this effect has been seen in the IDR NF [4]. Introducing a 16% Gaussian energy spread for a 2 GeV stored muon energy, we can completely determine the evolution of the polarization as a function of time (Fig. 1). If we weight the polarization as a function of time against the decay length of the muon, we find the average polarization to be 2×10^{-4}. This is much less than the uncertainties we could realistically measure cross sections, so polarization is ignored.

REFERENCES

[1] Alain Blondel. Muon polarisation in the neutrino factory. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 451(1):131 – 137, 2000.

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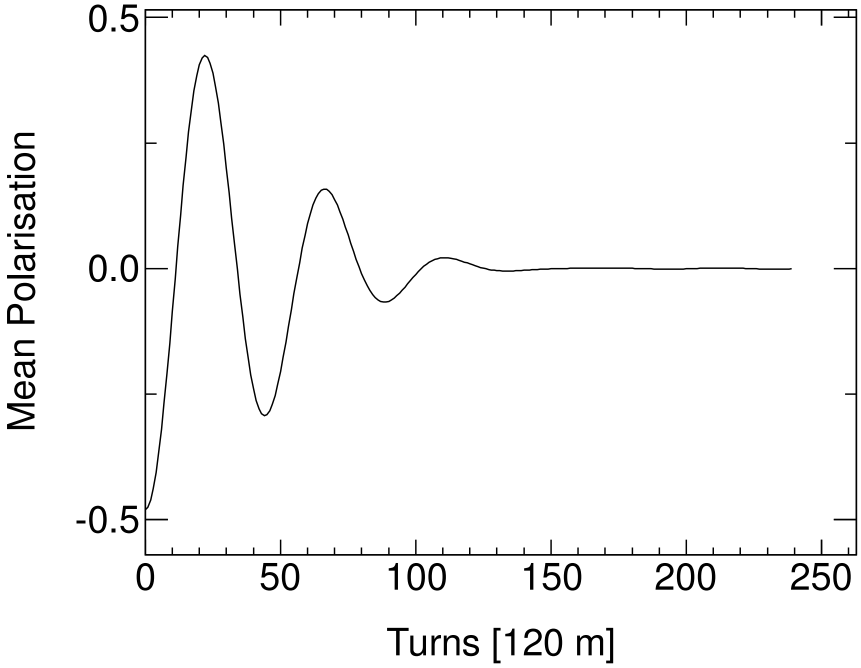

Figure 1. The muon polarization for an FFAG ring if the initial muons has -48% initial polarization and an energy distribution of 2 GeV ± 16%. The decay-weighted average is 2×10^{-4} under this assumption.