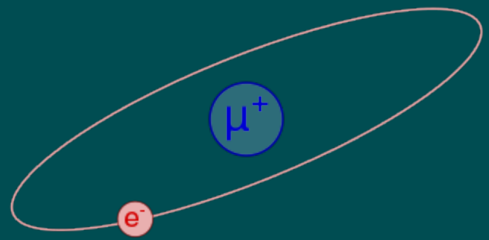

Physics Motivation for the Muon Program

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Wayne State University

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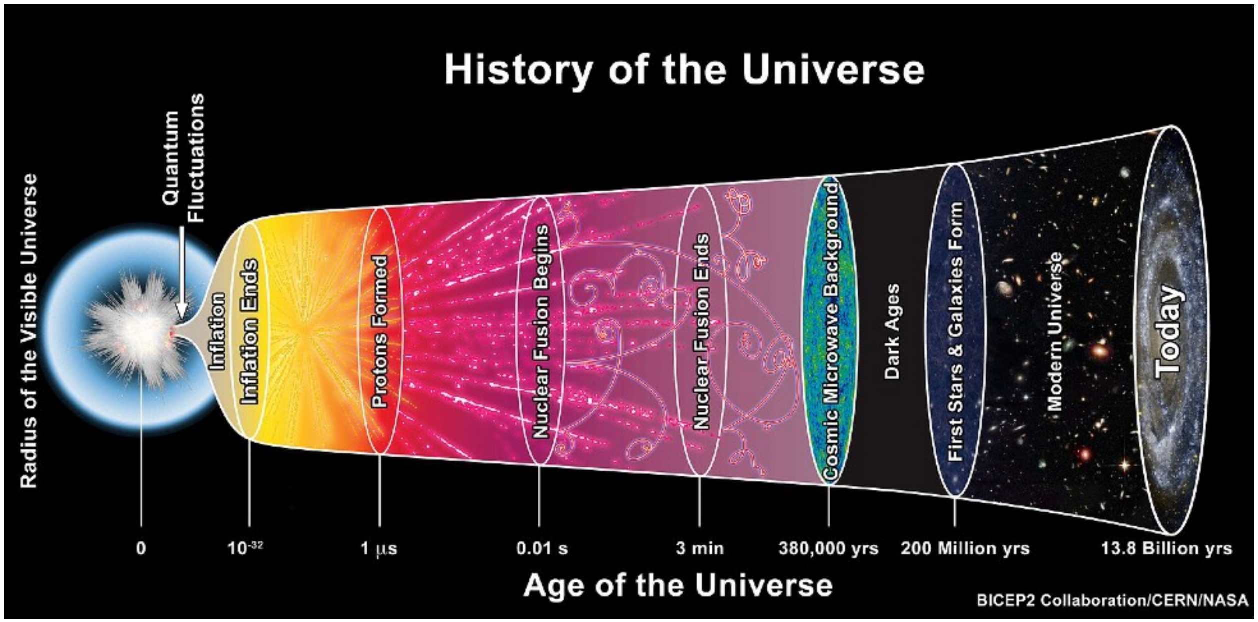
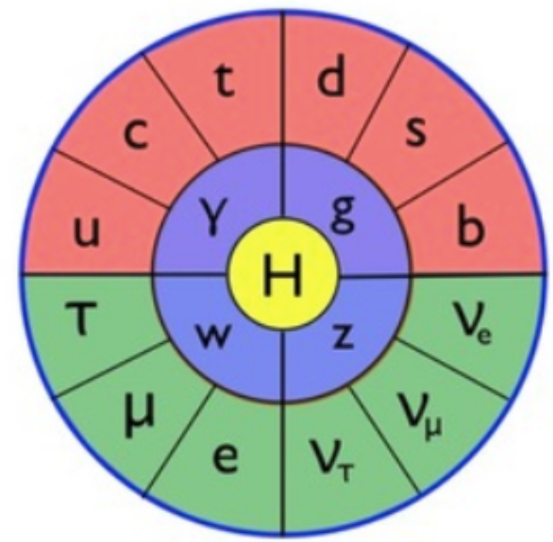
Fundamental physics: building the Universe

★ Is it possible to build the Universe using the Standard Model as a tool?
- no, but maybe it can tell us where to look for the new tools



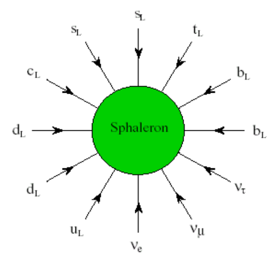


★ The era of "guaranteed discoveries" is over (top quark, electroweak breaking)
- new experiments designed to study rare decays or perform precision studies of various processes might point us in the right direction

Fundamental physics: building the Universe

★ Standard Model satisfies Sakharov's conditions for baryogenesis

✓ Baryon (and lepton) number - violating processes
to ***generate*** asymmetry



$\Delta B = 3,\ \Delta L = 3,$
$B - L$ conserved

✓ Universe that evolves out of thermal equilibrium
to ***keep*** asymmetry from ***being washed out***

✓ "Microscopic CP-violation"
to ***keep*** asymmetry from ***being compensated in the "anti-world"***

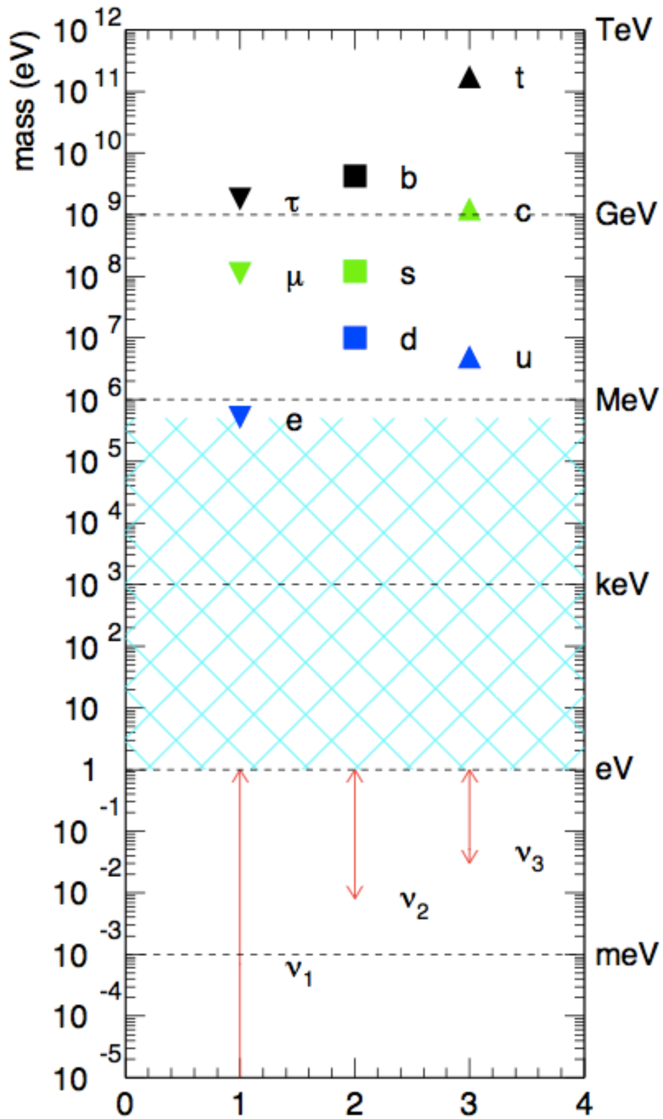
- but there are still issues preventing it to succeed (not enough CP-violation via CKM mechanism, order of the phase transition, …)

★ What about New Physics?

- no new strongly-interacting particles so far at the LHC (SUSY?)
- neutrinos oscillations: ν's have mass and so CLFV transitions are guaranteed
- use sphaleron mechanism: baryogenesis via leptogenesis   Fukugita, Yanagida
- new sources of CP-violation in the lepton sector

Should we look for New Physics in the charged lepton sector? Muons?

Fundamental physics with muons: flavor problem



★ SM and BSM Flavor problem

★ Flavor problem: patterns of masses of particles

- quarks

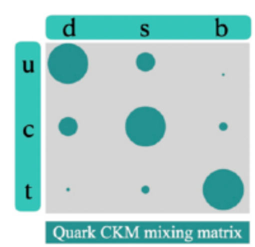
$$\frac{m_d}{m_u} \simeq 2 \,, \quad \frac{m_s}{m_d} \simeq 21 \,,$$
$$\frac{m_t}{m_c} \simeq 267 \,, \quad \frac{m_c}{m_u} \simeq 431 \,, \quad \frac{m_t}{m_u} \simeq 1.2 \times 10^5 \,.$$

- leptons

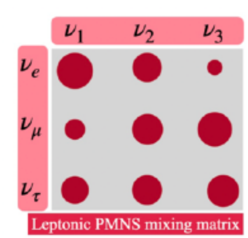
$$\frac{m_\tau}{m_\mu} \simeq 17 \,, \quad \frac{m_\mu}{m_e} \simeq 207 \,.$$

★ Flavor problem: pattern of fermion mixing

- why is the quark mixing matrix so different from the neutrino mixing matrix?



vs



S. Cao, et al.

★ Flavor problem: nature of neutrino mass

Fundamental physics with muons: flavor problem

★ Flavor problem: flavor-changing neutral currents (FCNC)

- there is no term in the SM Lagrangian that leads to FCNC effects: one loop
- quarks: massive quarks and non-zero mixing parameters automatically lead to FCNC processes: $b \to s\gamma, c \to u\ell\bar{\ell}, B^0 - \bar{B}^0$-mixing, etc.
- leptons: massive neutrinos and non-zero mixing parameters automatically lead to FCNC processes: $\mu \to e\gamma, \mu \to eee, \mu A \to eA$, etc.

★ Flavor problem: patterns of masses of particles and neutrino mass

- there could be a mechanism generating mass patterns (Froggatt-Nielsen, etc.)…

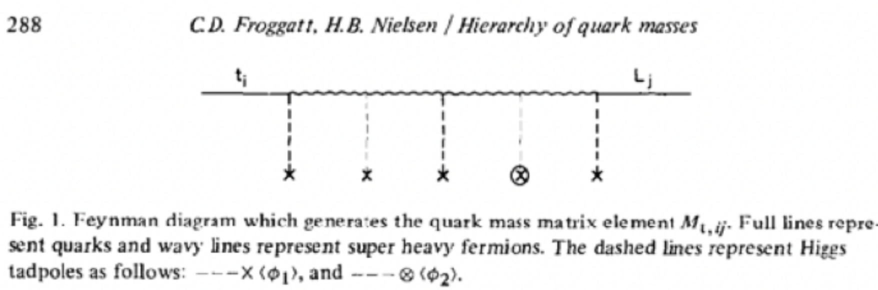

288 C.D. Froggatt, H.B. Nielsen / Hierarchy of quark masses

Fig. 1. Feynman diagram which generates the quark mass matrix element $M_{t,ij}$. Full lines represent quarks and wavy lines represent super heavy fermions. The dashed lines represent Higgs tadpoles as follows: $---\times \langle\phi_1\rangle$, and $---\otimes \langle\phi_2\rangle$.

- … or maybe not (a "just so" solution?)

Fundamental physics with muons: flavor violation

★ Example of the common origin of the neutrino masses and CLFV transitions

- consider a model with a triplet Higgs, e.g., a left-right model

$$-\mathcal{L}_{\text{Yukawa}} = \overline{\psi}'_{iL}\left(G_{ij}\phi + H_{ij}\widetilde{\phi}\right)\psi'_{jR} + \frac{i}{2}F_{ij}\left(\psi'^T_{iL}C\tau_2\Delta_L\psi'_{jL} + \psi'^T_{iR}C\tau_2\Delta_R\psi'_{jR}\right) + \text{h.c.}$$

with $\psi'_{iL,R} = \begin{pmatrix} \nu'_{iL,R} \\ e'_{iL,R} \end{pmatrix}$ and $\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$

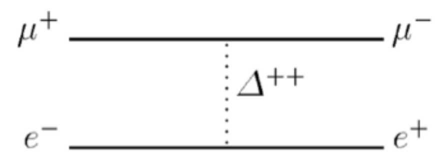
- this Lagrangian leads to the Majorana masses for the neutrinos

Pati, Salam; Mohapatra, Pati; Senjanovic, et al, Schechter and Valle; K. Kiers et al

$$-\mathcal{L}_{\text{Majorana}} = \frac{1}{2\sqrt{2}}\left(\overline{\nu'^c_L}Fv_Le^{i\theta_L}\nu'_L + \overline{\nu'^c_R}Fv_R\nu'_R\right) + \text{h.c.}$$

- … and both $\Delta L_\mu = 1$ (FCNC decays) and $\Delta L_\mu = 2$ (muonium oscillations) transitions

$$\mathcal{H}_\Delta = -\frac{g_{ee}g^*_{\mu\mu}}{8M^2_\Delta}(\overline{\mu}_L\gamma_\alpha e_L)(\overline{\mu}_L\gamma^\alpha e_L) + H.c.$$



Chang, Keung (89); Schwartz (89); Conlin, AAP (21); Han, Tang, Zhang (21)

Fundamental physics with muons: flavor violation

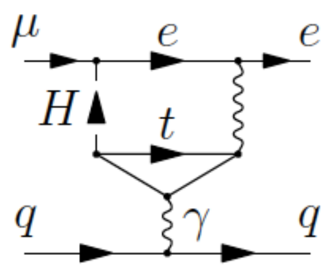
★ Leptonic FCNC could be generated by New Physics

★ Ex.1 FCNC Higgs decays H → μe, τe, etc.: $Y_{ij} = \frac{m_i}{v}\delta_{ij} + \frac{v^2}{\sqrt{2}\Lambda^2}\hat{\lambda}_{ij}$

Harnik, Kopp, Zupan

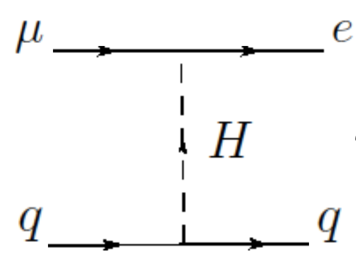
★ FCNC Higgs model & muon conversion/quarkonium decays

e.g.



$\sim \mathcal{O}(\alpha^2)$

Barr-Zee type



$\sim \mathcal{O}(m_u \text{ or } m_d)$

tree level
(note suppression of light quark couplings)

Process	Coupling	Bound
$h \to \mu e$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$< 5.4 \times 10^{-4}$
$\mu \to e\gamma$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$< 2.1 \times 10^{-6}$
$\mu \to eee$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$\lesssim 3.1 \times 10^{-5}$
$\mu\,\mathrm{Ti} \to e\,\mathrm{Ti}$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$< 1.2 \times 10^{-5}$

Calibbi, Signorelli

Fundamental physics with muons: flavor violation

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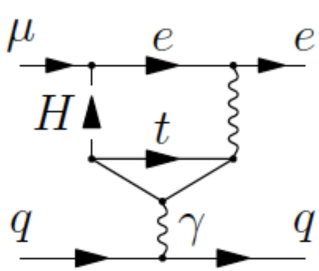
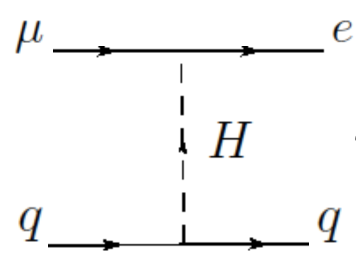


$\sim \mathcal{O}(\alpha^2)$

Barr-Zee type



$\sim \mathcal{O}(m_u \text{ or } m_d)$

tree level
(note suppression of light quark couplings)

★ Ex.2 Exceptional couplings of (flavor-diagonal) NP to third generation $\mathcal{H}_{\rm NP} = G\bar{b}'_L\gamma^\lambda b'_L\bar{\tau}'_L\gamma_\lambda\tau'_L$ -> flavor “anomalies”

Glashow, Guadagnoli, Lane

★ Ex.3 Leptoquarks -> flavor “anomalies”

★ Leptons and New Physics: choose muons (long lifetime and large mass)

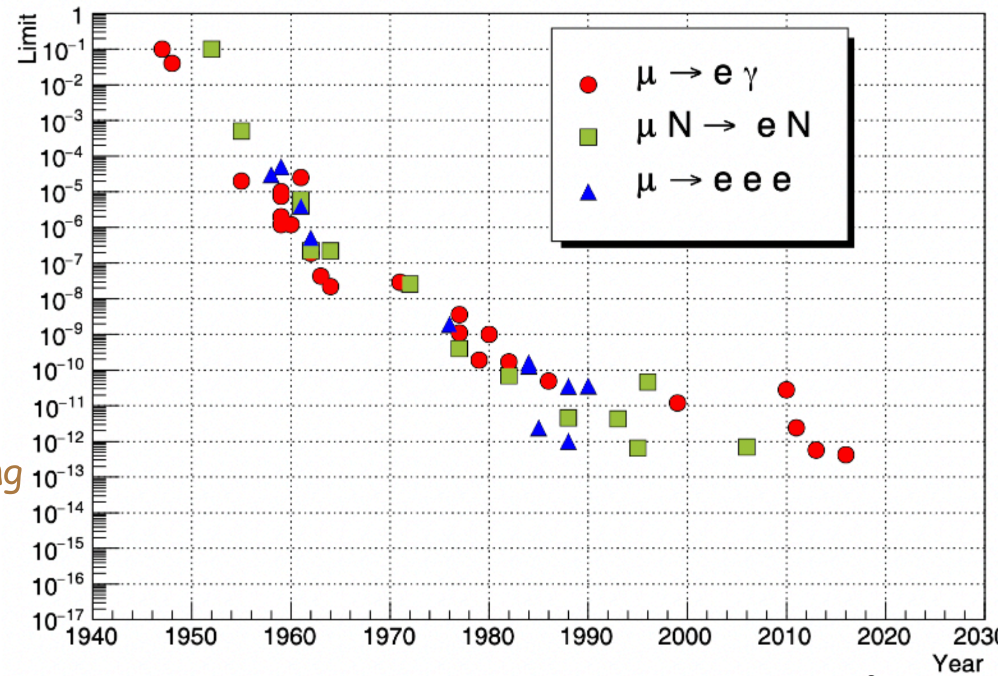
Fundamental physics with muons: flavor violation

★ Muons can help solving the most fundamental problems in particle physics!

★ Possible experimental searches of LFCNC

- lepton-flavor violating processes
  - $\mu \to e\gamma$, $\tau \to e\gamma$, etc.
  - $\mu \to eee$, $\tau \to \mu ee$, etc.
  - $\mu^+e^- \to e^-\mu^+$ (muonium oscillations)
  - $Z^0 \to \mu e$, τe, etc.
  - $H \to \mu e$, τe, etc.
  - K^0 (B^0, D^0, ...) $\to \mu e$, τe, etc.
  - $\mu^- + (A, Z) \to e^- + (A, Z)$
- lepton number and lepton-flavor violating processes
  - $(A, Z) \to (A, Z\pm 2) + e^\mp e^\mp$
  - $\mu^- + (A, Z) \to e^+ + (A, Z-2)$

LORENZO CALIBBI and GIOVANNI SIGNORELLI



★ Decays are highly suppressed in the Standard Model: $$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_i U^*_{\mu i} U_{ei} \frac{m^2_{\nu_i}}{M^2_W}\right|^2 < 10^{-54}$$

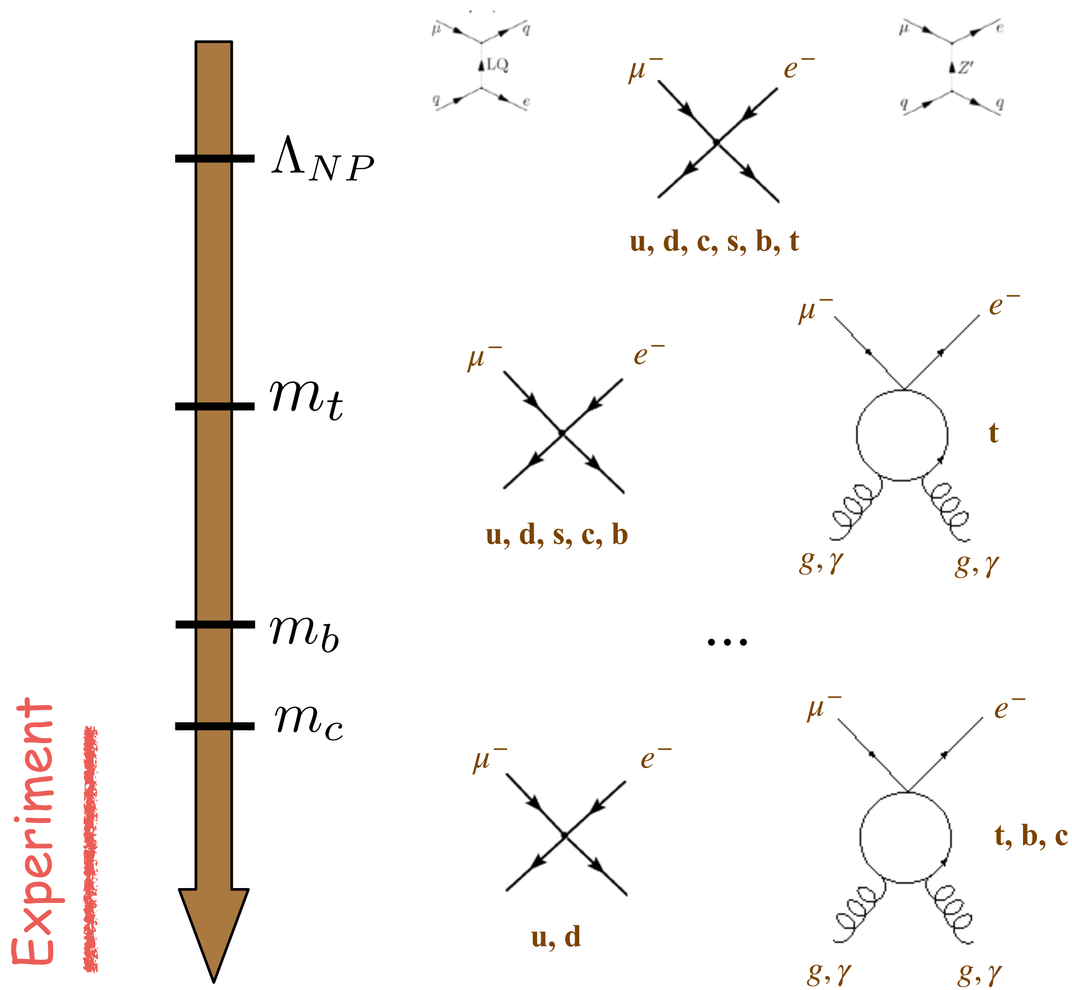
★ But: no trivial FCNC vertices in the Standard Model: sensitive tests of New Physics!

(Number of possible models) > (number of model builders). How do we proceed?

Flavor violation and effective Lagrangians

★ Modern approach to flavor physics calculations: effective field theories

- ★ It is important to understand ALL relevant energy scales for the problem at hand



New Physics generates lepton FCNC

Scales associated with heavy SM particles (quarks, leptons)

...

heavy quarks decouple

...

Scales associated with experiment

Effective Lagrangians and New Physics

★ Effective Lagrangians parameterize New Physics without specifying a model

- write out all terms consistent with symmetries and according to power counting
- match to possible UV completions/compute experimental observables

$$
\begin{aligned}
\mathcal{L}_{\ell_1 \to \ell_2 \nu_2 \bar{\nu}_1} = -\frac{4G_F}{\sqrt{2}} \Big[& g^S_{RR} \left(\overline{\ell_2}_R \nu_{\ell_2 L}\right) \left(\overline{\nu_{\ell_1}}_L \ell_{1R}\right) + g^S_{RL} \left(\overline{\ell_2}_R \nu_{\ell_2 L}\right) \left(\overline{\nu_{\ell_1}}_R \ell_{1L}\right) \\
& + g^S_{LR} \left(\overline{\ell_2}_L \nu_{\ell_2 R}\right) \left(\overline{\nu_{\ell_1}}_L \ell_{1R}\right) + g^S_{LL} \left(\overline{\ell_2}_L \nu_{\ell_2 R}\right) \left(\overline{\nu_{\ell_1}}_R \ell_{1L}\right) \\
& + g^V_{RR} \left(\overline{\ell_2}_R \gamma^\alpha \nu_{\ell_2 R}\right) \left(\overline{\nu_{\ell_1}}_R \gamma_\alpha \ell_{1R}\right) + g^V_{RL} \left(\overline{\ell_2}_R \gamma^\alpha \nu_{\ell_2 R}\right) \left(\overline{\nu_{\ell_1}}_L \gamma_\alpha \ell_{1L}\right) \\
& + g^V_{LR} \left(\overline{\ell_2}_L \gamma^\alpha \nu_{\ell_2 L}\right) \left(\overline{\nu_{\ell_1}}_R \gamma_\alpha \ell_{1R}\right) + g^V_{LL} \left(\overline{\ell_2}_R \gamma^\alpha \nu_{\ell_2 R}\right) \left(\overline{\nu_{\ell_1}}_L \gamma_\alpha \ell_{1L}\right) \\
& + \frac{g^T_{RL}}{2} \left(\overline{\ell_2}_R \sigma_{\alpha\beta} \nu_{\ell_2 L}\right) \left(\overline{\nu_{\ell_1}}_R \sigma^{\alpha\beta} \ell_{1L}\right) + \frac{g^T_{LR}}{2} \left(\overline{\ell_2}_L \sigma_{\alpha\beta} \nu_{\ell_2 R}\right) \left(\overline{\nu_{\ell_1}}_L \sigma^{\alpha\beta} \ell_{1R}\right) + h.c. \Big],
\end{aligned}
$$

- which for $\mu \to e\nu\bar{\nu}$ (muon decay) leads to

$$
\begin{aligned}
\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} & \left[F\left(\frac{m_e^2}{m_\mu^2}\right) + 4\eta \frac{m_e}{m_\mu} G\left(\frac{m_e^2}{m_\mu^2}\right) - \frac{32}{3}\frac{m_e^2}{m_\mu^2}\left(\rho - \frac{3}{4}\right)\left(1 - \frac{m_e^4}{m_\mu^4}\right) \right] \\
& \times \left(1 + \frac{3}{5}\frac{m_\mu^2}{m_W^2}\right) \left[1 + \frac{\alpha(m_\mu)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right],
\end{aligned}
$$

- where ρ and η are the Michel parameters

$$
\rho = \frac{3}{16}\left[\left|g^S_{RR}\right|^2 + \left|g^S_{LL}\right|^2 + \left|g^S_{RL} - 2g^T_{RL}\right|^2 + \left|g^S_{LR} - 2g^T_{LR}\right|^2 + \frac{3}{4}\left(\left|g^V_{RR}\right|^2 + \left|g^V_{LL}\right|^2\right) \right],
$$

$$
\eta = \frac{1}{2}\mathrm{Re}\left[g^V_{RR} g^{S*}_{LL} + g^V_{LL} g^{S*}_{RR} + g^V_{RL}\left(g^{S*}_{LR} + 6g^{T*}_{LR}\right) + g^V_{LR}\left(g^{S*}_{RL} + 6g^{T*}_{RL}\right) \right],
$$

Flavor violation and effective Lagrangians

★ Systematic approach: Standard Model Effective Field Theory (SMEFT)

- effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} Q^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} Q_i^{(6)} + \ldots$$

with the Weinberg operator $Q^{(5)}$

$$Q^{(5)} = \epsilon_{jk}\epsilon_{mn} H^j H^m \left(L_p^k\right)^T \mathcal{C} L_r^n$$

and lots (59+5) of $Q_i^{(6)}$ operators

- the strategy of identifying an NP model involves fitting C_i from experimental data and/or matching of $\mathscr{L}$ to UV-completed NP models

TABLE 2.3 Operators with H^n, sets X^3, H^6, H^4D^2, and $\psi^2 H^3$.

X^3		H^6 and H^4D^2		$\psi^2 H^3 +$ h.c.	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_H	$\left(H^\dagger H\right)^3$	Q_{eH}	$\left(H^\dagger H\right)\left(\overline{L}_p e_r H\right)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{H\Box}$	$\left(H^\dagger H\right)\Box\left(H^\dagger H\right)$	Q_{uH}	$\left(H^\dagger H\right)\left(\overline{Q}_p u_r \widetilde{H}\right)$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	Q_{HD}	$\left(H^\dagger D^\mu H\right)^* \left(H^\dagger D_\mu H\right)$	Q_{dH}	$\left(H^\dagger H\right)\left(\overline{Q}_p d_r H\right)$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

TABLE 2.4 Operators with H^n, sets X^2H^2, $\psi^2 XH$, and $\psi^2 H^2 D$.

X^2H^2		$\psi^2 XH +$ h.c.		$\psi^2 H^2 D$	
Q_{HG}	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$\left(\overline{L}_p \sigma^{\mu\nu} e_r\right) \tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$\left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\overline{L}_p \gamma^\mu L_r\right)$
$Q_{H\widetilde{G}}$	$H^\dagger H \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$\left(\overline{L}_p \sigma^{\mu\nu} e_r\right) H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$\left(H^\dagger i \overleftrightarrow{D}_\mu^I H\right)\left(\overline{L}_p \tau^I \gamma^\mu L_r\right)$
Q_{HW}	$H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$\left(\overline{Q}_p \sigma^{\mu\nu} T^A u_r\right) \widetilde{H} G_{\mu\nu}^A$	Q_{He}	$\left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\overline{e}_p \gamma^\mu e_r\right)$
$Q_{H\widetilde{W}}$	$H^\dagger H \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$\left(\overline{Q}_p \sigma^{\mu\nu} u_r\right) \tau^I \widetilde{H} W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$\left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\overline{Q}_p \gamma^\mu Q_r\right)$
Q_{HB}	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$\left(\overline{Q}_p \sigma^{\mu\nu} u_r\right) \widetilde{H} B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$\left(H^\dagger i \overleftrightarrow{D}_\mu^I H\right)\left(\overline{Q}_p \tau^I \gamma^\mu Q_r\right)$
$Q_{H\widetilde{B}}$	$H^\dagger H \widetilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$\left(\overline{Q}_p \sigma^{\mu\nu} T^A d_r\right) H G_{\mu\nu}^A$	Q_{Hu}	$\left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\overline{u}_p \gamma^\mu u_r\right)$
Q_{HWB}	$H^\dagger \tau^I H W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$\left(\overline{Q}_p \sigma^{\mu\nu} d_r\right) \tau^I H W_{\mu\nu}^I$	Q_{Hd}	$\left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\overline{d}_p \gamma^\mu d_r\right)$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$\left(\overline{Q}_p \sigma^{\mu\nu} d_r\right) H B_{\mu\nu}$	Q_{Hud}	$i\left(\widetilde{H}^\dagger D_\mu H\right)\left(\overline{u}_p \gamma^\mu d_r\right)$

TABLE 2.5 Four-fermion operators, classes $(\overline{L}L)(\overline{L}L)$, $(\overline{R}R)(\overline{R}R)$, and $(\overline{L}L)(\overline{R}R)$.

$(\overline{L}L)(\overline{L}L)$		$(\overline{R}R)(\overline{R}R)$		$(\overline{L}L)(\overline{R}R)$	
Q_{ll}	$\left(\overline{L}_p \gamma^\mu L_r\right)\left(\overline{L}_s \gamma^\mu L_t\right)$	Q_{ee}	$\left(\overline{e}_p \gamma^\mu e_r\right)\left(\overline{e}_s \gamma^\mu e_t\right)$	Q_{le}	$\left(\overline{L}_p \gamma^\mu L_r\right)\left(\overline{e}_s \gamma^\mu e_t\right)$
$Q_{qq}^{(1)}$	$\left(\overline{Q}_p \gamma^\mu Q_r\right)\left(\overline{Q}_s \gamma^\mu Q_t\right)$	Q_{uu}	$\left(\overline{u}_p \gamma^\mu u_r\right)\left(\overline{u}_s \gamma^\mu u_t\right)$	Q_{lu}	$\left(\overline{L}_p \gamma^\mu L_r\right)\left(\overline{u}_s \gamma^\mu u_t\right)$
$Q_{qq}^{(3)}$	$\left(\overline{Q}_p \gamma^\mu \tau^I Q_r\right)\left(\overline{Q}_s \gamma^\mu \tau^I Q_t\right)$	Q_{dd}	$\left(\overline{d}_p \gamma^\mu d_r\right)\left(\overline{d}_s \gamma^\mu d_t\right)$	Q_{ld}	$\left(\overline{L}_p \gamma^\mu L_r\right)\left(\overline{d}_s \gamma^\mu d_t\right)$
$Q_{lq}^{(1)}$	$\left(\overline{L}_p \gamma^\mu L_r\right)\left(\overline{Q}_s \gamma^\mu Q_t\right)$	Q_{eu}	$\left(\overline{e}_p \gamma^\mu e_r\right)\left(\overline{u}_s \gamma^\mu u_t\right)$	Q_{qe}	$\left(\overline{Q}_p \gamma^\mu Q_r\right)\left(\overline{e}_s \gamma^\mu e_t\right)$
$Q_{lq}^{(3)}$	$\left(\overline{L}_p \gamma^\mu \tau^I L_r\right)\left(\overline{Q}_s \gamma^\mu \tau^I Q_t\right)$	Q_{ed}	$\left(\overline{e}_p \gamma^\mu e_r\right)\left(\overline{d}_s \gamma^\mu d_t\right)$	$Q_{qu}^{(1)}$	$\left(\overline{Q}_p \gamma^\mu Q_r\right)\left(\overline{u}_s \gamma^\mu u_t\right)$
		$Q_{ud}^{(1)}$	$\left(\overline{u}_p \gamma^\mu u_r\right)\left(\overline{d}_s \gamma^\mu d_t\right)$	$Q_{qu}^{(8)}$	$\left(\overline{q}_p \gamma^\mu T^A q_r\right)\left(\overline{u}_s \gamma^\mu T^A u_t\right)$
		$Q_{ud}^{(8)}$	$\left(\overline{u}_p \gamma^\mu T^A u_r\right)\left(\overline{d}_s \gamma^\mu T^A d_t\right)$	$Q_{qd}^{(1)}$	$\left(\overline{q}_p \gamma^\mu q_r\right)\left(\overline{d}_s \gamma^\mu d_t\right)$
				$Q_{qd}^{(8)}$	$\left(\overline{Q}_p \gamma^\mu T^A Q_r\right)\left(\overline{d}_s \gamma^\mu T^A d_t\right)$

TABLE 2.6 Four-fermion operators, classes $(LR)(RL)$, and B (baryon-number) violating.

$(\overline{L}R)(\overline{R}L)$		B-violating	
Q_{ledq}	$\left((\overline{L}_p^j e_r\right)\left(\overline{d}_s Q_t^j\right)$	Q_{duq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[\left(Q_s^{\gamma j}\right)^T C L_t^k\right]$
$Q_{quqd}^{(1)}$	$\left((\overline{Q}_p^j u_r\right)\epsilon_{jk}\left(\overline{Q}_s^k d_t\right)$	Q_{qqu}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$\left((\overline{Q}_p^j T^A u_r\right)\epsilon_{jk}\left(\overline{Q}_s^k T^A d_t\right)$	$Q_{qqq}^{(1)}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\epsilon_{mn}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(1)}$	$\left((\overline{L}_p^j e_r\right)\epsilon_{jk}\left(\overline{Q}_s^k u_t\right)$	$Q_{qqq}^{(3)}$	$\epsilon^{\alpha\beta\gamma}\left(\tau^I\epsilon\right)_{jk}\left(\tau^I\epsilon\right)_{mn}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(3)}$	$\left((\overline{L}_p^j \sigma_{\mu\nu} e_r\right)\epsilon_{jk}\left(\overline{Q}_s^k \sigma^{\mu\nu} u_t\right)$	Q_{duu}	$\epsilon^{\alpha\beta\gamma}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

Flavor violation and effective Lagrangians

★ Radiative FCNC decays of leptons $\ell_1 \to \ell_2 + \gamma$

- the most general amplitude is

$$A_{\ell_1 \to \ell_2 \gamma}(p, p') = \frac{i}{m_{\ell_1}} \overline{u}_{\ell_2}(p') \left[A_L P_L + A_R P_R\right] \sigma_{\mu\nu} q^\nu u_{\ell_1}(p) \epsilon^{*\mu},$$

- which leads to the decay rate

$$\Gamma(\ell_1 \to \ell_2 \gamma) = \frac{m_{\ell_1}}{16\pi} \left(|A_L|^2 + |A_R|^2\right)$$

with $$A_R = A_L^* = \sqrt{2}\, \frac{v m_i^2}{\Lambda^2} \left(c_W C_{eB}^{fi} - s_W C_{eW}^{fi}\right) \equiv \sqrt{2}\, \frac{v m_i^2}{\Lambda^2} C_\gamma^{fi}$$

Effective coupling (example)	Bounds on Λ (TeV) (for $\lvert C_{ij}^6\rvert = 1$)	Bounds on $\lvert C_{ij}^6\rvert$ (for $\Lambda = 1$ TeV)	Observable
$C_{e\gamma}^{\mu e}$	6.3×10^4	2.5×10^{-10}	$\mu \to e\gamma$
$C_{e\gamma}^{\tau e}$	6.5×10^2	2.4×10^{-6}	$\tau \to e\gamma$
$C_{e\gamma}^{\tau\mu}$	6.1×10^2	2.7×10^{-6}	$\tau \to \mu\gamma$
$C_{\ell\ell,ee}^{\mu eee}$	207	2.3×10^{-5}	$\mu \to 3e$
$C_{\ell\ell,ee}^{e\tau ee}$	10.4	9.2×10^{-5}	$\tau \to 3e$
$C_{\ell\ell,ee}^{\mu\tau\mu\mu}$	11.3	7.8×10^{-5}	$\tau \to 3\mu$
$C_{(1,3)H\ell}^{\mu e}$, $C_{He}^{\mu e}$	160	4×10^{-5}	$\mu \to 3e$
$C_{(1,3)H\ell}^{\tau e}$, $C_{He}^{\tau e}$	≈ 8	1.5×10^{-2}	$\tau \to 3e$
$C_{(1,3)H\ell}^{\tau\mu}$, $C_{He}^{\tau\mu}$	≈ 9	$\approx 10^{-2}$	$\tau \to 3\mu$

Teixeira; Feruglio, Paradisi, Pattori

Other interesting modes that probe similar couplings: $\ell_1 \to \ell_2\gamma\gamma$, $\ell_1 \to 3\ell_2$, and others

Flavor violation and effective Lagrangians

★ Similarly, for purely leptonic FCNC decays $\ell_i \to \ell_j \ell_k \ell_l$

- the most general decay rate is [with (a): $\ell_1 \to 3\ell_2$ and (b): $\tau^\pm \to e^\pm \mu^+ \mu^-$ and $\tau^\pm \to \mu^\pm e^+ e^-$]

$$\Gamma(\ell_i \to \ell_j \ell_k \ell_l) = \frac{\kappa_c m_{\ell_1}^5}{32\ (192\pi^3)\Lambda^4}\Big[X_\gamma + 4\left(|C_{VLL}|^2 + |C_{VRR}|^2 + |C_{VLR}|^2 + |C_{VRL}|^2\right)$$

$$+ |C_{SLL}|^2 + |C_{SRR}|^2 + |C_{SLR}|^2 + |C_{SRL}|^2 + +48\left(|C_{TL}|^2 + |C_{TR}|^2\right)\Big]$$

with

$$X_\gamma^{(a)} = -\frac{16ev}{m_i}\mathrm{Re}\left[C_{\gamma L}^*\left(2C_{VLL} + C_{VLR} - \frac{1}{2}C_{SLR}\right) + C_{\gamma R}^*\left(2C_{VRR} + C_{VRL} - \frac{1}{2}C_{SRL}\right)\right]$$

$$+ \frac{64e^2v^2}{m_i^2}\left(\log\frac{m_i^2}{m_f^2} - \frac{11}{4}\right)\left(|C_{\gamma L}|^2 + |C_{\gamma R}|^2\right),$$

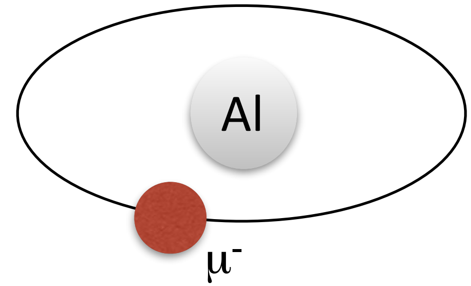
$$X_\gamma^{(a)} = -\frac{16ev}{m_i}\mathrm{Re}\left[C_{\gamma L}^*\left(C_{VLL} + C_{VLR}\right) + C_{\gamma R}^*\left(C_{VRR} + C_{VRL}\right)\right]$$

$$+ \frac{32e^2v^2}{m_i^2}\left(\log\frac{m_i^2}{m_f^2} - 3\right)\left(|C_{\gamma L}|^2 + |C_{\gamma R}|^2\right).$$

TABLE 3.1 Matching of SM EFT Wilson coefficients in leptonic LFV decays of leptons. Here $X = L$ or R.

	Class (a): $\ell_i \to 3\ell_j$	Class (b): $\ell_i \to \ell_j 2\ell_k$	Class (c): $\ell_i^\pm \to \ell_j^\mp \ell_k^\pm \ell_k^\pm$
C_{VLL}	$2\left[(2s_W^2 - 1)\left(C_{\ell H}^{(1)ji} + C_{\ell H}^{(3)ji}\right) + C_{\ell\ell}^{jikk}\right]$	$(2s_W^2 - 1)\left(C_{\ell H}^{(1)ji} + C_{\ell H}^{(3)ji}\right) + C_{\ell\ell}^{jijj}$	$2C_{\ell\ell}^{kikj}$
C_{VRR}	$2\left(2s_W^2 C_{eH}^{ji} + C_{ee}^{jijj}\right)$	$2s_W^2 C_{eH}^{ji} + C_{ee}^{jikk}$	$2C_{ee}^{kikj}$
C_{VLR}	$2s_W^2\left(C_{\ell H}^{(1)ji} + C_{\ell H}^{(3)ji}\right) + C_{\ell e}^{jijj}$	$2s_W^2\left(C_{\ell H}^{(1)ji} + C_{\ell H}^{(3)ji}\right) + C_{\ell e}^{jikk}$	$C_{\ell e}^{kikj}$
C_{VRL}	$(2s_W^2 - 1)C_{eH}^{ji} + C_{\ell e}^{jjji}$	$(2s_W^2 - 1)C_{eH}^{ji} + C_{\ell e}^{jkki}$	$C_{\ell e}^{kjki}$
C_{SLR}	$-2\left[(2s_W^2 - 1)C_{eH}^{ji} + C_{\ell e}^{jjji}\right]$	$-2C_{\ell e}^{jkki}$	$-2C_{\ell e}^{kjki}$
C_{SRL}	$-2\left[2s_W^2\left(C_{\ell H}^{(1)ji} + C_{\ell H}^{(3)ji}\right) + C_{\ell e}^{jijj}\right]$	$-2C_{\ell e}^{jikk}$	$-2C_{\ell e}^{kikj}$
C_{SXX}	0	0	0
C_{TX}	0	0	0
$C_{\gamma L}$	$\sqrt{2}C_\gamma^{ij*}$	$\sqrt{2}C_\gamma^{ij*}$	0
$C_{\gamma R}$	$\sqrt{2}C_\gamma^{ji}$	$\sqrt{2}C_\gamma^{ji}$	0

★ Employ bound states: μ conversion experiment



★ take low energy muons (~ 30 MeV) a stop them in a target A(Z,A-Z): muons cascade to atomic 1s state

★ Binding energy and orbit radius for muonic hydrogen-like state

$$E_b = -\frac{Z^2 m e^4}{8n^2} \sim \frac{Z^2 m}{n^2}$$

$$r = \frac{n^2}{Z\pi m e^2} \sim \frac{n^2}{Zm}$$

muonic atom is 200x stronger bound
radius is 200x smaller

★ The radial wave function for the hydrogen-like system: $R_{nl} \sim r^{\ell} Z^{3/2}$

overlap probability: $p \sim r^{2\ell} Z^3$

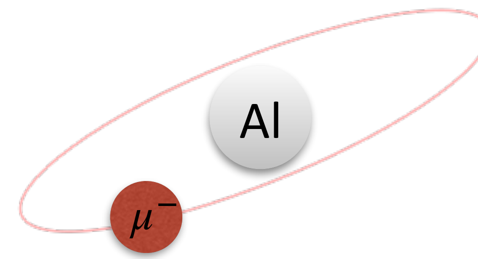
large overlap for an s-wave and high-Z nucleus

Measure $$R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$$ to probe NP

Fundamental physics with muons: flavor violation

- How effective is this approach compared to scattering/decays?
  - let's compute effective luminosity
  - recall that

$$L = \Phi \rho_T \ell = N \rho_T \frac{\ell}{t} = N \rho_T v$$



  - in this "experiment" probability density is given by the 1s wave function
  - … and we need to take into account the fact that muon decays
  - Then Luminosity = (density)(velocity)(flux of muons)(lifetime)

$$L_{\text{eff}} = |\psi(0)|^2 \times \alpha Z \times \Phi_\mu \times \tau_\mu = \frac{m_\mu^3 Z^4 \alpha^4}{\pi} \Phi_\mu \tau_\mu$$

  - For Al target (Z=13), flux of $\Phi_\mu = 10^{10}$ muons/sec and $\tau_\mu = 2\ \mu$sec

Bernstein, Czarnecki

$$L_{\text{eff}} = 10^{48} \text{cm}^{-2}\ \text{sec}^{-1}$$

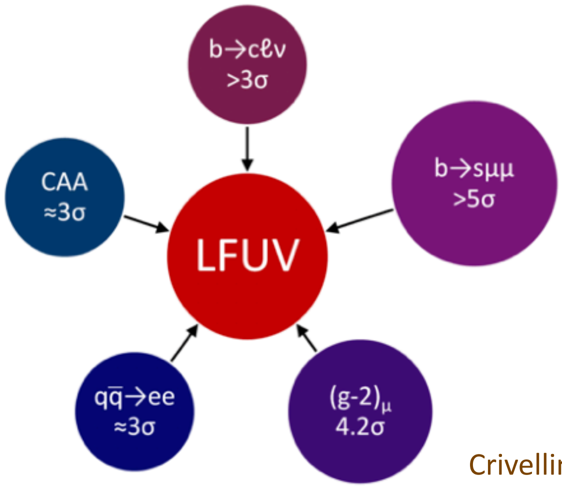
Recall the luminosity of the modern flavor experiment $L \sim 10^{34} - 10^{35}$ cm^{-2}sec^{-1}!

Muons and recent experimental anomalies

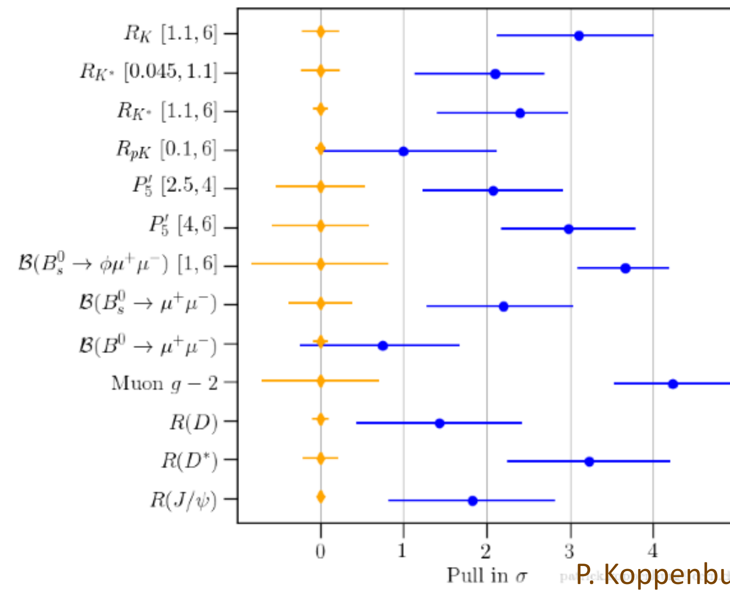
Are there any indications that NP might affect processes with muons?

Muons and recent experimental anomalies

★ Many experimental anomalies involve interactions with muons and taus

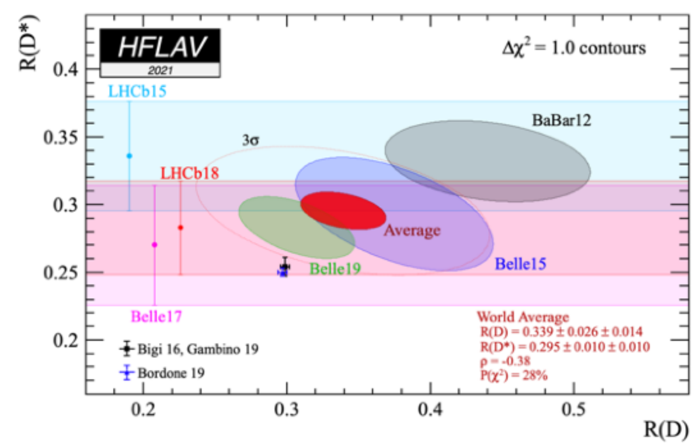


Crivellin, Hoferichter



P. Koppenburg

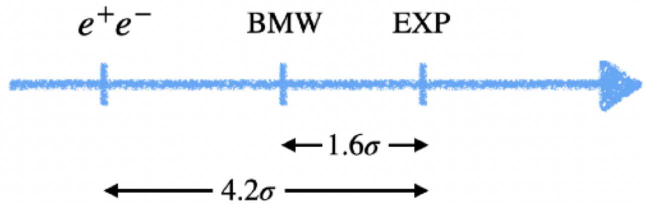
- other lepton-flavor conserving processes
  - magnetic properties: muon g-2
    - currently a discrepancy theory/exp
  - electric properties: muon EDM
    - probes CP-violation in leptons
  - muonic hydrogen
    - proton size/QED/New Physics



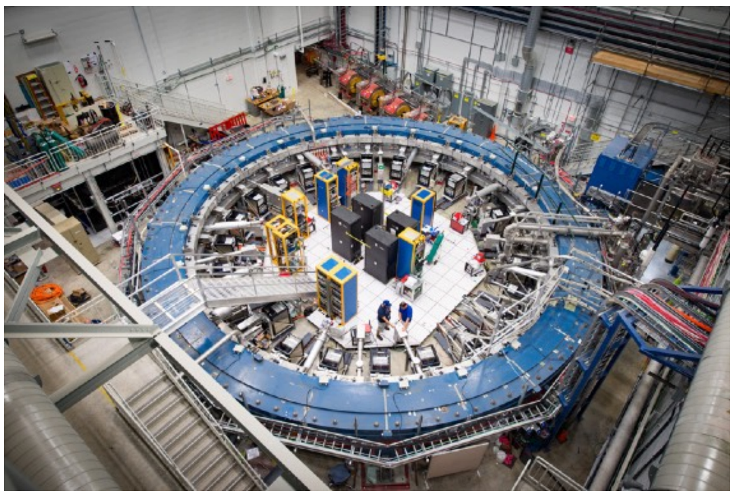
Muons and recent experimental anomalies

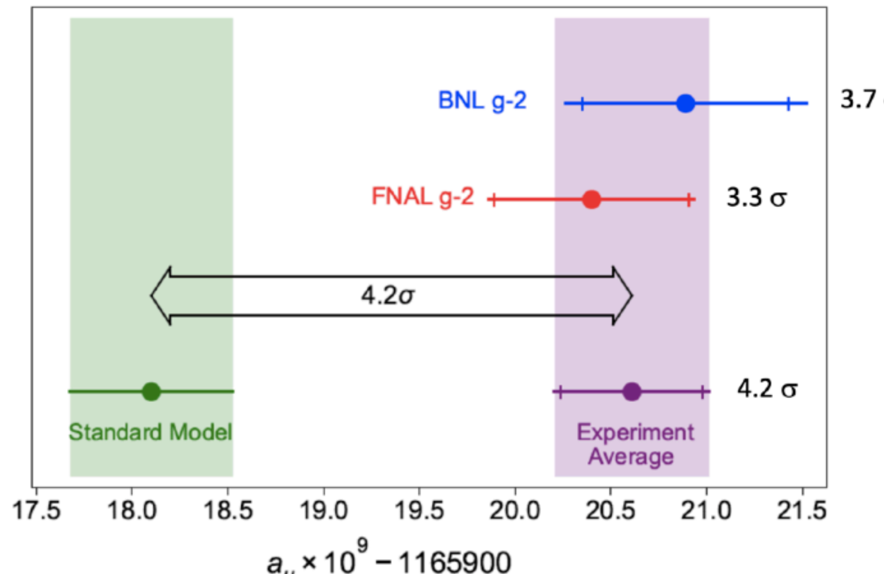
★ Muon's magnetic properties (g-2): $a_\mu = (g-2)/2$ with $\vec{\mu} = g\dfrac{e}{2m}\vec{s}$

discrepancy



Di Luzio, et al.





FNAL (g-2): $a_\mu(\mathrm{Exp}) = 116592061(41) \times 10^{-11}$

$a_\mu(\mathrm{Theory}) = 116591810(43) \times 10^{-11}$

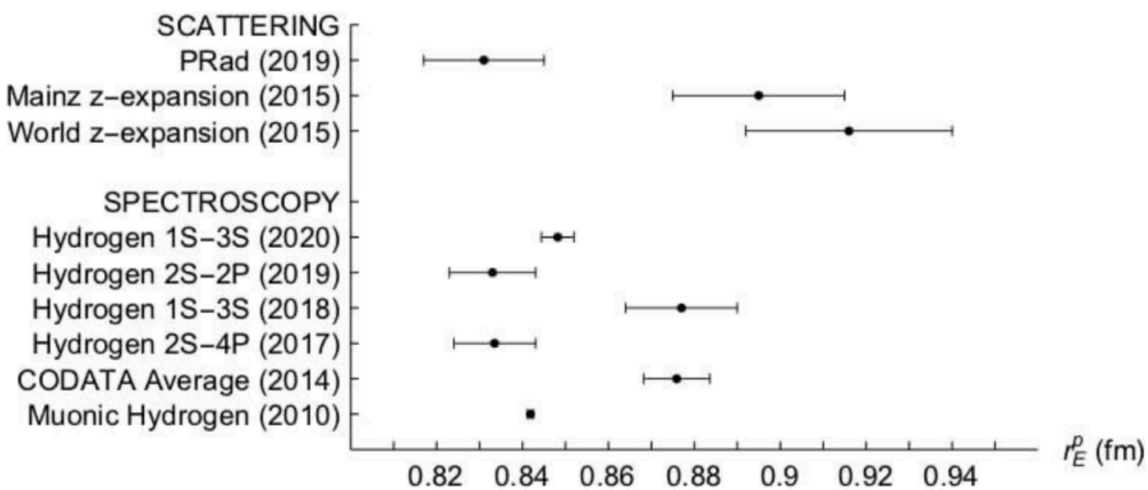
$a_\mu(\mathrm{BMW}) = 116591954(55) \times 10^{-11}$

Are there possible New Physics particles that are responsible for this difference?

Muons and recent experimental anomalies

★ Proton's radius from muonic hydrogen: possible New Physics?

★ Level splittings (e.g. Lamb shift) are sensitive to the charge radius of the proton



★ They are also sensitive to QED radiative corrections

★ Are there possible light New Physics particles that are responsible for this difference?

Barger et al, PRL 106 (2011) 153001

Remove proton radius issue from the problem: atomic physics with muonium?

Conclusions and things to take home

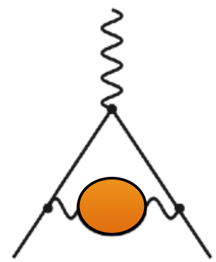
- There is no indication from high energy studies where the NP show up
  - this makes indirect searches the most valuable source of information
- Muons are ideal tools to probe fundamental physics
  - flavor-conserving quantities (g-2, EDM)
  - flavor-changing neutral current decays
  - flavor oscillations (muonium-antimuonium conversion)
  - muon transitions already probe the LHC energy domain and can do better!
- New experimental facilities are needed


EXTRA!

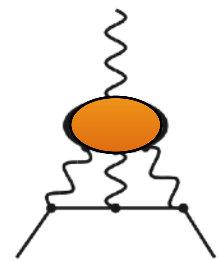
Theoretical issues with g-2

- Independent lattice computations of HVP
- Data-driven estimates of hadronic vacuum polarization (HVP)
  - discrepancy between KLOE and BaBar data used in HVP

Leading order



Vacuum polarization

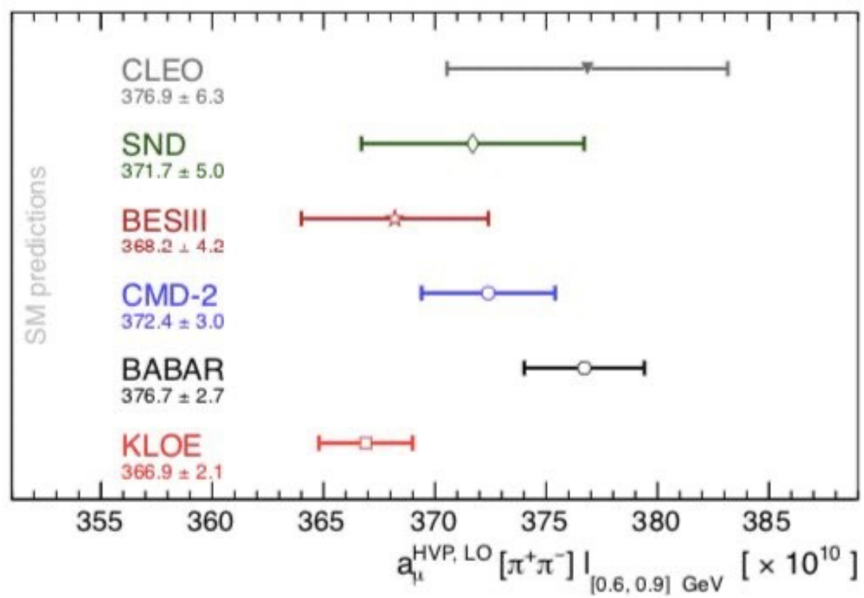


Light-by-light scattering

$$a_\mu^{hvp} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

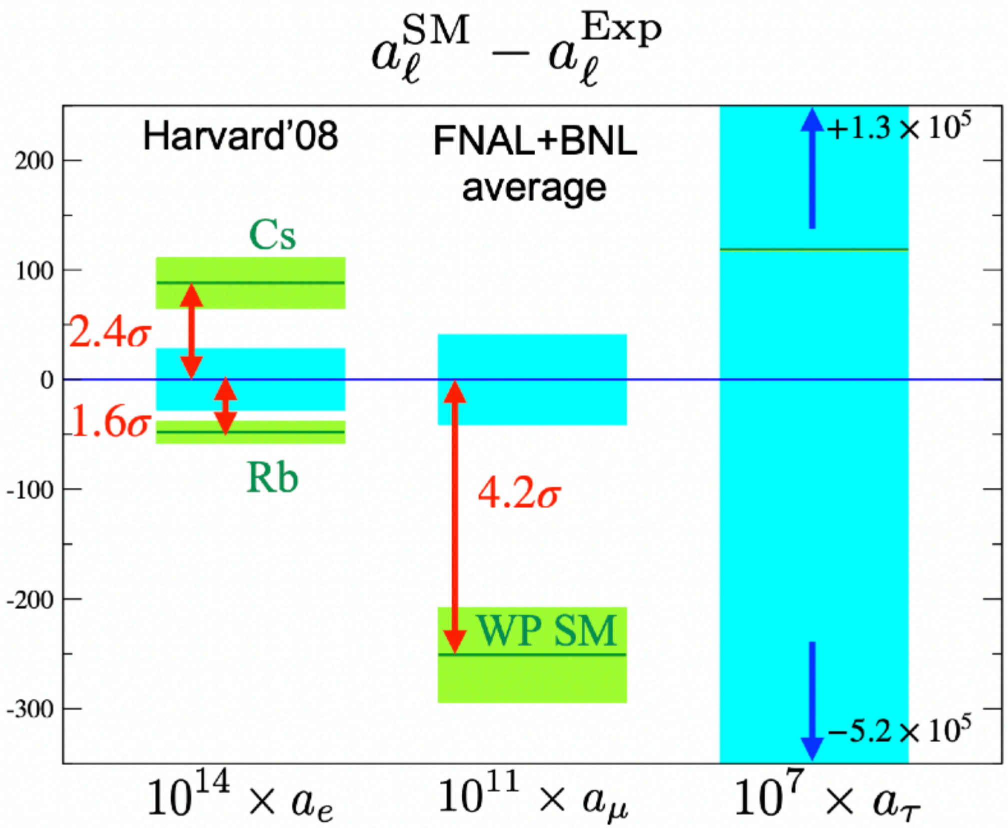
$$\frac{1}{12\pi} R(s) = \frac{1}{12\pi} \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)m_\mu^2}{x^2 m_\mu^2 + (1-x)s}$$



  - need radiative return Belle II data to eliminate the discrepancy
  - τ-decay data is not currently used: Belle II + lattice?

Summary of leptonic anomalous magnetic moments



Sensitivity to heavy new physics:

$$a_\ell^{\rm NP} \sim \frac{m_\ell^2}{\Lambda^2}$$

$$(m_\mu/m_e)^2 \sim 4\times 10^4$$

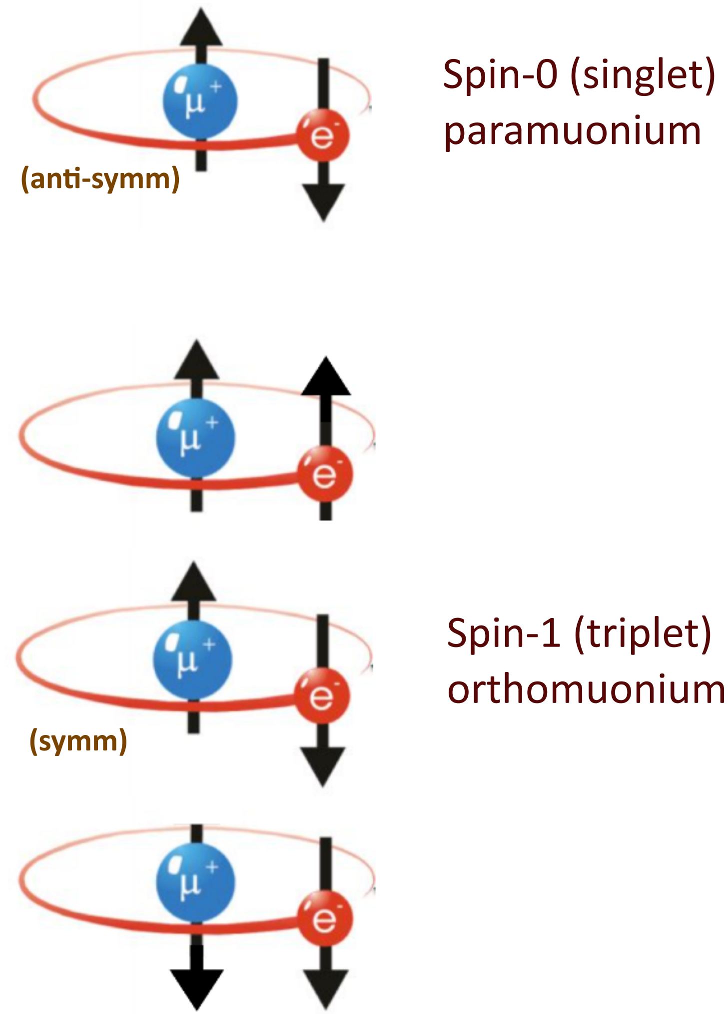
Cs: α from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: α from Paris group [Morel et al, Nature 588, 61–65(2020)]

A. El-Khadra (talk at LP21)

Muonium: just like hydrogen, but simpler!

- Muonium: a bound state of μ^+ and e^-
  - $(\mu^+\mu^-)$ bound state is a *true muonium*
- Muonium lifetime $\tau_{M_\mu} = 2.2\ \mu s$
  - main decay mode: $M_\mu \to e^+e^-\bar{\nu}_\mu\nu_e$
  - annihilation: $M_\mu \to \bar{\nu}_\mu\nu_e$
- Muonium's been around since 1960's
  - used in chemistry
  - QED bound state physics, etc.
  - New Physics searches (oscillations)



Hughes (1960)

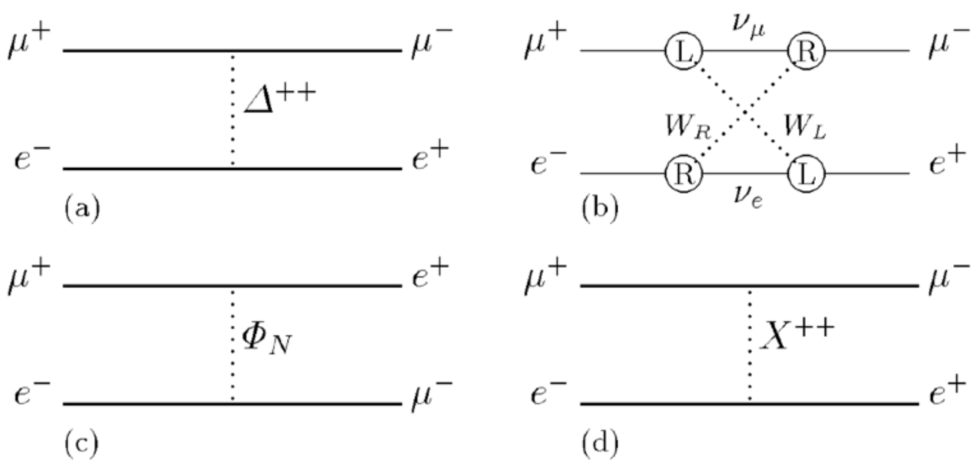
The masses of singlet and triplet are almost the same!

Muonium oscillations: just like $B^0\bar{B}^0$ mixing, but simpler!

★ Lepton-flavor violating interactions can change $M_\mu \to \overline{M}_\mu$

Pontecorvo (1957)
Feinberg, Weinberg (1961)

- Such transition amplitudes are tiny in the Standard Model
  - … but there are plenty of New Physics models where it can happen



$$\sim (\bar{\mu}\Gamma e)\,(\bar{\mu}\Gamma e)$$

effective operator

  - theory: compute transition amplitudes for ALL New Physics models!
  - experiment: produce M_μ but see for decay products of $\overline{M}_\mu$

Effective Lagrangians and particular models

- Effective Lagrangian approach encompasses all models
  - lets look at an example of a model with a doubly charged Higgs Δ^{--}
  - this is common for the left-right models, etc.

$$\mathcal{L}_R = g_{\ell\ell} \overline{\ell}_R \ell^c \Delta + H.c.,$$

  - integrate out Δ^{--} to get

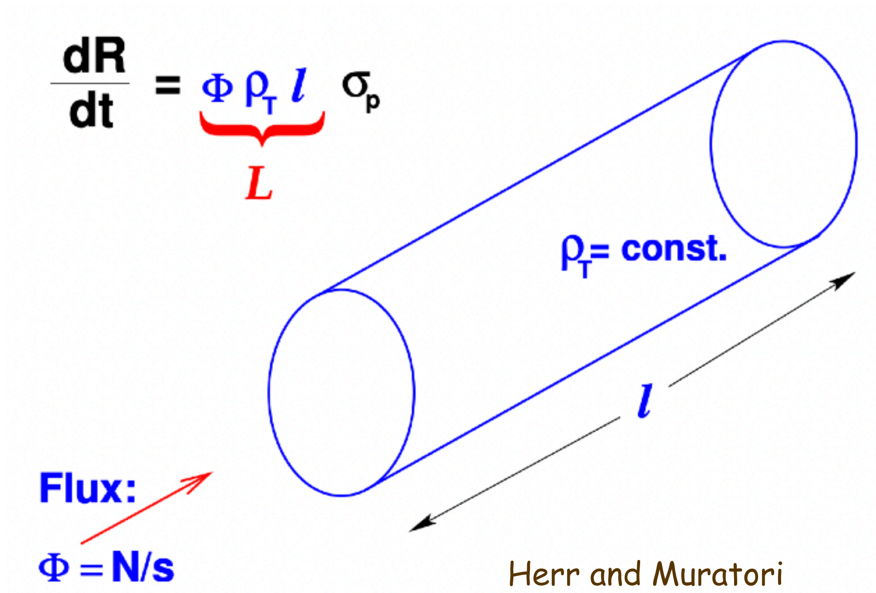
$$\mathcal{H}_\Delta = \frac{g_{ee} g_{\mu\mu}}{2 M_\Delta^2} \left(\overline{\mu}_R \gamma_\alpha e_R\right) \left(\overline{\mu}_R \gamma^\alpha e_R\right) + H.c.,$$

  - match to $\mathscr{L}_{\text{eff}}^{\Delta L=2}$ to see that $M_\Delta = \Lambda$ and

$$C_2^{\Delta L=2} = g_{ee} g_{\mu\mu}/2.$$

★ Need a lot of muons: high luminosity experiments

– Number of events/second



	Energy (GeV)	$\mathcal{L}$ $cm^{-2}s^{-1}$
SPS ($p\bar{p}$)	315x315	6 10^{30}
Tevatron ($p\bar{p}$)	1000x1000	50 10^{30}
HERA (e^+p)	30x920	40 10^{30}
LHC (pp)	7000x7000	10000 10^{30}
LEP (e^+e^-)	105x105	100 10^{30}
PEP (e^+e^-)	9x3	3000 10^{30}
KEKB (e^+e^-)	8x3.5	10000 10^{30}
eRHIC		10^{33}-10^{35}

– … or another way

$$L = \Phi \rho_T \ell = N \rho_T \frac{\ell}{t} = N \rho_T v$$

Bound states: muon conversion

★ Examples of nuclei suitable for muon conversion experiments

Nucleus	$R_{\mu e}(Z)$ / $R_{\mu e}(Al)$	Bound lifetime	Atomic Bind. Energy(1s)	Conversion Electron Energy	Prob decay >700 ns
Al(13,27)	1.0	.88 μs	0.47 MeV	104.97 MeV	0.45
Ti(22,~48)	1.7	.328 μs	1.36 MeV	104.18 MeV	0.16
Au(79,~197)	~0.8-1.5	.0726 μs	10.08 MeV	95.56 MeV	negligible

J. Miller, 2006

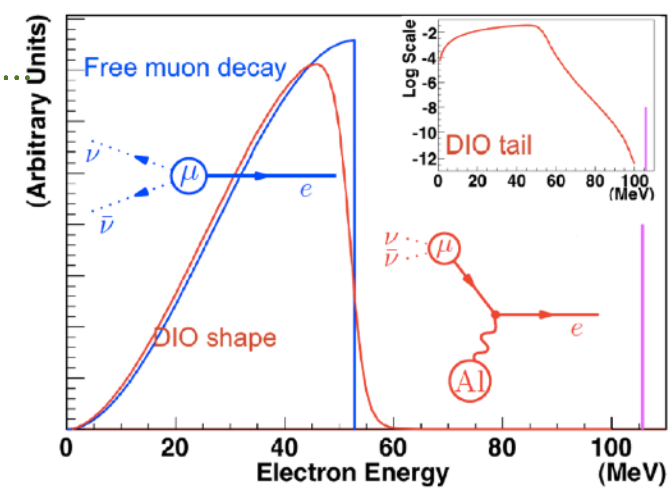
★ The experiment is tricky

✓ Muon conversion gives monoenergetic electrons...
✓ ... yet, there are other sources of electrons as well!

$$\begin{cases} \mu^- \to e^- + \bar{\nu}_e + \nu_\mu & \text{- decay (40\%)} \\ \mu^- + Al \to X + \nu_\mu & \text{- capture (60\%)} \\ \mu^- + Al \to e^- + Al & \text{- conversion} \end{cases}$$

SINDRUM II (PSI), 2006 : $R_{\mu e} < 7 \times 10^{-13}$

M2e goal : $R_{\mu e} < \text{a few} \times 10^{-17}$



Czarnecki, Marciano, Tormo