Rare Muon Processes in Radiative Neutrino Mass Models

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Project X Physics Study

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Based on:

Two–loop neutrino mass generation through leptoquarks

Nucl. Phys. B 841, 130 (2010) (with K. S. Babu)

Radiative neutrino mass generation through vectorlike quarks

Phys. Rev. D 85, 073005 (2012) (with K. S. Babu)

Outline

- Motivation
- Two–loop neutrino mass model via leptoquarks
  - Predictions
    θ_{13}, mass hierarchy
  - Low-energy phenomena
    $\mu \to e\gamma$, $\mu \to 3e$, $\mu - e$ conversion in nuclei, muon $g - 2$
- Radiative neutrino mass model with vectorlike quarks
- Conclusions

What we know about neutrinos

- The best fit of neutrino mass and mixing parameters:

Quantity	Value
$\Delta m^2_{21}(\mathrm{eV}^2)$	$(7.59 \pm 0.21) \times 10^{-5}$
$\Delta m^2_{31}(\mathrm{eV}^2)$	$(2.53^{+0.13}_{-0.08}) \times 10^{-3}$ (NH) $-(2.4^{+0.1}_{-0.07}) \times 10^{-3}$ (IH)
$\sin^2\theta_{12}$	$0.320^{+0.015}_{-0.017}$
$\sin^2\theta_{23}$	$0.49^{+0.08}_{-0.05}$ $0.53^{+0.08}_{-0.07}$
$\sin^2\theta_{13}$	$0.026^{+0.003}_{-0.004}$ $0.027^{+0.003}_{-0.004}$

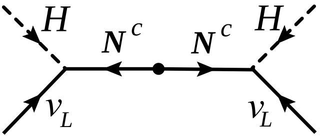
Forero, Tórtola, Valle (2012)

- The origin of neutrino mass, type of hierarchy, the *CP*-violating parameter, and whether neutrinos are Dirac or Majorana are still unknown.

The origin of neutrino mass (seesaw mechanism)

- Adding right-handed neutrino N^c which transforms as singlet under $SU(2)_L$,

$$\mathcal{L} = f_\nu \,(L \cdot H)\, N^c + \tfrac{1}{2} M_R N^c N^c$$



- Integrating out the N^c, $\Delta L = 2$ operator is induced:

$$\mathcal{L}_{\rm eff} = -\frac{f_\nu^2}{2} \frac{(L \cdot H)\,(L \cdot H)}{M_R}$$

- Once H acquires VEV, neutrino mass is induced:

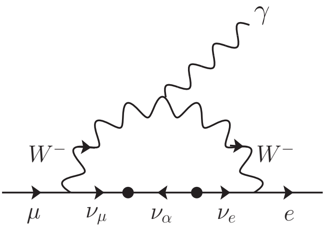
$$m_\nu \simeq f_\nu^2 \frac{v^2}{M_R}$$

- For $f_\nu v \simeq 100$ GeV, $M_R \simeq 10^{14}$ GeV.

Minkowski (1977)
Yanagida (1979)
Gell-Mann, Ramond, Slansky (1980)
Mohapatra & Senjanovic (1980)

Lepton flavor violation

- Neutrino mass and mixing can generate rare muon decay.
- The induced branching ratio is extremely small, i.e. $\mathrm{BR}(\mu \to e\gamma) \sim 10^{-50}$.
- The current limit: $\mathrm{BR}(\mu \to e\gamma) < 2.4 \times 10^{-12}$.
- Rare muon decay could probe new physics at TeV scale.



$$\mathcal{M} \sim \frac{m_\nu^2}{M_W^2}$$

Cheng & Li (1977)
Petcov (1977)
Marciano & Sanda (1977)
Shrock & Lee (1977)

Radiative neutrino mass generation

- One alternative is radiative neutrino mass generation, in which neutrino mass is absent at tree level but arises at loop level.
- The smallness of neutrino mass is caused by loop and chiral suppressions.
- The new physics scale could be at TeV.
- Some LFV processes may be observable.

$\Delta L = 2$ operators

$$
\begin{aligned}
\mathcal{O}_1 &= L^i L^j H^k H^l \epsilon_{ik}\epsilon_{jl} \\
\mathcal{O}_2 &= L^i L^j L^k e^c H^l \epsilon_{ij}\epsilon_{kl} \\
\mathcal{O}_3 &= L^i L^j Q^k d^c H^l \epsilon_{ij}\epsilon_{kl}, \quad L^i L^j Q^k d^c H^l \epsilon_{ik}\epsilon_{jl}\} \\
\mathcal{O}_4 &= \{L^i L^j \bar{Q}_i \bar{u^c} H^k \epsilon_{jk}, \quad L^i L^j \bar{Q}_k \bar{u^c} H^k \epsilon_{ij}\} \\
\mathcal{O}_5 &= L^i L^j Q^k d^c H^l H^m \bar{H}_i \epsilon_{jl}\epsilon_{km} \\
\mathcal{O}_6 &= L^i L^j \bar{Q}_k \bar{u^c} H^l H^k \bar{H}_i \epsilon_{jl} \\
\mathcal{O}_7 &= L^i Q^j \bar{e^c} \bar{Q}_k H^k H^l H^m \epsilon_{il}\epsilon_{jm} \\
\mathcal{O}_8 &= L^i \bar{e^c} \bar{u^c} d^c H^j \epsilon_{ij} \\
\mathcal{O}_9 &= L^i L^j L^k e^c L^l e^c \epsilon_{ij}\epsilon_{kl}
\end{aligned}
$$

Babu & Leung (2001)

de Gouvea & Jenkins (2008)

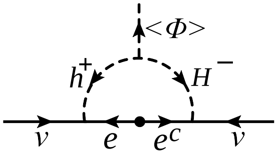
Operator $\mathcal{O}_2$

- Introducing a singly charged scalar and extra scalar doublet,

$$\mathcal{L} = f_{ij} L_i^a L_j^b h^+ \epsilon_{ab} + \mu H^a \Phi^b h^- \epsilon_{ab} + \text{h.c.}$$

Zee (1980)

- Neutrino mass arises at one-loop.



- The minimal version of this model in which only one Higgs doublet couples to fermions yields

$$m_\nu = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix}, \quad m_{ij} \simeq \frac{f_{ij}}{16\pi^2} \frac{(m_i^2 - m_j^2)}{\Lambda}$$

It requires $\theta_{12} \simeq \pi/4 \rightarrow$ ruled out by neutrino data.

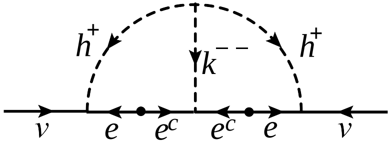
Koide (2001)
Frampton *et al.* (2002)
He (2004)

Operator $\mathcal{O}_9$

- Introducing singly charged and doubly charged scalars to break lepton number

$$\mathcal{L} = f_{ij} L_i^a L_j^b h^+ \epsilon_{ab} + g_{ij} e_i^c e_j^c k^{--} + \mu h^+ h^+ k^{--} + \text{h.c.}$$

Zee (1985); Babu (1988)



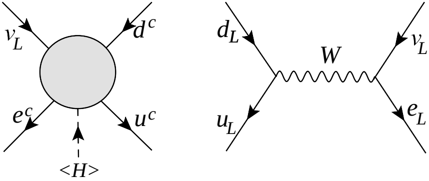
$$(m_\nu)_{\text{largest}} \simeq \frac{f^2 g}{(16\pi^2)^2} \frac{m_\tau^2}{\Lambda}$$

- One of the neutrinos is (nearly) massless.
- $\Lambda \sim 1$ TeV for $f \sim g \sim 0.1$.
- Fit the current data.

  Babu & Macesanu (2002); M. Nebot *et al.* (2008)

Operator $\mathcal{O}_8$

- Operator $\mathcal{O}_8 = L_i H_j d^c \bar{u^c} \bar{e^c} \epsilon_{ij}$ induces neutrino mass at two–loop:



$$m_\nu \sim \frac{m_\tau m_b m_t v}{(16\pi^2)^2 \Lambda^3}$$

- The new scale is at TeV.
- Leptoquarks are needed.

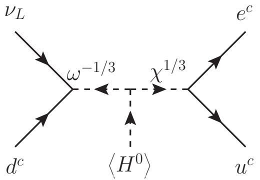
Model

- Introducing new interactions under $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group:

$$\mathcal{L} = Y_{ij} L_i^\alpha d_j^c \Omega^\beta \epsilon_{\alpha\beta} + F_{ij} e_i^c u_i^c \chi^{-1/3} + \mu \Omega^\dagger H \chi^{-1/3} + \text{h.c.}$$

$$\Omega \equiv \begin{pmatrix} \omega^{2/3} \\ \omega^{-1/3} \end{pmatrix} \ (3, 2, 1/6); \quad \chi^{-1/3} \sim (3, 1, -1/3)$$

- The simultaneous presence of these three terms will break lepton number.

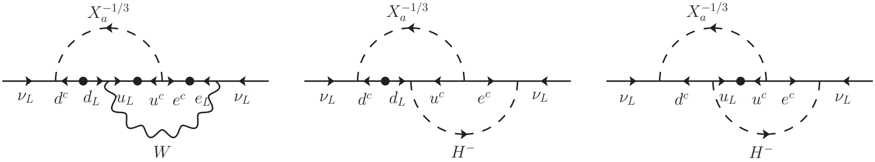


- The μ parameter will cause mixing between $\chi^{-1/3}$ and $\omega^{-1/3}$,

$$\begin{pmatrix} m_\omega^2 & \mu v \\ \mu v & m_\chi^2 \end{pmatrix} \Rightarrow \sin 2\theta = \frac{2\mu v}{M_2^2 - M_1^2}$$

$$M_{1,2}^2 = \frac{1}{2}\left[m_\omega^2 + m_\chi^2 \mp \sqrt{(m_\omega^2 - m_\chi^2)^2 + 4\mu^2 v^2}\right]$$

Neutrino mass model



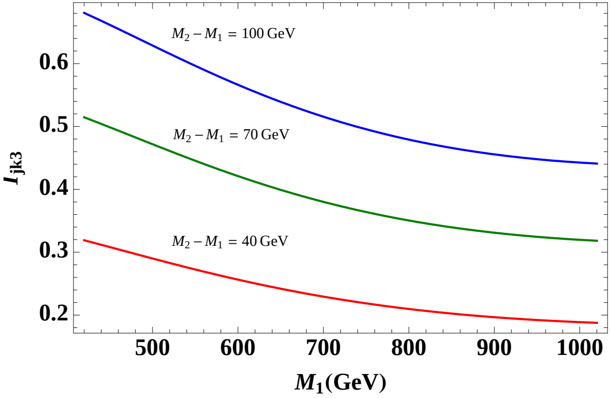
$$(M_\nu)_{ij} \quad = \quad \hat{m}_0 Y_{ik}(D_d)_k(V^T)_{kl}(D_u)_l(F^\dagger)_{lj}(D_\ell)_j I_{jkl} + \text{transpose},$$

$$\hat{m}_0 \quad = \quad \left(\frac{3g^2 \sin 2\theta}{(16\pi^2)^2}\right)\left(\frac{m_t m_b m_\tau}{M_1^2}\right)$$

$$D_u \quad = \quad \text{diag.}\left[\frac{m_u}{m_t}, \frac{m_c}{m_t}, 1\right], \;\; D_d = \text{diag.}\left[\frac{m_d}{m_b}, \frac{m_s}{m_b}, 1\right], \;\; D_\ell = \text{diag.}\left[\frac{m_e}{m_\tau}, \frac{m_\mu}{m_\tau}, 1\right]$$

Neutrino mass model

$$I_{jkl} = \sum_{a=1}^{2} (-1)^a \frac{M_1^2}{M_a^2 - m_{d_k}^2} \int_0^1 dx \int_0^\infty dt \; t \left(1 + \frac{t}{4m_W^2}\right) \frac{1}{t + M_W^2} \frac{1}{t + m_{e_j}^2}$$
$$\times \ln \left[\frac{x(1-x)t + x m_{u_l}^2 + (1-x) M_a^2}{x(1-x)t + x m_{u_l}^2 + (1-x) m_{d_k}^2} \right]$$



Neutrino mass matrix

$$M_\nu \simeq m_0 \begin{pmatrix} 0 & \frac{1}{2}\frac{m_\mu}{m_\tau}xy & \frac{1}{2}y \\ \frac{1}{2}\frac{m_\mu}{m_\tau}xy & \frac{m_\mu}{m_\tau}xz & \frac{1}{2}z + \frac{1}{2}\frac{m_\mu}{m_\tau}x \\ \frac{1}{2}y & \frac{1}{2}z + \frac{1}{2}\frac{m_\mu}{m_\tau}x & 1 + w \end{pmatrix}$$

$$x \equiv \frac{F^*_{23}}{F^*_{33}}, \quad y \equiv \frac{Y_{13}}{Y_{33}}, \quad z \equiv \frac{Y_{23}}{Y_{33}}; \quad w \equiv \frac{F^*_{32}}{F^*_{33}}\frac{Y_{32}}{Y_{33}}\left(\frac{m_c}{m_t}\right)\left(\frac{m_s}{m_b}\right)\frac{I_{jk2}}{I_{jk3}}$$

$$m_0 = 2\ \hat{m}_0 F^*_{33} Y_{33} I_{jk3}; \quad (M_\nu)_{11} \simeq y\frac{F^*_{13}}{F^*_{33}}\frac{m_e}{m_\tau}m_0$$

- This mass matrix has normal hierarchy structure.
- The (1,1) entry is highly suppressed, i.e. $\ll 0.01$ eV.
- w may be significant for $M_{LQ} < 800$ GeV.

Predictions for $w \ll 1$

- For $w \ll 1$,

$$M_\nu \simeq m_0 \begin{pmatrix} 0 & \frac{1}{2}\frac{m_\mu}{m_\tau}xy & \frac{1}{2}y \\ \frac{1}{2}\frac{m_\mu}{m_\tau}xy & \frac{m_\mu}{m_\tau}xz & \frac{1}{2}z + \frac{1}{2}\frac{m_\mu}{m_\tau}x \\ \frac{1}{2}y & \frac{1}{2}z + \frac{1}{2}\frac{m_\mu}{m_\tau}x & 1 \end{pmatrix}$$

- $\det M_\nu = 0$, together with $(M_\nu)_{11} \simeq 0$,

$$m_1 \simeq 0, \;\; \alpha \simeq 0, \;\; \beta \simeq 2\delta + \pi$$

$$\tan^2\theta_{13} \simeq \sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}}\sin^2\theta_{12}$$

$$\sin^2 2\theta_{13} \simeq 0.16$$

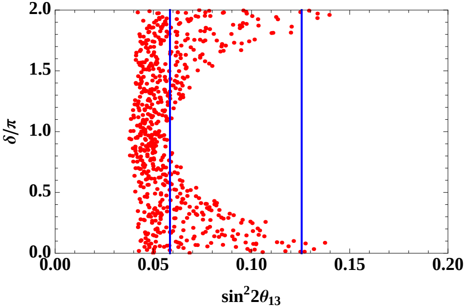
which differs by 4.2σ from Daya Bay result.

Predictions for $w \gg 1$

- For $w \gg 1$,

$$w \equiv \frac{F_{32}^*}{F_{33}^*}\frac{Y_{32}}{Y_{33}}\left(\frac{m_c}{m_t}\right)\left(\frac{m_s}{m_b}\right)\frac{I_{jk2}}{I_{jk3}} \gg 1 \quad \rightarrow \quad |F_{33}Y_{33}| \ll |F_{32}Y_{32}|$$

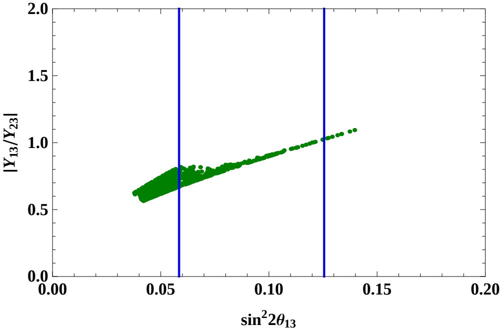
- This could generate $(M_\nu)_{13} \simeq (M_\nu)_{11} \simeq 0$.
  Glashow, Frampton, & Marfatia (2002)
  Xing (2002)



- The value of θ_{13} is consistent with current measurements (the blue lines correspond to 2σ allowed value from Daya Bay).

Predictions for $w \gg 1$

- Requiring $|F_{32} Y_{32}| \leq 1$ implies that LQ cannot be heavier than 800 GeV.

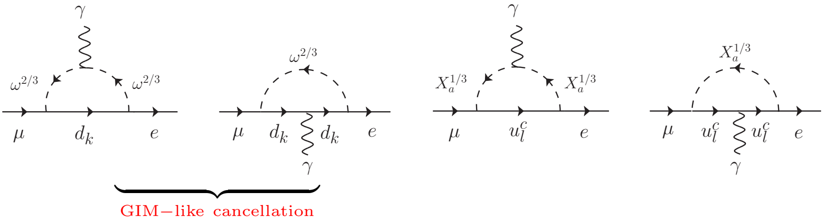


- The fit gives $|Y_{13}| \sim |Y_{23}|$.

- The smallest value of Y_{23} corresponding to $|F_{23}| \sim 1$ can be found from neutrino mass fit.

  $\rightarrow$ the rate of some LFV can be predicted.

$\mu \to e\gamma$

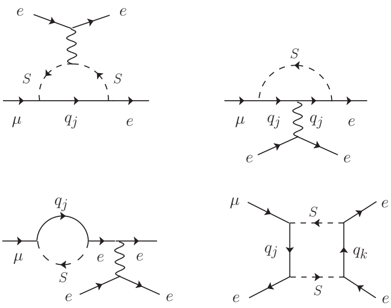


$$\left|F(x_{d_i})\frac{Y_{1i}Y_{2i}}{M_\omega^2}\right|^2 + \left|H(x_{u_i})\frac{F_{1i}F_{2i}}{M_\chi^2}\right|^2 < \frac{7.8\times 10^{-20}}{\text{GeV}^4}; \quad x_{d_i} \equiv \frac{m_{d_i}^2}{M_\omega^2}; \quad x_{u_i} \equiv \frac{m_{u_i}^2}{M_\chi^2}$$

$$F(x) = -\frac{x}{12}\frac{(1-x)(5+x)+2(2x+1)\ln x}{(1-x)^4}; \quad H(x) = -\frac{1}{12}\frac{(1-x)(5x+1)+2x(2+x)\ln x}{(1-x)^4}$$

- For $\omega^{2/3}$ mediated process, the two graphs in the limit $m_{d_k} \ll M_{LQ}$ cancel out because $Q_{\omega^{2/3}} = -2Q_d$.
- From neutrino mass fitting, only Y_{13}, Y_{23} are constrained but not F_{13}.
  $\to$ This model cannot predict the lowest rate of $\mu \to e\gamma$.

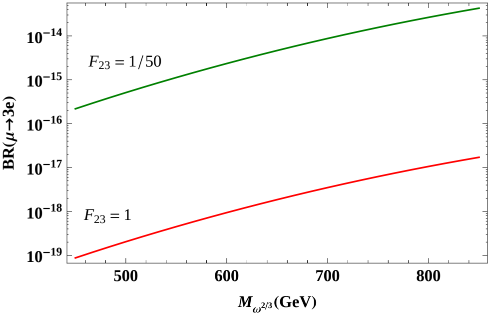
$\mu \to 3e$



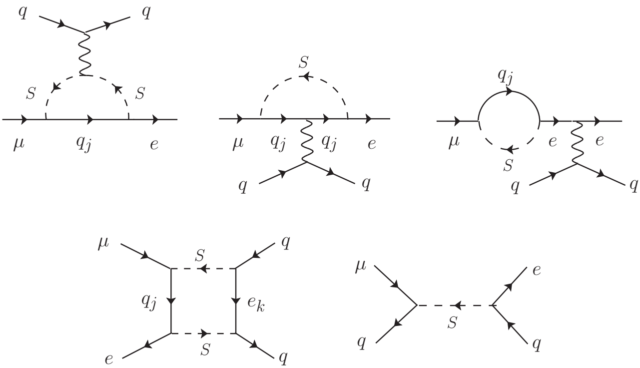
$S \equiv \omega^{2/3}(\chi^{1/3})$ if $q_j \equiv d_j(u_j^c)$

The wavy lines indicate γ and Z.

Since now photon is off-shell, the rate can be predicted.



$\mu - e$ conversion in nuclei



- Consider a muon is "trapped" inside an atom.
- In standard muon decay: $\mu \to e\nu_\mu\overline{\nu}_e$.
- Due to inverse beta decay: $\mu + N(A, Z) \to \nu_\mu + N(A, Z-1)$.
- This model can generate $\mu + N(A, Z) \to e + N(A, Z)$,

$\mu - e$ conversion in nuclei

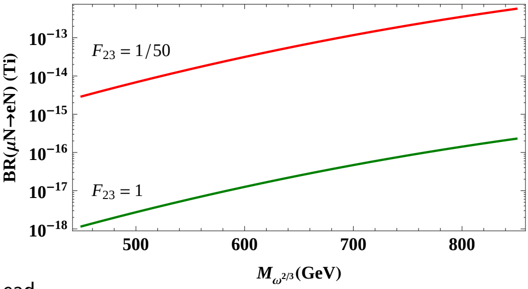
Element	BR	Constraint
$^{48}\mathrm{Ti}$	$< 4.3 \times 10^{-12}$	$\left\lvert \frac{a_j^L Y_{1j}^* Y_{2j}}{m_\omega^2} + \frac{a_j^R F_{1j} F_{2j}^*}{m_\chi^2} \right\rvert^2 + \left\lvert \frac{b_j^L Y_{1j}^* Y_{2j}}{m_\omega^2} + \frac{b_j^R F_{1j} F_{2j}^*}{m_\chi^2} \right\rvert^2 < \frac{5.2\times 10^{-16}}{\mathrm{GeV}^4}$
$^{208}\mathrm{Pb}$	$< 4.6 \times 10^{-11}$	$\left\lvert \frac{a_j^L Y_{1j}^* Y_{2j}}{m_\omega^2} + \frac{a_j^R F_{1j} F_{2j}^*}{m_\chi^2} \right\rvert^2 + \left\lvert \frac{b_j^L Y_{1j}^* Y_{2j}}{m_\omega^2} + \frac{b_j^R F_{1j} F_{2j}^*}{m_\chi^2} \right\rvert^2 < \frac{9.7\times 10^{-14}}{\mathrm{GeV}^4}$

$$
\begin{aligned}
a_j^L &= (2A - Z)\left[\frac{8\pi^2\delta_{1j}}{3} - \frac{(Y^\dagger Y)_{11}}{4}\right]; \quad a_j^R = 2Ze^2\tilde{h}_j; \quad b_j^L = 2Ze^2\tilde{g}_j \\
b_j^R &= (A + Z)\left[\frac{8\pi^2\delta_{1j}}{3} - \frac{(F^\dagger F)_{11}}{4}\right] + 2Ze^2 F_4(x_{u_j}) \\
&\quad + 8\sqrt{2}G_F m_\chi^2 F_5(x_t)\left(\tfrac{3}{4}A - Z\sin^2\theta_W\right)\delta_{3j} \\
\tilde{g}_{12} &= \tfrac{1}{27}\left(2 - 3\ln\frac{m_\mu^2}{m_\omega^2}\right); \quad \tilde{h}_{1,2} = \tfrac{1}{54}\left(5 - 12\ln\frac{m_\mu^2}{m_\chi^2}\right); \quad F_5(x) = -\frac{x}{2}\,\frac{1 - x + \ln x}{(1-x)^2} \\
g_3 &= \frac{-4 + 9x - 5x^3 + 2(2x^3 + 3x - 2)\ln x}{36(1-x)^4} \\
h_3 &= \frac{(x-1)(10 + x(x-17)) + 2(x^3 + 6x - 4)\ln x}{36(1-x)^4}
\end{aligned}
$$

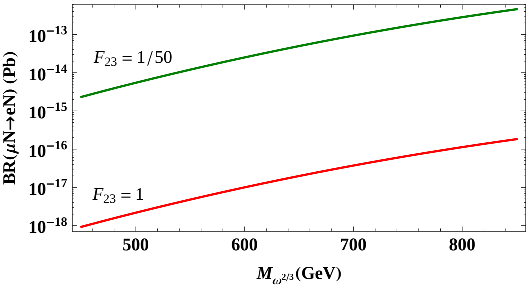
Kuno & Okada (1999); Chiang *et al.* (1993); Czarnecki & Marciano (1998)

$\mu - e$ conversion in nuclei (predictions)

- Titanium



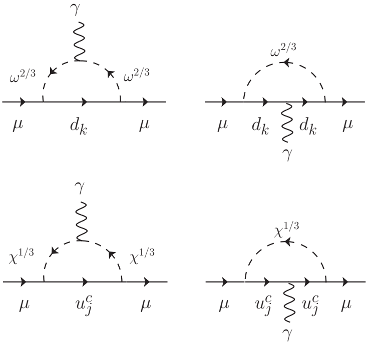
- Lead



The predictions of this model are within the future sensitivity of Project X and COMET.

Muon $g-2$

- $a_\mu \equiv (g-2)_\mu/2$.



- Since for $\omega^{2/3}$ mediating process there is cancellation, the dominant contribution comes from $\chi^{1/3}$ mediating process. For $|F_{2j}| \sim 1$,

$$(\Delta a_\mu)^{\text{NP}} = 25 \times 10^{-11} \left(\frac{400 \text{ GeV}}{M_\chi} \right)^2$$

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 249(87) \times 10^{-11}$$

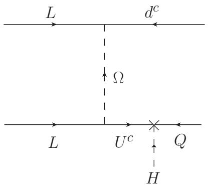
Aoyama, Hayakawa, Kinoshita, & Nio (2012)

Neutrino mass model with vectorlike quarks

- Introducing LQ doublet and charge 2/3 iso-singlet vectorlike quark:

$$\Omega \equiv \begin{pmatrix} \omega^{2/3} \\ \omega^{-1/3} \end{pmatrix}, \quad U^c, \overline{U^c}$$

$$\mathcal{L} \quad = \quad g_{ij} L_i d^c_j \Omega + h_i L_i \overline{U^c} \tilde{\Omega} - f_i Q_i U^c H + \lambda |\Omega^a H^b \epsilon_{ab}|^2 + \mathrm{h.c.}$$



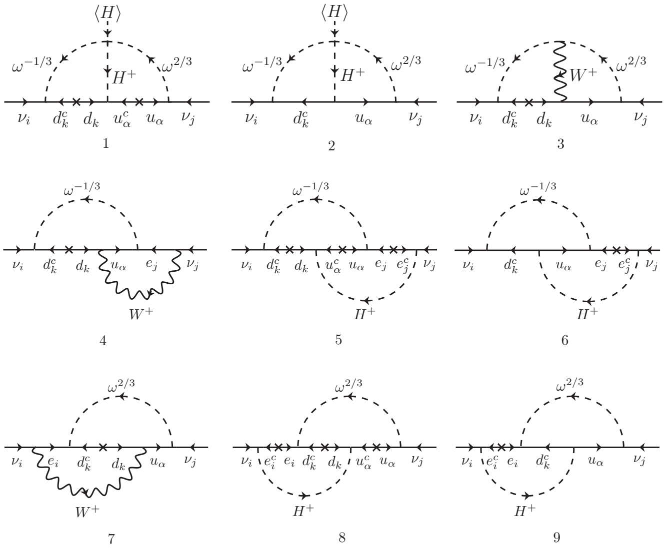
- The $\Delta L = 2$ operator is generated:

$$L^i L^j Q^k d^c H^l \epsilon_{ij} \epsilon_{kl}$$

- This scenario can be embedded into MSSM + 10 + $\overline{10}$ of SU(5).

Moroi & Okada (1992); Babu, Gogoladze, & Kolda (2004); Babu *et al.* (2008)

The two-loop diagrams (in R_ξ gauge):



Neutrino Mass Matrix

$$M_\nu = \begin{pmatrix} M_{11} & \frac{1}{2}\frac{h_1}{h_2}M_{22} + \frac{1}{2}\frac{h_2}{h_1}M_{11} & \frac{1}{2}\frac{h_1}{h_3}M_{33} + \frac{1}{2}\frac{h_3}{h_1}M_{11} \\ \frac{1}{2}\frac{h_1}{h_2}M_{22} + \frac{1}{2}\frac{h_2}{h_1}M_{11} & M_{22} & \frac{1}{2}\frac{h_2}{h_3}M_{33} + \frac{1}{2}\frac{h_3}{h_2}M_{22} \\ \frac{1}{2}\frac{h_1}{h_3}M_{33} + \frac{1}{2}\frac{h_3}{h_1}M_{11} & \frac{1}{2}\frac{h_2}{h_3}M_{33} + \frac{1}{2}\frac{h_3}{h_2}M_{22} & M_{33} \end{pmatrix}$$

$$M_{ii} = h_i \sum_{k=1}^{3} g_{ik} F_k; \qquad F_k = m_0 \sum_{\alpha=1}^{4} V^*_{\alpha 4} V_{\alpha k} (D_d)_k I_{\alpha k}$$

$$D_d \equiv \mathrm{diag}\left(\frac{m_d}{m_b}, \frac{m_s}{m_b}, 1\right); \qquad m_0 = \frac{3}{2}\frac{g^2 m_b}{(16\pi^2)^2}$$

- det $M_\nu = 0$
  $\longrightarrow$ one of the neutrinos is massless
- Both normal and inverted hierarchies can be accommodated here

Fitting

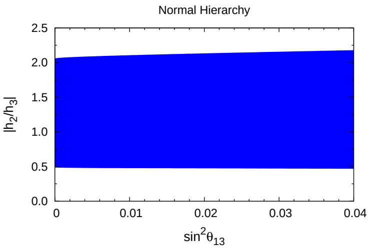
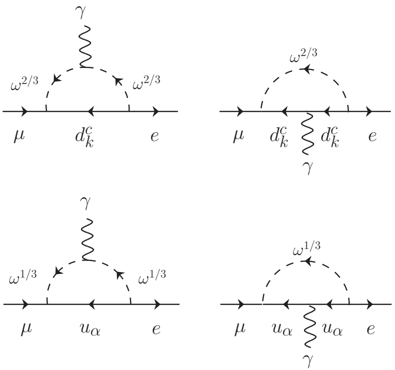


Figure: The allowed value of $\frac{h_2}{h_3}$ in NH (left panel) and IH (right panel).

- NH predicts $\text{BR}\ (\tau \to e\gamma) \simeq 5 \times \text{BR}\ (\mu \to e\gamma)$.
- The current upper limit, $\text{BR}\ (\mu \to e\gamma)_{\text{exp}} \leq 2.4 \times 10^{-12}$.
- If $\text{BR}\ (\tau \to e\gamma)$ is found to be near its current upper limit (10^{-8}), the NH scenario is ruled out.

$\mu \to e\gamma$



$$\mathrm{BR}(\mu \to e\gamma) = \frac{27\alpha}{16\pi G_F^2} \left| F(x_{d_i}) \frac{g_{1i}^* g_{2i}}{M_2^2} + H(x_{u_\alpha}) V_{\alpha 4}^* V_{\alpha 4} \frac{h_1^* h_2}{M_1^2} \right|^2$$

$$F(x) = -\frac{x}{12} \frac{(1-x)(5+x) + 2(2x+1)\ln x}{(1-x)^4}$$

$$H(x) = -\frac{1}{12} \frac{(1-x)(5x+1) + 2x(2+x)\ln x}{(1-x)^4}$$

$$x_{d_i} \equiv \frac{m_{d_i}^2}{M_2^2} \qquad x_{u_\alpha} \equiv \frac{m_{u_\alpha}^2}{M_1^2}$$

Conclusions

- In model with leptoquarks, the value of θ_{13} is predicted to be consistent with the current measurements.
- This model also predicts that the neutrino mass hierarchy to be normal.
- Some muon processes such as $\mu - e$ conversion are predicted to be within the forthcoming experiments sensitivity, while $\mu \to e\gamma$ may be unobserved.

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Thank you