Skyrme suppression of $n - \bar{n}$ oscillations

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Turning a Lagrangian−level source of baryon−number violation (BNV) into a testable prediction involves both perturbative and non−perturbative physics. The perturbative piece comes simply from the coefficient of the BNV operator and depends on the dimension of the operator and the scale where it enters. The non−perturbative piece is the hadronic matrix element. For example,

$$|\langle 0|\mathcal{O}_{\Delta B=1}|p\rangle|^2, \quad |\langle \bar{n}|\mathcal{O}_{\Delta B=2}|n\rangle|$$

for proton decay ($\Delta B = 1$) or n-$\bar{n}$ oscillations ($\Delta B = 2$), respectively. While the non−perturbative piece can be estimated using naive dimensional analysis, a more precise prediction requires a model of hadron dynamics.

One interesting, successful way to describe baryons is as 'Skyrmions', topological configurations that are permitted in the chiral lagrangian once a stabilizing term is added [1, 2]. However, being topological objects, true Skyrmions are forbidden from decaying and are therefore not a useful laboratory for studying BNV processes. This can be remedied by removing the centre of the Skyrmion and replacing it with a volume of free, massless quarks, a description knows as the chiral bag model [3–5]. The chiral bag model was introduced to combine the benefits of the Skyrme model – exact chiral symmetry – with a more accurate UV description of QCD. While not the original motivation for the chiral bag model, by cutting off the Skyrmion at short distance, we destroy the topological protection of the configuration and decays will occur.

Compared with the original Skyrme model, chiral bag models have an additional parameter, the separation scale r_{bag} where the two descriptions are matched; outside of r_{bag} the proton is described as a twisted, nearly topological pion configuration, while inside the bag the proton consists of massless quarks with wave-functions determined by the value of the Skyrmion at the boundary, $\theta(r_{\text{bag}})$. Normalizing such that the configuration has unit baryon number and shifting all dimensionful quantities into an overall prefactor, $\theta(r_{\text{bag}}) \approx \pi$. The radius r_{bag} should be thought of as an artificial parameter, much like a factorization scale. Calculated to sufficient accuracy, physical quantities should be independent of this scale – an issue we will return to at the end[1].

The decay of chiral bag Skyrmions was the topic of Ref. [7]. There, we modelled the decay of the Skymion through a dilation $x' = x/\lambda$. At $\lambda = 1$ we have the original configuration, while at large λ the entire Skyrmion portion of the nucleon has been sucked into the bag and there is nothing preventing decay. For a given r_{bag}, the energy is minimized for a certain partition of energy between the bag fermions and the Skyrmion, so as we increase λ we encounter a potential barrier. The tunnelling rate through this barrier, performed via an instanton calculation, gives the decay rate. The potential barrier $V(\lambda)$ has three components:

- The Skyrme potential $V_{\text{Sky}}(\lambda)$.
- The Casimir energy of the bag fermions. The energy levels of the interior fermions are sensitive, via boundary conditions, to the value of the Skymion at r_{bag}. As $\theta(r_{\text{bag}})$ changes with λ, the energy levels shift continuously, leading to a λ-dependent Casimir energy.
- When $S(r_{\text{bag}})$ crosses $\pi/2$ the lowest energy eigenvalue of the interior fermions hits zero and there is a zero-mode in the spectrum. Without a source of baryon number violation in the theory, this zero-mode would make the decay impossible. However, in the presence of an external BNV source, the zero-mode

[1] The baryon number of the chiral bag setup is a well known example of r_{bag} independence. As shown by Jaffe and Goldstone [6], the baryon number of the quarks in the bag exactly makes up for the piece cut out of the Skyrme portion, with no dependence on r_{bag}

is removed and tunnelling is possible – just as 't Hooft showed in the context of axial $U(1)$ transitions in QCD [8].

We find suppression of 10^{-3} for $r_{\rm bag} = 0.3$, shrinking to 10^{-5} for $r_{\rm bag} = 0.1$.

How does this picture change for n-$\bar{n}$ oscillations? The changes are more subtle than one might imagine, and are best shown by comparing the potentials within the chiral bag setup for proton decay and n-$\bar{n}$ oscillations. For proton decay, we cared about the transition only through the barrier, going from $B = 1$ to $B = 0$. For this purpose, the configuration we derived was a 'bounce' solution (shown in red in Fig. 1). Following the usual instanton techniques and summing a series of bounces, we obtain the decay rate itself, not an amplitude. To oscillate from a neutron to an anti-neutron, it is not sufficient to tunnel through the same barrier as in proton decay – we want to be in a configuration with $B = 1$ (or -1), not one with $B = 0$! Since the neutron and the antineutron have the same mass, the transition is between states of equal energy and therefore a true tunnelling configuration is needed, i.e., a trajectory that starts at $t = 0$ as a neutron and ends at $t = \infty$ as an antineutron (or vice versa). Unlike the bounce case, a tunnelling transition yields an amplitude, not a probability, so the suppression from tunnelling must be squared in order to get the effect in the rate.

The second difference concerns the the potential itself. In proton decay, the BNV operator involves three quarks, while in the n-$\bar{n}$ case it involves six. In our calculation, the BNV operators are needed to annihilate

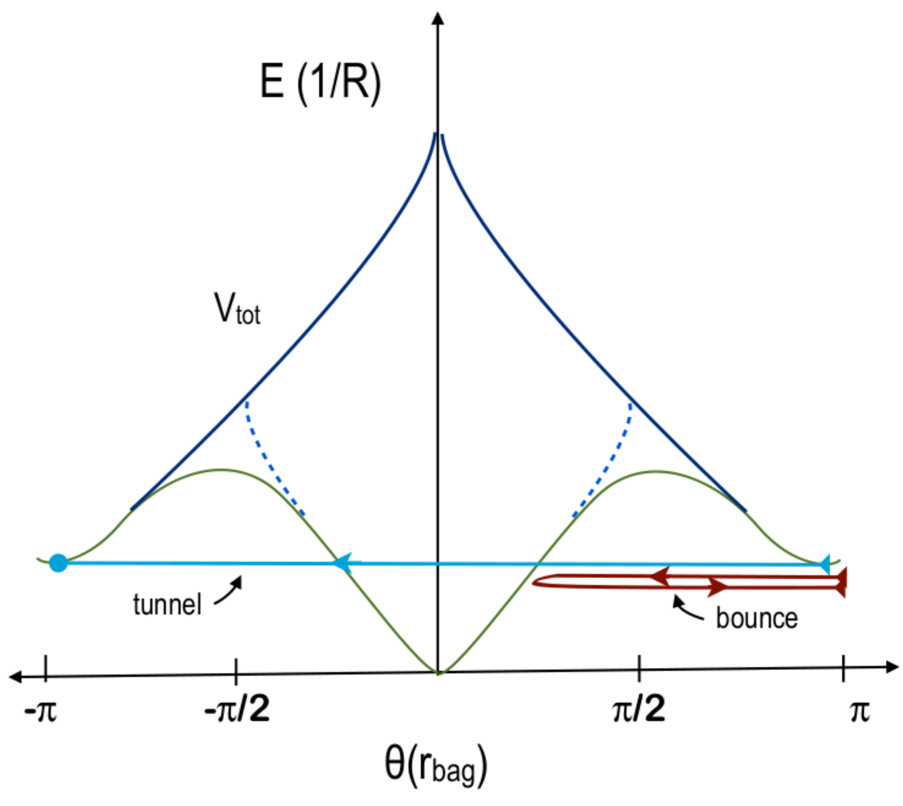

FIG. 1: Cartoon showing the potential barriers and tunnelling trajectories from proton decay and n-$\bar{n}$ oscillation in the chiral bag setup. The horizontal axis is the value of the Skyrmion at the bag surface $\theta(\lambda r_{\rm bag})$, varying from $\sim \pi$ (a baryon), to $\sim -\pi$ (an anti-baryon). The green solid curve is the potential from the Skyrme portion alone, the dashed line includes the Casimir contribution, and the solid blue line is the potential including the energy of valence modes that have been lifted out of the Dirac sea. The bounce trajectory appropriate for proton decay is shown in red, while a n-$\bar{n}$ tunnelling configuration is shown in light blue.

zero modes created as we squeeze the Skyrmion part of the nucleon into the bag. As described above, we get three zero modes as soon as the Skyrmion boundary value passes $\pi/2$. This is sufficient for proton decay, but not for an n-$\bar{n}$ oscillations. To have an n-$\bar{n}$ transition we need six zero modes, three to go from $B = 1$ to $B = 0$ and three more to go from $B = 0$ to $B = -1$. For this to occur the Skyrmion must be squished further, to $S(r_{\rm bag}) \sim 0$. Further squeezing of the Skyrmion means a larger barrier, and therefore a dramatically reduced rate. For $r_{\rm bag} = 0.3$ we find a suppression of 10^{-5} in the n-$\bar{n}$ amplitude, or 10^{-10} in the rate!

One weakness of our approach is the sensitivity to the scale $r_{\rm bag}$. However, we expect that this dependence will weaken when the calculation is performed beyond the leading order approximations we employed (namely, the adiabaticity of the transition), just as the sensitivity of cross sections to the factorization scale is lessened once higher order terms are included.

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