

Kaon rare decays in the warped extra dimensional model

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Outline

1. Introduction on the warped extra dimensional model:

- ◆ Gauge hierarchy problem
- ◆ Flavor sector: connection between the flavor puzzle and new sources of flavor violation

2. Kaon Rare decays

- ◆ $K \rightarrow \pi \nu \nu$
- ◆ $K_L \rightarrow \pi \ell \ell$
- ◆ $K_L \rightarrow \mu \mu$
- ◆ Correlations between these decays
- ◆ Correlations with other observables (ϵ' , $B_s \rightarrow \mu \mu$)

3. Conclusions

Main references:

Blanke, Buras, Duling, Gemmler, S.G, JHEP 0903 (2009) 108

Bauer, Casagrande, Haisch, Neubert, JHEP 1009 (2010) 017

The Hierarchy Problem in a Warped Metric

Randall, Sundrum, 1999

The gauge hierarchy problem

Huge hierarchy between the fundamental gravity scale M_{pl} & the EW scale Λ_{EWSB}

Even if $\frac{\Lambda_{EWSB}}{M_{pl}} \approx 10^{-16}$ is imposed at tree-level, loop corrections push $\Lambda_{EWSB} \sim M_{pl}$

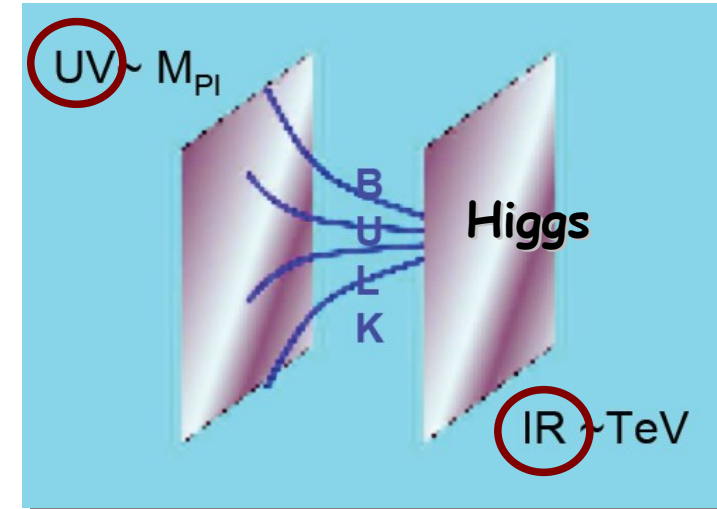
$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad 0 \leq y \leq L$$

Fundamental Planck scale on the **UV brane**: M_{fund}

Energy scale in the bulk: $M_{fund} \times e^{-ky}$

On the **IR brane** where the Higgs lives: $M_{fund} \times e^{-kL} \sim \text{TeV}$

for $kL \sim 30$



Geometrical solution of the gauge hierarchy problem

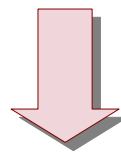
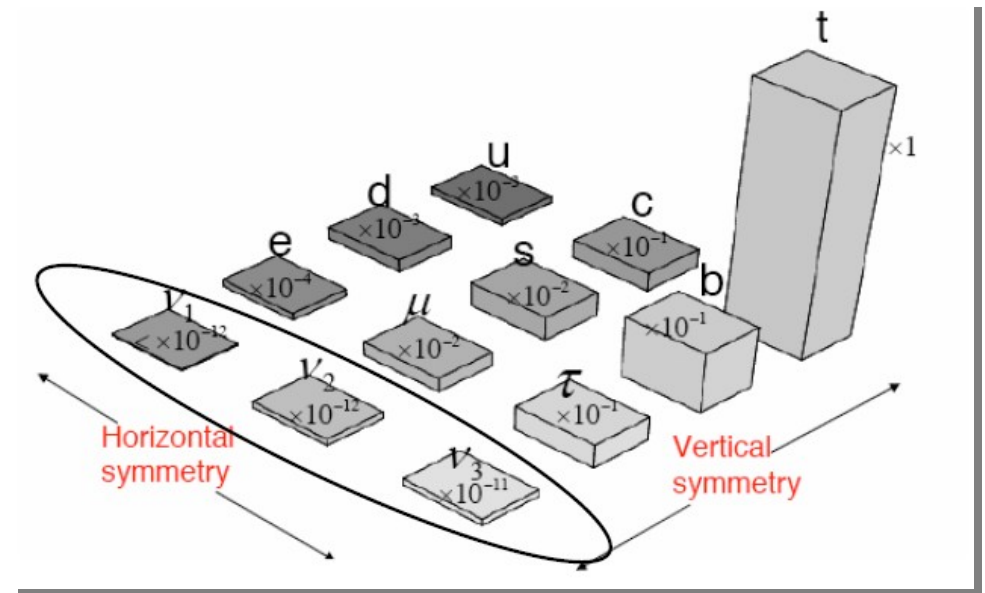
The Flavor Problem

Still a problem of Hierarchies



$$V_{CKM} = \begin{pmatrix} \blacksquare & \blacksquare & \square \\ \blacksquare & \blacksquare & \square \\ \square & \square & \blacksquare \end{pmatrix}$$

+



$$Y_D = \text{diag}(m_d, m_s, m_b)/v$$

$$Y_U = V_{CKM}^\dagger (m_u, m_c, m_t)/v$$

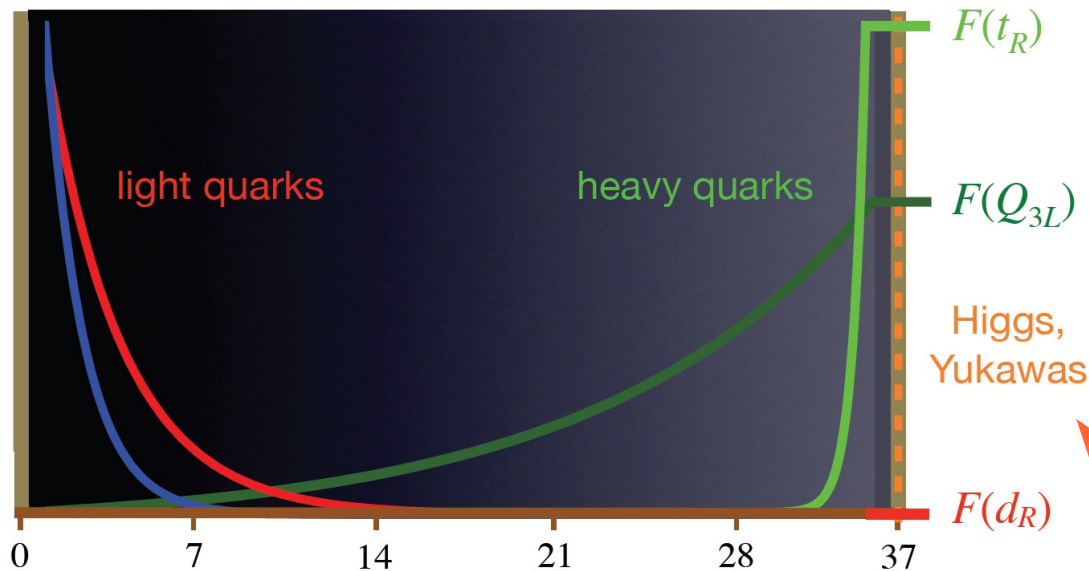
Very hierarchical
Why?

The Flavor Problem

Fermions in the bulk \rightarrow suggestive theory of flavor

UV brane

IR brane



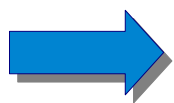
Fermion zero mode shape function

$$f^{(0)}(y, \mathbf{c}) \propto e^{(\frac{1}{2} - \mathbf{c})ky}$$

bulk mass

4-D Yukawa couplings: $Y_{ij} \propto \int_0^L \frac{dy}{L^{3/2}} \lambda_{ij} h(y) f_L^{(0)}(y, c^i) f_R^{(0)}(y, c^j)$

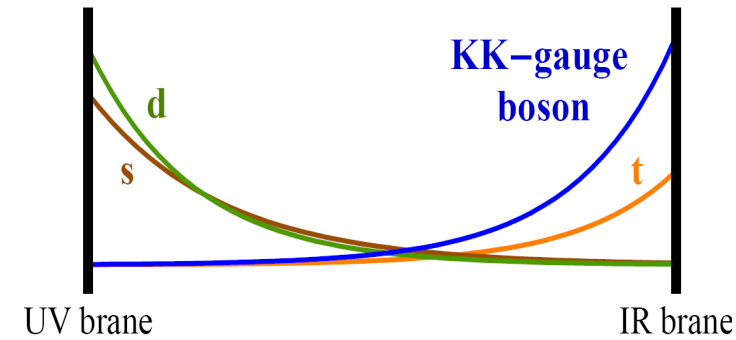
5-D Yukawa couplings (complex)



Slightly different \mathbf{c} parameters of $O(1)$ lead to large hierarchies in Y_{ij} , even for anarchical 5-D Yukawa couplings

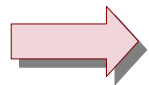
New Sources of Flavor Violation

- ◆ KK tower of **heavy gauge bosons**
...that are all **localized** towards the **IR brane**



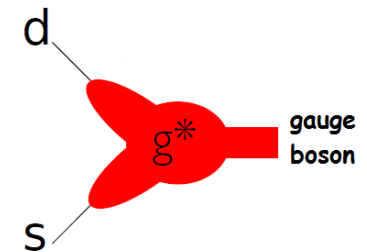
- ◆ Zero mode of the Z-boson
...that have in first approximation a **flat shape function**.

Perturbations induced through the mixing with KK excitations/other gauge bosons



Their couplings to SM fermions are **non-universal**
...because couplings to SM fermions depend on their localization

$$\Delta_{L,R} \propto \int_0^L dy e^{ky} \left[f_{L,R}^{(0)}(y, c_\Psi^i) \right]^2 g(y)$$



Rotation to mass eigenstates:

non universalities



off-diagonal terms

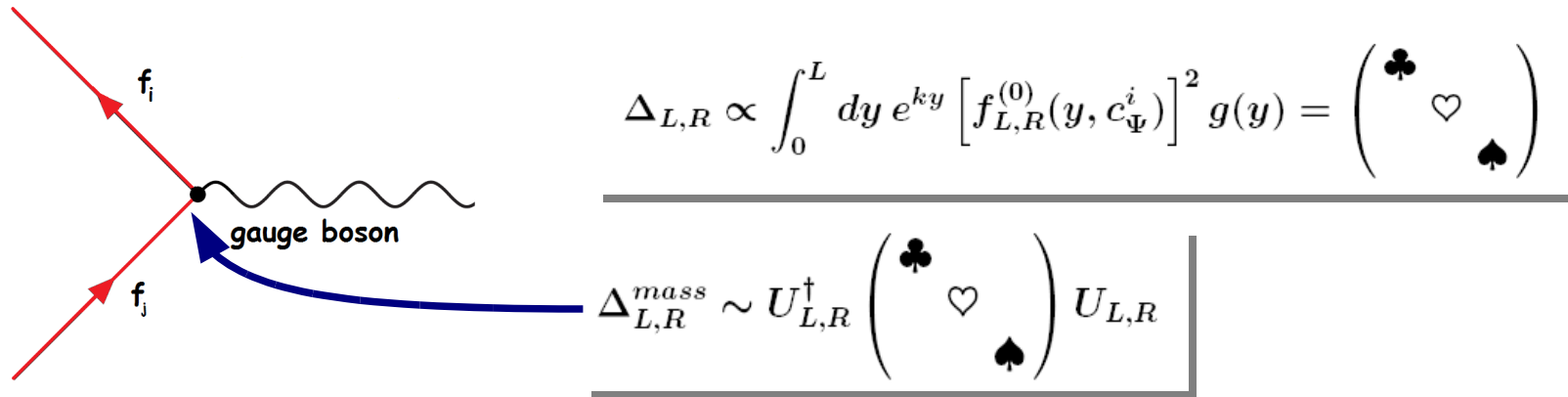
Flavor Changing Neutral Currents at Tree Level

$$\Delta_{L,R}^{mass} \sim U_{L,R}^\dagger \begin{pmatrix} \Delta_{L,R} & & \\ & \clubsuit & \\ & & \heartsuit \\ & & & \spadesuit \end{pmatrix} U_{L,R}$$

New sources of flavor and CP violation beyond CKM: **model is non-MFV**

RS-GIM Mechanism

Agashe, Perez, Soni,
hep-ph/0406101



Resulting FCNC couplings depend on same **exponentially small overlap integrals** $F(Q_L), F(q_R)$ that generate fermion masses

Flavor off-diagonal couplings **proportional to the mass splittings**: $\Delta_{L,R}^{ij} \propto (m_i - m_j)U_{ij}$

$$m_d \sim m_s, m_u \sim m_c \implies \clubsuit \sim \heartsuit$$



Suppression of FCNCs which involve the first two generation quarks

$$m_t \gg m_{u,c} \implies \text{RS-GIM mechanism broken by the large top mass}$$

Important for Kaon physics

WED with Custodial Protection

The most part of the results presented are for

Agashe et al., 0605341

Csaki et al., 0308038

Scale of KK excitations:

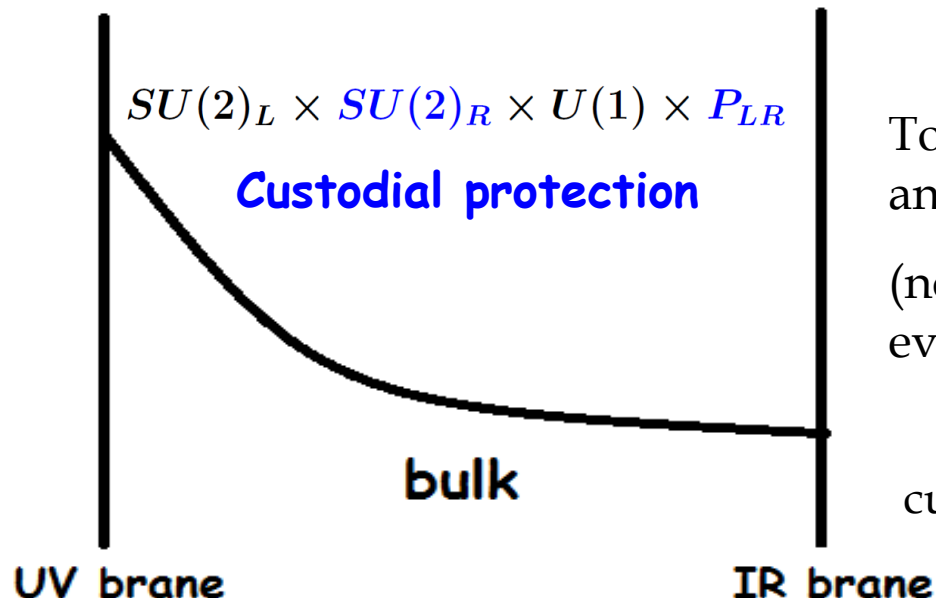
$$M_{KK} \approx 2.45ke^{-kL}$$

We fix:

$$M_{KK} = 2.5 \text{ TeV}$$



KK excitations directly accessible at the LHC



To protect **T parameter**
and **$Z\bar{b}_L b_L$ coupling**

(not too large NP contribution
even with low M_{KK} scale)

In the model without
custodial protection, typically

$$M_{KK} \geq 10\text{TeV}$$

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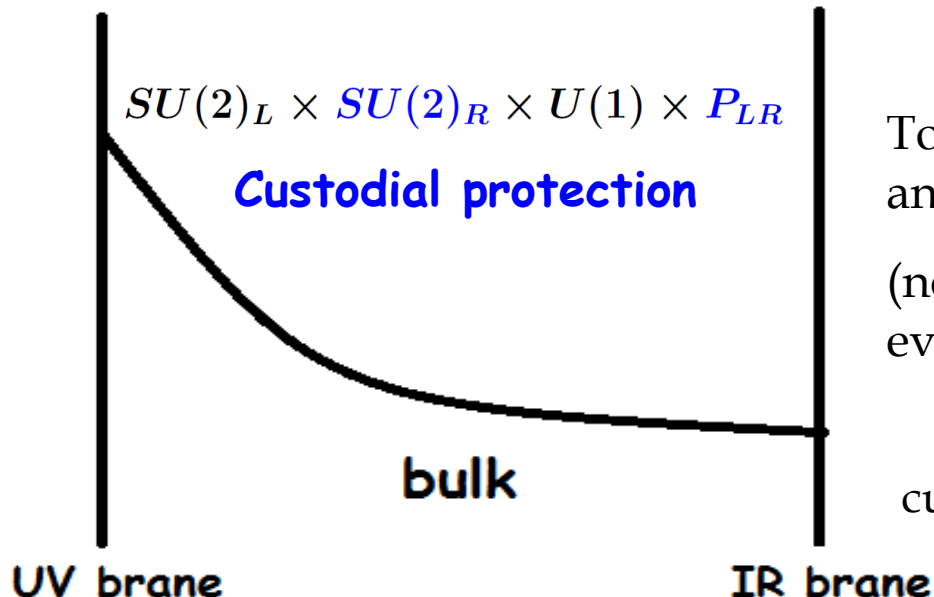
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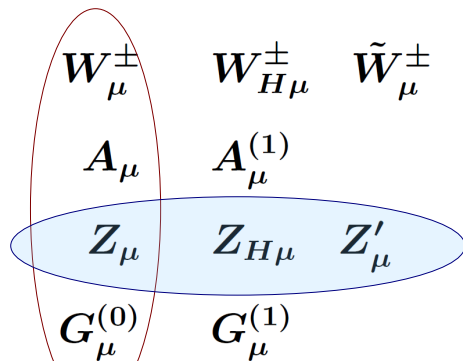
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Gauge bosons



SM gauge bosons

Gauge bosons important for Kaon rare decays

Quarks

Left-handed down quarks (all three generations) are eigenstates of P_{LR}

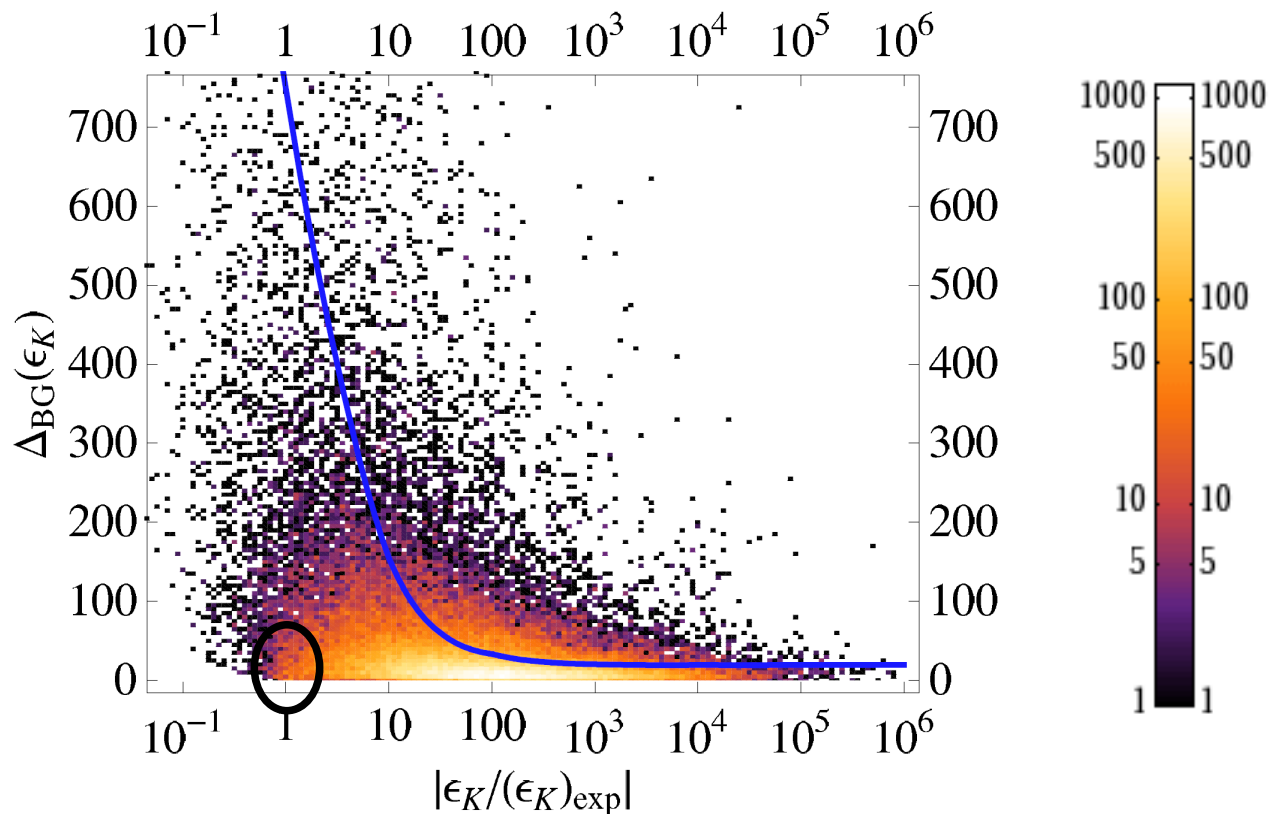


Small NP effects in $Z\bar{d}_L^i d_L^j$ couplings

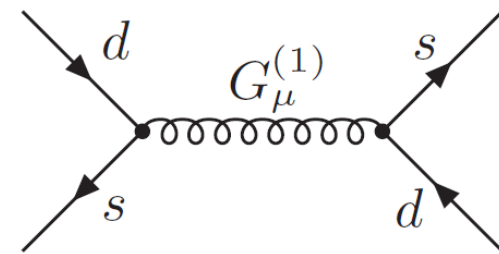
ϵ_K : Still a Challenging Observable

Obtained for $M_{KK} \sim 3\text{TeV}$

See also Csaki, Falkowski, Weiler, 2008



Blanke, Buras, Duling, SG, Weiler, 2009



$$Q_2^{LR} = (\bar{s}P_L d) (\bar{s}P_R d)$$

Very important contribution from the scalar operator

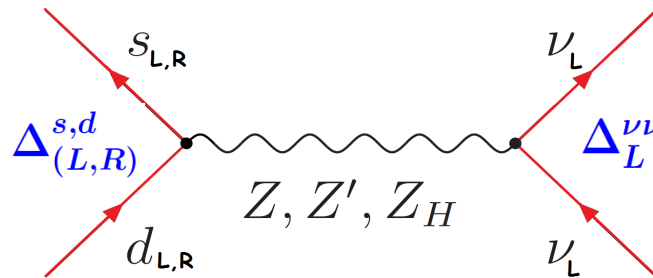
[See Uli's talk](#)

Fitzpatrick, Perez, and Randall, 2007
Santiago, 2008

Some degree of **fine tuning** OR **additional flavor symmetries** would be required if we want to keep a relatively low NP scale ($M_{KK} \sim \text{few TeV}$)

$K \rightarrow \pi \nu \nu$: Theory

Cleanest window into
 $s \rightarrow d$ transitions



$$\mathcal{H}_{\text{eff}} \propto \left[V_{ts}^* V_{td} (X(x_t) + X_{i,L}^{\text{NP}}) + V_{cs}^* V_{cd} X_{\text{NNL}}(x_c) \right] (\bar{s}_L \gamma_\mu d_L) (\bar{\nu}_L \gamma_\mu \nu_L) + V_{ts}^* V_{td} (X_{i,R}^{\text{NP}}) (\bar{s}_R \gamma_\mu d_R) (\bar{\nu}_L \gamma_\mu \nu_L)$$

SM operator

Since K and π are pseudoscalars: $\langle \pi | (\bar{s} \gamma_\mu \gamma_5 d) | K \rangle = 0$

CP conserving process: $\text{BR}(K^+ \rightarrow \pi^+ \bar{\nu} \nu) \propto |V_{ts}^* V_{td} X|^2$

CP violating process: $\text{BR}(K_L \rightarrow \pi^0 \bar{\nu} \nu) \propto \text{Im}(V_{ts}^* V_{td} X)^2$

$$X \sim X(x_t) + X_{i,L}^{\text{NP}} + X_{i,R}^{\text{NP}} + \frac{V_{cs}^* V_{cd}}{V_{ts}^* V_{td}} X_{\text{NNL}}(x_c)$$

$$X_{i,(L,R)}^{\text{NP}} \propto \frac{\Delta_L^{\nu\nu} \Delta_{(L,R)}^{s,d}}{V_{ts}^* V_{td} M_{Z_i}^2}$$

Naively expected
larger effects in Kaon
rare decays

than in B rare decays

$$|V_{ts}^* V_{td}| \sim 3 \cdot 10^{-4}$$

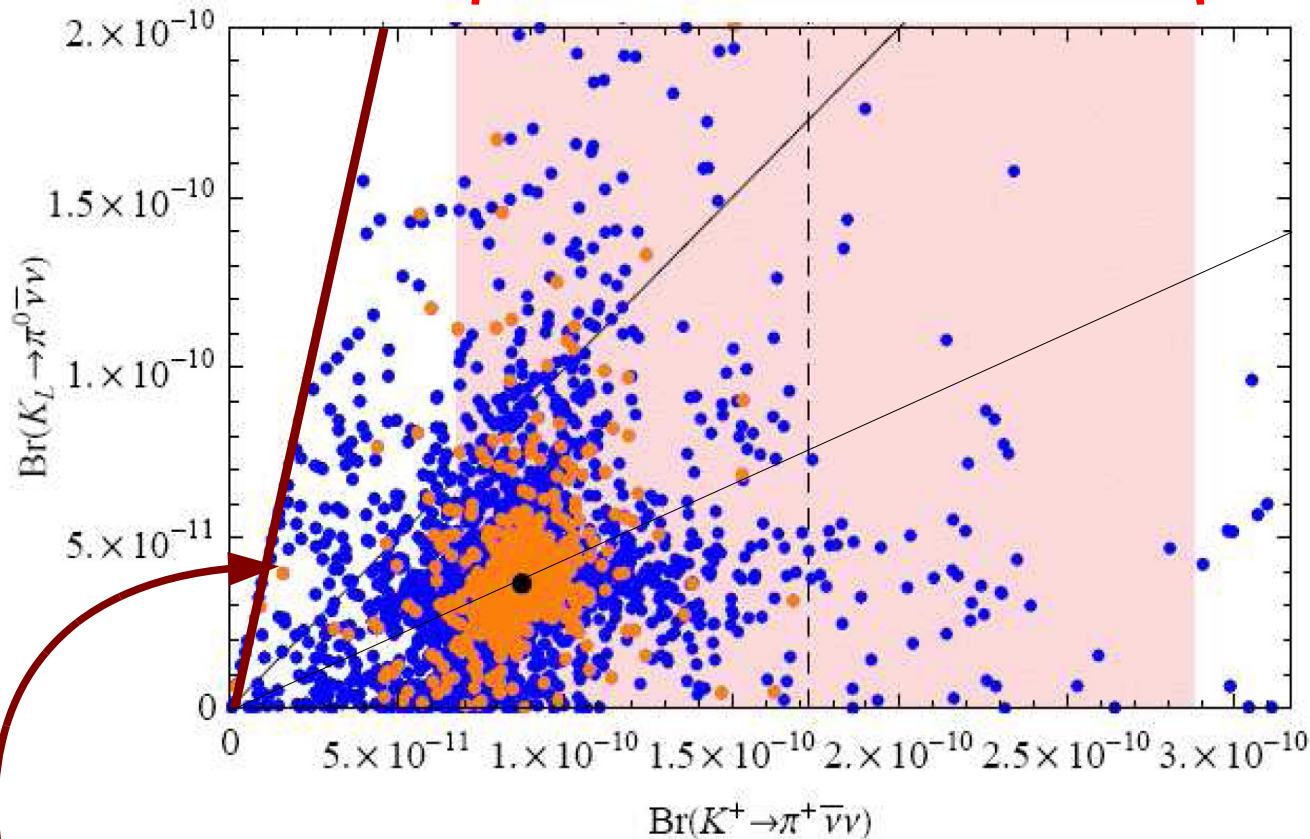
$$|V_{td}^* V_{tb}| \sim 9 \cdot 10^{-3}$$

$$|V_{ts}^* V_{tb}| \sim 4 \cdot 10^{-2}$$

$K \rightarrow \pi \nu \nu$: Numerics

Blanke, Buras, Duling,
Gemmler, S.G., 2009

E949 measurement



The weak phase can
take any value
↓
The two BRs are
basically **uncorrelated**

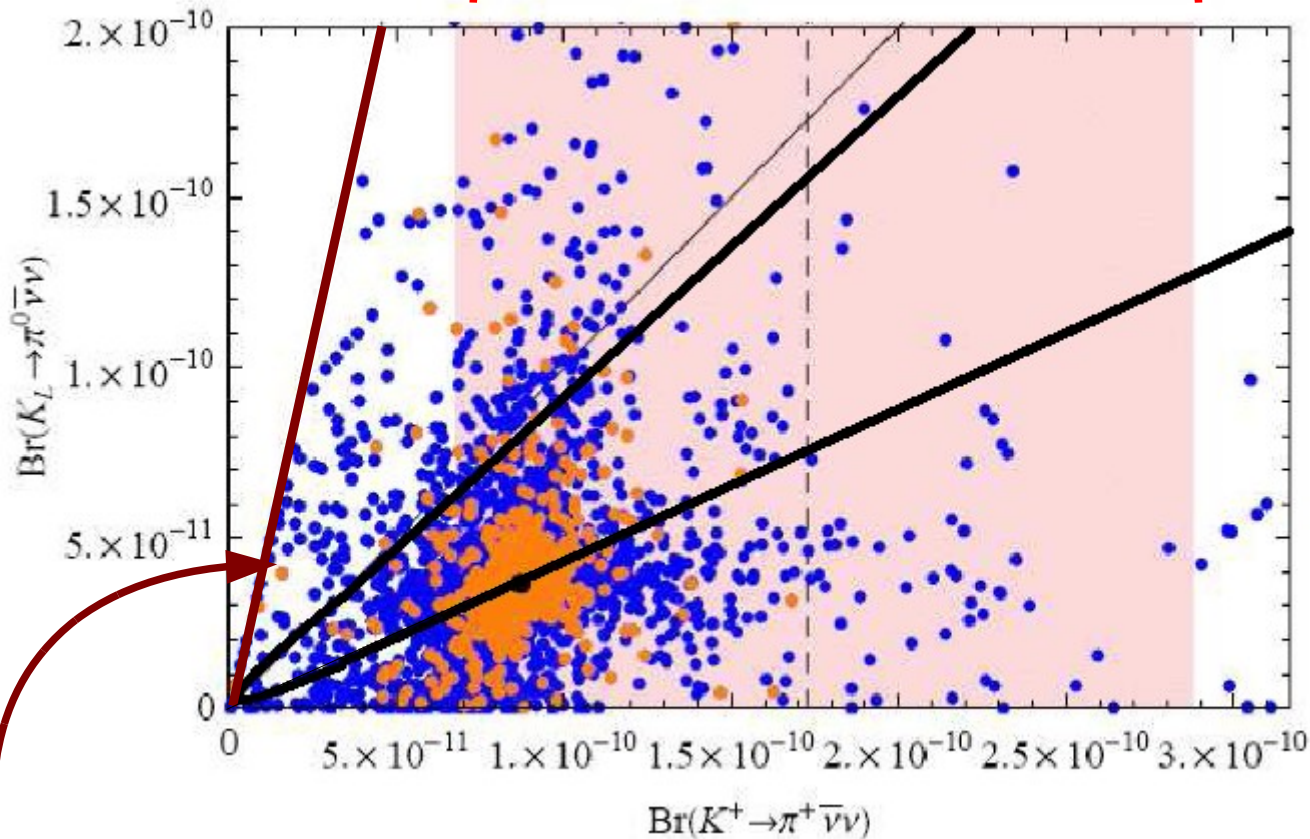
Grossman-Nir bound: $\text{BR}(K_L \rightarrow \pi^0 \bar{\nu}\nu) < 4.3 \text{BR}(K^+ \rightarrow \pi^+ \bar{\nu}\nu)$

Grossman, Nir, 1997

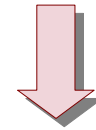
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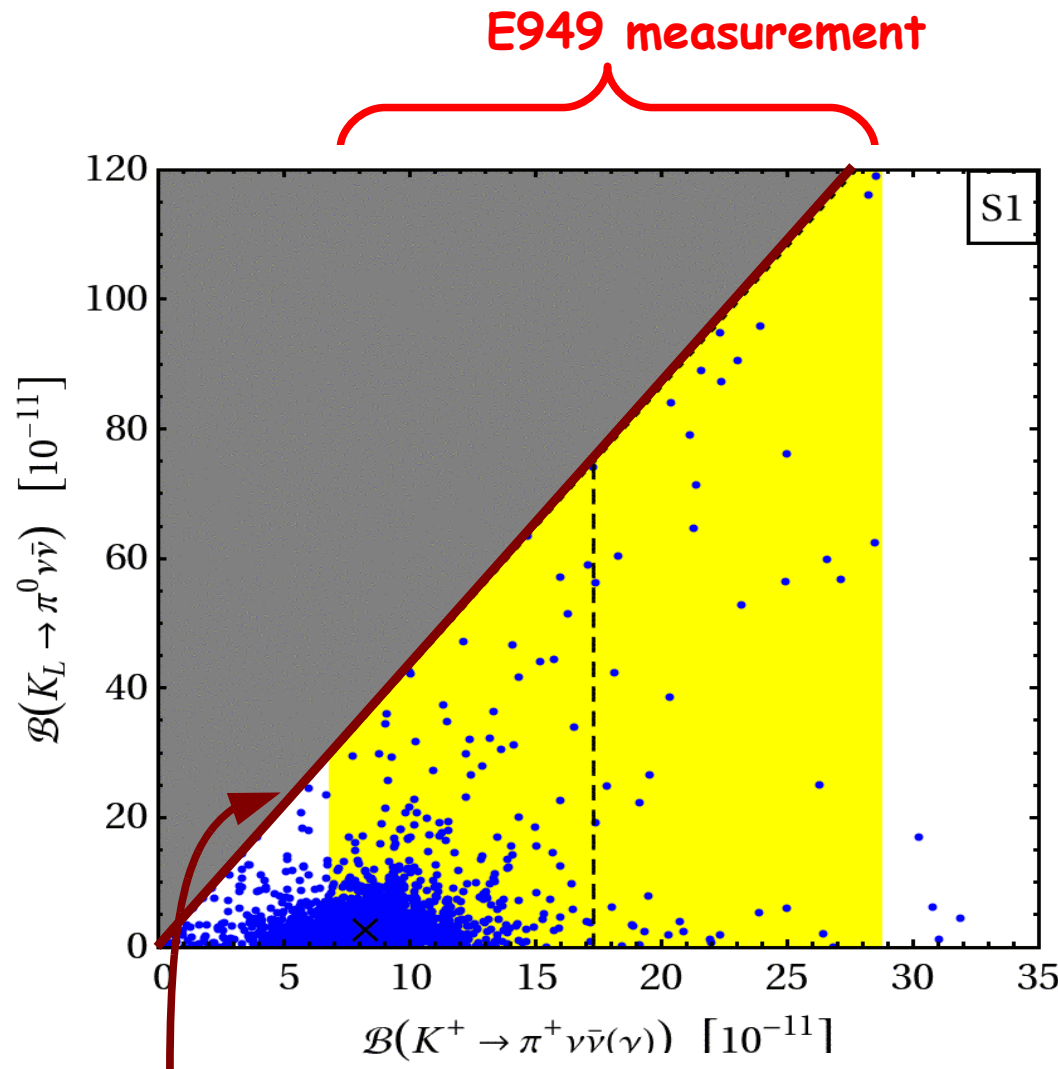
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Grossman, Nir, 1997

$K \rightarrow \pi \nu \nu$: Numerics

Bauer, Casagrande,
Haisch, Neubert, 2009



Model without
custodial protection
 $SU(2)_L \times U(1)$

Large effects possible also
for larger M_{KK} scale
($\sim(20-30)\text{TeV}$)

Grossman-Nir bound: $\text{BR}(K_L \rightarrow \pi^0 \bar{\nu} \nu) < 4.3 \text{BR}(K^+ \rightarrow \pi^+ \bar{\nu} \nu)$

Grossman, Nir, 1997

Constraint from $\varepsilon' / \varepsilon_K$

Constraints imposed in the plots of before:

- DF=2 observables (in particular ε_K);
- Quark masses and mixing angles (within 2σ)

What about the constraint from $\varepsilon' / \varepsilon_K$?

Measures the direct CP violation
in the decay $K \rightarrow \pi\pi$

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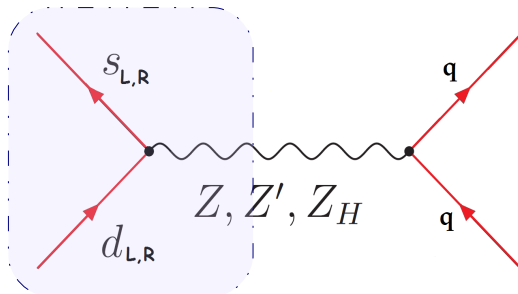
Study performed in the RS model without custodial protection

Bauer, Casagrande,
Haisch, Neubert, 2009

[See Uli's talk](#)

QCD and EW penguins, as well as chromomagnetic dipole operators, in principle affected by NP

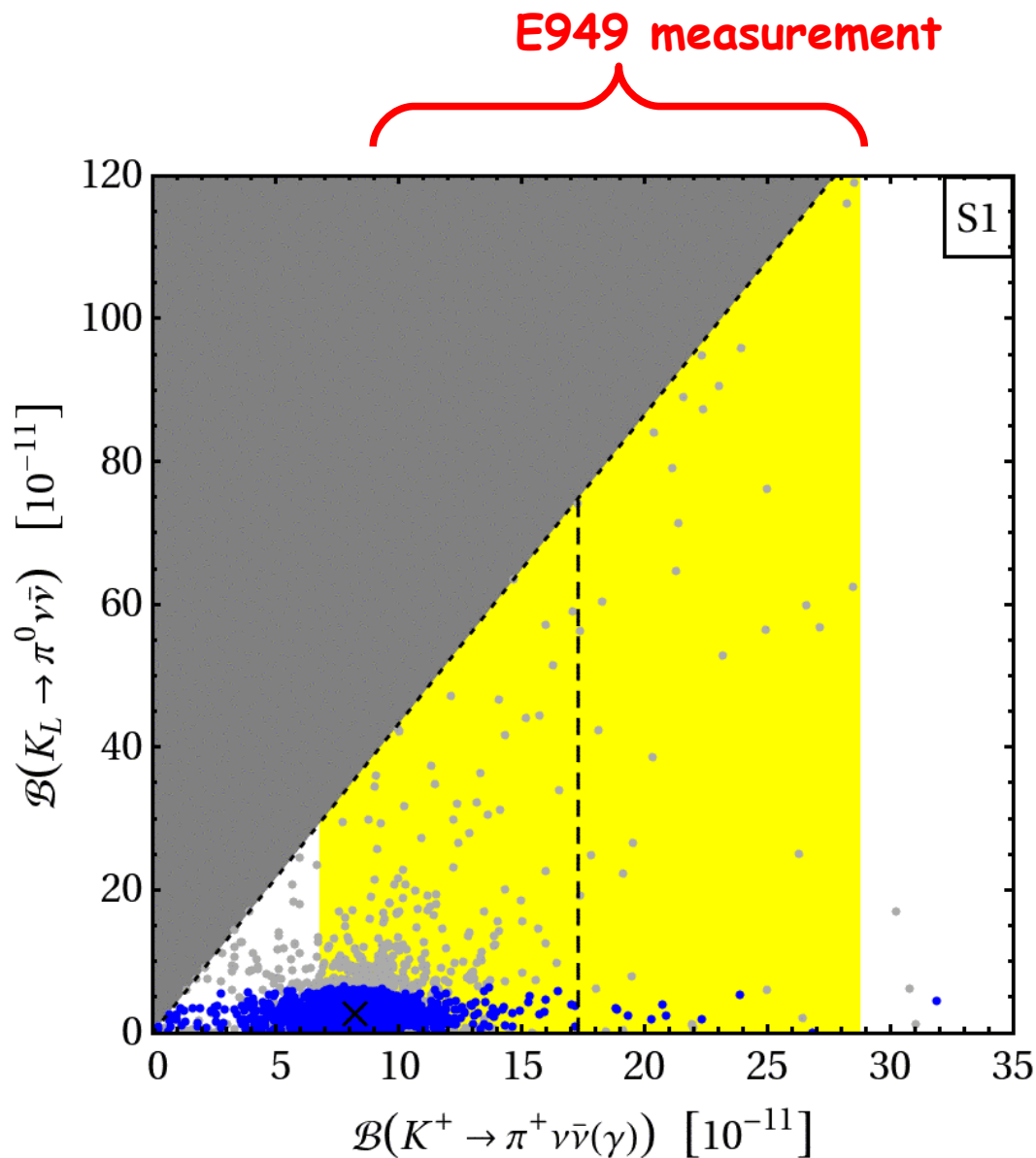
Main NP contribution



Correlation with the CP violating decay
 $K_L \rightarrow \pi^0 \bar{\nu} \nu$

Constraint from $\varepsilon'/\varepsilon_K$

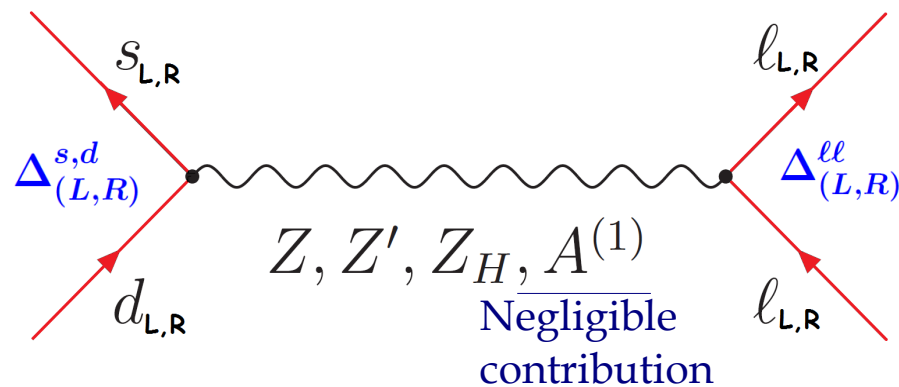
Bauer, Casagrande,
Haisch, Neubert, 2009



Inverse correlation between
 $\varepsilon'/\varepsilon_K$ and $K_L \rightarrow \pi^0 \bar{\nu} \nu$

Very large NP effects in
 $K_L \rightarrow \pi^0 \bar{\nu} \nu$ are disfavored

$K \rightarrow \pi |^+ |^-$: Theory



SM operators

$$\mathcal{H}_{\text{eff}} \propto V_{ts}^* V_{td} \left[Y_{i,R}^{\text{NP}} (\bar{s}_R \gamma_\mu d_R) (\bar{\ell}_L \gamma_\mu \ell_L) + (Y(x_t) + Y_{i,L}^{\text{NP}}) (\bar{s}_L \gamma_\mu d_L) (\bar{\ell}_L \gamma_\mu \ell_L) \right. \\ \left. + Z_{i,R}^{\text{NP}} (\bar{s}_R \gamma_\mu d_R) (\bar{\ell}_R \gamma_\mu \ell_R) + (Z(x_t) + Z_{i,L}^{\text{NP}}) (\bar{s}_L \gamma_\mu d_L) (\bar{\ell}_R \gamma_\mu \ell_R) \right]$$

Since K and π are pseudoscalars: $\langle \pi | (\bar{s} \gamma_\mu \gamma_5 d) | K \rangle = 0$

CP violating process:

$$\text{BR}(K_L \rightarrow \pi^0 \ell^+ \ell^-) \propto F(\text{Im}(V_{ts}^* V_{td} Y_A), \text{Im}(V_{ts}^* V_{td} Y_V))$$

$$\begin{cases} Y_A \sim Y(x_t) - Z(x_t) + Y_{i,L}^{\text{NP}} - Z_{i,L}^{\text{NP}} + Y_{i,R}^{\text{NP}} - Z_{i,R}^{\text{NP}} \\ Y_V \sim Y(x_t) + Z(x_t) + Y_{i,L}^{\text{NP}} + Z_{i,L}^{\text{NP}} + Y_{i,R}^{\text{NP}} + Z_{i,R}^{\text{NP}} \end{cases}$$

In principle also the scalar and pseudoscalar operators would contribute

$$Q_S = (\bar{s}d)(\bar{\ell}\ell)$$

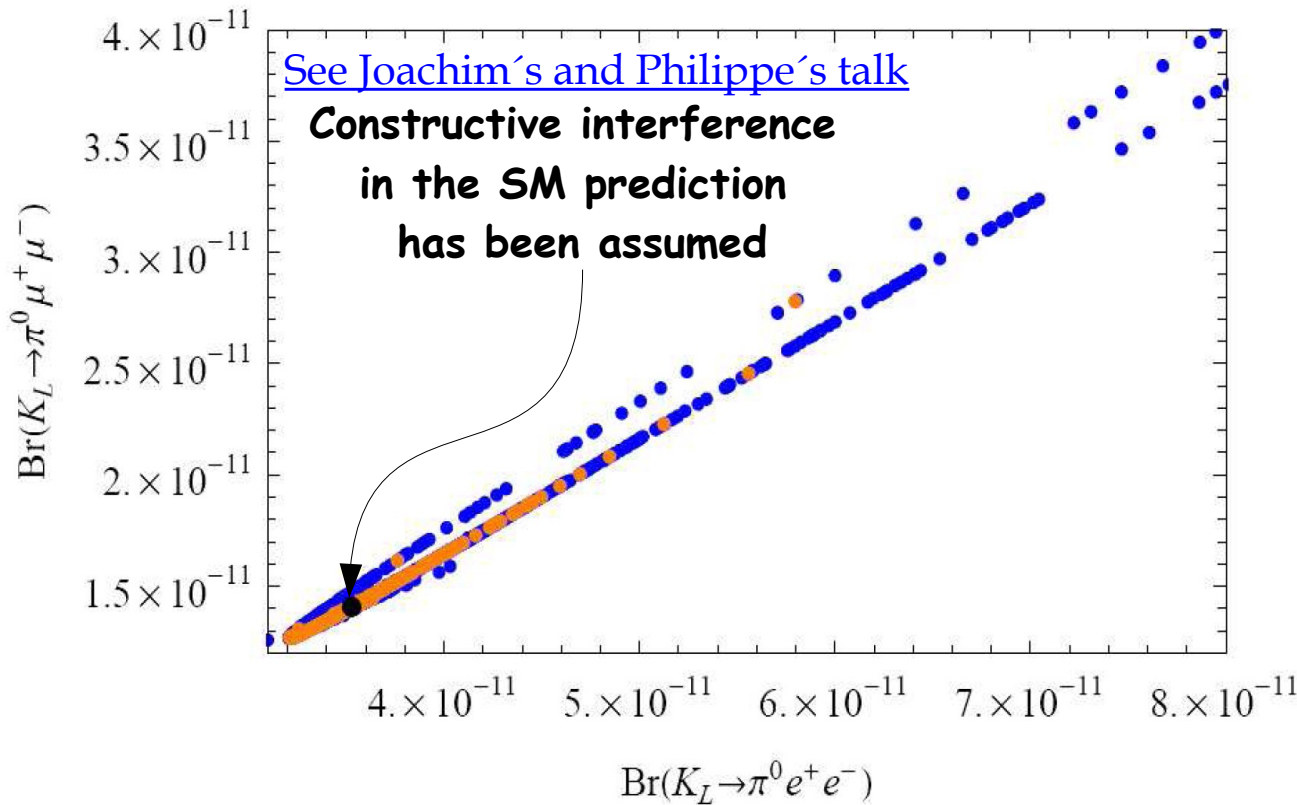
$$Q_P = (\bar{s}d)(\bar{\ell}\gamma_5\ell)$$

Their contribution is however very suppressed

$K \rightarrow \pi |^+|^-$: Numerics

$$\left\{ \begin{array}{l} \text{BR}(K_L \rightarrow \pi^0 \mu^+ \mu^-)_{\text{SM}} = (1.4 \pm 0.3, 0.9 \pm 0.2) \cdot 10^{-12} \\ \text{BR}(K_L \rightarrow \pi^0 e^+ e^-)_{\text{SM}} = (3.5 \pm 0.9, 1.6 \pm 0.6) \cdot 10^{-12} \end{array} \right.$$

Blanke, Buras, Duling,
Gemmler, S.G., 2009



Angle between the two branches is **small** since it is determined by

$$Y_V/Y_A \sim 1-4\text{Sin}(\theta_W)^2 \sim 0.08$$

(for theories with NP effects only in **Z-penguins** and **negligible scalar and photon penguin contributions**)

[See Uli's talk](#)

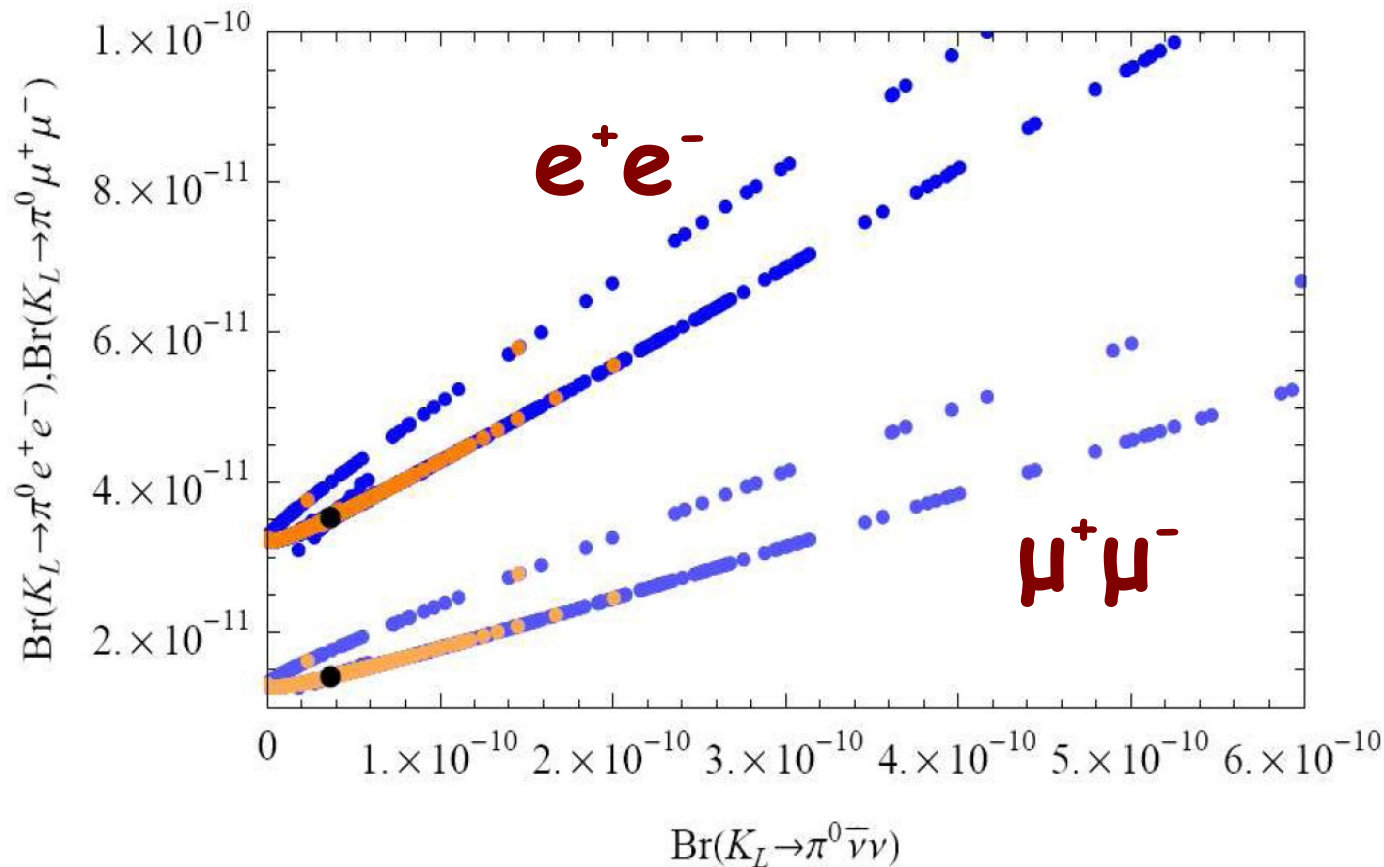
Similar results obtained in the non custodial model

Bauer, Casagrande,
Haisch, Neubert, 2009

(even a larger enhancement is possible)

$K \rightarrow \pi^+ l^-$ & $K \rightarrow \pi \nu \nu$ Correlation

Blanke, Buras, Duling,
Gemmler, S.G., 2009



Result of the interplay
of the coupling of Z
with leptons & neutrinos

Measurement of both
decays is a good test
of the operator
structure of the model

Similar results obtained in the non custodial model

Bauer, Casagrande,
Haisch, Neubert, 2009

(even a larger enhancement is possible)

$K \rightarrow \mu^+ \mu^-$: Theory

Same effective Hamiltonian as for the $K \rightarrow \pi \pi$ decay.

$$\mathcal{H}_{\text{eff}} \propto V_{ts}^* V_{td} \left[Y_{i,R}^{\text{NP}} (\bar{s}_R \gamma_\mu d_R) (\bar{l}_L \gamma_\mu l_L) + (Y(x_t) + Y_{i,L}^{\text{NP}}) (\bar{s}_L \gamma_\mu d_L) (\bar{l}_L \gamma_\mu l_L) \right. \\ \left. + Z_{i,R}^{\text{NP}} (\bar{s}_R \gamma_\mu d_R) (\bar{l}_R \gamma_\mu l_R) + (Z(x_t) + Z_{i,L}^{\text{NP}}) (\bar{s}_L \gamma_\mu d_L) (\bar{l}_R \gamma_\mu l_R) \right]$$

However...

$\text{BR}(K \rightarrow \mu^+ \mu^-)$ is dominated by long distance contributions

Extraction of the short distance part from the data is subject to considerable uncertainties

$$\text{BR}(K_L \rightarrow \mu^+ \mu^-)_{\text{SD}} < 2.5 \times 10^{-9} \quad \text{Isidori, Unterdorfer, 2004}$$

The scalar contribution is again negligible

Since $\langle 0 | (\bar{s} \gamma_\mu d) | K_L \rangle = \langle 0 | (\bar{\mu} \gamma_\mu \mu) | \mu^+ \mu^- \rangle = 0$ only $(\bar{s} \gamma_\mu \gamma_5 d) (\bar{\mu} \gamma_\mu \gamma_5 \mu)$ contributes

CP conserving process:

$$\text{BR}(K_L \rightarrow \mu^+ \mu^-) \propto F(\text{Re}(V_{ts}^* V_{td} \tilde{Y}_A))$$

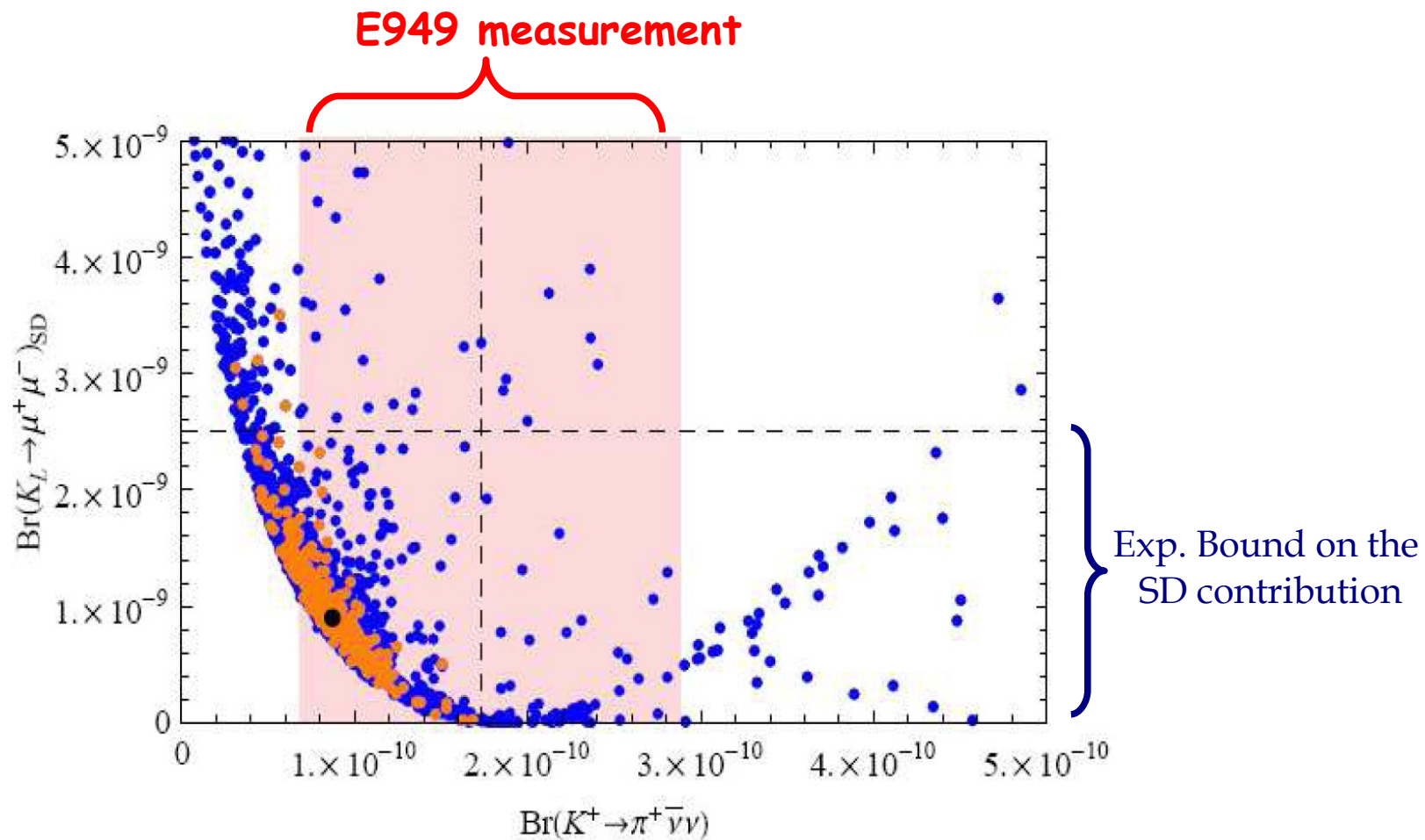
$$\tilde{Y}_A \sim Y(x_t) - Z(x_t) + Y_{i,L}^{\text{NP}} - Z_{i,L}^{\text{NP}} - Y_{i,R}^{\text{NP}} + Z_{i,R}^{\text{NP}}$$

Any correlation with the other CP conserving process?

$$\text{BR}(K^+ \rightarrow \pi^+ \bar{\nu} \nu)$$

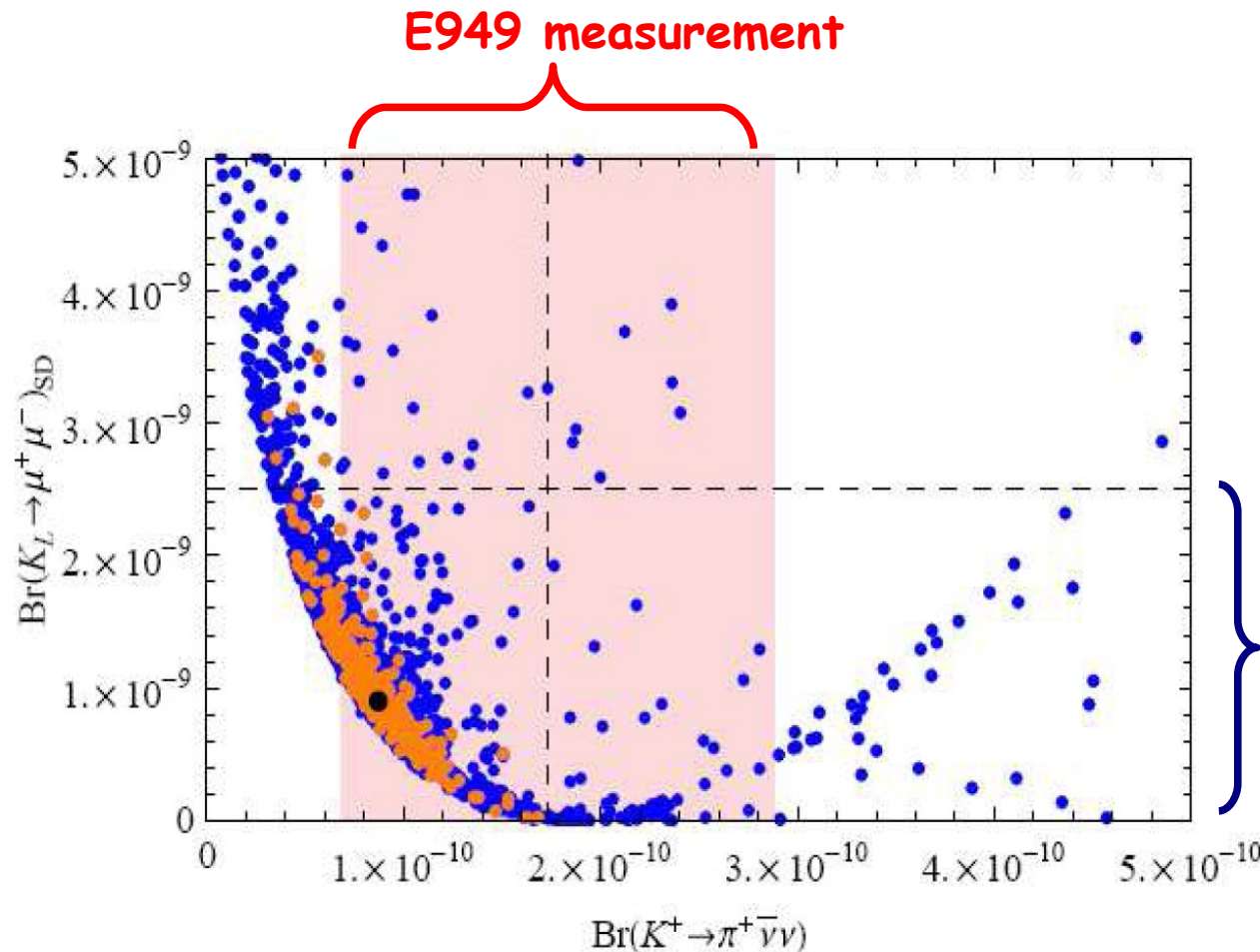
$K \rightarrow \mu^+ \mu^-$: Numerics

Blanke, Buras, Duling,
Gemmler, S.G., 2009



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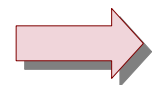
$K^+ \rightarrow \pi^+ \bar{\nu} \nu$
measures the coupling $Z^\mu (\bar{s} \gamma_\mu d)$

$K_L \rightarrow \mu^+ \mu^-$
measures the coupling $Z^\mu (\bar{s} \gamma_\mu \gamma_5 d)$

Exp. Bound on the
SD contribution

Main NP effects coming from
 $Z^\mu (\bar{s}_R \gamma_\mu d_R) \sim Z^\mu (\bar{s} \gamma_\mu (1 \oplus \gamma_5) d)$

To compare with the SM Z-penguin
 $Z^\mu (\bar{s}_L \gamma_\mu d_L) \sim Z^\mu (\bar{s} \gamma_\mu (1 \ominus \gamma_5) d)$



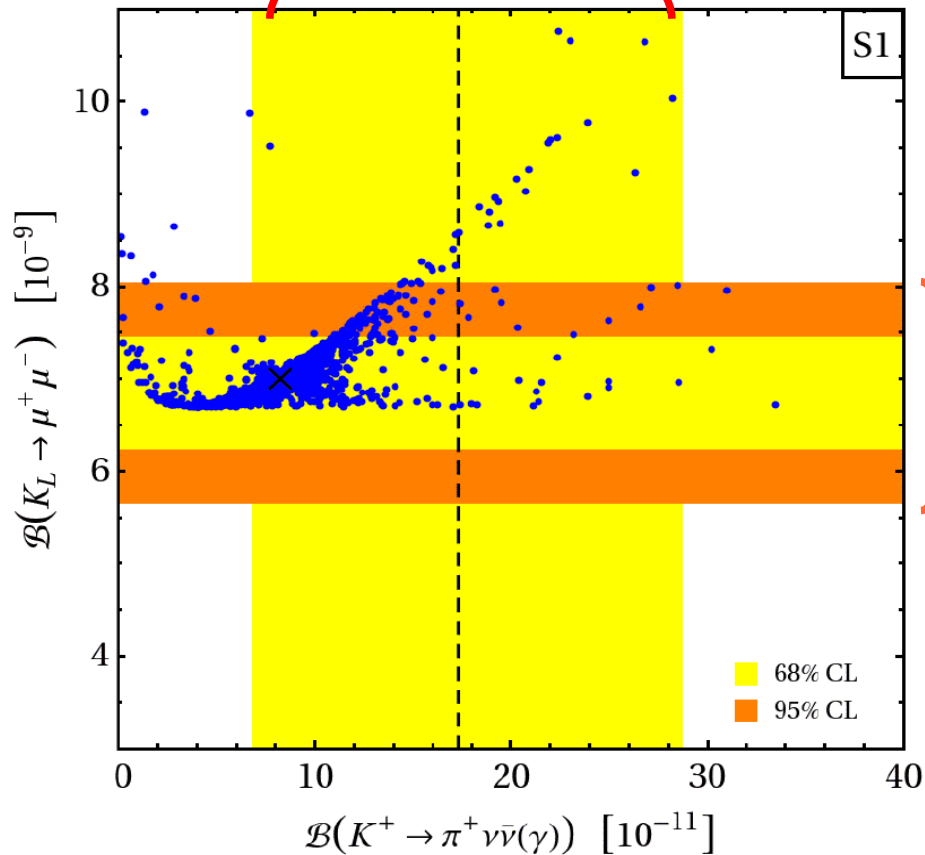
Inverse correlation

$K \rightarrow \mu^+ \mu^-$: Numerics

Bauer, Casagrande,
Haisch, Neubert, 2009

Model without
custodial protection

E949 measurement



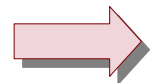
Allowed range
for the total
 $\text{BR}(K_L \rightarrow \mu^+ \mu^-)$

$K^+ \rightarrow \pi^+ \bar{\nu} \nu$
measures the coupling $Z^\mu (\bar{s} \gamma_\mu d)$

$K_L \rightarrow \mu^+ \mu^-$
measures the coupling $Z^\mu (\bar{s} \gamma_\mu \gamma_5 d)$

Main NP effect and SM Z-penguin

$$Z^\mu (\bar{s}_L \gamma_\mu d_L) \sim Z^\mu (\bar{s} \gamma_\mu (1 \ominus \gamma_5) d)$$



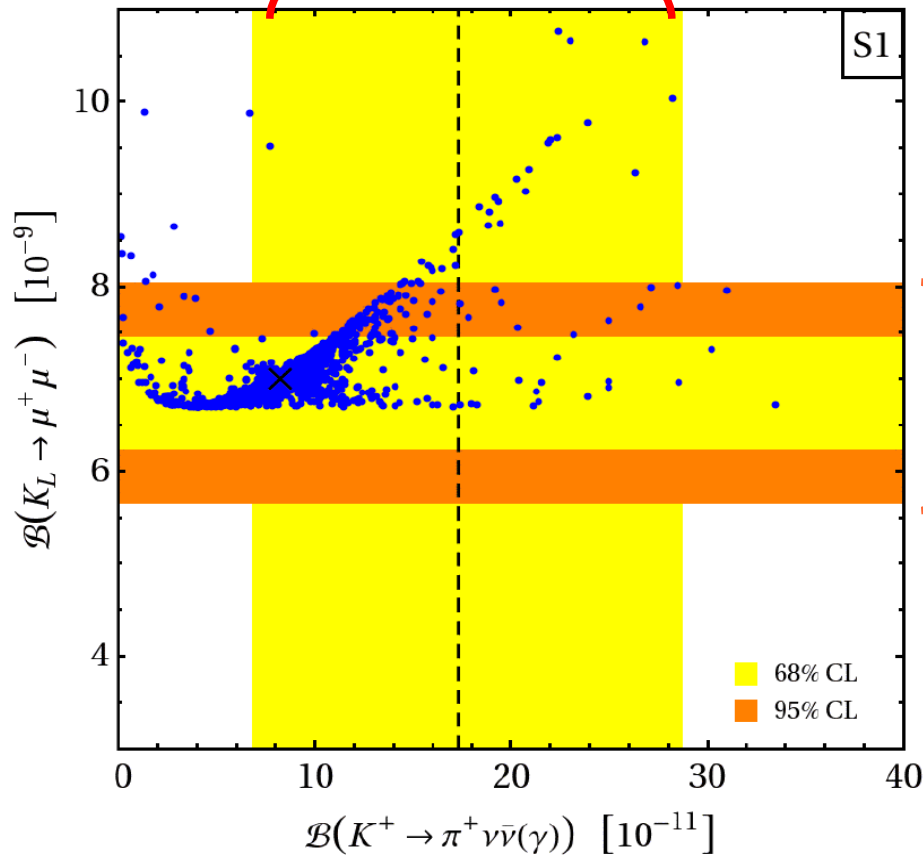
Positive correlation

$K \rightarrow \mu^+ \mu^-$: Numerics

Bauer, Casagrande,
Haisch, Neubert, 2009

Model without
custodial protection

E949 measurement



Allowed range
for the total
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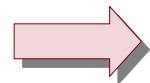
$K^+ \rightarrow \pi^+ \bar{\nu} \nu$
measures the coupling $Z^\mu (\bar{s} \gamma_\mu d)$

$K_L \rightarrow \mu^+ \mu^-$
measures the coupling $Z^\mu (\bar{s} \gamma_\mu \gamma_5 d)$

Clear test of the handedness
of NP flavor violating interactions

Main NP effect and SM Z-penguin

$$Z^\mu (\bar{s}_L \gamma_\mu d_L) \sim Z^\mu (\bar{s} \gamma_\mu (1 \ominus \gamma_5) d)$$

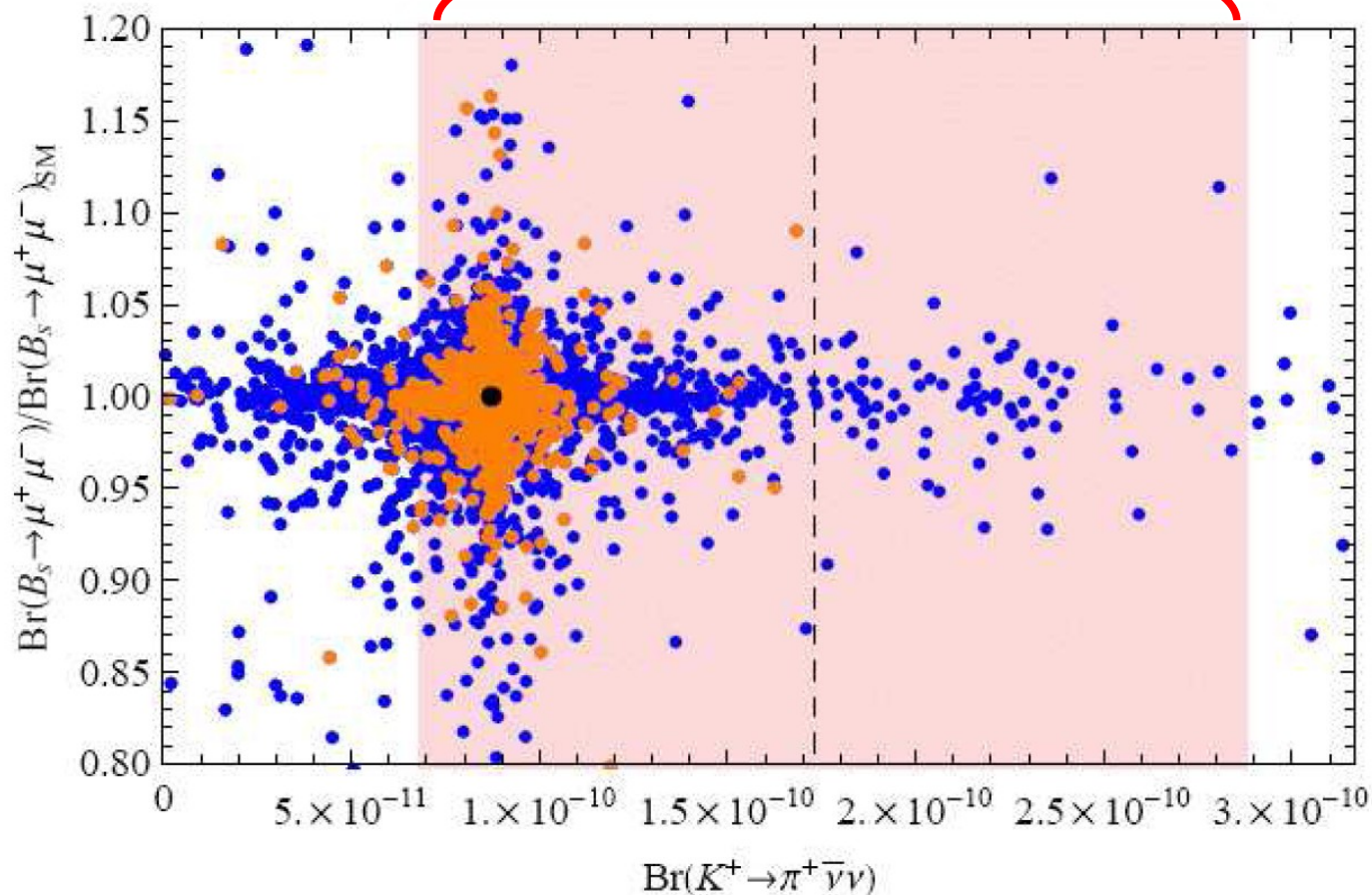


Positive correlation

Correlations between K and B Rare Decay?

E949 measurement

Blanke, Buras, Duling,
Gemmler, S.G., 2009



Rather smaller NP
effects in $B_s \rightarrow \mu\mu$

Simultaneous large
effects in both
observables is unlikely

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{exp}} \lesssim 1.5 \times \text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} \quad \text{LHCb collaboration, 1203.4493}$$

Conclusions

Warped extra dimensional models generically predict possible **sizeable NP contributions in rare kaon decays**

(contrary to the prediction for B rare decays, that are more and more constrained by the present experiments)

The measurement of **Kaon rare decays can unveil the flavor properties** of this set of NP models

i.e. testing the correlations between the several kaon rare decays could offer a **clear test of the type of NP arising in flavor transitions**

Ex: {

- Which is the **handedness** of the NP flavor violating interactions?
 $\text{BR}(K^+ \rightarrow \pi^+ \bar{\nu} \nu)$ vs. $\text{BR}(K_L \rightarrow \mu^+ \mu^-)$
- Are the **scalar** flavor violating interactions important?
 $\text{BR}(K_L \rightarrow \pi^0 \bar{e} e)$ vs. $\text{BR}(K_L \rightarrow \pi^0 \bar{\mu} \mu)$

$K \rightarrow \pi |^+|^-$: Some more Theory

$$\text{BR}(K_L \rightarrow \pi^0 \ell^+ \ell^-) \propto C_{\text{dir}}^\ell \left(\pm \right) C_{\text{int}}^\ell |a_S| + C_{\text{mix}}^\ell |a_S|^2 + C_{\gamma\gamma}^\ell + C_S^\ell$$

Constructive/destructive
interference

$$\left\{ \begin{array}{l} C_{\text{dir}}^\ell = G(\text{Im}(V_{ts}^* V_{td} \mathbf{Y}_A)^2, \text{Im}(V_{ts}^* V_{td} \mathbf{Y}_V)^2) \\ C_{\text{int}}^\ell = \tilde{G}(\text{Im}(V_{ts}^* V_{td} \mathbf{Y}_V)) \end{array} \right.$$

Probably one should take the + sign
(more investigation needed)

Buchalla, D'Ambrosio, Isidori, 2003

Friot, Greynat, De Rafael, 2004

Comparing the Several NP Contributions

- FCNC couplings to left-handed quarks

$$\Delta_L^{ij}(Z_H) : \Delta_L^{ij}(Z') : \Delta_L^{ij}(Z) \sim \mathcal{O}(10^4) : \mathcal{O}(10^3) : 1$$

- FCNC couplings to right-handed quarks

$$\Delta_R^{ij}(Z_H) : \Delta_R^{ij}(Z') : \Delta_R^{ij}(Z) \sim \mathcal{O}(10^2) : \mathcal{O}(10^2) : 1$$

- Flavor conserving couplings with leptons

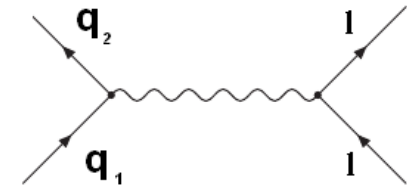
$$\Delta_{L,R}^{\nu\nu,\ell\ell}(Z_H) : \Delta_{L,R}^{\nu\nu,\ell\ell}(Z') : \Delta_{L,R}^{\nu\nu,\ell\ell}(Z) \sim \mathcal{O}(10^{-1}) : \mathcal{O}(10^{-1}) : 1$$

- Propagator of the gauge bosons

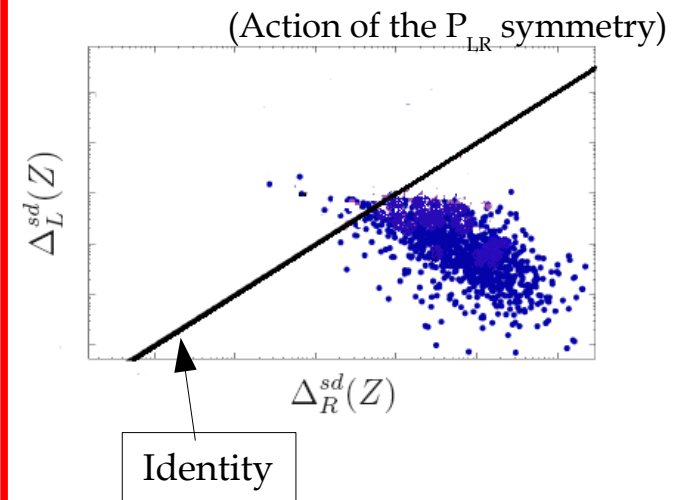
$$M_Z^2/M_{(Z_H,Z')}^2 \sim \mathcal{O}(10^{-3})$$



Z_H coupled with left-handed down-quarks
 ($\approx Z$ coupled with left-handed down-quarks)
 &
 Z coupled with right-handed down-quark
are dominant



Example:



Z coupled with right-handed quark is the dominant contribution