Rare K Decays in the Standard Model

Joachim Brod

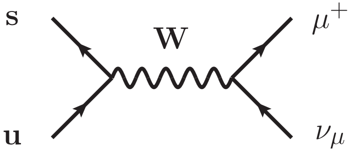
UNIVERSITY OF Cincinnati

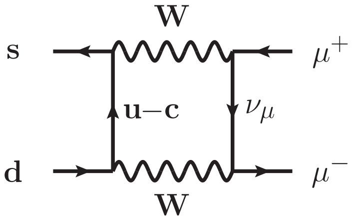
2012 Project X Physics Study (PXPS12)
14 - 23 June 2012, Fermi National Accelerator Laboratory

“City of the Future”

Why are rare K decays rare?

- $\mathrm{Br}(K^+ \to \mu^+ \nu_\mu) = 0.64$
- $\mathrm{Br}(K_L \to \mu^+ \mu^-) = 6.84 \times 10^{-9}$





- Flavour-changing neutral currents (FCNC) are highly suppressed in the Standard Model [Glashow, Iliopoulos, Maiani '70]
- Prediction of the charm-quark mass [Gaillard, Lee '74]

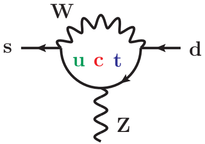
Rare K decays – overview

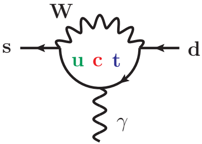
- $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$
- $K_L \to \pi^0 \ell^+ \ell^-$ $(\ell = e, \mu)$
- radiative decays $(K \to \pi\gamma\gamma, \dots)$
- $K_L \to \mu^+\mu^-$
- $K_L \to \pi\pi$
- $K \to \mu e$, ...

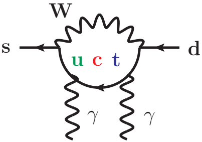
Rare K decays – overview

- $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$
- $K_L \to \pi^0\ell^+\ell^-$ ($\ell = e, \mu$) (→ More details in Philippe's talk)
- radiative decays ($K \to \pi\gamma\gamma, \ldots$) (→ Philippe's talk)
- $K_L \to \mu^+\mu^-$ (SM prediction difficult)
- $K_L \to \pi\pi$ ($\to \epsilon_K, \epsilon'/\epsilon$)
- $K \to \mu e, \ldots$ (≈ 0 in the SM)

GIM and Inami Lim







$$X_0(x) \xrightarrow{x\to\infty} x$$
$$X_0(x) \xrightarrow{x\to 0} x \log x$$

$$D_0(x) \xrightarrow{x\to\infty} \log x$$
$$D_0(x) \xrightarrow{x\to 0} \log x$$

Further suppression by $1/m_t$

$\lambda_i \equiv V_{is}^* V_{id}$

CKM unitarity $\Rightarrow \lambda_u + \lambda_c + \lambda_t = 0$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

How we compute

High scales:

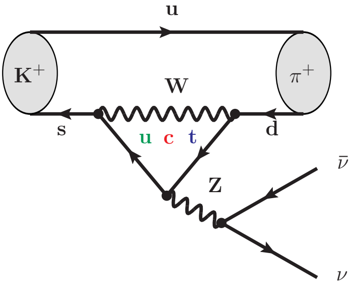
- Effective Hamiltonian $\mathcal{H}^{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{\text{CKM}} \sum C_i(\mu) Q_i$
- Advantage: short-distance information explicit
- Disadvantage: partonic picture
  - hadronic matrix elements from lattice

Low scales:

- Chiral perturbation theory
- Advantage: hadronic picture
- Disadvantage: short-distance information implicit

Contents

$K \to \pi\nu\bar{\nu}$: The main points



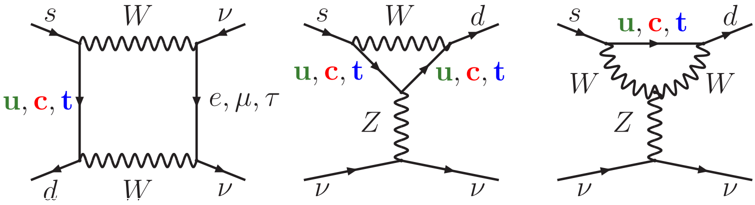
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- FCNC process
- Top-quark dominated, but parametrically suppressed
- Experimentally challenging ($\to$ NA62, Project X)
- Very precise theoretical predictions possible

Why are we able to make Precise Predictions?

- Short-distance (SD) contributions (Wilson coefficients) can be calculated precisely in perturbation theory
- Semileptonic decays $\Rightarrow$ extract matrix elements via isospin symmetry from $K_{\ell 3}$ decays [Marciano, Parsa '96]
- quadratic GIM suppresses long-distance (LD) contributions
- Error mainly parametric – can be reduced in the future

$K \to \pi\nu\bar{\nu}$: Effective Hamiltonian



$$\mathcal{H}^{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_w} \left[\lambda_t \left(\boxed{F(x_t)} - \boxed{F(x_u)} \right) + \lambda_c \left(\boxed{F(x_c)} - \boxed{F(x_u)} \right) \right] Q_\nu$$

Quadratic GIM:
($\lambda \equiv |V_{us}|$)

$\boxed{\lambda^5 \frac{m_t^2}{M_W^2}}$ $\boxed{\lambda \frac{m_c^2}{M_W^2} \log \frac{m_c^2}{M_W^2}}$ $\boxed{\lambda \frac{\Lambda_{\text{QCD}}^2}{M_W^2}}$

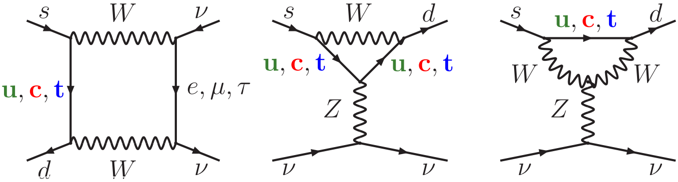
Matrix elements – isospin

$$A(K^+ \to \pi^+ \nu \bar{\nu}) \propto (\lambda_t F_t + \lambda_c F_c) \langle \pi^+ | (\bar{s} d)_{V-A} | K^+ \rangle (\bar{\nu} \nu)_{V-A}$$

$K \to \pi \nu \nu$		$K \to \pi \ell \nu$
$\langle \pi^+ \vert (\bar{s} d)_{V-A} \vert K^+ \rangle$	$=$	$\sqrt{2} \times \langle \pi^0 \vert (\bar{s} u)_{V-A} \vert K^+ \rangle$
Unknown		Well measured

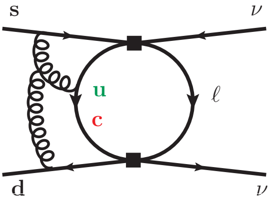
Isospin-breaking (plus QED radiative) corrections have been computed to (N)NLO in chiral perturbation theory
[Mescia, Smith '07]

$K^+ \to \pi^+ \nu \bar{\nu}$: Branching Ratio



$$B\left(K^+ \to \pi^+ \nu \bar{\nu}(\gamma)\right) = \kappa_+ (1 + \Delta_{\mathrm{EM}}) \\ \times \left| \frac{\lambda_t \boxed{X_t(m_t^2)} + \lambda^4 \lambda_c \left(\boxed{P_c(m_c^2)} + \boxed{\delta P_{c,u}} \right)}{\lambda^5} \right|^2 .$$

$K^+ \to \pi^+\nu\bar{\nu}$: Theoretical Status



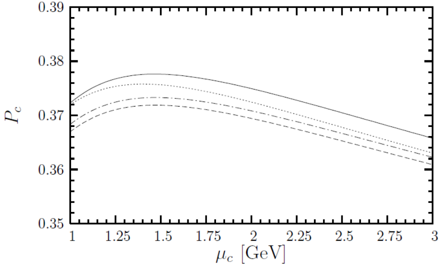
- X_t: Scale dependence reduced to $\pm 1\%$ (NLO QCD)
  [Misiak, Urban '99; Buchalla, Buras '99]
- P_c: Scale dependence reduced to $\pm 2.5\%$ (NNLO QCD)
  [Buras, Gorbahn, Haisch, Nierste '06]
- $\delta P_{c,u}$ enhances branching ratio by 6%
  [Falk, Lewandowski, Petrov '01; Isidori, Mescia, Smith '05]

$K^+ \to \pi^+ \nu\bar{\nu}$: Electroweak Corrections

$$\mathcal{H}^{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \boxed{\sin^2\theta_w}} \left[\lambda_t \left(X_t + \boxed{X_t^{(EW)}} \right) + \lambda_c \left(P_c + \boxed{P_c^{(EW)}} \right) \right] Q_\nu$$

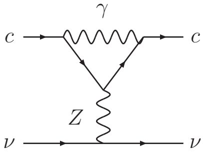
- Match precision achieved in matrix elements
- Fix input parameters:
  - $\sin^2\theta_w(\text{on-shell}) = 0.2233$
  - $\sin^2\theta_w(\overline{\text{MS}}) = 0.2315$
- Naively, $\mathcal{O}(4\%)$ uncertainty in $\sin^2\theta_w$
- P_c has a large QED log

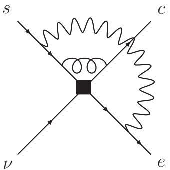
$K^+ \to \pi^+ \nu\bar{\nu}$: Electroweak Corrections to P_c

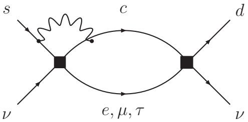


P_c increases by up to 2%

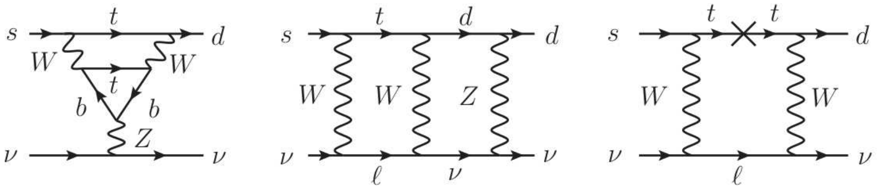
[Brod, Gorbahn '08]

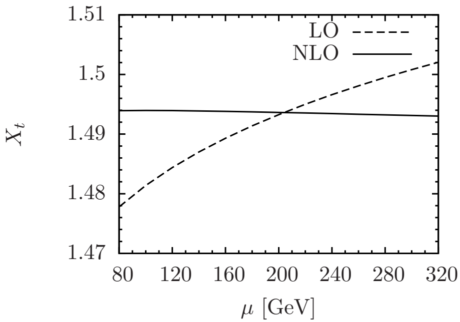


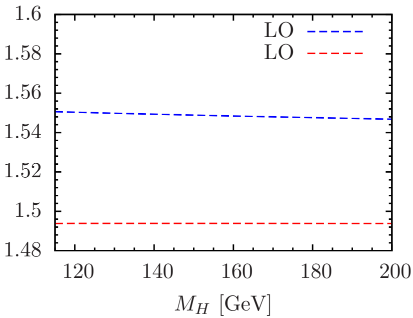




$K^+ \to \pi^+ \nu\bar{\nu}$: Electroweak Corrections to X_t

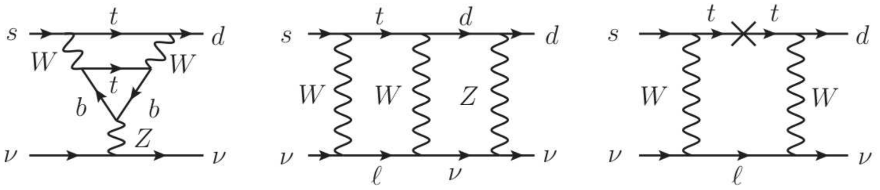


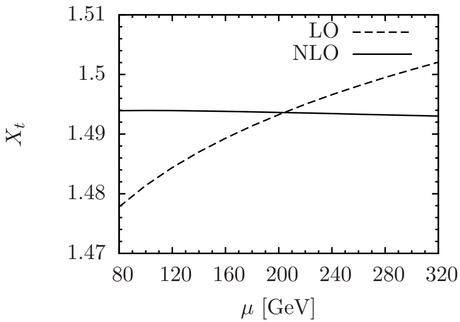


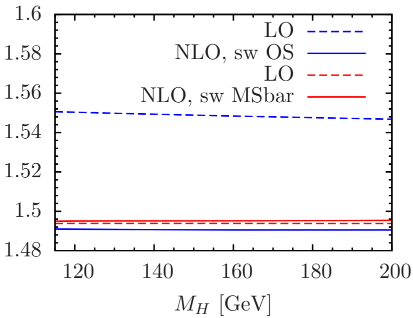


[Brod, Gorbahn, Stamou '11]

$K^+ \to \pi^+ \nu\bar{\nu}$: Electroweak Corrections to X_t

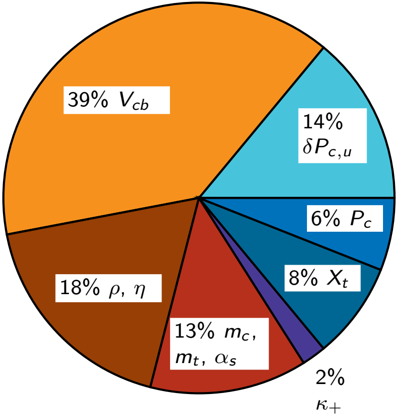






[Brod, Gorbahn, Stamou '11]

$K^+ \to \pi^+ \nu \bar{\nu}$: Error Budget



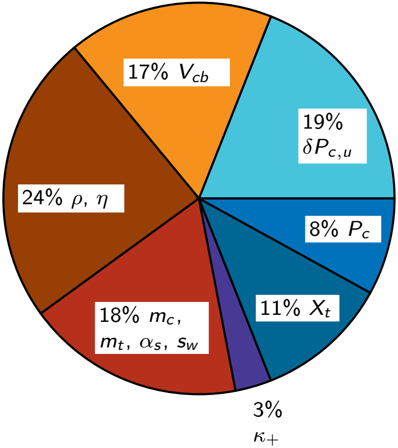
- main parametric error V_{cb}
- Improve on $\delta P_{c,u}$ by a lattice calculation [Isidori et al. '06]

$$\mathrm{Br}^{\mathrm{th}}\left(K^+\right) = 7.81(75)(29) \times 10^{-11}$$

$$\mathrm{Br}^{\mathrm{exp}}\left(K^+\right) = \left(17.3^{+11.5}_{-10.5}\right) \times 10^{-11}$$

[E787, E949 '08]

$K^+ \to \pi^+\nu\bar{\nu}$: Error Budget – $\delta V_{cb}/V_{cb} = 1\%$



$$\mathrm{Br}^{\mathrm{th}}\left(K^+\right) = 7.81(37)(29) \times 10^{-11}$$

$$\mathrm{Br}^{\mathrm{exp}}\left(K^+\right) = (17.3^{+11.5}_{-10.5}) \times 10^{-11}$$

[E787, E949 '08]

$K_L \to \pi^0 \nu\bar{\nu}$: Overview

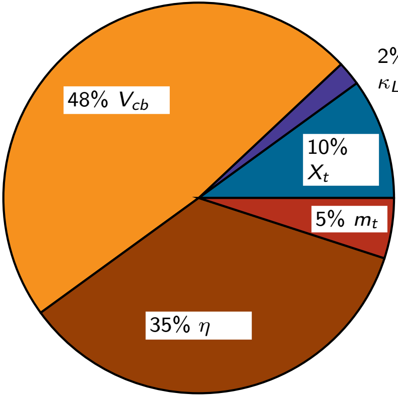
$K_L \to \pi^0 \nu\bar{\nu}$ is almost purely CP-violating:

$$\mathrm{DCPV} : \mathrm{ICPV} : \mathrm{CPC} = 1 : 10^{-2} :\lesssim 10^{-4}$$ [Buchalla, Isidori '98]

$\mathrm{Im}(\lambda_c) = 0 \quad \Rightarrow \quad$ completely top-quark dominated

$$\mathrm{Br}\left(K_L \to \pi^0 \nu\bar{\nu}\right) = \kappa_L \left[\frac{\mathrm{Im}(\lambda_t)}{\lambda^5} \boxed{X_t(m_t^2)} \right]^2$$

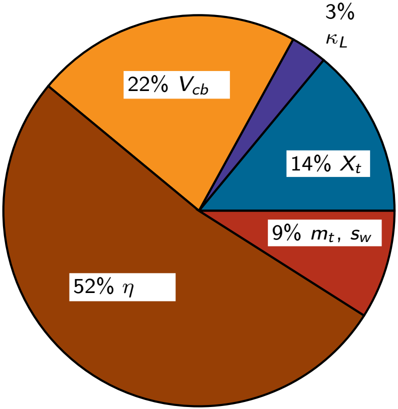
$K_L \to \pi^0 \nu\bar{\nu}$: Error Budget



$$\mathrm{Br}^{\mathrm{th}}(K_L) = 2.43(39)(6) \times 10^{-11}$$

$$\mathrm{Br}^{\mathrm{exp}}(K_L) < 2.6 \times 10^{-8}$$ [E391a '08]

$K_L \to \pi^0 \nu\bar{\nu}$: Error Budget – $\delta V_{cb}/V_{cb} = 1\%$

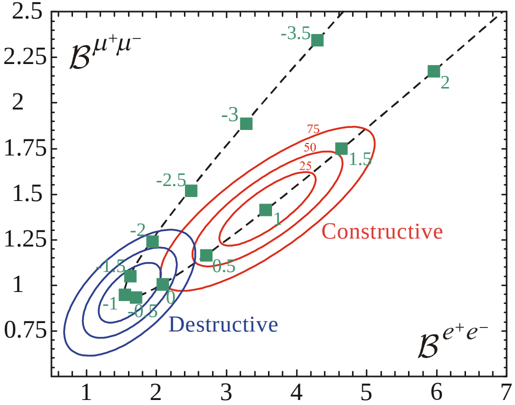


$$\mathrm{Br}^{\mathrm{th}}(K_L) = 2.43(25)(6) \times 10^{-11}$$

$$\mathrm{Br}^{\mathrm{exp}}(K_L) < 2.6 \times 10^{-8}$$ [E391a '08]

Contents

$K_L \to \pi^0 \ell^+ \ell^-$: Motivation

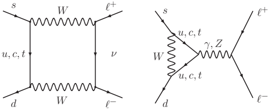


[Mescia, Smith, Trine '06]

$K_L \to \pi^0 \ell^+ \ell^-$: Three Contributions

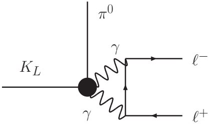
$$Q_{7V[A]} = (\bar{s}_L \gamma_\mu d_L)(\bar{\ell}\gamma^\mu[\gamma_5]\ell)$$

DCPV:



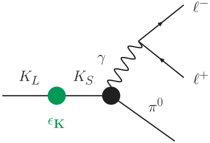
- NLO QCD
  (scale dependence $< 1.5\%$)
  [Buchalla et al. '95]

CPC:



- Estimate from $K_L \to \pi^0 \gamma\gamma$
  [Isidori et al. '04]

ICPV:



- Estimate from ϵ_K, $K_S \to \pi^0 \ell^+ \ell^-$
  [D'Ambrosio et al. '98, Mescia et al. '06]
- Sign of interference with Q_{7V}?
  [Buchalla et al. '03, Friot et al. '04; Bruno et al. '93]

$K_L \to \pi^0 \ell^+ \ell^-$: Branching Ratio

[Mertens, Smith '11]

$$B^{\text{theo}}(K_L \to \pi^0 e^+ e^-) = 3.23\,^{+0.91}_{-0.79}\,[1.37\,^{+0.55}_{-0.43}] \times 10^{-11}$$

$$B^{\text{theo}}(K_L \to \pi^0 \mu^+ \mu^-) = 1.29\,^{+0.24}_{-0.23}\,[0.86\,^{+0.18}_{-0.17}] \times 10^{-11}$$

$$B^{\text{exp}}(K_L \to \pi^0 e^+ e^-) < 28 \times 10^{-11}$$ KTEV [hep-ex/0309072]

$$B^{\text{exp}}(K_L \to \pi^0 \mu^+ \mu^-) < 38 \times 10^{-11}$$ KTEV [hep-ex/0001006]

Uncertainty completely dominated by $K_S \to \pi^0 \ell^+ \ell^-$ measurement!

Contents

$K_L \to \mu^+\mu^-$

Short-distance contribution:

- NLO QCD (top)
  [Buchalla, Buras '99; Misiak, Urban '99]
- NNLO QCD (charm)
  [Gorbahn, Haisch '06]

Long-distance part hard to compute, estimate from $K_L \to \gamma\gamma$, $K_L \to \gamma\ell^+\ell^-$, $K_L \to \mu^+\mu^- e^+ e^-$
[Isidori, Unterdorfer '03]

Still useful constraint:

$$\boxed{\mathrm{Br}(K_L \to \mu^+\mu^-)_{\mathrm{SD}} < 2.5 \times 10^{-9}}$$

Contents

ϵ_K: Introduction

ϵ_K parametrizes indirect CP violation in the neutral kaon system

$$\langle K^0 | \mathcal{H}^{|\Delta S|=2} | \overline{K^0} \rangle \qquad \frac{\mathrm{Im}\langle (\pi\pi)_{I=0} | K^0 \rangle}{\mathrm{Re}\langle (\pi\pi)_{I=0} | K^0 \rangle}$$

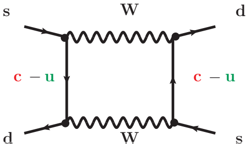
$$\boxed{\epsilon_K = \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})}{\Delta m_K} + \xi \right)}$$

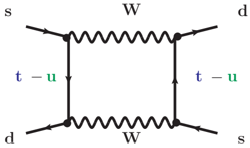
From experiment

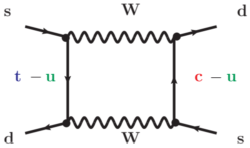
- $\xi = -1.63(19)(20) \times 10^{-4}$ [Blum et al. '11]
- $B_K = 0.737(20)$ [Aubin et al.; Aoki et al.; Dürr et al. '10; Bae et al.; '11]

ϵ_K: Effective Hamiltonian

$$H^{|\Delta S|=2} \propto \left[\lambda_c^2 \eta_{cc} \boxed{S(x_c)} + \lambda_t^2 \eta_{tt} \boxed{S(x_t)} + \lambda_c \lambda_t \eta_{ct} \boxed{S(x_c, x_t)} \right] Q^{|\Delta S|=2}$$







$\eta_{cc} = 1.87(76)$
@ NNLO

[Brod, Gorbahn '11]

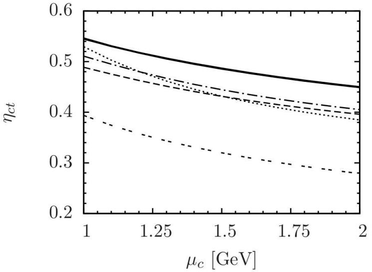
$\eta_{tt} = 0.5765(65)$
@ NLO

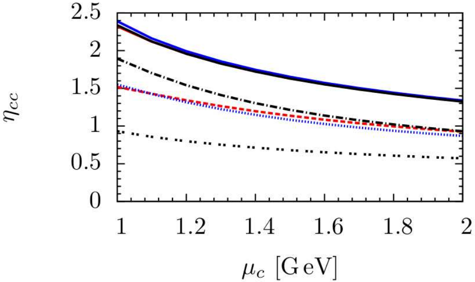
[Buras et al. '90]

$\eta_{ct} = 0.496(47)$
@ NNLO

[Brod, Gorbahn '10]

ϵ_K: Scale dependence

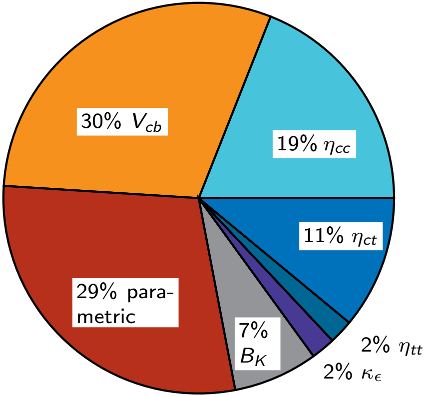




- Compute NNLO lattice matching
  [Gorbahn, Jäger, work in progress]
- Dynamical charm on the lattice

ϵ_K: Error Budget

$$\epsilon_K \propto \kappa_\epsilon B_K |V_{cb}|^2 \xi_s \sin\beta \times \left[|V_{cb}|^2 \xi_s \cos\beta \eta_{tt} S(x_t) + \eta_{ct} S(x_c, x_t) - \eta_{cc} S(x_c) \right]$$



$\epsilon_K \overset{\text{theo}}{=} 1.81(28) \times 10^{-3}$ [Brod, Gorbahn '11]

$\epsilon_K \overset{\text{exp}}{=} 2.228(11) \times 10^{-3}$ [PDG '10]

Conclusion

	SM	Experiment
$K^+ \to \pi^+ \nu\bar{\nu}$	$7.81(75)(29) \times 10^{-11}$	$(1.73\,^{+1.15}_{-1.05}) \times 10^{-10}$ E787 E949
$K_L \to \pi^0 \nu\bar{\nu}$	$2.43(39)(6) \times 10^{-11}$	$< 2.6 \times 10^{-8}$ E391a
$K_L \to \pi^0 e^+ e^-$	$(3.23\,^{+0.91}_{-0.79}) \times 10^{-11}$	$< 28 \times 10^{-11}$ KTEV
$K_L \to \pi^0 \mu^+ \mu^-$	$(1.29\,^{+0.24}_{-0.23}) \times 10^{-11}$	$< 38 \times 10^{-11}$ KTEV
$K_L \to \mu^+ \mu^-$	$< 2.5 \times 10^{-9}$(SD)	$(6.84 \pm 0.11) \times 10^{-9}$ PDG
ϵ_K	$1.81(28) \times 10^{-3}$	$2.228(11) \times 10^{-3}$ PDG
$K_L \to e^\pm \mu^\mp$	≈ 0	$< 4.7 \times 10^{-12}$ B871