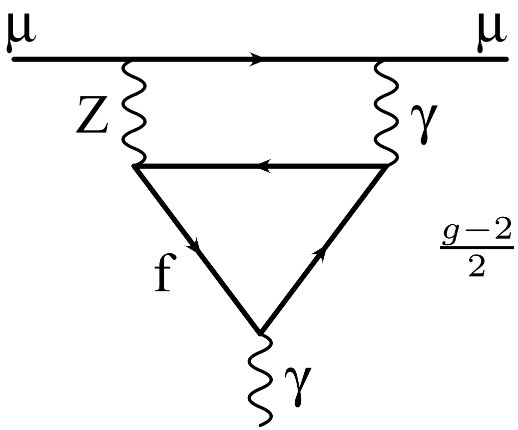
The anomaly triangle and muon $g - 2$

SANTI PERIS

San Francisco State U. and U. Autonoma de Barcelona

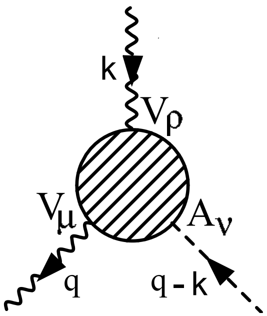
2-loop EW contribution to $g-2$

Kukhto et al. '92
SP, Perrottet, de Rafael '95
Czarnecki, Krause, Marciano '95, '96
Knecht, SP, Perrottet, de Rafael '02
Czarnecki, Marciano,Vainshtein '03



$$\frac{g-2}{2} \propto \frac{\alpha}{\pi}\frac{G_\mu}{8\pi^2}\frac{m_\mu^2}{\sqrt{2}} \int_{m_\mu^2}^{\infty} dQ^2 \underbrace{\left[w_L(Q^2) + \frac{M_Z^2}{M_Z^2+Q^2}\, w_T(Q^2) \right]}_{w_L=2\ w_T=2\frac{N_c}{Q^2}\quad (one-loop,\ m_f=0)}$$

$$\frac{g-2}{2}|_{e,u,d} \sim 2\times 10^{-11}$$



$Q^2=-q^2$, "Gluon-irreducible" quark triangle

$$W_{\mu\nu\rho}(q,k) = T_f^{(3)} Q_f^2 \left[w_L(Q^2)\, q_\nu\, \epsilon_{\mu\rho\alpha\beta} q^\alpha\, k^\beta + w_T(Q^2)\, k^\sigma \left(q^2 \epsilon_{\mu\nu\rho\sigma} + q_\nu \epsilon_{\mu\rho\lambda\sigma} q^\lambda + q_\mu \epsilon_{\rho\nu\lambda\sigma} q^\lambda \right) \right] + \mathcal{O}(k^2)$$

- $w_L(Q^2)$ related to the chiral anomaly $\Longrightarrow \sum_{\nu,e,u,d}\left(T_f^{(3)} Q_f^2\right)\, w_L = 0.$
- $w_T(Q^2)$ not but...

Theorem

Vainshtein '02

Knecht, SP, Perrottet, de Rafael, '03

In the massless limit, to all orders in α_s:

$$\boxed{w_L(Q^2) = 2\, w_T(Q^2)}$$

and, since anomaly does not get renormalized: $w_L = 2\frac{N_c}{Q^2}$ exact! (Adler,Bardeen '69; Witten '83)

$\Rightarrow$ neither does $2w_T = 2\frac{N_c}{Q^2}$, to all orders in α_s.

Using $L_\mu^{(3)} = \sum_{\ell=\nu,e} \overline{\ell_L}\gamma_\mu T^{(3)}\ell_L + \sum_{q=u,d} \overline{q_L}\gamma_\mu T^{(3)} q_L$, etc...in $SU(2)_L \times U(1)_Y$:

$$Q^2 \left[w_L(Q^2) - 2w_T(Q^2)\right]_{quarks} \propto \int d^4x d^4y\, e^{iqx}\, (y-x)_\lambda \epsilon^{\mu\nu\rho\lambda} \underbrace{\left\langle T\left\{L_\mu^{(3)}(x) V_\nu^{(Y)}(y) R_\rho^{(Y)}(0)\right\}\right\rangle}_{=0,\ \text{(Pert. Theory)}}$$

- i.e., $w_L - 2w_T$ has no pert. contributions in α_s, it is like, e.g., $\langle LR\rangle = \langle VV - AA\rangle$.

Non-perturbative effects

1) Adler-Bardeen-Witten : $w_L(Q^2) = 2\frac{N_c}{Q^2}$ (exact for all Q !)

2) However, for $w_T(Q^2)$:

- Large Q^2:

$$2w_T(Q^2) \approx 2\frac{N_c}{Q^2}\left(1 + \underline{NO\ \ \alpha_s}\right) + (const.)\,\alpha_s\,\chi\frac{\langle\overline{\psi}\psi\rangle^2}{Q^6} + \mathcal{O}(1/Q^8)$$

Magnetic susceptibility, $\chi = \frac{\Pi_{VT}(0)}{\langle\overline{\psi}\psi\rangle}$, very poorly known.

- Small Q^2:

$$2w_T(Q^2) \approx (const.)\ C_{22}^{(p^6)} + \mathcal{O}(Q^2) \quad , \quad C_{22}^{(p^6)} \sim 1/M^2_{Hadron} \quad \text{(unknown)}$$

Chiral Pert. Theory, $\mathcal{L}_{eff}$ (parity-odd):

(Ebertshauser, Fearing, Scherer '01; Bijnens, Girlanda, Talavera '02, Kampf, Moussallam '09)

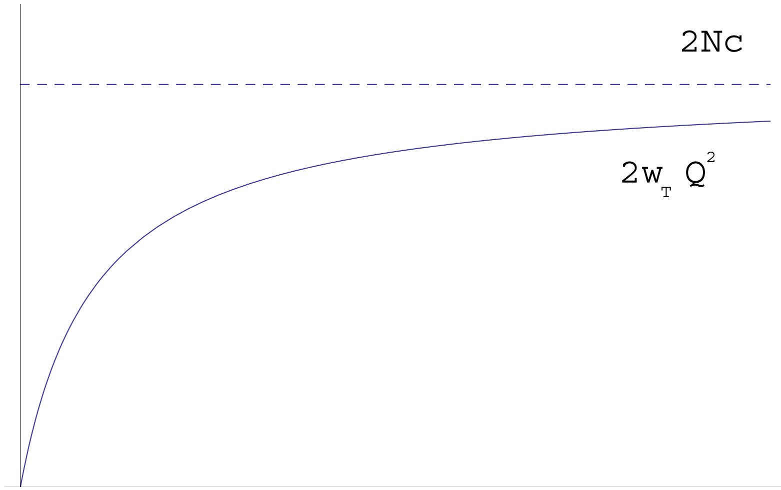
$$\mathcal{L}_{\mathcal{O}(p^6)} = C_{22}^{(p^6)}\,\epsilon_{\mu\nu\alpha\beta}\underbrace{\mathrm{Tr}\left(u^\mu\left\{\nabla_\gamma f_+^{\gamma\nu}, f_+^{\alpha\beta}\right\}\right)}_{\pi,\eta,\ldots} + \ldots \quad ; \quad SU(N_F)_L\times SU(N_F)_R \to SU(N_F)_V$$

Non-perturbative effects (II)

Very roughly,

$$2\, w_T(Q^2)\, Q^2 \quad \sim \quad \overbrace{2N_c}^{Anomaly} \quad \frac{Q^2}{Q^2 + \Lambda^2_{Hadron}}$$

i.e.



Conjectures

- Conjecture 1: $w_L(Q^2) - 2\, w_T(Q^2) = -2\, \frac{N_c}{f_\pi^2}\, \Pi_{LR}(Q^2)$ (Son-Yamamoto '10)

in wide class of "AdS/QCD" models (chiral limit, $N_c \to \infty$)
(not without caveats, e.g. OPE is exponential; wrong chiral limit in pert. theory)

(Knecht, SP, de Rafael '11)

Chiral log's respect this relation in $SU(2) \times SU(2) \times U(1)$ ($m_{u,d} = 0, m_s \neq 0$)
(Gorsky, Kopnin,Krikun, Vainshtein '12)

$$-\,64\pi^2 \quad c_{13}^{(p^6)}(\mu) \;=\; -\;\frac{N_c}{f_\pi^2} \quad \ell_5^{(p^4)}(\mu)$$

However, they don't in $SU(3) \times SU(3)$ ($m_{u,d,s} = 0$) (Knecht, SP , '12 (unpublished))

$$128\pi^2 \quad C_{22}^{(p^6)}(\mu) \;\neq\; -\;\frac{N_c}{f_\pi^2} \quad L_{10}^{(p^4)}(\mu)$$

??

- Conjecture 2: $\chi = -\frac{N_c}{4\pi^2 f_\pi^2} \sim -9\ \mathrm{GeV}^{-2}$ (Magnetic susceptibility) ??

(Vainshtein '02):

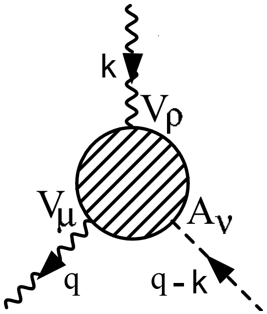
Other results: $\chi \sim -3\ \mathrm{GeV}^{-2}$, sum rules, VMD, (Ioffe, Fadin, Lipatov '10; Balitsky et al. '85; Belyaev et al. '84; Ball et al. '02)

Another Perturbative Surprise

Up to now, special kinematic configuration in $\langle VVA \rangle$.

Jegerlehner,Tarasov '06

However, it has been found at two loops for arbitrary momenta that :



$$W_{\mu\nu\rho}(q,k) \quad = \quad W_{\mu\nu\rho}(q,k)|_{one-loop}\ (1 + \underbrace{\mathcal{O}(\alpha_s)}_{=0\ !!}\ )$$

i.e., no renormalization, not just for the anomaly, but for the whole triangle !

Given the non-trivial momentum dependence,

can this be just a coincidence ?

could this be true to all orders in α_s ?

Summary

- VVA triangle is a very interesting theoretical laboratory for QCD

- Even though most results obtained in chiral limit: can lattice help/check ?

- The LbL $\longleftrightarrow \langle VVA \rangle$ connection:

  (Melnikov, Vainshtein '04; Prades, de Rafael, Vainshtein '09)

  $k_1 \approx k_2 \gg k_3$