Pion and kaon physics

Jack Laiho
University of Glasgow

Project X Workshop
Fermilab June 18, 2012

Introduction

Pion and Kaon physics has become a precision science.

I will cover:

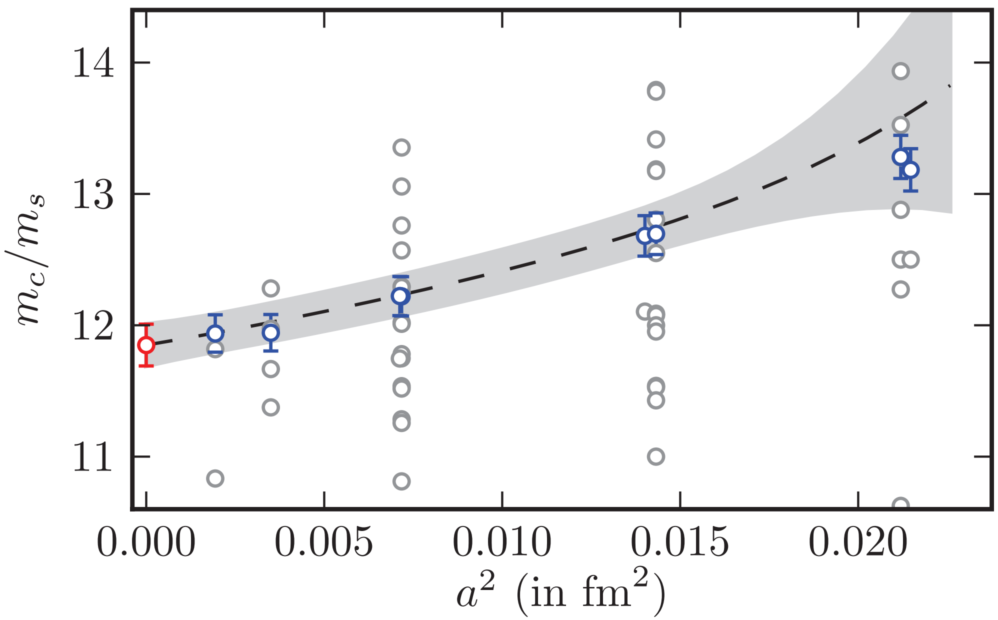
- light quark masses and decay constants
- $K \to \pi \ell \nu$ form factors
- kaon mixing: B_K
- V_{cb}

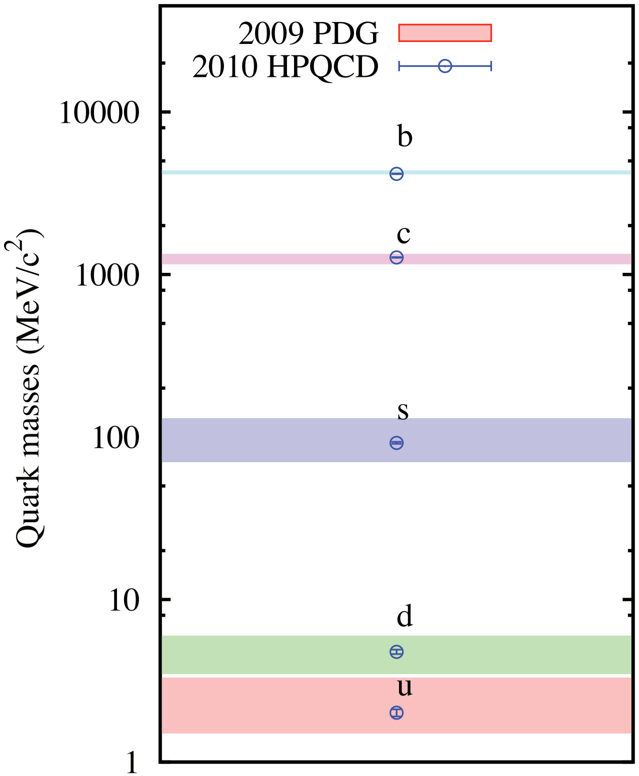
The Lattice calculations

Group	N_f	action	a(fm)	$m_\pi L$	$m_\pi^{\min}$ (MeV) sea/val
ETMC	2	Twisted Mass	0.05-0.10 fm	$\gg 1$	$280/280$
MILC	2+1	(Asqtad) staggered	0.045-0.12 fm	> 4	$250/180$
RBC/UKQCD	2+1	Domain Wall	0.085-0.11 fm	> 4	$290/210$
JLQCD	2+1	Overlap	0.11 fm	≥ 2.7	$310/310$
PACS-CS	2+1	Clover	0.09 fm	≥ 2.0	$140/140$
BMW	2+1	Clover	0.054-0.125 fm	≥ 4	$135/135$
ALV	2+1	DW on MILC	0.06-0.12 fm	> 3.5	$250/210$
HPQCD	2+1	HISQ on MILC	0.045-0.15 fm	≥ 3.7	$360/310$

(In staggered calculations, the sea pion mass quoted is the rms value. The valence pion mass quoted is the taste-goldstone.)

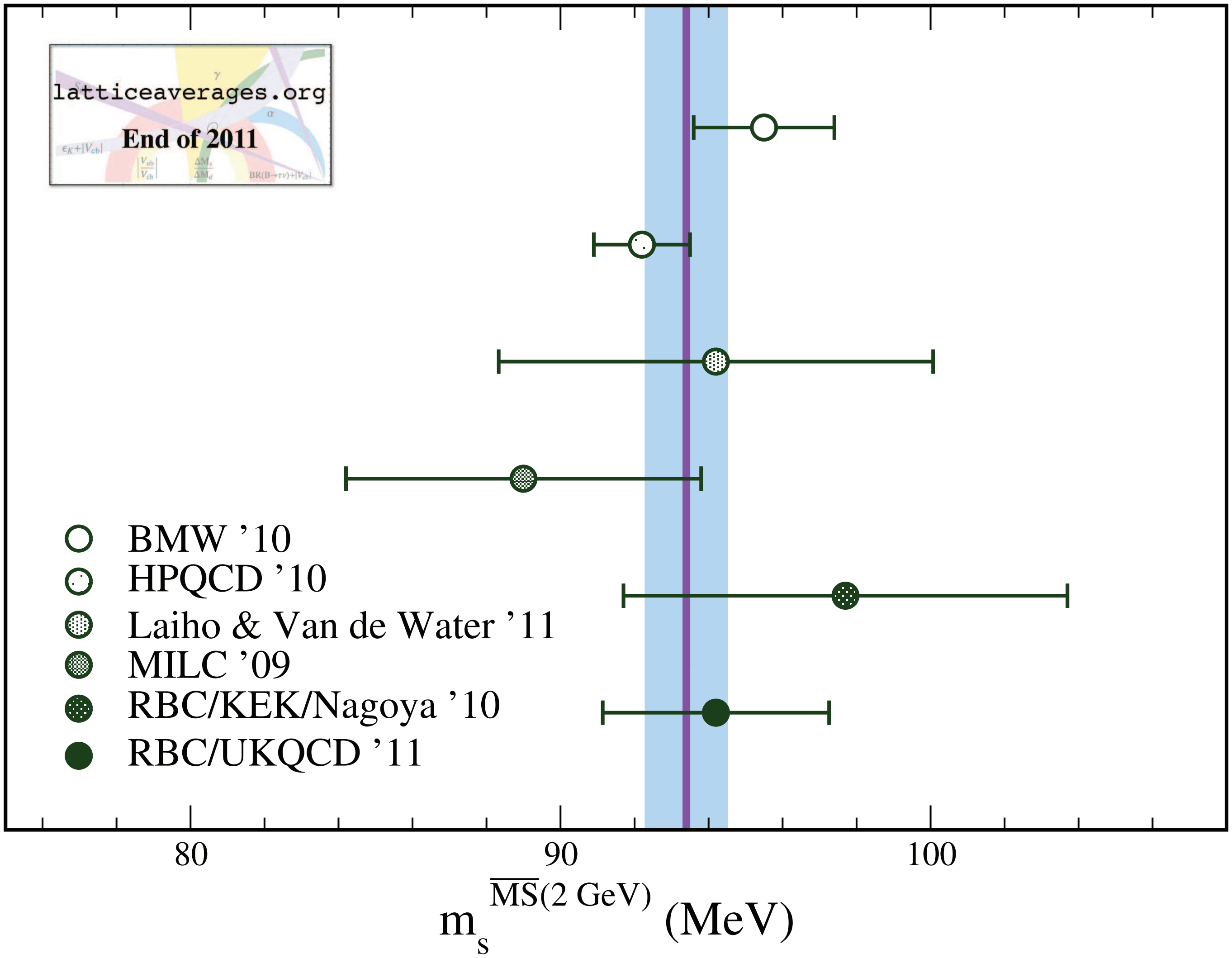
m_c/m_s and light quark masses



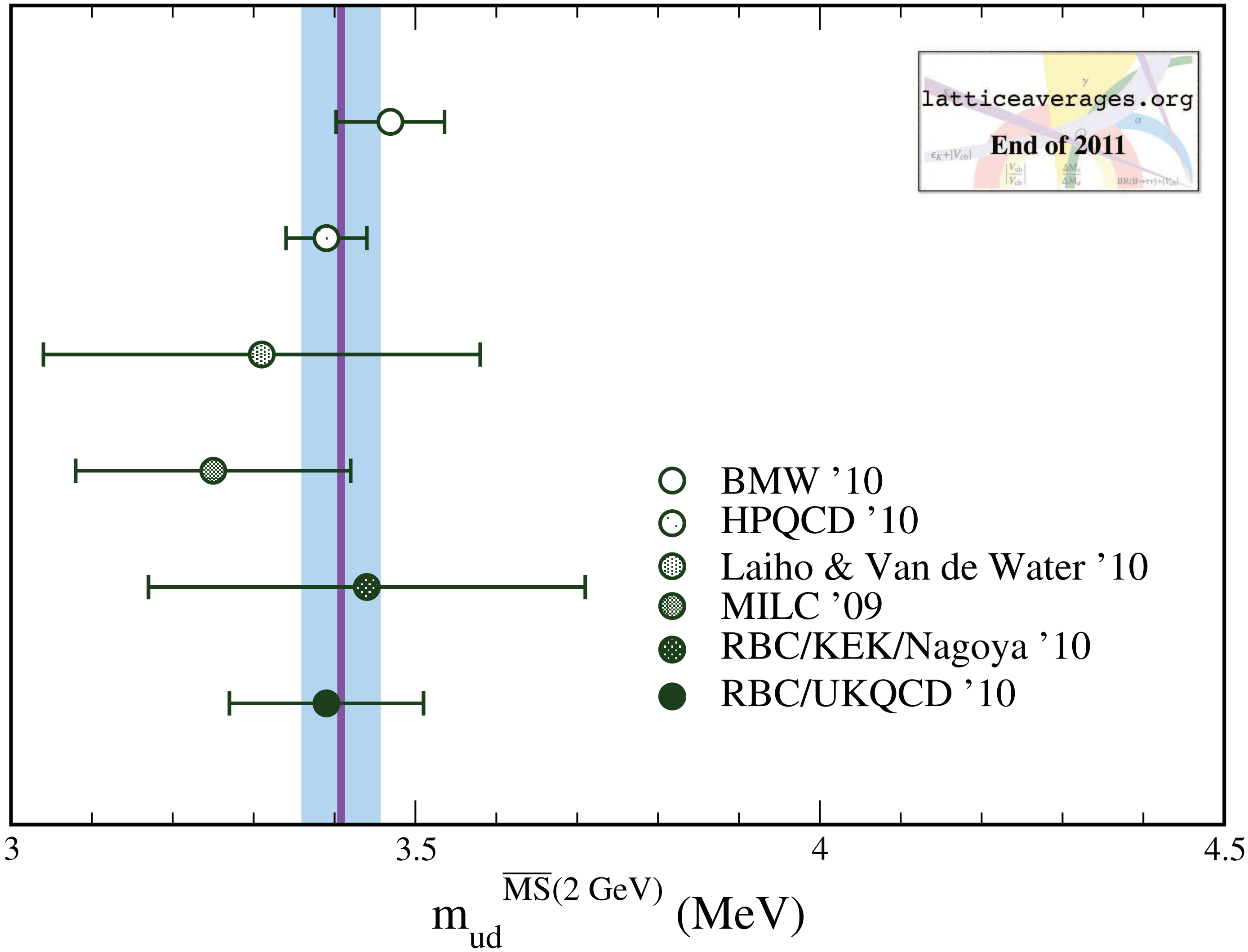


HPQCD uses ratio of m_c/m_s "to cascade the accuracy of the heavy quark mass down to the light quarks." Very fine MILC lattices, down to 0.045 fm, and the HISQ formalism for valence quarks, allow a precise determination of this ratio. ETMC obtain a consistent result for m_c/m_s with a 2 flavor calculation.

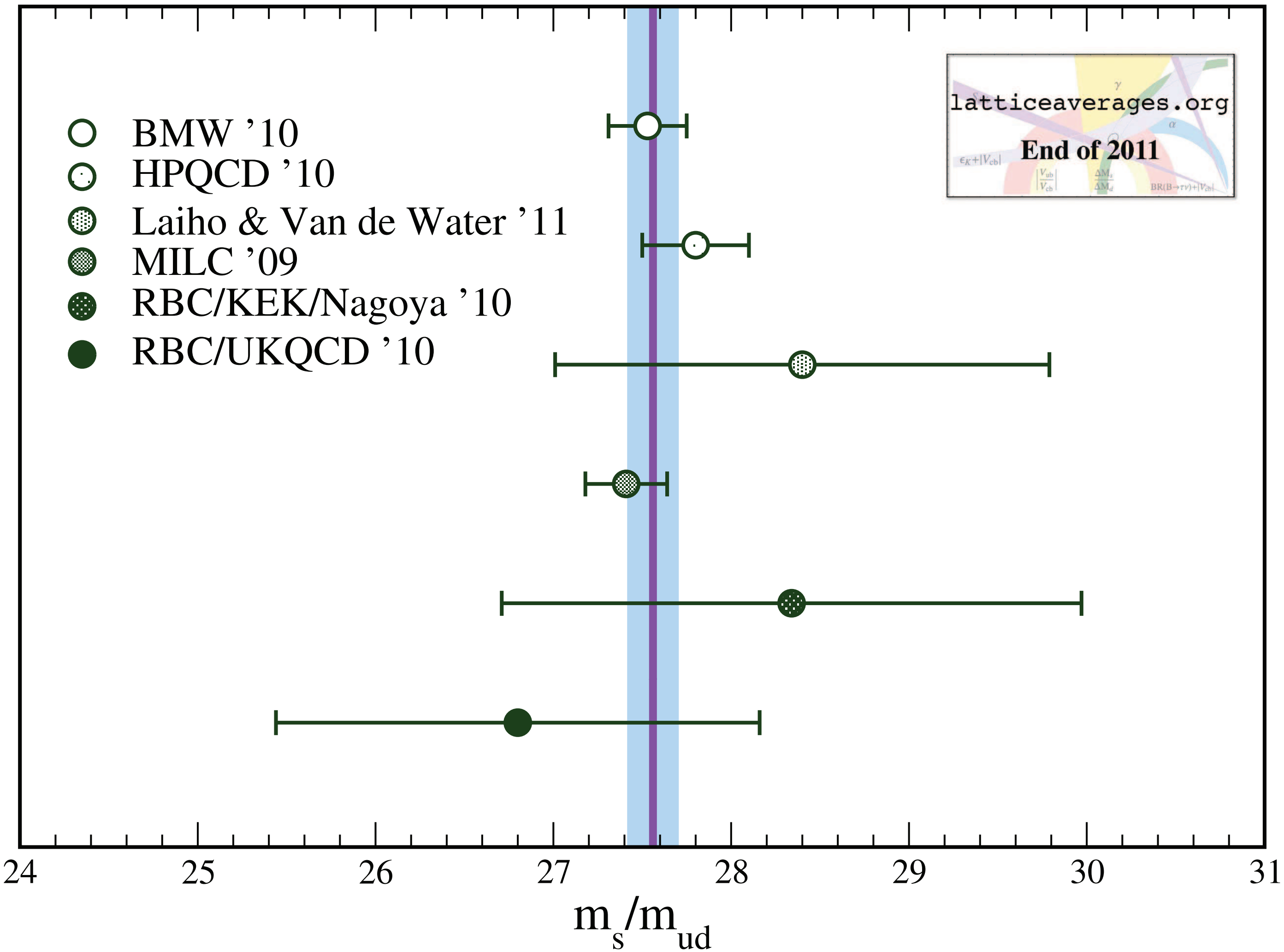
Strange quark mass



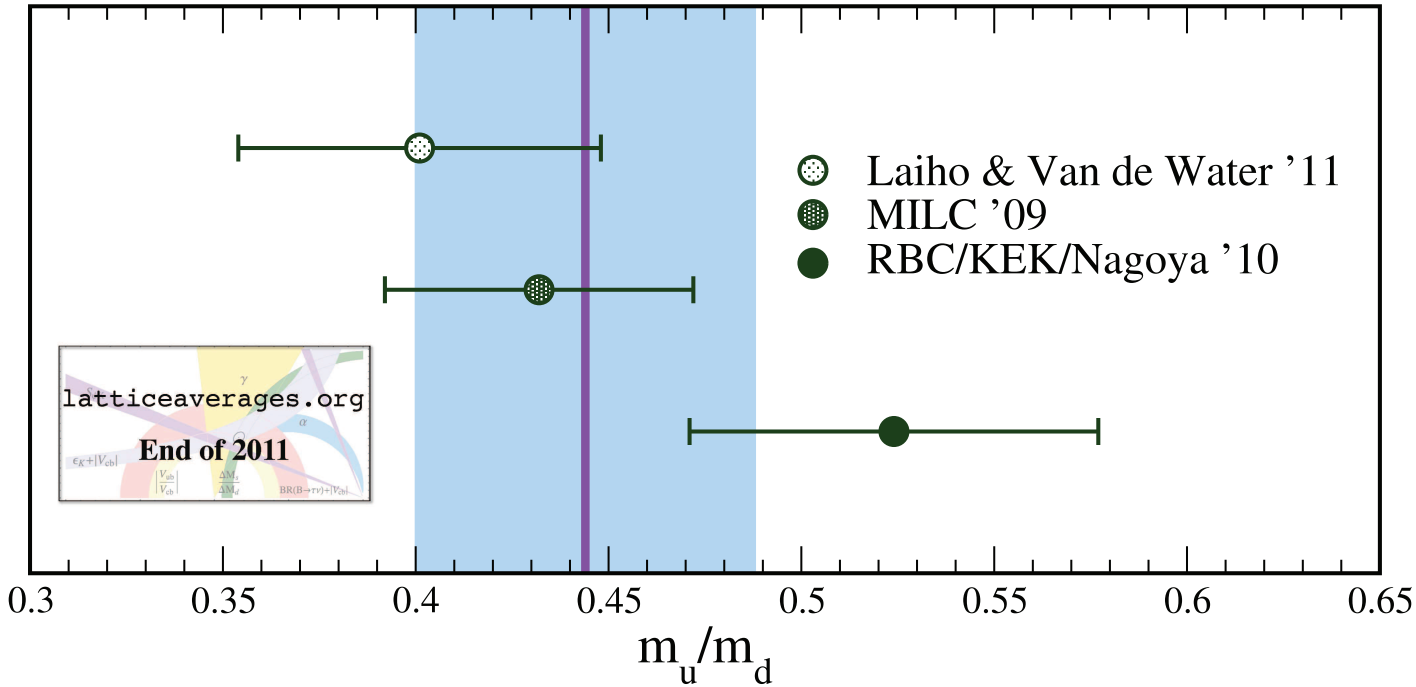
Light-quark mass



Quark mass ratio

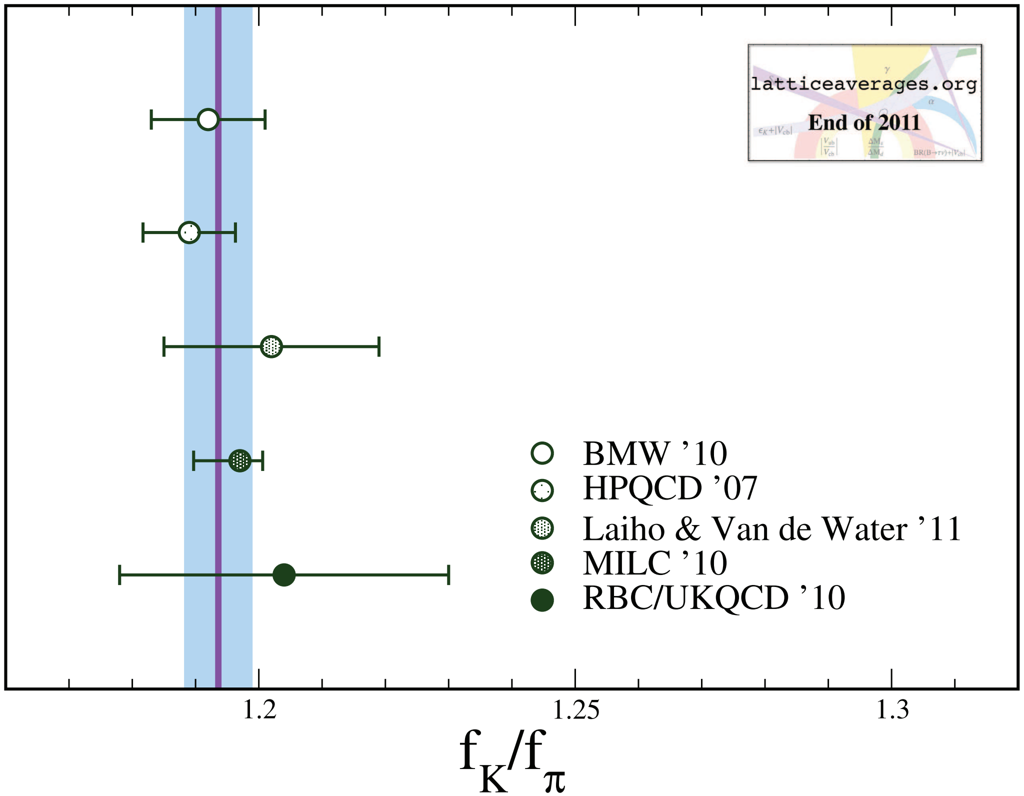


Light quark mass ratio



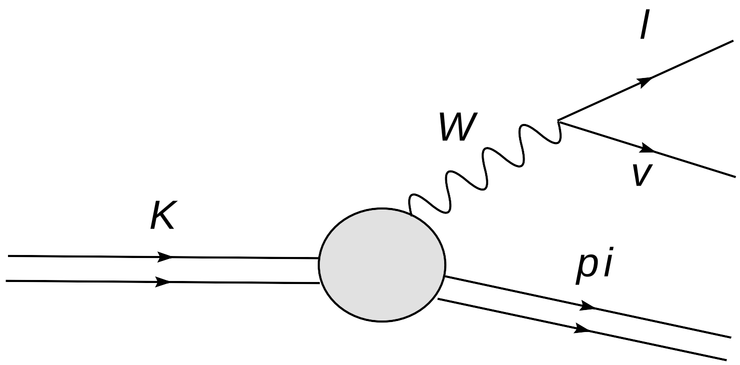
Errors inflated by ~ 1.4 due to somewhat low confidence level. Still $\sim 10\sigma$ from zero.

f_K/f_π



$$\frac{\Gamma(K \to \ell\overline{\nu}_\ell)}{\Gamma(\pi \to \ell\overline{\nu}_\ell)} = \left(\frac{|V_{us}|}{|V_{ud}|}\right)^2 \left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K \left(1 - \frac{m_\ell^2}{m_K^2}\right)^2}{m_\pi \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2} \left[1 + \frac{\alpha}{\pi}(C_K - C_\pi)\right] \qquad (1)$$

$K \to \pi \ell \nu$



$$\Gamma_{K\ell 3} = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{\mathrm{EW}} (|V_{us}| f_+^{K^0\pi^-}(0))^2 I_{K\ell} (1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi})^2, \quad (2)$$

where $S_{EW} = 1.0232(3)$ is the short-distance electroweak correction, C_K is a Clebsch-Gordan coefficient, $f_+^{K^0\pi^-}(0)$ is the form factor at zero momentum transfer, and $I_{K\ell}$ is a phase-space integral that is sensitive to the momentum dependence of the form factors. The quantities $\delta_{\mathrm{EM}}^{K\ell}$ and $\delta_{\mathrm{SU(2)}}^{K\pi}$ are long-distance EM corrections and isospin breaking corrections, respectively.

RBC/UKQCD

New calculation with twisted boundary conditions to directly simulate at $q^2 = 0$ (arXiv:1004.0886).

No interpolation in momentum transfer necessary.

New estimation of chiral extrapolation errors, and slightly different choice of extrapolation method. RBC/UKQCD obtain

$$f_+^{K\pi}(0) = 0.9599(34)(^{+31}_{-43})(14) \quad (3)$$

where the first error is statistical, the second is due to the chiral extrapolation, and the third is an estimate of discretization effects.

ETMC

Uses multiple lattice spacings, several light quark masses corresponding to pions as light as 260 MeV, and twisted boundary conditions.

The calculation is 2 flavor, i.e. the strange quark is quenched, but there is an estimate for the error due to quenching the strange quark using χPT through NLO.

FNAL/MILC

No number yet, but work is in progress.

Uses method developed by HPQCD for $D \to K\ell\nu$ to get result for $f_+^{K\pi}(0)$

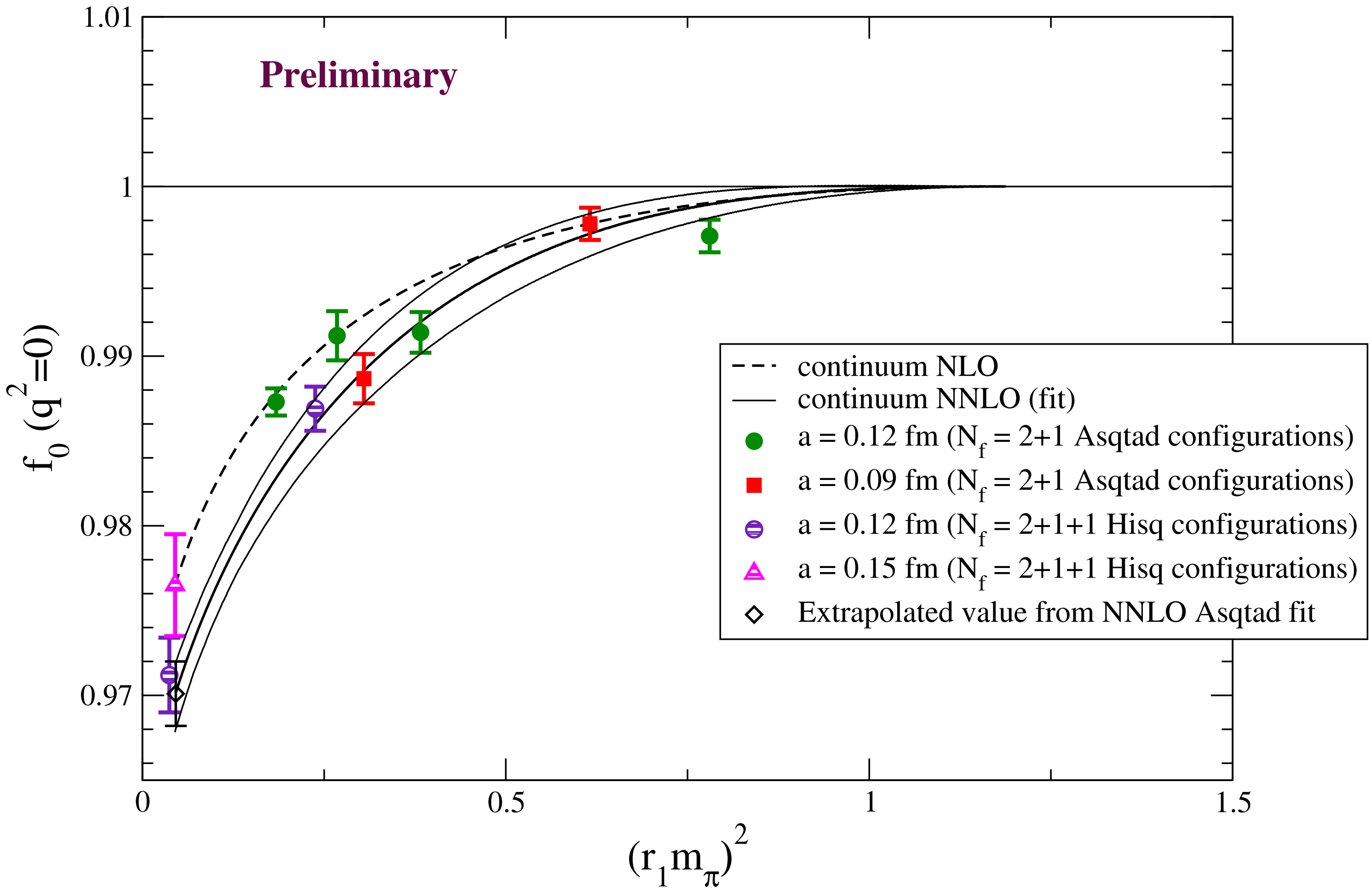
$$f_+(0) = f_0(0) = \frac{m_s - m_q}{m_K^2 - m_\pi^2} \langle\pi|S|K\rangle|_{q^2=0} \qquad (4)$$

No renormalization is required.

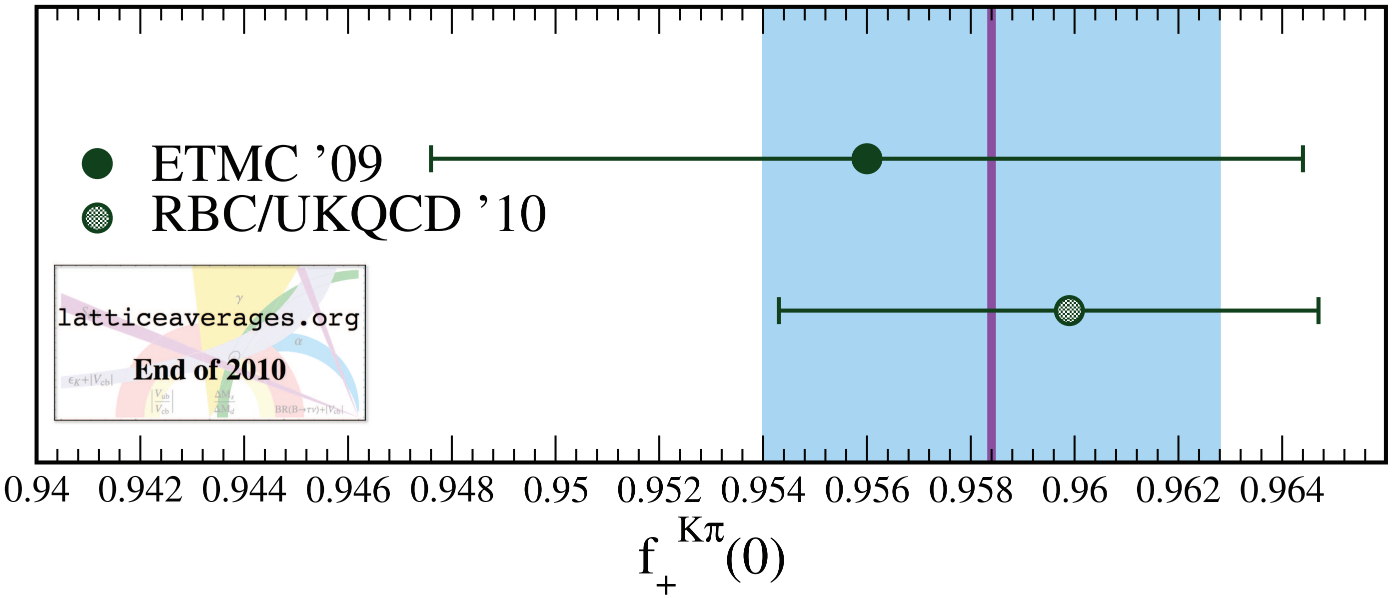
Avoids the use of non-local vector currents. It does not require multiple three-point correlators to form various double ratios.

Disadvantage: One only gets $f_0(q^2)$ for $q^2 \neq 0$, but this is still sufficient to determine $|V_{us}|$.

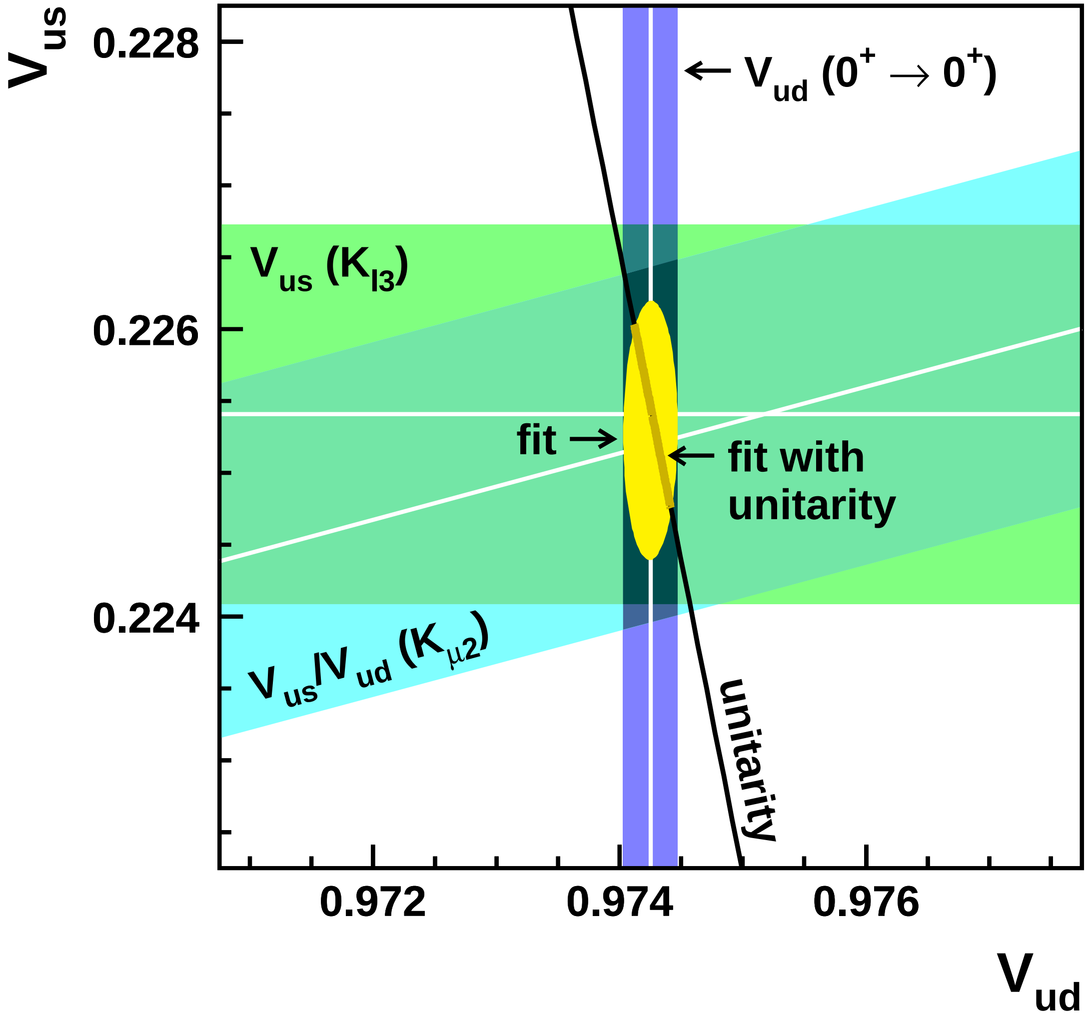
Early look


Preliminary
1.01
1
0.99
0.98
0.97
0
0.5
1
1.5
$f_0\,(q^2=0)$
$(r_1 m_\pi)^2$
continuum NLO
continuum NNLO (fit)
a = 0.12 fm (N_f = 2+1 Asqtad configurations)
a = 0.09 fm (N_f = 2+1 Asqtad configurations)
a = 0.12 fm (N_f = 2+1+1 Hisq configurations)
a = 0.15 fm (N_f = 2+1+1 Hisq configurations)
Extrapolated value from NNLO Asqtad fit

$K \to \pi\ell\nu$

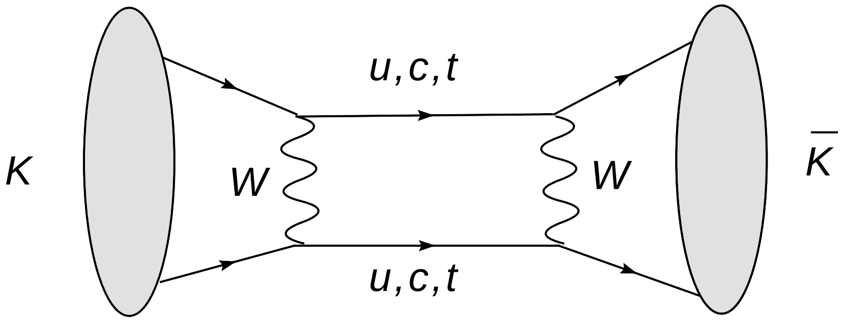

ETMC '09
RBC/UKQCD '10
latticeaverages.org
End of 2010
0.94
0.942
0.944
0.946
0.948
0.95
0.952
0.954
0.956
0.958
0.96
0.962
0.964
$f_+^{K\pi}(0)$

First row unitarity constraint



Plot from Flavianet.

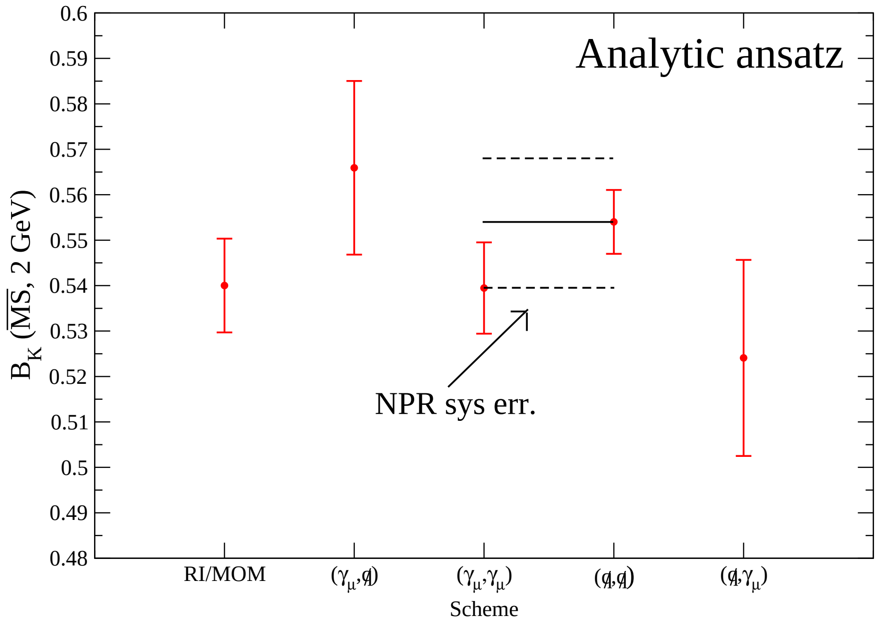
B_K



$$|\epsilon_K| = C_\epsilon \kappa_\epsilon B_K A^2 \overline{\eta} \{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho})\}$$

where C_ϵ is a collection of experimentally determined parameters, κ_ϵ represents long-distance corrections and a correction due to the fact that $\phi_\epsilon \neq 45$ degrees, the $\eta_i S_0$ are perturbative coefficients, the terms in blue are CKM matrix elements in Wolfenstein parameterization.

RBC/UKQCD



Multiple RI-SMOM schemes with non-exceptional momenta are used to determine the matching factor. Different schemes have different one-loop truncation errors, so the perturbative matching error is reduced by taking an average over results from different schemes.

$$B_K(\overline{MS}, 2\ \text{GeV}) = 0.546(7)(16)(3)(14) \quad (5)$$

where errors are: statistical, chiral extrapolation, finite volume, renormalization.

JL and Van de Water

Published first result for B_K with 2+1 flavors and a continuum extrapolation [Aubin, JL, Van de Water, PRD 81 014507 (2010)].

Used domain wall quarks in the valence sector and improved staggered (MILC asqtad) in the sea. Two lattice spacings and non-perturbative renormalization for B_K.

Now Ruth and I are working to improve on this result.

- Adopting new renormalization scheme of RCB/UKQCD with non-exceptional momentum.
- Also added statistics, lighter quark masses, and finer lattice spacing.

SBW

Mixed action approach:

HYP-smeared staggered fermions on MILC Asqtad lattices using 4 lattice spacings down to 0.045 fm.

One-loop perturbative matching.

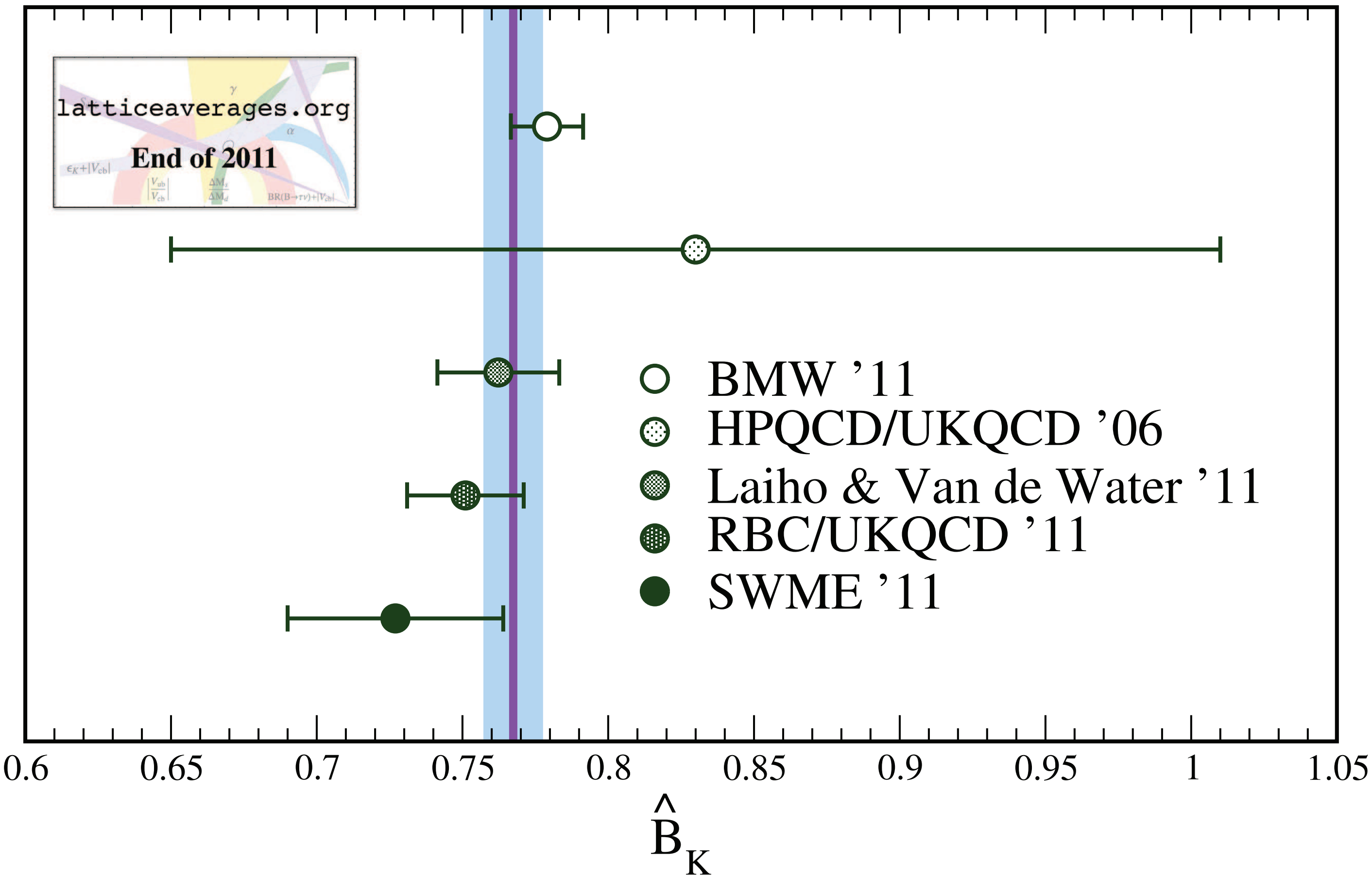
$SU(2)$ chiral perturbation theory is used. This provides much simpler extrapolation formulas than in the $SU(3)$ case, where many new staggered parameters enter.

Preliminary results:

$$\widehat{B}_K = 0.727(4)(37) \tag{6}$$

where errors are statistical and the sum of systematic errors in quadrature.

B_K



latticeaverages.org
End of 2011
BMW '11
HPQCD/UKQCD '06
Laiho & Van de Water '11
RBC/UKQCD '11
SWME '11
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
1.05
$\hat{B}_K$

Importance of $|V_{cb}|$

$|V_{cb}|$ is needed to constrain the apex of the unitarity triangle from kaon mixing (along with B_K). Given that

$$A = \frac{|V_{cb}|}{\lambda^2} \tag{7}$$

has $\approx 2\%$ error, we see that this contributes a 9% error to ϵ_K because it appears in the formula below to the fourth power.

$$|\epsilon_K| = C_\epsilon B_K A^2 \overline{\eta} \{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho})\}$$

Given the progress in B_K, it is essential to improve $|V_{cb}|$ to make full use of the experimental information in ε_K.

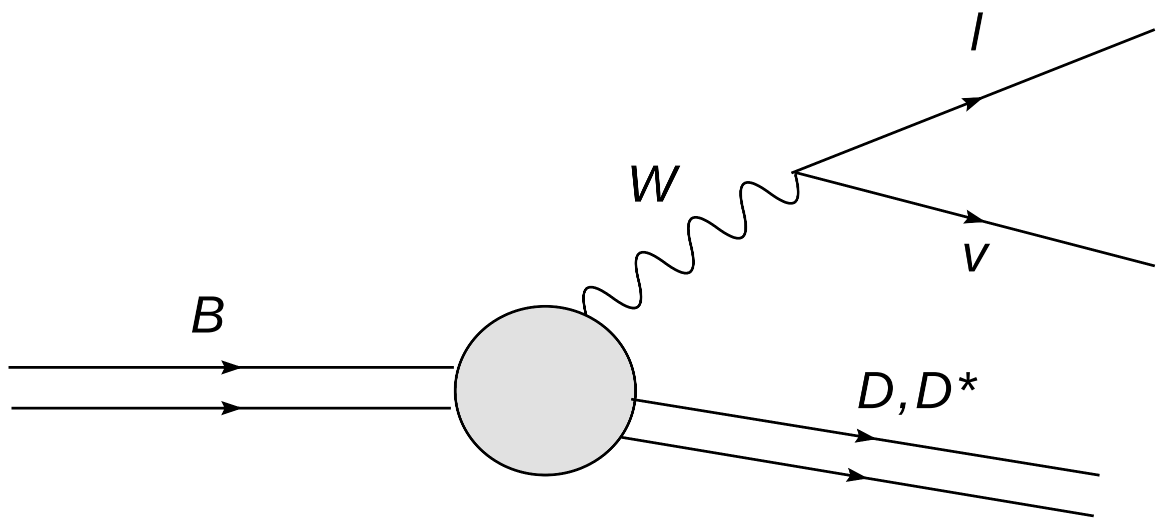
Methods for extracting $|V_{cb}|$

- Inclusive $b \to c\ell\nu$ can be calculated using the OPE and perturbation theory. Requires non-perturbative input from experiment: moments of inclusive form factor $\overline{B} \to X_c \ell \overline{\nu}_\ell$ as a function of minimum electron momentum. Theoretical uncertainties from truncating the OPE and PT, and also perhaps from duality violations.

- Exclusive $B \to D\ell\nu$ has an $\sim 4\%$ experimental error in the zero-recoil point. No problem in principle of going to small recoil on the lattice.

- Exclusive $B \to D^*\ell\nu$ is experimentally cleaner ($\sim 1.7\%$ experimental error at zero-recoil).

Charmed B semileptonic decays



Vertex proportional to $|V_{cb}|$. In order to extract it, nonperturbative input is needed.

Obtaining V_{cb} from $\overline{B} \to D^* l \overline{\nu}_l$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2 - 1} \times |V_{cb}|^2 \mathcal{G}(w) |\mathcal{F}_{B\to D^*}(w)|^2 \quad (8)$$

where $\mathcal{G}(w)|\mathcal{F}_{B\to D^*}|^2$ contains a combination of form-factors that must be computed non-perturbatively. $w = v' \cdot v$ is the velocity transfer from initial (v) to final state (v').

History of $B \to D^*\ell\nu$

2008, first unquenched 2+1 calculation, JL et. al.(FNAL/MILC) PRD79:014506

Used a single double ratio at $w = 1$.

2010, update JL et. al. (FNAL/MILC)

Quadrupled statistics, smaller lattice spacings, completely new data set with retuned parameters.

Heavy quarks on the lattice

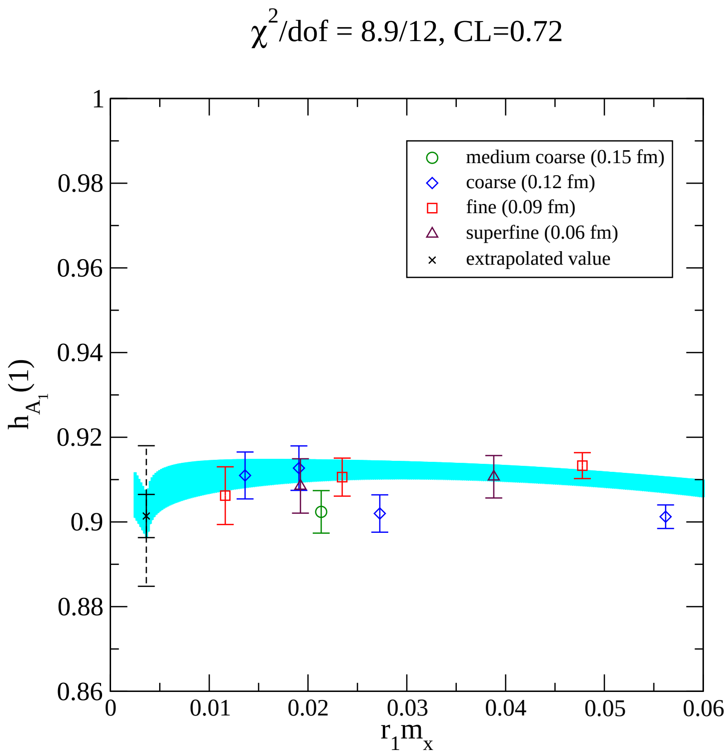
The lattice cut-off is smaller than the heavy quark masses for realistic lattices.
A solution: heavy quark effective theory(HQET)

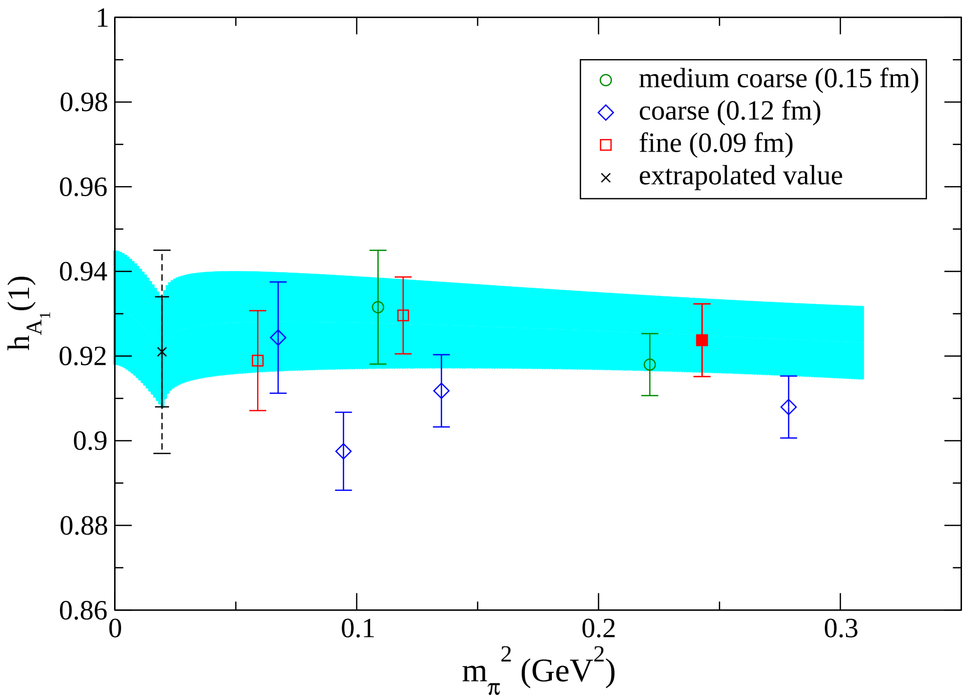
Fermilab Method:

Continuum QCD $\longrightarrow$ Lattice gauge theory
(using HQET)

- Requires tuning parameters of the lattice action. Can be systematically improved by adding higher dimensional operators to the action.
- The currents and 4-quark operators must also be matched to continuum QCD. Typically this is done using lattice perturbation theory.

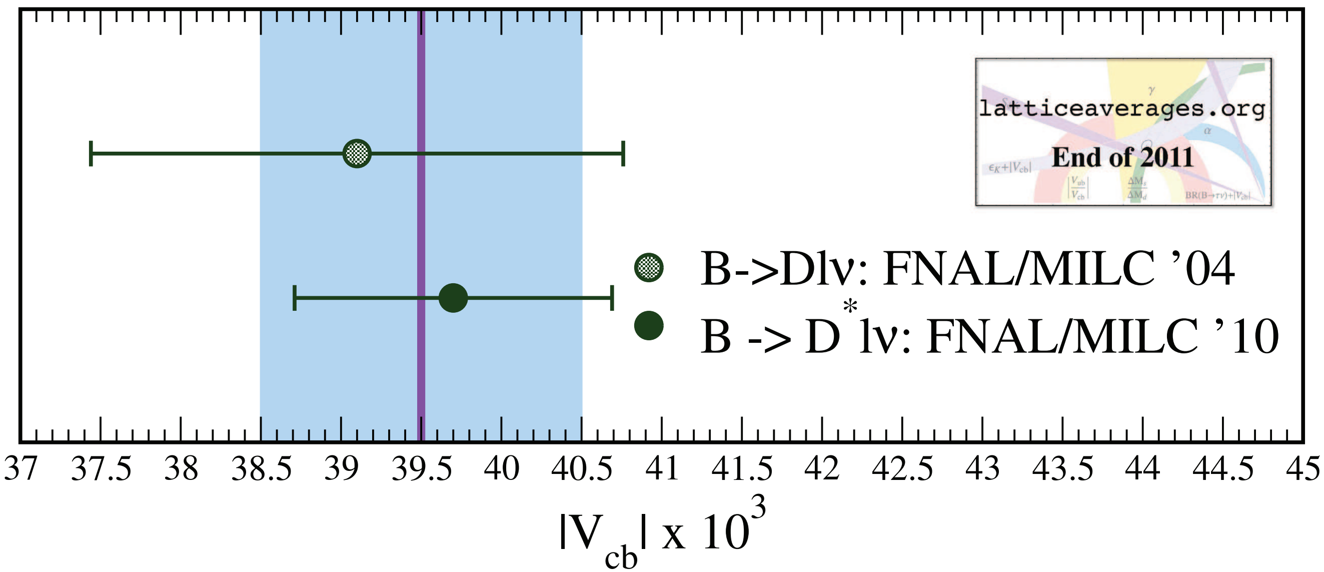
New Data $B \to D^*$ Extrapolation





New (2010) vs Old (2008), FNAL/MILC

$|V_{cb}|$



$|V_{cb}| = (39.5 \pm 1.0) \times 10^{-3}$. Still a 1.6σ discrepancy with the inclusive (non-lattice) method for determining $|V_{cb}|$.

$B \to D\ell\nu$ at non-zero recoil

No published unquenched result.

Errors in experiment are large at zero-recoil, so we work at non-zero recoil on the lattice.

z parameter expansion provides a theoretically constrained parameterization of the entire q^2 (or w) range and allows an extrapolation to the whole q^2 range with minimal model dependence.

Situation similar to the case for $B \to \pi\ell\nu$, so we apply the same methodology.

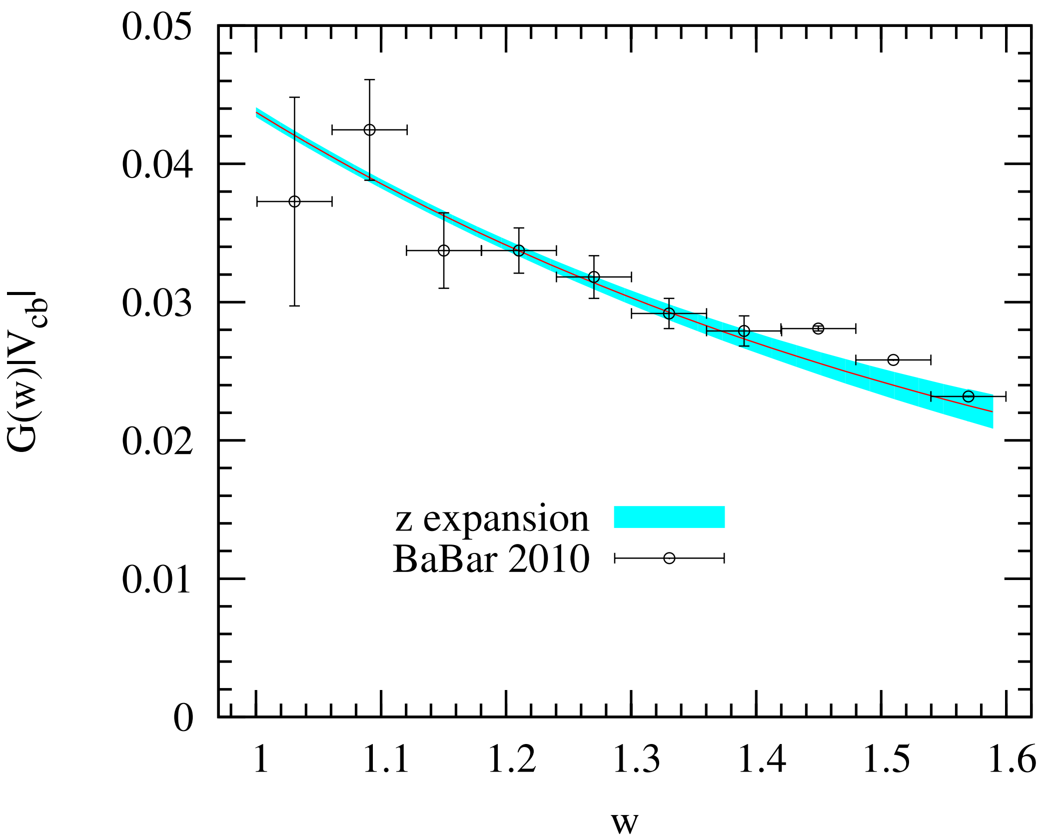
$|V_{cb}|$ from $B \to D\ell\nu$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} \times |V_{cb}|^2 |\mathcal{G}_{B\to D}(w)|^2 \quad (9)$$

where $w = v' \cdot v$ is the velocity transfer from initial (v) to final state (v'), and where

$$\mathcal{G}_{B\to D}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w). \quad (10)$$

$B \to D\ell\nu$ at non-zero recoil



A comparison of the form factor shape using lattice calculations and the z expansion to BaBar data assuming $|V_{cb}| = 41.4 \times 10^{-3}$. More data and analysis in progress for precision determination of $|V_{cb}|$.

Prospects for improvement

Present and projected errors on lattice quantities.

Quantity	CKM	expt. now	lattice now	2014 lat.	2020 lat.	non-lattice method
f_K/f_π	V_{us}	0.2%	0.6%	0.3%	0.1%	-
$f_{K\pi}(0)$	V_{us}	0.2%	0.5%	0.2%	0.1%	1% (ChPT)
$B \to D^*\ell\nu$	V_{cb}	1.8%	1.8%	0.8%	$< 0.5\%$	$< 2\%$ (Incl. $b \to c$)
$B \to D\ell\nu$	V_{cb}	4%	2%	$< 2\%$	$< 0.5\%$	$< 2\%$ (Incl. $b \to c$)

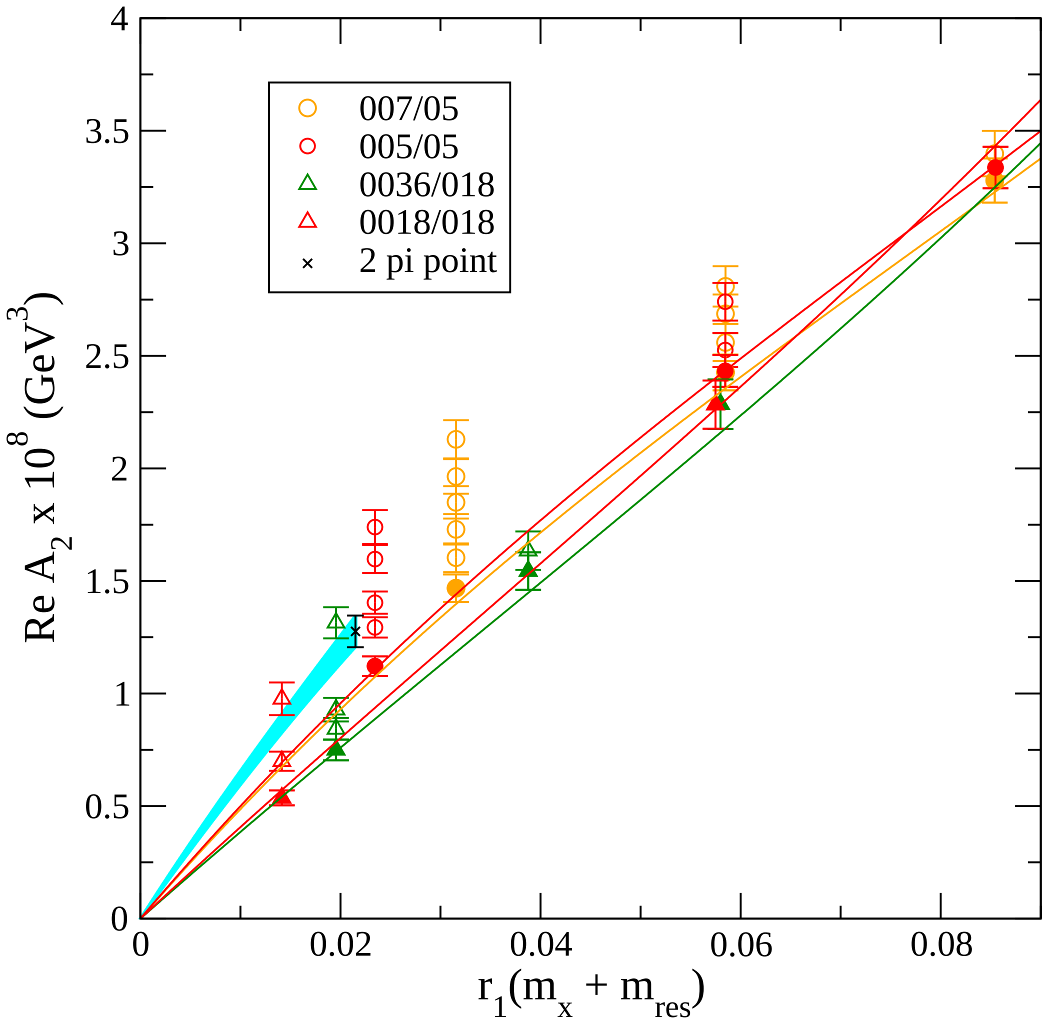
Conclusions

Simple quantities from many different groups using different methods can be calculated with controlled systematic errors.

Many quantities are now precision calculations, and results are in good agreement. Prospects for improvement are excellent!

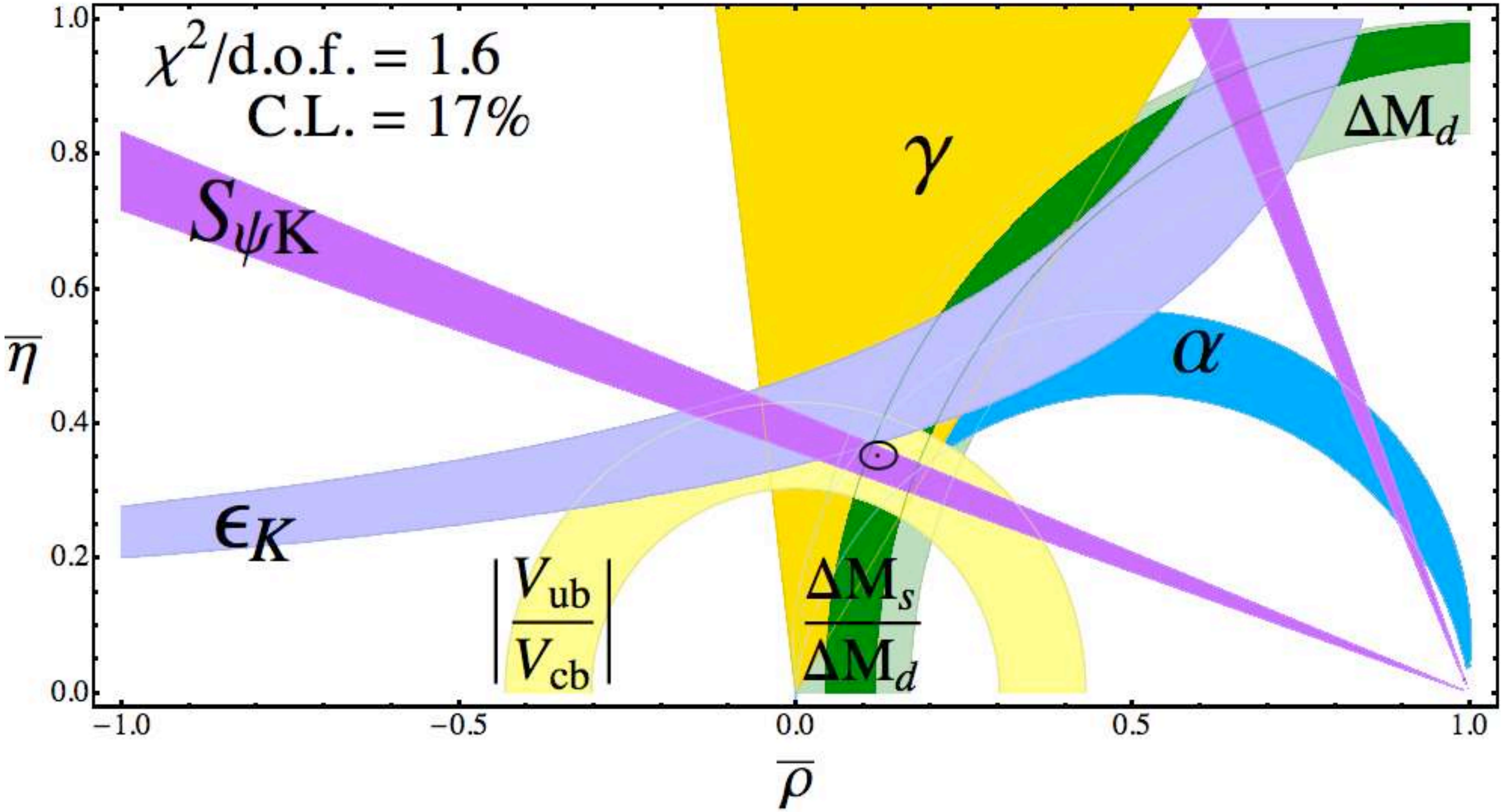
Backup Slides

$K \to \pi\pi$, $\Delta I = 3/2$, $(27, 1)$

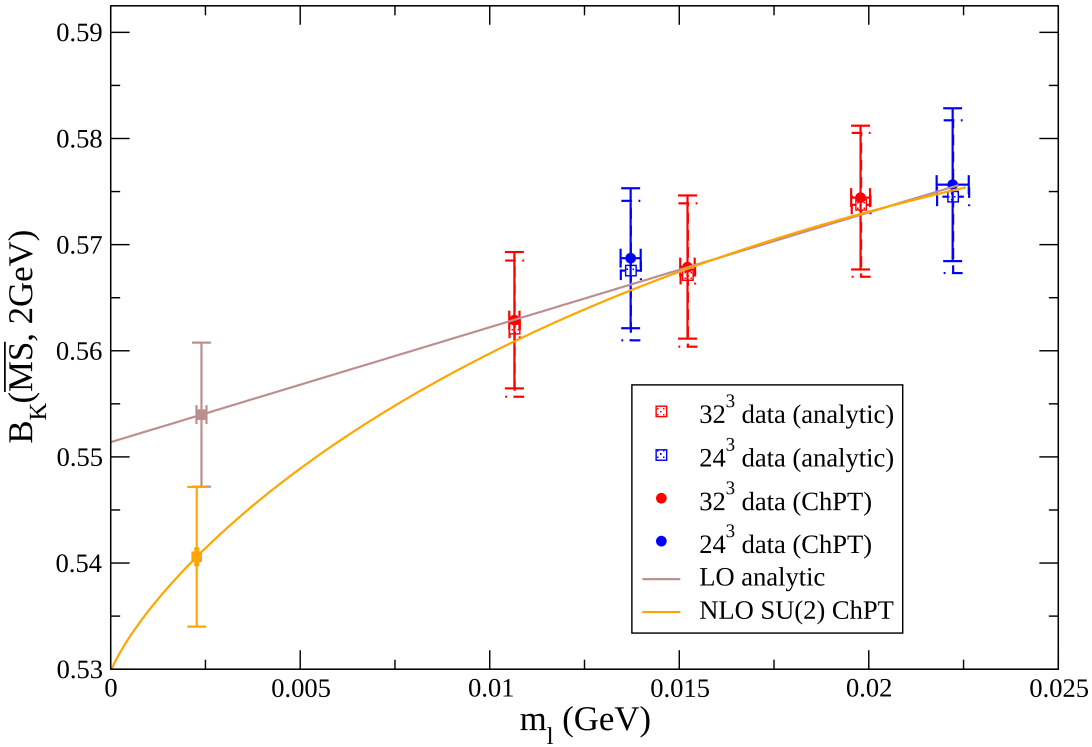


Motivation

2.4σ tension between lattice B_K value and preferred value from CKM fit with B_K omitted: $\widehat{B}_K = 0.725 \pm 0.027$ versus $(\widehat{B}_K)_{\rm fit} = 0.98 \pm 0.10$.



RBC/UKQCD



New treatment of chiral extrapolation, where $SU(2)$ chiral extrapolation result is averaged with linear extrapolation result. Motivated by absence of curvature in lattice data, and the tendency for the $SU(2)$ fit to undershoot f_π. (Talk by Chris Kelly)