Neutron Electric Dipole Moment in the Standard Model and beyond from Lattice QCD

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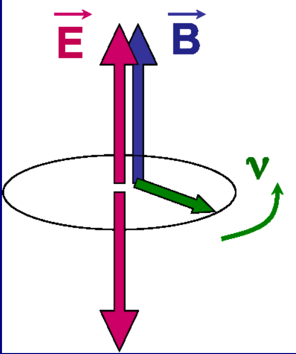
Introduction
Dipole Moments



$$H = -d\,\vec{E}\cdot\vec{S} - \mu\,\vec{B}\cdot\vec{S}$$

- Spin precesses in Electric and Magnetic Fields.
- Precession Frequency depends on E through EDM d.
- Change in Precession Frequency on flipping E measures EDM.

$\mu \vec{B} \cdot \vec{S}$ is even under C, P, and T

$\vec{B}$, $\vec{S}$ are parity even:	$\vec{B} \longleftrightarrow +\vec{B}$	$\vec{S} \longleftrightarrow +\vec{S}$
$\vec{B}$, $\vec{S}$ are time reversal odd:	$\vec{B} \longleftrightarrow -\vec{B}$	$\vec{S} \longleftrightarrow -\vec{S}$
$\vec{B}$, $\vec{S}$ are charge conjugation even:	$\mu \vec{B} \longleftrightarrow +\mu \vec{B}$	$\vec{S} \longleftrightarrow +\vec{S}$

$d \vec{E} \cdot \vec{S}$ term violates P, T, and CP

violates Parity:	$\vec{E} \longleftrightarrow -\vec{E}$	$\vec{S} \longleftrightarrow +\vec{S}$
violates Time reversal:	$\vec{E} \longleftrightarrow +\vec{E}$	$\vec{S} \longleftrightarrow -\vec{S}$
conserves Charge conjugation:	$d \vec{E} \longleftrightarrow +d \vec{E}$	$\vec{S} \longleftrightarrow +\vec{S}$

Introduction
Sakharov Conditions for Baryogenesis

Without CP violation, freezeout ratio: $n_B/n_\gamma \approx 10^{-20}$.

Kolb and Turner, *Front. Phys.* **69** (1990) 1.

Observed baryon asymmetry: $n_B/n_\gamma = 6.1^{+0.3}_{-0.2} \times 10^{-10}$.

WMAP + COBE 2003

$\Rightarrow$ Either asymmetric initial conditions or baryogenesis!

Sakharov Conditions for Baryogenesis

- Baryon Number violation
- C, CP and T violation
- Out of equilibrium evolution

Sakharov, *Pisma Zh. Eksp. Teor. Fiz.* **5** (1967) 32.

Introduction
Standard Model CP Violation

Two sources of CP violation in the Standard Model.

- Complex phase in CKM quark mixing matrix.
  - Too small to explain baryon asymmetry
  - Gives a tiny ($\sim 10^{-32}$ e-cm) contribution to nEDM

Dar `arXiv:hep-ph/0008248`.

- Effective $\Theta G\tilde{G}$ interaction from QCD instantons
  - Effects suppressed at high energies
  - nEDM limits constrain $\Theta \lesssim 10^{-10}$ unnaturally small

Crewther *et al.*, *Phys. Lett.* **B88** (1979) 123.

Need $\not{CP}$ from BSM to explain baryogenesis.

- Could give rise to large EDM

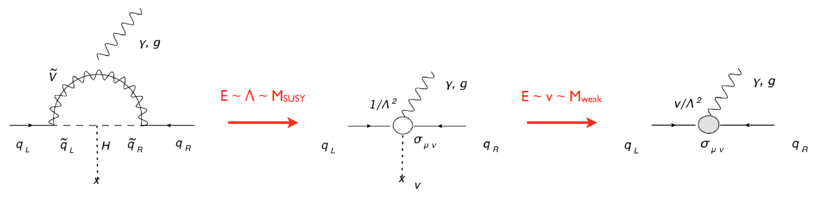
Introduction
Effective Field Theory

Parameterize BSM $\not{CP}$ using an effective field theory at the weak scale. Two important dimension six operators are the Electric and Chromoelectric dipole moments of the quark.

$$\begin{aligned}\mathcal{S} &= \mathcal{S}_{QCD}^{\text{CP Even}} - i\,\Theta \frac{g^2}{16\pi^2}\int d^4x\, G^{\mu\nu}\tilde{G}_{\mu\nu} \\ &+ \frac{i\,e\,d_u^\gamma}{\Lambda_{\text{BSM}}^2}\bar{Q}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}\tilde{H}U + \frac{i\,e\,d_d^\gamma}{\Lambda_{\text{BSM}}^2}\bar{Q}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}HD \\ &+ \frac{i\,g_3\,d_u^G}{\Lambda_{\text{BSM}}^2}\bar{Q}\sigma_{\mu\nu}\gamma_5\lambda^A G^{\mu\nu\,A}\tilde{H}U + \frac{i\,g_3\,d_d^G}{\Lambda_{\text{BSM}}^2}\bar{Q}\sigma_{\mu\nu}\gamma_5\lambda^A G^{\mu\nu\,A}HD \\ &+ \cdots\,.\end{aligned}$$

The two quark dipole moments are generated at 3-loops in the standard model and give tiny nEDM ($\sim 10^{-34}$ e-cm).

They are generated at one loop in BSM.



Expected contribution can be as large as experimental limit of $\sim 2.9 \times 10^{-26}$e-cm.

Baker *et al.*, *Phys. Rec. Lett.* **97** (2006) 131801.

Matrix Elements

Model expectations

Model analysis estimate of the neutron electric dipole moment:

$$
\begin{aligned}
d_n \approx \frac{8\pi^2}{M_n^3} \Bigg[& -\frac{2m_*}{3} \frac{\partial\langle\bar{q}\sigma q\rangle_F}{\partial F} \left(\bar{\Theta} + g_s \frac{\langle\bar{q}G\sigma q\rangle}{2\langle\bar{q}q\rangle} \sum \frac{d_q^G}{m_q} \right) \\
& + \frac{\langle\bar{q}q\rangle}{3} (4\, d_d^\gamma - d_u^\gamma) \\
& + g_s \frac{\langle\bar{q}G\sigma q\rangle}{6\langle\bar{q}q\rangle} \left(4\, d_d^G \frac{\partial\langle\bar{d}\sigma d\rangle_F}{\partial F} - d_u^G \frac{\partial\langle\bar{u}\sigma u\rangle_F}{\partial F} \right) \Bigg] \\
\approx & \left(\frac{4}{3} d_d^\gamma - \frac{1}{3} d_u^\gamma \right) - \frac{2e\langle\bar{q}q\rangle}{M_n f_\pi^2} \left(\frac{2}{3} d_d^G + \frac{1}{3} d_u^G \right),
\end{aligned}
$$

assuming the first term vanishes by Peccei-Quinn mechanism.

Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119

Numerically,

$$d_n(\bar{\Theta}) \approx (1 \pm 0.5)\frac{|\langle \bar{q}q\rangle|}{(225\mathrm{MeV})^3}\bar{\Theta}\,(2.5\times 10^{-16}\ \text{e-cm})$$

$$d_n(d_q^{\gamma,G}) \approx -d_n(\bar{\Theta}=\Theta_{\rm ind}) + (1 \pm 0.5)\frac{|\langle \bar{q}q\rangle|}{(225\mathrm{MeV})^3}\left[1.1\,(d_d^G + 0.5\,d_u^G)\,e + 1.4\,(d_d^\gamma - 0.25\,d_u^\gamma)\right],$$

where

$$\Theta_{\rm ind} \approx (3.1\times 10^{-17}\mathrm{cm})^{-1}\sum \frac{\mathrm{d_q^G}}{\mathrm{m_q(MeV)}}\frac{|\mathrm{m_0^2}|}{(0.8\mathrm{GeV})^2}$$

is the value at the minimum of the Peccei Quinn potential.

Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119

Note that the quark dipole moments violate chirality, and, hence, are expected to be of the order

$$\kappa_q = \frac{m_q}{16\pi^2 M_\Lambda^2} = 1.3 \times 10^{-25}\text{e-cm}\frac{m_q}{1\text{MeV}}\left(\frac{1TeV}{M_\Lambda}\right)^2 .$$

Rough estimates of the other dimension 6 operators:

Weinberg Operator:

$$|d_n(w)| \approx (4.4 \times 10^{-22}\,\text{e-cm}) \left.\frac{w}{(1\text{TeV})^{-2}}\right|_{\mu=1\text{GeV}}$$

Four-quark Operators:

$$|d_n(C)| \approx (1.2 \times 10^{-24}\,\text{e-cm}) \left.\frac{C_{bd} + C_{db}}{(1\text{TeV})^{-2}}\right|_{\mu=m_b}$$

Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119

Matrix Elements
Lattice Basics

With $\not{CP}$, we can extract nEDM in two ways.

- As the difference of the energies of spin-aligned and anti-aligned neutron states in an electric field E:

$$d_n = \frac{1}{2}\left(M_{n\downarrow} - M_{n\uparrow}\right)\big|_{E=E\uparrow}$$

- By extracting the CP violating form factor of the electromagnetic current.

$$\langle n|J_\mu^{\mathrm{EM}}|n\rangle \sim \frac{F_3(q^2)}{2M_n}\bar{n}\, q_\nu \sigma^{\mu\nu}\gamma_5\, n$$

$$d_n = \lim_{q^2\to 0}\frac{F_3(q^2)}{2M_n}$$

Difficult to perform simulations with complex ($\not{CP}$) action

Expand and calculate correlators with the $\not{CP}$ term:

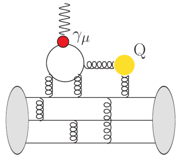
$$\begin{aligned}\langle\, C^{\not{CP}}(x,y,\ldots)\,\rangle_{CP+\not{CP}} &= \int [\mathcal{DA}] \exp\left[-\int d^4x(\mathcal{L}^{CP} + \mathcal{L}^{\not{CP}})\right] \\ &\quad \times C^{\not{CP}}(x,y,\ldots) \\ &\approx \int [\mathcal{DA}] \exp\left[-\int d^4x \mathcal{L}^{CP}\right] \\ &\quad \times \left(-\int d^4x \mathcal{L}^{\not{CP}}\right) C^{\not{CP}}(x,y,\ldots) \\ &= -\langle\, C^{\not{CP}}(x,y,\ldots)\, \mathcal{L}^{\not{CP}}(p_\mu = 0)\,\rangle_{CP}\end{aligned}$$

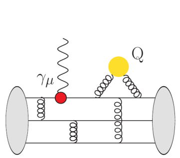
Matrix Elements

Topological charge

To find the contribution proportional to $\bar{\Theta}$, use $\int d^4x G\tilde{G} = Q$, the topological charge. Need to calculate the correlation between the electric current and the topological charge.

$$\left\langle n \left| (\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d) Q \right| n \right\rangle = \frac{1}{2}\langle n \left| (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) Q \right| n \rangle + \frac{1}{6}\langle n \left| (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) Q \right| n \rangle$$



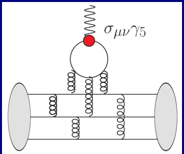


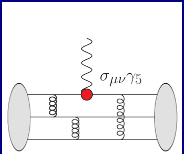
Matrix Elements

Quark Electric Dipole Moment

Expectation value of the quark electric dipole moment is calculated by taking its matrix element in the neutron state.

$$\left\langle n \left| d_u^\gamma \, \bar{u}\sigma^{\mu\nu} u + d_d^\gamma \, \bar{d}\sigma^{\mu\nu} d \right| n \right\rangle \quad = \\ \frac{d_u^\gamma + d_d^\gamma}{2} \left\langle n \left| \bar{u}\sigma^{\mu\nu} u + \bar{d}\sigma^{\mu\nu} d \right| n \right\rangle + \frac{d_u^\gamma - d_d^\gamma}{2} \left\langle n \left| \bar{u}\sigma^{\mu\nu} u - \bar{d}\sigma^{\mu\nu} d \right| n \right\rangle$$





Matrix Elements
Quark Chromoelectric Moment

The interaction of the quark chromoelectric moment with J_μ is a 4-pt function. We simplify using Feynman-Hellmann Theorem.

$$\left\langle n \left| J_\mu \int d^4x (d_u^G \, \bar{u}\sigma^{\nu\kappa}u + d_d^G \, \bar{d}\sigma^{\nu\kappa}d) \, \tilde{G}_{\nu\kappa} \right| n \right\rangle$$

$$= \frac{\partial}{\partial A_\mu} \left\langle n \left| \int d^4x (d_u^G \, \bar{u}\sigma^{\nu\kappa}u + d_d^G \, \bar{d}\sigma^{\nu\kappa}d) \, \tilde{G}_{\nu\kappa} \right| n \right\rangle_E$$

where the subscript E refers to the correlator calculated in the presence of a external electric field.

Lattice QCD
Renormalization

Non-perturbative renormalization of lattice operators:

- Topological charge is well studied and understood.
- Electric current and Quark Elecric Dipole moment operators are quark bilinears: well understood renormalization procedure.
- Quark Chromoelectric Moment operator mixes with the Topological charge; need to study simultaneous running. This is related to the influence of Chromoelectric moment on the PQ potential for Θ.

Lattice QCD
State of the Art

Neutron electric dipole moment from

- Topological charge:
  - Preliminary results from lattie calculations
  - Discussed in previous talk
- Quark Electric Dipole Moment:
  - Connected diagrams: estimates exist.
  - Disconnected diagrams: not yet calculated.
- Quark Chromoelectric Dipole Moment: not yet calculated

Lattice QCD
Needed Calculations

Exploratory calculations needed before one can estimate various errors and resource requirements.
We will perform preliminary calculations using

- Previously generated 2+1+1 flavor HISQ lattices
- Use Clover valence quarks

and study

- Statistical signal
- Chiral behavior
- Dependence on lattice spacing
- Excited state contamination

Lattice QCD
Outlook

- The connected diagram for Quark Electric Dipole Moment (same as tensor charge of the nucleon) will soon reach 20% precision.
- The calculation of the disconnected diagrams, and matrix elements of other operators discussed need more study.
- nEDM insensitive to neglected EM and isospin-breaking
- Modern calculations include dynamical charm