Lattice Calculations of Neutron-Antineutron Matrix Elements

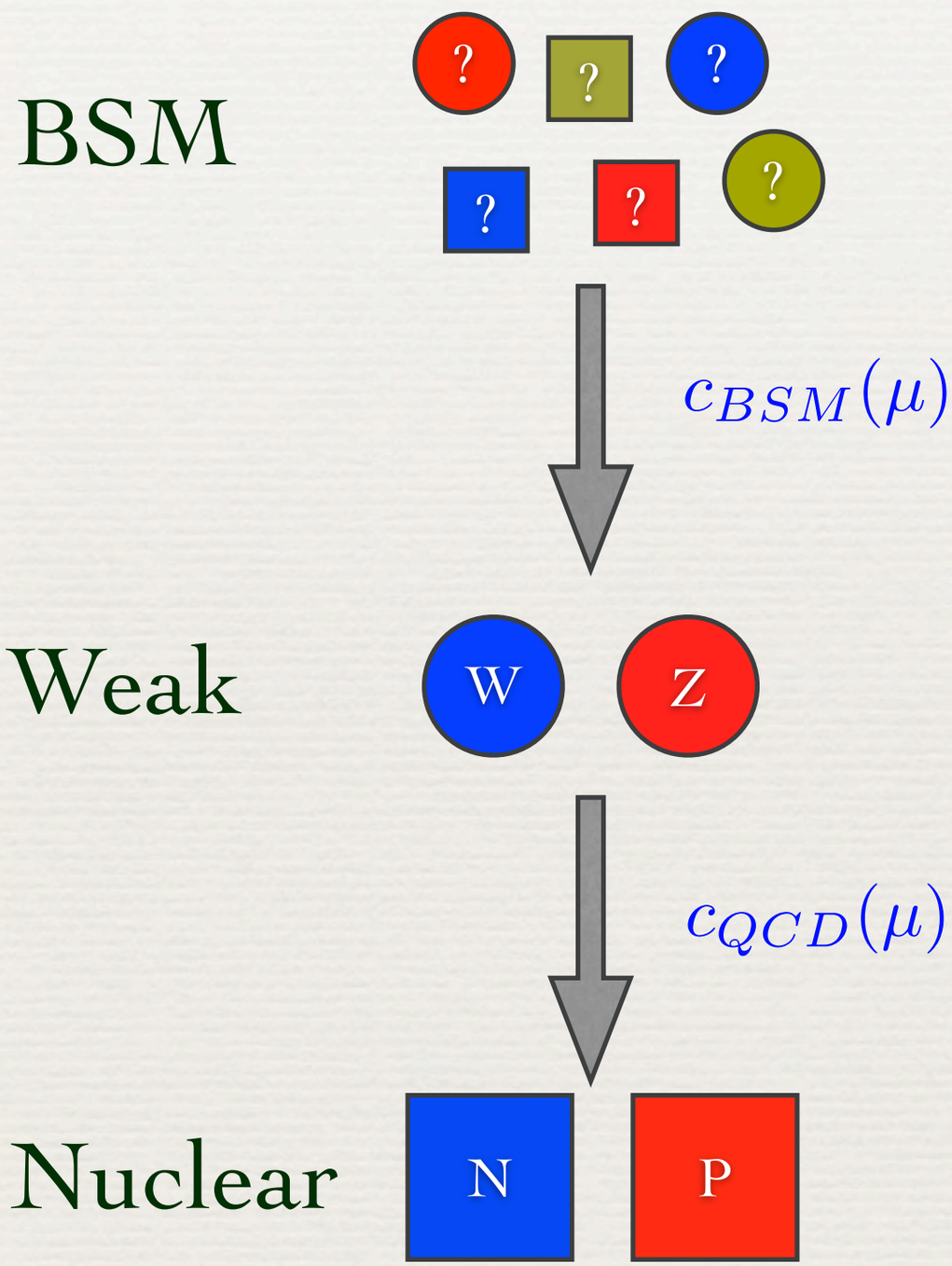
Michael I. Buchoff

Lawrence Livermore National Laboratory

In collaboration with Chris Schroeder and Joe Wasem

LLNL-PRES-561415

Origin of Oscillations



- Running of BSM interaction to nuclear scale

$$\frac{1}{\tau_{n\overline{n}}} = \delta m = c_{BSM}(\mu) c_{QCD}(\mu) \langle \overline{n} | \mathcal{O} | n \rangle$$

Where Lattice Can Help

- Is BSM running non-perturbative?
  - Model-dependent (assume pert. models for now)
- Is QCD running non-perturbative?
  - Should be checked (pert. running reasonable)
- What is neutron-antineutron matrix element?
  - Inherently non-perturbative question
- What is effect in nuclei?
  - Very interesting, VERY hard question

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Six-quark Operators

Rao, Shrock (1982)

Three pairs of quarks:

1. $u^T C u$ or $u^T C d$ or $d^T C d$

2. $u_L^T C d_L$ or $u_R^T C d_R$

3. $$\Gamma^s_{ijklmn} = \epsilon_{mik}\epsilon_{njl} + \epsilon_{nik}\epsilon_{mjl} + \epsilon_{mjk}\epsilon_{nil} + \epsilon_{njk}\epsilon_{mil}$$

$$\Gamma^a_{ijklmn} = \epsilon_{mij}\epsilon_{nkl} + \epsilon_{nij}\epsilon_{mkl}$$

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$$\chi_i = L, R$$

1. $$\mathcal{O}^1_{\chi_1\chi_2\chi_3} = (u^T_{i\chi_1} C u_{j\chi_1})(d^T_{k\chi_2} C d_{l\chi_2})(d^T_{m\chi_3} C d_{n\chi_3})\Gamma^s_{ijklmn}$$

2. $$\mathcal{O}^2_{\chi_1\chi_2\chi_3} = (u^T_{i\chi_1} C d_{j\chi_1})(u^T_{k\chi_2} C d_{l\chi_2})(d^T_{m\chi_3} C d_{n\chi_3})\Gamma^s_{ijklmn}$$

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18 Independent Operators

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Caswell, Milutinovic, Sejanovic (1983)

18 Independent Operators → 14 Indep. Operators

Six-quark Operators

Rao, Shrock (1982)

If invariant under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$:

(Six Operators)

$$\mathcal{P}_1 = \mathcal{O}^1_{RRR}$$
$$\mathcal{P}_2 = \mathcal{O}^2_{RRR}$$
$$\mathcal{P}_3 = \mathcal{O}^3_{RRR}$$
$$\mathcal{P}_4 = 2\mathcal{O}^3_{LRR}$$
$$\mathcal{P}_5 = 4\mathcal{O}^3_{LLR}$$
$$\mathcal{P}_6 = 4(\mathcal{O}^1_{LLR} - \mathcal{O}^2_{LLR})$$

- Matrix elements cannot be calculated perturbatively

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(Four Operators)

Caswell, Milutinovic, Sejanovic (1983)

$$\mathcal{P}_6 = -3\mathcal{P}_5$$
$$\mathcal{P}_2 - \mathcal{P}_1 = 3\mathcal{P}_3$$

- Matrix elements cannot be calculated perturbatively

Current Understanding of Matrix Elements

MIT bag Model: (Rao, Shrock 1982)

- -Model dependent estimation
- -Results roughly consistent with DA
- -No QCD input

Lattice Motivation:

- -Numerical QCD calculation
- -Quantification of uncertainties
- -Pinpoint target sensitivity for experiment
- -Large enhancements/suppressions?

Lattice Calculation

Correlation Functions via path integral:

$$C_{\mathcal{O}} = \langle \mathcal{O} \rangle = \int d[U]\ \mathcal{O}\ \det(D_{lat}(U)) e^{-S_G(U)}$$

$$C_{NN}(t) = \langle \overline{N}(t) N(0) \rangle \to |\langle N|n\rangle|^2 e^{-m_n t}$$

$$C_{\overline{N}\overline{N}}(t) = \langle N(t) \overline{N}(0) \rangle \to |\langle \overline{N}|\overline{n}\rangle|^2 e^{-m_n t}$$

$$C_{\overline{N}\mathcal{O}N}(t_1, t_2) = \langle N(t_1) \mathcal{O}(0) \overline{N}(t_2) \rangle \to \langle \overline{N}|\overline{n}\rangle \langle N|n\rangle e^{-m_n(t_1+t_2)} \langle n|\mathcal{O}|\overline{n}\rangle$$

$$\mathcal{R} = \frac{C_{\overline{N}\mathcal{O}N}(t_1, t_2)}{C_{\overline{N}\overline{N}}(t_1 + t_2)} \left[\frac{C_{NN}(t_1) C_{\overline{N}\overline{N}}(t_2) C_{\overline{N}\overline{N}}(t_1 + t_2)}{C_{\overline{N}\overline{N}}(t_1) C_{NN}(t_2) C_{NN}(t_1 + t_2)} \right]^{\frac{1}{2}} \to \langle \overline{n}|\mathcal{O}|n\rangle$$

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Prob. Dis.

$$C_{NN}(t) = \langle \overline{N}(t) N(0) \rangle \to |\langle N|n\rangle|^2 e^{-m_n t}$$

$$C_{\overline{NN}}(t) = \langle N(t) \overline{N}(0) \rangle \to |\langle \overline{N}|\overline{n}\rangle|^2 e^{-m_n t}$$

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Lattice Contractions

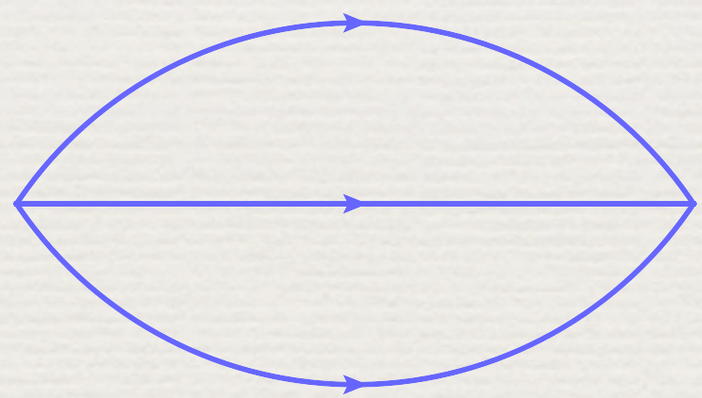
Propagator Contractions:

$$\overline{q}_{i'}^{\alpha'}(y)\ q_i^{\alpha}(x) = S_{i'i}^{\alpha'\alpha}(y,x) \qquad S^\dagger = \gamma_5 S \gamma_5$$

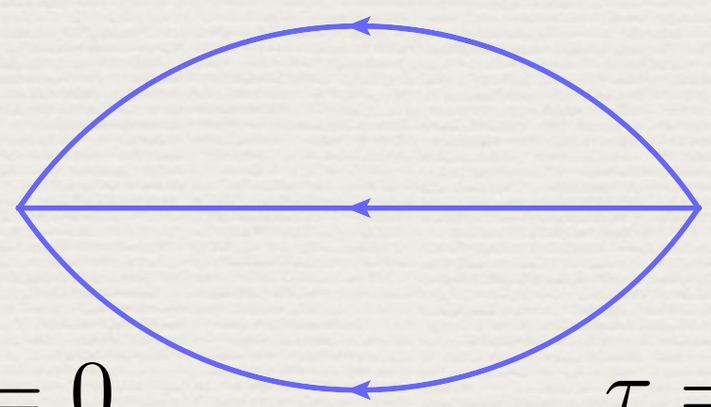
All Points

One Point

$C_{NN}(t)$



$C_{\overline{NN}}(t)$

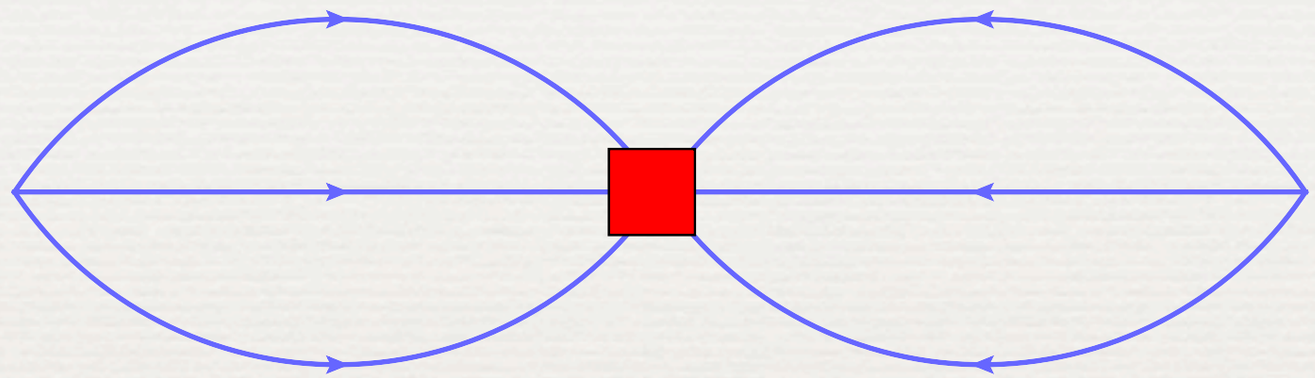


$\tau = 0$ $\tau = t$

Lattice Contractions

$\tau = -t_1$ $\tau = 0$ $\tau = t_2$

$C_{\overline{N}\mathcal{O}N}(t_1, t_2)$



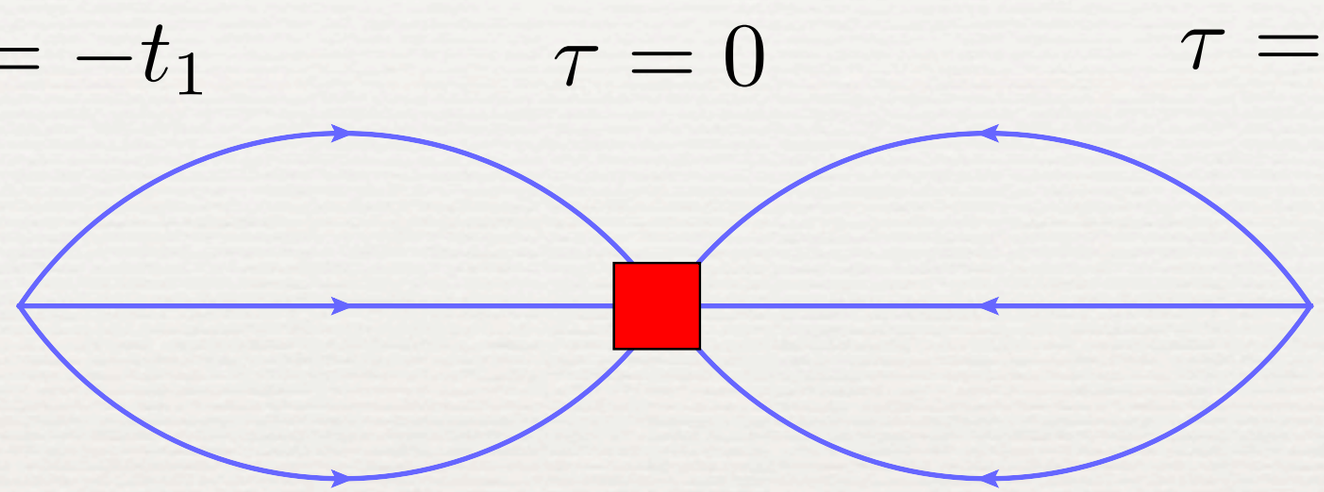
No Dis. Diagrams

1 Propagator

Two measurements
ALL time insertion

Lattice Contractions

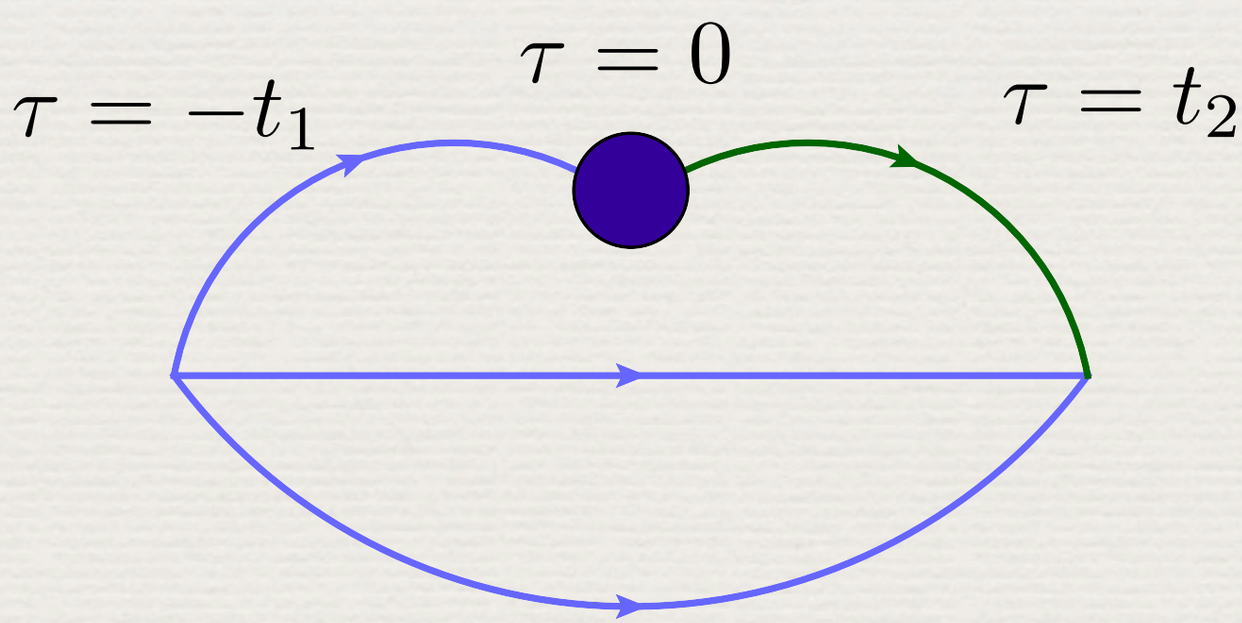
$C_{\overline{N}\mathcal{O}N}(t_1, t_2)$



No Dis. Diagrams

1 Propagator → Two measurements ALL time insertion

Typical 3-point



Often Dis. Diagrams

2 Propagators → One measurements One time insertion

Executive Summary

- Advantages of Neutron-Antineutron calculations

  For same cost:

  More Statistics

  All Operator Insertions

  No Quark Loop or Disconnected Diagrams

- Disadvantages of Neutron-Antineutron calculations

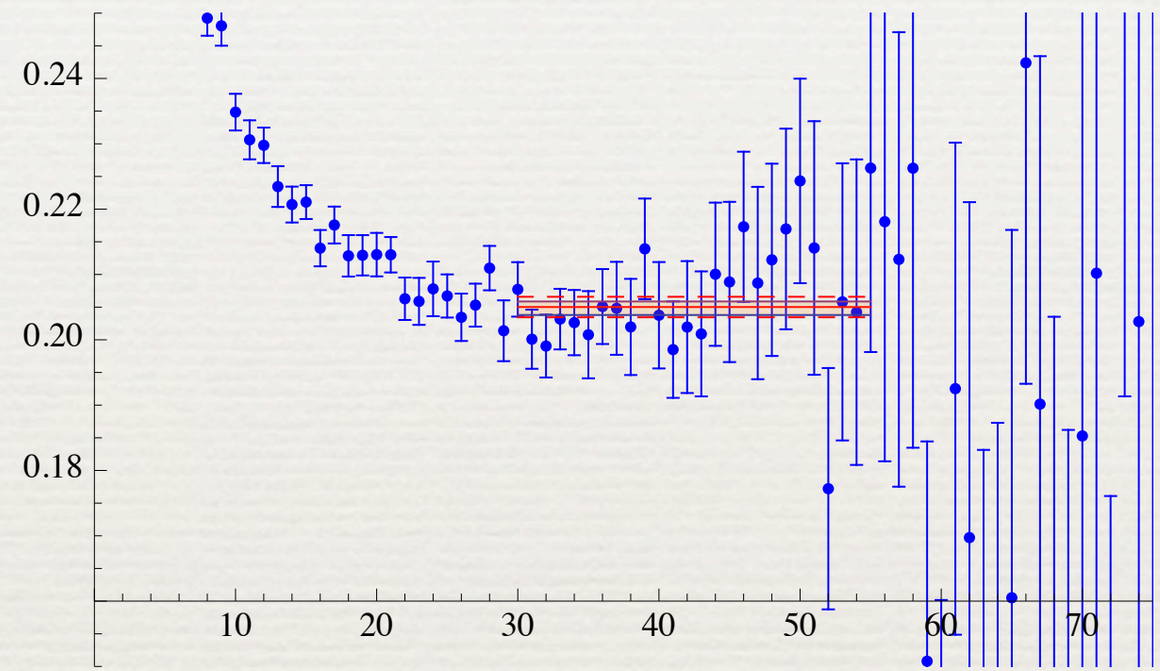
  More Propagator Multiplications → Potentially Worse Signal

Lattice Details

- $32^3 \times 256$ anisotrpoic clover-Wilson lattices
- $m_\pi \sim 390$ MeV
- $a_t \sim 0.04$ fm, $a_s \sim 0.125$ fm
- $L \sim 4$ fm
- 159 configurations (every 4th trajectory)
- 7268 propagators

Nucleon Effective Mass

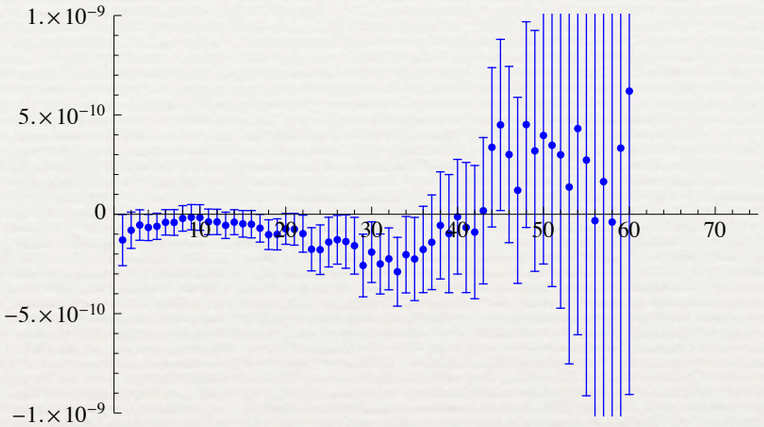
$$\frac{C_{NN}(t+1)}{C_{NN}(t)} \to m_n$$



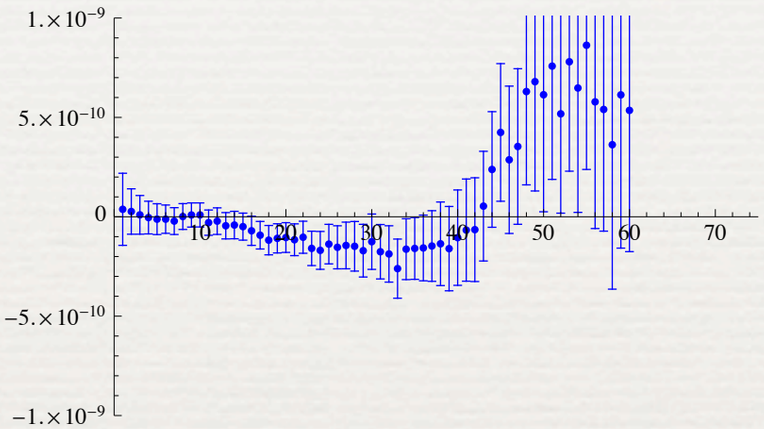
$$M_N = 1.148(\pm 0.0088)(+0.0048)(-0.0068)\ \mathrm{GeV}$$

N-NBar Matrix Element

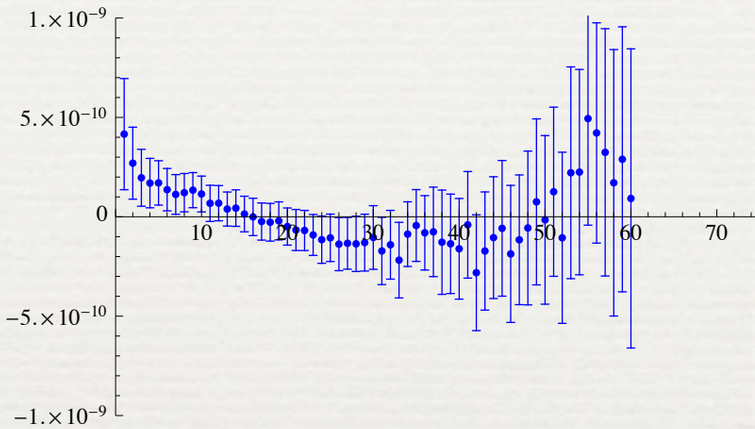
$t_1 = 5$



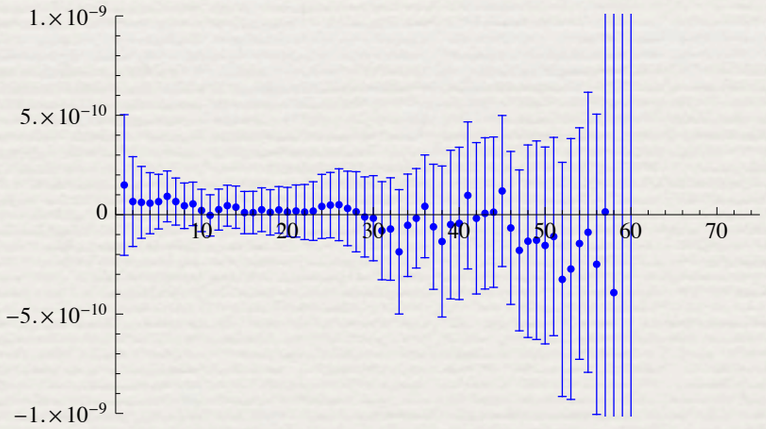
$t_1 = 10$



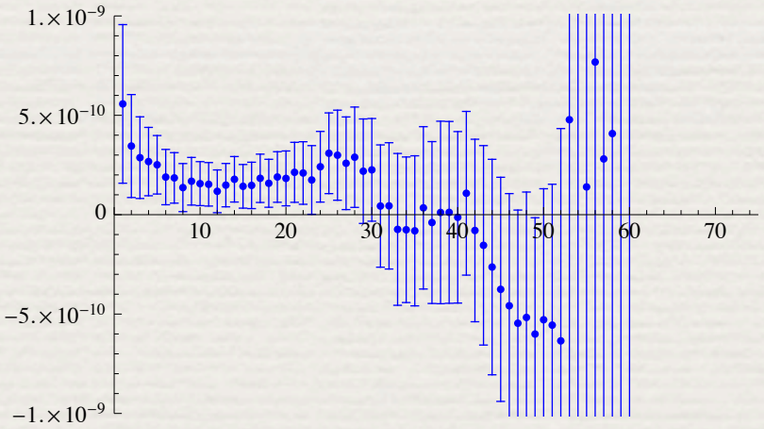
$t_1 = 15$



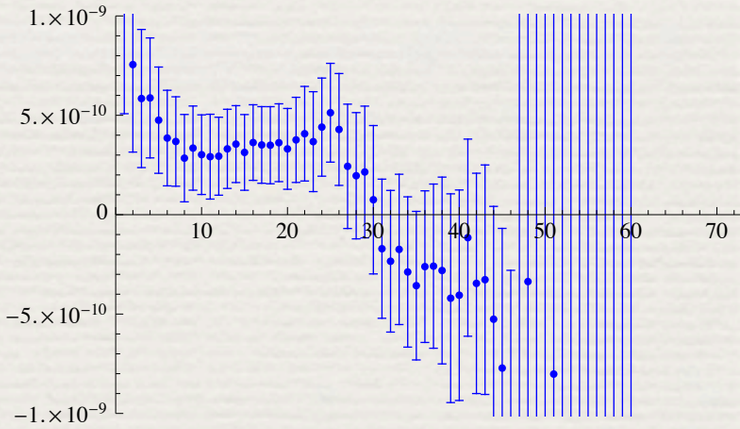
$t_1 = 20$



$t_1 = 25$



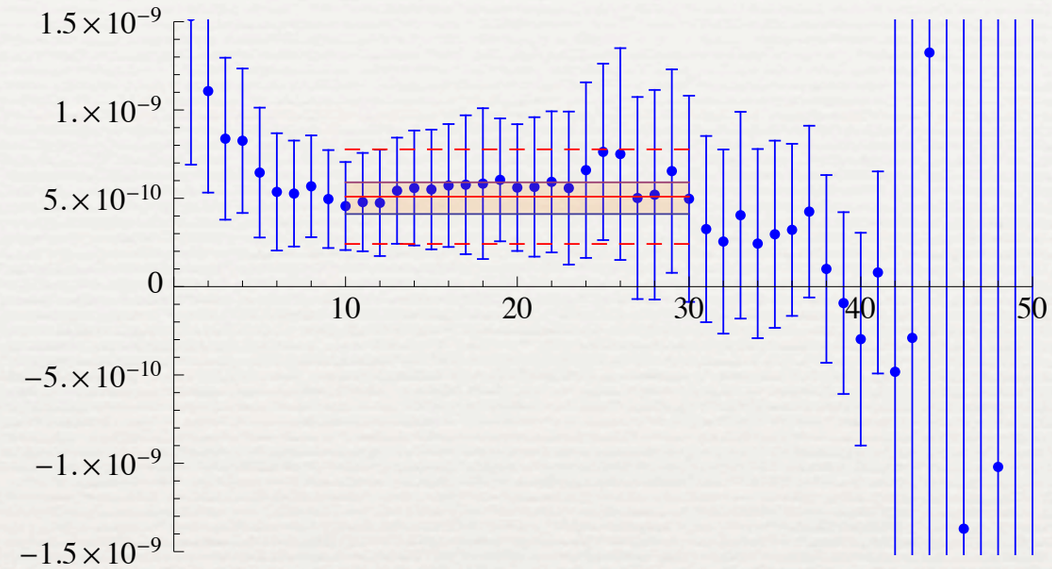
$t_1 = 30$



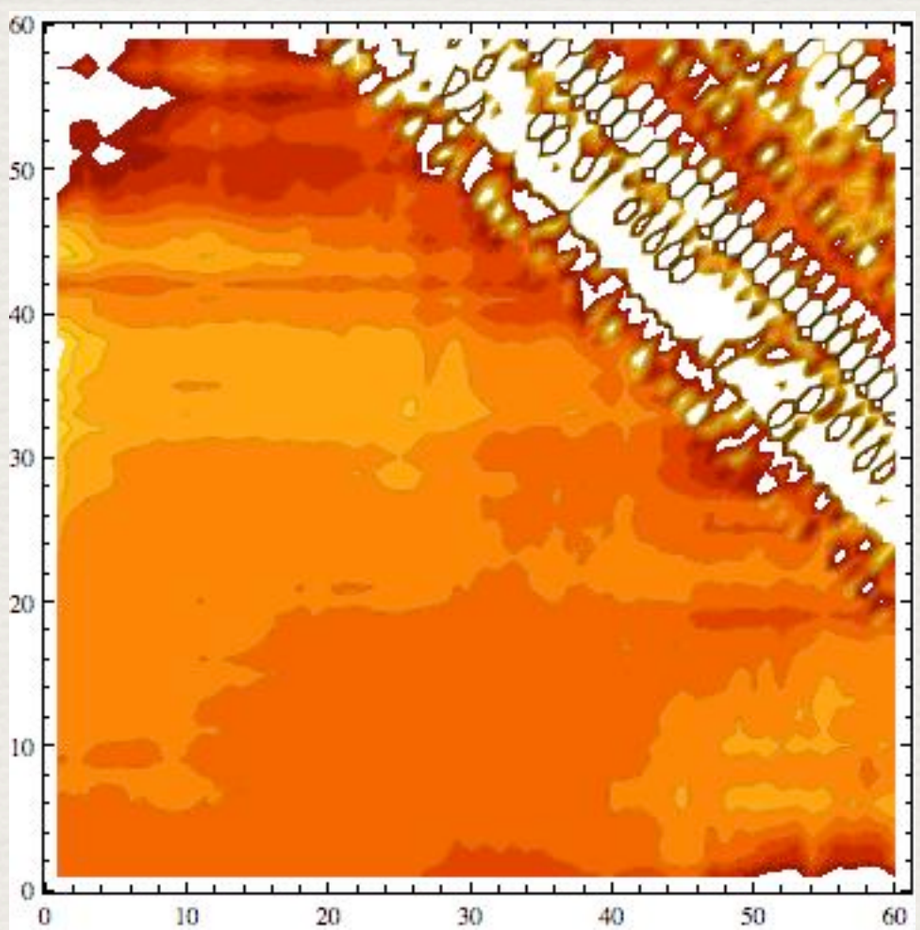
$$\mathcal{R} \rightarrow \langle \overline{n} | \mathcal{O} | n \rangle$$

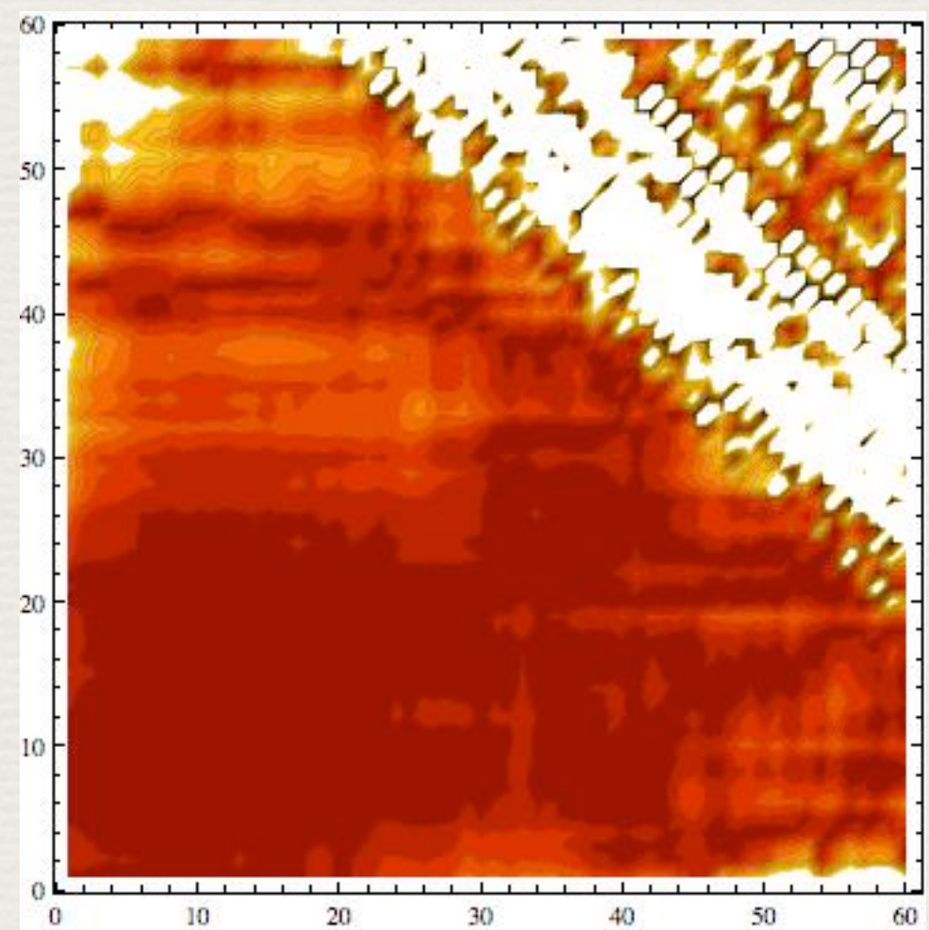
N-NBar Matrix Element

$$\mathcal{R} \to \langle \overline{n} | \mathcal{O} | n \rangle$$



$$\langle \overline{n} | \mathcal{O}^1_{RRR} | n \rangle = 1.57(\pm 0.85)(+0.25)(-0.30) \ \times 10^{-5} \ \mathrm{GeV}^6$$





VERY Preliminary Results

	Lattice	MIT Bag Model
$\langle\overline{n}\|\mathcal{P}_1\|n\rangle$	$1.57(\pm 0.85)(+0.25)(-0.30)$	-6.56
$\langle\overline{n}\|\mathcal{P}_2\|n\rangle$	$-0.20(\pm 0.14)(+0.14)(-0.12)$	1.64
$\langle\overline{n}\|\mathcal{P}_3\|n\rangle$	$-0.24(\pm 0.26)(+0.10)(-0.07)$	2.73
$\langle\overline{n}\|\mathcal{P}_4\|n\rangle$	$-0.02(\pm 0.39)(+0.07)(-0.18)$	-6.36
$\langle\overline{n}\|\mathcal{P}_5\|n\rangle$	$0.34(\pm 0.82)(+0.27)(-0.57)$	9.64
$\langle\overline{n}\|\mathcal{P}_6\|n\rangle$	$-2.07(\pm 1.10)(+1.28)(-0.77)$	-28.92
	$\times 10^{-5}\ \mathrm{GeV}^6$	$\times 10^{-5}\ \mathrm{GeV}^6$

Systematic Effects

- Unphysical Pion Mass
  - No clear chiral extrapolation
  - Real-world dynamics could differ
- Unphysical discretization effects
  - Most violent case should not occur
  - Beneficial to quantify
- Excited State Contamination
  - Range of operator insertions help some
- Volume Effects

Future Outlook

Currently in progress:

- Independent analysis checks
- L = 20, 390 MeV pions
- L = 32, 240 MeV pions

Feasible in the next year or two:

- Physical Point Calculation
- Chiral Fermion Calculation