New methods for HLbL

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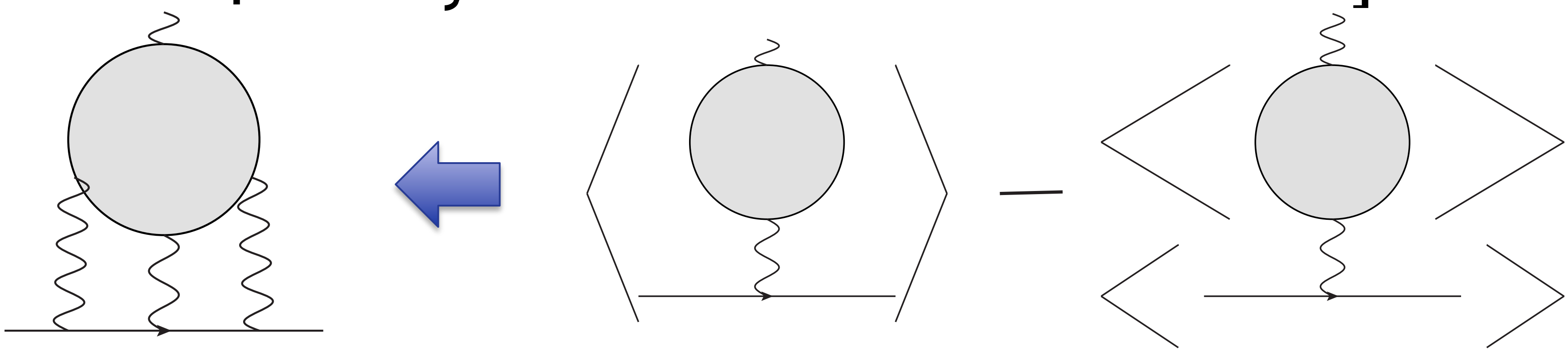
Outline

- A-slash SeqSrc method
- QED reweighting
- All Mode Averaging (AMA)

Aslash SeqSrc

Motivation

- Light-by-Light only needs the part of O(α^3)
- Currently O(α), O(α^2), and unwanted O(α^3) are subtracted (T. Blum's talk)
  [M. Hayakawa et.al PoS LAT2005 353]



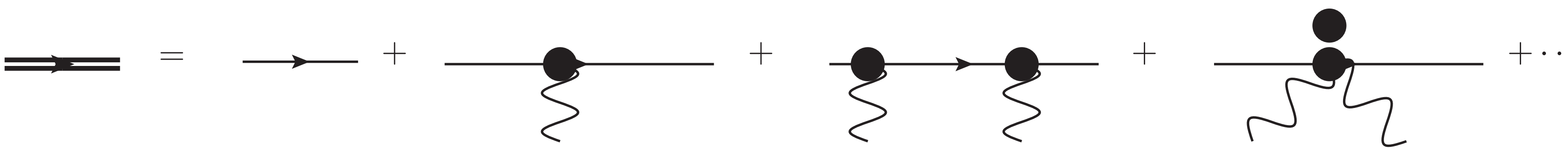
- QED perturbative expansion works

→ Order by Order Feynman diagram calculation on lattice : Aslash SeqSrc method

Aslash SeqSrc

- Quark propagator with QED charge, S(e)

$$S(e) = S + ieS\not{A}S - e^2 S(0)\not{A}S\not{A}S - e^2 S\not{A}\not{A}S + \cdots$$



- Each term could be computed by the sequential source method :

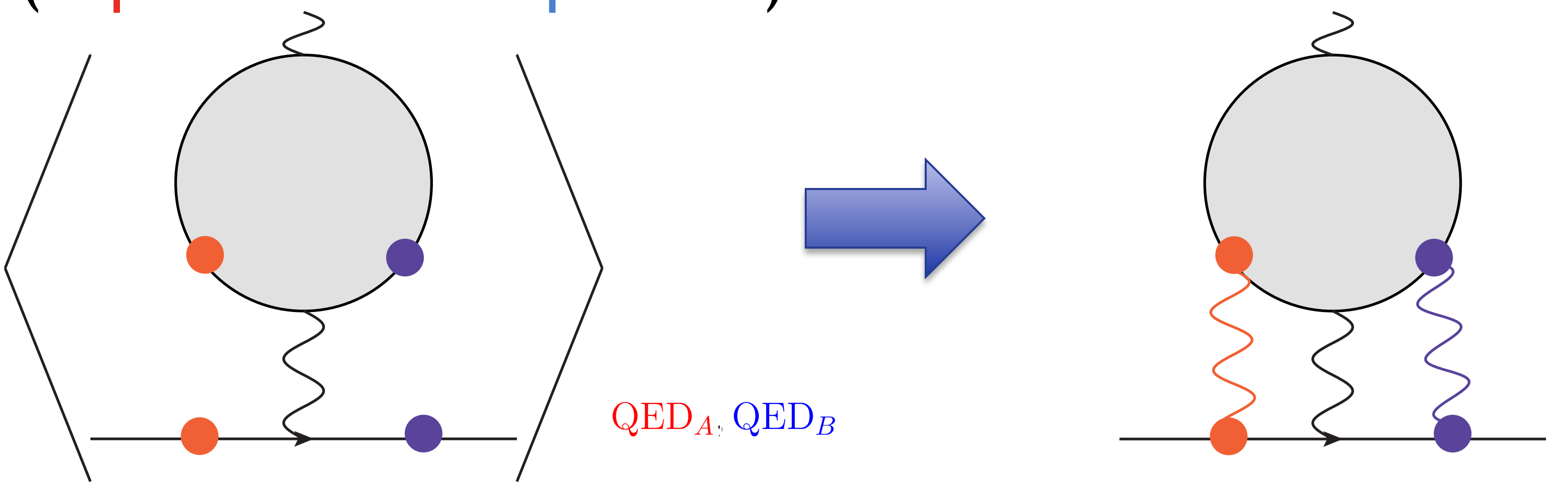
$$DX_0 = b, \quad DX_1 = \sum_x \not{A}(x) X_0(x) \to X_1 = S\not{A}Sb$$

- $\not{A}$ is the conserved vector current with photon field contracted :

$$\not{A}(x) = \sum_\mu \mathcal{V}_\mu(x) A_\mu(x)$$

Aslash SeqSrc for LbL

- Insert two Aslash for each of quark and lepton
- Use statistically independent photon field (A-photon and B-photon)



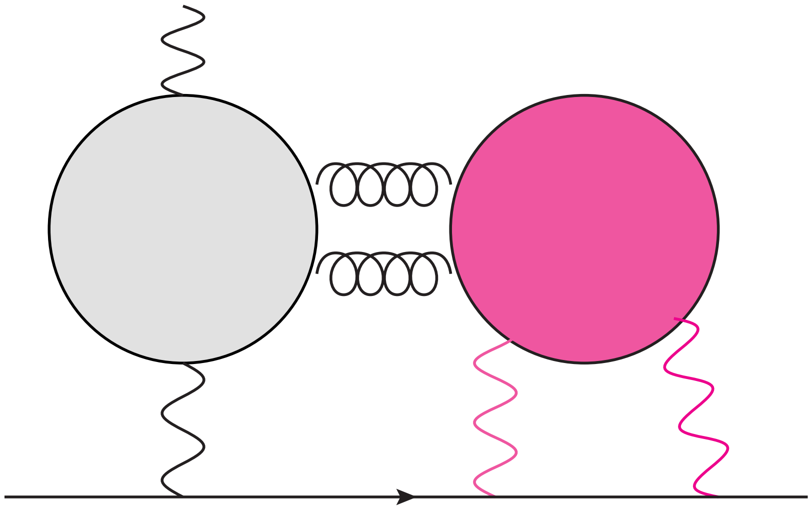
- An alternative to the subtraction method
- Explicitly free from lower/higher orders in alpha
- Could recycle low modes of pure QCD propagators

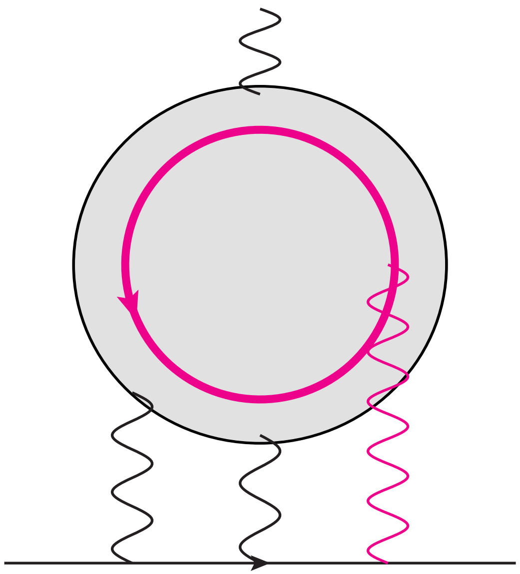
QED reweighting

T. Ishikawa et. al.
“Full QED+QCD low-energy constants through reweighting”
arXiv:1202.6018

Disconnected diagrams in HLbL

- Missing disconnected diagrams





- The second quark loop could be automatically evaluated as sea quark effect, if the sea quark electric charge effect is taken into account

  → QED reweighting (or dynamics QCD+QED)

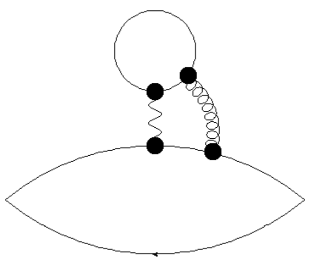
QED reweighting

- Full QED (+QCD) from quenched QED (+QCD)
  [Duncan et. al. PRD72 094509(2005)]

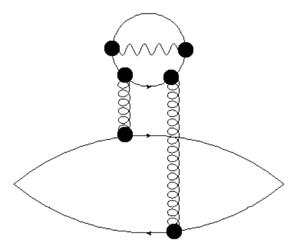
by computing the reweighting factor:

$$w[U_{\mathrm{QCD}}, A] = \frac{\det D[U_{\mathrm{QCD}} \times e^{iqeA}]}{\det D[U_{\mathrm{QCD}}]}$$

on the dynamical QCD configuration



$$O(e^2)$$



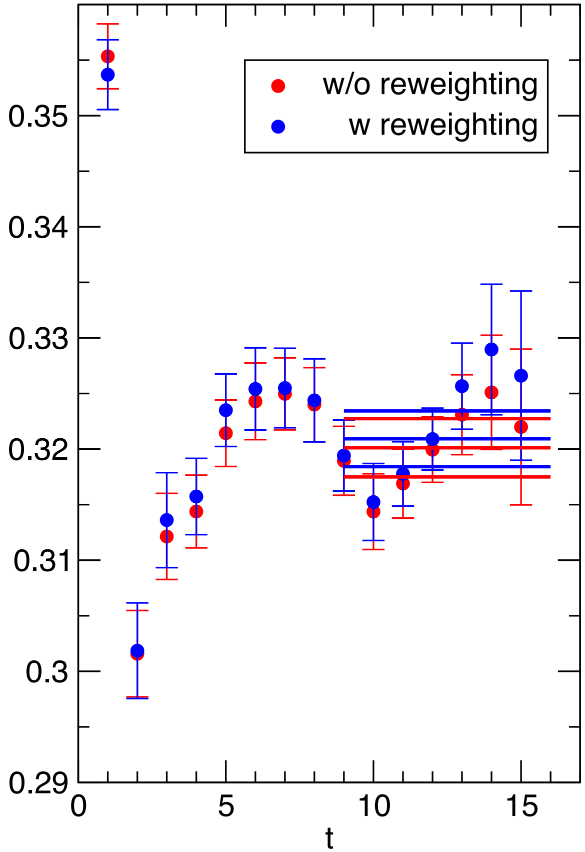
- Stochastic eval. via Root trick [T.Ishikawa et. al. 2007]

$$\det \Omega = (\det \Omega^{1/n})^n = \prod_{i=1}^{n} \langle e^{-\xi_i^\dagger (\Omega^{-1/n} - 1)\xi_i} \rangle_{\xi_i}$$

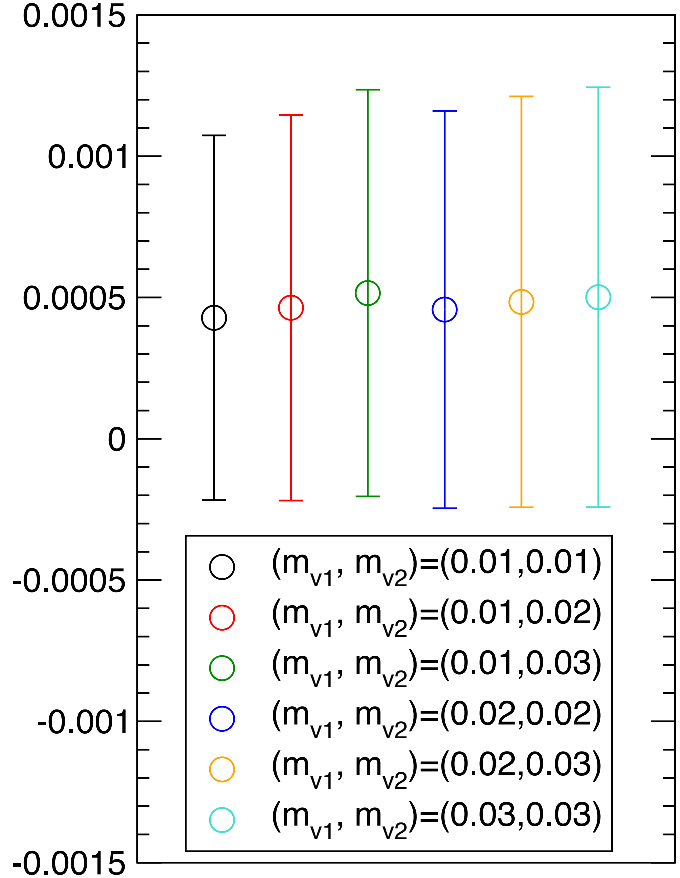
QED reweighting result [T. Ishikawa]

- PS meson mass

$$m_{\rm PS}^{eff} = -\ln(C(t+1)/C(t))$$



$$m_{\rm PS}^2[e_S = e_{phys}] - m_{\rm PS}^2[e_S = 0]$$



ChPT + gamma fit
[T. Ishikawa]

- Fitting e_s e_v term in squared PS mass

$$
\begin{aligned}
&\Delta M^2_{PS(v_1 v_2)} \\
=\ & M^2_{PS(v_1 v_2)}(e_S \neq 0) - M^2_{PS(v_1 v_2)}(e_S = 0) \\
=\ & e_S e_V \frac{C}{F_0^4} \frac{1}{8\pi^2} \left\{ \left(\chi_{v_1 u} \ln \frac{\chi_{v_1 u}}{\mu^2} q_u + \chi_{v_1 d} \ln \frac{\chi_{v_1 d}}{\mu^2} q_d + \chi_{v_1 s} \ln \frac{\chi_{v_1 s}}{\mu^2} q_s \right) \right. \\
& \left. - \left(\chi_{v_2 u} \ln \frac{\chi_{v_2 u}}{\mu^2} q_u + \chi_{v_2 d} \ln \frac{\chi_{v_2 d}}{\mu^2} q_d + \chi_{v_2 s} \ln \frac{\chi_{v_2 s}}{\mu^2} q_s \right) \right\} (q_{v_1} - q_{v_2}) \\
& -12 e_S^2 Y_1 \overline{q}^2 \chi_{v_1 v_2} + O(e_S e_V^3, e_S^2 e_V^2, e_S^3 e_V, e_S^4).
\end{aligned}
$$

	$SU(3)$ ChPT		$SU(2)$ ChPT	
	uncorr	**corr**	**uncorr**	**corr**
$10^7 C$ (qQED)	2.2(2.0)	–	18.3(1.8)	–
$10^7 C$	8.4(4.3)	8.3(4.7)	20(14)	15(21)
$10^2 Y_1$	-5.0(3.6)	-0.4(5.6)	–	–
$10^2 \mathcal{Y}_1$	-3.1(2.2)	-0.2(3.4)	-3.0(2.2)	-0.2(3.4)
$10^4 \mathcal{J}$	–	–	-2.6(1.6)	-3.3(2.8)
$10^4 \mathcal{K}$	–	–	-3.1(6.9)	-3.7(7.8)

All Mode Averaging
- a class of error reduction technique -

E. Shintani, TI, and RBC in preparation

State of Obvious

- Many interesting physics are limited by statistical error

$$\mathrm{err} \approx C \times \frac{1}{\sqrt{N_{\mathrm{meas}}}}$$

- Do more number of measurements, N_{meas}

- Change to observable with smaller fluctuation, C

- Covariant Approximation Averaging (CAA)
  Combine the above using
  - symmetries of the lattice action
  - (crude) approximations

Covariant Approximation Averaging (CAA)

- Original observable $\mathcal{O}$

- Covariant approximation of the observable $\mathcal{O}^{(\mathrm{appx})}$ under a lattice symmetry $g \in G$

$$\langle \mathcal{O}^{(\mathrm{appx})} \rangle = \langle \mathcal{O}^{(\mathrm{appx}),g} \rangle$$

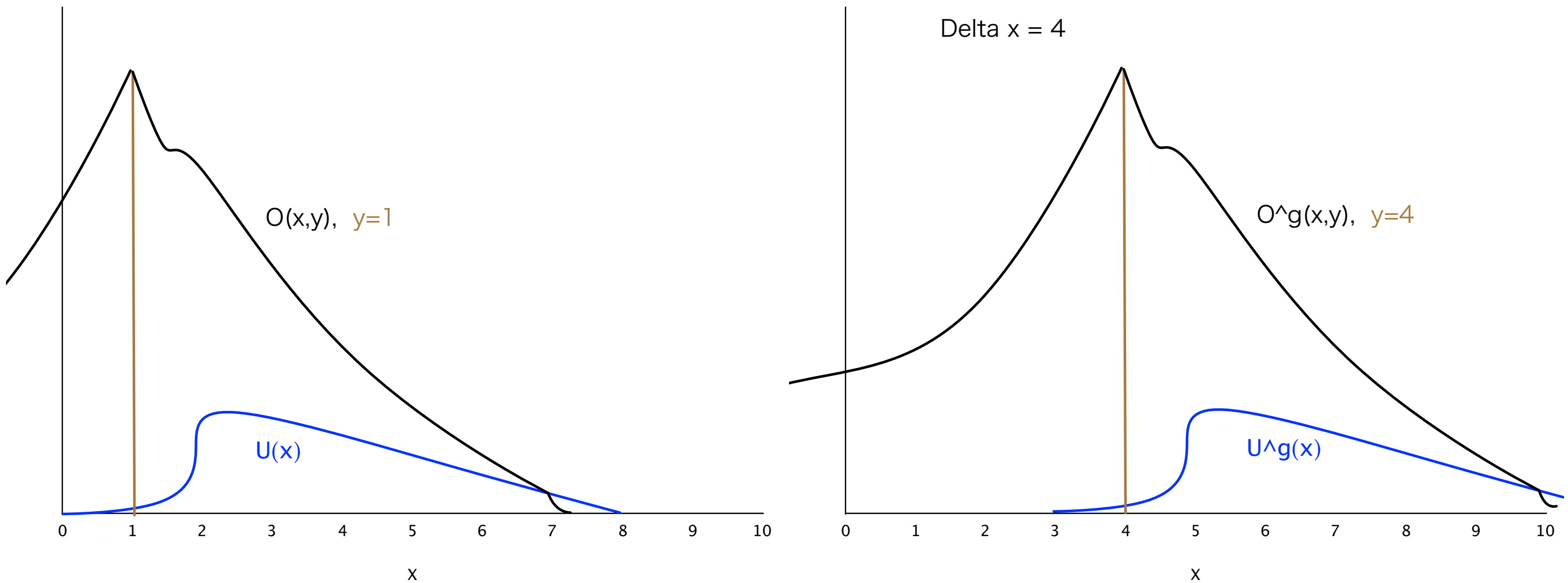
- Unbiased improved estimator

$$\mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G} \sum_{g \in G} \mathcal{O}^{(\mathrm{appx}),g}$$

Covariant approximation

- $O^{(appx)}$ needs to be precisely (to the numerical accuracy required) covariant under the symmetry of lattice action to avoid systematic errors.



One should check in the code using explicitly shifted gauge configuration

Why expect improvements ?

$$\mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g}$$

- $O^{(\mathrm{imp})}$ has smaller error, smaller C

  <= accuracy of approximation controls error,

  shouldn't be too accurate (0.1% is good enough)

- N_G suppresses the bulk part of noise cheaply

$$\mathrm{err} \approx C \times \frac{1}{\sqrt{N_{\mathrm{meas}}}}$$

Valence version of Hasenbushing in HMC

Examples of covariant approximations

- Low mode approximation used in the Low Mode Averaging (**LMA**)

L. Giusti et al (2004), see also T. DeGrand et al. (2004)

accuracy control : # of eigen mode

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_{\lambda} v_\lambda f(\lambda) v_\lambda^\dagger,$$

$$f(\lambda) = \frac{1}{\lambda}\theta(\lambda_{\mathrm{cut}} - |\lambda|)$$

Examples of Covariant Approximations (contd.)

- All Mode Averaging **AMA**

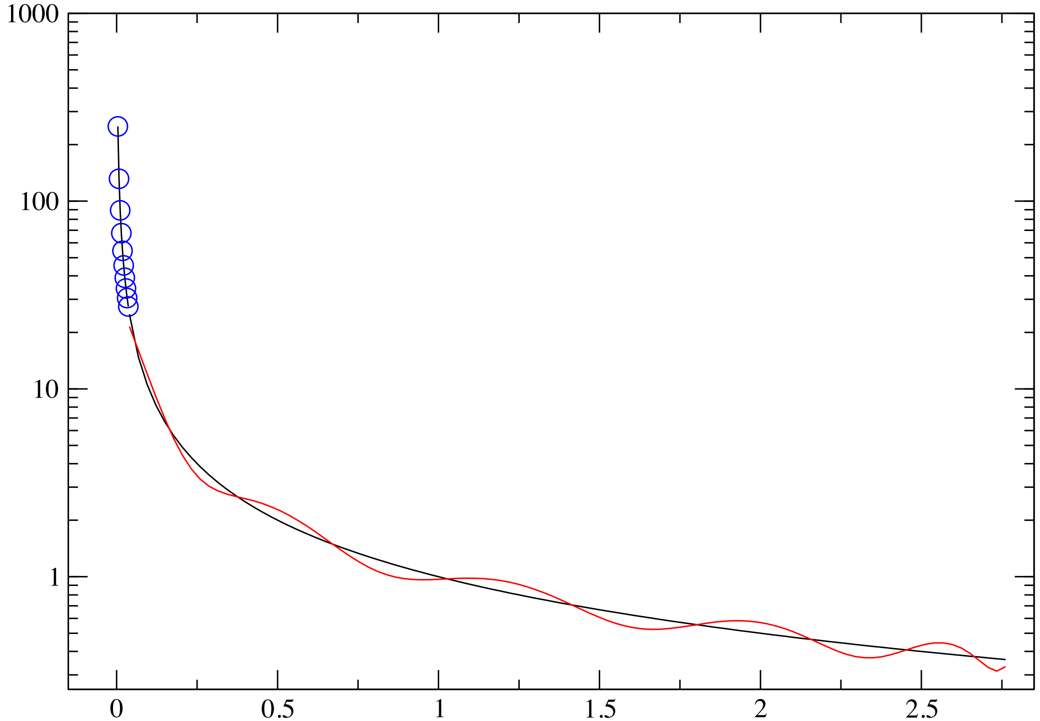
  Sloppy CG or Polynomial approximations

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_\lambda v_\lambda f(\lambda) v_\lambda^\dagger,$$

$$f(\lambda) = \begin{cases} \frac{1}{\lambda}, & |\lambda| < \lambda_{\mathrm{cut}} \\ P_n(\lambda) & |\lambda| > \lambda_{\mathrm{cut}} \end{cases}$$

$$P_n(\lambda) \approx \frac{1}{\lambda}$$

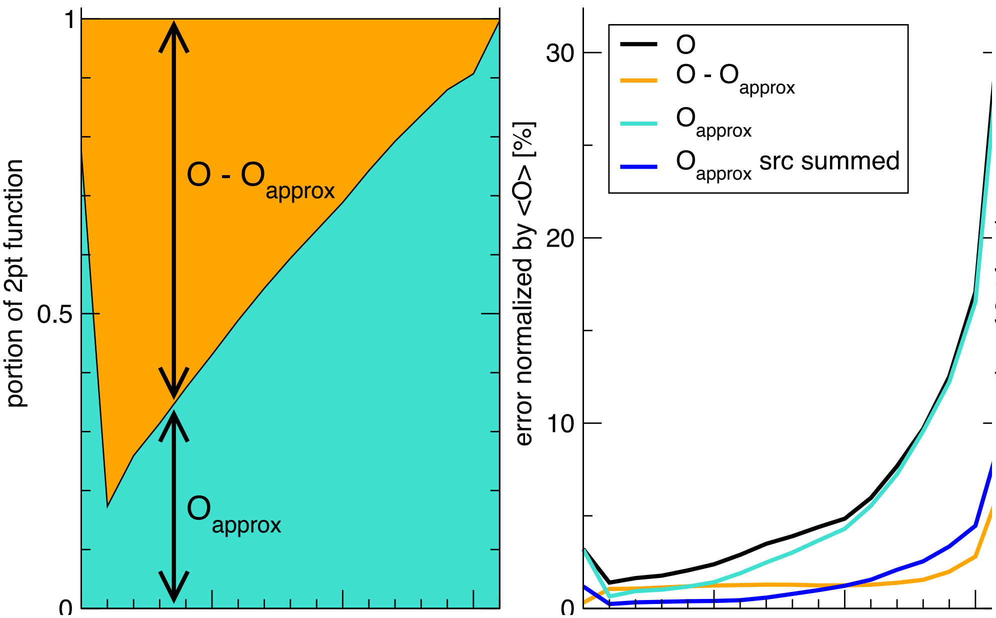


accuracy control :

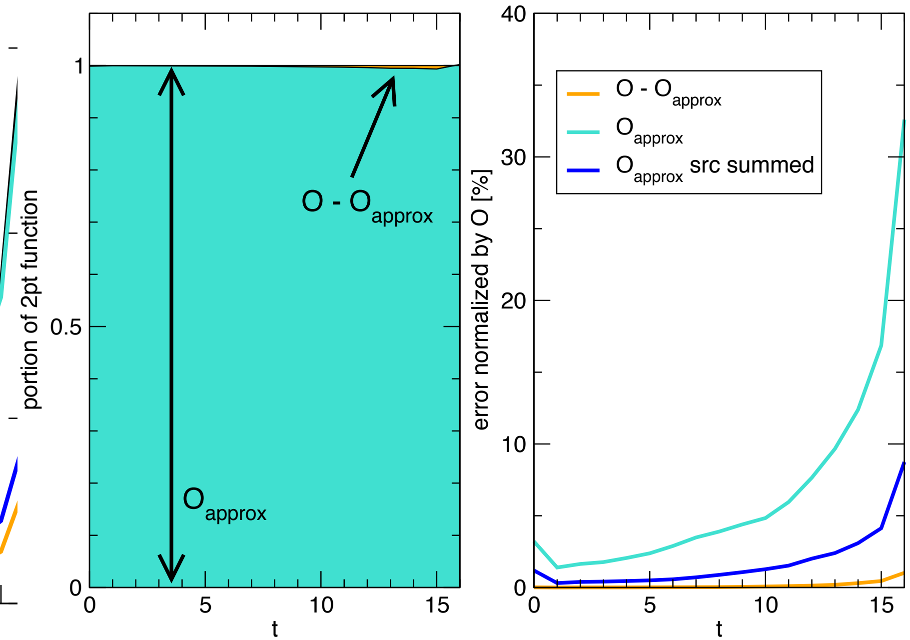
- low mode part : # of eig-mode
- mid-high mode : degree of poly.

AMA in USQCD Static-light
[PI Tomomi Ishikawa]

16^3x64x16, 20 conf, 100 eigenvectors

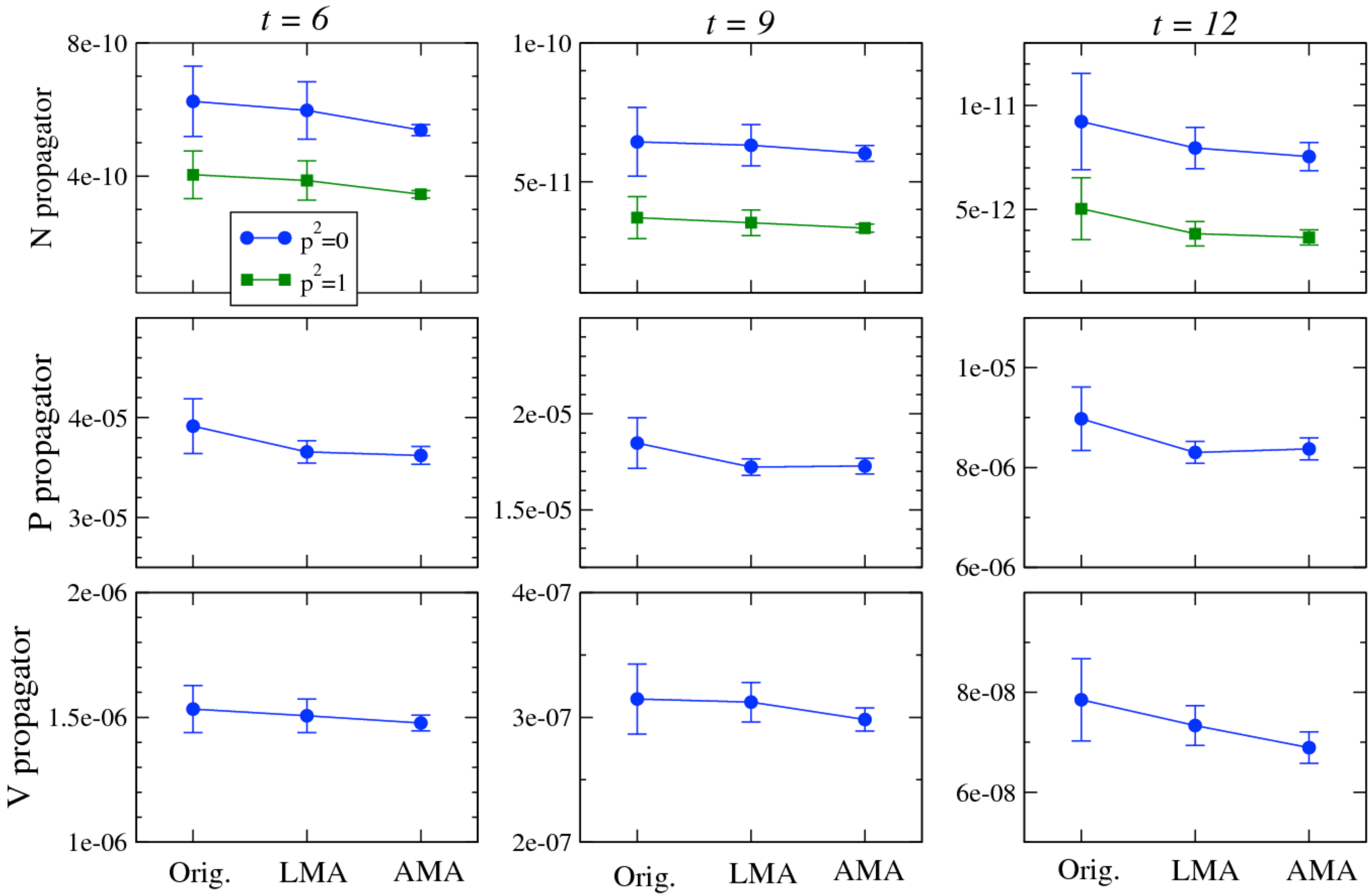


LMA

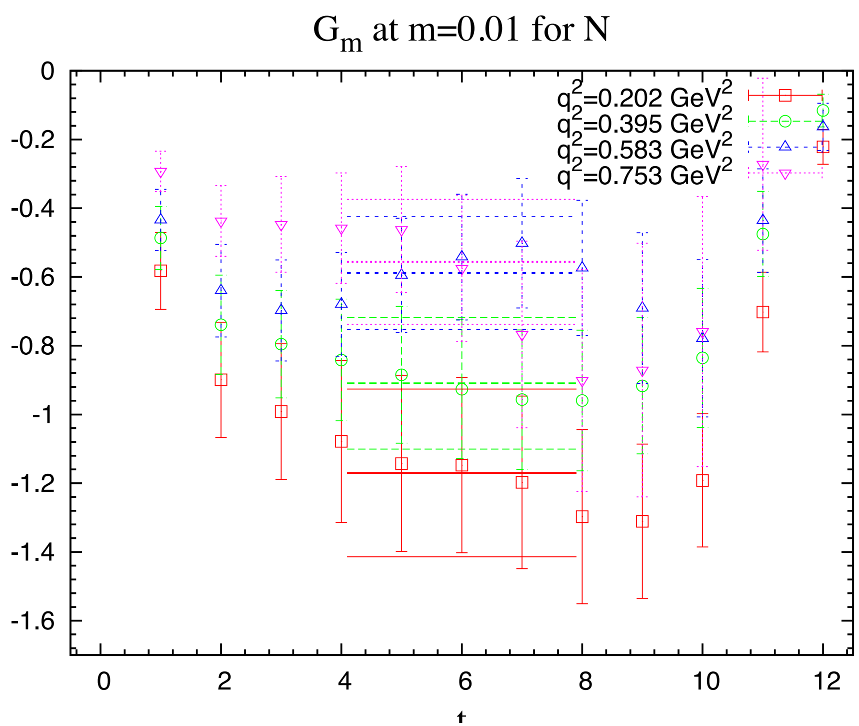


AMA

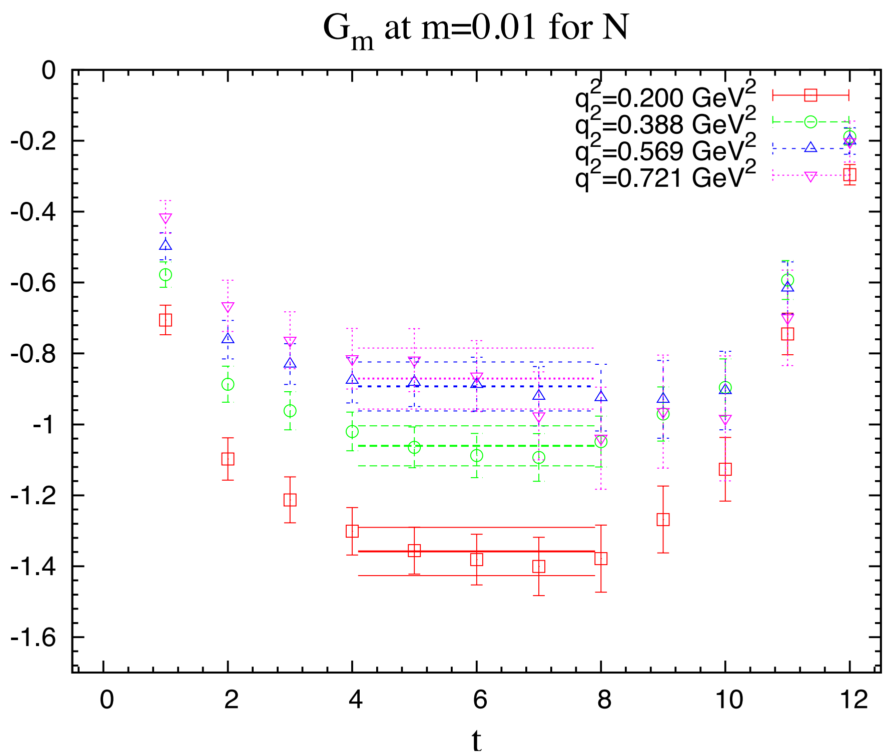
AMA results for hadron 2pt functions [E. Shintani]



Nucleon Magnetic formfactor [E. Shintani]



Original CG



AMA

Examples of Covariant Approximations (contd.)

- Less expensive (parameters of) fermions :
  - Larger mf
  - Smaller Ls DWF
  - Mobius
  - even staggered or Wilson …..
- Different boundary conditions
- More than one kinds of approximation (c.f. multi mass Hasenbushing)

Strongly depends on Observables / Physics (YMMV)

Would work better for EXPENSIVE observables and/or fermion, potentially a game changer ?

Other related/similar techniques

- LMA

  L. Giusti, P. Hernandez, M. Laine, P. Weisz and H. Wittig, JHEP 0404, 013 (2004)
  see also H. Neff, N. Eicker, T. Lippert, J. W. Negele and K. Schilling, Phys. Rev. D 64 (2001) 114509 and T. DeGrand and S. Schaefer, Comput. Phys. Commun. 159 (2004) 185

  works for low mode dominant quantities

- Truncated Solver Method (TSM)

  G. Bali, S. Collins, A. Schaefer, Comput. Phys. Commun. 181 (2010) 1570

  uses stochastic noise to avoid systematic error

- All-to-all propagator

  J.Foley, K.Juge, A. O'Cais, M. Peardon, S. Ryan, J-I. Skullerud, Comput.Phys.Commun. 172 (2005) 145

  uses stochastic noise
  could use CAA as a part of A2A

Summary

- Aslash method will be useful for both HLbL and QCD+QED simulations (isospin breaking studies)
- QED reweighting is an option for disconnected diagram of HLbL
- AMA : Statistical error reduction technique

Other technical details

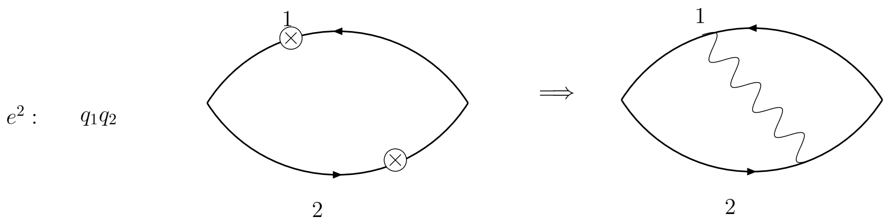
- Implicitly Restarted Lanczos with Polynomial acceleration and spectrum shifts for DWF and staggered in CPS++ [E. Shintani, T. Blum, TI].
- Eigen Vector compression / decompression

- Sea Electric Charge is now controlled by QED reweighting

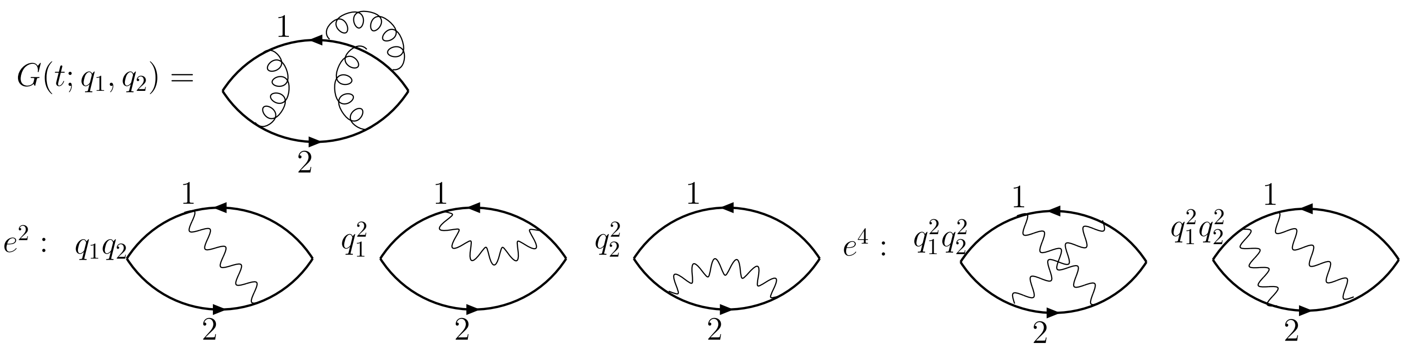
  [T. Ishikawa et. al. arXiv:1202.6018]
- Aslash-SeqSrc method

- $\not{A}$-Sequential source method. Compute each term of propagator in the e expansion.

$$S(e) = S(0) + ieS(0)\not{A}S(0) - e^2S(0)\not{A}S(0)\not{A}S(0) - e^2S(0)(\not{A})^2S(0)\cdots$$



make the contraction to desired orders of wanted diagrams piece by piece.



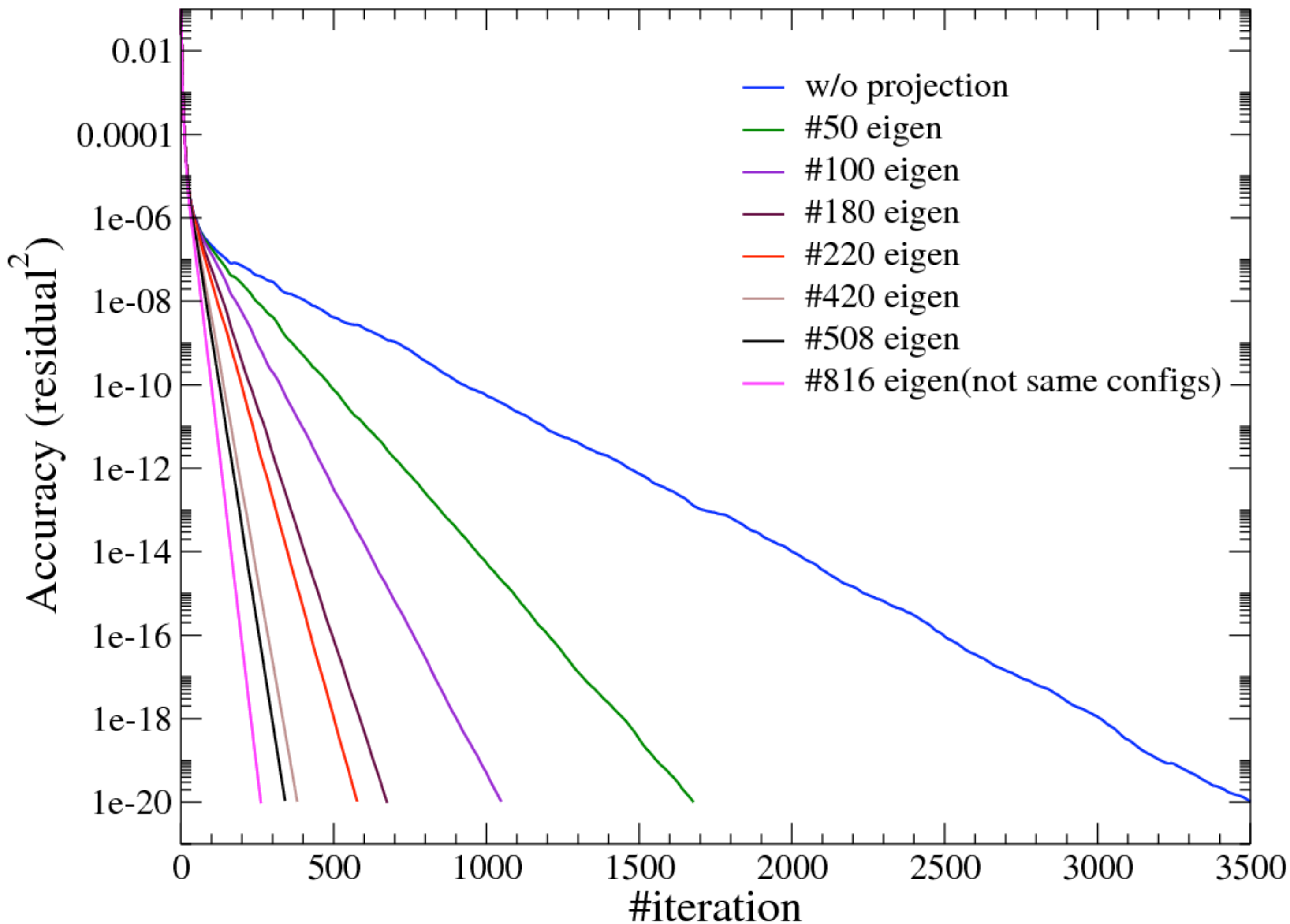
** No $\mathcal{O}(e^{2n+1})$ noise to disturb $\mathcal{O}(e^{2n})$, can skip diagrams of lower orders than the target.*

** Value of q and e could be determined off-line.*

** # of solves are equal or less up to $\mathcal{O}(e^2)$, compared to the original methods, needs five solves ($q = 0, \pm 2e/3, \mp e/3$).*

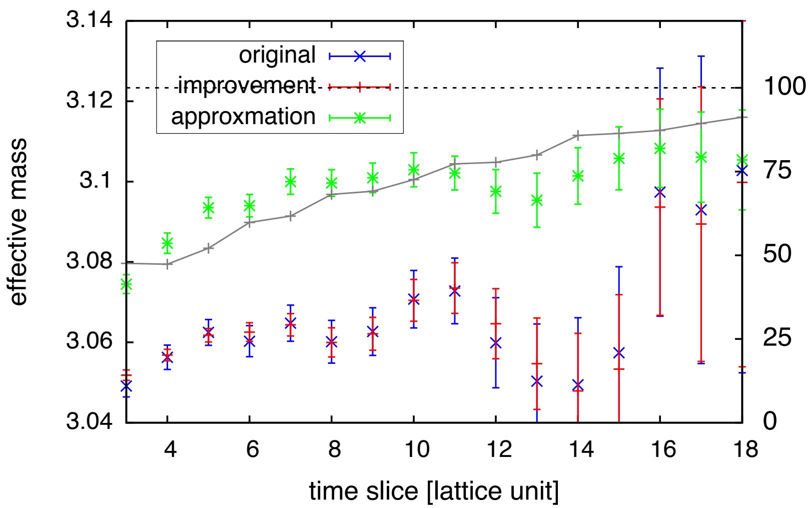
** Could use the $e = 0$ Eigen values/vectors.*

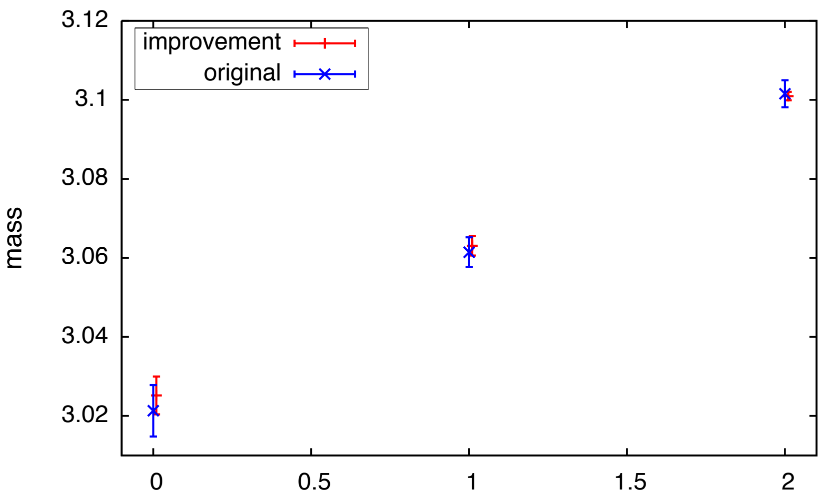
Deflation using low eigenmodes from Lanczos



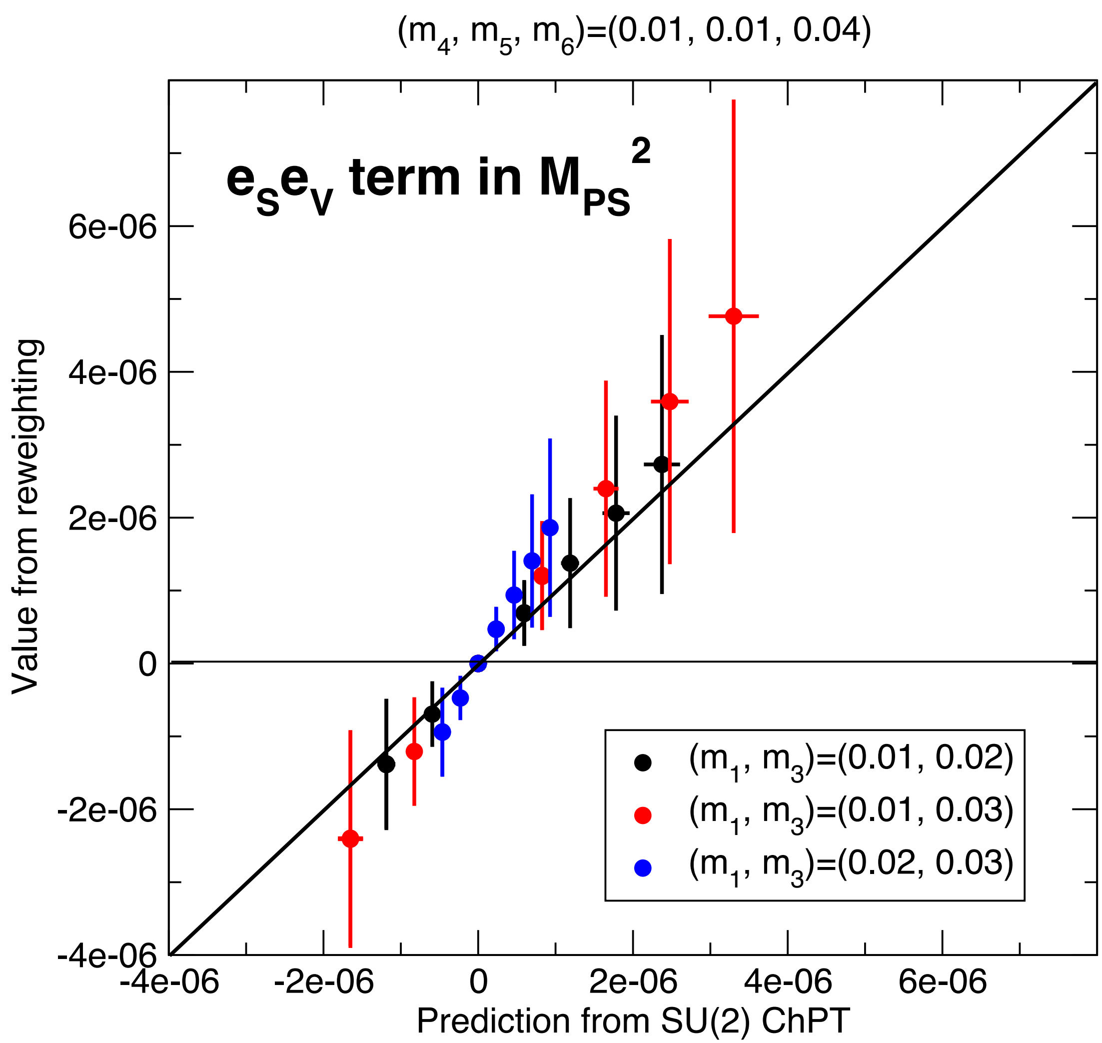
Larger mass as CAA [Taichi Kawanai]

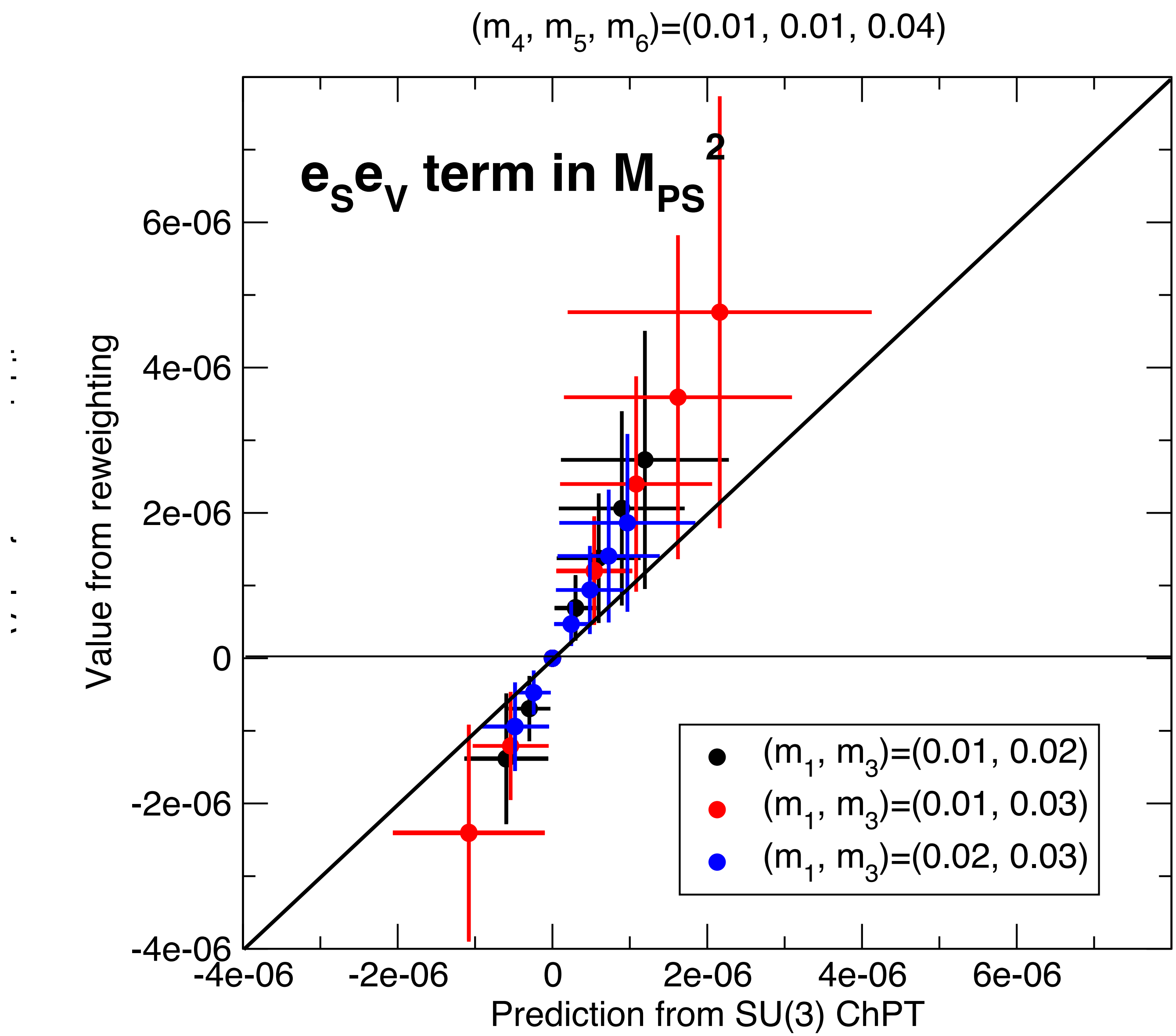
24^3x64x16, 20 config ,
mf=0.01 (target) mf=0.04 “approximation”





QED sea charge effect [Tomomi Ishikawa]





(m4, m5, m6)=(0.01, 0.01, 0.04)
eSeV term in MPS2
Value from reweighting
Prediction from SU(3) ChPT
6e-06
4e-06
2e-06
0
-2e-06
-4e-06
-4e-06
-2e-06
0
2e-06
4e-06
6e-06
(m1, m3)=(0.01, 0.02)
(m1, m3)=(0.01, 0.03)
(m1, m3)=(0.02, 0.03)