Probing Neutrino Models in Extra Dimensions with Muon to electron conversion

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Plan

- why $\mu - e$ conversion and ex-dim neutrino model
- a toy model: Zee model with split fermion
  (PRD 2011, with YTC and SCL)
- conclusion

Most of the SM parameters relate to Flavor Physics

- 21(+2) out of 27(+2) SM free parameters relate to the fermion masses.
- Extra dimension provides a new frame work for Yukawa
- ADD, RS, SF, UED, deconstruction....
- Neutrino models involve extra-dimension alleviate some problems in 4D theories
- Just like quark mixing, nonzero neutrino masses imply LFV

some common features of ex-dim model

- fermion masses and mixing $\Leftrightarrow$ WF overlapping in ex-dim
- in mass basis, KK gauge boson has tree-level LFV couplings
- usually, $\mu \to 3e$, μ-e conversion, and the like are much bigger than $\mu \to e\gamma$
- tree vs loop

effective operator analysis-1

$$\frac{\mathcal{L}_{\text{eff}}}{\sqrt{2}G_F} = \bar{e}(s - p\gamma^5)\mu\sum_q \bar{q}(s_q - p_q\gamma^5)q$$
$$+ \bar{e}\gamma^\alpha(v - a\gamma^5)\mu\sum_q \bar{q}\,\gamma_\alpha(v_q - a_q\gamma^5)q$$
$$+ \frac{1}{2}\bar{e}(t_s + t_p\gamma^5)\sigma^{\alpha\beta}\mu\sum_q \bar{q}\,\sigma_{\alpha\beta}q + \text{H.c.}$$

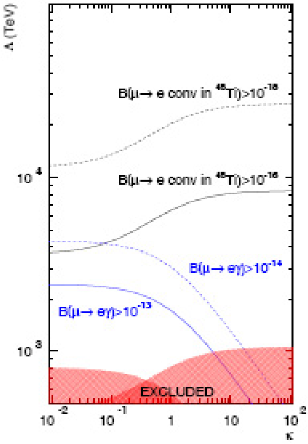
$$B_{\text{conv}} = \frac{p_e E_e G_F^2 F_p^2 m_\mu^3 \alpha^3 Z_{\text{eff}}^4}{2\pi^2 Z\Gamma_{\text{capt}}}\{|4eA_L Z + (s-p)S_N$$
$$+ (v-a)Q_N|^2 + |4eA_R Z + (s+p)S_N$$
$$+ (v+a)Q_N|^2\}$$

S_N, Q_N: the coherent scalar and vector coupling of nuclei N.

effective operator analysis-2

$$\mathcal{L} = \frac{m_\mu}{(1+\kappa)\Lambda^2}\bar{\mu}_R \sigma_{\mu\nu} e_L F^{\mu\nu} + \frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L \gamma_\mu e_L \left(\sum_{q=u,d} \bar{q}_L \gamma^\mu q_L\right)$$

Mu2e Proposal



The original 4D Zee Model

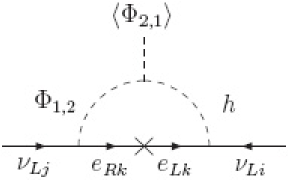
- In addition to SM Φ_1, one more $SU(2)_L$ Higgs doublet Φ_2 and an extra $SU(2)_L$ singlet charged Higgs h.
- The lagrangian

$$\begin{aligned}\mathcal{L}_{Zee} &= -f^1_{ab}\bar{\Psi}_{aL}\Phi_1 e_{bR} - f^2_{ab}\bar{\Psi}_{aL}\Phi_2 e_{bR} \\ &\quad -M_{12}\Phi_1 i\tau_2\Phi_2 h^* - f^h_{ab}\overline{\Psi^{\mathbf{c}}_{aL}} i\tau_2\Psi_{bL} h + H.c.\,,\end{aligned}$$

  a, b: the generation indices.
- This lepton number violating coupling term $\overline{\Psi^c}\Psi h$ is the key for generating the effective neutrino Majorana masses.

Its (good) consequences

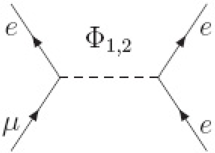
- No need of ν_R.
- No tree level masses. Neutrino masses generated at 1-loop.



- A natural and economical explanation of the smallness of neutrino masses.

FCNC

- Well known phenomenological problem in 2HDM.



- The charged lepton mass matrix,

$$\mathcal{M}^e_{ab} = \frac{1}{\sqrt{2}}(y^1_{ab} v_1 + y^2_{ab} v_2)$$

can be diagonalized by bi-unitary transformation:

$$V_L^\dagger \mathcal{M}^e_{ab} V_R = \text{diag}\{m_e, m_\mu, m_\tau\}$$

- In mass basis, both Yukawa $(V_L^\dagger y^{1,2} V_R)_{ab}$ are not flavor diagonal.

Zee-Wolfenstein model

- Wolfenstein proposed that only Φ_1 couples to the lepton, $f^2_{ab} = 0$.
- Additional $L_e - L_\mu - L_\tau$ symmetry gives inverted hierarchy and the bi-maximum mixings

$$\mathcal{M}_\nu \sim \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

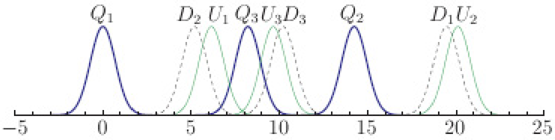
- The Zee-Wolfenstein model has been studied extensively in the past thirty years and found to be inconsistent with the neutrino data.
- However, the leading order pattern is very close to the data.

Not ruled out but not pretty either

- However, the **original** Zee model is **NOT** ruled out.
- Still possible to accommodate the observed neutrino data if both Higgs doublets couple to the leptons.
- But suffer from having too many arbitrary parameters (21 unknown complex Yukawa in the original Zee model for 3 generations) and the fine tuning problem to avoid the persist FCNC.

A New Paradigm for studying the flavor physics

- Flavor Problem $\Leftrightarrow$ Geometry in extra dimension
- Split fermion model as an example:
  - Linear displacement between left-handed and right-handed fermions in the fifth dimension becomes exponentially suppressed 4D Yukawa.
  - A realistic configuration to fit quark masses and mixings



Model Setup

- The space-time is $M_4 \times S_1/Z_2$ orbifold. The fifth-dim coordinate $0 \leq y \leq \pi R$, R radius of the S_1
- Z_2 transforms $y \leftrightarrow -y$, every bulk filed must be even or odd under this transformation.
- We assign Φ_1, Φ_2, h to be $+, -, -$ under this Z_2

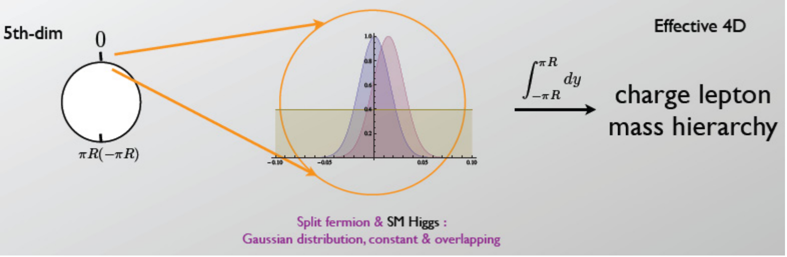
$$\Phi_1 = \frac{\Phi^{(0)}}{\sqrt{2\pi R}} + \sum_n \frac{\Phi_1^{(n)}}{\sqrt{\pi R}} \cos\frac{ny}{R}$$

$$\Phi_2 = \sum_n \frac{\Phi_2^{(n)}}{\sqrt{\pi R}} \sin\frac{ny}{R}\, , h = \sum_n \frac{h^{(n)}}{\sqrt{\pi R}} \sin\frac{ny}{R}$$

- The KK mode masses are

$$M_n^2 = M_0^2 + \frac{n^2}{R^2}$$

- Assumption: all SM chiral fermions are Gaussian distributed in y, universal width $1/\mu \equiv \sigma$, $\sigma \ll R$ ($\sigma/R = 5 \times 10^{-4}$ for numerical), and $c_i^{L/R}$ the peak location of fermion-i.



- After SSB, $\langle \Phi_1^{(0)} \rangle = v/\sqrt{2}$,

$$\mathcal{M}_{ab}^{e} = \hat{f}_{ab}^{1} \frac{v}{\sqrt{2}} \exp\left[\frac{-(c_a^L - c_b^R)^2}{2\sigma^2}\right] .$$

Model Setup

- The relevant Lagrangian for 5D Zee model is given by

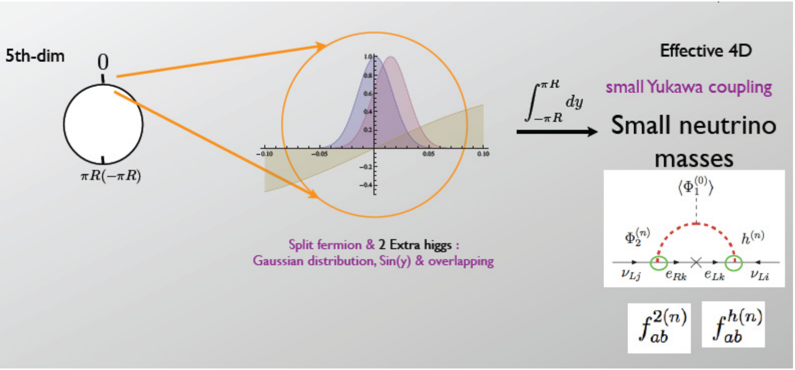
$$\begin{aligned}\mathcal{L}_{5DZee} &= -\sqrt{2\pi R}\hat{f}^{1}_{ab}\overline{\hat{\Psi}_{aL}}\hat{\Phi}_1\hat{e}_{bR} - \sqrt{2\pi R}\hat{f}^{2}_{ab}\overline{\hat{\Psi}_{aL}}\hat{\Phi}_2\hat{e}_{bR} \\ &\quad -\sqrt{2\pi R}\hat{f}^{h}_{ab}\overline{\hat{\Psi}^{c}_{aL}}i\tau_2\hat{\Psi}_{bL}\hat{h} - \frac{\kappa}{\sqrt{2\pi R}}\hat{\Phi}_1 i\tau_2\hat{\Phi}_2\hat{h}^* + H.c.\,,\end{aligned}$$

  $\hat{f}$'s and κ are dimensionless.

- κ acts as M_{12} in the original Zee model and it controls the overall neutrino masses.

Model Setup

- Effective 4D Yukawa are determined by overlapping as well:



- Assuming that all 5D Yukawa couplings, $\hat{f}^1_{ab}$, are of the same order, the CL mass hierarchy becomes a problem of finding the solution of the SF peak locations $\{c^R_1, c^R_2, c^R_3, c^L_1, c^L_2, c^L_3\}$ in the fifth dimension.
  21 complex Yukawa in 4D Zee $\Rightarrow$ 6 c's + $1/R$.
- The mass matrix can be diagonalized by a bi-unitary transformation,

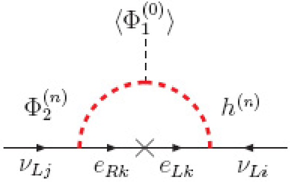
$$\mathrm{diag}\{m_e, m_\mu, m_\tau\} = U_L^\dagger \mathcal{M}^e U_R\,.$$

  In CL mass basis, the Yukawa couplings become $f^{2(n)} \Rightarrow U_L^\dagger f^{2(n)} U_R$, and $f^{h(n)} \Rightarrow U_L^T f^{h(n)} U_L$.

- 7B people in the world. Everyone is unique even we share 99.9% of DNA.


- But it's easy to differentiate 2 diff species. (98.5% of DNA are common in human and chimps)


Neutrino Mass Generation



- The neutrino mass is generated at 1-loop level. For $1/R \gg M_{\Phi_2,0}, M_{h,0}$, it is

$$\mathcal{M}^{\nu}_{ij} \simeq \frac{1}{16\pi^2} \sum_{n=1}^{\infty} \sum_{k} \left(\frac{\kappa v R}{2\sqrt{2}\pi} \right) \frac{m_k}{n^2} \left[\left(f_{ik}^{2(n)} \right)^* f_{kj}^{h(n)} + \left(f_{jk}^{2(n)} \right)^* f_{ki}^{h(n)} \right]$$

- In large $1/R$ limit, m_ν is NOT sensitive to $M_{\Phi_2,0}, M_{h,0}$.

4 sets are all Inverted hierarchy.

Configuration	κ (cubic-higgs coupling)	c_1^R	c_2^R	c_3^R	c_1^L	c_2^L	c_3^L
I	0.389	10.112	2.989	9.592	14.350	13.954	6.060
II	1.054	9.789	9.570	10.557	5.715	13.498	5.201
III	0.169	9.416	8.956	18.602	5.881	13.249	13.591
IV	0.974	1.371	8.159	17.663	12.595	12.106	4.346

TABLE I: The four viable configurations which can accommodate charge lepton and neutrino data in the same time. The split fermion location c's are in the unit of $\sigma(= 5 \times 10^{-4} R)$.

Configuration	m_e(MeV)	m_μ(MeV)	m_τ(GeV)	$\sin^2(2\theta_{12})$	$\sin^2(2\theta_{23})$	θ_{13} (rad)	(deg)
I	3.1 ± 1.5	120(22)	1.73(31)	0.79(24)	0.43(26)	0.11(8)	6.3(46)
II	6.3 ± 3.0	119(20)	2.49(48)	0.84(18)	0.72(24)	0.16(11)	9.2(63)
III	0.64(12)	122(22)	1.70(31)	0.76(27)	0.56(27)	0.33(20)	19(11)
IV	0.49(10)	78(14)	2.25(43)	0.83(20)	0.93(08)	0.13(7)	7.4(40)

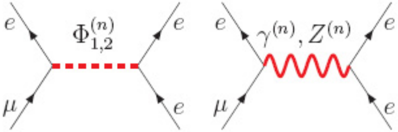
TABLE II: Charged lepton masses and neutrino mixings in the 4 viable configurations

- Only inverted hierarchical masses (in *meV*) are found.

Configuration	m_1^ν	m_2^ν	m_3^ν	$\lvert m_{ee}^\nu \rvert$
I	38 ± 13	46 ± 14	1.4 ± 1.3	14 ± 7
II	41 ± 16	45 ± 15	5.1 ± 4.2	6 ± 3
III	40 ± 16	45 ± 16	6.2 ± 5.0	8 ± 4
IV	39 ± 16	49 ± 15	5 ± 7	9 ± 5

Phenomenology

- Only Z_2-even $\Phi_1^{(0)}$ gets nonzero VEV, SM Higgs Yukawa is always flavor diagonal. The LFV in the 5D Zee model is $\sim 10^{-2}$ smaller than the 4D Zee model.
- KK gauge boson or scalar have tree-level FCNC couplings.
- tree-level LFV processes will be much larger than the loop induced ones, e.g. $Br(\mu \to 3e) \gg Br(\mu \to e\gamma)$.



KK gauge boson dominates

- Exp: $Br(\mu \to 3e) < 10^{-12}$ and $Br(\tau \to l_1 l_2 l_3) < 3 \times 10^{-8}$
- Summing up the contrib. of the first 200 KK photon and Z.

Decay mode	Conf. I	Conf. II	Conf. III	Conf. IV
$Br(\mu^- \to e^+e^-e^-)$	$4(2) \times 10^{-13}$	$1.6(6) \times 10^{-13}$	$2(1) \times 10^{-13}$	$1.3(7) \times 10^{-13}$
$Br(\tau^- \to e^+e^-e^-)$	$1.9(9) \times 10^{-11}$	$9(6) \times 10^{-14}$	$1.5(1.5) \times 10^{-14}$	$1.3(1.3) \times 10^{-18}$
$Br(\tau^- \to \mu^+\mu^-e^-)$	$1.0(5) \times 10^{-11}$	$5(3) \times 10^{-14}$	$1.0(9) \times 10^{-14}$	$1.2(1.2) \times 10^{-18}$
$Br(\tau^- \to e^+e^-\mu^-)$	$4(3) \times 10^{-13}$	$3.0(2.8) \times 10^{-14}$	$2.8(2.6) \times 10^{-13}$	$3(2) \times 10^{-13}$
$Br(\tau^- \to \mu^+\mu^-\mu^-)$	$7(6) \times 10^{-13}$	$5.3(5.0) \times 10^{-14}$	$7(6) \times 10^{-13}$	$1.1(6) \times 10^{-12}$

- Doubly suppressed rare tau decays $\tau \to e^-\mu^+e^-$, $e^-e^+\mu^-$, $\mu^-e^+\mu^-$, $\mu^-\mu^+e^-$ not considered.

LFV at precision

- No obvious pattern among $Br(\mu \to e^- e^+ e^-)$, $Br(\tau \to \mu^- \mu^+ \mu^-)$, and $Br(\tau \to e^- e^+ e^-)$ across the four configurations we found. They will provide a handle to distinguish different geography in the split fermions scenario.
- All four configuration give $\Lambda_{\mu \to e} \sim 4 \times 10^3$ TeV $\Rightarrow$ could be probed at the proposed Mu2e experiment.
- This shows the importance of improving of the current LFV experimental bounds. They will provide crucial information to decipher the origin of flavor physics.

LFV at high energy

- LFV decay $V^{(1)} \to l_i^+ l_j^-$ is proportional to $\left|g_{L,ij}^{V_1}\right|^2 + \left|g_{R,ij}^{V_1}\right|^2$.
- Since $\sin^2\theta_W = 0.23 \sim 1/4$, $\left|-1/2 + \sin^2\theta_W\right| \sim \left|\sin^2\theta_W\right|$, hence LFV branching ratios of the $\gamma^{(1)}$ and $Z^{(1)}$ are proportional to each other mode by mode.
- If LFV dominated by the first KK gauge boson, $V^{(1)}$, be it the $\gamma^{(1)}$ or $Z^{(1)}$, one has

$$\frac{Br(V^{(1)} \to \tau e)}{Br(V^{(1)} \to \mu e)} : \frac{Br(V^{(1)} \to \tau\mu)}{Br(V^{(1)} \to \mu e)} \sim \frac{Br(\tau \to 3e)}{Br(\mu \to 3e)} : \frac{Br(\tau \to 3\mu)}{Br(\mu \to 3e)}.$$

Conclusion

- flavor may have ex-dim origin
- intense frontier is complementary to high energy frontier. ex-dim could be probed indirectly at low energy
- in particular, LFV provides invaluable information of the origin of neutrino masses
- generally speaking, $\mu \to 3e$ and $\mu - e$ conversion is favored in ex-dim models.