Optics Techniques in Muon Cooling Channels

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Overview

- Beam Optics Basics
- Issues for Muon Cooling Channels
- Example of Optics-based Muon Cooling Channel Design: PIC – Twin Helix Channel
- Uses of Muon Cooling in PX Era

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Beam Optics Overview

- Define Particle Coordinates:

$$\vec{z}\left(x,a,y,b,l,\delta\right)$$

- Taylor Map Calculates Change in Coordinates:

$$\vec{z}_f = \left(M + N\right)\vec{z}_i$$

Linear terms can be represented as a matrix

$$\begin{pmatrix} x_f \\ a_f \\ y_f \\ b_f \\ l_f \\ \delta_f \end{pmatrix} = \begin{pmatrix} x\,|\,x & x\,|\,a & 0 & 0 & x\,|\,l & x\,|\,\delta \\ a\,|\,x & a\,|\,a & 0 & 0 & a\,|\,l & a\,|\,\delta \\ 0 & 0 & y\,|\,y & y\,|\,b & 0 & 0 \\ 0 & 0 & b\,|\,y & b\,|\,b & 0 & 0 \\ l\,|\,x & l\,|\,a & 0 & 0 & l\,|\,l & \delta\,|\,l \\ \delta\,|\,x & \delta\,|\,a & 0 & 0 & l\,|\,\delta & \delta\,|\,\delta \end{pmatrix} \circ \begin{pmatrix} x_i \\ a_i \\ y_i \\ b_i \\ l_i \\ \delta_i \end{pmatrix}$$

Non-linear terms can be calculates as a Taylor series expansion in terms of the initial particle coordinates:

$$x_f = \left(x\,|\,xx\right)x_i^2 + \left(x\,|\,xa\right)x_i a_i + \left(x\,|\,aa\right)a_i^2 + \ldots..$$

$$+(x\,|\,xxx)x_i^3 + (x\,|\,xxa)x_i^2 a_i + \ldots\ldots$$

Uses of Transfer Maps

Terms in the linear map contain crucial optical information about your beam channel

Magnification terms

(x|a) = 0 for point to point imaging
(a|x) = 0 for parallel to parallel imaging

$$
\begin{pmatrix}
x|x & x|a & 0 & 0 & x|l & x|\delta \\
a|x & a|a & 0 & 0 & a|l & a|\delta \\
0 & 0 & y|y & y|b & 0 & 0 \\
0 & 0 & b|y & b|b & 0 & 0 \\
l|x & l|a & 0 & 0 & l|l & \delta|l \\
\delta|x & \delta|a & 0 & 0 & l|\delta & \delta|\delta
\end{pmatrix}
$$

Determinant of the matrix will be less than one for a system with cooling in 6-D phase space

Non-zero for dispersion

Nonlinear Optics

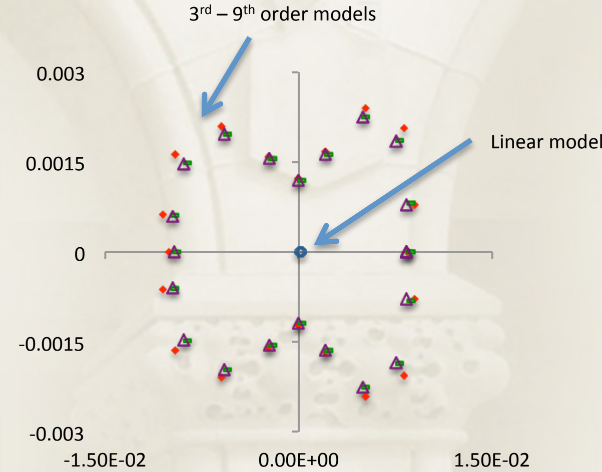
- Chromatic Aberrations:
  - energy dependent
  - ex. (x|ad) is a second order aberration dependent on initial angle and energy spread relative to reference particle orbit
- Geometric Aberrations:
  - angular and position dependent only
  - ex. (x|xxx) is the third order aberration dependent on the cube of initial position offset from the reference orbit

Impact of Nonlinear Aberrations

Simulation codes such as COSY Infinity can be used to calculate aberration coefficients and show impact on the channels optics:

Second Order Aberration Map

-0.5980590E-04	0.9763552E-05	0.000000	0.000000	-0.5401955E-04	2000000
0.1001248E-04	0.7760842E-02	0.000000	0.000000	0.2169705E-03	1100000
$\boxed{0.1995436E-02}$	-0.3226334E-02	0.000000	0.000000	-0.1005011E-01	$\boxed{0200000}$
0.000000	0.000000	-0.1046741E-03	-0.7584133E-04	0.000000	1010000
0.000000	0.000000	0.3367245E-04	-0.3062197E-01	0.000000	0110000
-0.2356046E-03	-0.2986482E-04	0.000000	0.000000	-0.2483222E-03	0020000
0.000000	0.000000	-0.1126418E-05	0.6798941E-02	0.000000	1001000
0.000000	0.000000	-0.4565814E-03	-0.4007927E-02	0.000000	0101000
0.3271621E-04	-0.6803578E-02	0.000000	0.000000	-0.1972945E-03	0011000
0.2292975E-03	-0.1728703E-03	0.000000	0.000000	-0.3555218E-02	0002000
0.2282534E-04	0.7349571E-03	0.000000	0.000000	0.7267789E-06	1000010
$\boxed{0.2061992E-02}$	-0.7999127E-03	0.000000	0.000000	0.2934764E-03	$\boxed{0100010}$
0.000000	0.000000	0.2216899E-04	0.3354879E-02	0.000000	0010010
0.000000	0.000000	-0.7371019E-03	-0.1125724E-02	0.000000	0001010
-0.2045517E-04	0.1356720E-03	0.000000	0.000000	-0.3927054E-04	0000020



Spot size after 20 cells for monochromatic point source, θ = 50 mrad,

Largest 2nd order aberrations effecting horizontal position are (x|aa) and (x|aδ)

Since muon beams can have large initial angular and energy spreads, these aberrations might dramatically impact final beam spot size

Minimizing Aberrations

- Aberrations traditionally minimized using higher order multipoles
  - sextupoles for 2nd order terms, octopoles of 3rd order terms etc.
- Symmetries can also be used to minimize certain aberrations
- Simulations at various orders can demonstrate which order terms are most critical
  - If simulation results converge above 5th order then aberration correction may focus on lower order terms

Optics Issues for Muon Cooling Channels

- Large phase space volume impacts aberration correction
  - Large initial angular spreads
  - Need large dynamic aperture
  - Energy spread
- Short muon lifetime
- Muon cooling channels can have complicated fields and non-standard elements that creating higher order effects
  - solenoid fringe fields
  - helical magnet channels
  - PIC Twin Helix
- Ionization cooling requires you to deal with material interactions
  - Multiple scattering
  - Energy straggling
  - Space charge interactions
  - Particle decays
- RF Cavities must be properly modeled

Parametric-Resonance Ionization Cooling

Parametric-resonance Ionization Cooling (PIC) offers an example of using an optics-based approach to designing a muon cooling channel

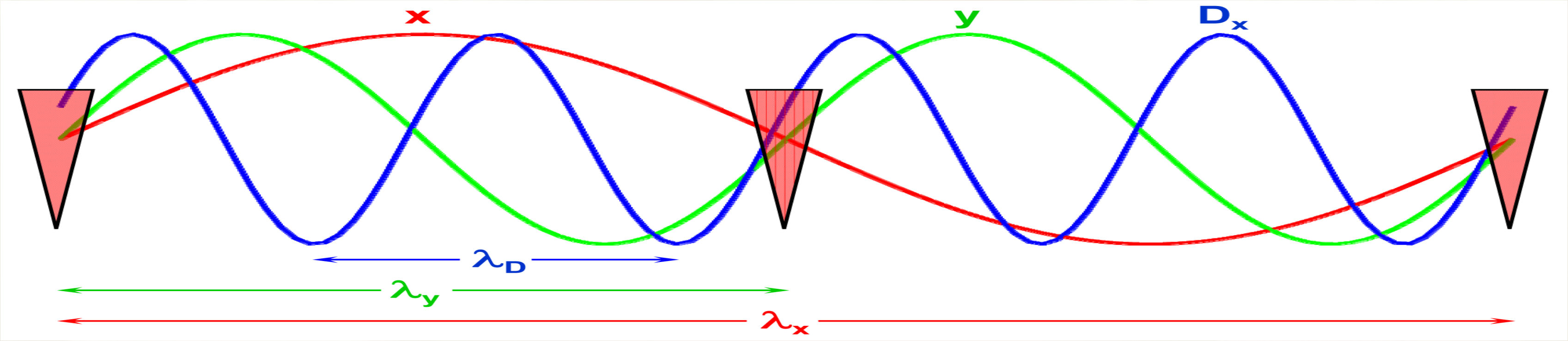
PIC is proposed as a final stage 6-D cooler for muon collider

Analytic calculations show PIC can improve cooling by about a factor of 10 over ionization cooling only

Theorize it! – Implement it? – Simulate it? – Optimize it?

How PIC works

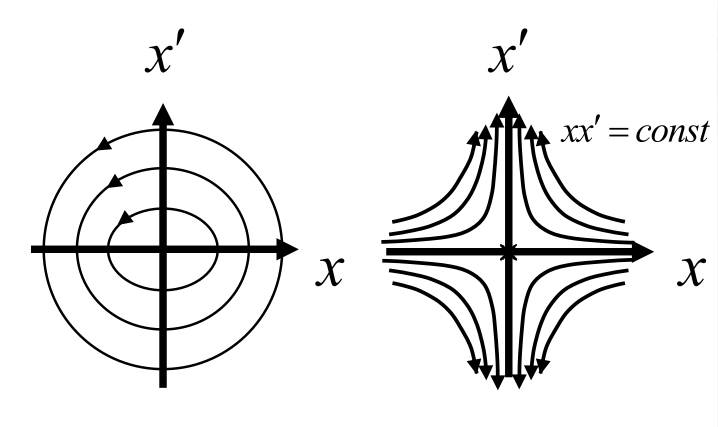
- Correlated optics maintains a stable reference orbit where the betatron tunes in the horizontal and vertical planes are integer multiples of the dispersion function for the system



Ex. $\lambda_y = 2\lambda_x = 4\lambda_D$

- ½ integer resonances are introduced to create a hyperbolic fixed point

- Wedge absorbers minimizes angular blowup while RF cavities are used to maintain the reference momentum



PIC in the Twin-Helix Channel

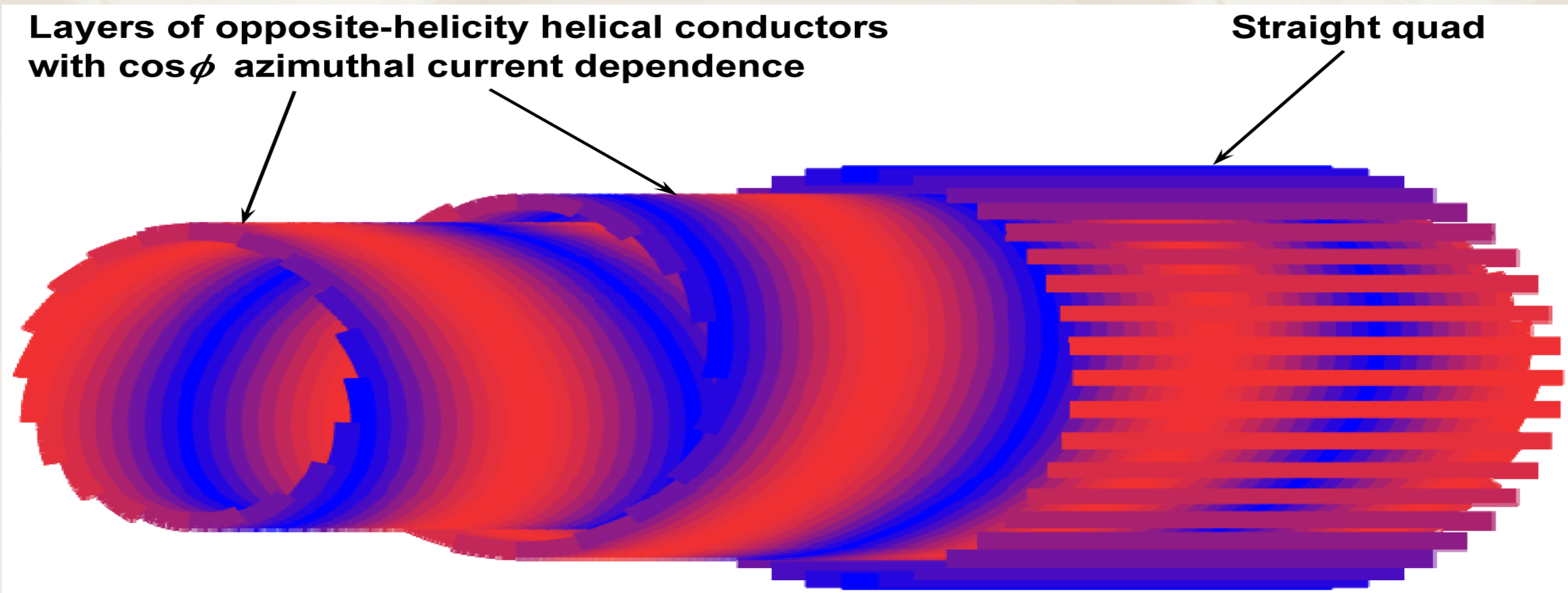
- One example implementing PIC criteria is the twin-helix channel
- The basic channel involves:
  - A pair of helical dipole harmonics of equal field gradient and equal but opposite helicities
  - A continuous quadrupole field is superimposed to maintain the correlated optics conditions
  - Two additional pairs of helical quadrupole harmonics (parametric lenses) induce the ½ integer resonances: one pair for the horizontal and the other for the vertical plane
  - Beryllium wedge absorbers placed every 4 meters
  - RF cavities are placed 3 cms after each absorber

Simulation Parameters

H. Dipole Field	1.63 T
H. Dipole wavelength	1 meter
Continuous Quadrupole Field	.72 T/m
H. Quadrupole Field (Horizontal Lenses)	.02 T/m
H. Quadrupole wavelength	2 meters
H. Quadrupole Field (Vertical Lenses)	.04 T/m
H. Quadrupole wavelength	1 meter
RF Voltage	-12.5 MeV
RF Frequency	201.25 MHz
RF Phase	30 Degrees
Wedge central thickness	2 cm
Wedge thickness gradient	30%

Implementation of Twin-Helix

- One proposed conceptual drawing of implementation of twin-helix channel using a combination of 2 helical conductor layers and a straight quadrupole.
- Colors indicate current variation in the conductors

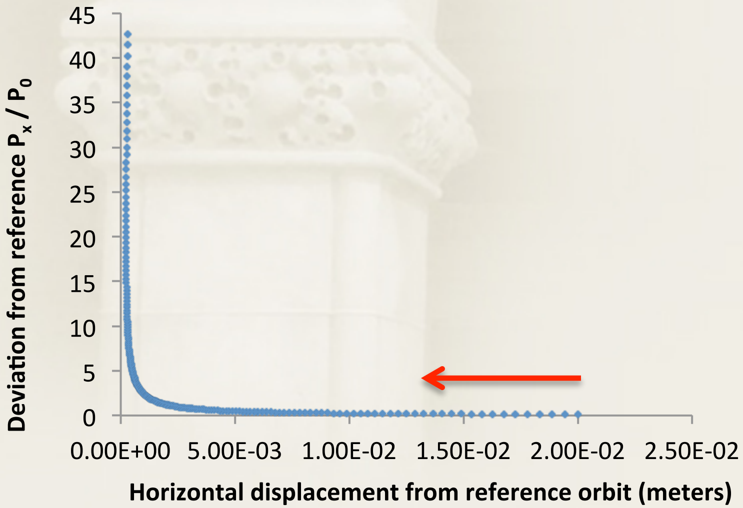


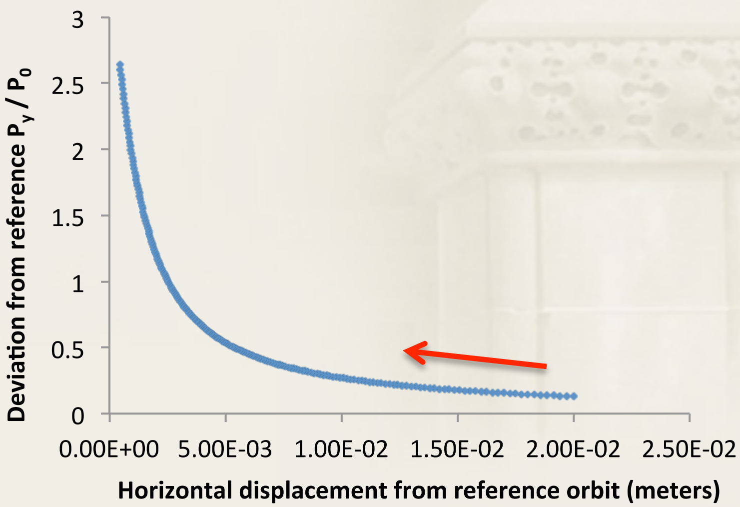
Hyperbolic Fixed Point

The basic twin-helix channel is simulated without wedge absorbers or energy restoring RF cavities

250 MeV/c μ^- launched offset from the reference orbit by 2 cm and 130 mrad in both planes

This test particle is tracked every 4 meters for 200 cells



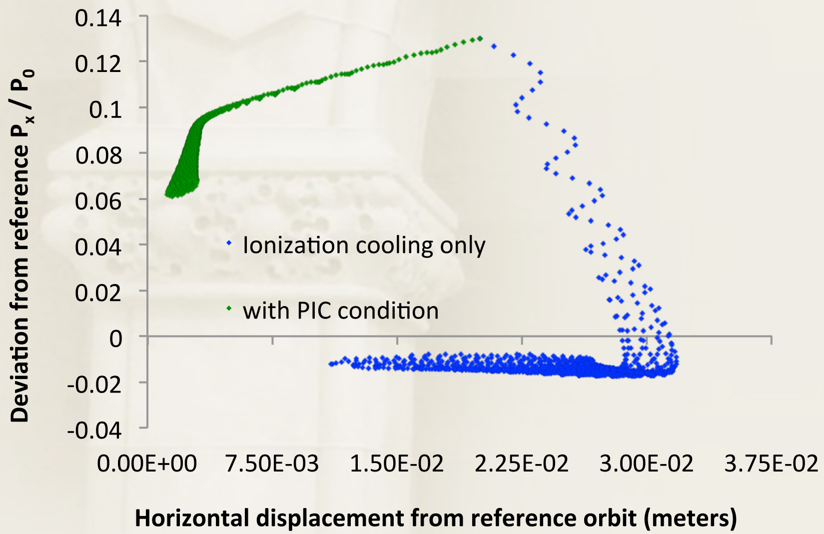


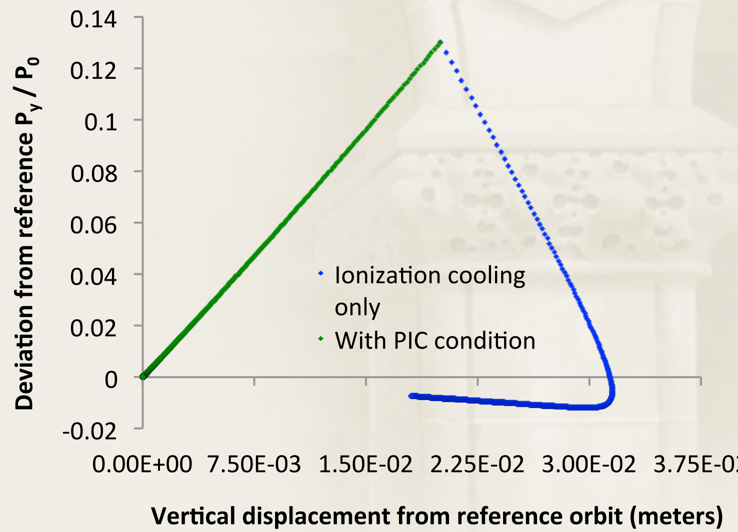
Ionization Cooling and PIC

Basic channel with wedge absorbers and energy-restoring RF cavities is simulated with and without parametric lenses

250 MeV/c μ^- launched offset from the reference orbit by 2 cm and 130 mrad in both planes

This test particle is tracked every 4 meters for 1000 cells





Addition of Stochastic Effects

- Tracking using transfer maps: $\vec{z}_f = \ldots \mathrm{M} \bullet \mathrm{M} \bullet \mathrm{M} \bullet \vec{z}_i$
- Adding stochastics "map": $\vec{z}_f = \ldots \Sigma \bullet \mathrm{M} \bullet \Sigma \bullet \mathrm{M} \bullet \Sigma \bullet \mathrm{M} \bullet \vec{z}_i$
- Stochastic "map" defined to produce these results:

$$\begin{pmatrix} x_f = x_i \\ a_f = a_i + \Delta(scattering) \\ y_f = y_i \\ b_f = b_i + \Delta(scattering) \\ l_f = l_i \\ \delta_f = \delta_i + \Delta(straggling) \end{pmatrix}$$

Multiple scattering modeled using PDG formula RMS98

$$\theta_{scatter} = \frac{13.6 MeV}{\beta c p} \sqrt{\frac{z}{\chi_0}} \left(1 + .028 \ln\left(\frac{z}{\chi_0}\right)\right)$$

Energy straggling modeled using Bohr approximation

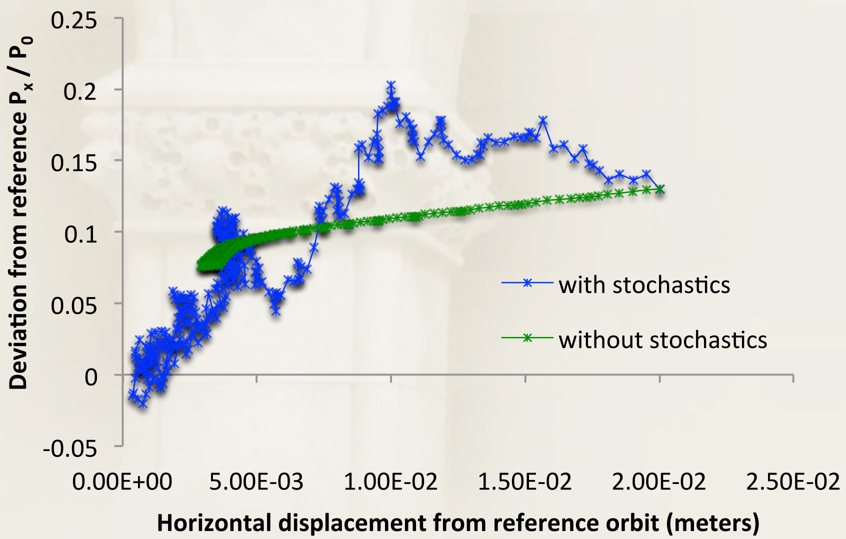
$$\Omega^2_{straggling}\left[KeV^2\right] \approx .26 Z_{absorber} z N_t \left[10^{18} atoms / cm^2\right]$$

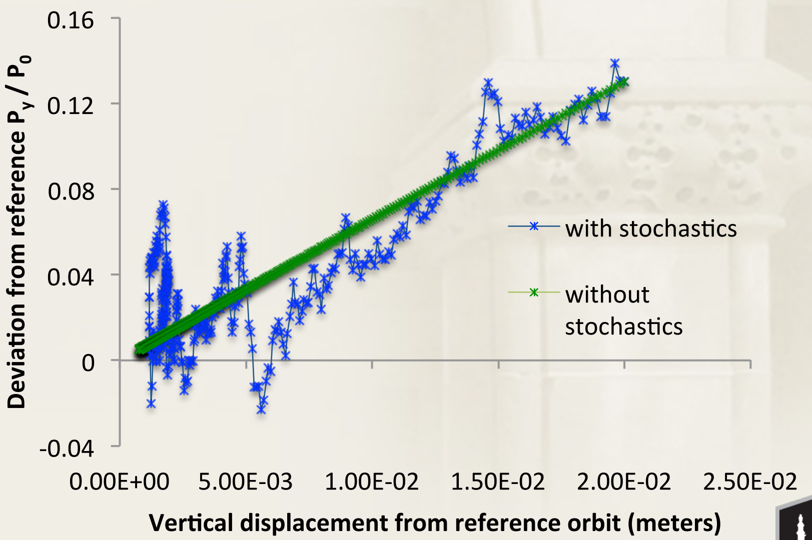
Stochastic Effects for Single Particle

Basic channel with wedge absorbers and energy-restoring RF cavities is simulated with parametric lenses – with and without the stochastic effects of multiple scattering and energy straggling

250 MeV/c μ^- launched offset from the reference orbit by 2 cm and 130 mrad in both planes

This test particle is tracked every 4 meters for 400 cells

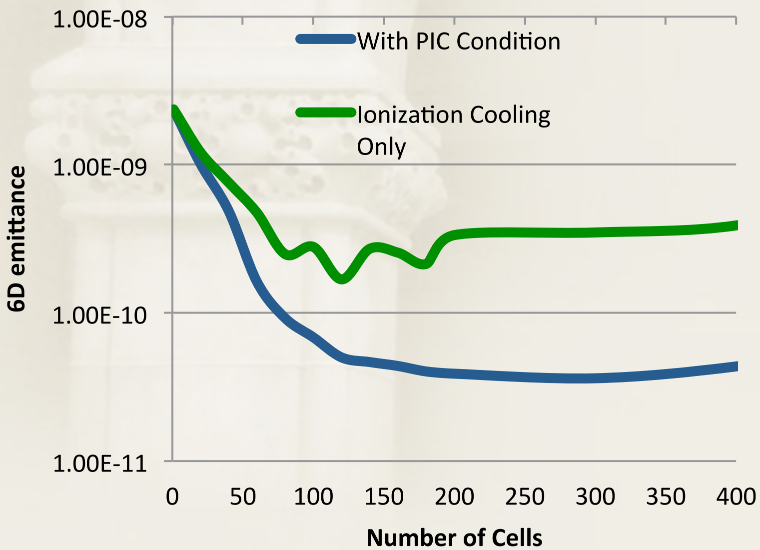


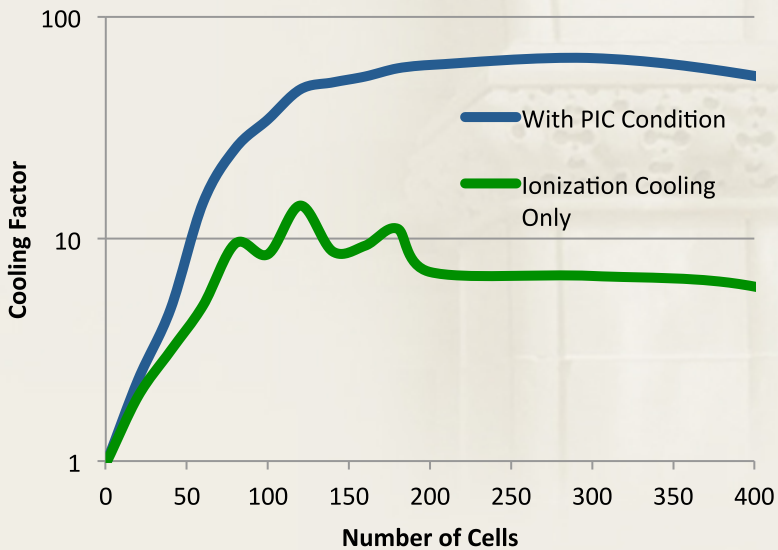


Cooling Factor Measurements

A distribution of test particles in the full simulation of the linear channel with stochastics. The initial distribution uses a sigma of 2 cm in offsets, 130 mrad in angles, and 1% spread in energy from the reference particle. The distribution is also spread over a bunch length of ± 3 cms relative to the reference particle.

Comparison of cooling factor (ratio of initial to final 6D emittance) with and without the PIC condition is consistent with theory indicating improved in cooling by ~ factor of 10

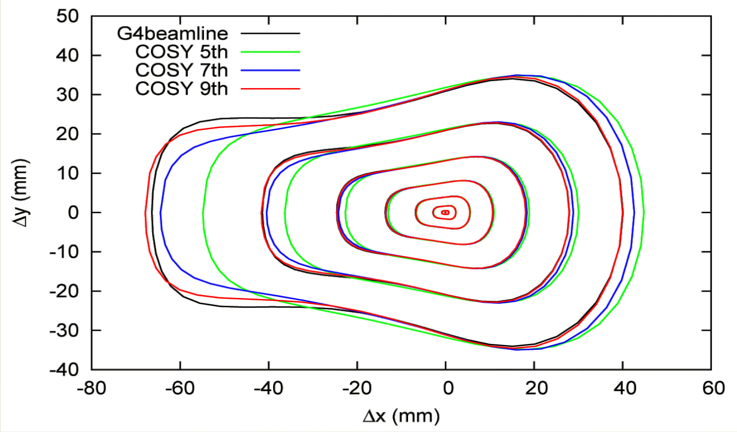




Evaluating Aberrations and Effects

- Linear Model sets a baseline for aberration corrections
  - "perfect" correction is the linear model
- Aberrations can be studied through maps
  - which aberrations at each order are largest
  - which aberrations will impact sensitivity of optics
- Aberration effects can be evaluated by order
  - at what order do optics results converge
  - correcting lower order aberrations can correct dependent higher order aberrations

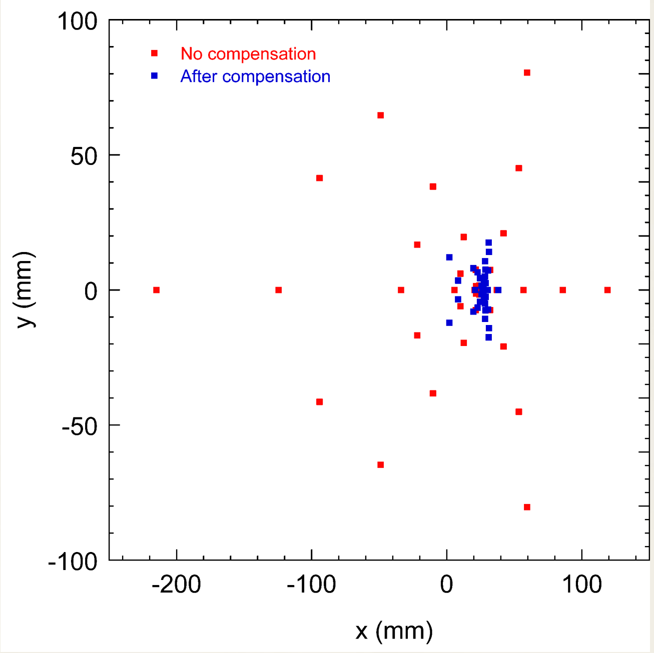
Aberrations effecting spot size for the λ_D=20 cm. twin-helix > 10^{-3} at 2nd and 3rd order	
(x|aa)	0.0015
(x|aδ)	0.0021
(x|aaa)	-0.0178
(x|abb)	-0.0061
(y|aab)	0.0061
(y|bbb)	0.0012



monochromatic point source, ±160 mrad, 2 helix periods

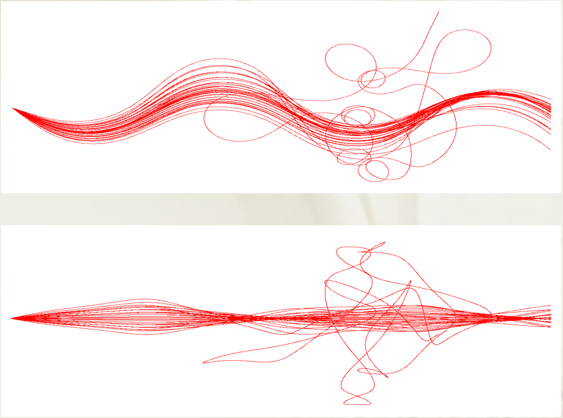
Effects of Aberration Correction

Correction of aberrations offers substantial improvement in dynamic aperture

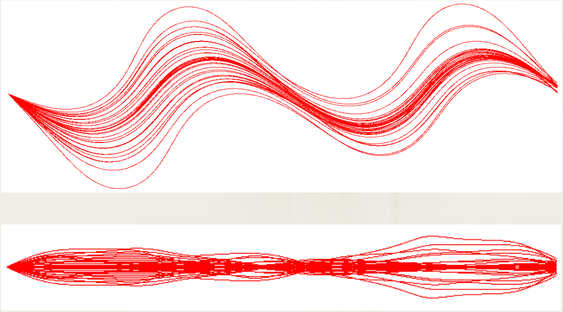


Basic twin-helix helical dipole and straight quadrupole

Initial beam distribution: azimuthal ϕ from 0 to 2π in steps of $\pi/4$, polar θ from 20 to 220 mrad in steps of 40 mrad



Additional helical quadrupole and straight sextupole and octopole fields are added to help correct aberrations



Uses for Muon Cooling in Project X Era

- M2e
  - better energy resolution
  - monochromatic beam = less signal loss
- Neutrino Factory -
  - increased survival of muons to storage ring
  - Reduce aperture for accelerating structure
  - Test cooling techniques for MC
- Muon Collider
  - increased luminosity