

# Lattice QCD: an Enabling Technology for Project X

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Andreas S. Kronfeld on behalf of the Lattice QCD WG

Tom Blum, Ruth Van de Water conveners

Christopher Aubin, Tanmoy Bhattacharya, Michael Buchoff, Norman Christ, Saul Cohen, Don Holmgren, Jack Laiho, Taku Izubuchi, Emanuele Mereghetti, Santiago Peris, Brad Plaster, Dru Renner, Eigo Shintani.

# One-Slide Review

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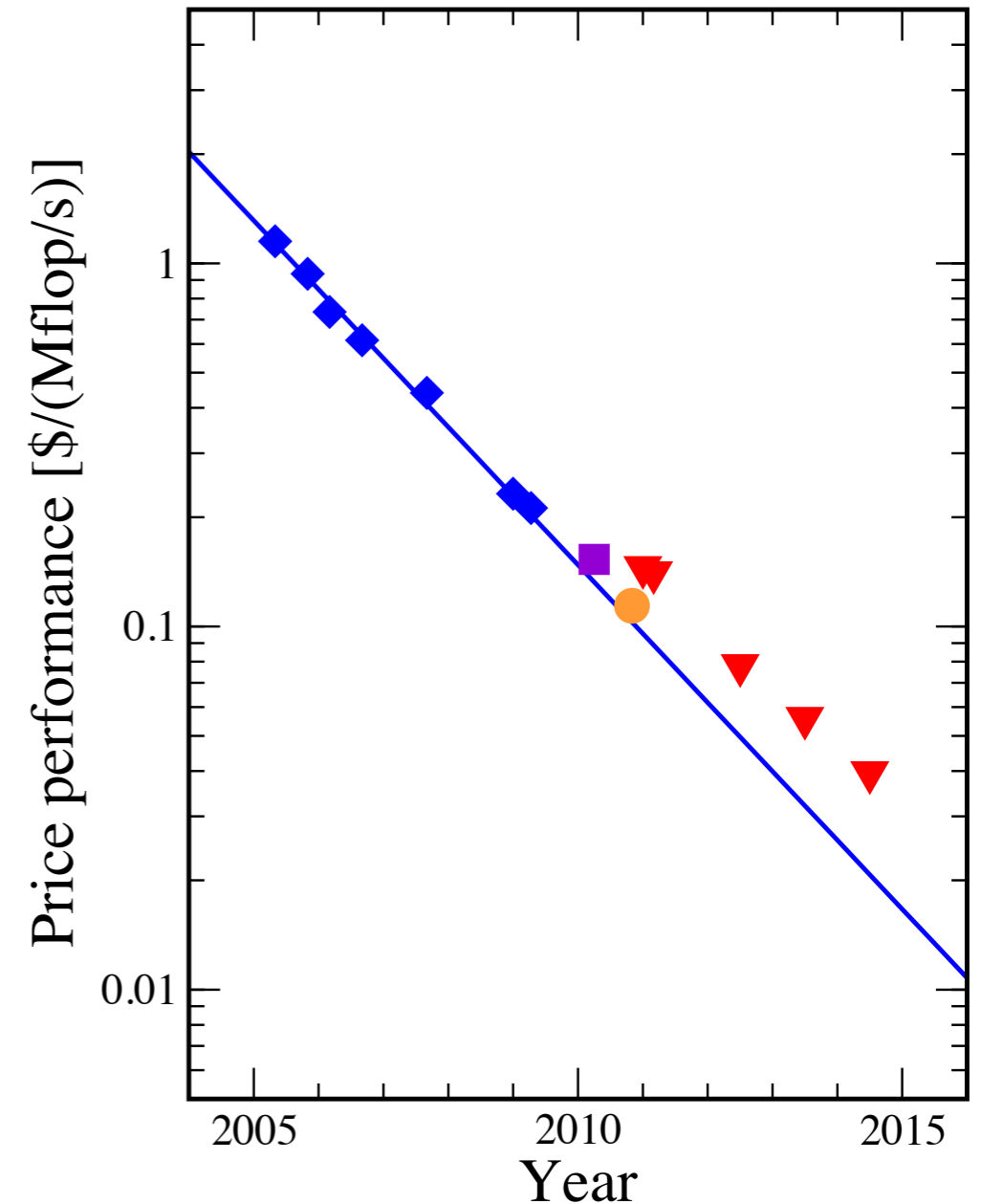
- **QCD** Lagrangian with gauge invariance on a Euclidean spacetime lattice.
- Compute functional integrals using a Monte Carlo with importance sampling.
- Analyze these numerical data to determine  $M(a(g^2), m_q, \theta; L), f(a(g^2), m_q, \theta; L)$  for a sequence of  $\{a, m_q, \theta, L\}$ .
- Pick  $1 + n_f + 1$  fiducial  $\{M, f\}$  to eliminate bare parameters (renormalization).
- Volume  $L < \infty$  either small (& correctable) effect or incisive tool.
- Remove discretization effects via fit to form from Symanzik EFT.
- Take  $m_q \rightarrow m_{u,d}$  with chiral EFT; control  $m_{Qa}$  with heavy-quark EFT.

The output is QCD with an understandable error.

# Computing

Don Holmgren

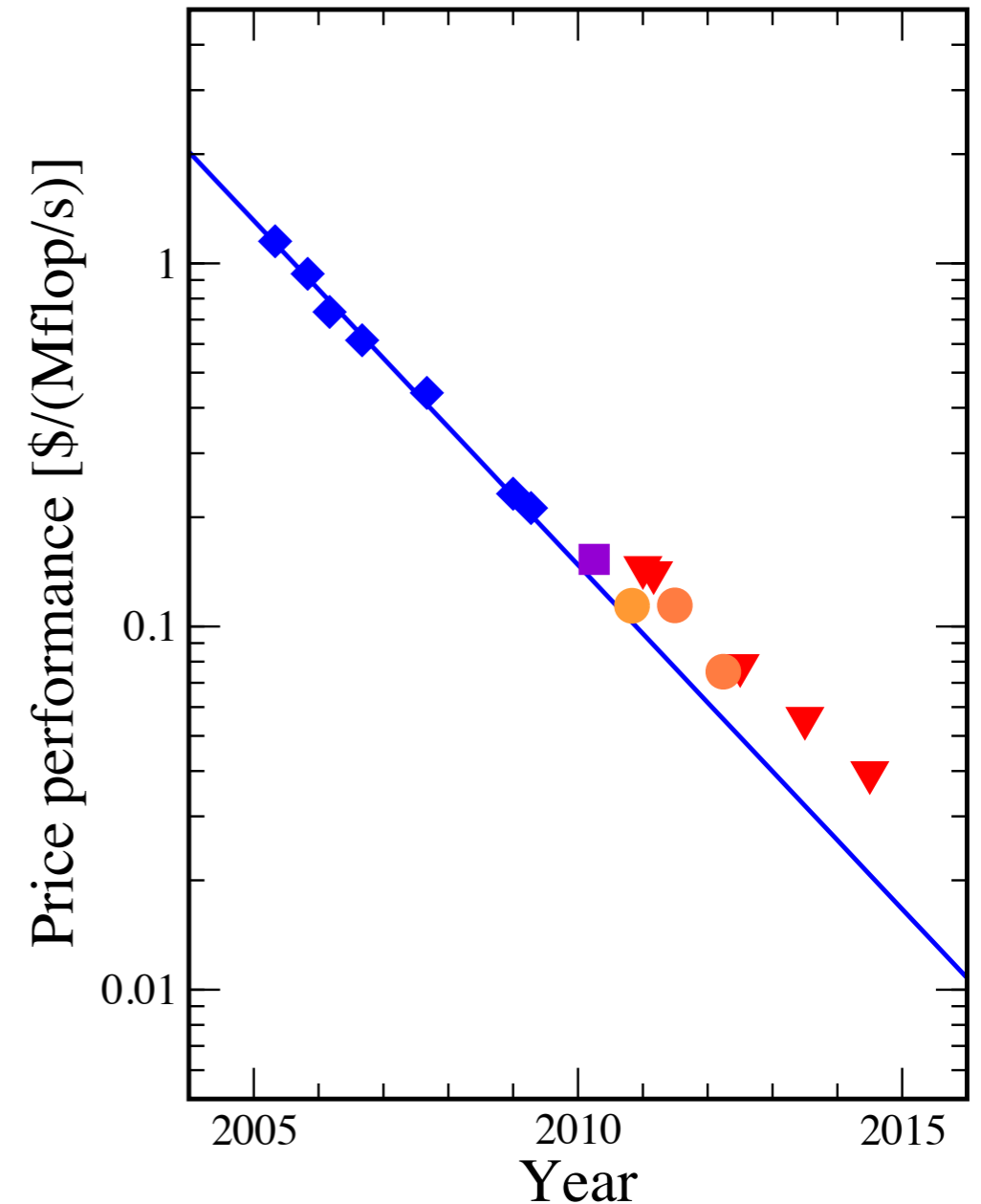
- Need computing of **high capability** and of **high capacity**.
- Leadership machines; dedicated clusters.
- Chipsets with high memory bandwidth.
- Low latency communications network.
- Moore-ish laws: multi-core, GPU, hybrid; rely on market.
- LQCD-ext hardware & ops: ~\$3.5M/FY.



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# Material pertaining to the physics working groups

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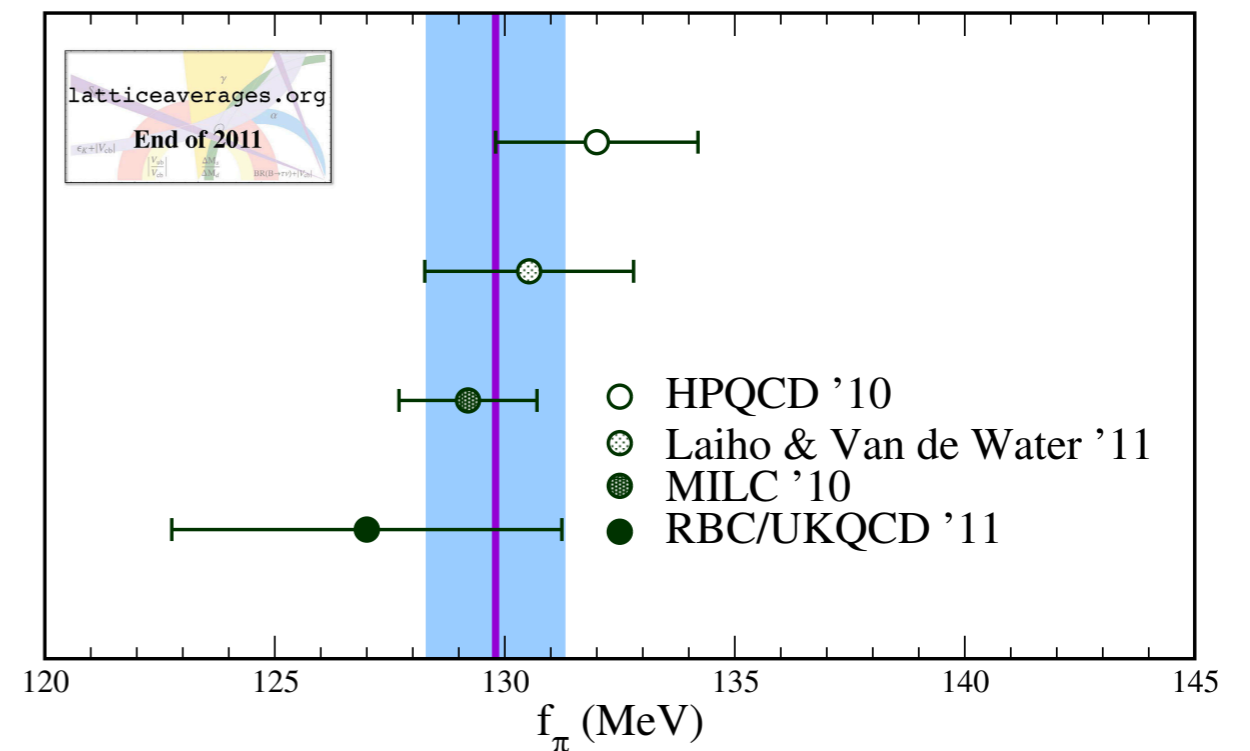
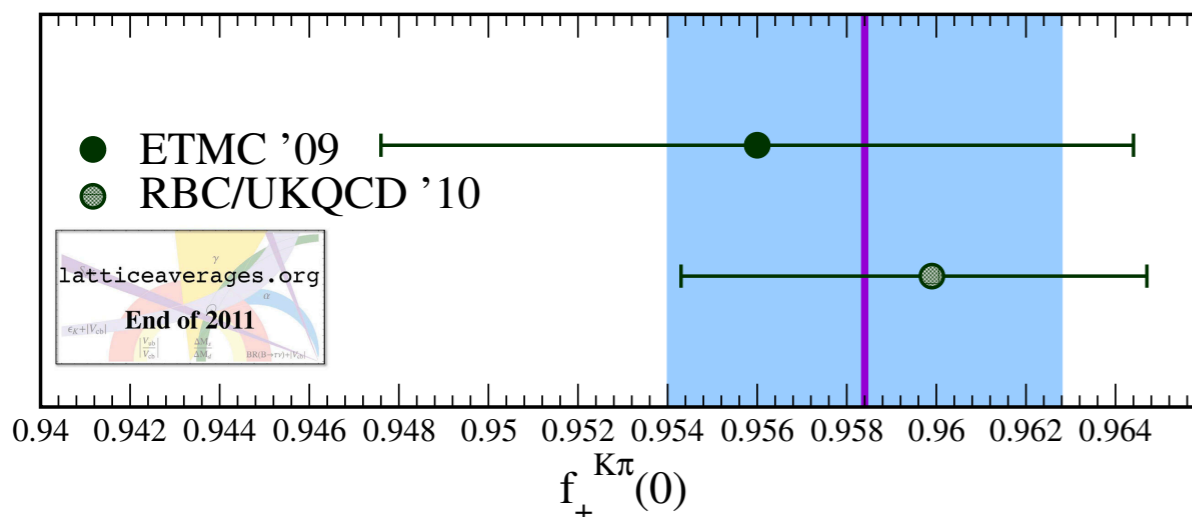
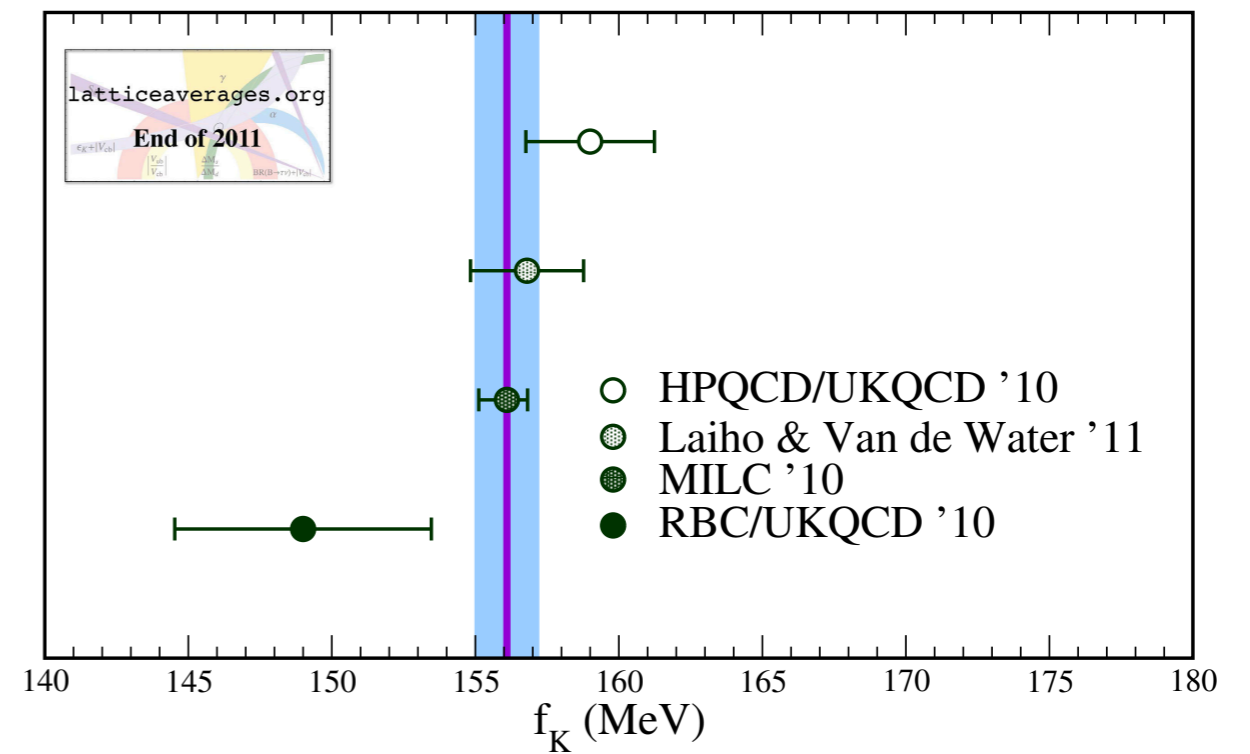
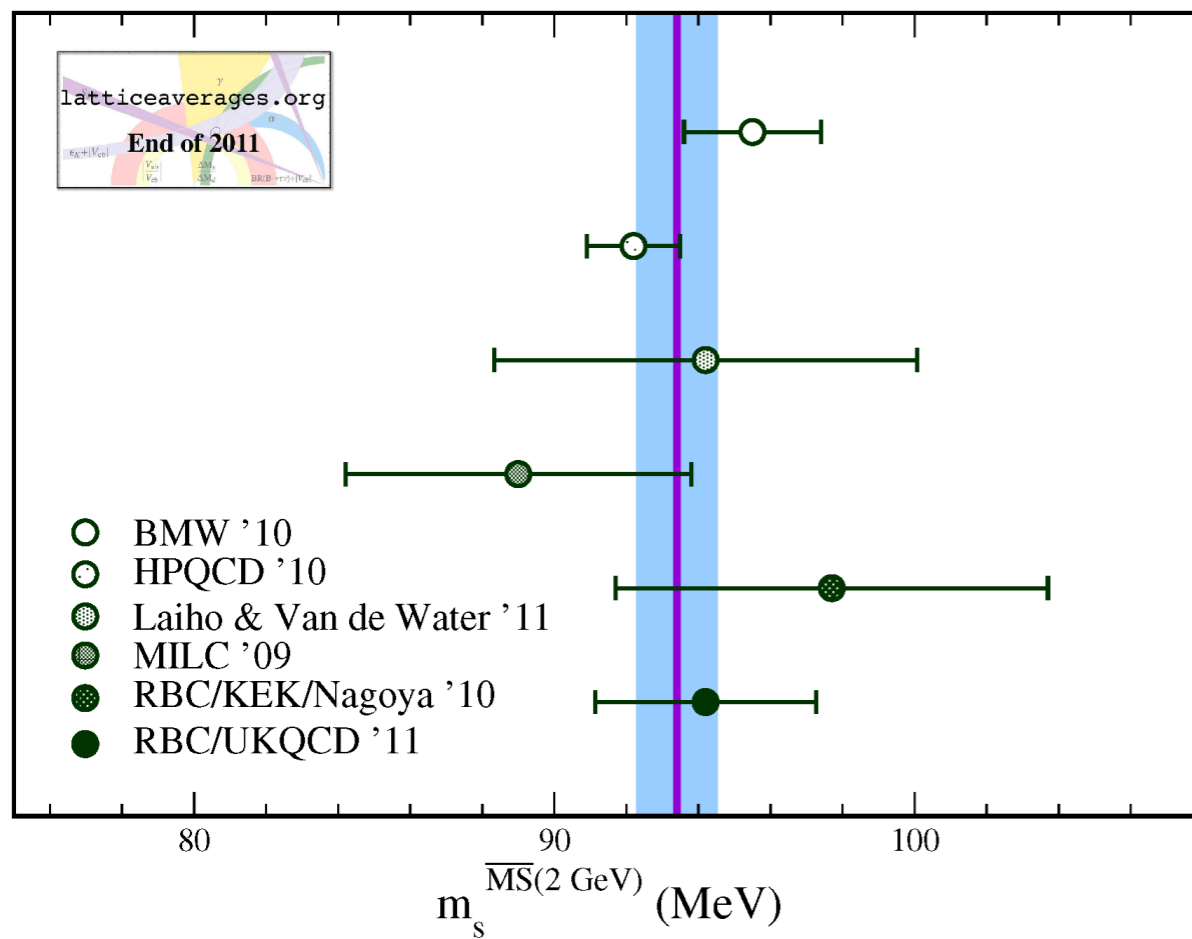
- Kaon experiments
- Hadronic physics
- Electric dipole moments (nucleon)
- Neutron-antineutron oscillations and nucleon decay
- Muon experiments
  
- Neutrino experiments

# Kaon Experiments

Jack Laiho  
Norman Christ

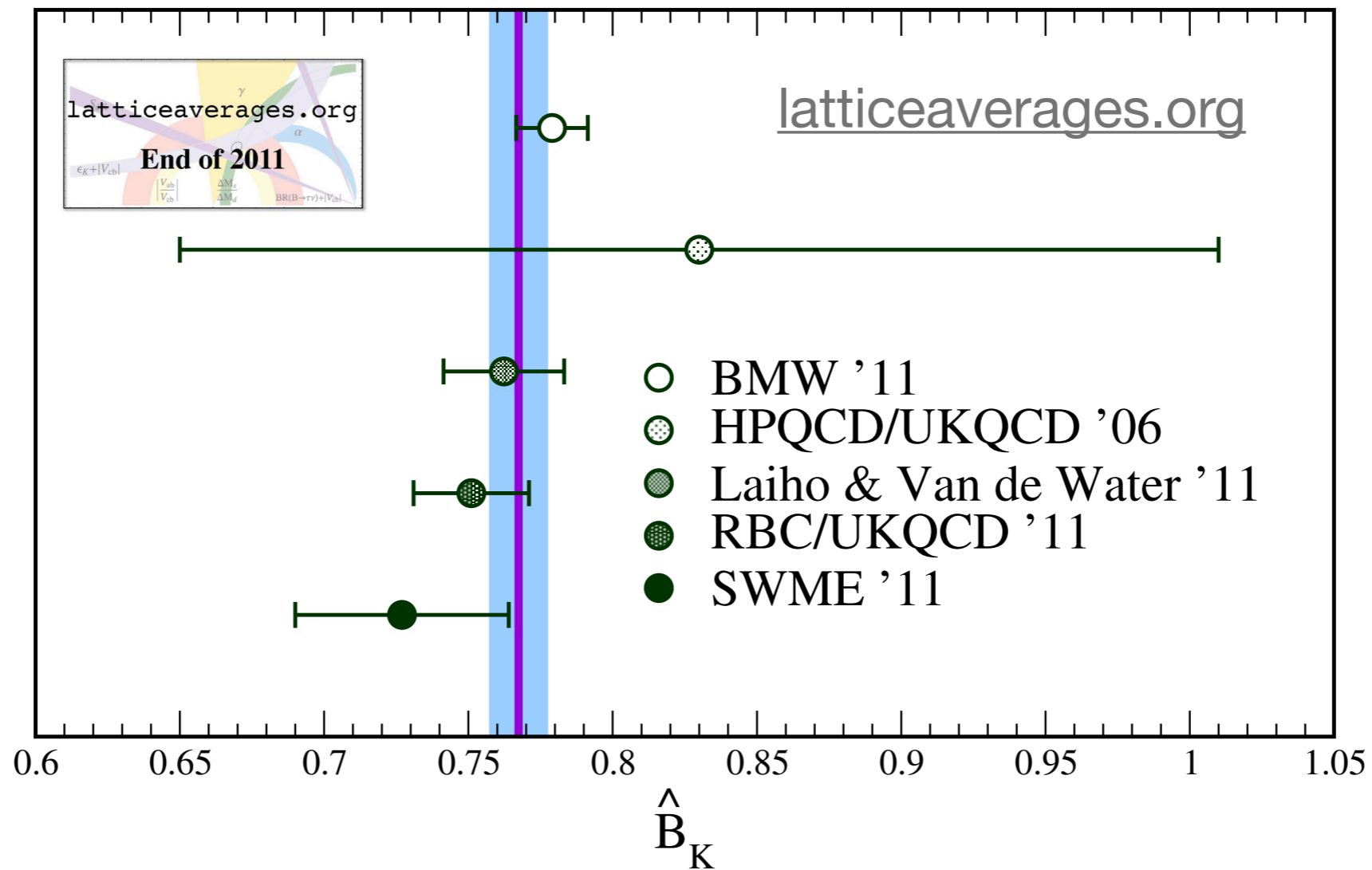
# Simple Kaon Matrix Elements

Jack Laiho





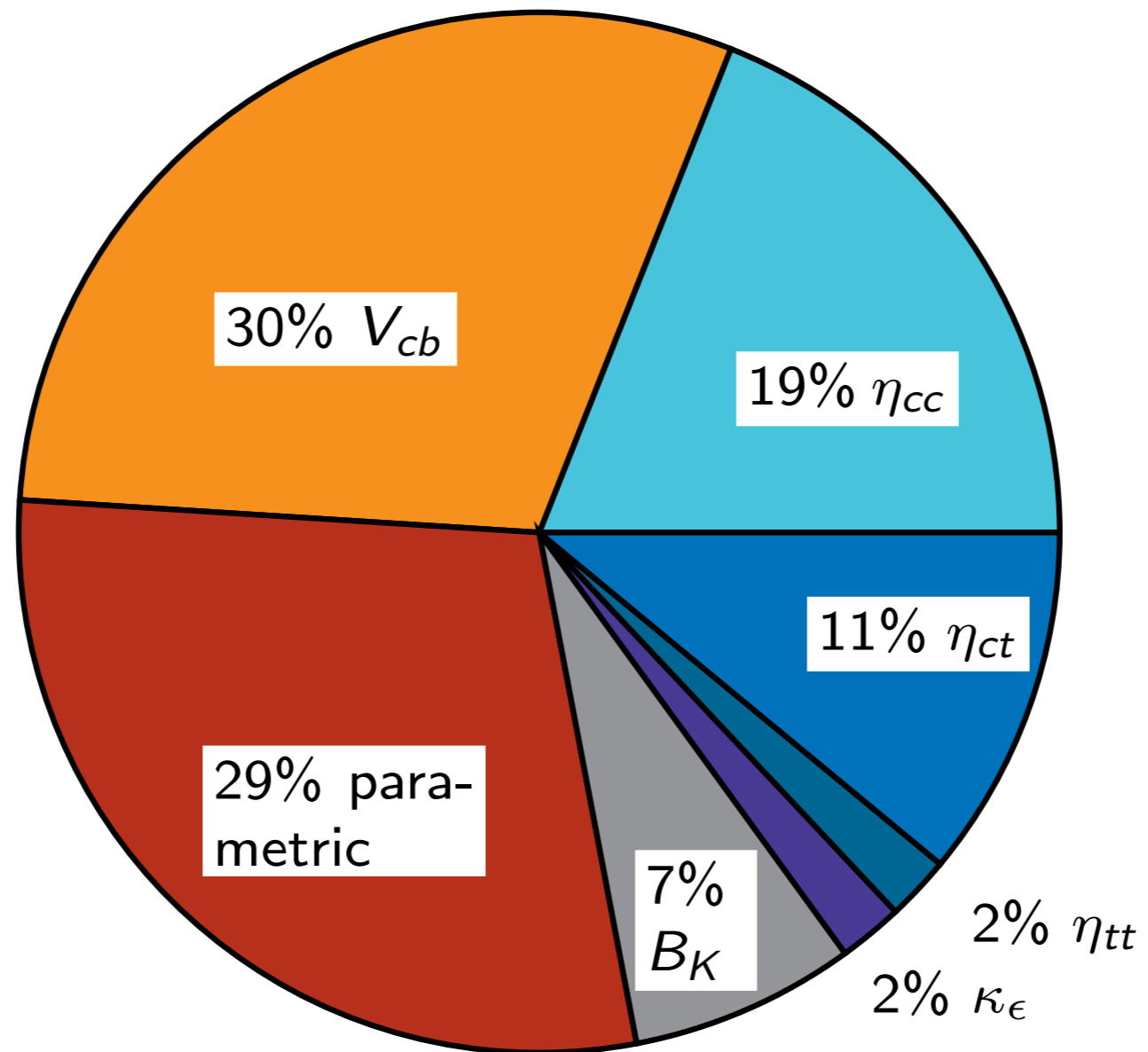
# Kaon Mixing: $B_K$



# Kaon Mixing: $B_K$

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Joachim Brod



# The Quest for $K \rightarrow \pi\pi$

Norman Christ

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- $K \rightarrow \pi\pi$  poses a GeV-scale puzzle ( $\Delta I=1/2$  rule) and a TeV-scale puzzle ( $\varepsilon'$ ).
- Need to understand the final-state  $\pi\pi$  interaction in a box with pbc.
- Lüscher quantization condition (1986):  $\phi(kL/2\pi) + \delta(k) = n\pi$ .
  - Extended to weak decay in 2001 (Lellouch and Lüscher).
- Explains how to use  $L$  dependence of energy levels to disentangle elastic phases shifts.
- Provides normalization of finite- $L$  transition matrix element.

- RBC/UKQCD have used this technology to solve the  $\Delta I=3/2$  amplitude:

$$A_2 = \left[ 1.44 \pm 0.06_{\text{stat}} \pm 0.26_{\text{syst}} - i(6.3 \pm 0.5_{\text{stat}} \pm 1.2_{\text{syst}})10^{-5} \right] 10^{-8} \text{ GeV}$$

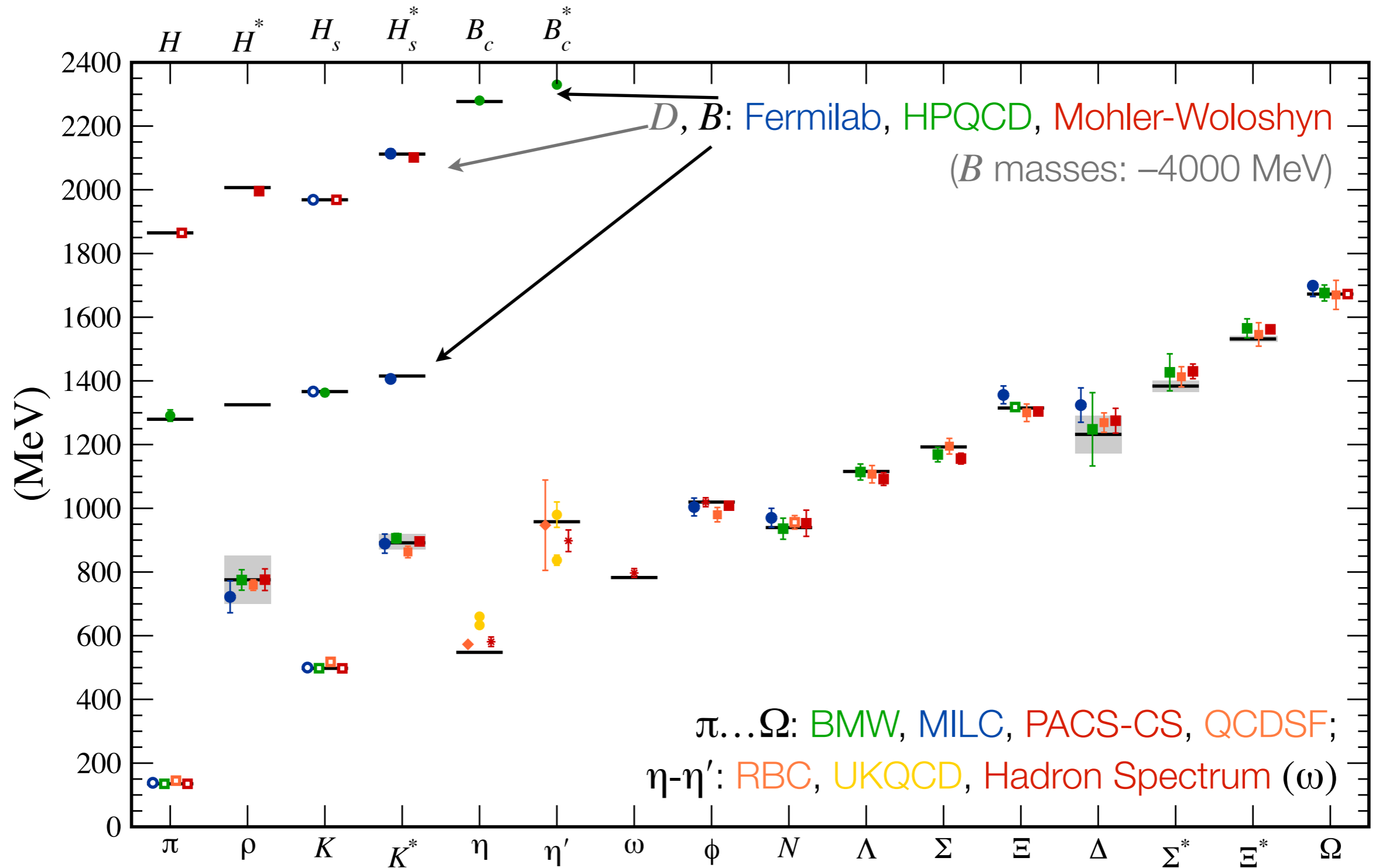
where discretization (one lattice spacing) is the largest error.

- The  $\Delta I=1/2$  amplitude contains much noisier “disconnected” diagrams.  
Exploratory calculation has  $(20 + i 50)\%$  error:  $\text{Re}(\epsilon'/\epsilon) = (2.0 \pm 1.7) \times 10^{-3}$
- Technology has wide application, e.g.,  $K_L-K_S$  mass difference.

# Hadronic Physics

# Spectroscopy: Low-Lying Hadrons

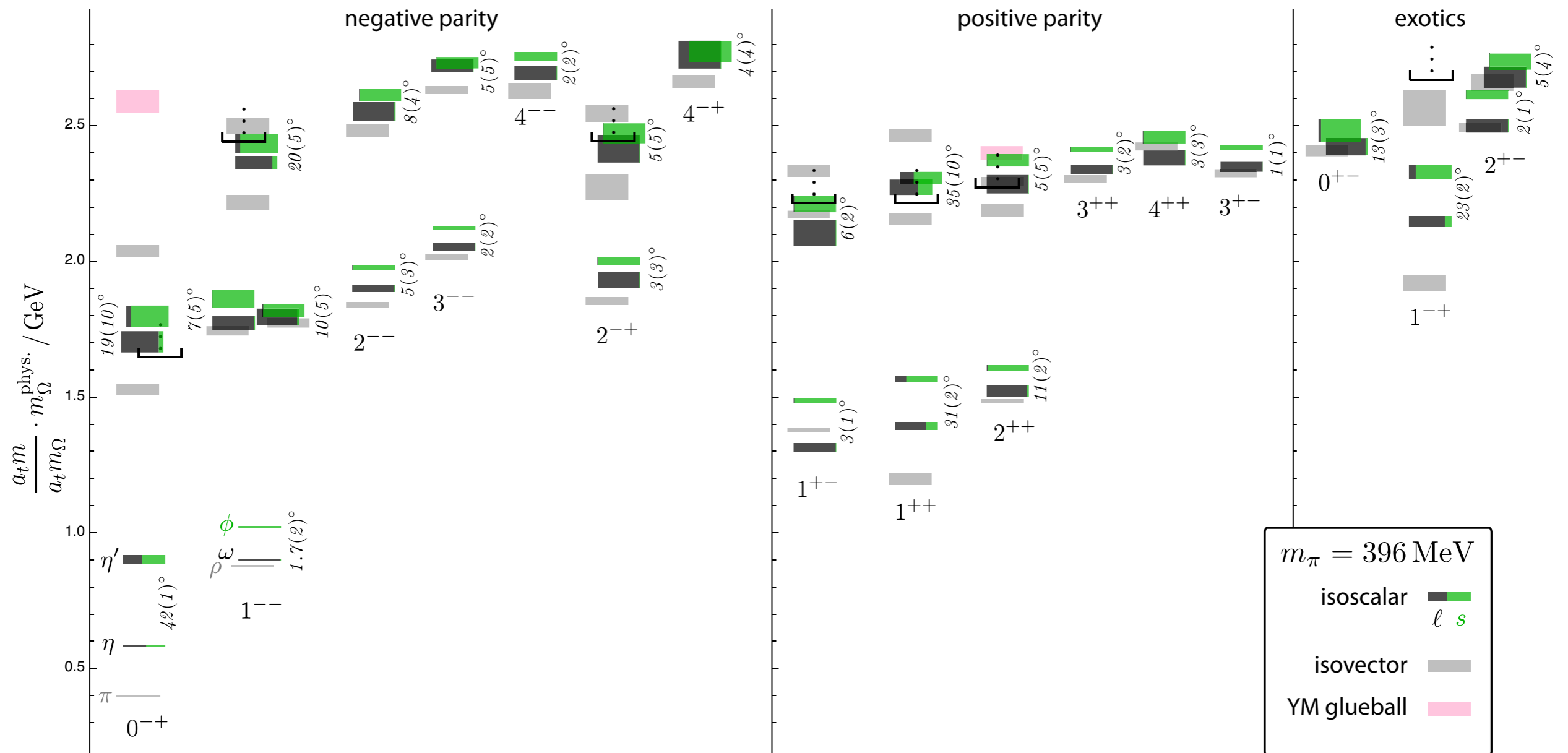
from [arXiv:1203.1204 \[hep-lat\]](https://arxiv.org/abs/1203.1204)



# Spectroscopy: Excited States

cf., Steve Godfrey

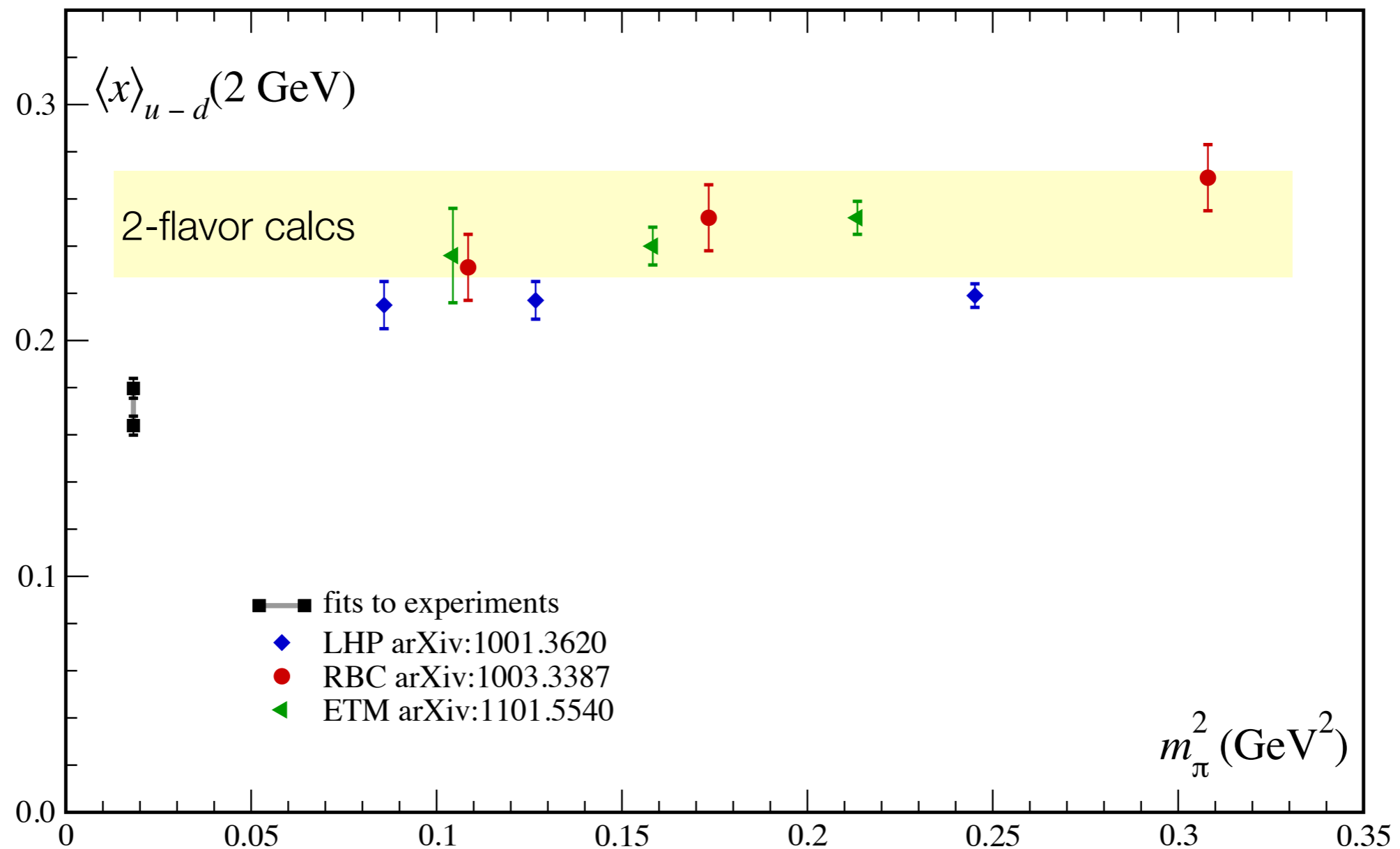
- Hadron Spectrum Collaboration [[arXiv:1102.4299 \[hep-lat\]](https://arxiv.org/abs/1102.4299)]:



# Hadron Structure

*cf.*, Paul Reimer

- Moments of parton densities:



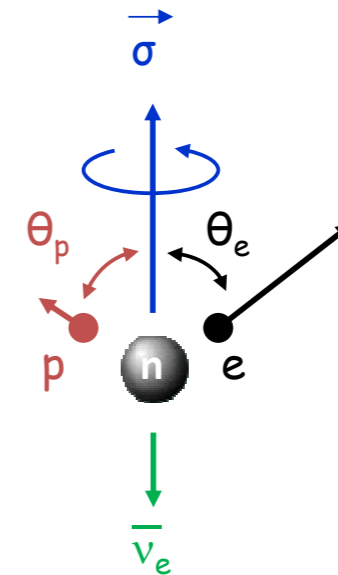
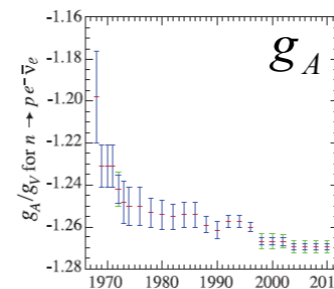
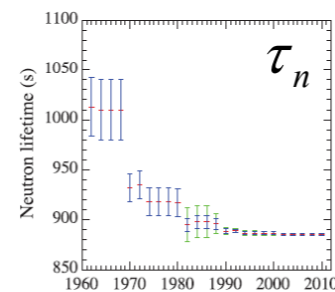
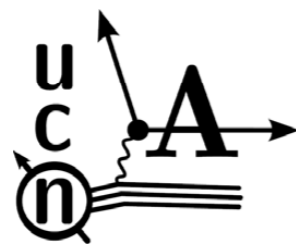


# Nucleon Structure in the Lab: $g_A$ and $g_V$

Brad Plaster

## High-Precision Measurements of $g_A$ and $g_V$ in Neutron and Nuclear $\beta$ -Decay

Brad Plaster, University of Kentucky



$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

FNAL Project X Physics Study

June 16, 2012

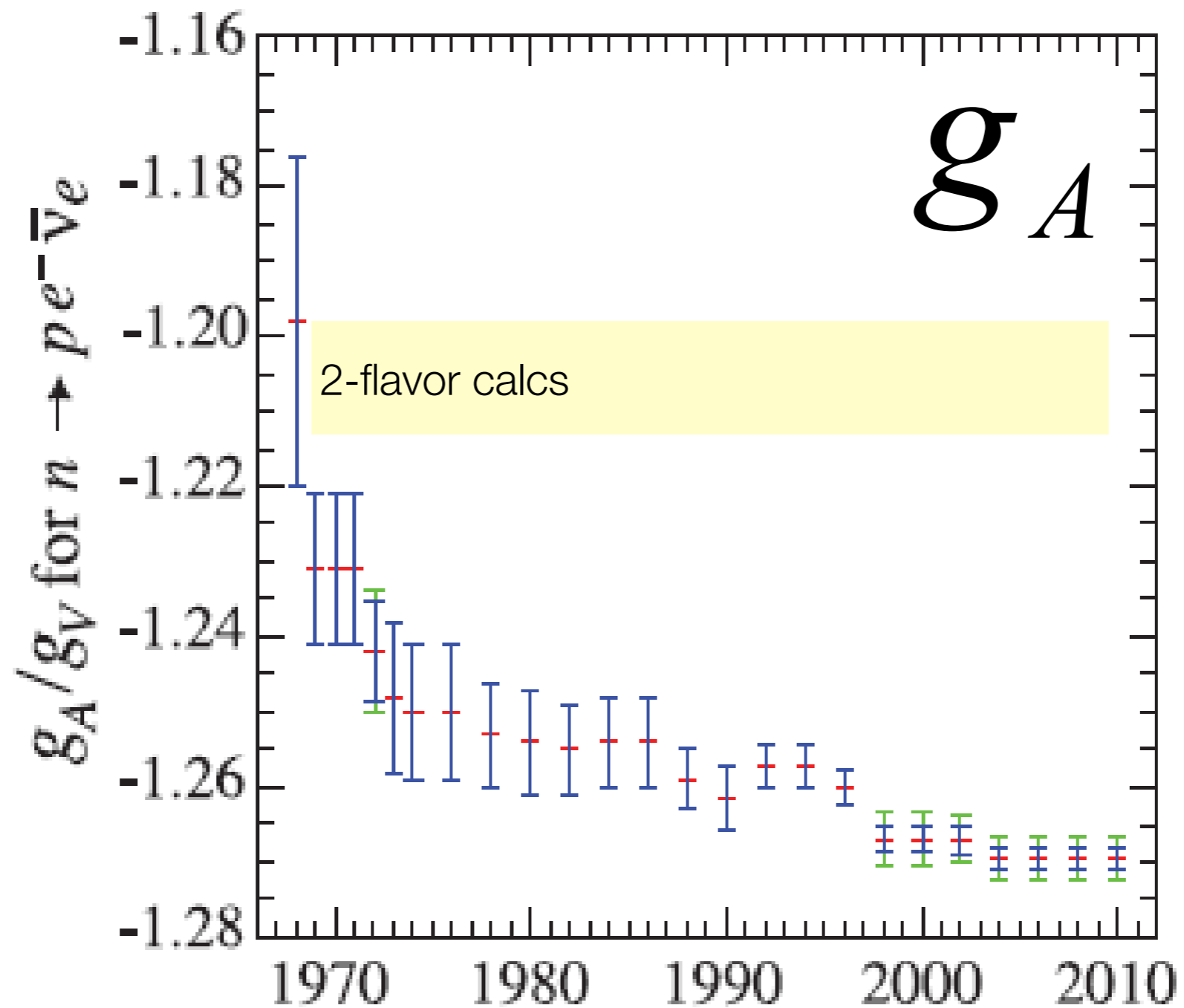


B. Plaster **UK**

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# Nucleon Structure in the Lab: $g_A$ and $g_V$

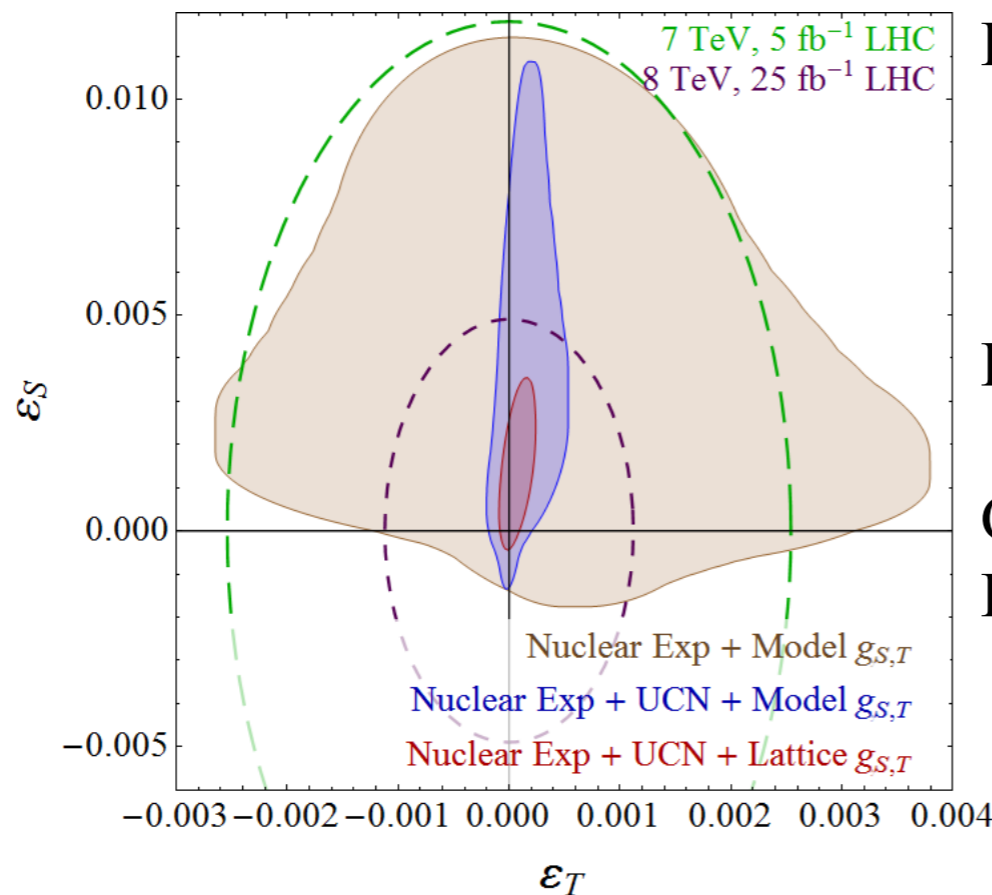
Brad Plaster



## High-Energy Constraints

§ Constraints from high-energy experiments?

LHC current bounds and near-term expectation



Estimated though effective  $L$

$$\mathcal{L} = -\frac{\eta_S}{\Lambda_S^2} V_{ud}(\bar{u}d)(\bar{e}P_L\nu_e) - \frac{\eta_T}{\Lambda_T^2} V_{ud}(\bar{u}\sigma^{\mu\nu}P_Ld)(\bar{e}\sigma_{\mu\nu}P_L\nu_e)$$

Looking at high transverse mass  
in  $e\nu + X$  channel

Compare with  $W$  background

Estimated 90% C.L. constraints on

$$\epsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$

HWL, 1112.2435; 1109.2542

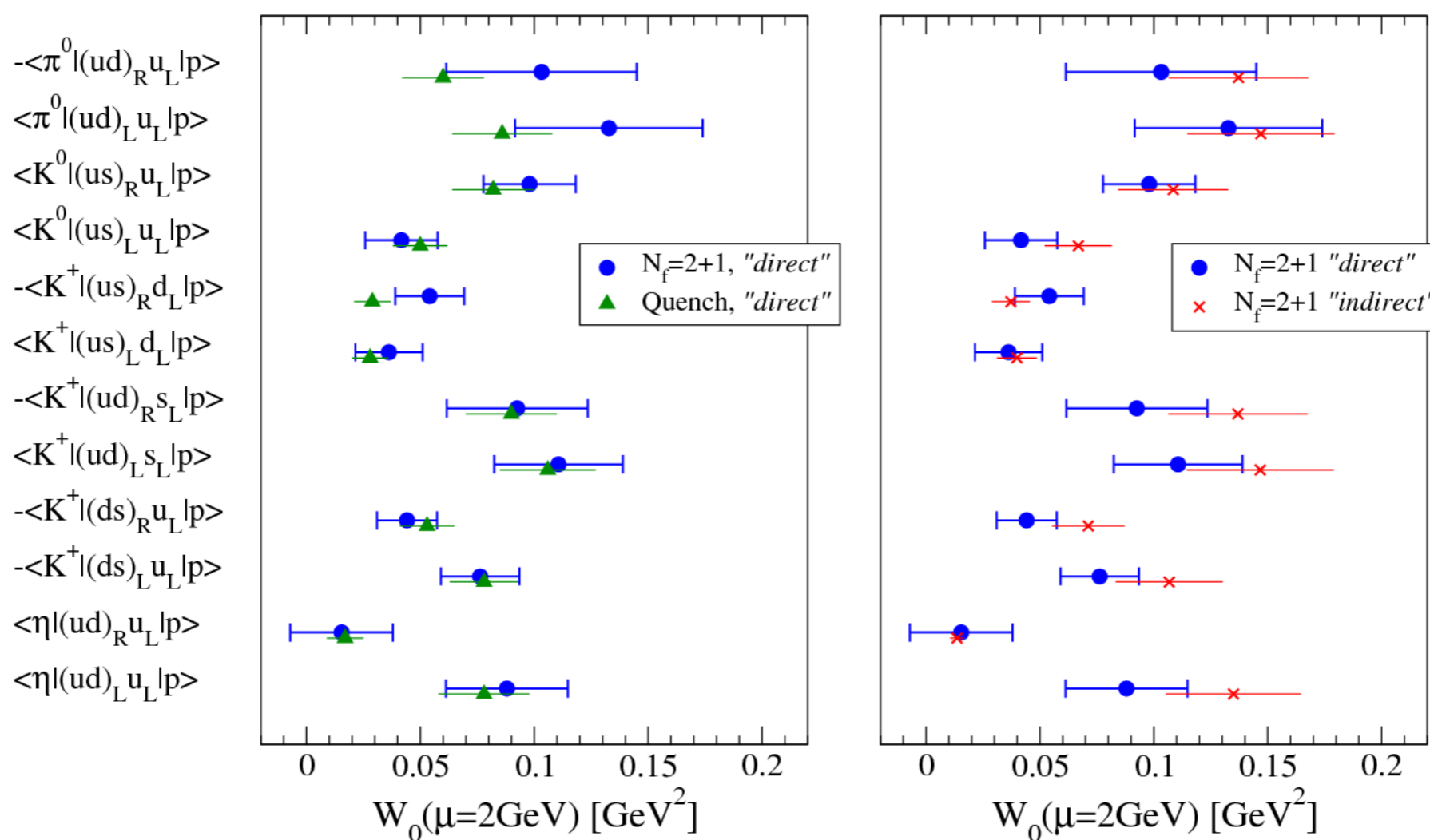
T. Bhattacharya et al, 1110.6448

# Neutron-Antineutron Oscillations & Nucleon Decay

Taku Izubuchi  
Michael Buchoff

## In full QCD

RBC/UKQCD in prep.



- There is no significant discrepancy between each results.
- Statistical and systematic errors are still large.

# Neutron-Antineutron Transitions

Michael Buchoff

## VERY Preliminary Results

	Lattice	MIT Bag Model
$\langle \bar{n}   \mathcal{P}_1   n \rangle$	$1.57(\pm 0.85)(+0.25)(-0.30)$	-6.56
$\langle \bar{n}   \mathcal{P}_2   n \rangle$	$-0.20(\pm 0.14)(+0.14)(-0.12)$	1.64
$\langle \bar{n}   \mathcal{P}_3   n \rangle$	$-0.24(\pm 0.26)(+0.10)(-0.07)$	2.73
$\langle \bar{n}   \mathcal{P}_4   n \rangle$	$-0.02(\pm 0.39)(+0.07)(-0.18)$	-6.36
$\langle \bar{n}   \mathcal{P}_5   n \rangle$	$0.34(\pm 0.82)(+0.27)(-0.57)$	9.64
$\langle \bar{n}   \mathcal{P}_6   n \rangle$	$-2.07(\pm 1.10)(+1.28)(-0.77)$	-28.92
	$\times 10^{-5} \text{ GeV}^6$	$\times 10^{-5} \text{ GeV}^6$

# Electric Dipole Moments

Emanuele Mereghetti  
Eigo Shintani  
Tanmoy Bhattacharya

# Nucleon Electric Dipole Moments

Tanmoy Bhattacharya

- Both strong and BSM CP violation contribute

$$\begin{aligned}
 d_n &\approx \frac{8\pi^2}{M_n^3} \left[ -\frac{2m_*}{3} \frac{\partial \langle \bar{q}\sigma q \rangle_F}{\partial F} \left( \bar{\Theta} + g_s \frac{\langle \bar{q}G\sigma q \rangle}{2\langle \bar{q}q \rangle} \sum \frac{d_q^G}{m_q} \right) \right. \\
 &\quad + \frac{\langle \bar{q}q \rangle}{3} (4d_d^\gamma - d_u^\gamma) \\
 &\quad \left. + g_s \frac{\langle \bar{q}G\sigma q \rangle}{6\langle \bar{q}q \rangle} \left( 4d_d^G \frac{\partial \langle \bar{d}\sigma d \rangle_F}{\partial F} - d_u^G \frac{\partial \langle \bar{u}\sigma u \rangle_F}{\partial F} \right) \right] \\
 &\approx \left( \frac{4}{3}d_d^\gamma - \frac{1}{3}d_u^\gamma \right) - \frac{2e\langle \bar{q}q \rangle}{M_n f_\pi^2} \left( \frac{2}{3}d_d^G + \frac{1}{3}d_u^G \right),
 \end{aligned}$$

- Need both neutron and proton to disentangle!



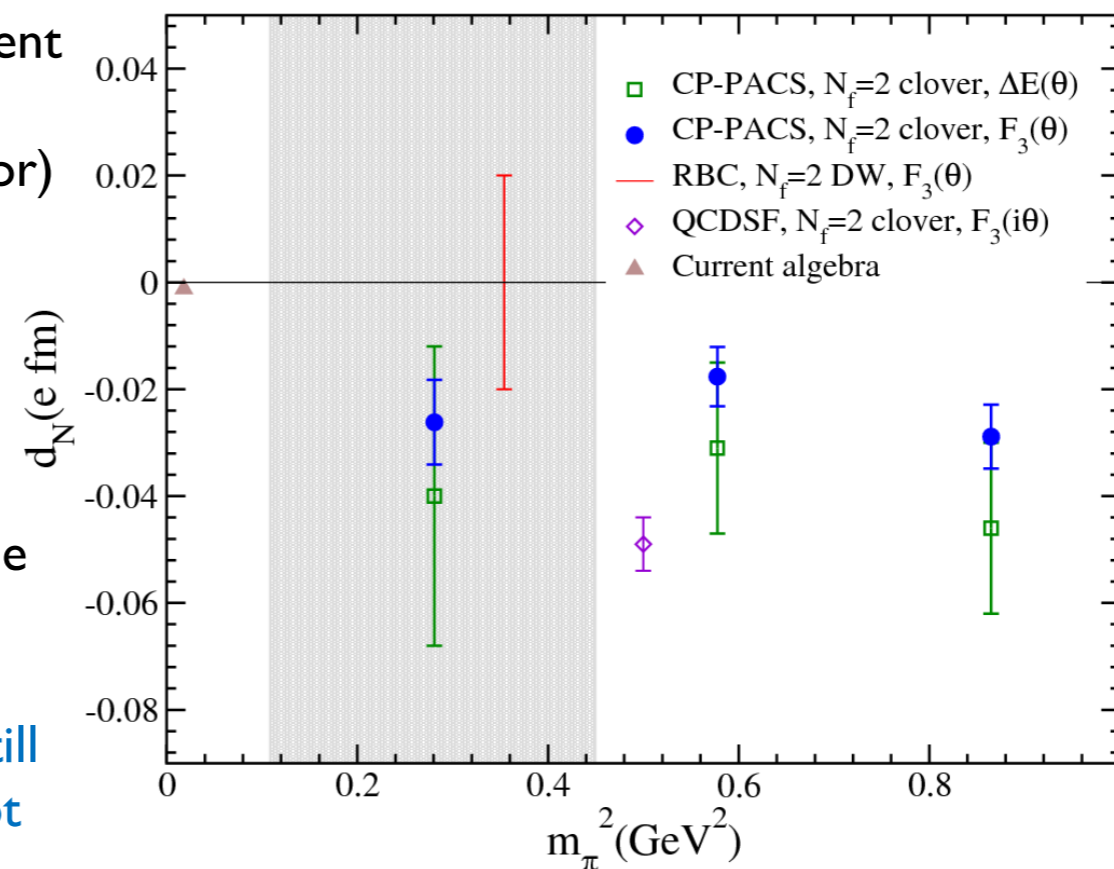
# Neutron EDM (in units of $\Theta$ )

Eigo Shintani

## Comparison of results

### ► Full QCD

- Lattice results are consistent within  $1\sigma$ . (not include systematic error)
- An order of magnitude larger than the results of current algebra.
- $N_f = 2+1$  DWF configs. (RBC/UKQCD) are available for near physical pion mass.
- Large statistical error is still problem. ( $O(100)$  stat. is not enough)

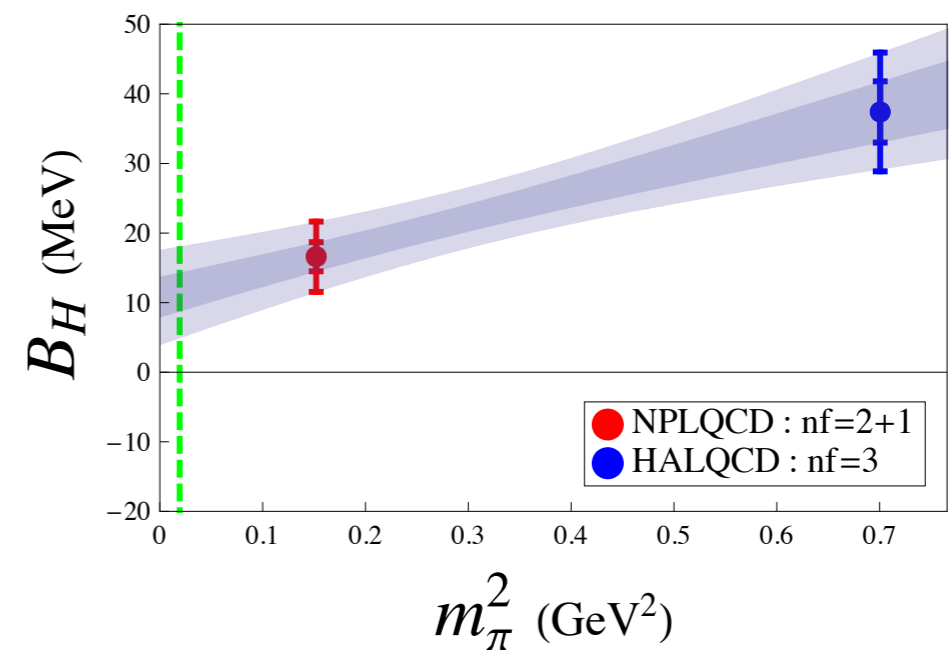
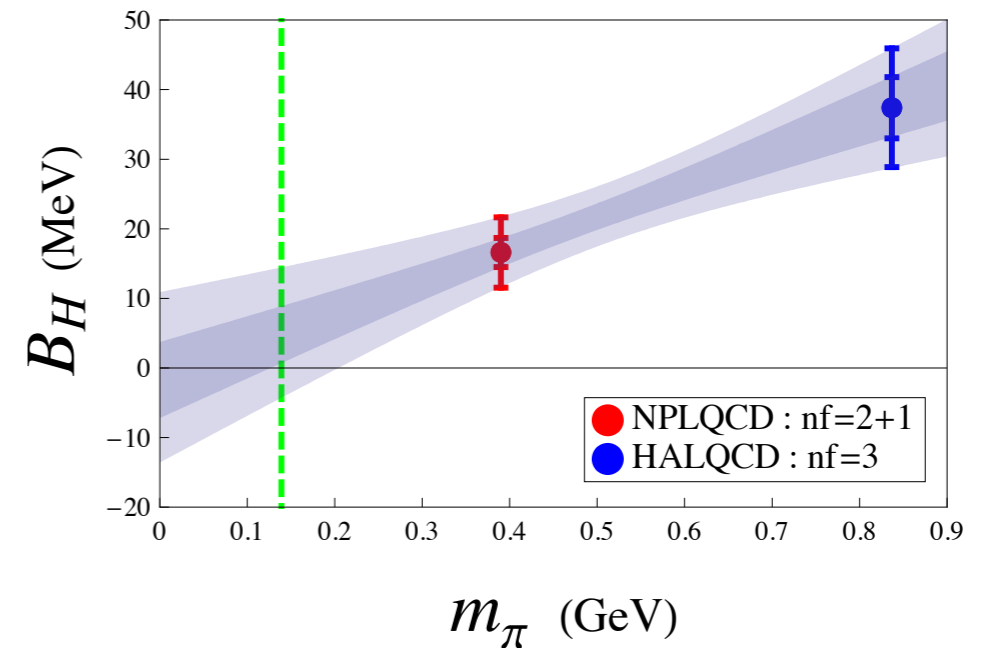


► 10

# Also the deuteron EDM, please

Emanuele Mereghetti

- The simplest nucleus is the deuteron,  $pn$ .
- Barely bound: fine tuning of QCD parameters.
- Do similar dibaryons exist?  
Conjectured  $H = \Lambda\Lambda$ .
- Recent lattice QCD calculations, with slightly unphysical quark masses, suggest that the  $H$  is indeed bound.

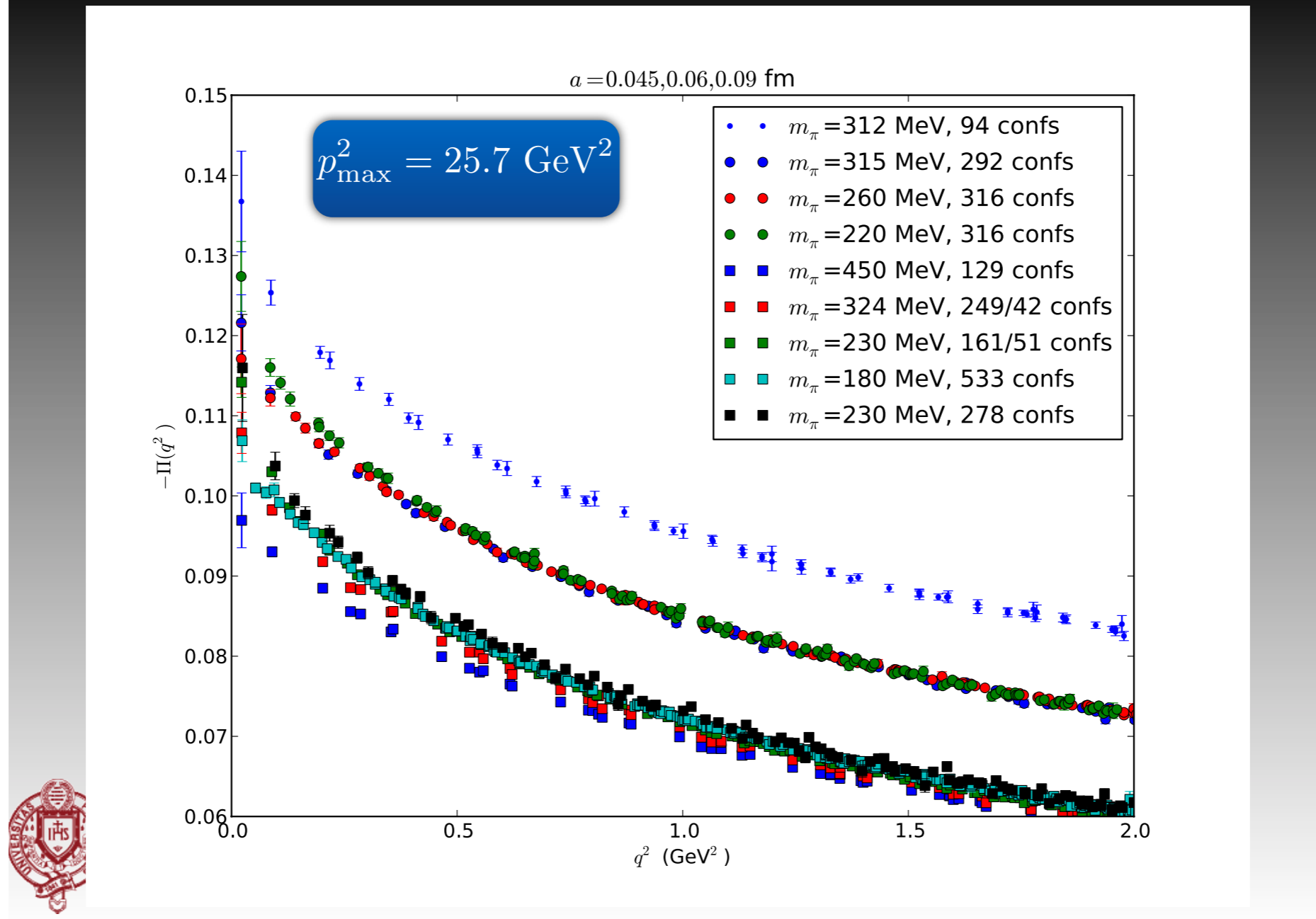


# Muon Experiments

# Muon $g-2$ : Hadronic Vacuum Polarization

Christopher Aubin

## All results (thus far)



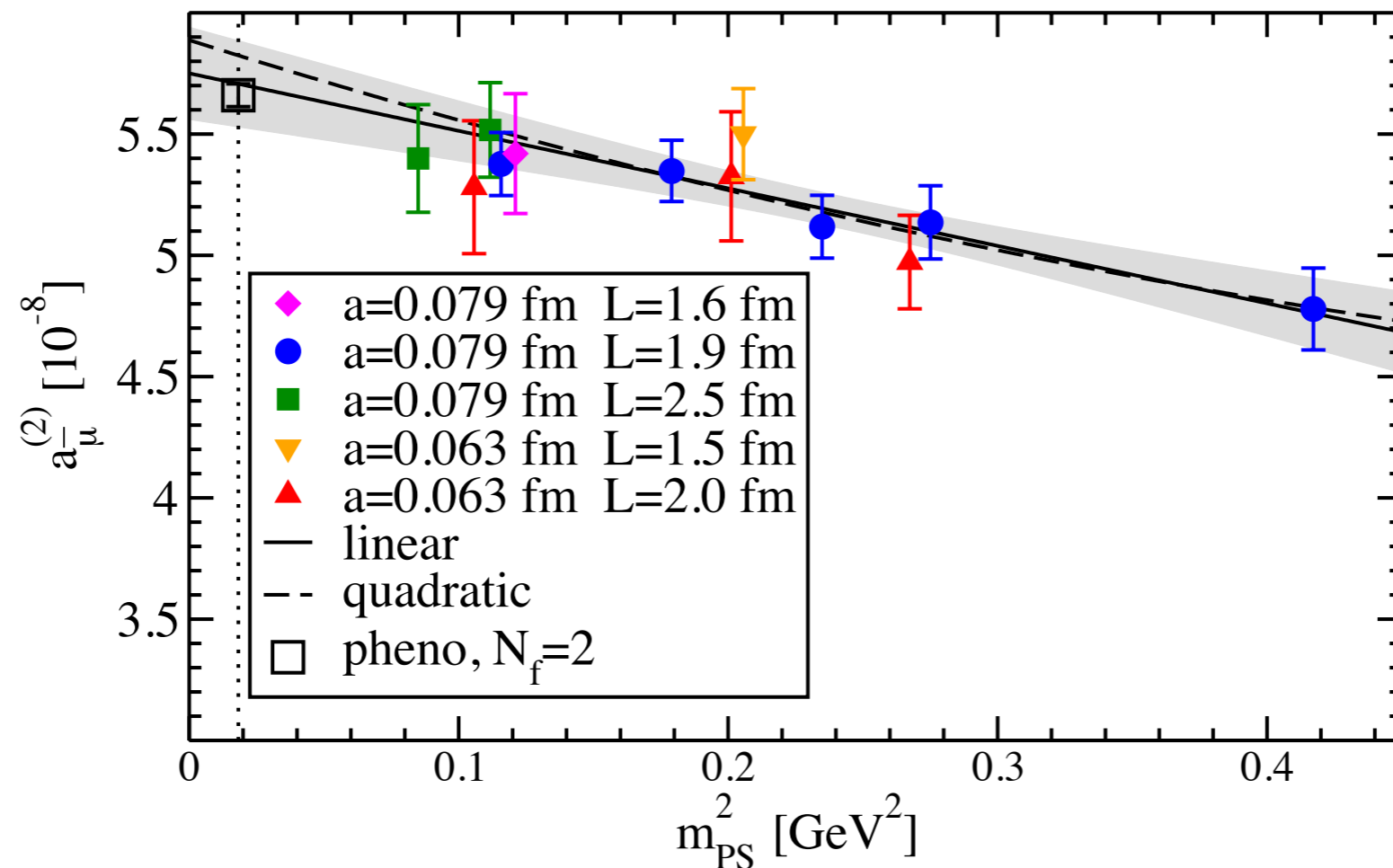
Sunday, June 17, 2012

# Muon $g-2$ : Hadronic Vacuum Polarization

Dru Renner

## Leading-order correction to $a_\mu$

modified method lead to reliable well-controlled calculation of  $a_\mu^{(2)}$



use of  $N_f = 2$  was the only substantially weak part of calculation

5/12

# Muon $g-2$ : Hadronic Light-by-Light

Tom Blum

Introduction  
The hadronic light-by-light (HLbL) contribution ( $\mathcal{O}(\alpha^3)$ )  
Summary/Outlook

$a_\mu(\text{HLbL})$  in 2+1f lattice QCD+QED (PRELIMINARY)

Signal may be emerging in the model ballpark:

- ▶  $F_2(0.18 \text{ GeV}^2) = (0.142 \pm 0.067) \times \left(\frac{\alpha}{\pi}\right)^3$
- ▶  $F_2(0.11 \text{ GeV}^2) = (0.038 \pm 0.095) \times \left(\frac{\alpha}{\pi}\right)^3$
- ▶  $a_\mu(\text{HLbL}/\text{model}) = (0.084 \pm 0.020) \times \left(\frac{\alpha}{\pi}\right)^3$

Lattice size  $24^3$ ,  $m_\pi = 329 \text{ MeV}$ ,  $m_\mu \approx 190 \text{ MeV}$

model value/error is “Glasgow Consensus” (arXiv:0901.0306 [hep-ph])



# Cousins of Muon $g-2$

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- Santi Peris talked about the relation between the triangle anomaly and the muon anomaly, revealing several mysteries.
- Saul Cohen talked about  $\pi^0 \rightarrow \gamma\gamma$ , which is closely related to the HLbL.
- Taku Izubuchi talked about several methods to improve the signal in HLbL:
  - broadly applicable to other problems (re-use propagators);
  - shows innovation in lattice technology, inspired by New  $g-2$  Experiment.

# Muon Capture

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- Coherent: low-energy limit of  $pn$  form factor.
- Incoherent contribution (quasi-informed guess)
  - low-energy scale  $\sim 100$  MeV
  - nuclear effective theory (Kaplan, Savage, Wise, ...), to reduce nuclear physics to nucleon-nucleon interactions
  - used, e.g., to assess strangeness content of neutron stars (NPLQCD).



# Neutrino Experiments

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- Deeply inelastic neutrino scattering → hadron structure (see above).
- Several other hadronic issues in neutrino physics:
  - neutrino cross sections;
  - matter effects;
  - $\pi$  and  $K$  production in target.
- Topics for the future.

# Outlook

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- Lattice QCD has made substantial progress with mesons, even in  $K \rightarrow \pi\pi$ .
- Nucleons, nucleons, nucleons: noisier in lattice QCD but key to Project X.
- New methodology (such as that being developed for  $g-2$  HLbL) should help.
- Matrix elements for proton decay and neutron properties inform bounds inferred from experiments.