# TMD distributions, saturation and the BFKL limit

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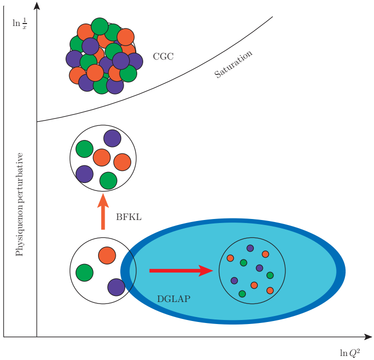
Low $x$, Saturation, Diffraction White Paper

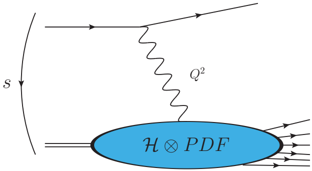
February 9, 2022

# QCD at moderate $x$

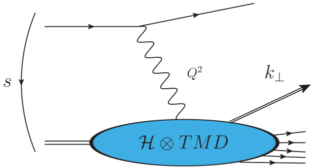
$$Q^2 \sim s$$



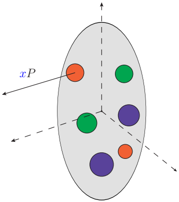
# Parton Distributions

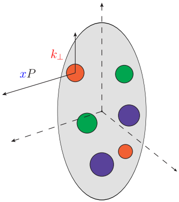



Parton Distribution Fonction (PDF)




Transverse Momentum Dependent distributions (TMD)



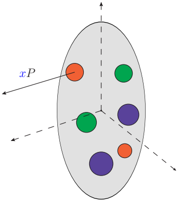
# Operator definition for parton distributions

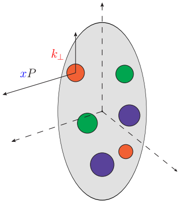
## Parton distribution function

$$\mathcal{F}(x) \propto \int dz^+ e^{ixP^- z^+} \left\langle P \left| F^{-i}(z^+) \left[z^+, 0^+\right] F^{-i}(0) \left[0^+, z^+\right] \right| P \right\rangle$$

## Transverse Momentum Dependent distribution

$$\mathcal{F}(x, k_\perp) \propto \int d^4 z \delta(z^-) e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P \left| F^{-i}(z) \mathcal{U}_{z,0} F^{-i}(0) \mathcal{U}_{0,z} \right| P \right\rangle$$



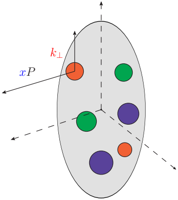
# Operator definition for parton distributions

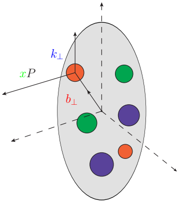
## TMD distribution

$$\mathcal{F}(x, k_\perp) \propto \int d^4z\delta(z^-)e^{ixP^-z^+ + i(k_\perp \cdot z_\perp)} \left\langle P \left| F^{-i}(z)\mathcal{U}_{z,0}F^{-i}(0)\mathcal{U}_{0,z} \right| P \right\rangle$$

## Generalized TMD distribution

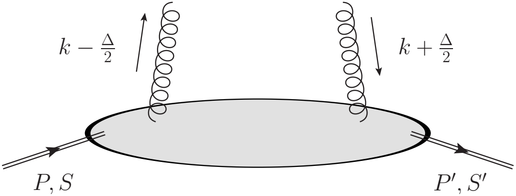
$$\mathcal{F}(x, k_\perp, \Delta) \propto \int d^4z\delta(z^-)e^{ixP^-z^+ + i(k_\perp \cdot z_\perp)} \left\langle P+\Delta \left| F^{-i}(z)\mathcal{U}_{z,0}F^{-i}(0)\mathcal{U}_{0,z} \right| P \right\rangle$$



# GTMD

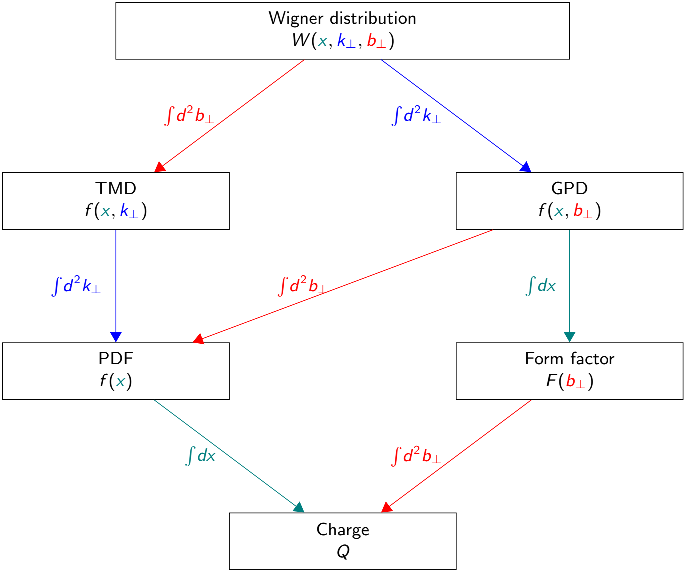
## Parametrization and coupling to the target hadron




$$\int d^4v\,\delta(v^-)e^{ix\bar{P}^-v^+ - i(\boldsymbol{k}\cdot\boldsymbol{v})}\langle P'S'|\mathrm{Tr}\left[F^{i-}(-\frac{v}{2})\mathcal{U}^{[+]}_{\frac{v}{2},-\frac{v}{2}}F^{i-}(\frac{v}{2})\mathcal{U}^{[-]}_{-\frac{v}{2},\frac{v}{2}}\right]|PS\rangle$$

$$= (2\pi)^3\frac{\bar{P}^-}{2M}\bar{u}_{P'S'}\left[F^g_{1,1} + i\frac{\sigma^{i-}}{\bar{P}^-}(\boldsymbol{k}^iF^g_{1,2} + \Delta^iF^g_{1,3}) + i\frac{\sigma^{ij}\boldsymbol{k}^i\Delta^j}{M^2}F^g_{1,4}\right]u_{PS}$$

# The family tree of parton distributions



# Leading twist gluon TMD distributions

| Hadron pol. \ Parton | Unpolarized | Circular | Linear |
|---|---|---|---|
| Unpolarized | $f_1^g$ | $\emptyset$ | $h_1^{\perp g}$ |
| Longitudinal | $\emptyset$ | $g_{1L}^g$ | $h_{1L}^{\perp g}$ |
| Transverse | $f_{1T}^{\perp g}$ | $g_{1T}^g$ | $h_1^g$, $h_{1T}^{\perp g}$ |

**PDF-spanning**

- Unpolarized $f_1^g$
- Helicity $g_{1L}^g$

**Naive $T$-even pure TMDs**

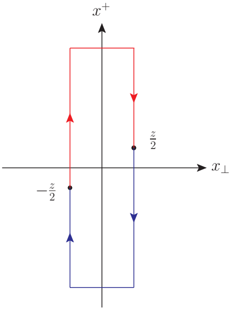
- Worm-gear $h_{1L}^{\perp g}, g_{1T}^g$
- Pretzelosity $h_{1T}^{\perp g}$
- Transversity $h_1^g$

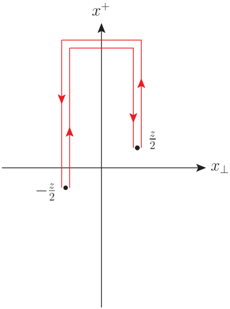
**Naive $T$-odd pure TMDs**

- Boer-Mulders $h_1^{\perp g}$
- Sivers $f_{1T}^{\perp g}$

# TMD gauge links

## "Non-universality" of gluon TMD distributions



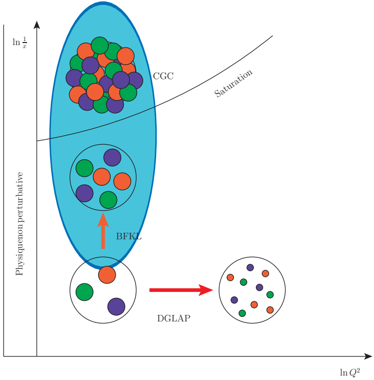




$$\mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[-]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right] \qquad \mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[+]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]$$

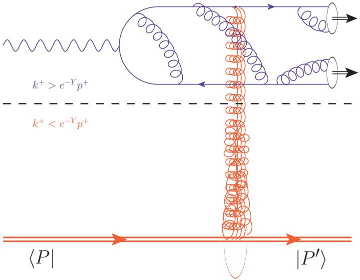
Even more possibilities for gluon TMD distributions!

# QCD at small $x$

$$Q^2 \ll s$$

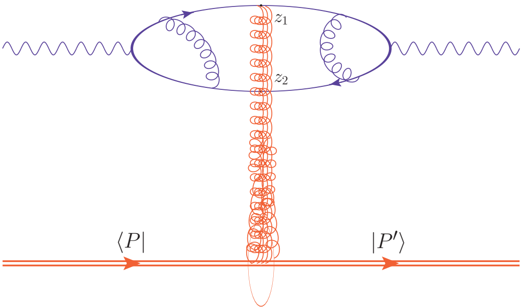


# QCD at large $s$ : semi-classical small $x$ effective theories




- Eikonal expansion
  $\sigma = \sigma_0 + \frac{1}{s}\sigma_1 + \ldots$
- Resummation of logarithms
  $\sigma_0 = \sum_n \left[A_n(\alpha_s \ln s)^n + \alpha_s B_n(\alpha_s \ln s)^n \ldots\right]$
- Renormalization group equation:
  Balitsky/Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner (B/JIMWLK) evolution for the shockwave operators.

# Factorized picture




Factorized amplitude

$$\mathcal{A}^{Y_c} = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2\, \Phi^{Y_c}(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U_{\vec{z}_1}^{Y_c} U_{\vec{z}_2}^{Y_c\dagger}) - N_c]|P\rangle$$

Dipole operator $\mathcal{U}_{ij}^{Y_c} = \frac{1}{N_c}\mathrm{Tr}(U_{\vec{z}_i}^{Y_c} U_{\vec{z}_j}^{Y_c\dagger}) - 1$

Written similarly for any number of Wilson lines in any color representation!

$Y_c$ independence: B-JIMWLK hierarchy of equations
[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

# The seemingly incompatible nature of the distributions

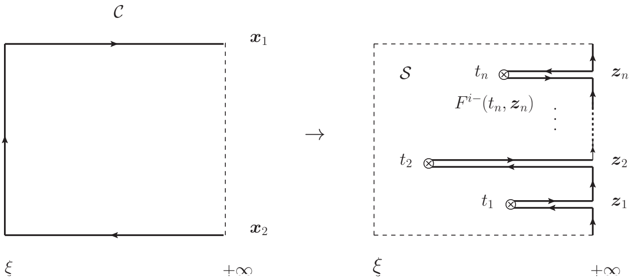
Two different kinds of gluon distributions

| Moderate $x$ distributions | Low $x$ distributions |
|---|---|
| GTMD, GPD, TMD, PDF... | Dipole scattering amplitude |
| $\langle P^{(\prime)} \vert F^{-i} W F^{-j} W \vert P \rangle$ | $\langle P^{(\prime)} \vert \mathrm{tr}(U_1 U_2^{\dagger}) \vert P \rangle$ |

# The Wilson line ↔ parton distribution equivalence

Most general equivalence: use the Non-Abelian Stokes theorem

[RB, Mehtar-Tani]



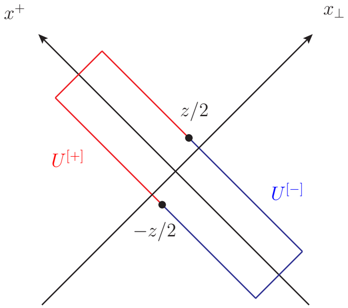

$$\mathcal{P}\exp\left[\oint_{\mathcal{C}} dx_\mu A^\mu(x)\right] = \mathcal{P}\exp\left[\int_{\mathcal{S}} d\sigma_{\mu\nu}\, W F^{\mu\nu} W^\dagger\right]$$

$$U_{x_{1\perp}} U^\dagger_{x_{2\perp}} = [\hat{x}_{1\perp}, \hat{x}_{2\perp}]$$

# An example: the dipole operator as a TMD distribution

[Dominguez, Marquet, Xiao, Yuan],[Hatta, Xiao, Yuan]

$$\int \frac{\mathrm{d}^2\boldsymbol{b}}{(2\pi)^2} \frac{\left\langle P \left| \mathrm{tr}\left(U_{\boldsymbol{b}+\frac{\boldsymbol{r}}{2}} U^\dagger_{\boldsymbol{b}-\frac{\boldsymbol{r}}{2}}\right) - N_c \right| P \right\rangle}{\langle P | P \rangle} = \alpha_s \int \mathrm{d}^2\boldsymbol{k} \frac{\mathrm{e}^{i(\boldsymbol{k}\cdot\boldsymbol{r})}}{\boldsymbol{k}^2} f^D\left(x=0,\boldsymbol{k}\right)$$




$$= \alpha_s \int \mathrm{d}^2\boldsymbol{k} \frac{\mathrm{e}^{i(\boldsymbol{k}\cdot\boldsymbol{r})}}{\boldsymbol{k}^2} \int \frac{\mathrm{d}\xi^+}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{\xi}}{(2\pi)^2} \mathrm{e}^{-i(\boldsymbol{k}\cdot\boldsymbol{\xi})} \times \frac{1}{P^-} \left\langle P \middle| \mathrm{tr}\left(WF^{i-}W^\dagger\right)_\xi \left(WF^{i-}W^\dagger\right)_0 \middle| P \right\rangle$$

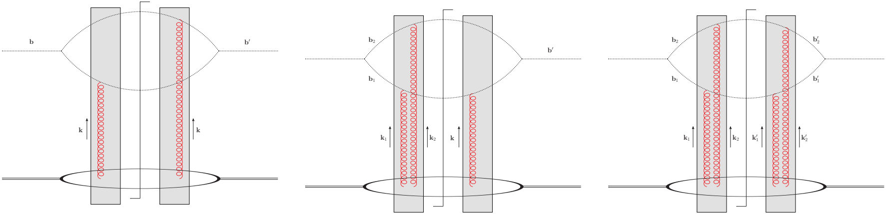
Dipole-type gluon TMD

$$f^D\left(x,\boldsymbol{k}\right) \equiv \frac{1}{P^-} \int \frac{\mathrm{d}\xi^+}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{\xi}}{(2\pi)^2} \mathrm{e}^{ixP^-\xi^+ - i(\boldsymbol{k}\cdot\boldsymbol{\xi})} \times \left\langle P \left| \mathrm{tr} F^{i-}\left(\xi\right) \mathcal{U}^{[-]}_{\xi,0} F^{i-}\left(0\right) \mathcal{U}^{[+]}_{0,\xi} \right| P \right\rangle_{\xi^-=0}$$

# Inclusive low $x$ cross section

## Inclusive low $x$ cross section = TMD cross section

[Altinoluk, RB, Kotko], [Altinoluk, RB]



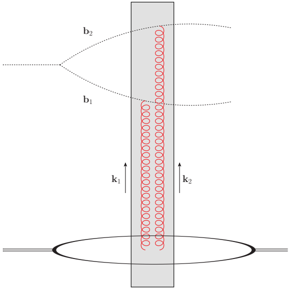

$$\sigma = \mathcal{H}_2^{ij}(\boldsymbol{k}) \otimes f_2^{ij}(x=0, \boldsymbol{k})$$
$$+ \mathcal{H}_3^{ijk}(\boldsymbol{k}, \boldsymbol{k}_1) \otimes f_3^{ijk}(x=0, x_1=0, \boldsymbol{k}, \boldsymbol{k}_1)$$
$$+ \mathcal{H}_4^{ijkl}(\boldsymbol{k}, \boldsymbol{k}_1, \boldsymbol{k}'_1) \otimes f_4^{ijkl}(x=0, x_1=0, x'_1=0, \boldsymbol{k}, \boldsymbol{k}_1, \boldsymbol{k}'_1)$$

All distributions are evaluated in the strict $x = 0$ limit

# Exclusive low $x$ cross section

## Exclusive low $x$ amplitude = GTMD amplitude

[Altinoluk, RB]




$$\mathcal{H}^{ij}(k_{1\perp}, k_{2\perp}) \otimes \left\langle P' \left| F^{-i} W F^{-j} W \right| P \right\rangle$$

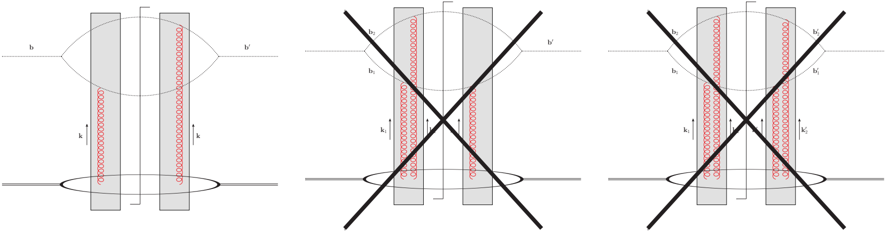
Every exclusive low $x$ process probes a Wigner distribution!

# The so-called dilute limit in terms of TMD distributions

# Inclusive low $x$ cross section

## First, take the Wandzura-Wilczek approximation

[Altinoluk, RB, Kotko]: matches iTMD cross sections




$$\sigma = \mathcal{H}_2^{ij}(k_\perp) \otimes \left\langle P \middle| F^{-i} W F^{-j} W \middle| P \right\rangle$$

$$+\mathcal{H}_3^{ijk}(k_\perp, k_{1\perp}) \otimes \left\langle P \middle| F^{-i} W g_s F^{-j} W F^{-k} W \middle| P \right\rangle$$

$$+\mathcal{H}_4^{ijkl}(k_\perp, k_{1\perp}, k'_{1\perp}) \otimes \left\langle P \middle| F^{-i} W g_s F^{-j} W g_s F^{-k} W F^{-l} W \middle| P \right\rangle$$

## BFKL distributions and genuine twist corrections

What is neglected in BFKL: 3- and 4-Reggeon matrix elements.

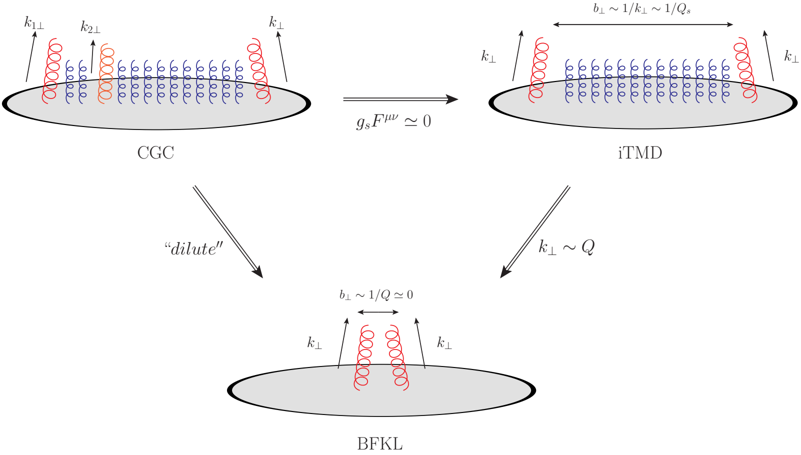
$$\langle P\,|RR|\,P\rangle\,,\quad \langle P\,|R(g_sR)R|\,P\rangle\,,\quad \langle P\,|R(g_sR)(g_sR)R|\,P\rangle$$

They are not perturbatively suppressed.

Suppression = WW approximation (unquantifiable)

# The dilute limit

## The dilute limit in terms of TMD distributions

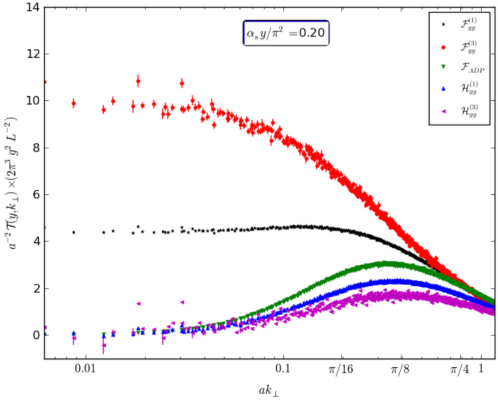



Two kinds of multiple scattering effects: higher genuine twists and higher kinematic twists

# Kinematic saturation

## "Saturation" from a TMD gauge link

Link length $\sim 1/|k_\perp|$, hence effect suppressed at large $k_\perp$

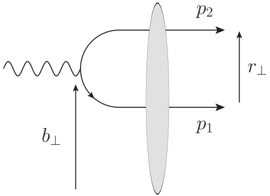



[Marquet, Petreska, Roiesnel ; Marquet, Roiesnel, Taels]

Unintegrated PDF: universal $(x = 0, k_\perp \gg Q_s)$ limit of TMDs

## Case study

A case study for saturation effects: probing a target with a dipole




$k_\perp = p_{1\perp} + p_{2\perp}$ Fourier conj to $b_\perp$
$P_\perp = (p_2^+ p_{1\perp} - p_1^+ p_{2\perp})/(p_1^+ + p_2^+)$
Fourier conj to $r_\perp$

- Twist corrections: $|k_\perp|/Q$ from the hard part, $Q_s/Q$ from higher twist operators.
- Kinematic saturation: multiple scatterings from the TMD gauge link $\propto Q_s/|k_\perp|$
- Genuine saturation: multiple scatterings from higher twist operators $Q_s/Q$
- "Old school" saturation: large dipole size $r_\perp \sim 1/Q_s$

## Examples from our contribution

### Examples we included so far

### Exclusive light meson electroproduction

Light meson wave functions peak at large dipoles $|r_\perp| \sim 1/m$

- $\rho^0$ **production**: full NLL amplitude for $\rho^0_L$ electroproduction is known $\Rightarrow$ Precision study of saturation at the EIC
  [RB, Grabovsky, Ivanov, Szymanowski, Wallon]
- $\pi^0$ **production**: Odderon exchange. Dominated by the gluon Sivers TMD even for unpolarized proton beams thanks to the CGC/GTMD correspondence $\Rightarrow$ transverse spin physics at small $x$
  [RB, Hatta, Szymanowski, Wallon]

# Examples from our contribution

## Examples we included so far

## Forward dijet electroproduction

- **Exclusive case**: known at NLL [RB, Grabovsky, Ivanov, Szymanowski, Wallon] and directly probes a gluon Wigner distribution [Hatta, Xiao, Yuan] $\Rightarrow$ Precision 5d proton imaging with saturation
- **Semi-inclusive case**: known at NLL [Caucal, Salazar, Venugopalan]. Probes genuine saturation in the back-to-back limit [RB, Mäntysaari, Salazar, Schenke]
- **Coherent vs incoherent diffractive cases**: in CGC models, probes the variance of the distribution of color fields in the target

## Examples from our contribution

### Examples we included so far

### Ultra Peripheral Collisions (UPC)

- **Exclusive $\eta_c$ or $\chi_c$ photoproduction**: same physics as $ep \to ep\pi^0$ to be probed at the LHC
- **Exclusive light meson photoproduction at large $t$**: $t$-scaling in BFKL vs in BK is a good measurement of non-linear effects and thus saturation [Fucilla, Li, Mäntysaari, Salazar, Szymanowski, Wallon, *in progress*]
- **Semi-inclusive photoproduction of a large $p_t$ meson**: easy(-ish) to compute at NLL with the known NLL impact factors
- **Exclusive photoproduction of a meson-photon pair**: clean GPD limit, 2-body final state so interesting probe for kinematic vs genuine saturation effects

## Summary

# Summary

- Our contribution focuses on the TMD/CGC equivalence and its consequences for small $x$ physics: spin and saturation can be reinterpreted very thoroughly
- There is a HUGE overlap with certain groups
- Suggestions to coordinate and/or merge contributions?