FFT analysis quick Thoughts

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Intro

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- The basic functional form of the interferometer clouds indicates that analysis in frequency domain is quite natural
- We started looking at this for two reasons
  - It might allow to judge quality of camera parameters independent of “physics knowledge”
  - Quite intuitive about effects of noise modulation, MTF etc

Analysis via 2D FFTs - Derivation

$$h(x,z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x^2+z^2)}{2\sigma^2}\right)(1+\cos(kx+\phi))$$

Start from function we are looking at

$$\begin{aligned} f(x) &= 1+\cos(kx+\phi) \\ g(x,z) &= \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x^2+z^2)}{2\sigma^2}\right) \end{aligned}$$

$$\tilde{H}(u,v) = \tilde{F}(u) * \tilde{G}(u,v)$$

Split into gaussian and cos parts, and apply convolution theorem

$$\tilde{F}(u) = \delta(u) + \frac{1}{2}e^{i\phi}\delta(u-k) + \frac{1}{2}e^{-i\phi}\delta(u+k)$$

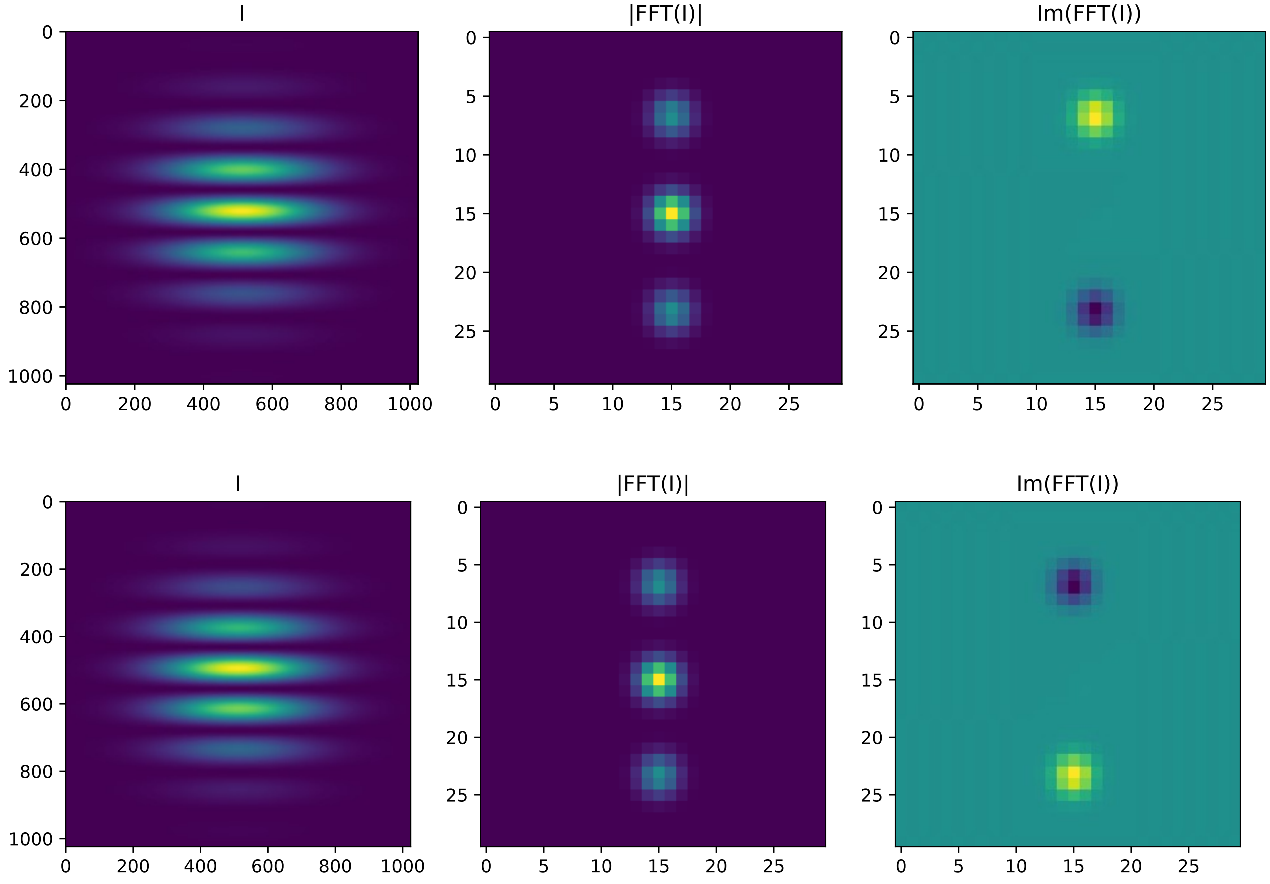
$$\tilde{H}(u,v) = \tilde{G}(u,v) + \frac{1}{2}e^{i\phi}\tilde{G}(u+k,v) + \frac{1}{2}e^{-i\phi}\tilde{G}(u-k,v)$$

Can then write down analytical 2D FT quite easily – three shifted Gaussians and a phase factor

$$\Im\left(\tilde{H}(u,v)\right) = \frac{1}{2}\sin(\phi)\tilde{G}(u+k,v) - \frac{1}{2}\sin(\phi)\tilde{G}(u-k,v)$$

Bottom line – can recover phase information by looking at imaginary part of this

2D FFT analysis demo



Examples:
-30 degree phase shift (top)

+50 degree phase shift (bottom)

If one were to change frequency or width of gaussian cloud, blobs in fourier space would shift relative to each other (not shown here)

Still working on best way to directly pull phase out of this but all information is there!

Shifted Cloud

The above derivation (and images) assumed a cloud perfectly centred in the image. It's easy to work out what happens when this is not the case

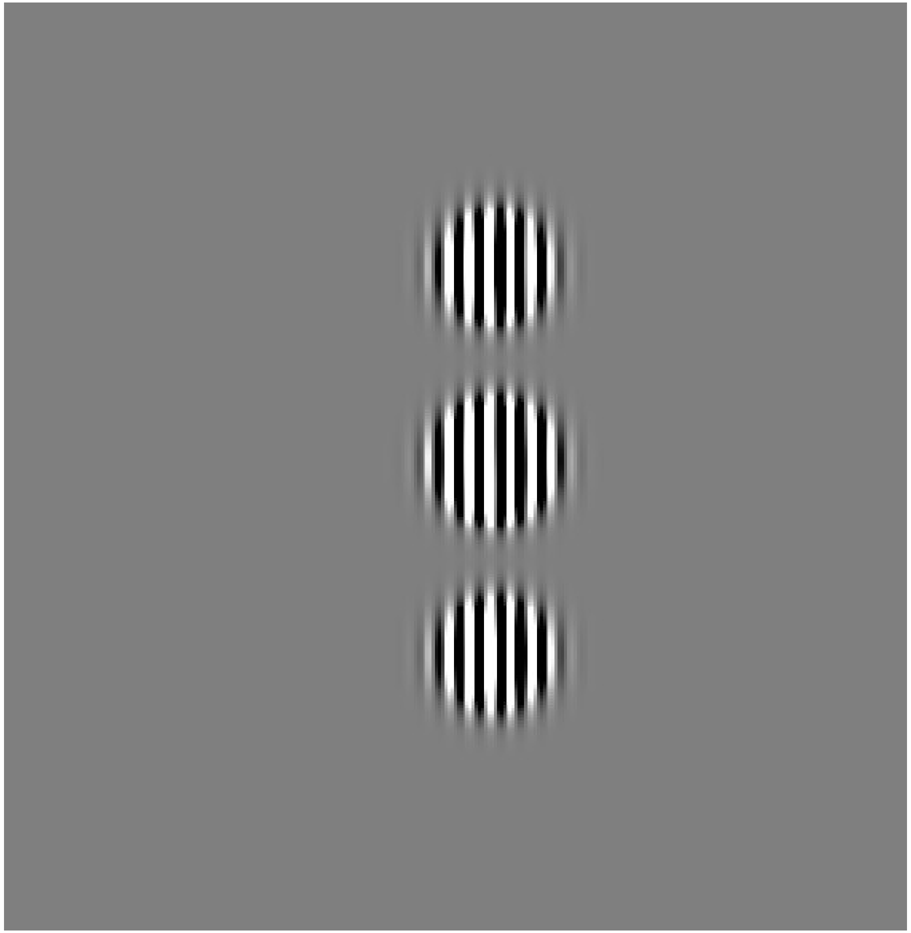
$$\mathcal{F}\{g(t-a)\} = e^{-i\omega a}\hat{G}(\omega)$$

$$\hat{H}_{\text{shift}}(u,v) = e^{-z_0 v}\hat{G}(u,v) + \frac{1}{2}e^{i\phi - \text{i}z_0 v}\hat{G}(u+k,v) + \frac{1}{2}e^{-i\phi - \text{i}z_0 v}\hat{G}(u-k,v)$$

The central "blob" will re-appear in the Fourier Transforms, and an oscillating phase offset is added.
If one knows the offset, it can be easily corrected. However, probably best to pre-align the images to get rid of this – **question: how consistent will the total offset be in our images?**

Shifted Cloud FFT

Imaginary part!



Multi Cloud

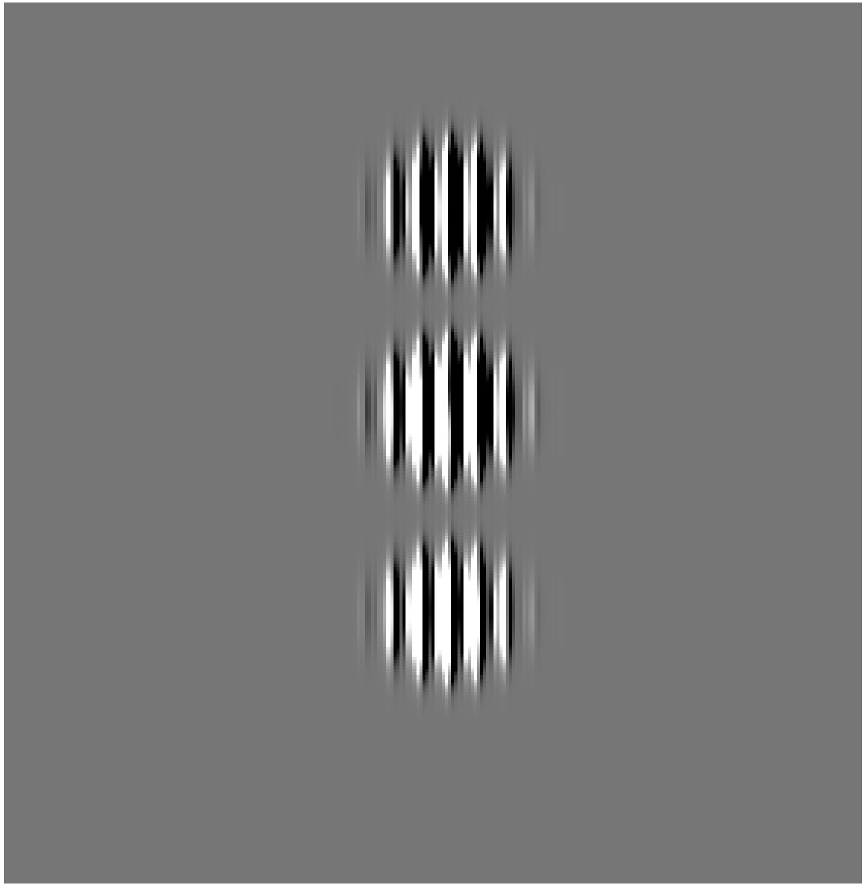
For multiple clouds, similar effects happen. For 4 clouds:

$$\hat{H}_{\text{shift}}(u,v) = e^{-\mathrm{iZv}}\hat{G}(u,v) + \frac{1}{2}e^{i(\phi_0+\phi_1+\phi_2+\phi_3)-\mathrm{iZv}}\hat{G}(u+k,v) + \frac{1}{2}e^{-i(\phi_0+\phi_1+\phi_2+\phi_3)-\mathrm{iZ}v}\hat{G}(u-k,v)$$

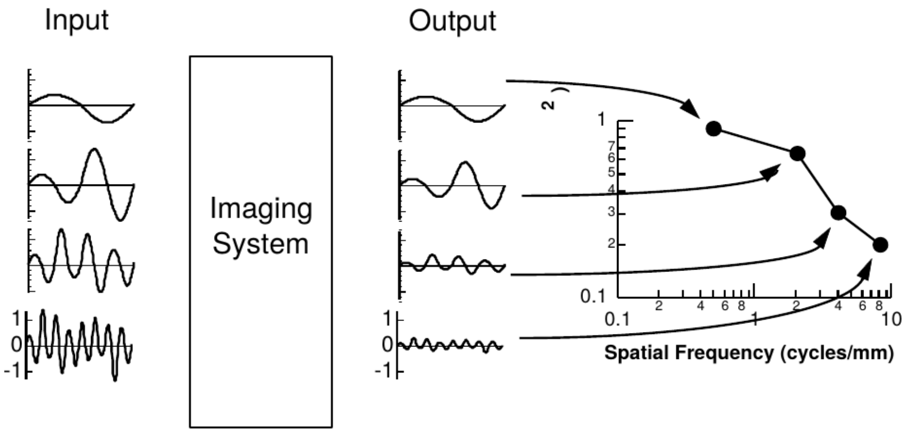
There does not seem a simple way to do direct phase extraction from this without pre-segmenting the image so there is one cloud in each. Again, this is practical if we know quite well the regions in advance and don't have to do some clever segmentation via computer vision. That again is possible, but probably negates much of the elegeance / simplicity of doing FFT analysis directly.

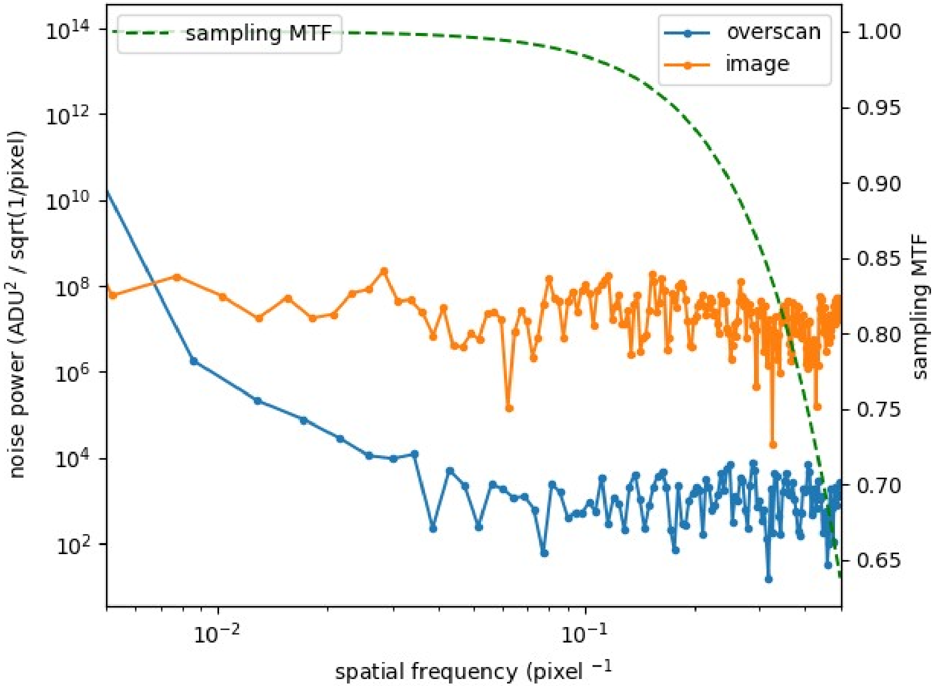
Multi Cloud example

Imaginary part!



Note on noise





When noise is added, the temporal → spatial transformation of information via sampling converts it ultimately into a “noise equivalent modulation limit”

Example: note NOT from a MAGIS simulation
Almost all readouts have a “white + 1/f” spectrum.

Noise effect on FFT

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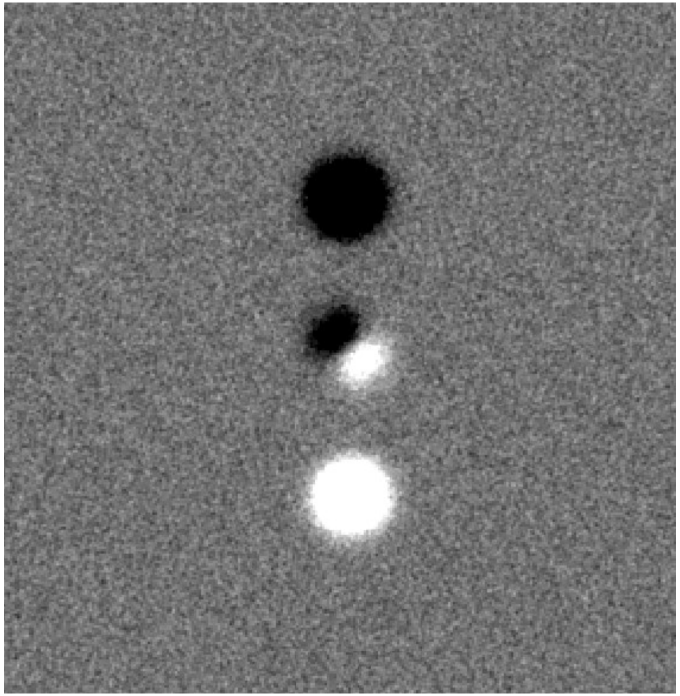
Imaginary part!



Imperfections in Cloud Wavefunction

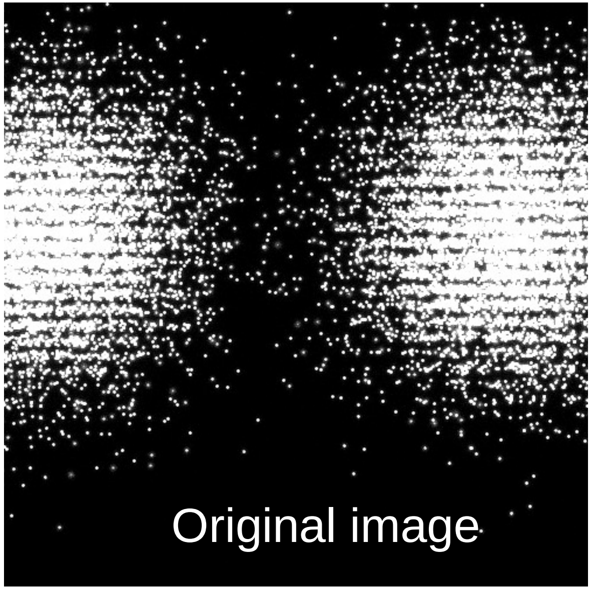
Obvious questions (and possible work around on next slide):

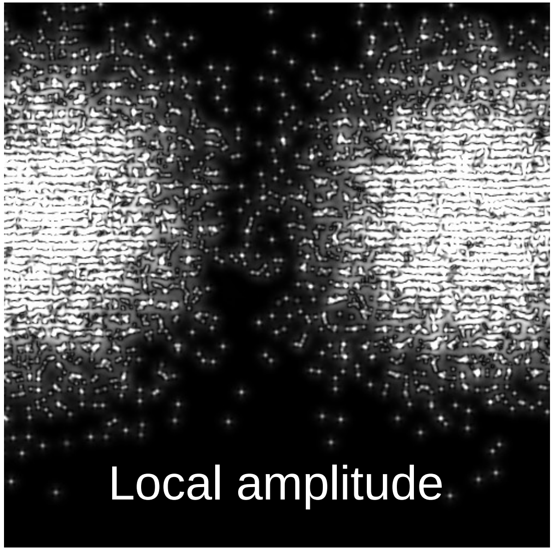
What if the fringing direction isn't aligned to the pixel direction?

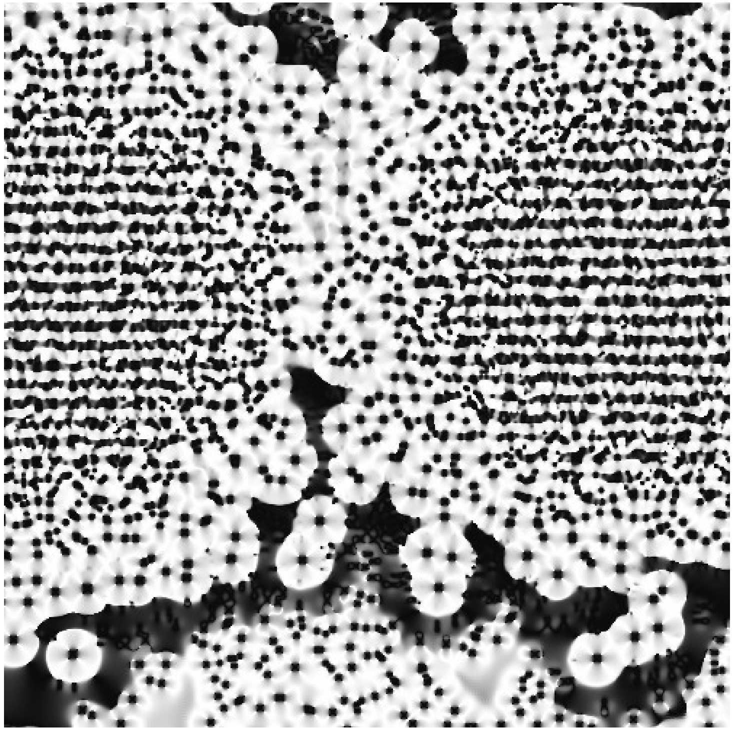
What about other (non-pure cosine mod) inclusions in the function?

Monogenic Signal Analysis

- Extension to FFT analysis – a 2D generalisation of the Hilbert transform (via the Riesz transform) allows us to obtain local amplitude and phase information
- First apply a log-Gabor filter in Fourier domain (eliminating noise modulation), then extract phase & amplitude (see right).
- Early stages, but promising


Original image


Local amplitude


Local phase