Latest from Models and Generators

Noemi Rocco

NuFact 2022: The 23 International Workshop on Neutrinos from Accelerators

July 30th — August 6th, 2022

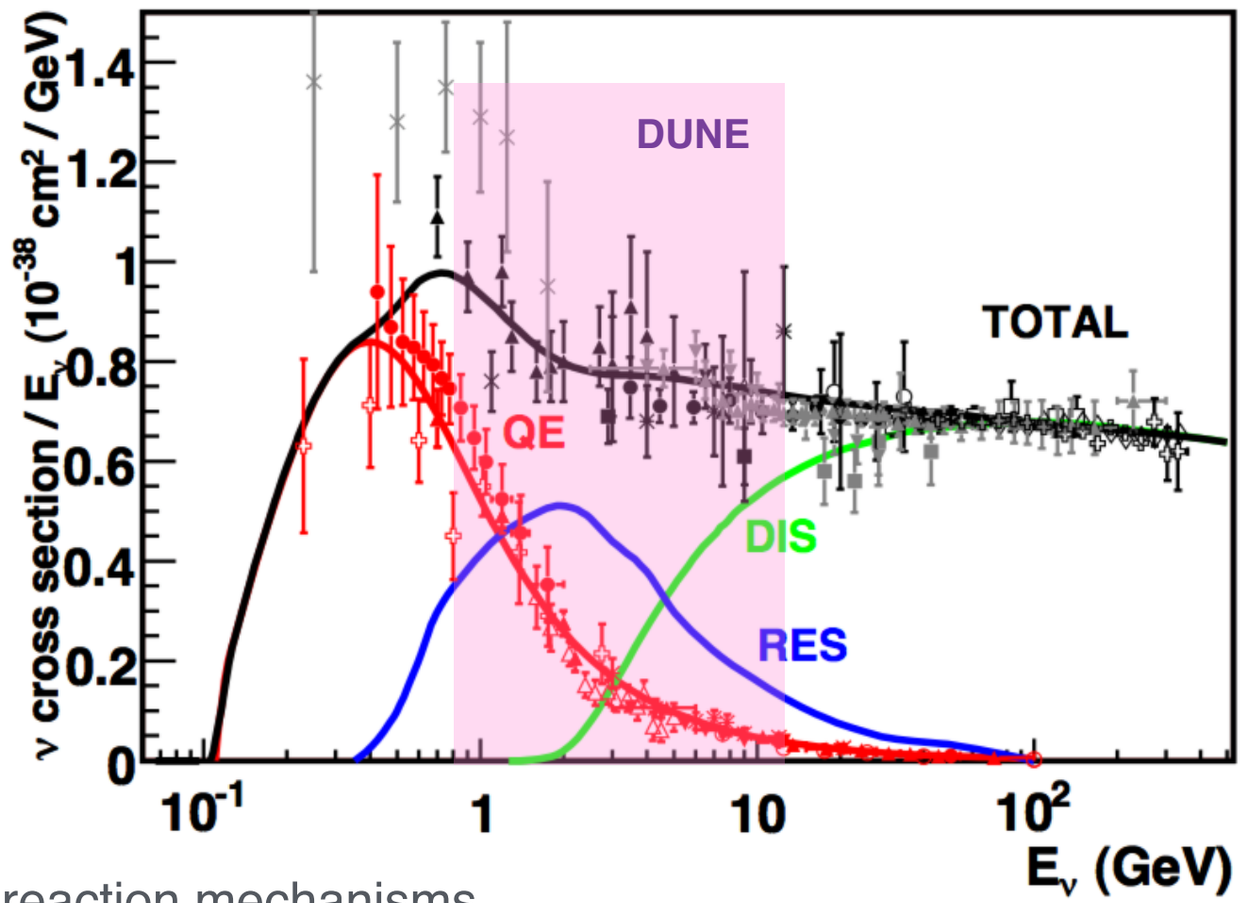
Cliff Lodge

Addressing Neutrino-Oscillation Physics

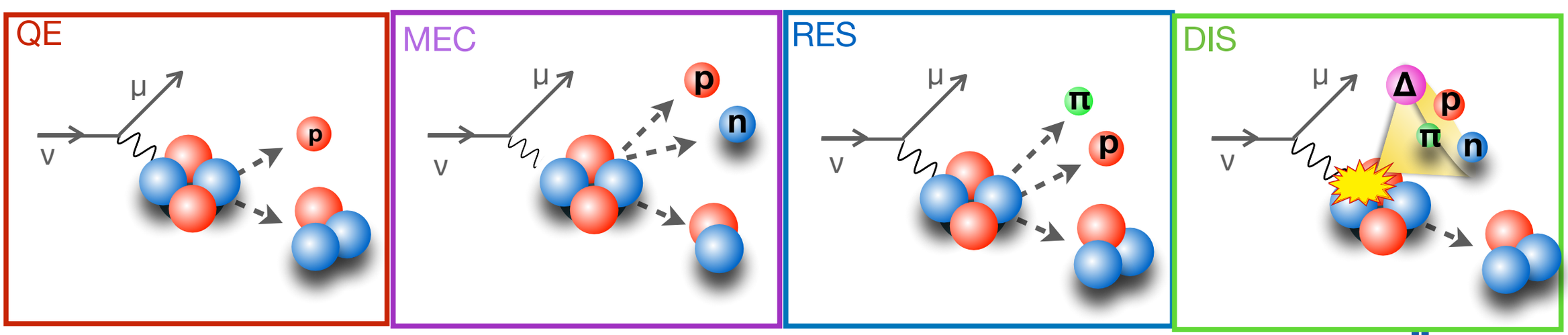
Unprecedented accuracy in the determination of **neutrino-argon cross section** is required to achieve design sensitivity to CP violation at DUNE

Current oscillation experiments report **large systematic uncertainties** associated with the neutrino- nucleus interaction models.

J.A. Formaggio and G.P. Zeller, Rev. Mod. Phys. 84 (2012)



Nuclei are complicated objects. Many different reaction mechanisms



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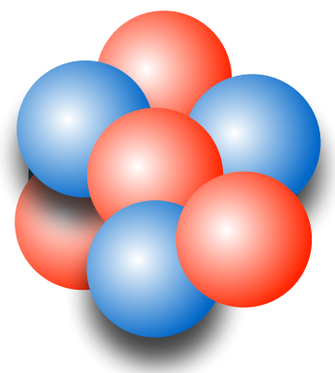
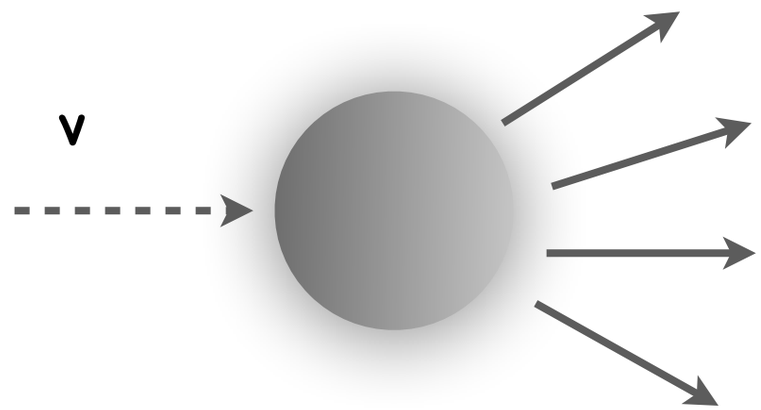
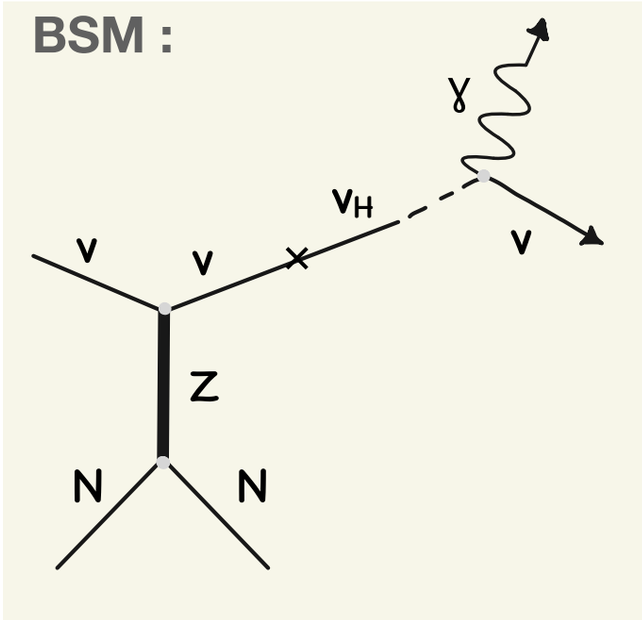
Addressing Neutrino-Oscillation Physics

Nuclear Physics:



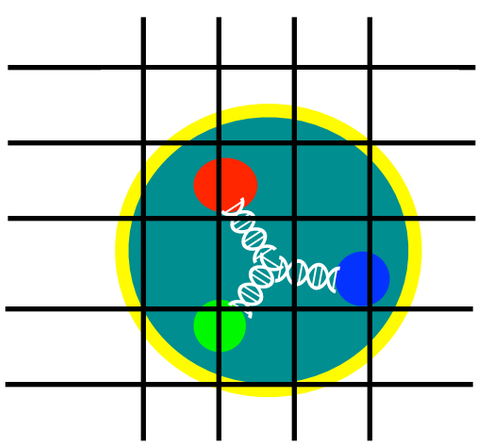


BSM :

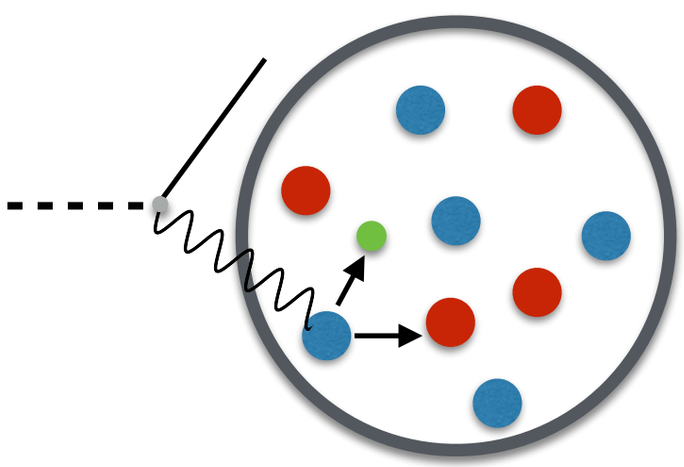


Simulate neutrino-nucleus interactions to untangle neutrino oscillations from the measured interactions

Lattice QCD :



Event Generator :



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Outline of the talk

- **Ab-initio calculations** virtually exact predictions in the QE region. Limited to low/intermediate energy transfer region

- **Factorization Approaches:** rely on approximations but they are able to tackle QE, dip and π-production regions.

- **Event generators:** interaction vertex+ intra-nuclear cascade used to propagate particles produced at the interaction vertex through the nucleus; comparison with electron scattering data

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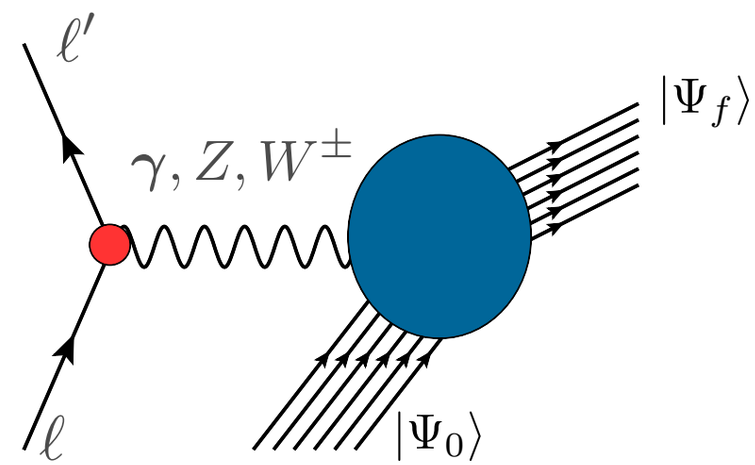
Theory of lepton-nucleus scattering

- The cross section of the process in which a lepton scatters off a nucleus is given by

$$d\sigma \propto L^{\alpha\beta} R_{\alpha\beta}$$



Leptonic Tensor: can include new physics models

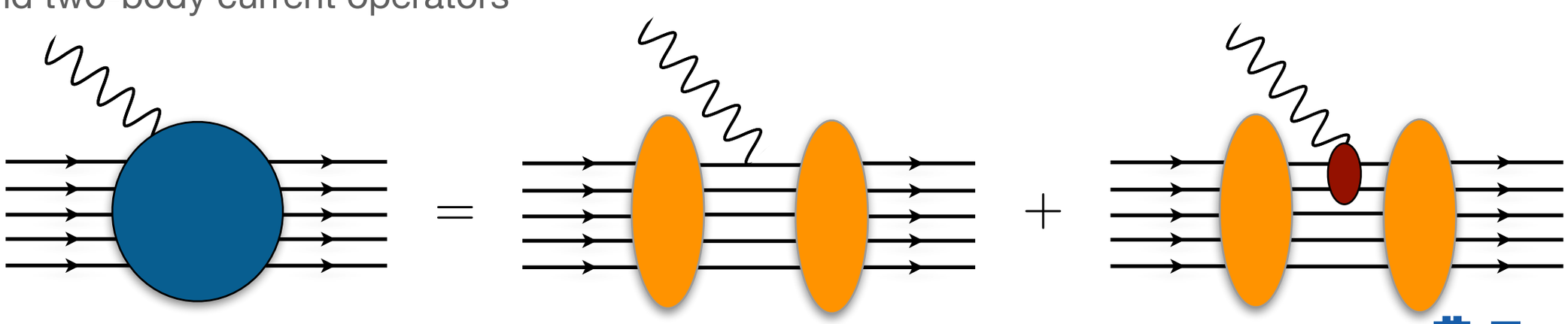
Hadronic Tensor: nuclear response function

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0|J^\dagger_\alpha(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

The initial and final wave functions describe many-body states:

$$|0\rangle = |\Psi_0^A\rangle \ , |f\rangle = |\Psi_f^A\rangle, |\psi_p^N, \Psi_f^{A-1}\rangle, |\psi_k^\pi, \psi_p^N, \Psi_f^{A-1}\rangle \ldots$$

One and two-body current operators



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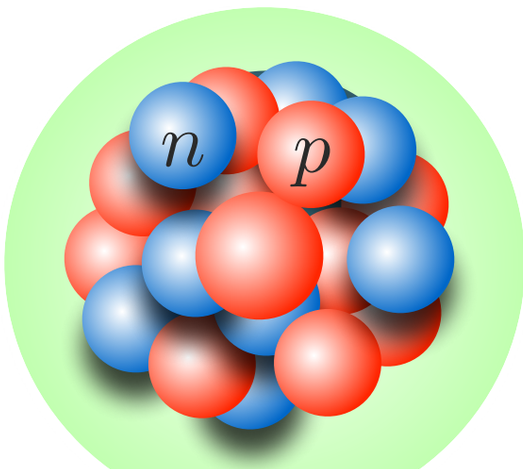
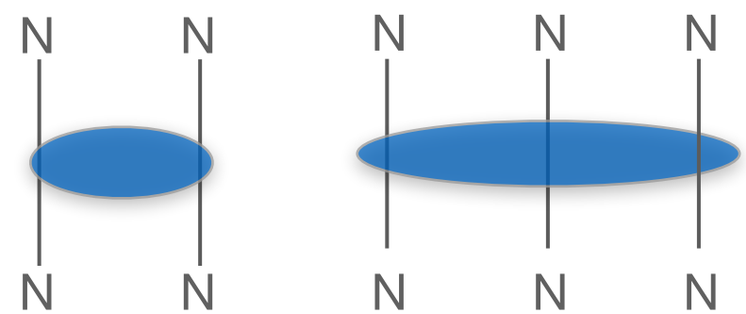
The basic model of nuclear theory

At low energy, the effective degrees of freedom are pions and nucleons:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$



1-body **2-body** **3-body**



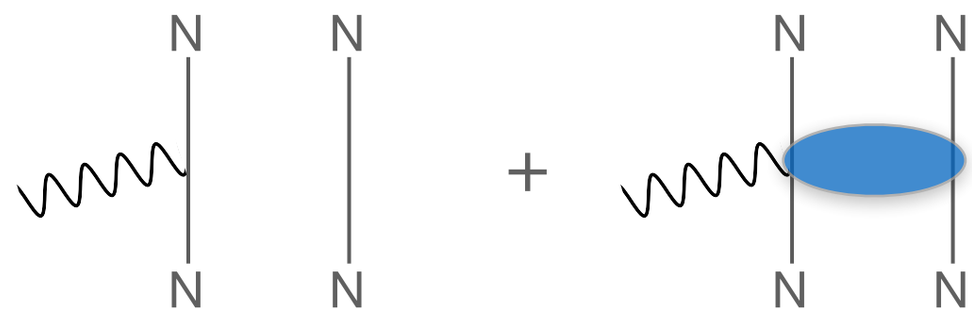
av18 + IL7

The electromagnetic current is constrained by the Hamiltonian through the **continuity equation**

$$\boldsymbol{\nabla} \cdot \mathbf{J}_{\mathrm{EM}} + i[H, J^0_{\mathrm{EM}}] = 0 \qquad [v_{ij}, j_i^0] \neq 0$$

The above equation implies that the current operator includes one and two-body contributions

$$J^\mu(q) = \sum_i j_i^\mu + \sum_{i<j} j_{ij}^\mu + \dots$$



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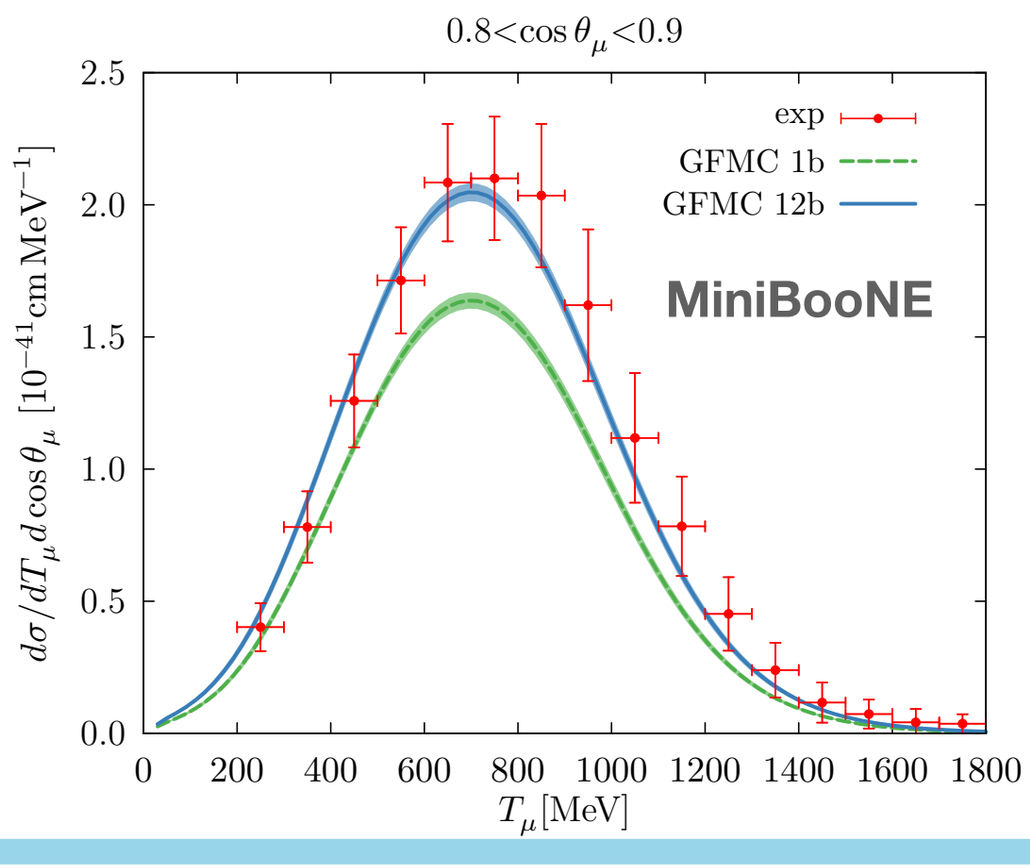
Cross sections: Green's Function Monte Carlo

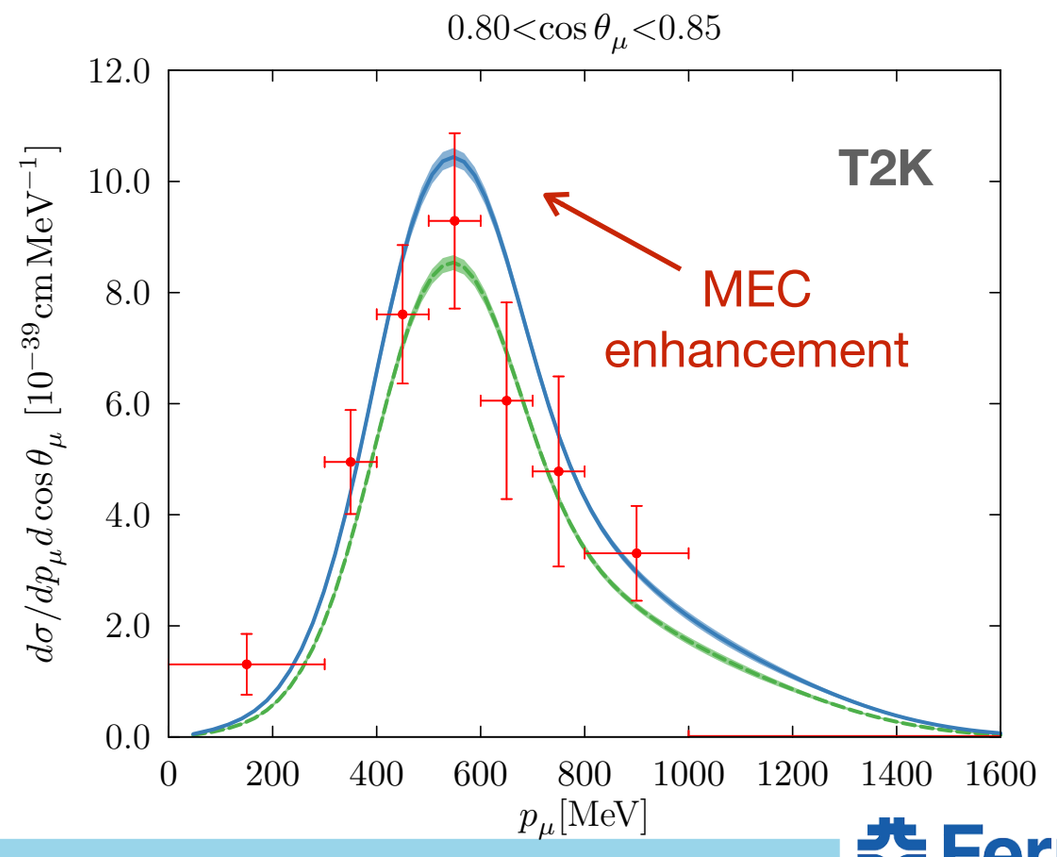
GFMC accurately obtain the properties of nuclei to ^{12}C using high performance computing

Exact results for ν-cross sections in the **quasi-elastic region** up to moderate values of **q**.

First microscopic calculation of neutrino-nucleus cross section

A. Lovato, NR et al , PRX. 10 (2020) 3, 031068





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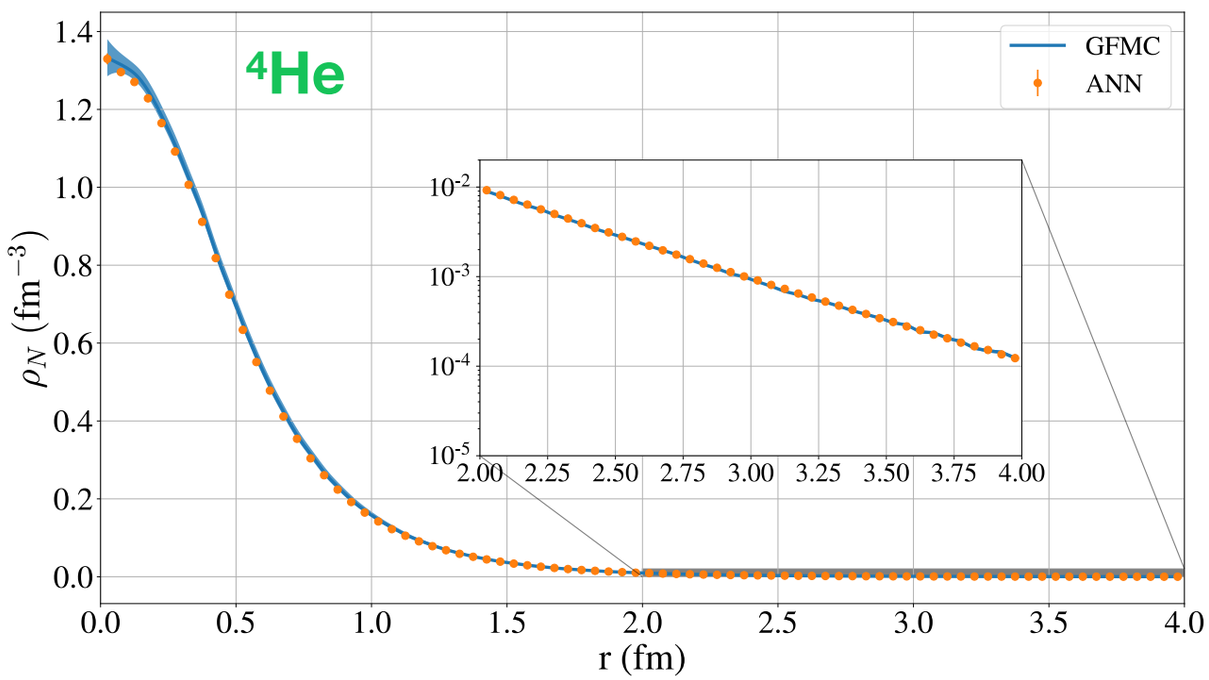
Green's Function Monte Carlo

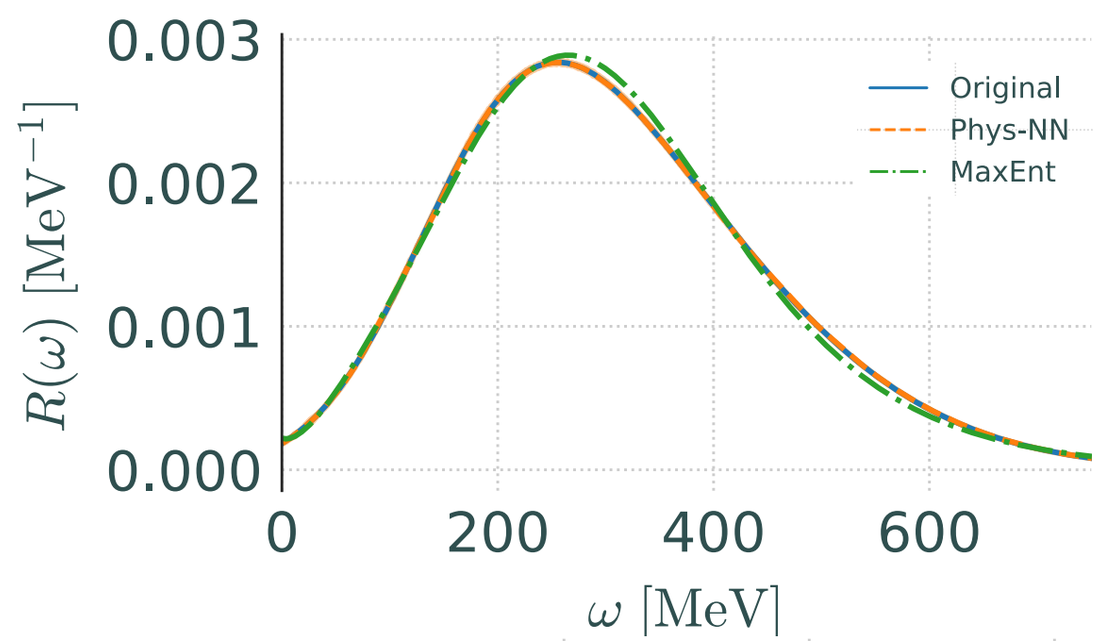


Machine learning methods allows to devise accurate **nuclear wave functions** suitable for quantum Monte Carlo calculations that **do not scale exponentially** with the number of nucleons

A. Lovato et al, PRL. 127 (2021)

A. Lovato et al, PRC 103 (2021) 3, 035502

Machine learning based approach to invert the integral transform (one and two peaks)



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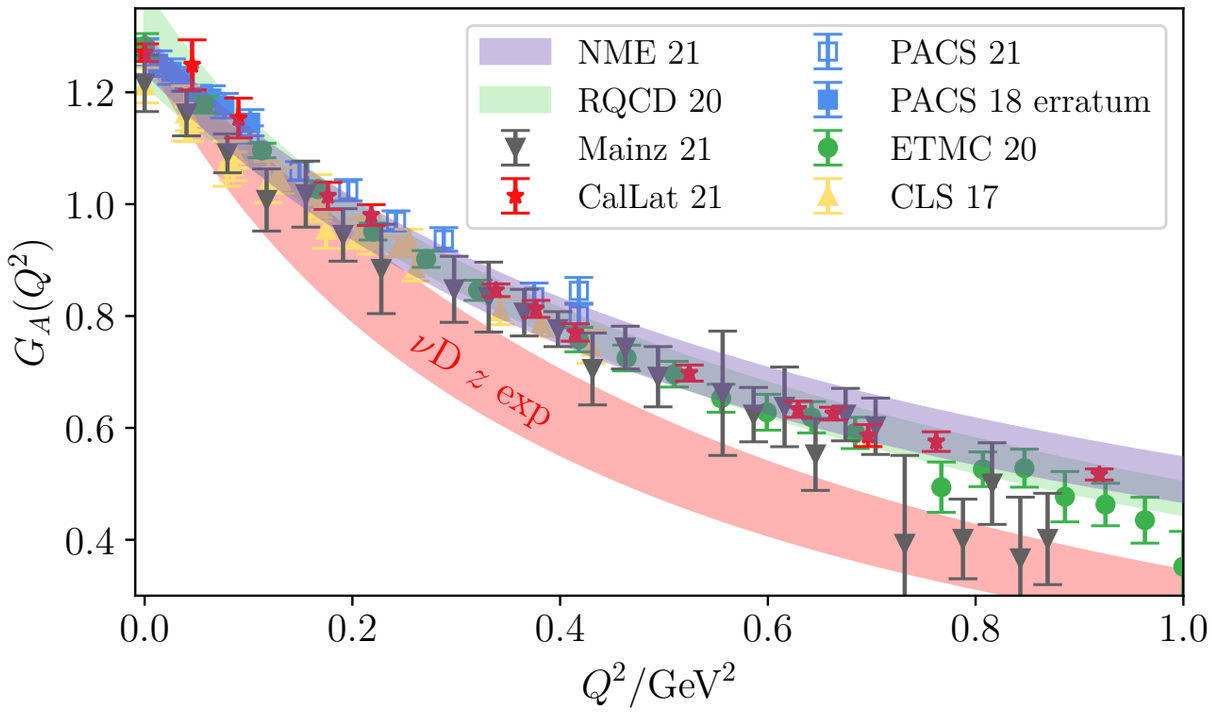
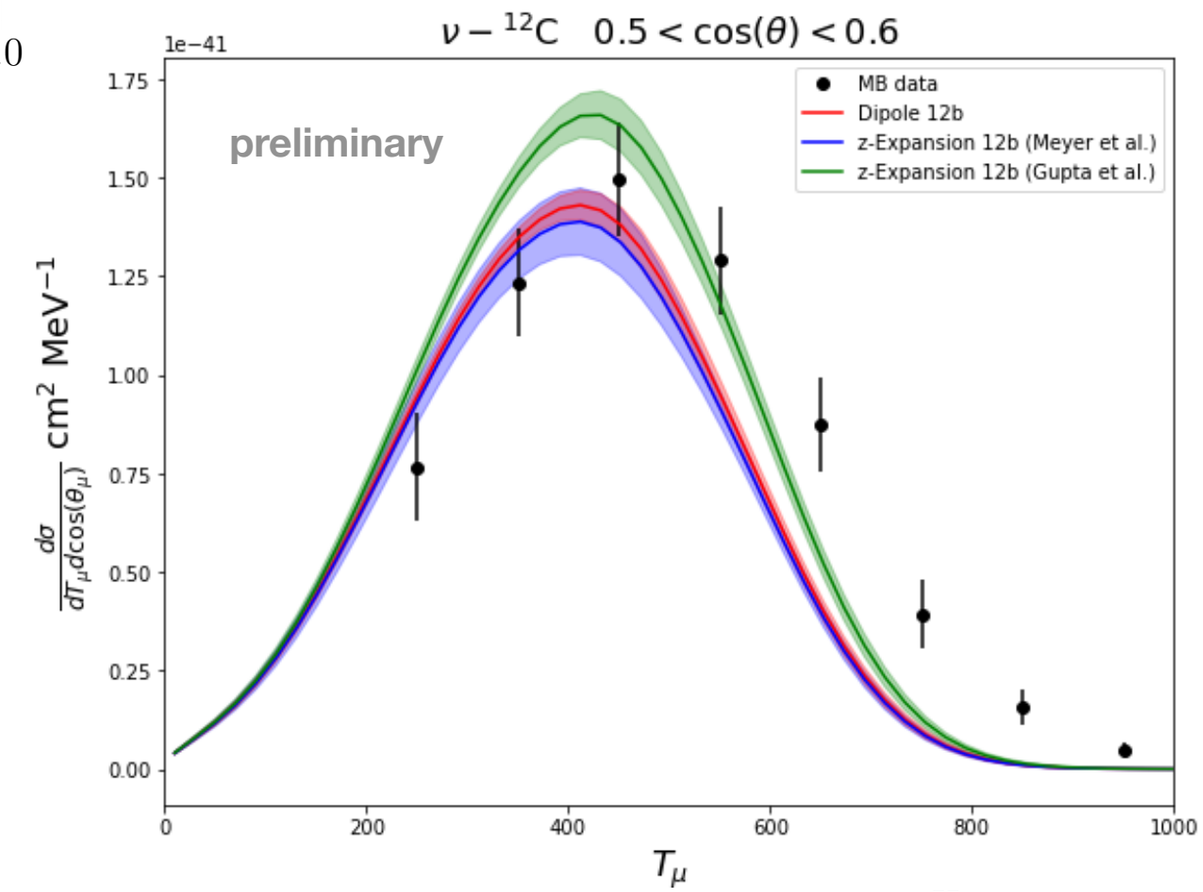
Cross sections: Green's Function Monte Carlo



J. Carlson, M. Wagman, et al, 2203.09030
Contribution to 2022 Snowmass Summer Study

Summary of current LQCD calculations of the nucleon axial form factor compared to *z-expansion*



Preliminary results on the dependence of the GFMC calculations from the axial form factors

D. Simons, N. Steinberg, A. Lovato, Y. Meurice, NR, M. Wagman, in prep

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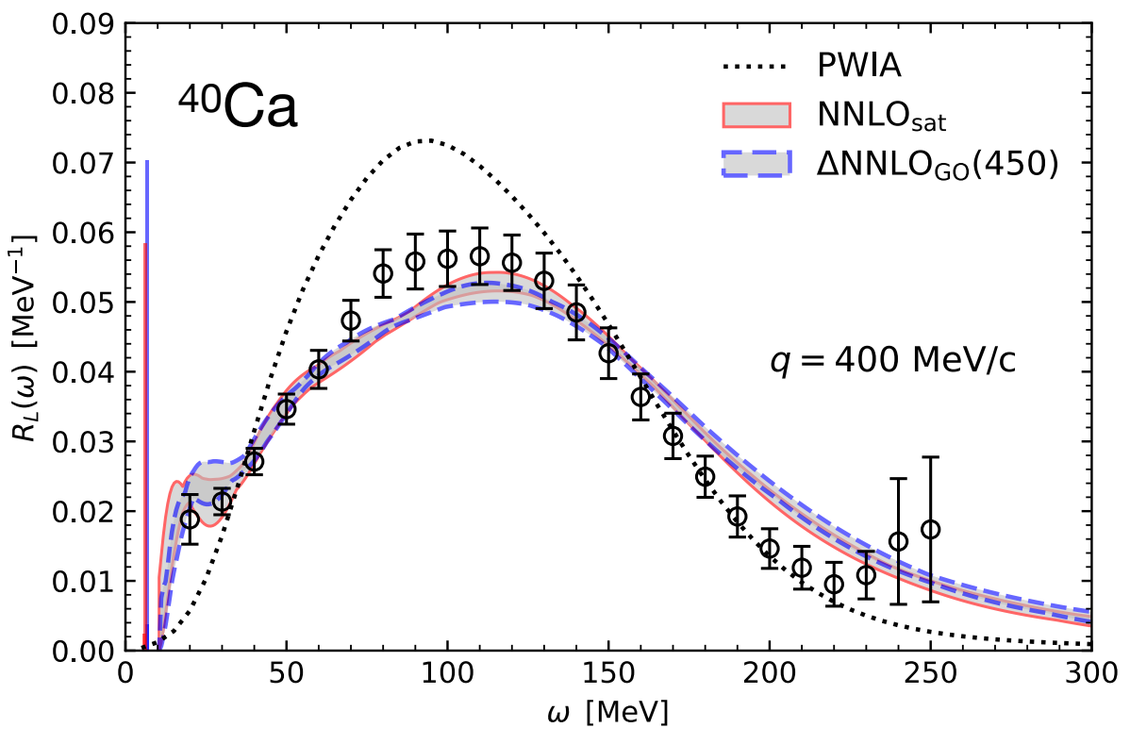
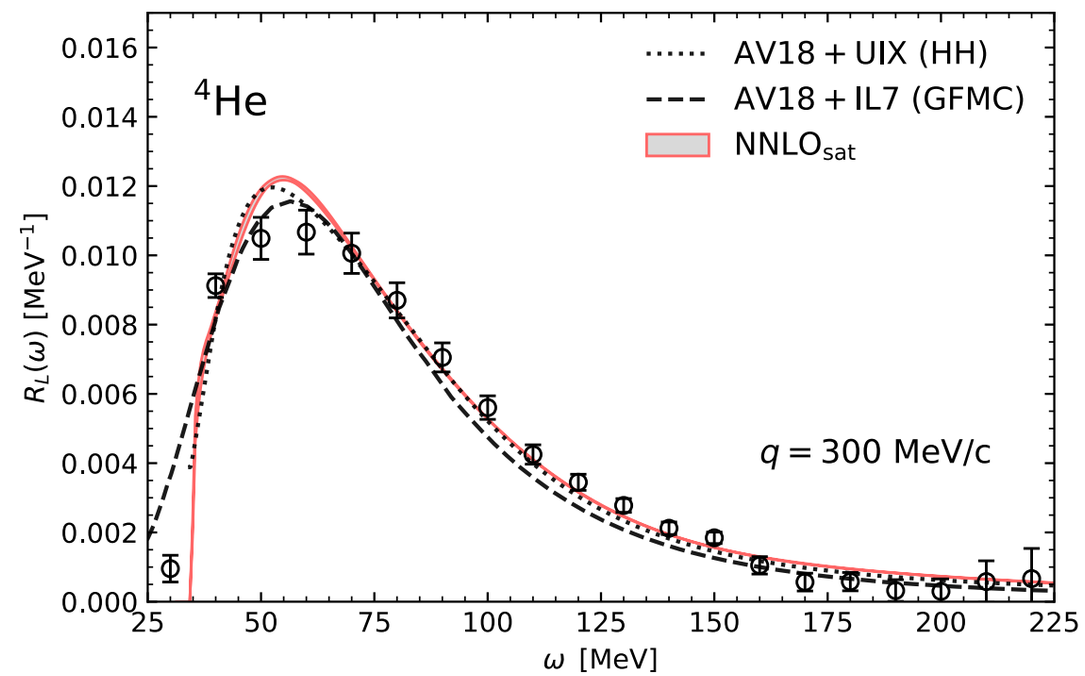
Coupled Cluster method

B. Acharya, J. E. Sobczyk and collaborators are working on developing these techniques:

- Correlations are created applying: $e^T|\Psi\rangle_{\mathrm{HF}}$
  - Polynomially scales with the number of nucleons, suitable to study very large nuclei





J. Sobczyk, B. Acharya et al:

PRL 127 (2021) 7, 072501
PRC 102 (2020) 6, 064312

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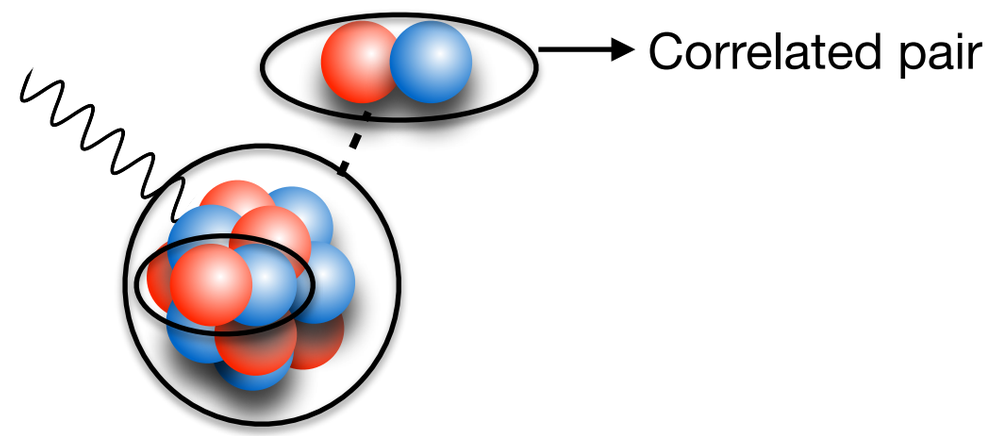
Short Time Approximation

Pastore et al. PRC101(2020) 044612

The Short Time Approximation:

- Based on Quantum Monte Carlo methods
- Based on Factorization
- Retains two body physics



$$R_\alpha(q,\omega) = \int \frac{dt}{2\pi} e^{i(\omega+E_i)t} \langle \Psi_i | J_\alpha^\dagger(\mathbf{q}) e^{-iHt} J_\alpha(\mathbf{q}) | \Psi_i \rangle$$

$$J_i^\dagger e^{-iHt} J_i + J_i^\dagger e^{-iHt} J_j + J_i^\dagger e^{-iHt} J_{ij} + J_{ij}^\dagger e^{-iHt} J_{ij}$$

$$H \sim t_i + \sum_{i<j} v_{ij}$$

- Response functions are given by the scattering from pairs of fully interacting nucleons that propagate into a correlated pair of nucleons

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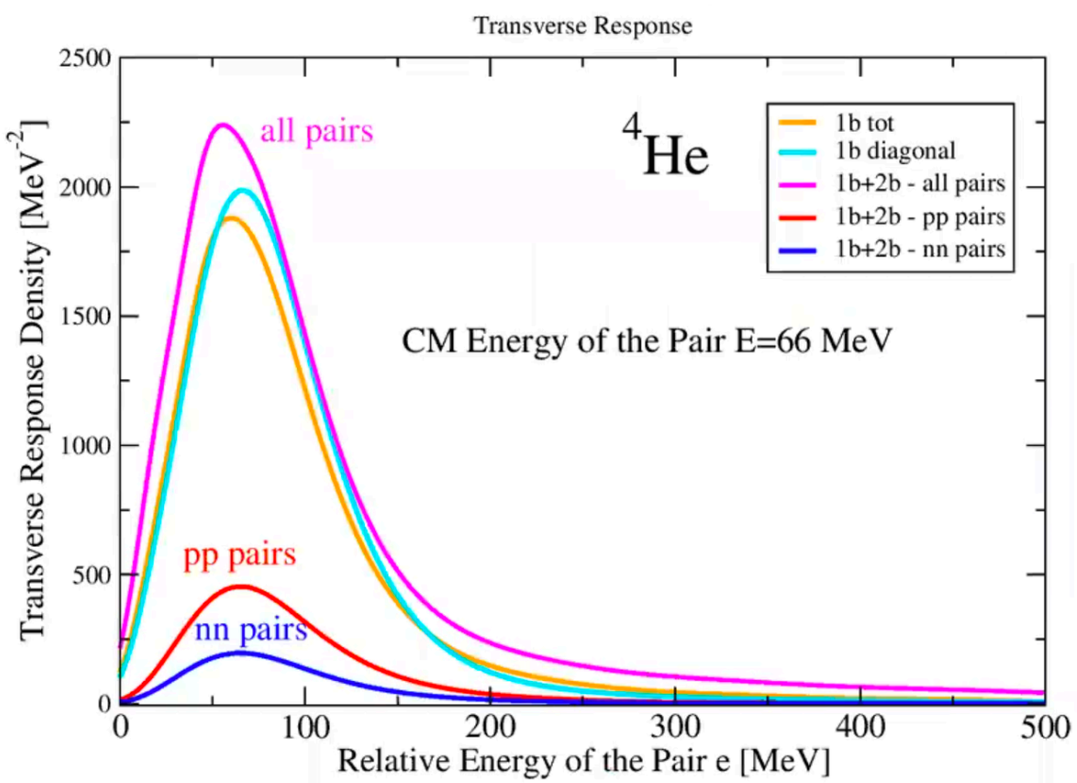
Short Time Approximation

Pastore et al. PRC101(2020) 044612

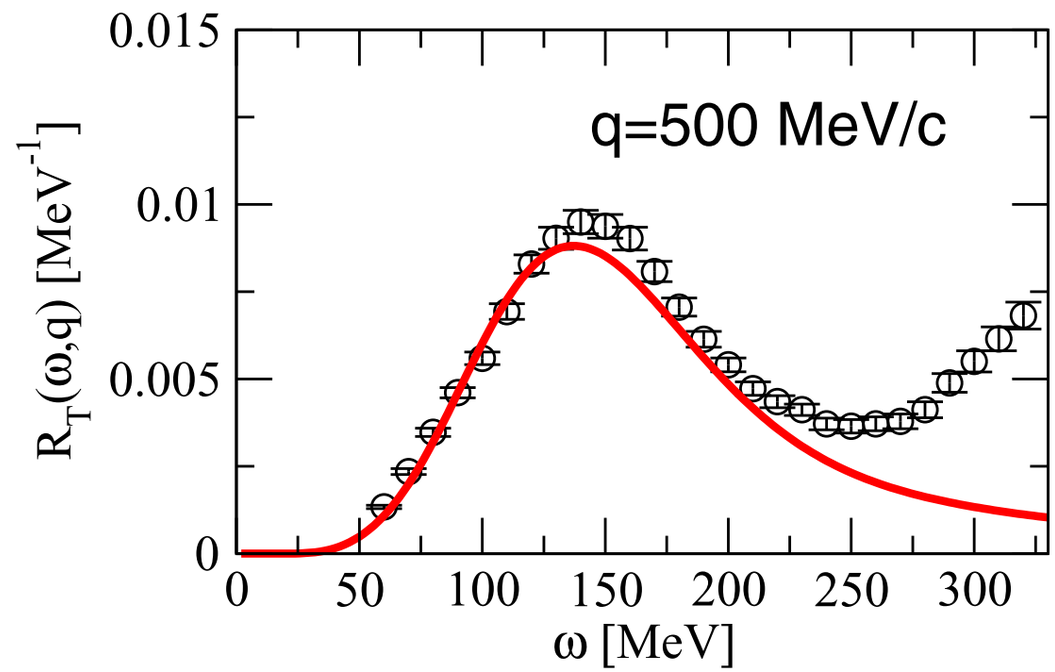
The Short Time Approximation:

- Provides access to exclusive kinematics in terms of nucleon-pair kinematics via the Response Densities

$$R(q,\omega) \sim \int \delta(\omega + E_0 - E_f) dP' dp' \mathcal{D}(p', P'; q)$$



Electromagnetic response of ^{4}He



- Select a particular kinematic, and assess the contributions from different particle identities

- Effort led by **L. Andreoli:** extend the predictions to ^{12}C

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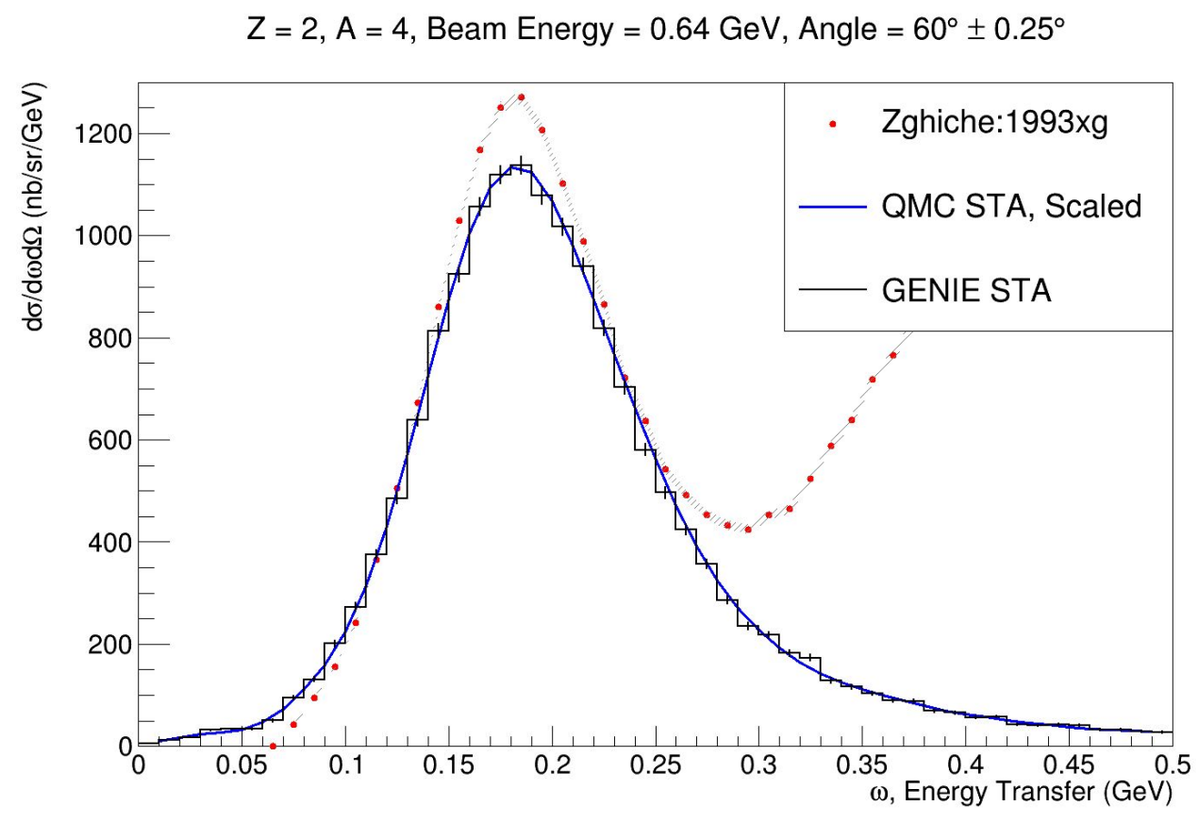
Short Time Approximation

Implementation into GENIE:

- The STA are used to obtain the electromagnetic cross sections
- Tables of response functions are interpolated for different values of the energy and momentum transfer to obtain the cross sections using scaling properties
- Ongoing work: extension to the electroweak sector as well as larger nuclei; preliminary results are available for ^{12}C

Barrow, Gardiner et al, PRD 103 (2021) 5, 052001



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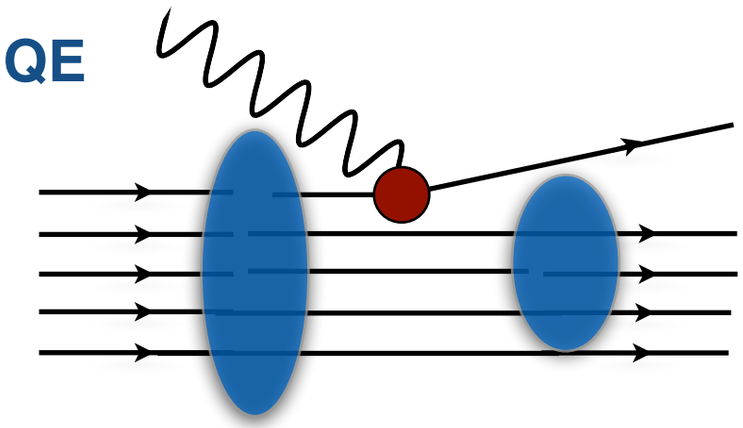
Spectral function approach

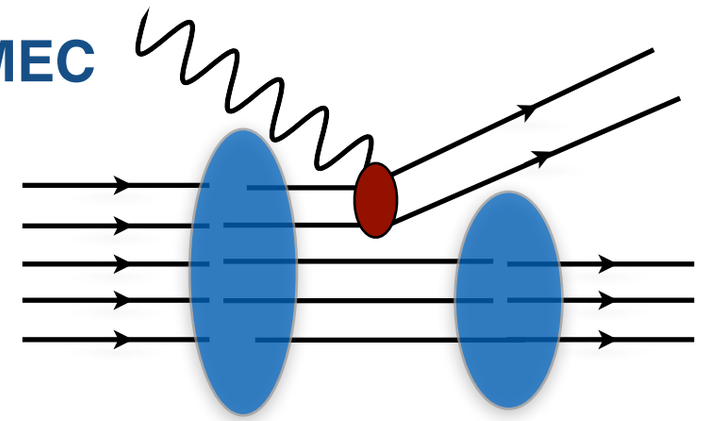
For sufficiently large values of |**q**|, the **factorization scheme** can be applied

QE



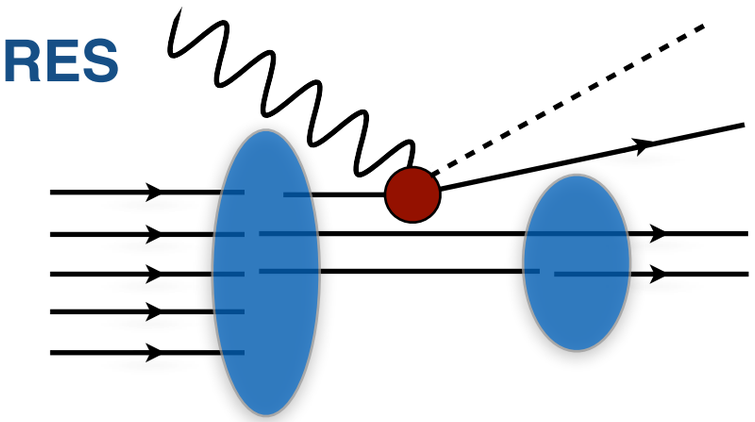
$$|f\rangle \to |p\rangle \otimes |f_{A-1}\rangle$$

MEC



$$|f\rangle \to |pp'\rangle_a \otimes |f_{A-2}\rangle$$

RES

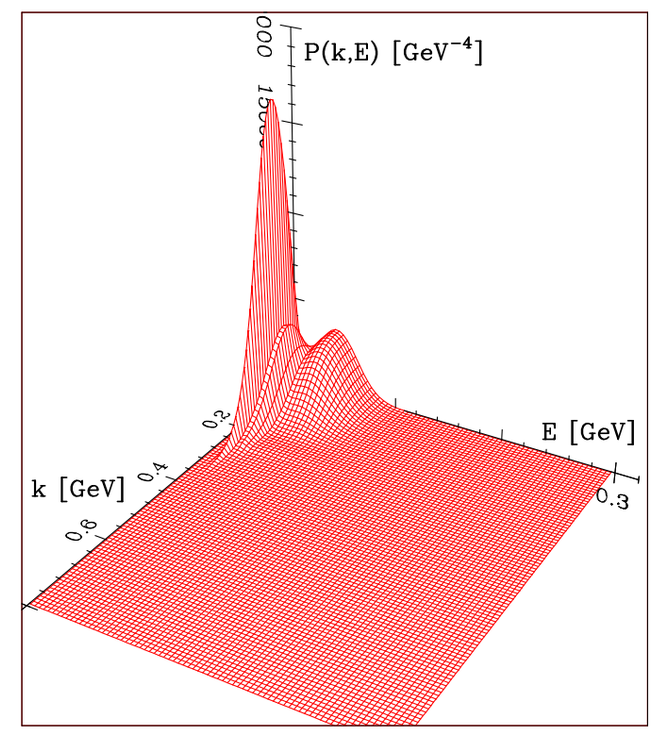


$$|f\rangle \to |p_\pi p\rangle \otimes |f_{A-1}\rangle$$

The intrinsic properties of the nucleus are described by the **Spectral Function**→ effective field theory and nuclear many-body methods

$$d\sigma_A = \int dE d^3k \; d\sigma_N P(\mathbf{k}, E)$$

- SF of light nuclei computed ab-initio within QMC



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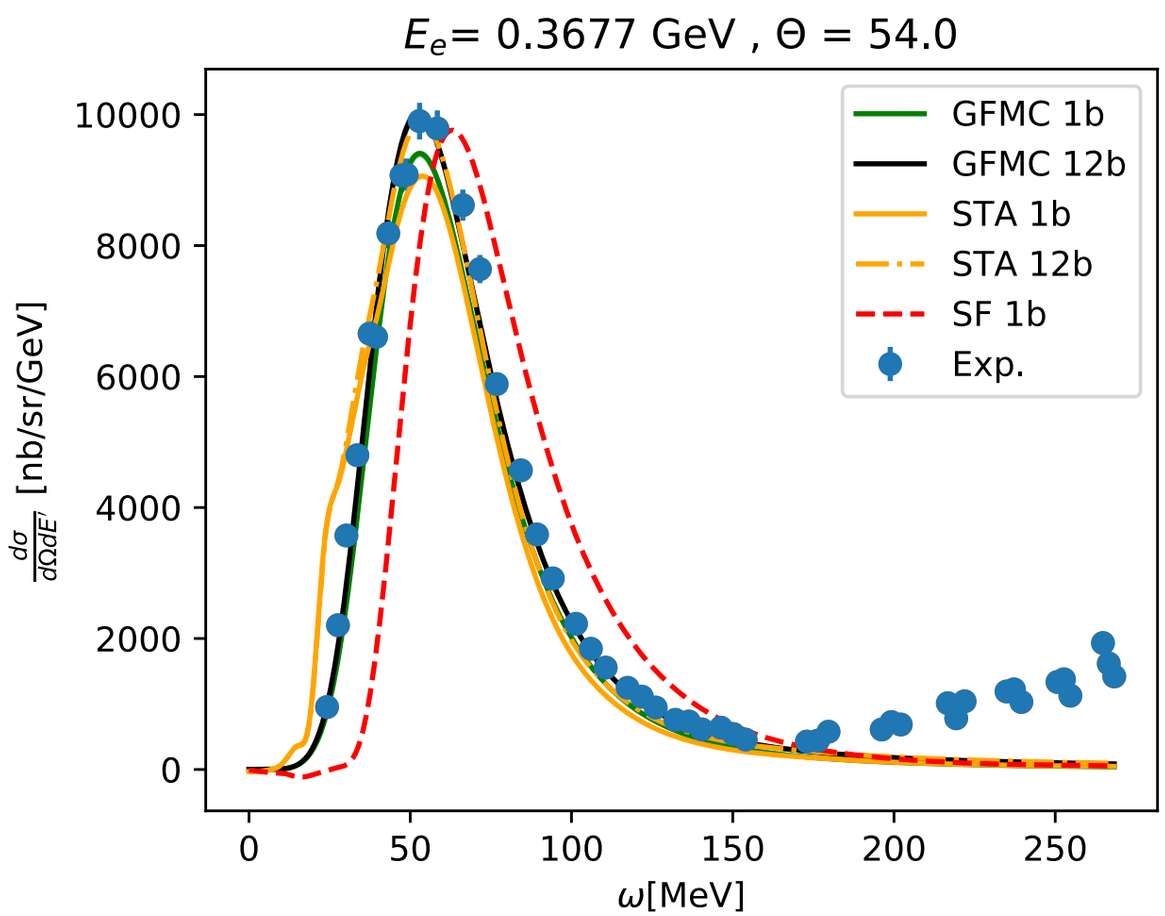
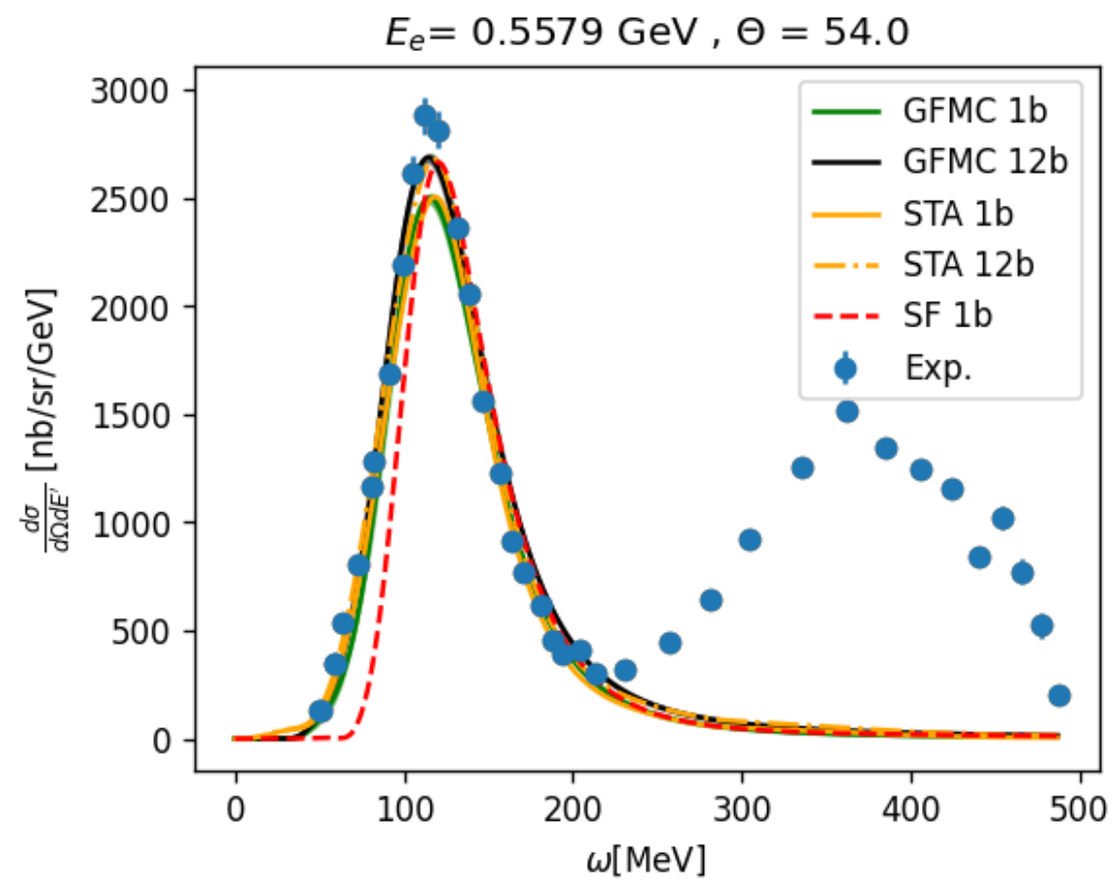
Comparing different many-body methods

- e -^{3}H: inclusive cross section

L. Andreoli, NR, et al, PRC 105 (2022) 1, 014002





- Comparisons among GFMC, SF, and STA approaches: first step to precisely **quantify the uncertainties** inherent to the factorization of the final state.

- Gauge the role of **relativistic effects** in the energy region relevant for neutrino experiments.

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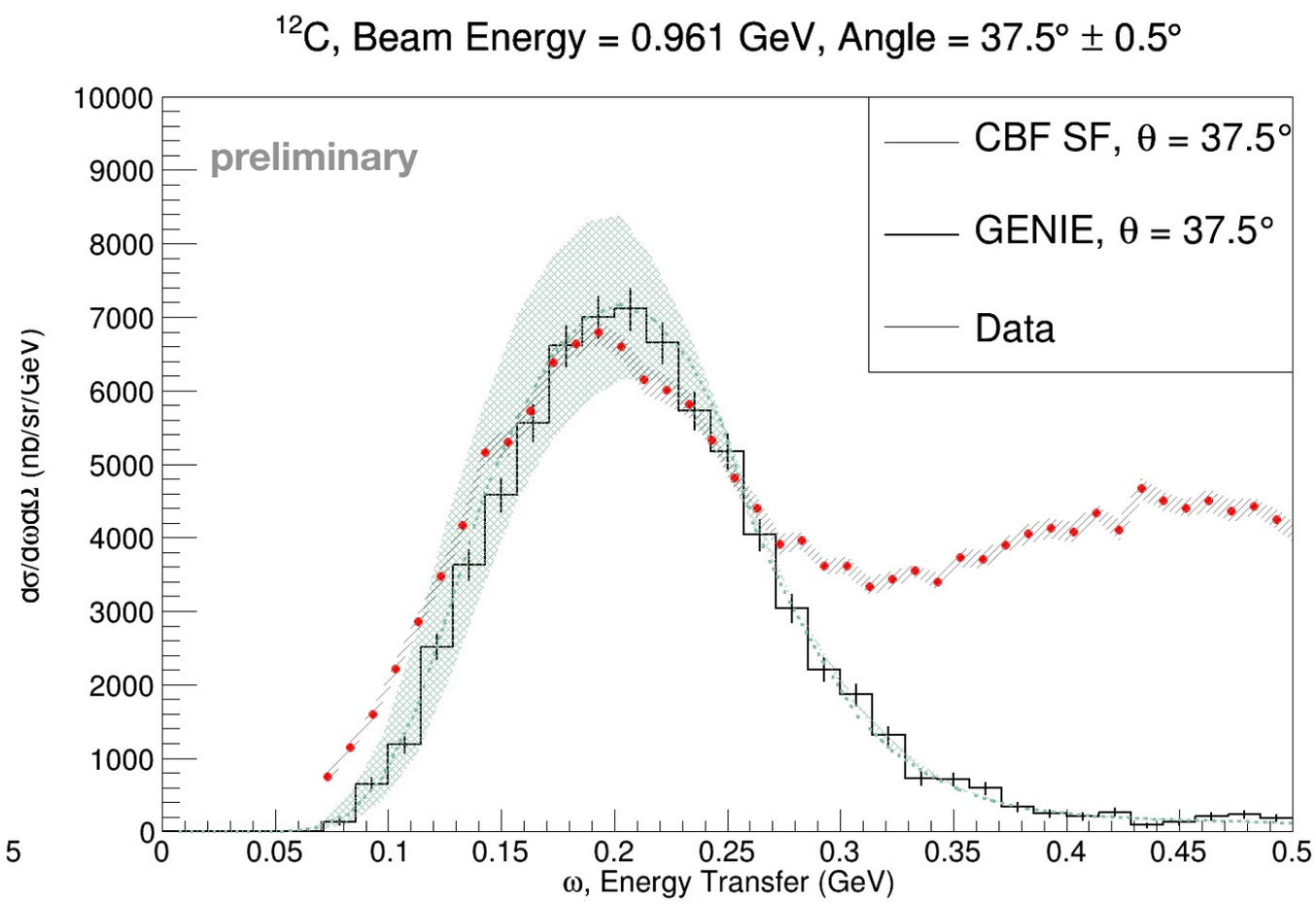
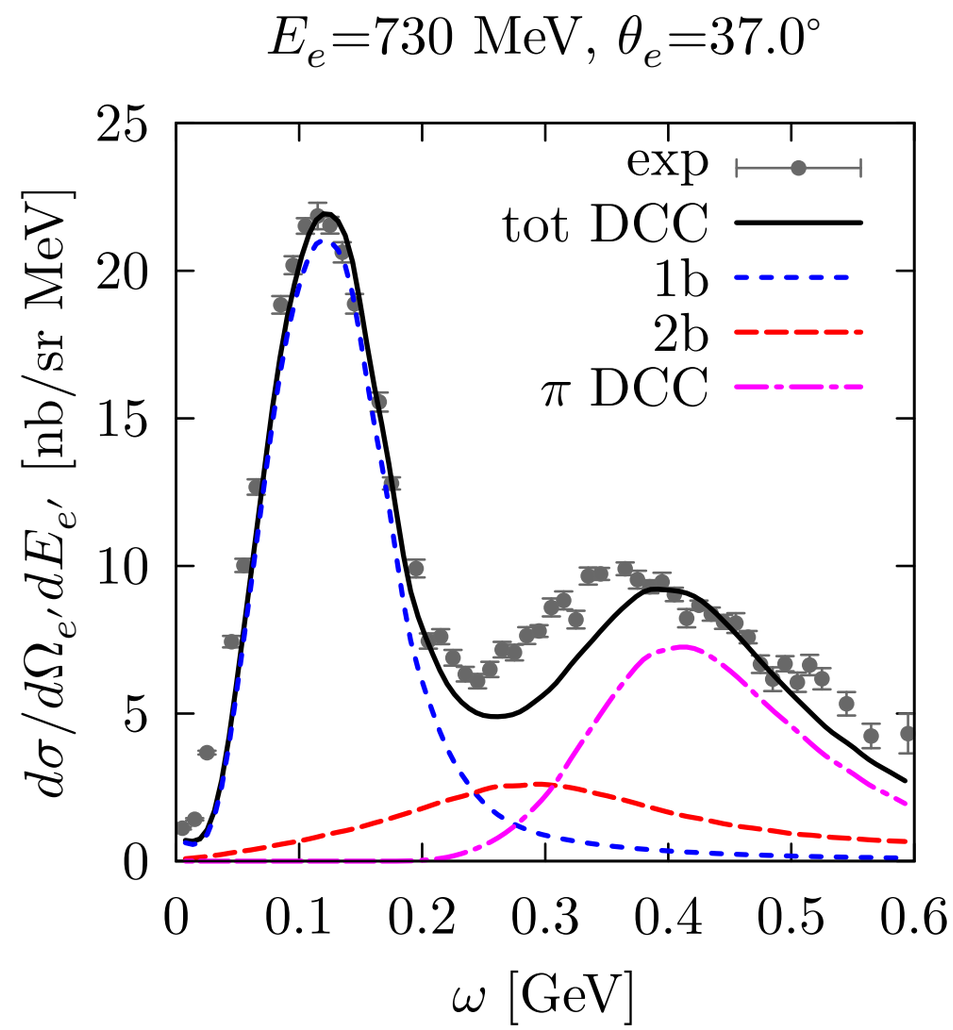
Cross sections e^-: Spectral function approach

- Effort lead by Noah Steinberg: **implementing the spectral function model in the GENIE event generator** developing an interface to easily incorporate the theory subroutines





- Good agreement with electron scattering data when all reaction mechanisms are included

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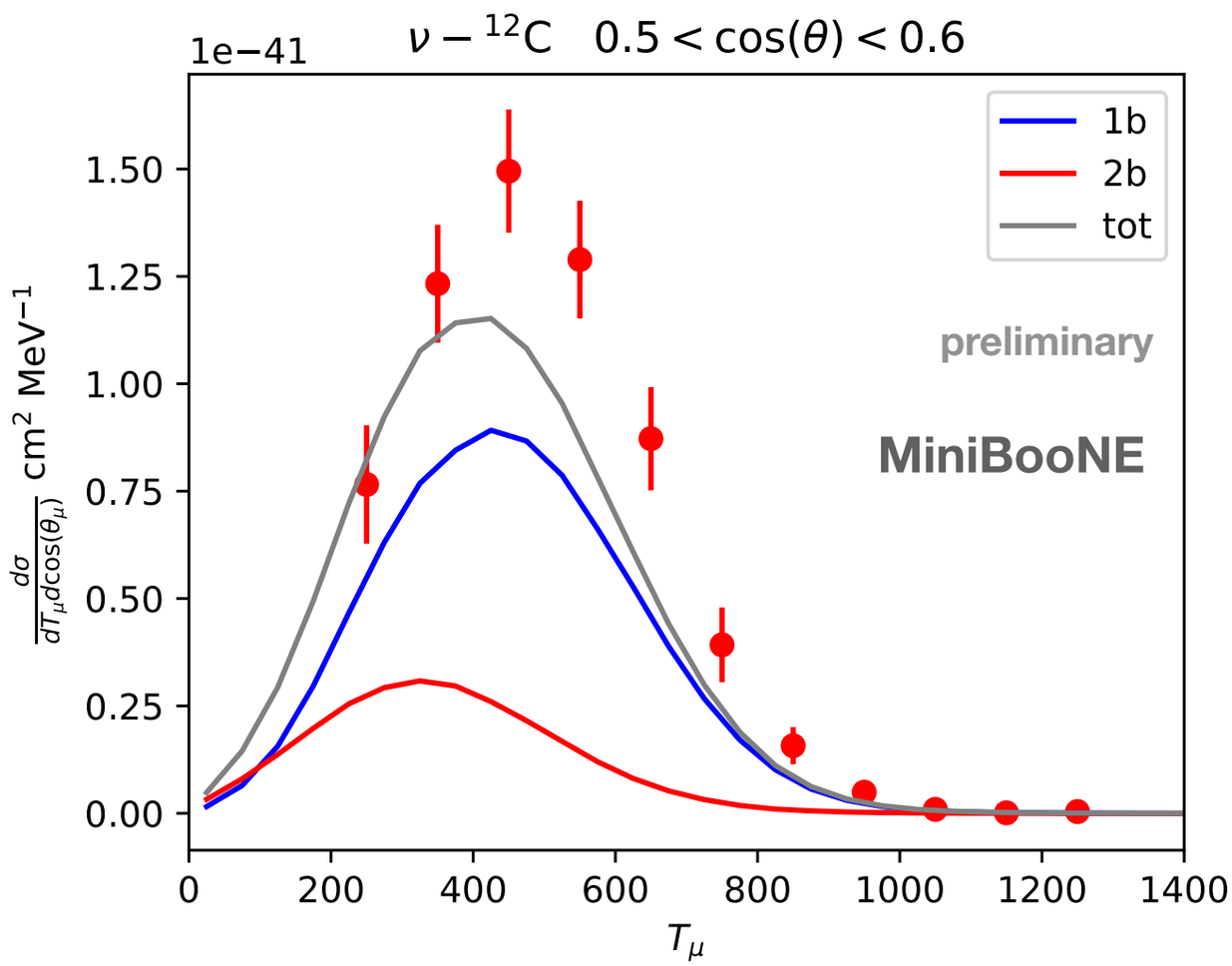
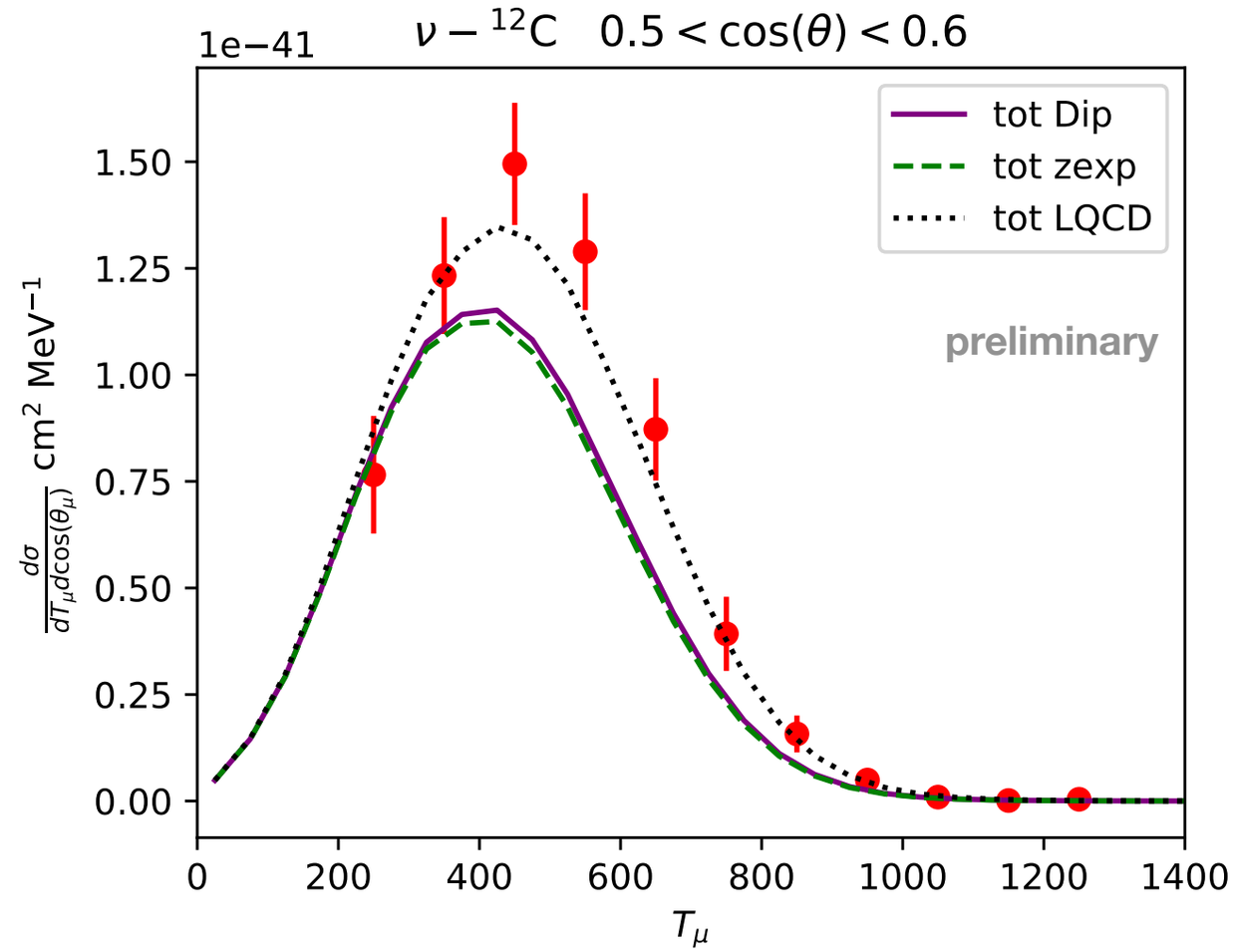
Cross sections ν: Spectral function approach

We computed the flux-folded doubly-differential cross section of ν -^{12}C scattering and compared with MiniBooNE data.





D. Simons, N. Steinberg, A. Lovato, Y. Meurice, NR, M. Wagman, in prep

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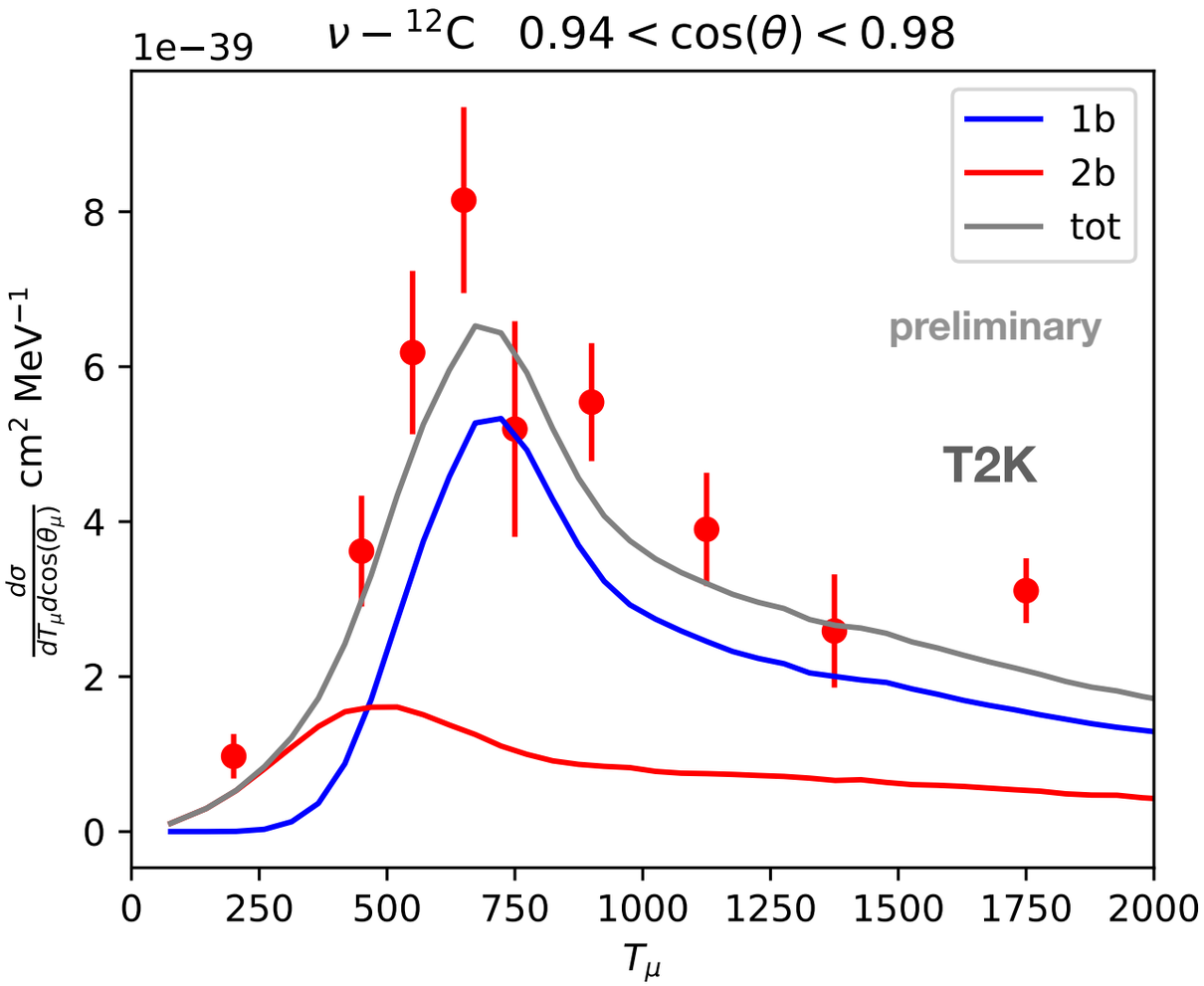
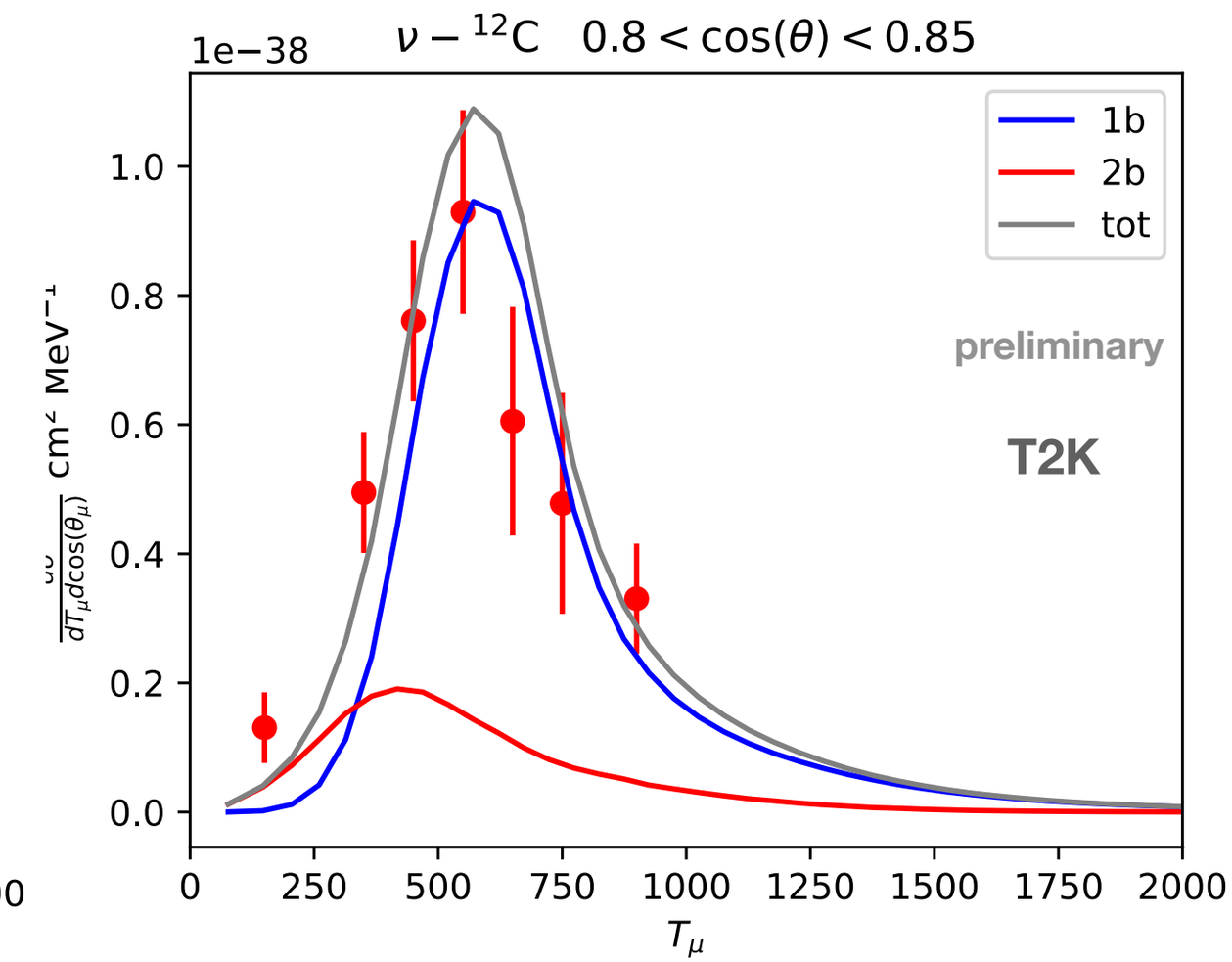
Cross sections ν: Spectral function approach

We computed the flux-folded doubly-differential cross section of ν -^{12}C scattering and compared with T2K data.





D. Simons, N. Steinberg, A. Lovato, Y. Meurice, NR, M. Wagman, in prep

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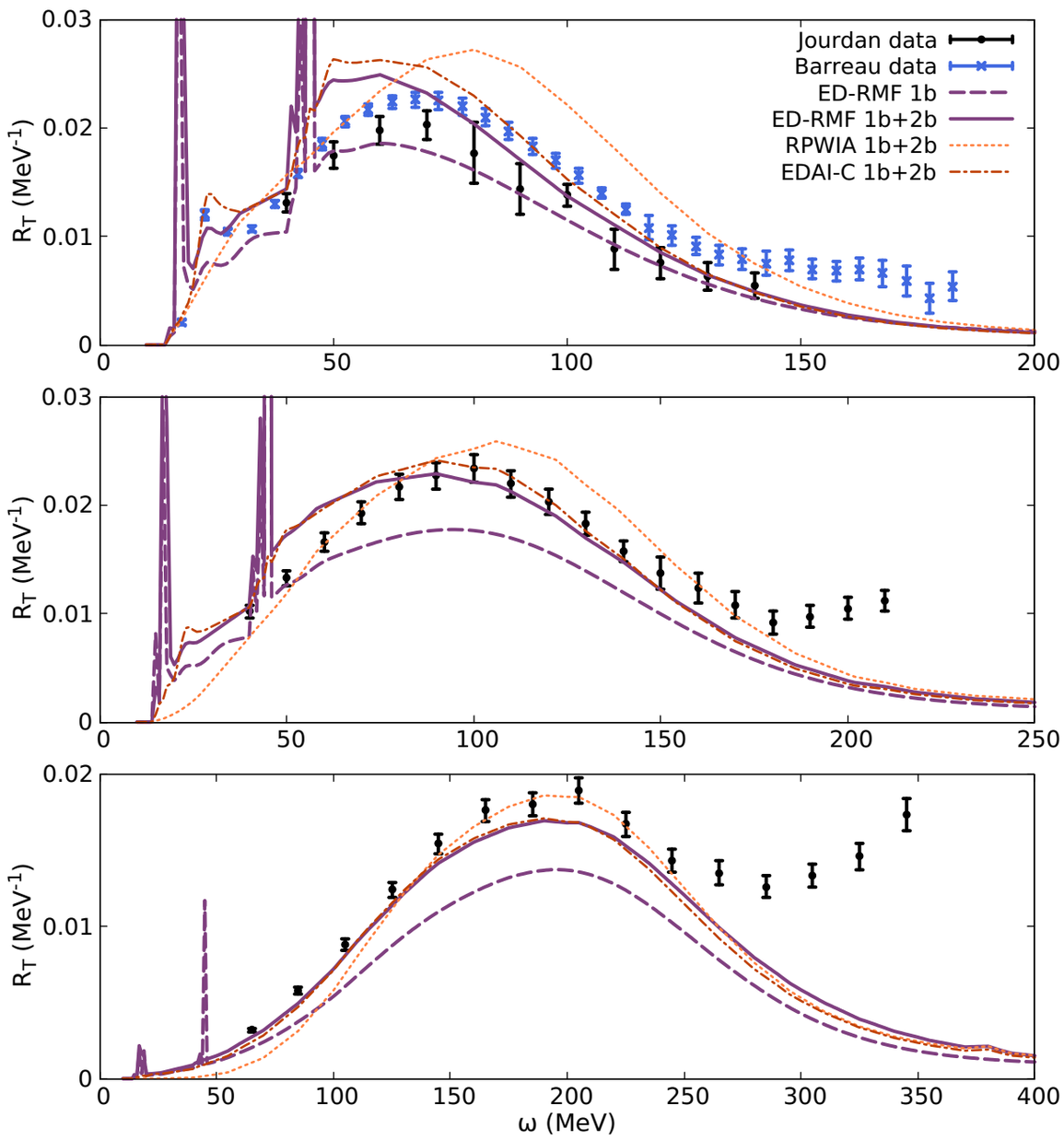
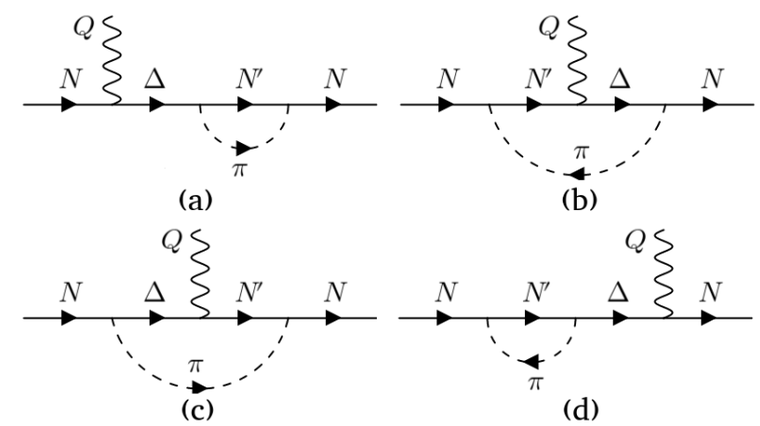
Effects of two-body currents within a RMF approach

T. Franco-Munoz, R. Gonzalez-Jimenez, and J.M. Udias, arXiv: 2203.09996



Hadronic current, with bound wave function obtained within a RMF approach

$$J^{\mu}_{\kappa,m_j,s} \propto \int d\mathbf{p}\overline{\Psi}^{s}(\mathbf{p}+\mathbf{q},\mathbf{p}_N)\mathcal{O}^{\mu}\Psi^{m_j}_{\kappa}(\mathbf{p}).$$



Observed a **significant enhancement** coming from **interference** between one- and two-body currents

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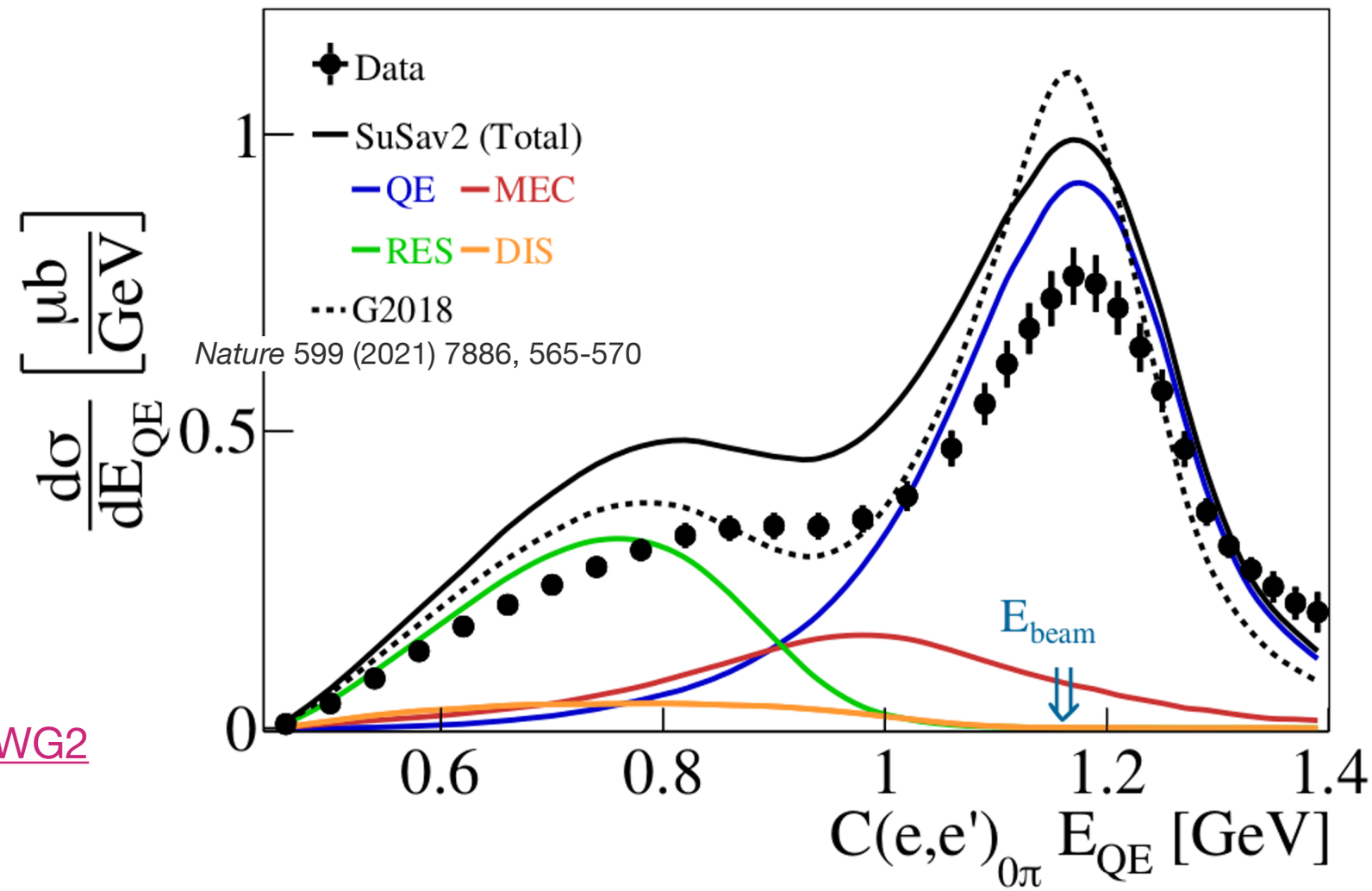
Uncertainty in Neutrino Interactions and Simulations

CLAS and e4v collaboration,
Nature 599 (2021) 7886, 565-570

$$E_{QE} = \frac{2M\epsilon + 2ME_l - m_l^2}{2\left(M - E_l + |k_l|\cos\theta_l\right)}$$

Used semi-exclusive electron scattering data to test the accuracy of interaction models and event generators used in oscillation analyses



See Afro's talk this afternoon WG2

The results indicate the **need** for **substantial improvement** in the **accuracy** of the neutrino interactions' models and simulations

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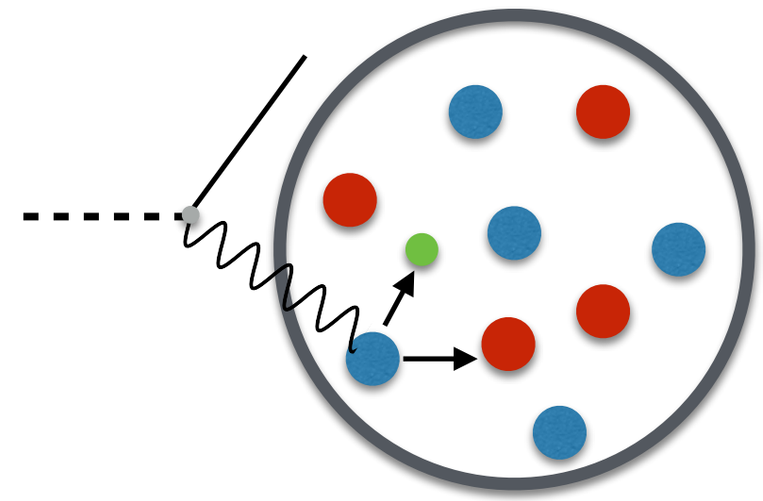
ACHILLES: A CHicago Land Lepton Event Simulator

The propagation of **nucleons** through the **nuclear medium** is crucial in the analysis of electron-nucleus scattering and neutrino oscillation experiments.



- Theory driven, modular event simulator
- Provide automated BSM calculations for neutrino experiments
- Uses realistic QMC nuclear calculations as inputs

J.Isaacson, W Jay, A. Lovato, P Machado, NR:

- *arXiv:2205.06378*
- *PRD 105 (2022) 9, 096006*
- *PRC 103 (2021) 1, 015502*

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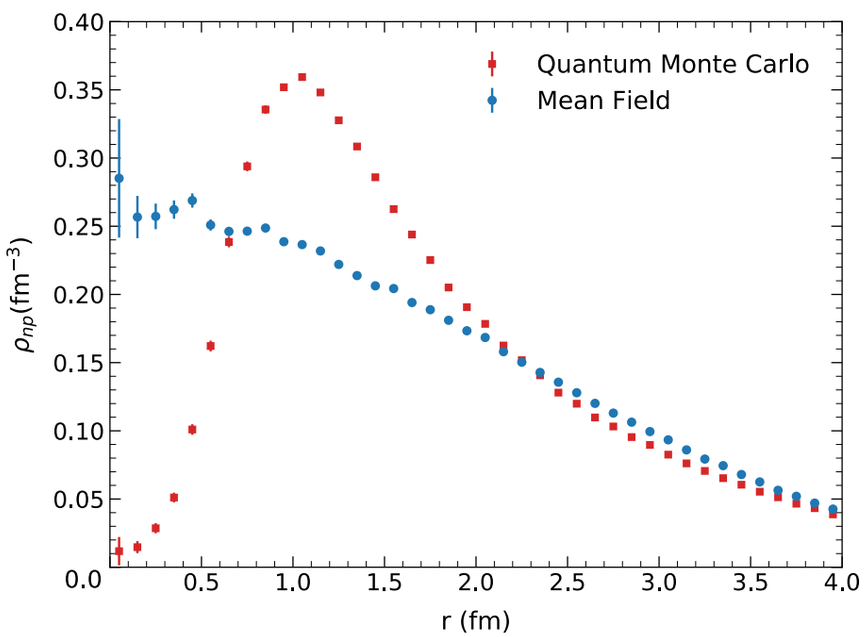
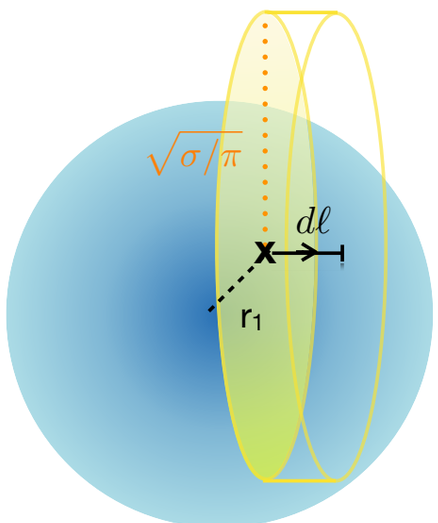
A Quantum Monte Carlo based cascade



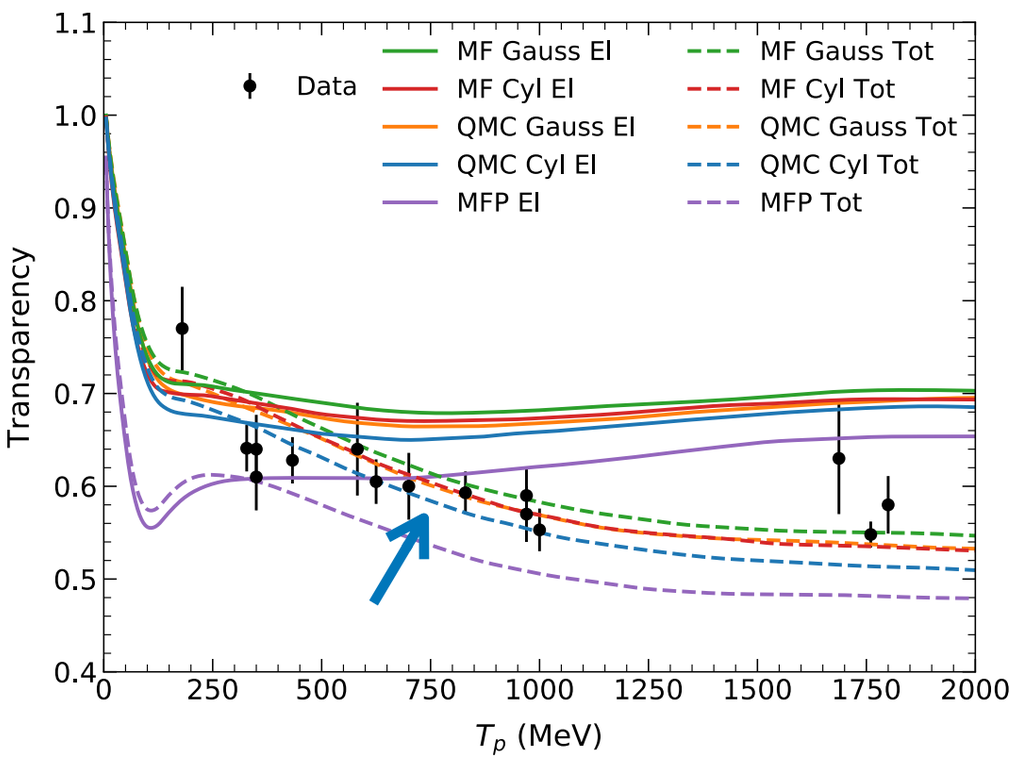
We investigated the role of nuclear effects in intra-nuclear cascade

*J.Isaacson, W Jay, A. Lovato, P Machado, NR, Phys. Rev. C **103**, 015502 (2021)*

The nucleons' positions are sampled from **36000 GFMC configurations**.



Check interaction: **accept-reject** test with a cylinder probability distribution.



- We computed different observables: p-^{12}C cross section, ^{12}C transparency obtained are in fair agreement with data
- The **nuclear transparency** yields the average probability that a struck nucleon leaves the nucleus without interacting with the spectator particles

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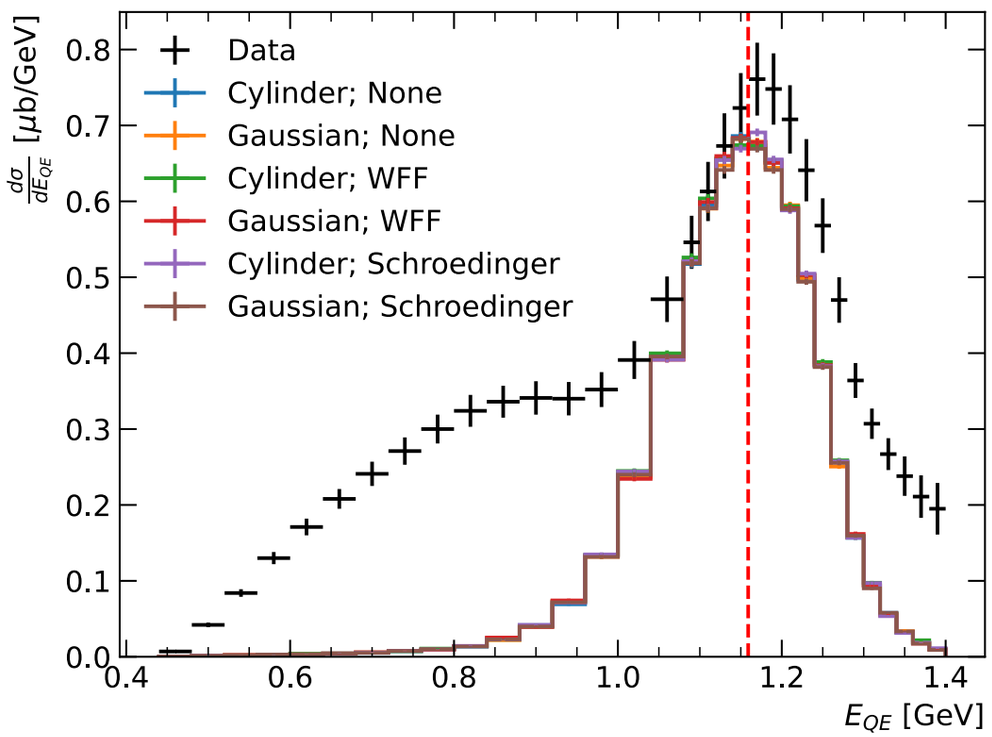
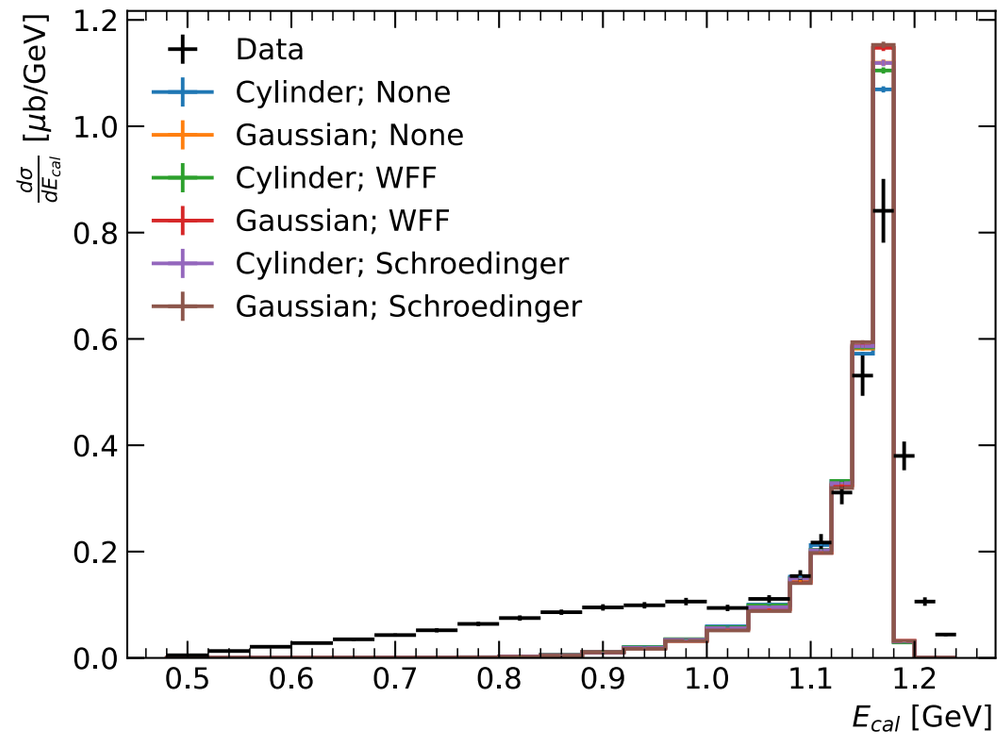
CLAS/e4ν collaboration





$$\mathrm{E}_{QE} = \frac{2\mathrm{M}\epsilon + 2\mathrm{M}\mathrm{E}_l - \mathrm{m}_l^2}{2\left(\mathrm{M} - \mathrm{E}_l + |\mathrm{k}_l|\cos\theta_l\right)}$$

- Mimics energy reconstruction techniques used in Cherenkov detectors

$$E_{\mathrm{cal}} = \sum_i (E_i + \epsilon_i)$$

- Mimics energy reconstruction techniques used in LArTPC detectors: ionization energy

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Summary and Future Prospects

- Estimate the **uncertainty of the theoretical calculation** can be achieved within ab-initio methods: limited to low/intermediate energies

- Include inputs from Lattice QCD to describe the elementary vertex and form factors is going to be crucial for precision physics

- Methods relying on some approximations are needed to go to large energies relevant for neutrino experiments

  See J.M. Franco-Patiño talk Thursday afternoon WG2

- Need for **substantial improvement** in the **accuracy** of the neutrino interactions' models and simulations has been highlighted. Different efforts in this direction: ACHILLES, STA and SF implementation in GENIE

  See S. Dolan talk this afternoon WG1-WG2

  See A. Nikolakopulos, S. Gardiner, talks Thursday afternoon WG2

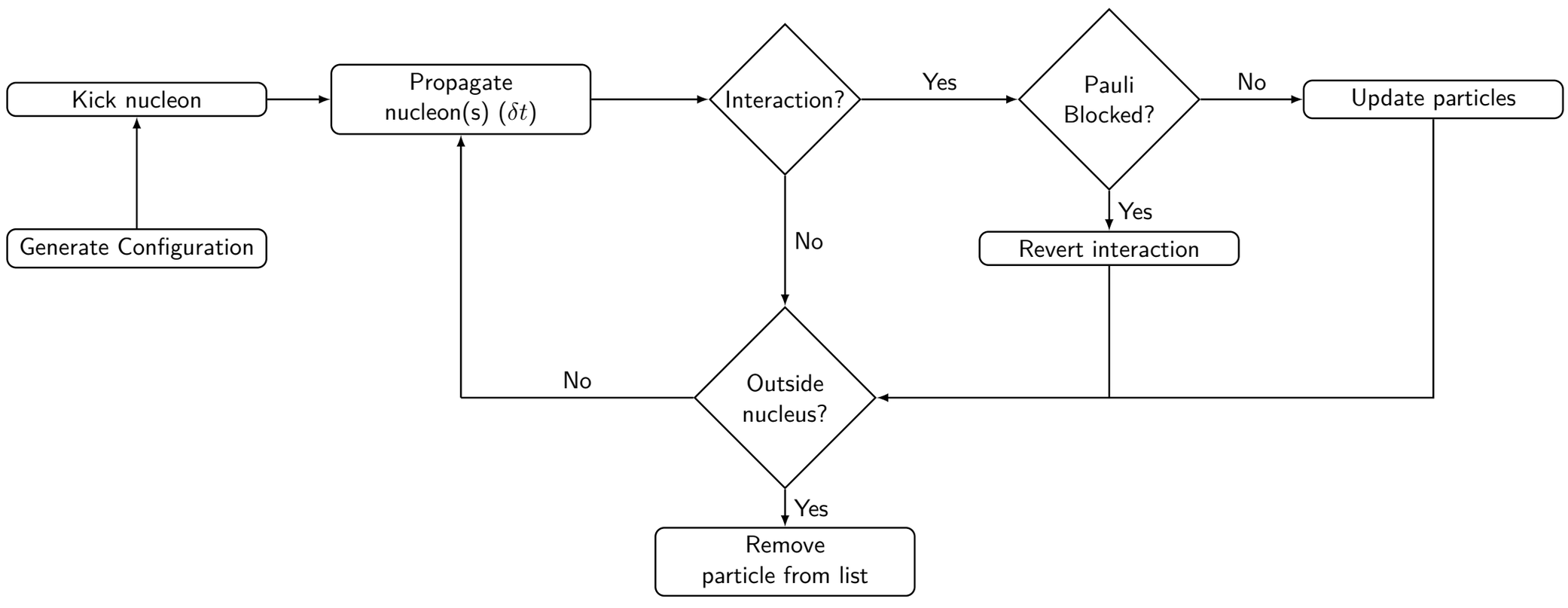
Thank you for your attention!

Algorithm Overview

$$|\mathcal{M}(\{k\} \to \{p\})|^2 \simeq \sum\!\!\!\!\!\!\int_{p'} |\mathcal{V}(\{k\} \to \{p'\})|^2 \times |\mathcal{P}(\{p'\} \to \{p\})|^2$$

- Primary Interaction
- Evolution out of the nucleus (intra-nuclear) cascade

Approximate as incoherent product of primary interaction and cascade



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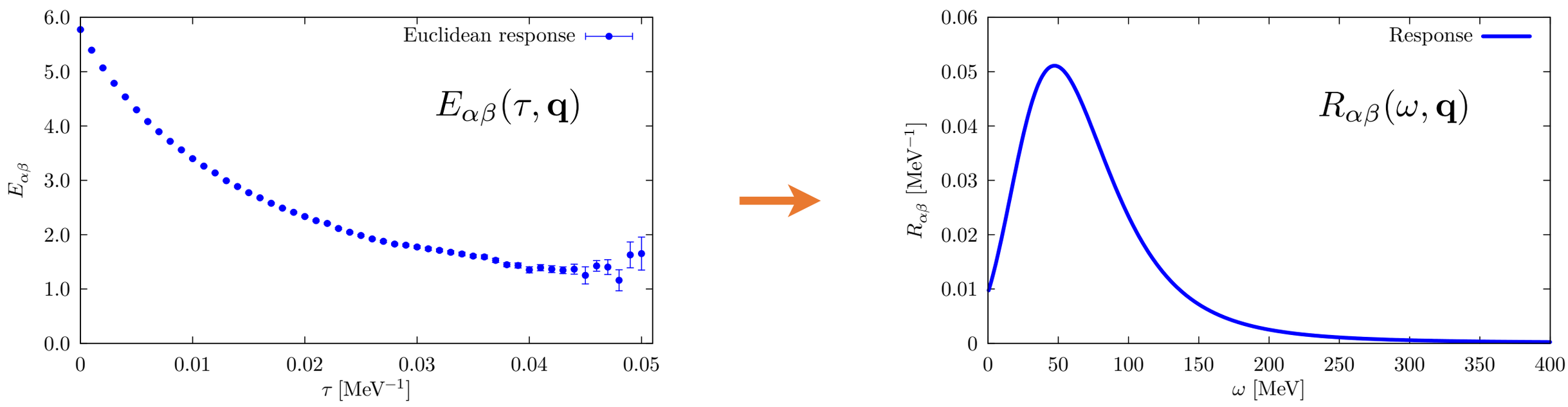
Green's Function Monte Carlo

Nuclear response function involves evaluating a number of transition amplitudes.
Valuable information can be obtained from the **integral transform of the response function**

$$E_{\alpha\beta}(\sigma, \mathbf{q}) = \int d\omega K(\sigma, \omega) R_{\alpha\beta}(\omega, \mathbf{q}) = \langle \psi_0 | J_\alpha^\dagger(\mathbf{q}) K(\sigma, H - E_0) J_\beta(\mathbf{q}) | \psi_0 \rangle$$

Inverting the integral transform is a complicated problem

A. Lovato et al, PRL117 (2016), 082501, PRC97 (2018), 022502



Same problem applies to different realm physics for example lattice QCD

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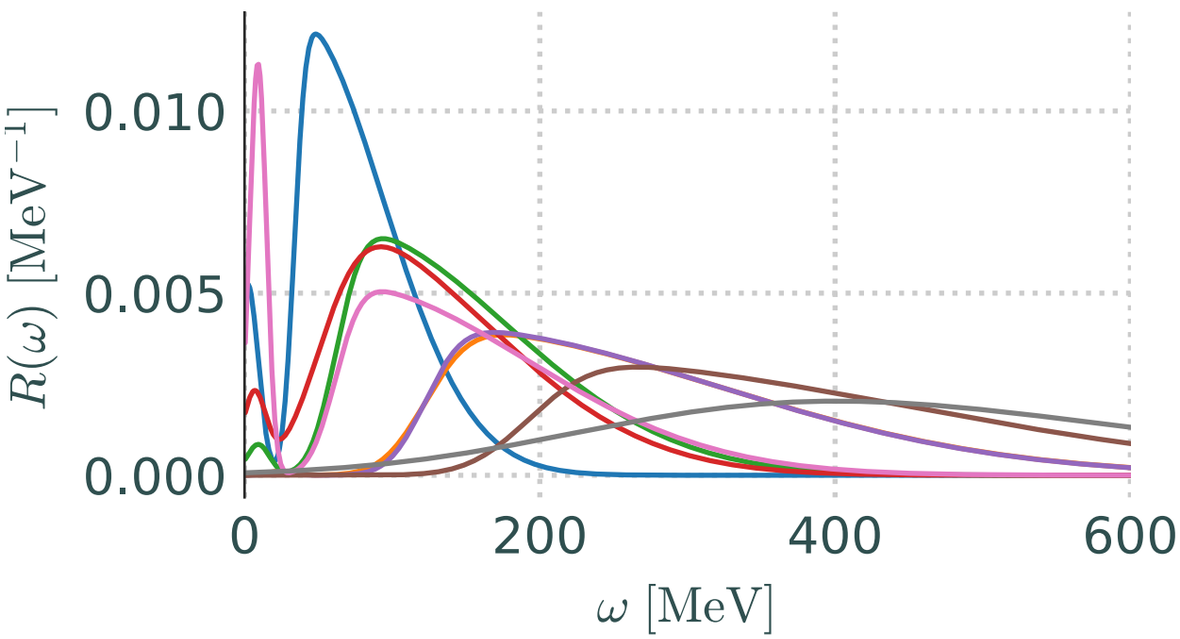
Machine learning-based inversion of R(q,ω)

Training data examples of response functions



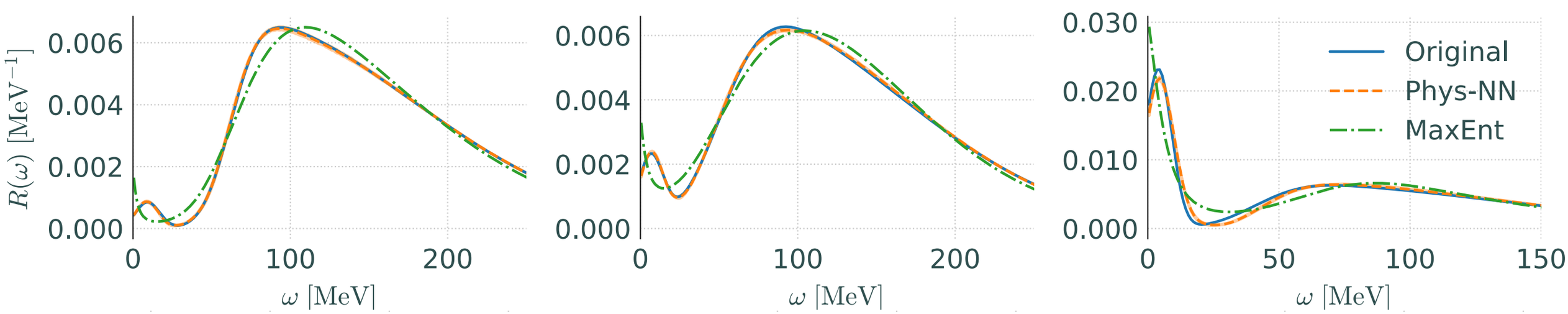
The parameters are found by the supervised learning approach solving

$$\min_{\theta} \frac{1}{|\mathbb{T}|} \sum_{k \in \mathbb{T}} \ell \left(E_k(\mathcal{T}), R_k(\Omega), \hat{R}_k(\Omega; \theta) \right)$$

Using a mini-batch gradient descent to minimize a loss function that is the sum of the response and Euclidean cost

$$\ell(E_k, R_k, \hat{R}_k) = \gamma_R S_R(R_k, \hat{R}_k) + \gamma_E \chi_E^2(E_k, \hat{R}_k)$$

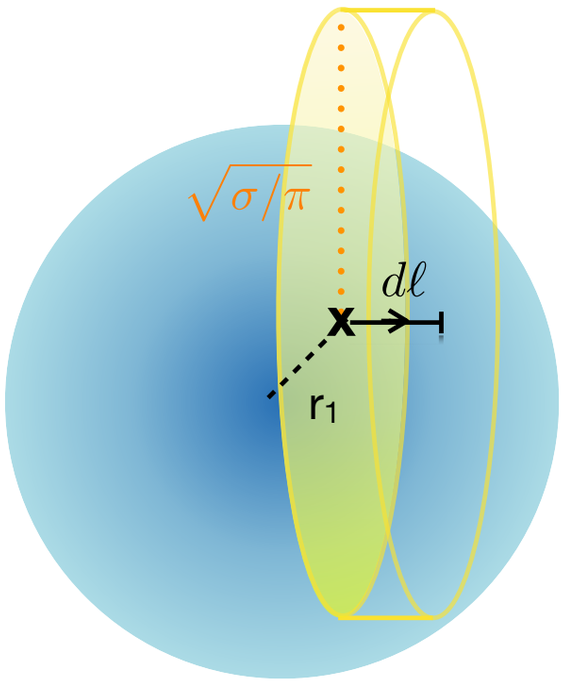
Comparison between the Phys-NN and MaxEnt reconstructions for the two-peak dataset



Best Median Worst

Probability of interaction

To check if an interaction between nucleons occurs an **accept-reject** test is performed on the **closest nucleon** according to a probability distribution.



We use a **cylinder probability distribution**, this mimics a more classical billiard ball like system where each billiard ball has a radius
In addition we consider a **gaussian probability distribution**

For benchmark purposes, we also implemented the **mean free path approach**, routinely used in event generators

$$P = \sigma \bar{\rho} d\ell$$

where a constant density is assumed $\rho(r_1) \sim \rho(r_1 + d\ell) \sim \bar{\rho}$

we sample a number $0 \leq x \leq 1$

$$\begin{cases} x < P & \checkmark \\ x > P & \times \end{cases}$$

✓ the interaction occurred, check Pauli blocking

✗ the interaction DID NOT occur

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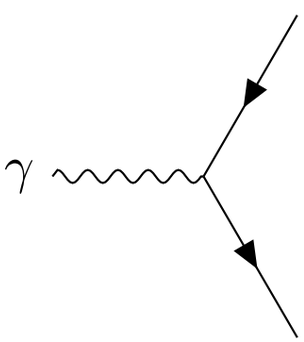
Universal FeynRules Output

- Python output files
- Contains model-independent files and model-dependent files
- Contains all information to calculate any tree level matrix element
- Has parameter file to adjust model parameters to scan allowed regions

Example QED ($e^+e^-\gamma$ Vertex):

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(iD^\mu\gamma_\mu - m\right)\psi$$

$$V_{e^+e^-\gamma} = ie\gamma^\mu =$$



[arXiv:1108.2040]