04th August 2022

NuFact 2022

Probing Light Mediators in the Radiative Emission of Neutrino Pair

In Collaboration with: Prof. Shao-Feng Ge (pronouns: He/Him/His)

Pedro S. Pasquini



Shanghai Jiao Tong University

Tsung-Dao Lee Institute

Neutrino Pair Emission

Proposed in "Neutrino Pair Emission from Excited Atoms," M. Yoshimura, Phys. Rev. D **75**, 113007 (2007)

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Excited atom \longrightarrow Ground State atom $+\gamma + \nu \bar{\nu}$

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Low E_{ν} but hard experimentally

Advantages

Disadvantages

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- Atomic transition: $\mathcal{O}(1) \, \text{eV}$
- Can measure the emitted photon
- Can be stimulated and coherently enhanced
- Tests ν physics in a different energy regime

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- 3γ background still a problem
- Exp. challenge to coherent enhancement
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Benefits are great and can provide interesting results! m_{ν} ordering and scale, non-unitary, BSM interactions and maybe neutrino mixing and nature

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Neutrino LASER?

Radiative Emission of Neutrino Pair (RENP)

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Usual Laser beam:

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— \bigcirc Excited (meta-stable) State (|e>)

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RENP:



—— Ground State (|g>)

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RENP:



E1, M1 forbid



—— Ground State (|g>)

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RENP:

Virtual State (|v>)



E1, M1 forbid



— Ground State (|g>)

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Neutrino LASER?



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Neutrino LASER?



Quantum effects enhance a lot

Stimulated Emission

Coherent Enhancement

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$$P \propto \left|\sum_{a} e^{i\mathbf{x}_{a}\cdot\mathbf{p}}\right|^{2} \approx \left|\int d^{3}\mathbf{x} e^{i\mathbf{x}\cdot\mathbf{p}}\right|^{2} \sim N_{a}^{2}$$

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Overall, there is a $N_{\gamma}N_a^2$ macroscopic enhancement! Current technology (probably) allows $\mathcal{O}(10)$ events/days of exposure

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How to extract ν physics?

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Emitted photon contains information of the neutrino pair

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Because of Stimulation: $|\mathbf{p}_{\gamma}^{\text{emitted}}| = \omega$ (trigger LASER frequency)

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Emitted photon contains information of the neutrino pair

Measure emitted photon spectral function *I*!

Because of Stimulation: $|\mathbf{p}_{\gamma}^{\text{emitted}}| = \omega$ (trigger LASER frequency)

The # of photons from de transition depends on ω .

Two important information: $I \equiv I(\omega)$ and ω_{ij}^{\max}

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From energy conservation: $\Delta E = \omega + E_{\overline{\nu}\nu}$

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Massive neutrinos:
$$\omega_{ij}^{\max} = \frac{\Delta E}{2} - \frac{(m_i + m_j)^2}{2\Delta E}$$

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Massive neutrinos:
$$\omega_{ij}^{ ext{max}} = rac{\Delta E}{2} - rac{(m_i + m_j)^2}{2\Delta E}$$

There is a total of 6 thresholds related to all combinations of $m_i, m_j, i, j = 1, 2, 3$.

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Threshold contain mass information



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- Threshold gives mass scale

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- Threshold gives mass scale
- Size gives mass hierarchy

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Under reasonable assumptions:

 3σ for $m_{
m lightest}$ and mass ordering

N. Song, R. B. Garcia et.al. Phys. Rev. D **93**, no.1, 013020 (2016)

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We can extract BSM physics

We can go further: BSM physics

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$$I(\omega) = \sum_{ij} \frac{\Delta_{ij}(\omega)}{(E_{vg} - \omega)^2} \Theta(\omega - \omega_{ij}^{\max}) \left[|a_{ij}|^2 I_{ij}^{(D)} - \delta_M \operatorname{Re}[a_{ij}^2] m_i m_j \right]$$
$$a_{ij} = U_{ei} U_{ej}^* + \frac{1}{2} (UU^{\dagger})_{ij}$$

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Require a large number of events...

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- Information on mixing angles
- Non-unitarity of U
- Even majorana phases

See: N. Song etal Phys. Rev. D **93**, no.1, 013020 (2016) and G. Y. Huang et al. Int. J. Mod. Phys. A **35**, no.01, 2050004 (2020)

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Caveat: Needs larger number of events ($\sim 10^3$ or larger). But current technology $O(10) \Rightarrow$ needs technological improvement

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$$\mathcal{L}_V = g^e \bar{e} \gamma^\mu \gamma_5 e Z'_\mu + \bar{\nu}_i \gamma^\mu (g^\nu_{L,ij} P_L + g^\nu_{R,ij} P_R) \nu_j Z'_\mu.$$

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$$\mathcal{L}_{V} = g^{e} \bar{e} \gamma^{\mu} \gamma_{5} e Z'_{\mu} + \bar{\nu}_{i} \gamma^{\mu} (g^{\nu}_{L,ij} P_{L} + g^{\nu}_{R,ij} P_{R}) \nu_{j} Z'_{\mu}.$$
$$\mathcal{L}_{S} = i y^{e}_{P} \bar{e} \gamma_{5} e \phi + \bar{\nu}_{i} (y^{\nu}_{S,ij} + i \gamma_{5} y^{\nu}_{P,ij}) \nu_{j} \phi + h.c.,$$

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- The mediator propagator:

$$rac{1}{q^2-M^2} \qquad \sim -rac{1}{M^2} \mbox{ for } q^2 \ll M^2$$

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- Also, light mediators, the effect (and bounds) will be enhanced, specially for m_ϕ^2 around the eV scale: $\frac{m_W^2}{q^2-m_{\phi,Z'}^2}\sim 10^{21}$

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Specially good for light mediators



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See S.-F. Ge & Pedro Pasquini Eur. Phys. J.C 82 (2022) 3, 208

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Still too small, but BSM physics can make $\mu_{
u}, \epsilon_{
u} \sim 10^{-11} \mu_B$

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Photon as mediator for RENP



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Notice



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Notice

(1) Shape is different for μ_{ν} and ϵ_{ν}



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(2) ij contribution starts when $\omega < \omega_{ij}^{\max}$



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It can also tell which element $(\mu_{\nu})_{ij}$ (or $(\epsilon_{\nu})_{ij}$) is non-zero!

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Compare with current constraints:

Solar/Accelerator:

$$(\nu + e \rightarrow \nu + e)$$

 $(\mu_{\alpha\beta}^{\text{eff}})^2 \equiv \sum_j |\sum_k U_{\alpha k}^* [(\mu_{\nu})_{jk} - i(\epsilon_{\nu})_{jk}]|^2$

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Compare with current constraints:

 $\begin{array}{ll} \text{Solar/Accelerator:} & \text{Stellar Cooling:} \\ (\nu + e \rightarrow \nu + e) & (\gamma^* \rightarrow \nu\nu) \\ (\mu_{\alpha\beta}^{\text{eff}})^2 \equiv \sum_j |\sum_k U_{\alpha k}^* \left[(\mu_{\nu})_{jk} - i(\epsilon_{\nu})_{jk} \right] |^2 & (\mu_{\nu}^{\odot})^2 \equiv \sum_{ij} |(\mu_{\nu})_{ij}|^2 + |(\epsilon_{\nu})_{ij}|^2 \end{array}$

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Has blind spots D. A. Sierra et al. Phys.Rev.D 105 (2022) 3, 035027

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Stellar Cooling: $(\gamma^* \to \nu\nu)$ $(\mu_{\nu}^{\odot})^2 \equiv \sum_{ij} |(\mu_{\nu})_{ij}|^2 + |(\epsilon_{\nu})_{ij}|^2$

Large system. uncertanties R. J. Stancliffe et al. A&A 586, A119 (2016)

Good Sensitivity!



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- Radiative emission of neutrino pair (RENP) is a novel and interesting idea.
- The RENP can obtain the neutrino mass scale and look for new physics.
- The low energy q^2 of RENP makes it specially powerful probe of light mediator.
- μ_{ν} and ϵ_{ν} can be thoroughly explored

Founding Support:

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Backup Slides

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$$\mathcal{I}_{Z'} = \sum_{ij} \frac{\Delta_{ij}(\omega)}{(E_{vg} - \omega)^2} \Theta(\omega - \omega_{ij}^{\max}) \left[\left(|a_{ij}^L|^2 + |a_{ij}^R|^2 - 2\delta_M \operatorname{Re}[a_{ij}^L a_{ij}^R] \right) I_{ij}^{(D)} + \left(\delta_M \operatorname{Re}\left[(a_{ij}^L)^2 + (a_{ij}^R)^2 \right] - 2\operatorname{Re}\left[a_{ij}^{L*} a_{ij}^R \right] \right) \right]$$

$$\mathcal{I}_{\phi} = \sum_{ij} \frac{\Delta_{ij}(\omega)}{(E_{vg} - \omega)^2} \Theta(\omega - \omega_{ij}^{\max}) \left[I_{ij}^{SM}(\omega) + \delta I_{ij}(\omega) \right]$$

$$\begin{split} \delta I_{ij} &= \frac{|y^e|^2 \omega^2}{m_e^2 G_{\rm F}^2} \frac{\left[|y^e_{S,ij}|^2 + (1 - \delta_M)|y^e_{P,ij}|^2\right] E_{eg}(E_{eg} - 2\omega) - |y^e_{S,ij}|^2 (m_i + m_j)^2 - (1 - \delta_M)|y^e_{P,ij}|^2 (m_i - m_j)^2}{24[E_{eg}(E_{eg} - 2\omega) - m_{\phi}^2]^2} \end{split} \tag{29} \\ &+ \frac{y^e \omega^2}{6\sqrt{2}G_F} \left\{ \frac{\operatorname{Re}\left[a_{ij}y^e_{S,ij}\right] (m_i - m_j) \left[1 - \frac{(m_i + m_j)^2}{E_{eg}(E_{eg} - 2\omega)}\right]}{m_e[E_{eg}(E_{eg} - 2\omega) - m_{\phi}^2]} - (1 - \delta_M) \frac{\operatorname{Im}\left[a_{ij}y^e_{P,ij}\right] (m_i + m_j) \left[1 - \frac{(m_i - m_j)^2}{E_{eg}(E_{eg} - 2\omega)}\right]}{m_e[E_{eg}(E_{eg} - 2\omega) - m_{\phi}^2]} \right\}. \end{split}$$

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Coupling	$\mathcal{L}_{ ext{new}}$	Non-Relativistic Transition	Туре
scalar	$y^e_S \bar{e} e$	$\langle f i angle$	E1
pseudo-scalar	$iy_P^e \bar{e} \gamma_5 e$	$rac{{f q}}{2m_e}\cdot \langle f {m \sigma} i angle$	M1
vector	$g_V^e \bar{e} \gamma^\mu e$	$(\langle f i angle, rac{\mathbf{q}}{2m_e}\cdot \langle f oldsymbol{\sigma}oldsymbol{\sigma} i angle)$	E1
axial-vector	$g^e_A \bar{e} \gamma^\mu \gamma_\mu e$	$(rac{\mathbf{q}}{2m_e}\cdot\langle f oldsymbol{\sigma} i angle,\langle f oldsymbol{\sigma} i angle)$	M1

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