Benchmarking intranuclear cascade models for neutrino scattering with relativistic optical potentials

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In collaboration with
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Benchmarking intranuclear cascade models for neutrino scattering with relativistic optical potentials

A. Nikolakopoulos,[1,2,*] R. González-Jiménez,[3] N. Jachowicz,[1] K. Niewczas,[1,4] F. Sánchez,[5] and J. M. Udías[3]

A direct comparison of RDWIA with relativistic optical potential (ROP) with intranuclear cascade (INC) model of the NEUT generator

In short:

The ROP removes nucleons that suffer inelastic FSI

The INC explicitly models inelastic FSI

Kinematic cuts on the INC results can be used to define an event sample that is equivalent to the ROP results

INC and ROP are found compatible for large energy only!

Broader context:

- Most precision experiments that measure interactions with nuclei apply stringent kinematic cuts to isolate observables of interest, minimize FSI, etc… e.g. exclusive (e,e'p), elastic p-A

- Theory to calculate/interpret these observables successfully are often specific to the problem/experimental setup at hand

- In accelerator-based neutrino experiments an exclusive measurement does not exist. Measurements are semi-inclusive! This means (in principle) all coupled final-states have to be described consistently → Currently done with INC

Some open questions:

- What is the minimum needed to describe a specific CC1p + X signal ? (depends on flux, kinematics, cuts)
- What is the 'correct' input for a cascade model ?

See e.g. [R. Gonzalez-Jimenez et al. PRC 105, 025502]
[J.W. Van Orden et al. PRC 100, 044620]
[J.M. Franco Patino Arxiv:2207.02086 (2022)] (Talk NuFACT)
[Isaacson et al. Arxiv:2205.06378 (2022)]
[B. Bourgouille et al. JHEP4 (2021) 004]

Intra-nuclear cascade model (INC) in NEUT

1.) Nucleon propagates in straight lines

2.) Check for interaction based on density and nucleon-nucleon CS

3.) Pauli-blocking: Reaction products must be above p_{Fermi}

4.) Track the created particles on the way and propagate them

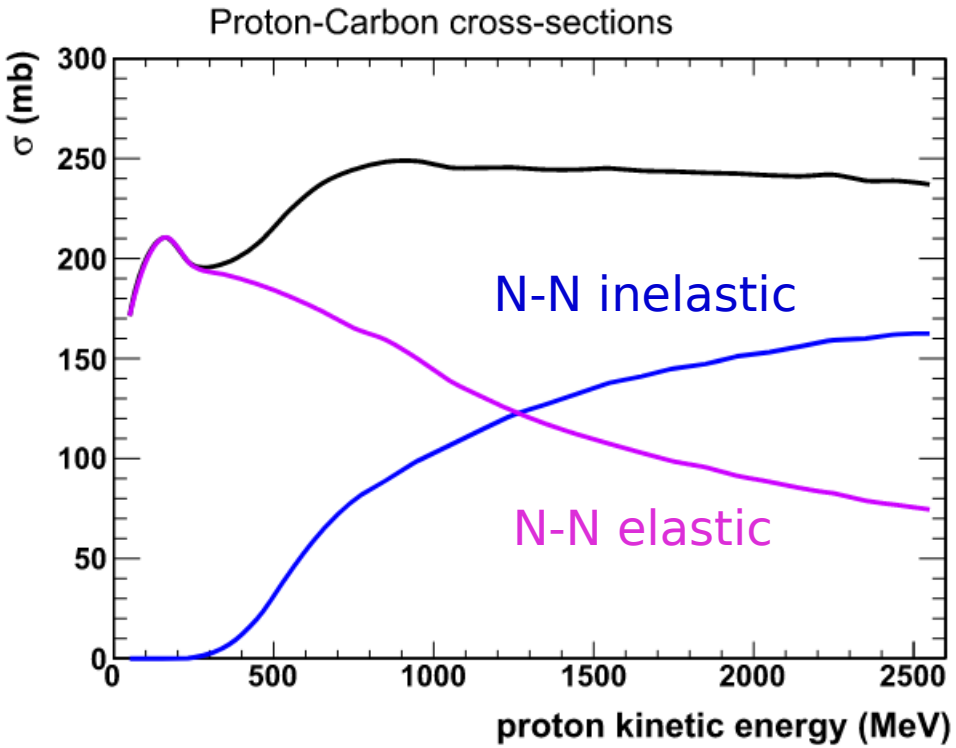
Eur. Phys. J. Spec. Top. (2021) 230:4469–4481

Why ?

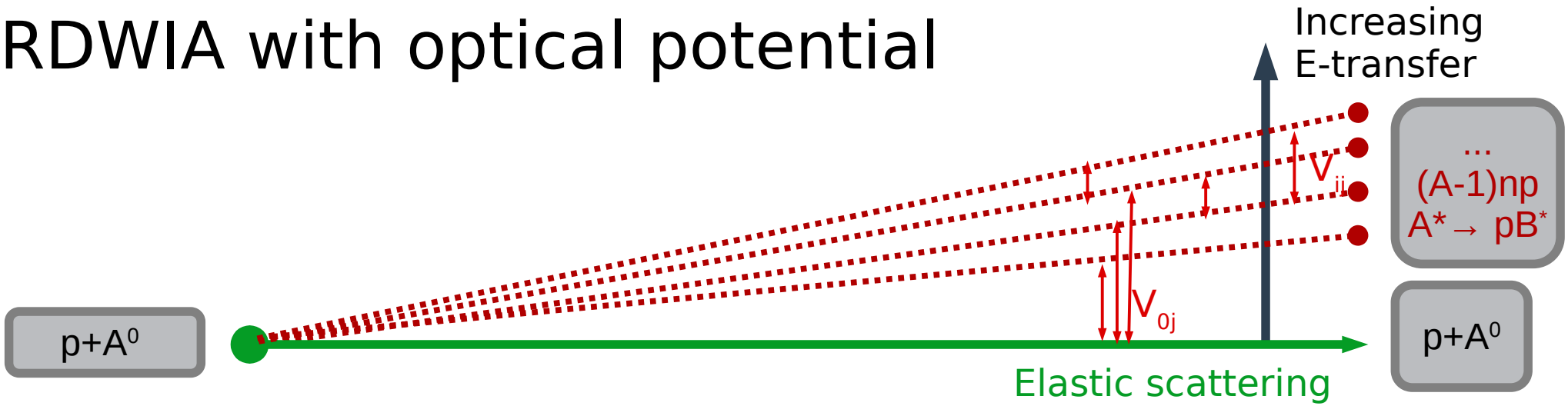
Treat the involved coupled-channels problem posed by FSI

Tradeoffs:

Factorized and classical approach
Expected to break down at low Energy



RDWIA with optical potential



Coupled channels problem, separate out elastic channel

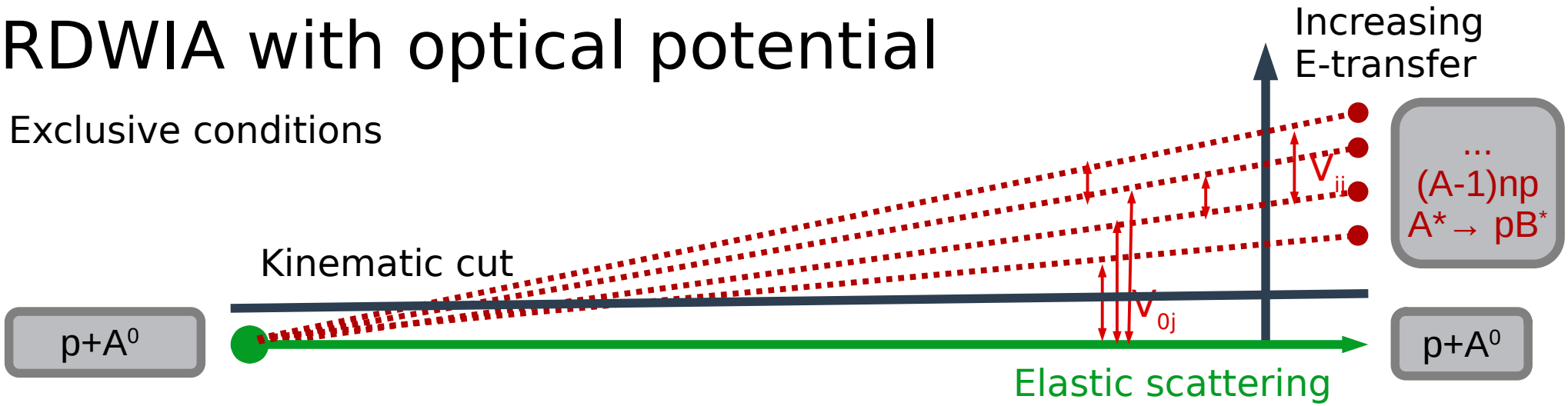
$$(H_{00} - E)|\phi_0\rangle = -V_{0j}|\phi_j\rangle \ (j > 0)$$

$$(H_{ij} - E)|\phi_j\rangle = -V_{j0}|\phi_0\rangle \ (i, j > 0)$$

$$H_{ij} = H^{free}\delta_{ij} + V_{ij}^{nA} + \epsilon_i \delta ij$$

RDWIA with optical potential

Exclusive conditions



Coupled channels problem → one-body problem with optical potential

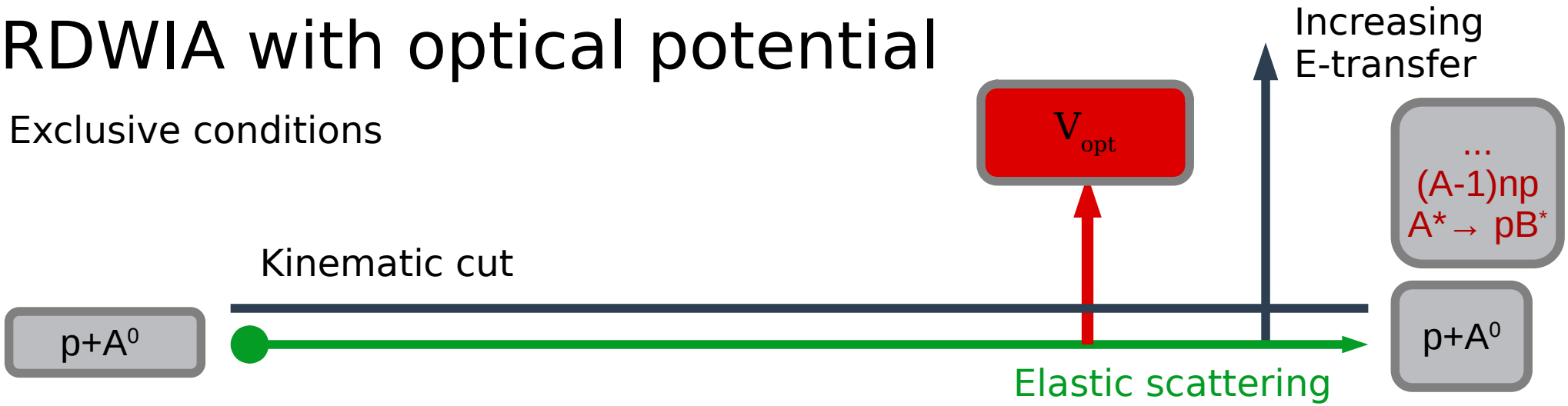
$$(H_{00} - E)|\phi_0\rangle = -V_{0j}|\phi_j\rangle \ (j > 0)$$

$$(H_{ij} - E)|\phi_j\rangle = -V_{j0}|\phi_0\rangle \ (i, j > 0)$$

$$\left[H^{free} + V_{00}^{nA} + V_{0j}\frac{1}{E - H_{ij} + i\eta}V_{j0} - E\right]|\phi_0\rangle$$

RDWIA with optical potential

Exclusive conditions



Coupled channels problem → one-body problem with optical potential

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$$(H_{ij} - E)|\phi_j\rangle = -V_{j0}|\phi_0\rangle \ (i, j > 0)$$

$$\left[H^{free} + V_{00}^{nA} + V_{0j}\frac{1}{E - H_{ij} + i\eta}V_{j0} - E\right]|\phi_0\rangle$$

$$\approx \left[H^{free} + \mathcal{V}^{opt} - E\right]|\phi_0\rangle$$

Empirical relativistic optical potential (ROP)

PHYSICAL REVIEW C VOLUME 47, NUMBER 1 JANUARY 1993

Global Dirac phenomenology for proton-nucleus elastic scattering

E. D. Cooper, S. Hama, and B. C. Clark
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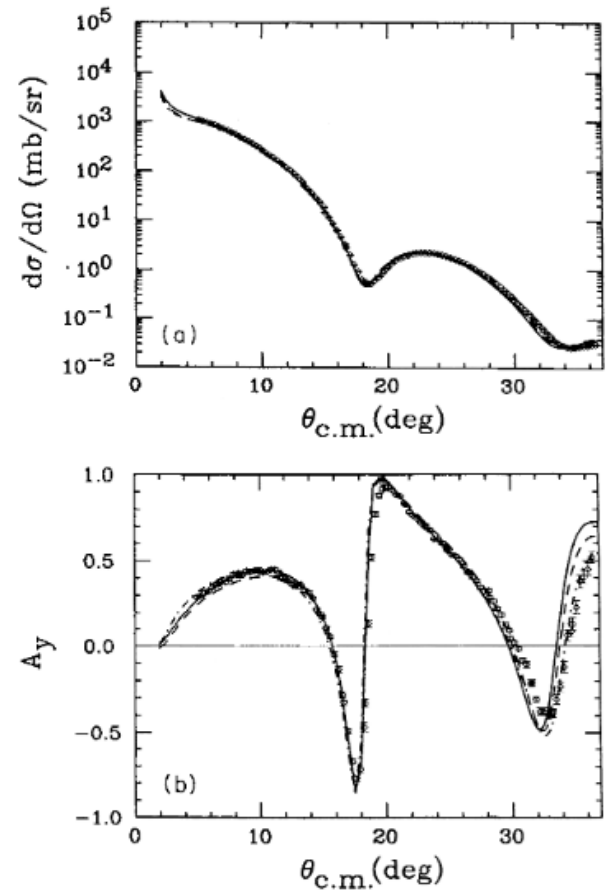
R. L. Mercer
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 31 August 1992)

		σ_R(mb)				
			EDAD-fit			
Target	T_p (MeV)	EDAI-fit	fit 1	fit 2	fit 3	Reference
^{12}C	29.00	420.2	435.5	433.1	422.7	[6]
	30.30	415.9	429.0	425.6	414.2	[7]
	49.00	358.8	363.0	348.4	327.7	[6]
	49.48	357.4	361.8	347.0	326.1	[8]
	61.40	323.3	335.6	317.0	294.8	[9]
	65.00	313.5	329.0	309.7	287.4	[10]
	122.00	202.2	269.0	254.4	230.5	[11]
	160.00	177.8	252.3	246.4	215.2	[11]
	200.00	177.6	243.0	243.9	205.0	[11–13]
	300.00	201.1	233.0	235.4	194.9	[14]
	398.00	215.8	227.4	218.6	199.1	[15]
	494.00	227.2	223.7	203.0	211.6	[16]
	797.50	238.4	235.3	209.9	250.0	[17,18]
	1040.00	198.6	259.4	243.8	232.2	[19,20]

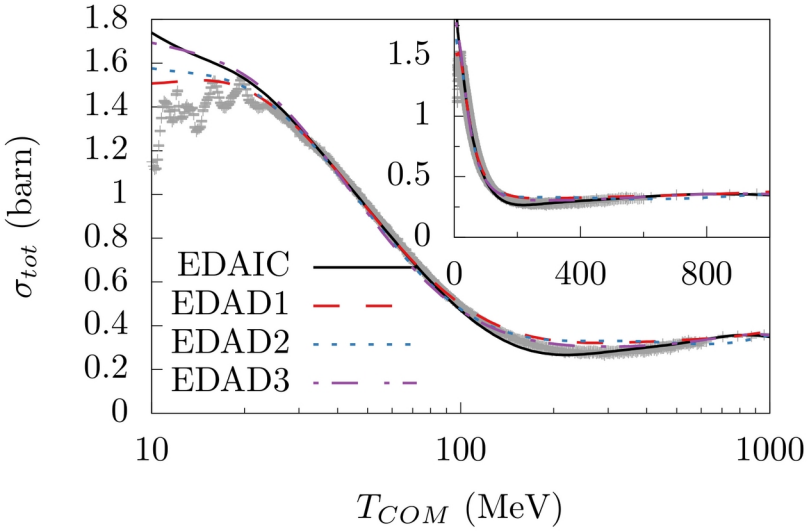


Energy-dependent potentials
fit to *elastic* proton-nucleus scattering

N–A scattering with INC and ROP

N-A data is used as benchmark/input to INC

Total neutron cross section with ROPs

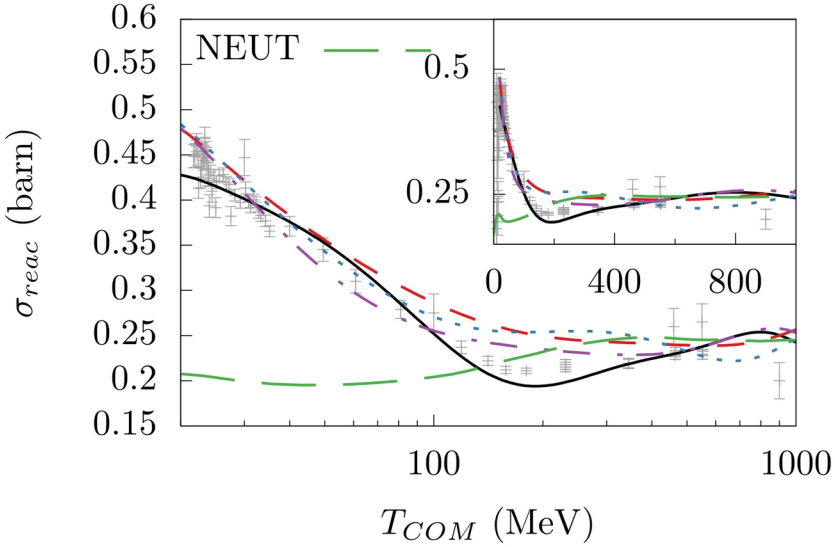


$$\sigma_{tot} = \sigma_{reac} + \sigma_{el}$$

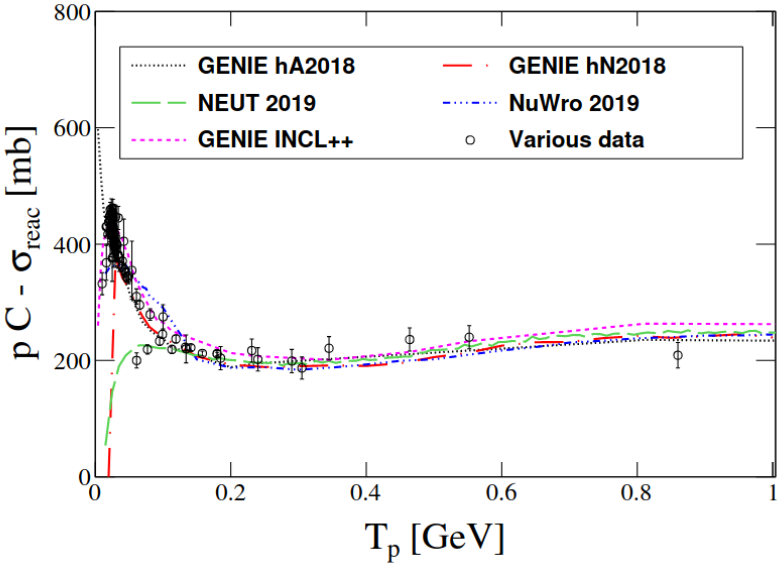
σ_{el} not modeled in (NEUT) INC

σ_{tot} Obtained in ROP with optical theorem

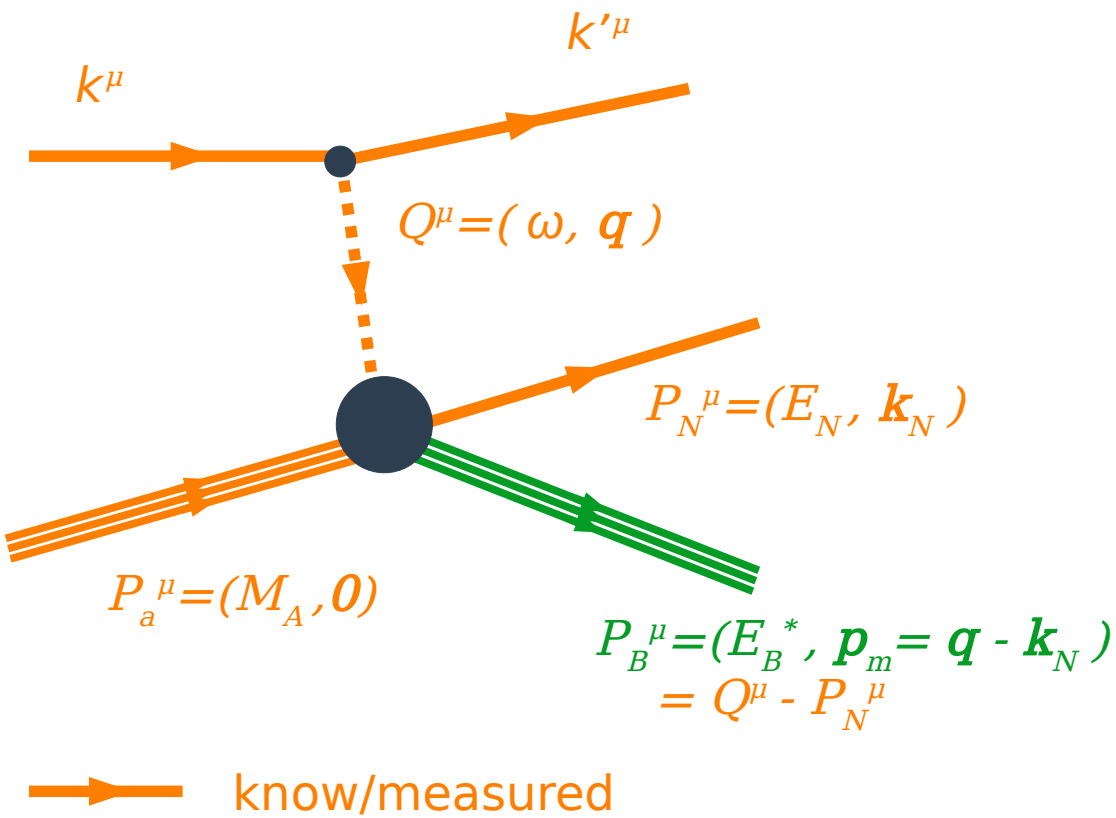
Proton reaction cross section ROPs and NEUT

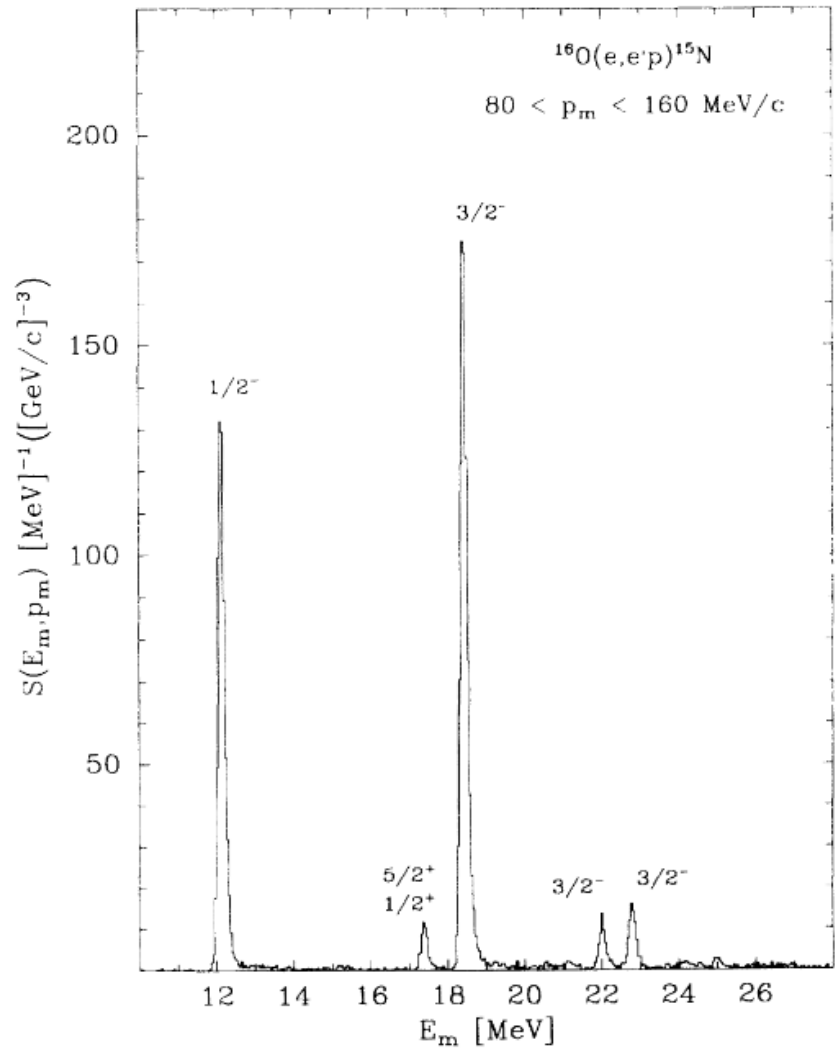


[Dytman et al. PRD104, 053006]



Exclusive electron scattering with ROP



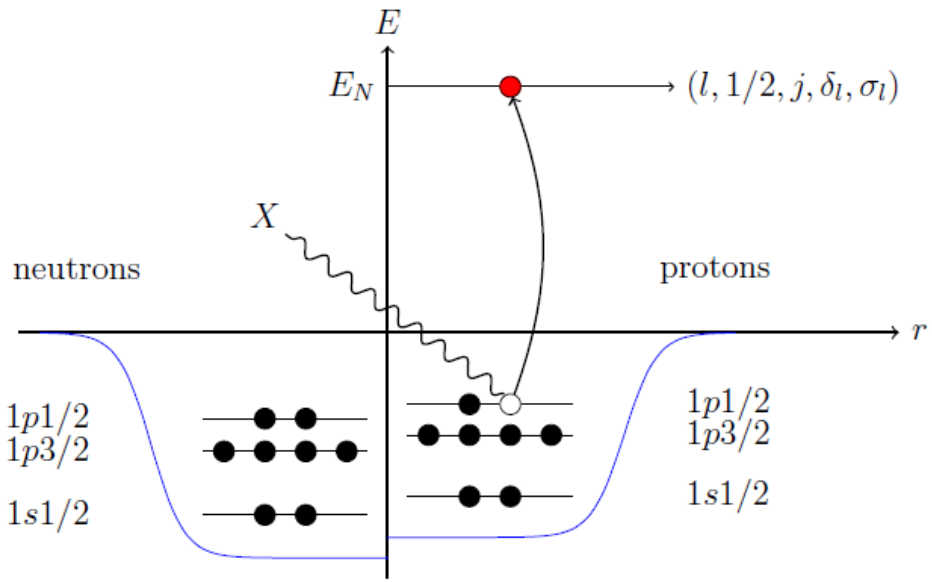


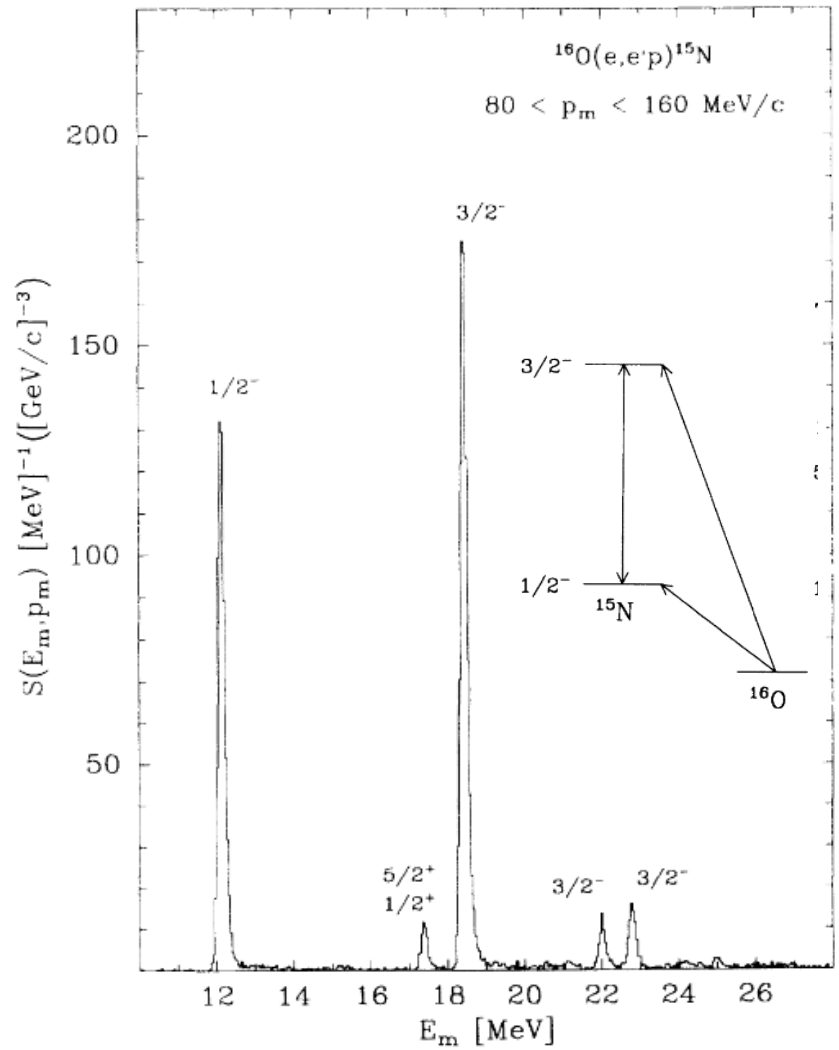
[M. Leuschner et al. PRC49, 955 (1994)]

Exclusive conditions: everything is known

Incoming energy is known
Never satisfied for accelerator neutrinos!

Independent particle model (IPM)





[M. Leuschner et al. PRC49, 955 (1994)]

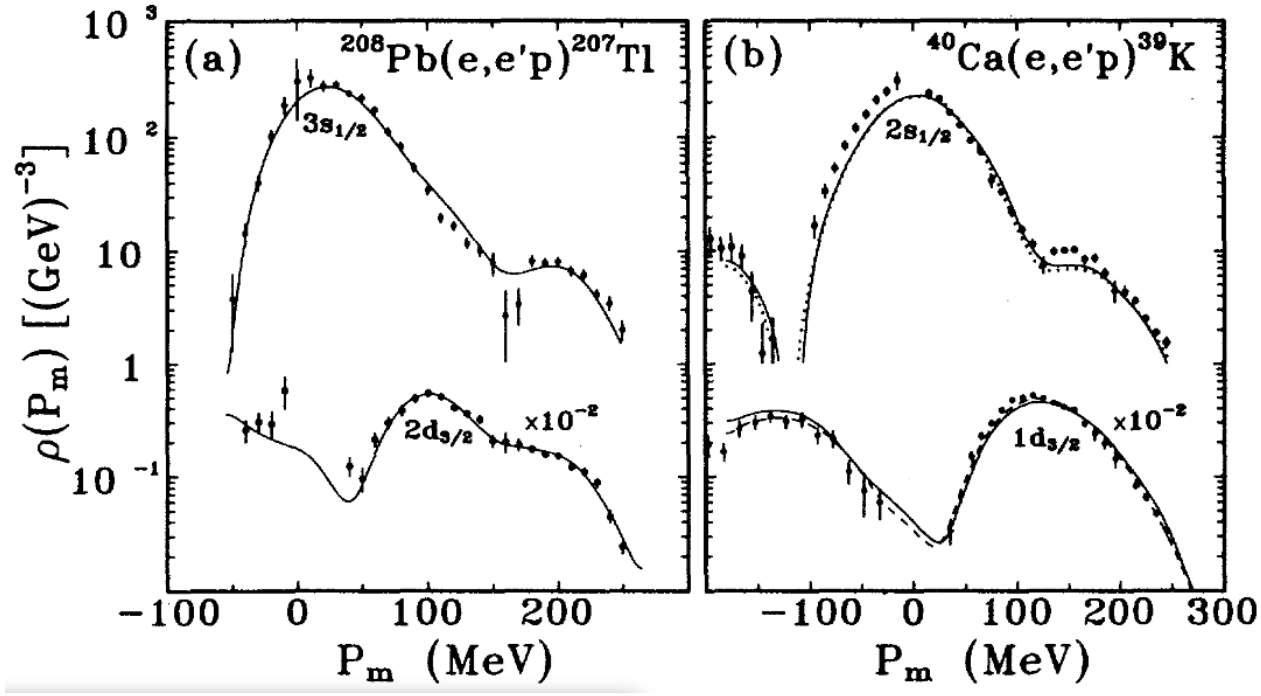
Quantum numbers:
- angular momentum
- Energy

(No momentum eigenstates)

Experimental data implies smaller occupations than IPM
→ Correlations **and FSI**

RDWIA with ROP for exclusive (e,e'p)

[Udias et al. PRC48, 2731]



[Meucci et al. PRC64, 014604]

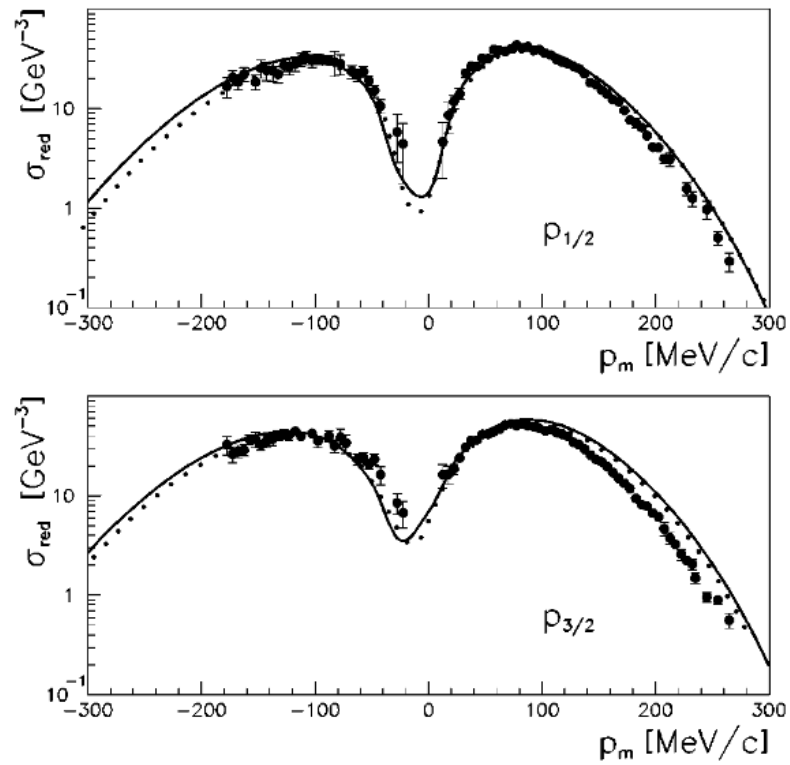


FIG. 11. The reduced cross section (σ_{red}) of the $^{16}O(e,e'p)$ reaction as a function of the recoil momentum p_m for the transitions to the $1/2^-$ ground state and to the $3/2^-$ excited state of ^{15}N, in

The optical potential removes nucleons that suffer **inelastic FSI → changes E_m**

The ROP also changes the observed p_m spectrum!

Direct comparison of INC and ROP

1. Input to NEUT INC

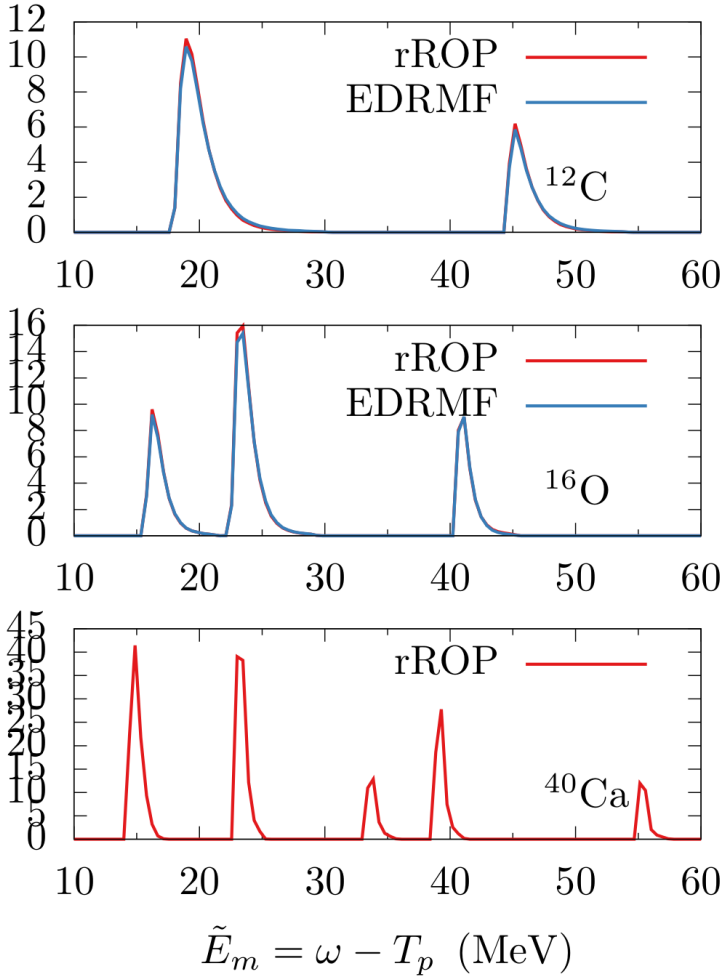
Events according to **unfactorized** fluxfolded five-fold differential nucleon knockout cross section

2. Kinematic cuts on NEUT result

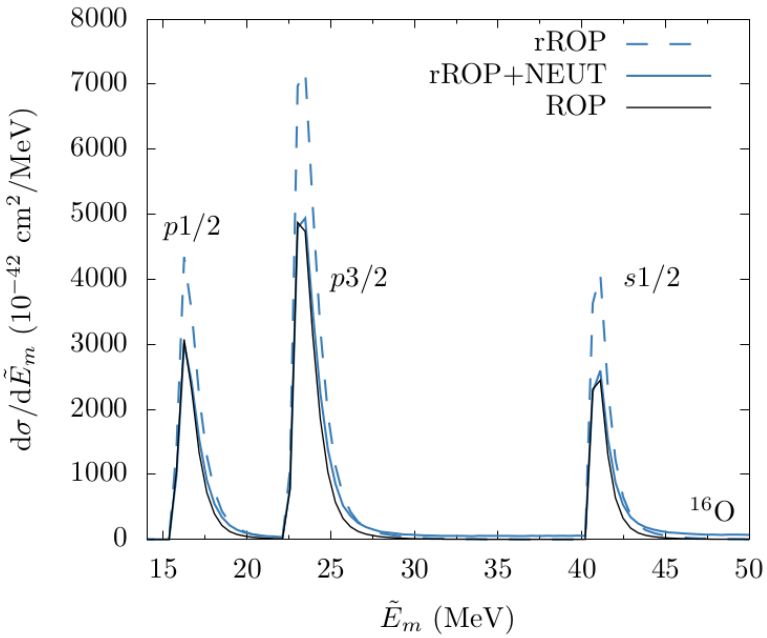
Select events with E_m corresponding to shell-model peaks

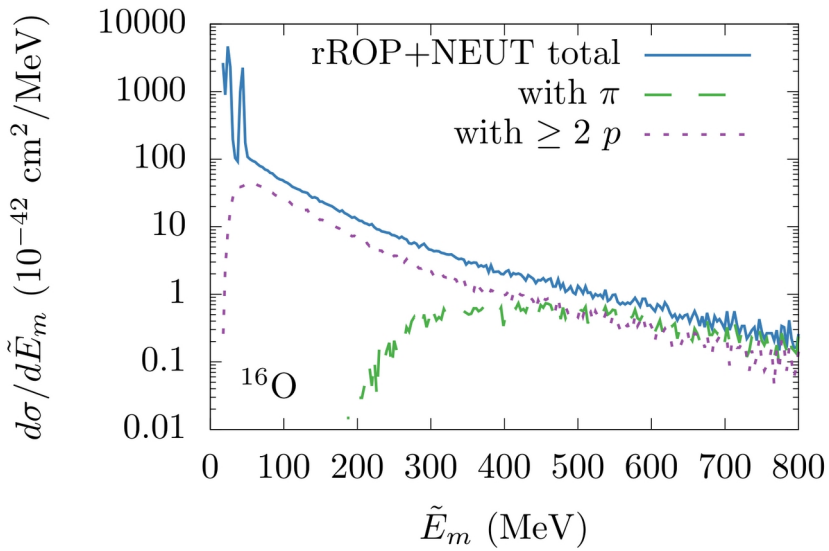
Signal with only 'elastic' FSI
That does not change E_m

Can be directly compared to ROP



NEUT Cascade with rROP input





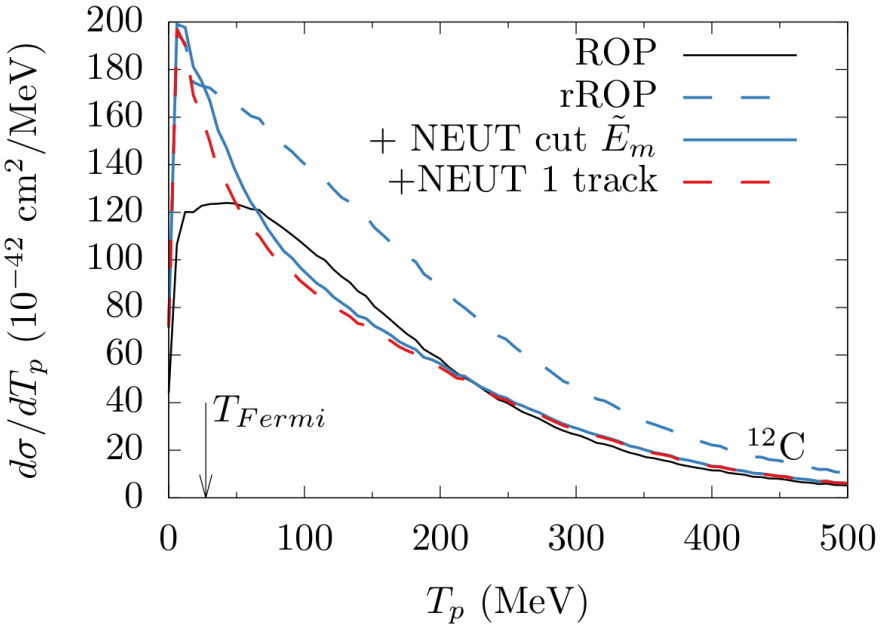
Simplified E_m contains recoil energy

$$\tilde{E}_m = E_i - E_l - T_p$$

T2K flux-folded calculations

- rROP+NEUT moves strength from shell model peaks to larger E_m
- Resulting strength of shell model peaks agrees with ROP predictions
- FSI leads to mostly large ~50 MeV shifts in missing energy

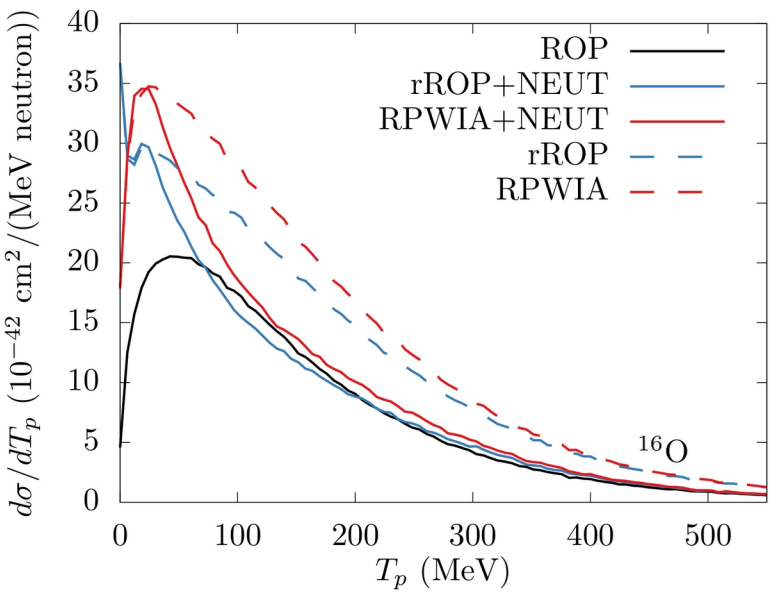
T_p distributions INC and ROP

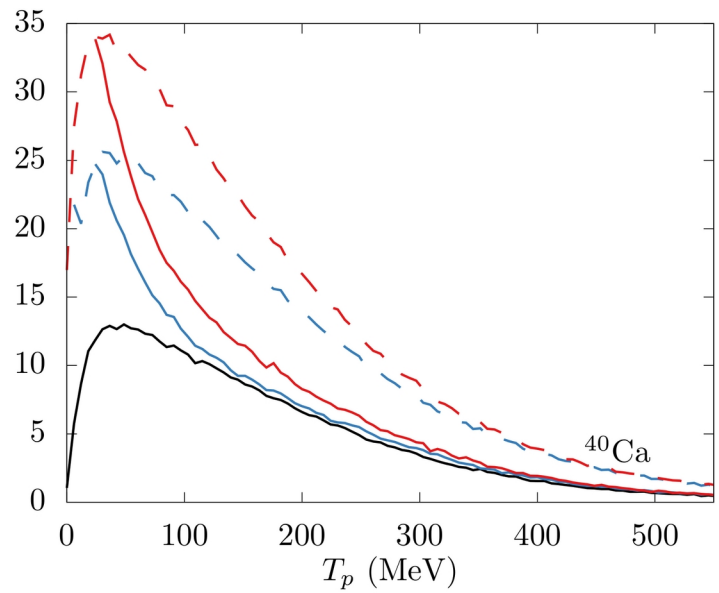


The INC agrees with the ROP for high T_p

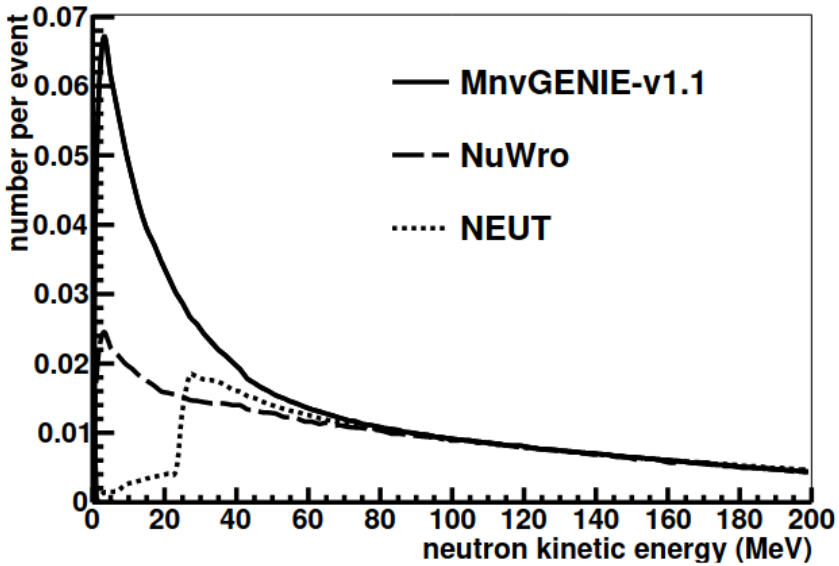
At T_p < 100 differences go up to 100% !

In NEUT low energy nucleons don't undergo FSI at all!





Large model dependence in INC's at low E



[MINERvA PRD100, 052002]

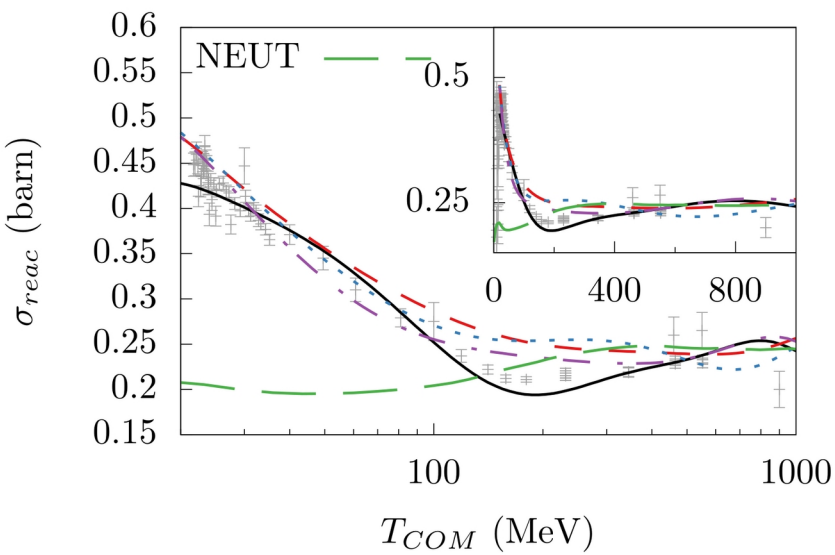
The INC agrees with the ROP for high T_p

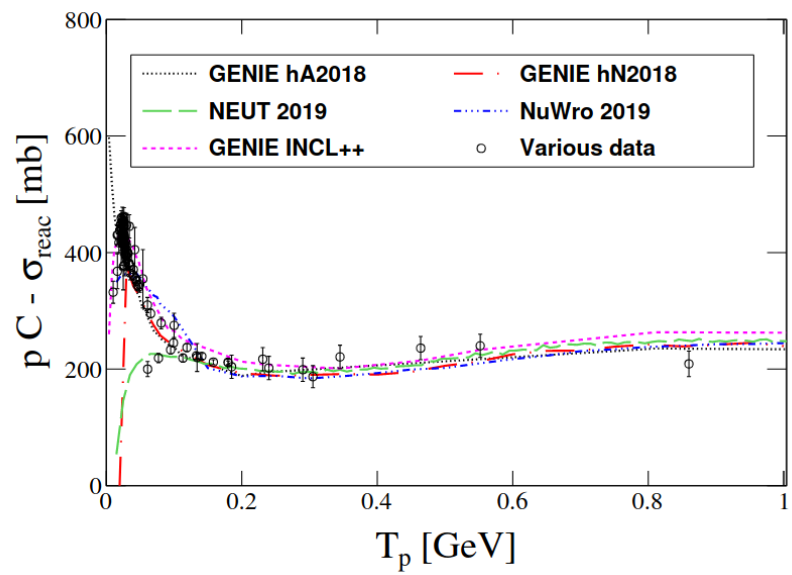
At $T_p < 100$ differences go up to 100% !

In NEUT low energy nucleons don't undergo FSI at all!

→ Will influence a.o. neutron rates

How do other INC perform ?
You can use these results!





[Dytman et al. PRD104, 053006]

Important issue: the input to the INC

1. Input to NEUT INC

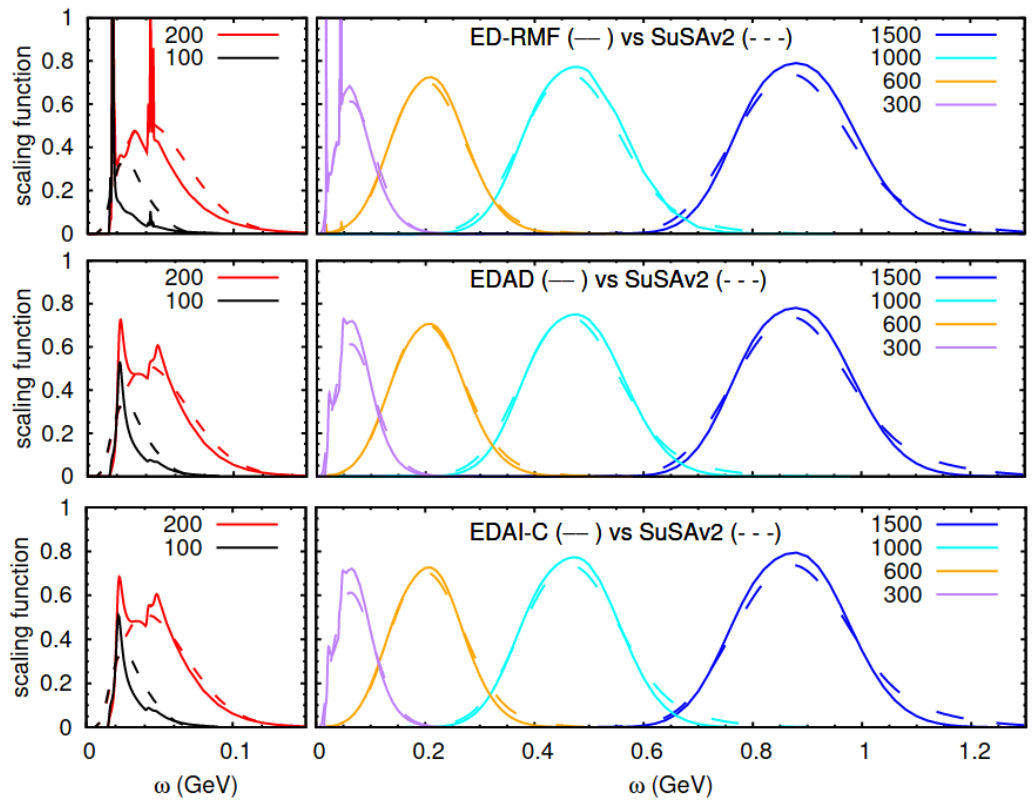
Events according to **unfactorized** fluxfolded five-fold differential nucleon knockout cross section

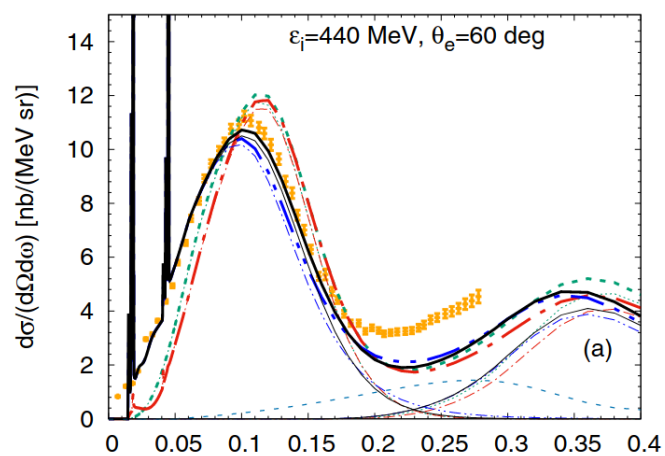
(computationally non-trivial work by RGJ)

RDWIA calculations with real final-state potentials

- Describe *inclusive* (e,e') [PRC 100 045501 (2019)]
- Similar to SuSAv2 [PRC 101 015503 (2020)]
- Not limited to inclusive CS! [PRC 105 025502 (2022)] [Arxiv:2207.02086 (2022)]

= J.M. Franco-Patino's talk

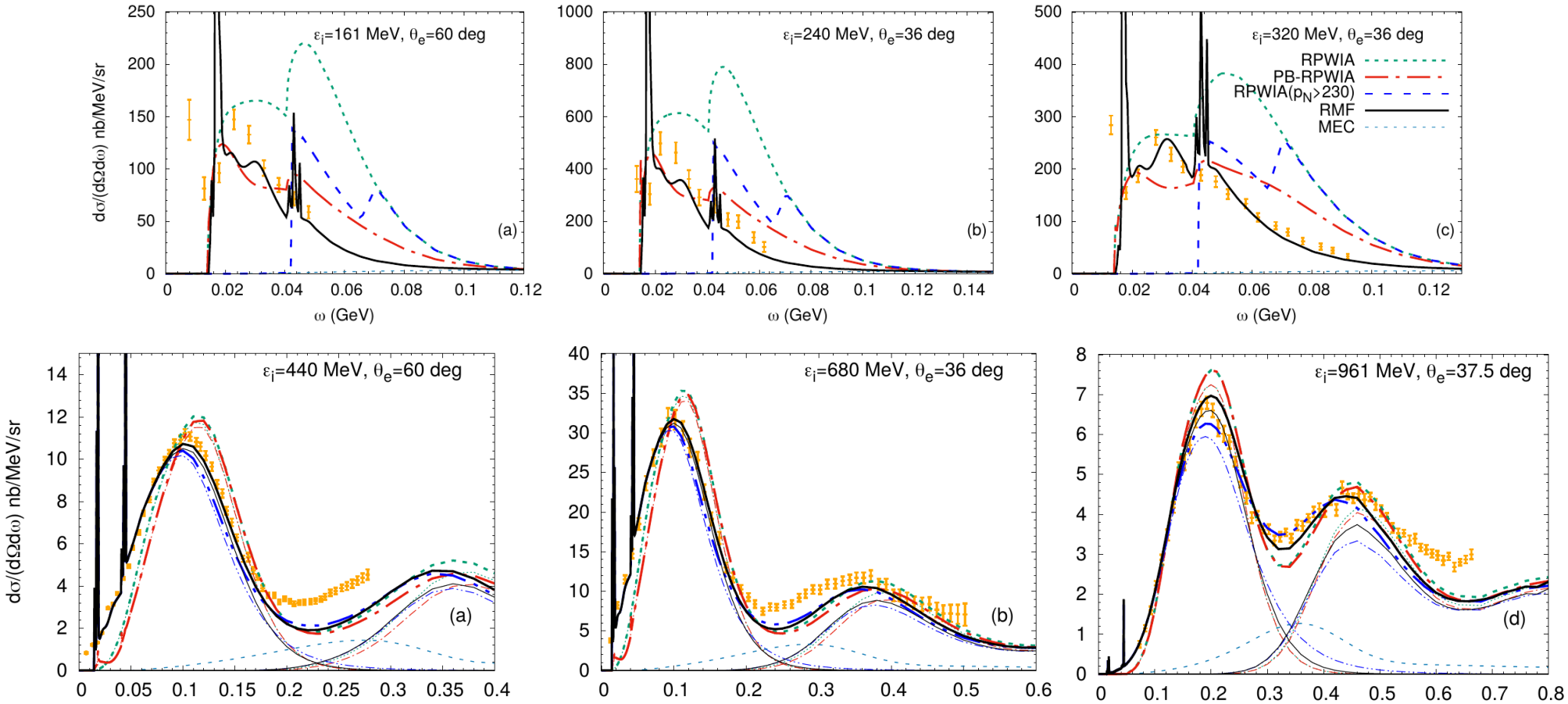




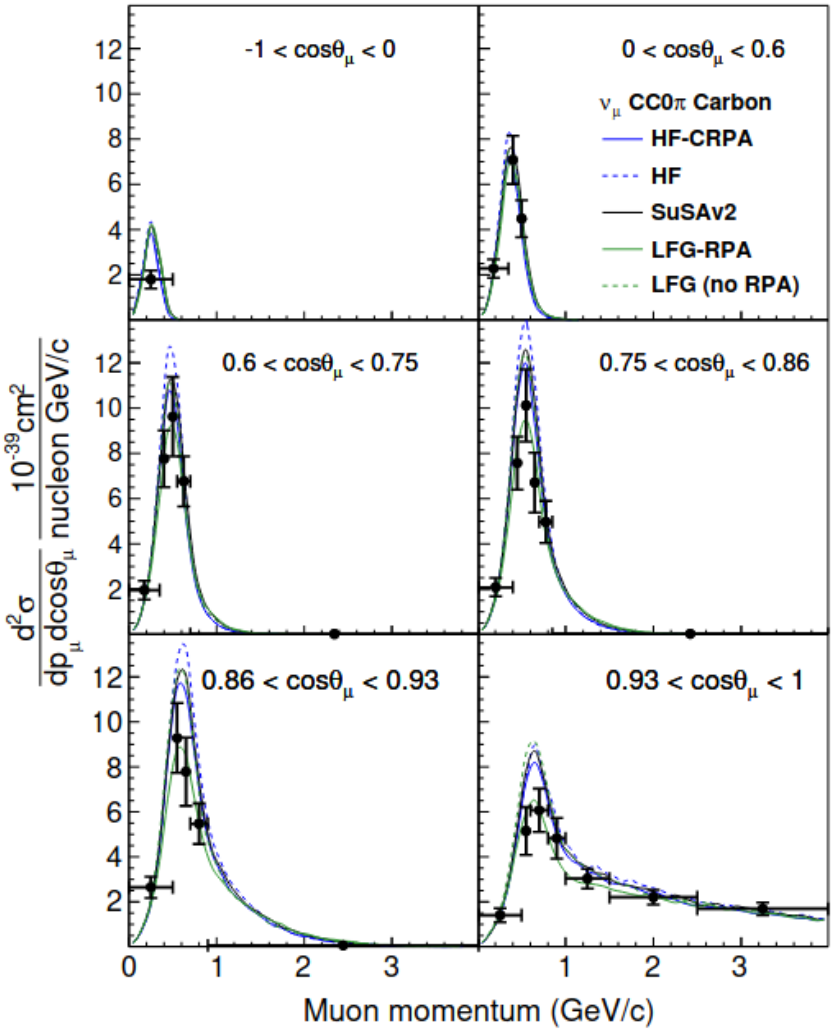
Important issue: the input to the INC

1. Input to INC includes some FSI!

FSI treatment is important to reproduce the inclusive cross section
In RDWIA, CRPA, effective SF, LFG+RPA, … some FSI included!
Cannot always (ever?) separate 'initial' and 'final'!



Inclusive cross sections in generators



Implementation of the CRPA model in the GENIE event generator and analysis of nuclear effects in low-energy transfer neutrino-nucleus interactions

S. Dolan,[1] A. Nikolakopoulos,[2] O. Page,[3] S. Gardiner,[4] N. Jachowicz,[2] and V. Pandey[5]

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[2] *Department of Physics and Astronomy, Ghent University, Proeftuinstraat 86, B-9000 Gent, Belgium*[†]
[3] *School of Physics, University of Bristol, Bristol BS8 1TL, United Kingdom*
[4] *Fermi National Accelerator Laboratory, Batavia, IL 60502, USA*
[5] *Department of Physics, University of Florida, Gainesville, FL 32611, USA*[‡]

(Dated: November 2, 2021)

Implementation of the SuSAv2-MEC 1p1h and 2p2h models in GENIE and analysis of nuclear effects in T2K measurements

S. Dolan,[1,2,3] G.D. Megias,[1,2,4] and S. Bolognesi[2]

[1] *IN2P3-CNRS, Laboratoire Leprince-Ringuet, Palaiseau 91120, France*
[2] *DPhP, IRFU, CEA Saclay, 91191 Gif-sur-Yvette, France*
[3] *CERN, European Organization for Nuclear Research, Geneva, Switzerland*
[4] *University of Tokyo, Institute for Cosmic Ray Research, Research Center for Cosmic Neutrinos, Kashiwa, Japan*

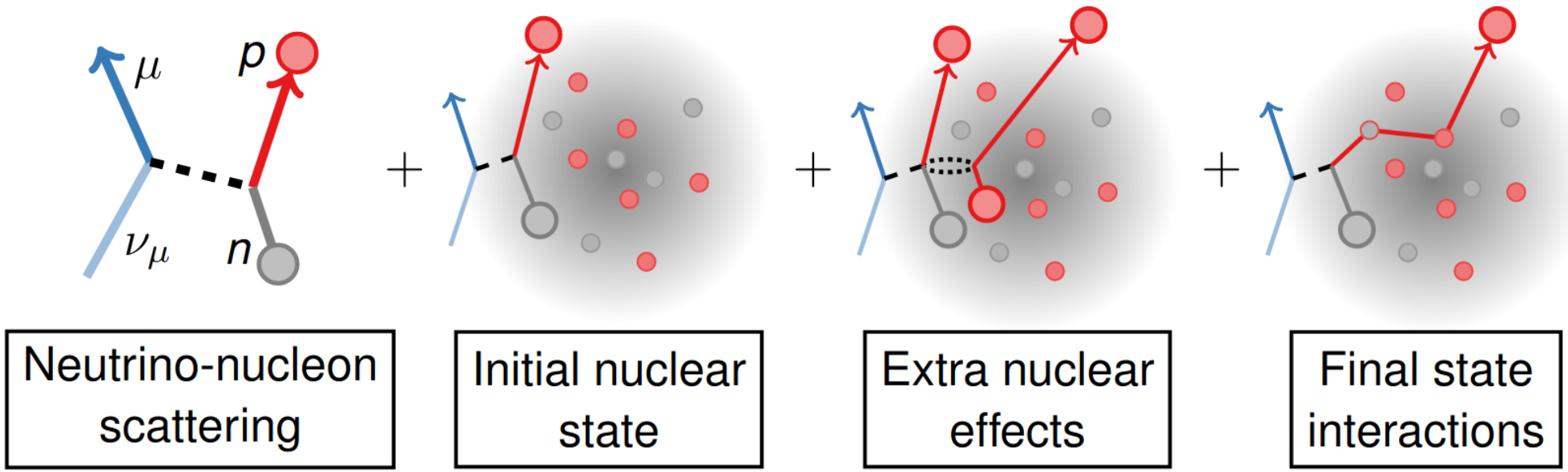
(Dated: February 21, 2020)

Implementation strategy

- Include inclusive responses
- Add nucleon final state in GENIE based on momentum distribution
- SuSAv2, LFG+RPA (Valencia), CRPA, SuSA-MEC, ...

Inclusive cross sections in generators

The idealized version of the event generator:



[Fig. From K. Niewczas]

The reality:

(part of) the initial-state, the ‘extra effects’ and the FSI are included

It is not possible to obtain the hadron information from inclusive CS!!

Nucleon variables in GENIE

Input to the generator is inclusive cross section:

$$\frac{\mathrm{d}\sigma(E_\nu)}{\mathrm{d}E_l \mathrm{d}\cos\theta_l} = G^2 \frac{k_l}{E_\nu} L_{\mu\nu} \int \mathrm{d}\Omega_N \sum_{n,\kappa} H^{\mu\nu}_{n,\kappa}(\omega, q, \Omega_N, E_{n,\kappa})$$

Lost nucleon information → Need to generate it in GENIE

1. Draw initial nucleon $\mathbf{p}_m$ from $p^2\, n(p)$ (e.g. LFG)

!! 2. Compute $E_m^2 = p_m^2 + M_N^2$

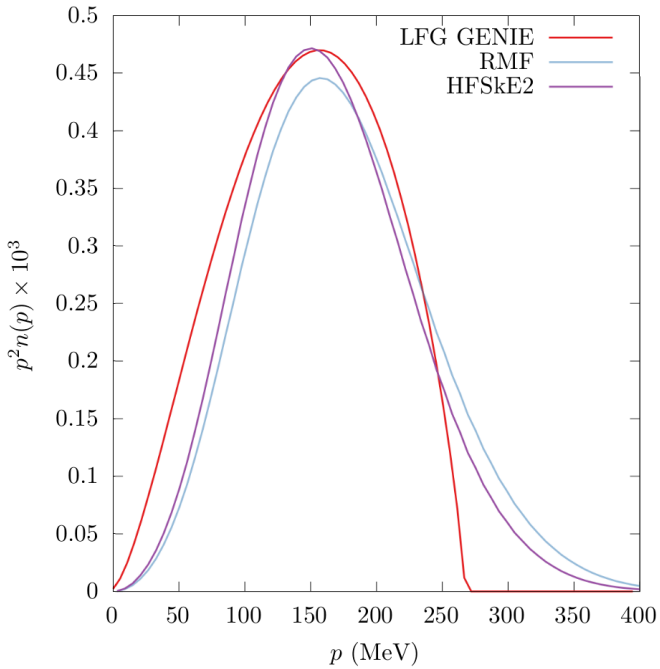
3. $E_N = E_m + \omega - E_b(q)$

4. $k_N^2 = E_N^2 - M_N^2$

!! $|\mathbf{p}_m + \mathbf{q}| \neq k_N = \sqrt{E_N^2 - M_N^2}$

→ $$\mathbf{k}_N = \frac{k_N}{|\mathbf{p}_m + \mathbf{q}|} (\mathbf{p}_m + \mathbf{q})$$

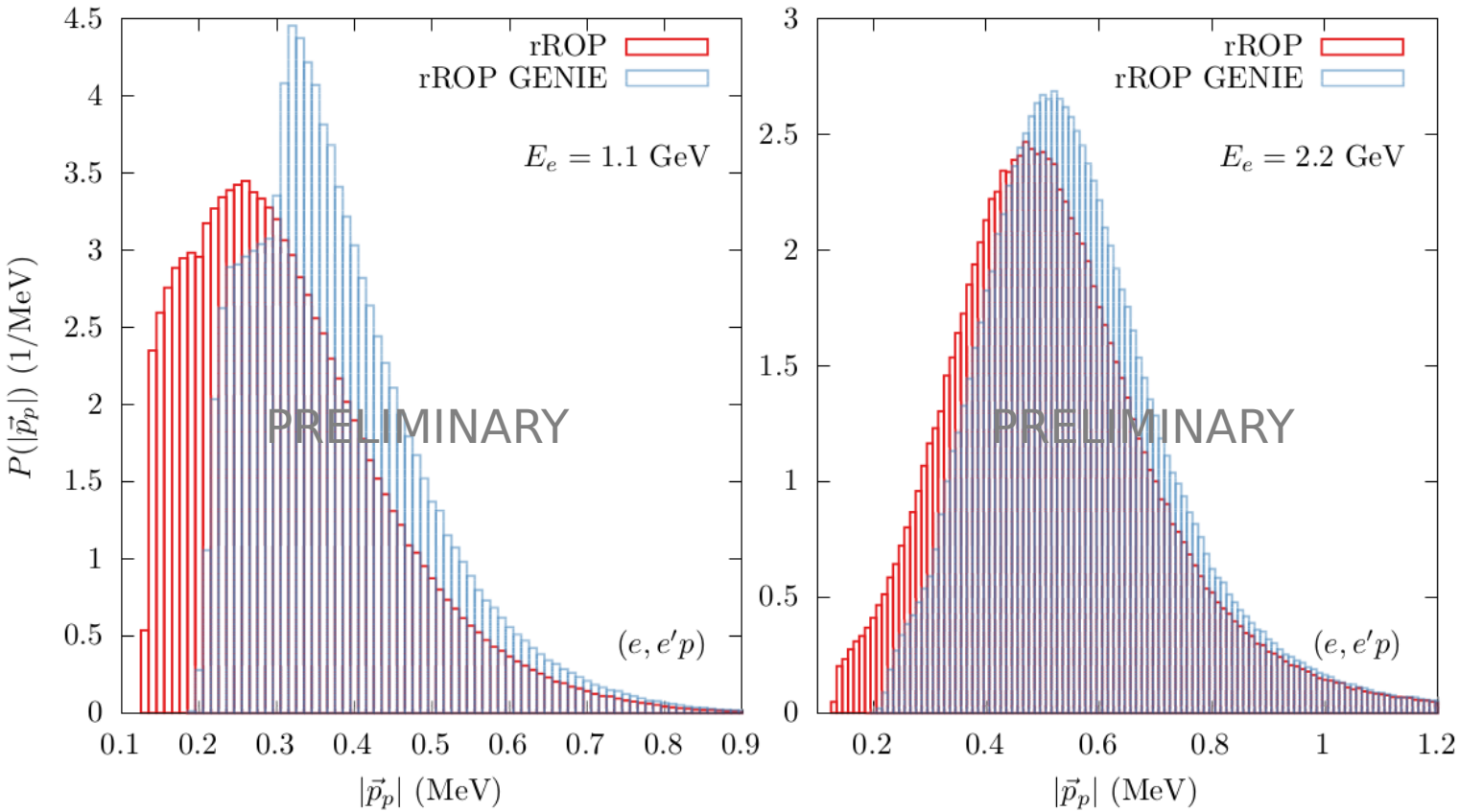
5. Give residual momentum to remnant



Nucleon variables in GENIE and RDWIA

Comparison of the nucleon kinematics

- The full 5-fold RDWIA calculation
- The GENIE algorithm for ‘adding’ the nucleon



(e,e’p) at fixed Incoming energy

$\Theta_{e'} > 10^{o}$

= *e4nu kinematics*

Nucleon variables in GENIE and RDWIA

Comparison of the nucleon kinematics

- The full 5-fold RDWIA calculation
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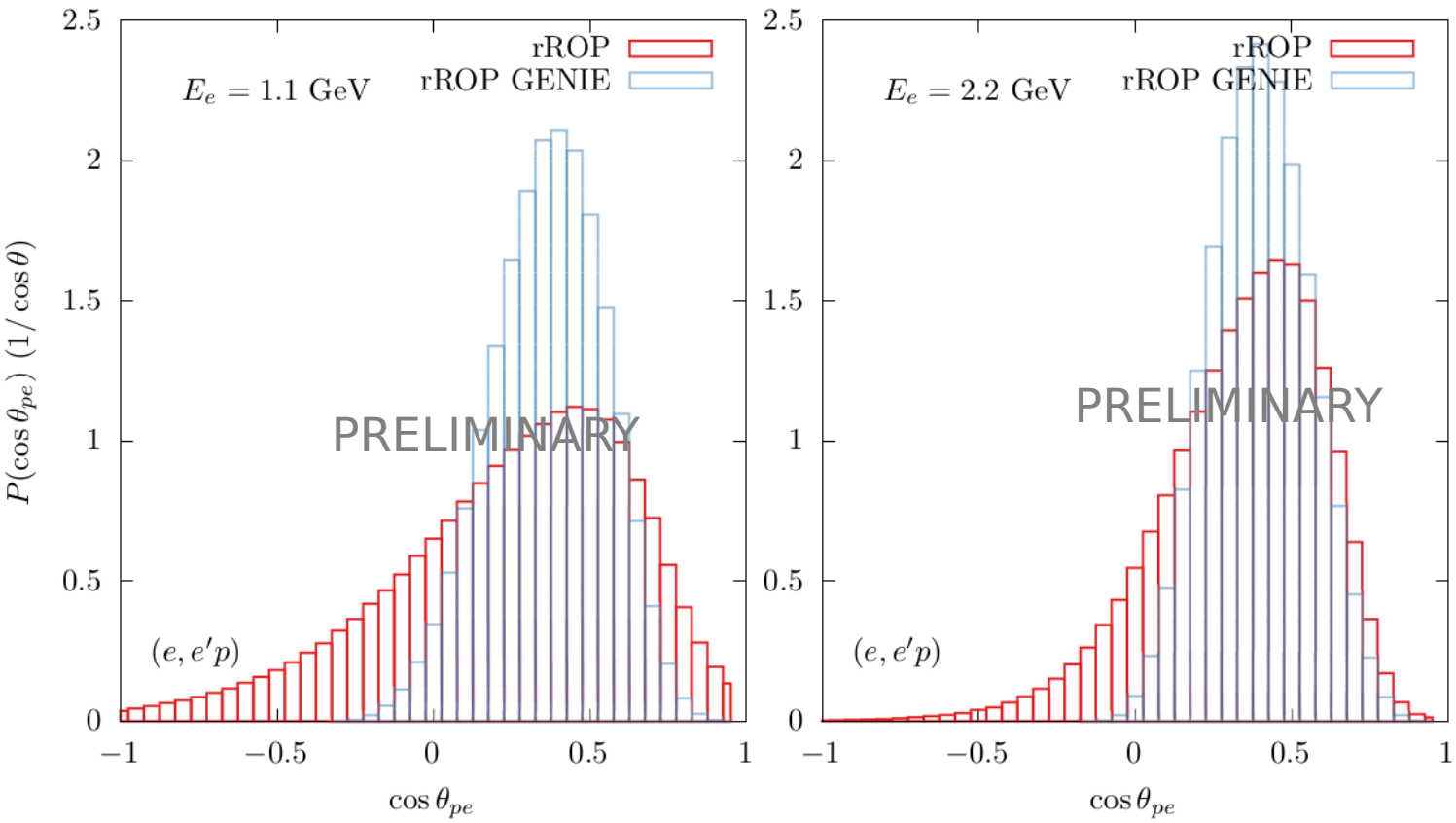


(e,e’p) at fixed Incoming energy

$\Theta_{e'} > 10^{o}$

= *e4nu kinematics*

Conclusions

- The ROP and INC approaches both use nucleon-nucleus scattering to constrain FSI, albeit in very different ways.

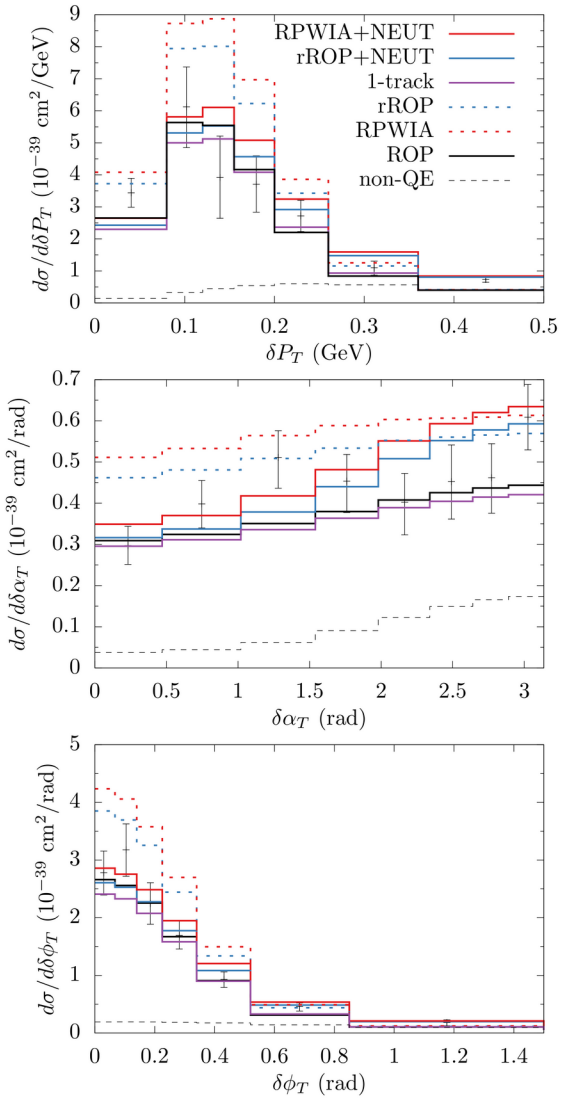
- A consistent comparison of the NEUT INC and optical potential approaches shows that there is quantitative agreement only at large kinetic energies.
- For small kinetic energy the differences are up to 100% !!

- The ROP should be more reliable in this comparison, but the true answer is unknown!

- Results of the generator will depend crucially on the input to the INC, but current implementations (necessarily) use unrealistic approximations

- **Unfactorized Events** for flux-averaged signals over the whole phase space can be generated
  → **You can use these for validation/error estimation/… of your own INC/simulation/…**

Transverse Kinematic Imbalance



Non-QE from [Bourguille et al. JHEP04(2021)004]

Because p_p > 450 MeV low energy differences are not seen

Effect of non-elastic FSI visible in P_T and α_T

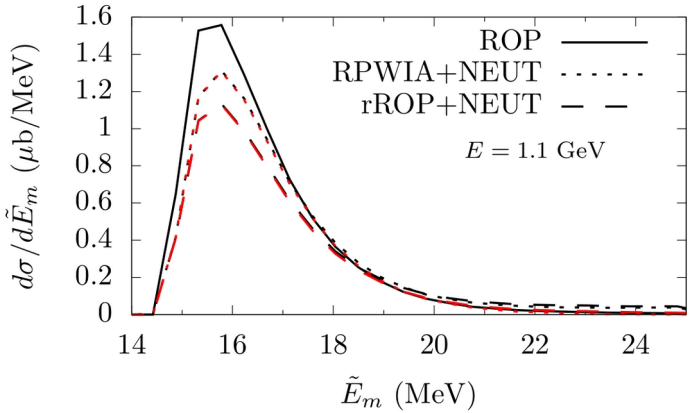
Large non-QE ‘background’ not separable from FSI effects

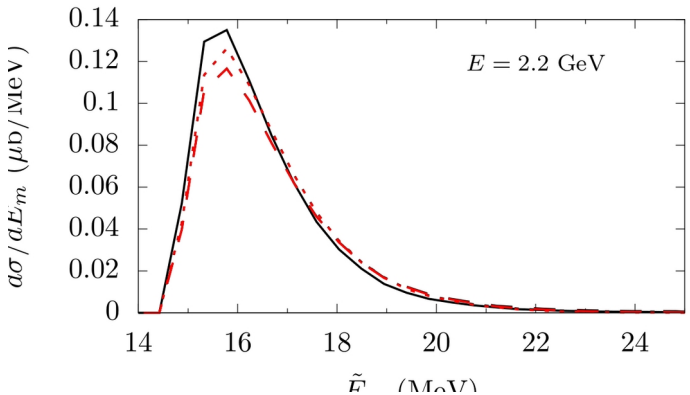
(e,e'p) results for CLAS e4nu kinematics

Fixed-E experiment (e,e'p) in CLAS

Range of nucleon and lepton kinematics

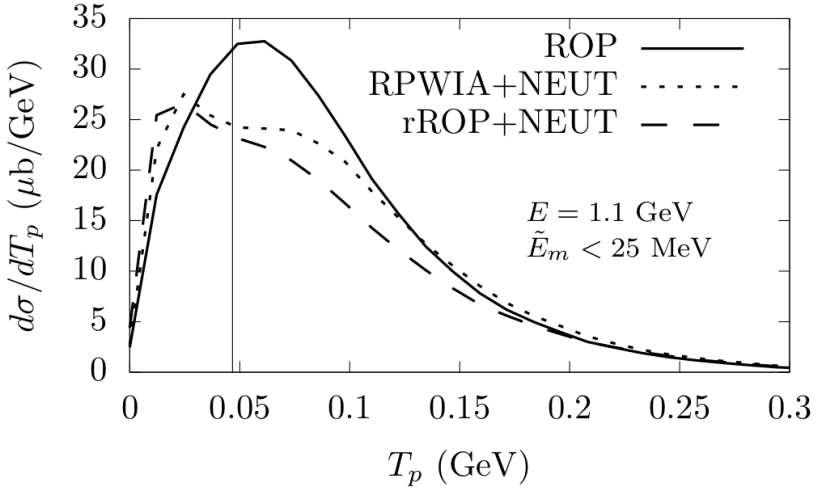
<u>Restrict E_m to probe specific interaction mechanisms</u>

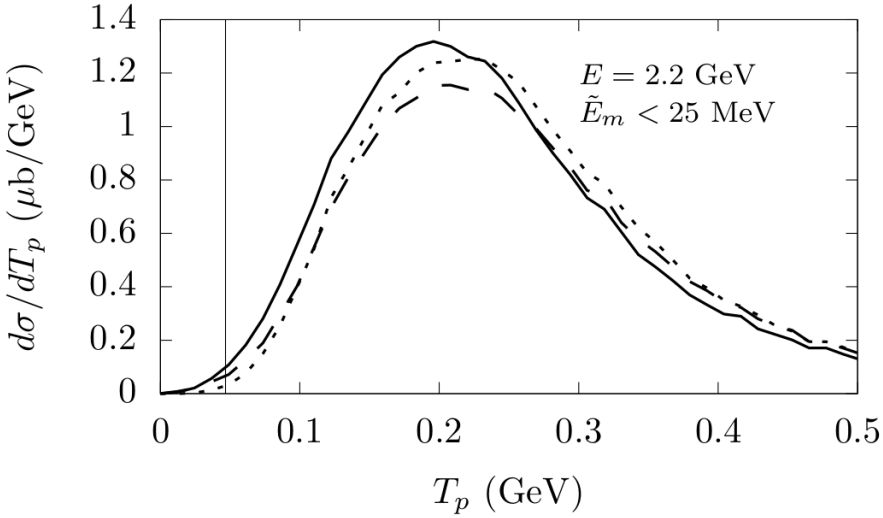




$\tilde{E}_m < 25$ MeV → No 2p2h, RES, inelastic FSI

But with full range of ω, q, T_N to study FSI





Degeneracy of interaction channels

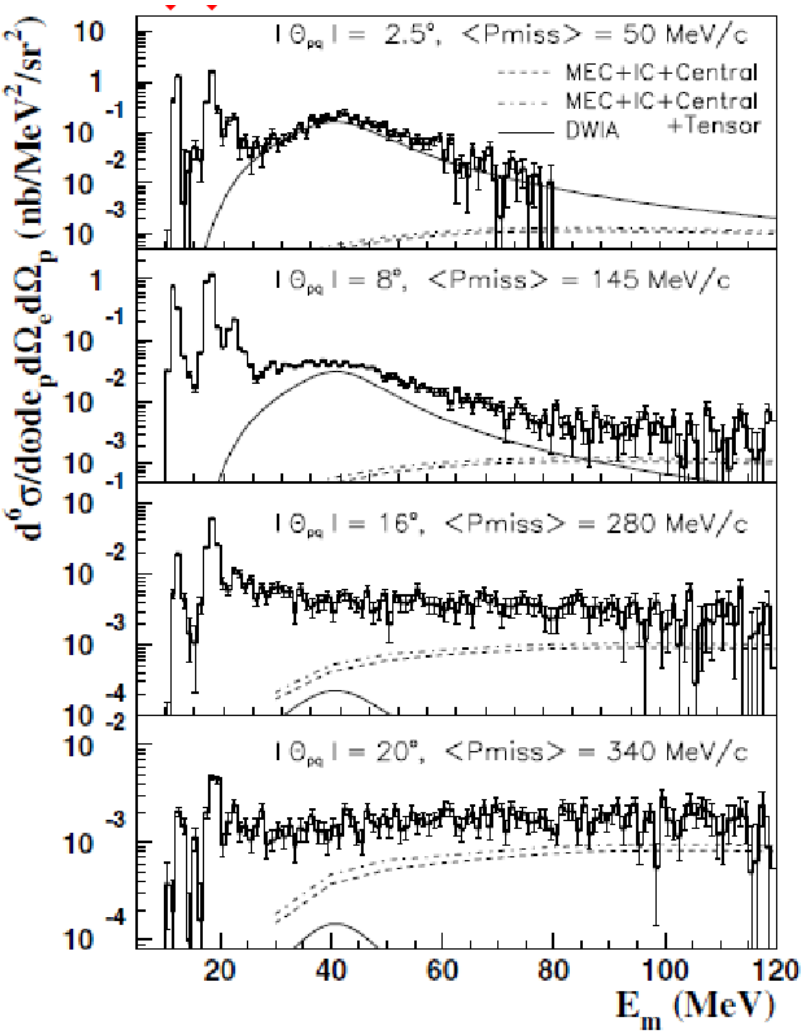
Is reduced, but not removed by fixing incoming energy

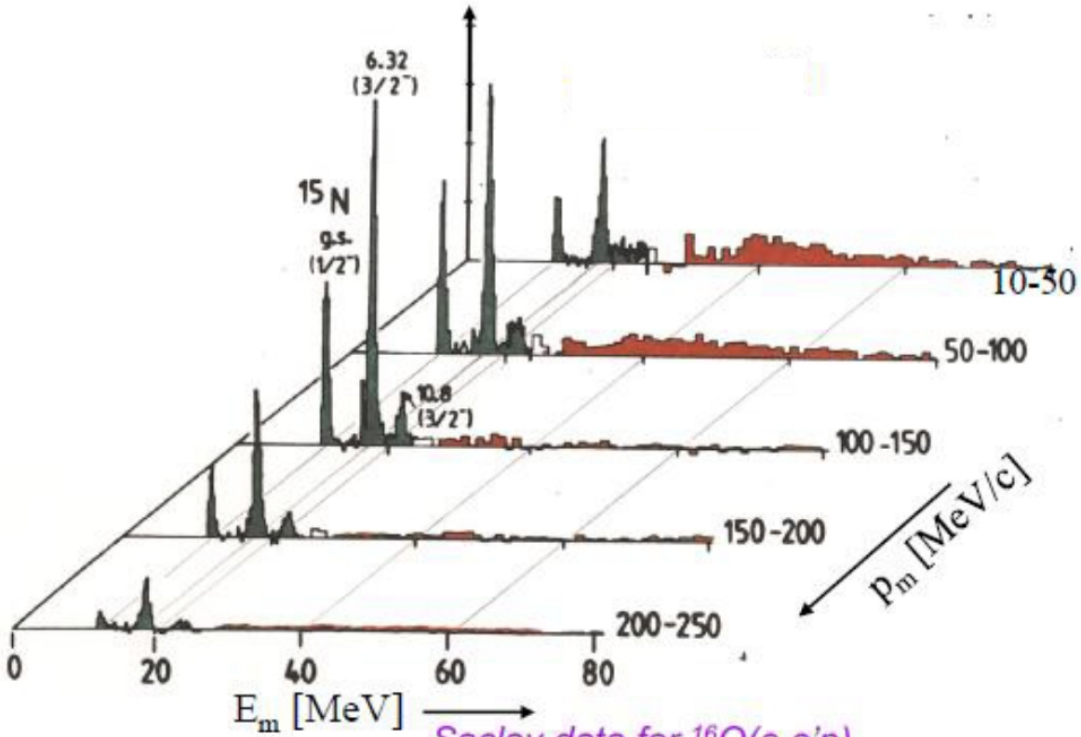
Is removed by restricting the energy/momentum of residual system

$$l + A \rightarrow l' + N + B$$

Restricted kinematics		Unrestricted kinematics
exclusive (e,e'p)	Fixed-E experiment (ν_l,l'p) or (e,e'p)	Accelerator (ν_l,l'p)
Fixed nucleon and lepton kinematics	Range of nucleon and lepton kinematics	Range of nucleon and lepton kinematics
Severely restrict E_m	<u>Restrict E_m to probe specific interaction mechanisms</u>	No E_m restriction possible
Learn nuclear structure and Reduce FSI and kinematic effects		

E_m (& p_m) : (e,e'p)





Saclay data for $^{16}O(e,e'p)$
[Mougey et al., Nucl. Phys. A335, 35 (1980)]

DWIA (1s): J. Kelly
2-nucleon : Ryckebusch et al.

Figures from talk by C. Giusti (INT Workshop INT-18-1a)

Relativistic mean field with nlsω interaction

$$\begin{aligned}\mathcal{L} &= \overline{\Psi}\left(i\gamma_\mu\partial^\mu - M\right)\Psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - U(\sigma) \\ &- \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\mathbf{R}_{\mu\nu}\mathbf{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &- g_\sigma\overline{\Psi}\sigma\Psi - g_\omega\overline{\Psi}\gamma_\mu\omega^\mu\Psi - g_\rho\overline{\Psi}\gamma_\mu\boldsymbol{\tau}\boldsymbol{\rho}^\mu\Psi - g_e\frac{1+\tau_3}{2}\overline{\Psi}\gamma_\mu A^\mu\Psi\,.\end{aligned}$$

Extension of the original σ-ω Walecka model (Ann. Phys.83,491 (1974)).

$$U(\sigma) = \tfrac{1}{3}g_2\sigma^3 + \tfrac{1}{4}g_3\sigma^4$$

$$\left[\hat{\alpha}\cdot\hat{\mathbf{p}} + \beta\left(m_N + S(r)\right) - \left(E - V(r)\right)\right]\psi = 0,$$

Where:

$$S(r) = g_\sigma\sigma(r)$$

$$V(r) = g_\omega\omega^0(r) + g_\rho\tau_3\rho_3^0(r) + e\frac{1+\tau_3}{2}A^0(r).$$

$$\begin{aligned}\left(\nabla^2 - m_\sigma^2\right)\sigma(r) &= g_\sigma\rho_s(r) + g_2\sigma^2(r) + g_3\sigma^3(r), \\ \left(\nabla^2 - m_\omega^2\right)\omega^0(r) &= g_\omega\rho_B(r), \\ \left(\nabla^2 - m_\rho^2\right)\rho_3^0(r) &= g_\rho\rho_I(r), \\ \nabla^2\sigma(r) &= -e\rho_e(r),\end{aligned}$$

Main approximations:

1) Mean-field approximation: $\omega_\mu \to \langle\omega_\mu\rangle$ $\sigma \to \langle\sigma\rangle$ $\boldsymbol{\rho}_\mu \to \langle\boldsymbol{\rho}_\mu\rangle$

2) Static limit:

$\partial^0\omega_0 = \partial^0\boldsymbol{\rho}_0 = \partial^0\sigma = 0$ $\omega_\mu = \delta_{\mu 0}\omega_0\,,\quad \boldsymbol{\rho}_\mu = \delta_{\mu 0}\boldsymbol{\rho}_0$

3) Spherical symmetry for finite nuclei:

$\omega_0 = \omega_0(r)$ $\boldsymbol{\rho}_0 = \boldsymbol{\rho}_0(r)$ $\sigma = \sigma(r)$