Oscillation and decay of neutrinos in matter: an analytic treatment

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and arXiv:2204.05803 [hep-ph]

In collaboration with Amol Dighe, Kaustav Chakraborty, Srubabati Goswami, SM Lakshmi

Objective

If neutrinos decay, the effective non-Hermitian Hamiltonian needs to be treated carefully due to subtle issues regarding its mass and decay components.

We derive **compact analytic expressions** for 2-flavor 3-flavor neutrino probabilities with:

- Invisible **decay** + **Oscillation** + **Explicit matter effects included.**

Useful for:

1. **Long-baseline** neutrino experiments
2. Atmospheric neutrino experiments
3. Reactor anti-neutrino experiments

The problem

- The inclusion of decay makes the effective Hamiltonian non-Hermitian

$$\mathscr{H} = H - i\Gamma/2 \qquad \Gamma_{ij} = 2\pi \sum_k \langle \nu_i | \mathscr{H}' | \phi_k \rangle \langle \phi_k | \mathscr{H}' | \nu_j \rangle \, \delta(E_k - E_\nu)$$

- The decay and the mass eigenstates need not be the same ⇒ *Mismatch*

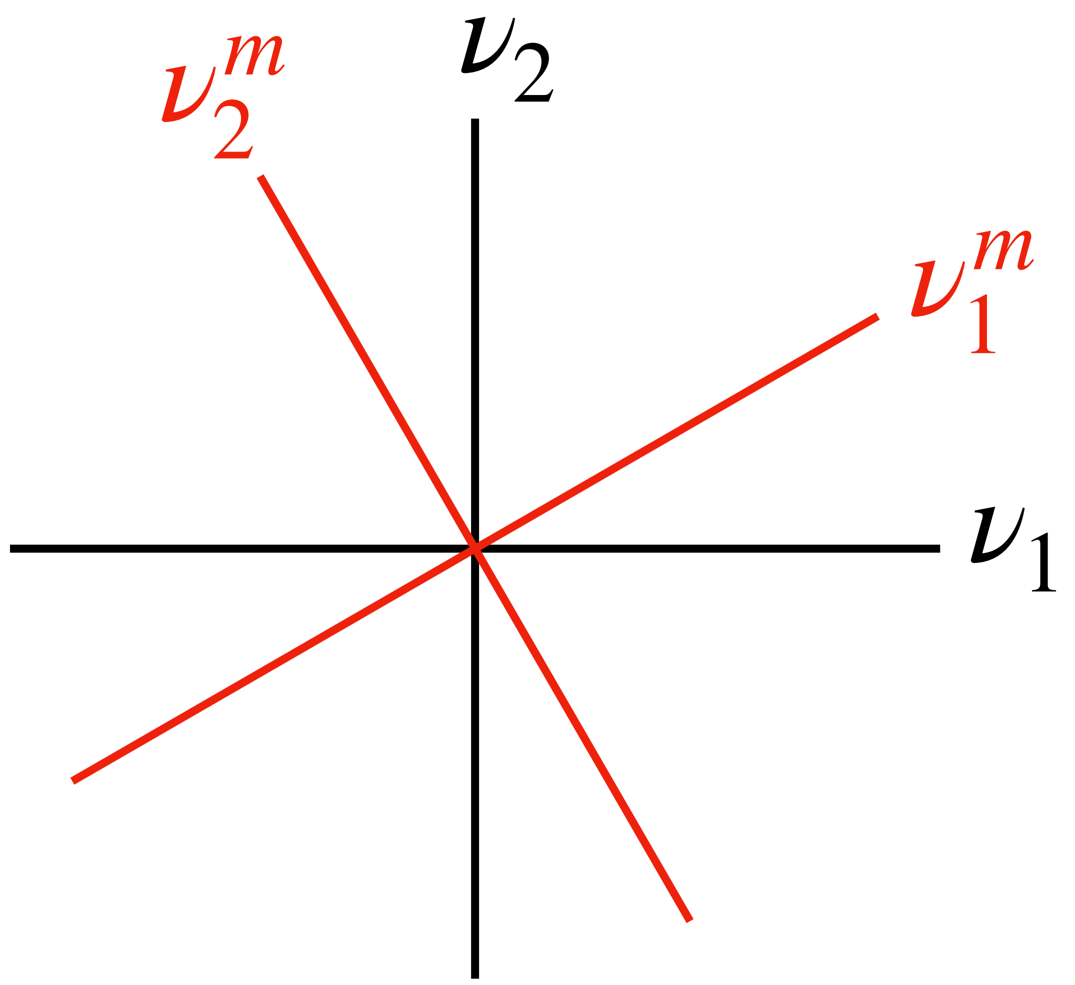
$$[H, \Gamma] \neq 0$$

- Even if there's no mismatch in vacuum, due to matter effects, the components will invariably become non-commuting.

Inevitability of the off-diagonal elements

- In the **2-flavor** approximation:

$$\mathscr{H}_m = \begin{pmatrix} a_1 - i\,b_1 & -\frac{1}{2} i\,\gamma\,e^{i\chi} \\ -\frac{1}{2} i\,\gamma\,e^{-i\chi} & a_2 - i\,b_2 \end{pmatrix}$$



- Even if only ν_2 in vacuum decays, with $\alpha_2 = m_2/\tau_2$, in matter, we get:

$$a_{1,2} = \frac{\tilde{m}^2_{1,2}}{2E} \quad , \quad b_{1,2} = \frac{\alpha_2}{4E}[1 \mp \cos[2(\theta - \theta_m)] \; ,$$

$$\chi = 0 \quad , \quad \gamma = \frac{\alpha_2}{2E} \sin[2(\theta - \theta_m)] \, .$$

- The **off-diagonal term** γ is **generated**, even though it was absent in vacuum.
- **Inevitable "mismatch"** in matter.
- We develop techniques using Zassenhaus (inverse BCH) expansion and Cayley-Hamilton theorem.

2 flavor expressions

- We use the inverse BCH (Zassenhaus) expansion to calculate the probabilities.
- The survival probability of a neutrino flavor is

$$\Delta_a \equiv a_2 - a_1, \quad \Delta_b \equiv b_2 - b_1$$

$$P_{\alpha\alpha} = \frac{e^{-(b_1+b_2)t}}{2}\left[(1+|A|^2)\cosh(\Delta_b t) + (1-|A|^2)\cos(\Delta_a t) - 2\mathrm{Re}(A)\sinh(\Delta_b t) + 2\mathrm{Im}(A)\sin(\Delta_a t)\right].$$

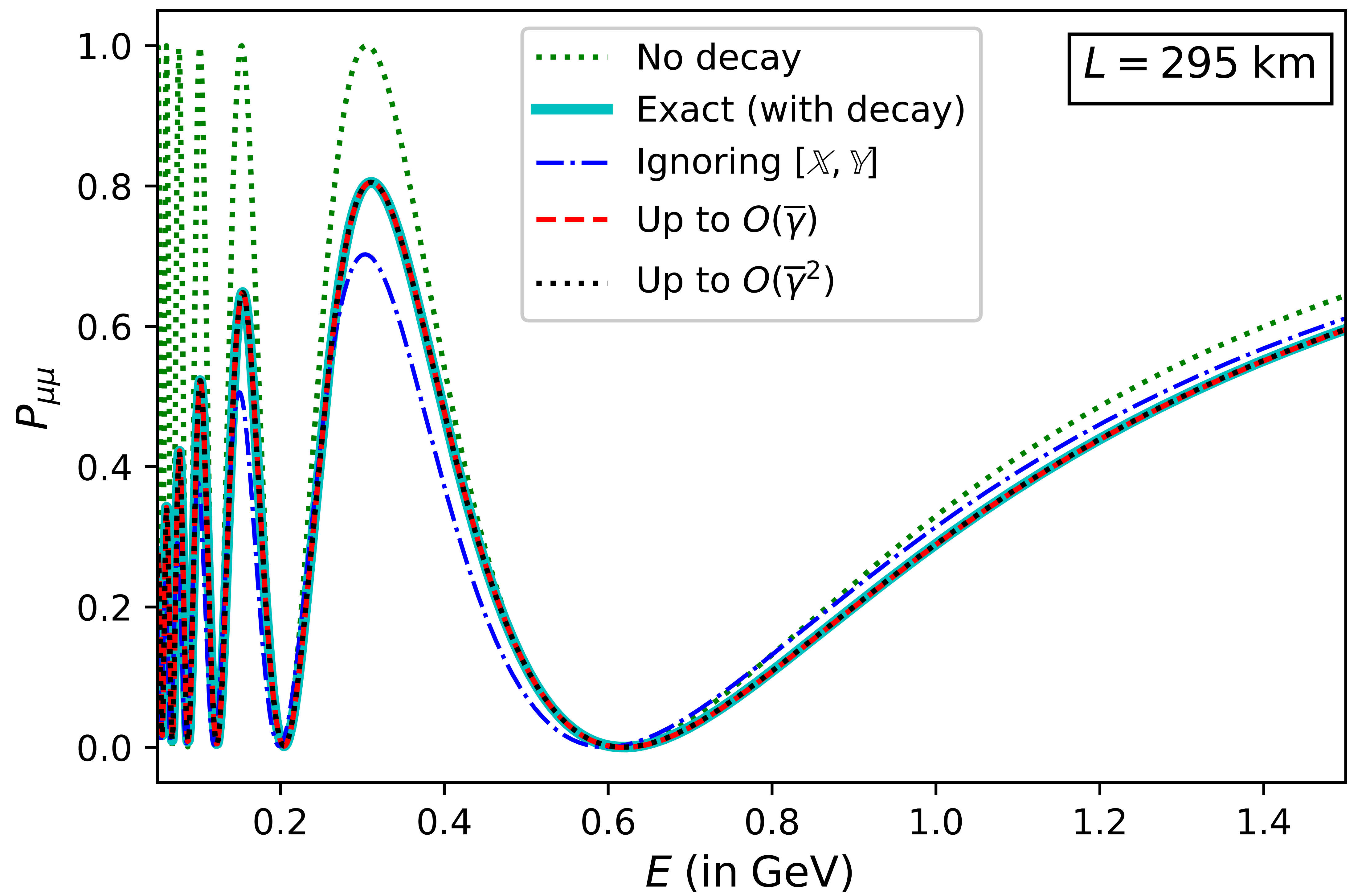
- The conversion probability is given by

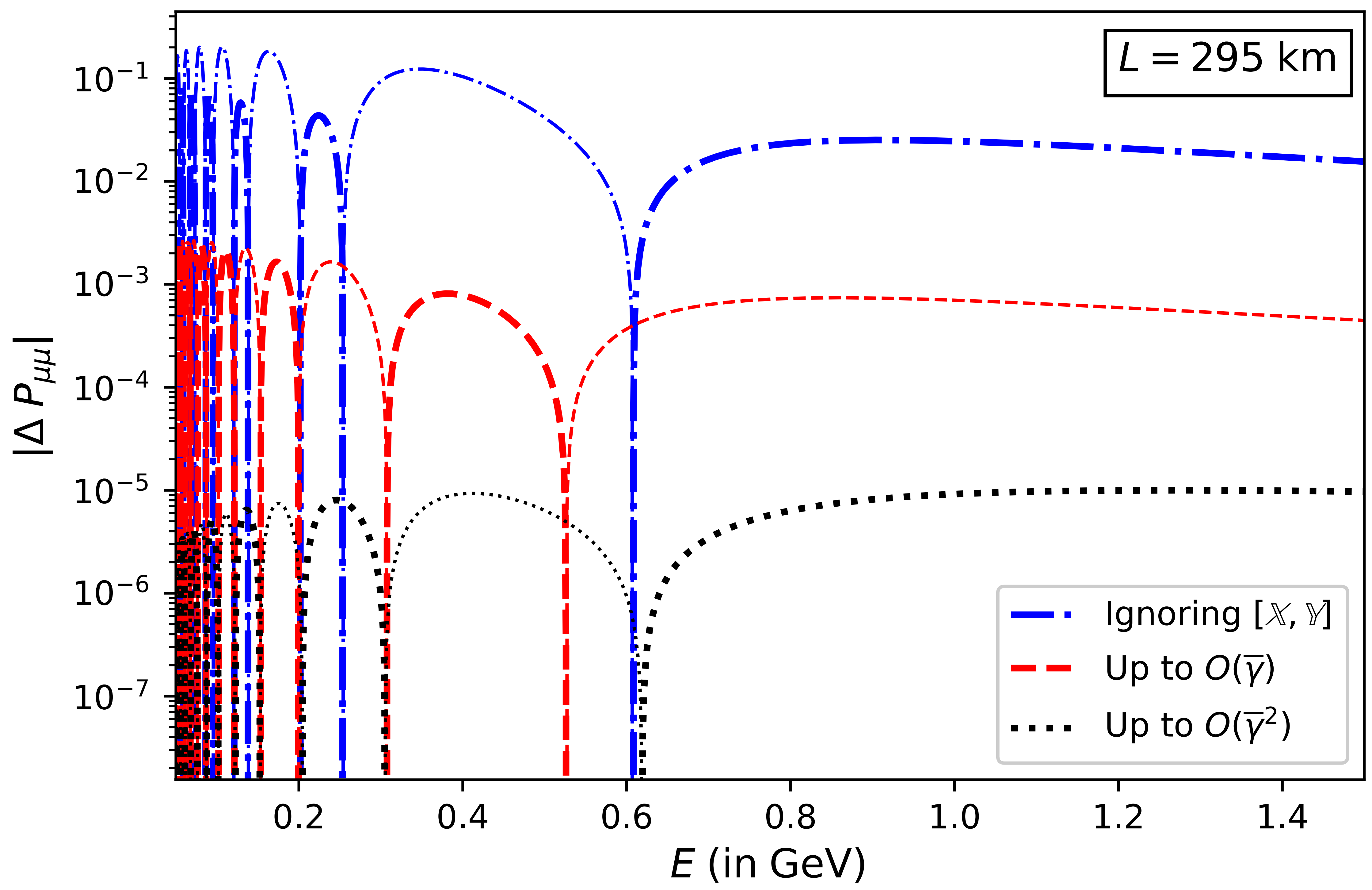
$$P_{\beta\alpha} = \frac{e^{-(b_1+b_2)t}}{2}|B(\chi)|^2\left[\cosh(\Delta_b t) - \cos(\Delta_a t)\right].$$

Term	Expression
$\mathrm{Re}(A)$	$-\cos 2\theta_m + \bar{\gamma}\bar{\Delta}_b \sin 2\theta_m \cos\chi$
$\mathrm{Im}(A)$	$-\bar{\gamma}\bar{\Delta}_a \sin 2\theta_m \cos\chi$
$\|A\|^2$	$\cos^2 2\theta_m - 2\bar{\gamma}\bar{\Delta}_b \sin 2\theta_m \cos 2\theta_m \cos\chi$
$\|B\|^2$	$\sin^2 2\theta_m + 2\bar{\gamma}\sin 2\theta_m\left(\bar{\Delta}_a \sin\chi + \bar{\Delta}_b \cos 2\theta_m \cos\chi\right)$

- Within the 2 flavor approximation, it is possible to get exact results as well.

2 flavor plots





$$\Delta P_{\mu\mu} \equiv P_{\mu\mu}(\text{analytical}) - P_{\mu\mu}(\text{exact})$$

$$e^{\mathbb{X}+\mathbb{Y}} \neq e^{\mathbb{X}} e^{\mathbb{Y}}$$

- $L = 295$ km, $E \sim 1$ GeV, $\Delta_a = 2.56 \times 10^{-3}$ eV$^2/(2E)$, $\theta_m = 45^\circ$, $(b_1, b_2, \gamma) = (3{,}6{,}8) \times 10^{-5}$ eV$^2/(2E)$, $\chi = \pi/4$.
- Now, let's move on to the more realistic 3-flavor scenario.

Case I: Decay of ν_3 only

- Strong constraints from solar neutrino data on ν_1 and ν_2 decay.

- Therefore, the special case where only ν_3 **mass eigenstate in vacuum decays**:

$$\mathcal{H}_f^{(\gamma_3)} = \frac{1}{2E_\nu} U \left[\Delta m^2_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} - i\, \Delta m^2_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \gamma_3 \end{pmatrix} \right] U^\dagger + \begin{pmatrix} V_{cc} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Here, γ_i is defined such that $\gamma_i \Delta m^2_{31} = m_i/\tau_i$.

$\alpha \equiv \Delta m^2_{21} / \Delta m^2_{31}$

- Current long-baseline constraints[†]: $\tau_3/m_3 > 1.5 \times 10^{-12}$ s/eV (3σ)

[†] arXiv:1805.01848

Case II: The general decay matrix Γ

- The solar neutrino constraint on decay is for neutrinos propagating in vacuum.
- For matter induced decay, the solar neutrino constraint may be relaxed.
- For the general decay matrix Γ we have:

$$\mathscr{H}_f^{(\Gamma)} = U\left[\frac{\Delta m_{31}^2}{2E_\nu}\begin{pmatrix}0 & 0 & 0\\ 0 & \alpha & 0\\ 0 & 0 & 1\end{pmatrix} - \frac{i}{2}\Gamma\right]U^\dagger + \begin{pmatrix}V_{cc} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},$$

$$\Gamma = \frac{\Delta m_{31}^2}{E_\nu}\begin{pmatrix}\gamma_1 & \frac{1}{2}\gamma_{12}e^{i\chi_{12}} & \frac{1}{2}\gamma_{13}e^{i\chi_{13}}\\ \frac{1}{2}\gamma_{12}e^{-i\chi_{12}} & \gamma_2 & \frac{1}{2}\gamma_{23}e^{i\chi_{23}}\\ \frac{1}{2}\gamma_{13}e^{-i\chi_{13}} & \frac{1}{2}\gamma_{23}e^{-i\chi_{23}} & \gamma_3\end{pmatrix}.$$

Formalism and scales

$$\alpha \approx 0.03 \simeq O(\lambda^2)\,, \qquad s_{13} \equiv \sin\theta_{13} \simeq 0.14 \simeq O(\lambda)\,.$$

- Decay has not been observed yet over the timescale of oscillations.
- Decay must be subleading to oscillation, i.e. decay length must be larger.
- Therefore, $\gamma_3 < O(1)$ and $\gamma_1, \gamma_2 < O(\alpha)$.

$$\boxed{\gamma_3 \sim O(\lambda)\,, \qquad \gamma_1\,, \gamma_2 \sim O(\lambda^3)\,.}$$

$\lambda \equiv 0.2$

Book-keeping parameter

- Decay matrix should be positive definite.

$$\boxed{\gamma_{12} \sim O(\lambda^3)\,, \qquad \gamma_{13}\,, \gamma_{23} \sim O(\lambda^2)\,.}$$

Probabilities expanded in s_{13}, α and γ_3

$$P_{\mu\mu}^{(0)} = 1 - \sin^2 2\theta_{23} \sin^2 \Delta - \frac{2}{A-1} s_{13}^2 \sin^2 2\theta_{23} \times \left(\sin\Delta \cos A\Delta \frac{\sin[(A-1)\Delta]}{A-1} - \frac{A}{2}\Delta \sin 2\Delta \right) - 4 s_{13}^2 s_{23}^2 \frac{\sin^2[(A-1)\Delta]}{(A-1)^2} + \alpha c_{12}^2 \sin^2 2\theta_{23} \Delta \sin 2\Delta + O(\lambda^3),$$

$$P_{\mu\mu}^{(\gamma_3)} = -\gamma_3 \Delta \left(\sin^2 2\theta_{23} \cos 2\Delta + 4 s_{23}^4\right) + \gamma_3^2 \Delta^2 \left(\sin^2 2\theta_{23} \cos 2\Delta + 8 s_{23}^4\right) + O(\lambda^3),$$

$$P_{\mu\mu}^{(\Gamma)} = \sin 2\theta_{23} \left(\gamma_{13} s_{12} \cos\chi_{13} - \gamma_{23} c_{12} \cos\chi_{23}\right) \times \sin 2\Delta + O(\lambda^3) \, .$$

with $P_{\mu e} = P_{e\mu}(\delta_{\mathrm{CP}} \to -\delta_{\mathrm{CP}}, \chi_{ij} \to -\chi_{ij})$.

$$A = \frac{2E_\nu V_{cc}}{\Delta m_{31}^2} \, , \, \Delta = \frac{\Delta m_{31}^2 L}{4E_\nu}$$

$$P_{\alpha\beta} = P_{\alpha\beta}^{(0)} + P_{\alpha\beta}^{(\gamma_3)} + P_{\alpha\beta}^{(\Gamma)}$$

$$P_{e\mu}^{(0)} = 4 s_{13}^2 s_{23}^2 \frac{\sin^2[(A-1)\Delta]}{(A-1)^2} + 2\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta - \delta_{\mathrm{CP}}) \times \frac{\sin[(A-1)\Delta]}{A-1} \frac{\sin A\Delta}{A} + O(\lambda^4) \, ,$$

$$P_{e\mu}^{(\gamma_3)} = -8\gamma_3 \, s_{13}^2 s_{23}^2 \, \Delta \, \frac{\sin^2[(A-1)\Delta]}{(A-1)^2} + O(\lambda^4) \, ,$$

$$P_{e\mu}^{(\Gamma)} = -4 s_{13} s_{23}^2 \left(\gamma_{23} \, s_{12} \sin[\delta_{\mathrm{CP}} + \chi_{23}] + \gamma_{13} \, c_{12} \sin[\delta_{\mathrm{CP}} + \chi_{13}]\right) \frac{\sin^2[(A-1)\Delta]}{(A-1)^2} + O(\lambda^4),$$

Key observations

- **The leading effect of ν_3 decay** at the muon neutrino **survival** channel.

- No matter effects in the muon neutrino survival channel.

- In the **conversion** channel, the effect of **off-diagonal decay terms are as important** as effects of γ_3.

- Matter dependence in the conversion channel decay terms.

Probabilities expanded in s_{13} and α, exact in γ_3

$$P_{\mu\mu} = \Bigg| c_{23}^2 + s_{23}^2 e^{-2i(1-i\gamma_3)\Delta} - 2i\alpha\, c_{12}^2 c_{23}^2\, \Delta + s_{13}^2 s_{23}^2 \Bigg(e^{-2iA\Delta} \frac{(1-i\gamma_3)^2}{[A-(1-i\gamma_3)]^2}$$
$$+ e^{-2i(1-i\gamma_3)\Delta} \Big[2iA\Delta\, [A-(1-i\gamma_3)] - (1-i\gamma_3) \Big] \frac{1-i\gamma_3}{[A-(1-i\gamma_3)]^2} \Bigg) \Bigg|^2 + O(\lambda^3)\,.$$

$$P_{\mu\mu}^{\text{leading}} = c_{23}^4 + s_{23}^4\, e^{-4\gamma_3\Delta} + 2 s_{23}^2 c_{23}^2 \cos(2\Delta) e^{-2\gamma_3\Delta}$$
$$= 1 - \sin^2 2\theta_{23} \sin^2\Delta - s_{23}^4 \left(1 - e^{-4\gamma_3\Delta}\right) - 2 s_{23}^2 c_{23}^2 \cos(2\Delta) \left(1 - e^{-2\gamma_3\Delta}\right)\,.$$

- Exact dependence on γ_3.
- The region of validity increases to $\alpha\Delta \lesssim 1$ from $\gamma_3\Delta \lesssim 1$.
- Valid at lower energies.

$$P_{e\mu} = s_{13}^2 s_{23}^2 \left(1 + e^{-4\gamma_3\Delta} - 2e^{-2\gamma_3\Delta} \cos[2(A-1)\Delta]\right) \frac{\gamma_3^2 + 1}{(A-1)^2 + \gamma_3^2}$$
$$+ \alpha\, s_{13}\, \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin A\Delta}{A}$$
$$\times \Bigg[\left(\sin\left[(A-2)\Delta + \delta_{\rm CP}\right] e^{-2\gamma_3\Delta} + \sin\left[A\Delta - \delta_{\rm CP}\right] \right) \frac{(A-1) - \gamma_3^2}{(A-1)^2 + \gamma_3^2}$$
$$+ \gamma_3 \left(\cos\left[A\Delta - \delta_{\rm CP}\right] - \cos\left[(A-2)\Delta + \delta_{\rm CP}\right] e^{-2\gamma_3\Delta} \right) \frac{A}{(A-1)^2 + \gamma_3^2} \Bigg] + O(\lambda^4)\,.$$

Analytic vs Numerical Comparison

- **How accurate are our expressions?**
- Let us define: $\boxed{\Delta P_{\alpha\beta} = P_{\alpha\beta}(\text{analytic}) - P_{\alpha\beta}(\text{numerical})\ .}$
- We plot for the values:

$$\Delta m^2_{21} = 7.37 \times 10^{-5}\ \text{eV}^2,\ \Delta m^2_{31} = 2.56 \times 10^{-3}\ \text{eV}^2,$$

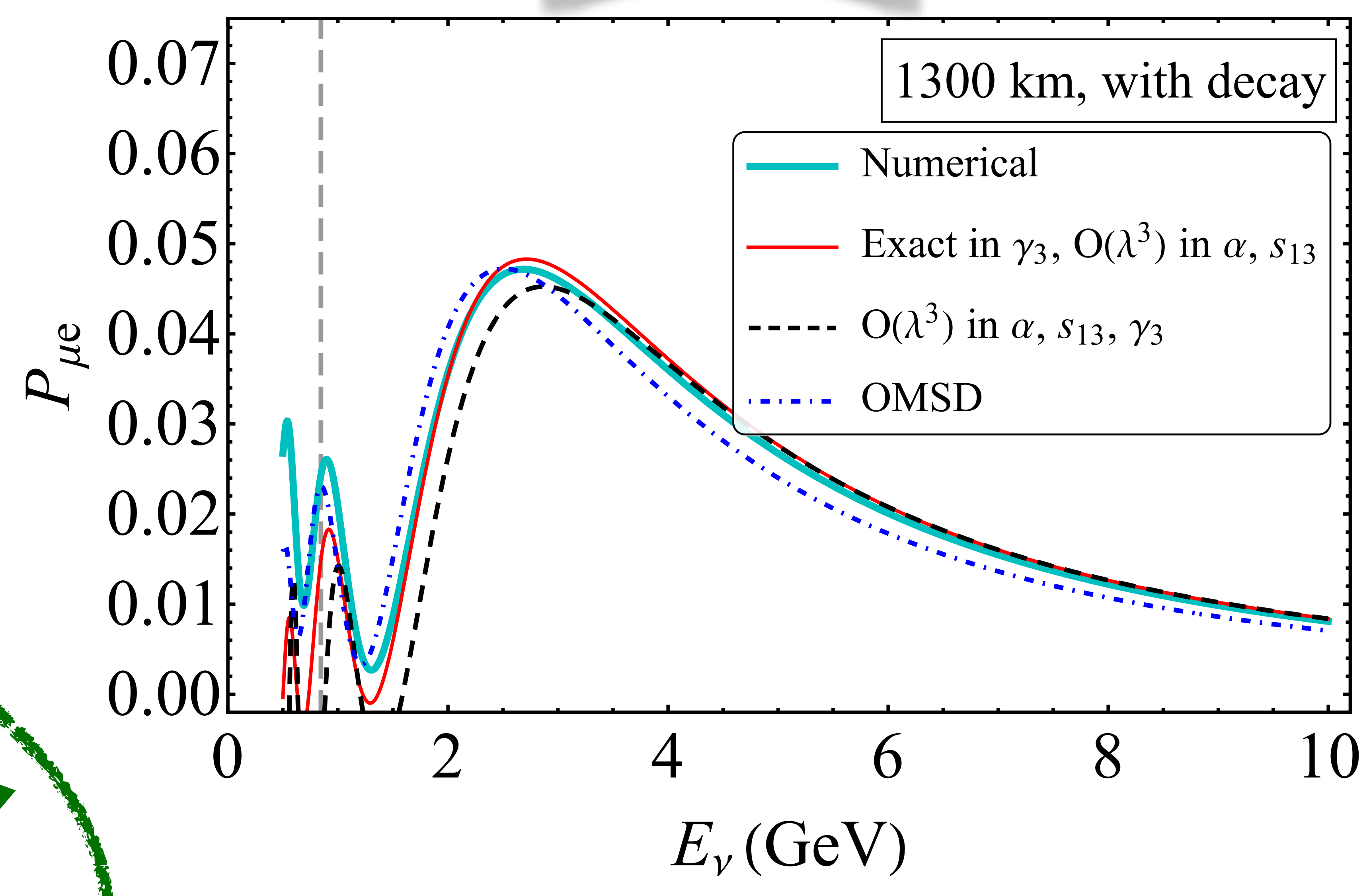
$$\theta_{12} = 33^\circ,\ \theta_{23} \simeq 45^\circ,\ \theta_{13} \simeq 8.5^\circ,\ \delta_{\text{CP}} = 0^\circ, \gamma_3 = 0.1$$

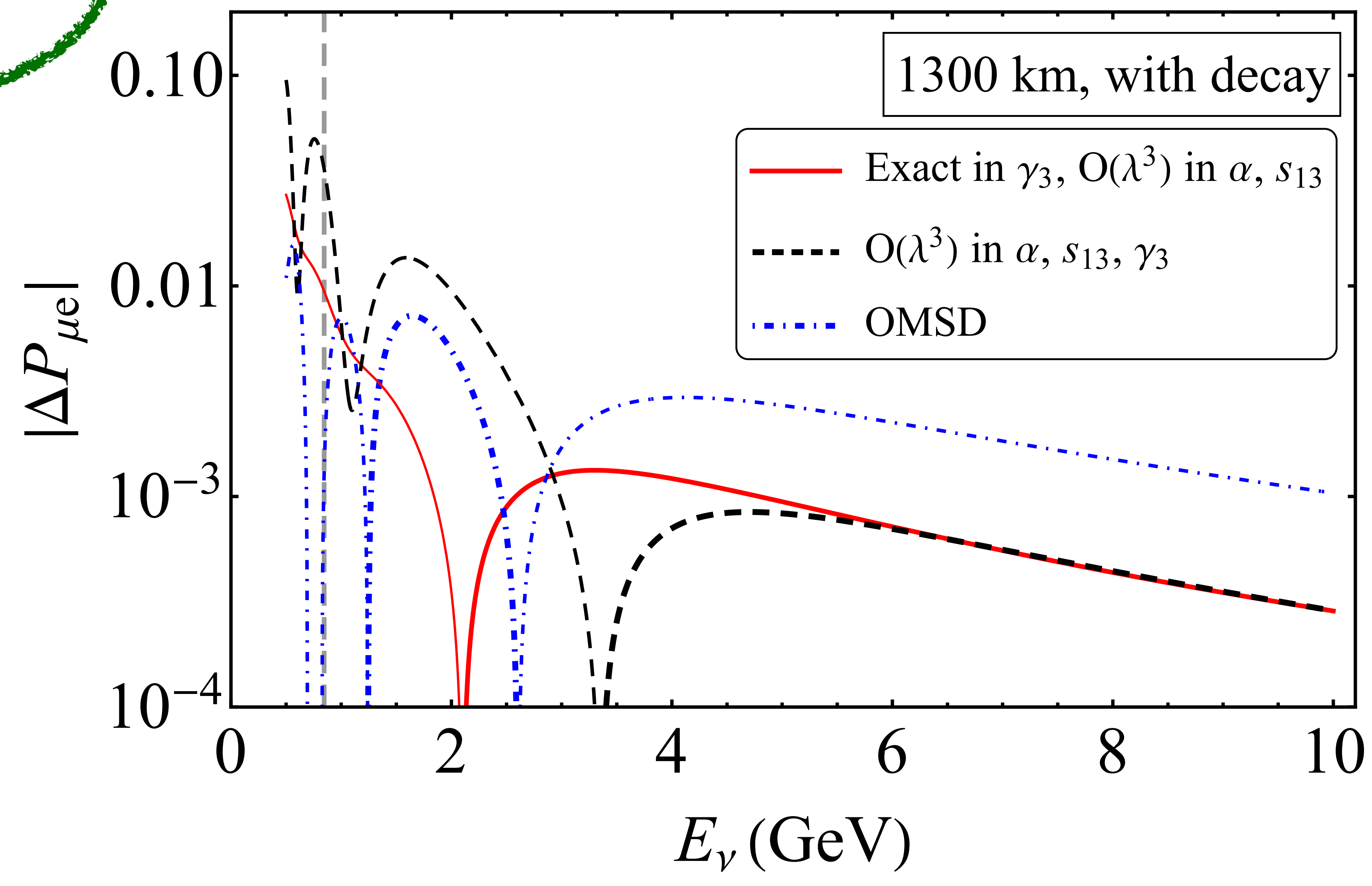
- For the expressions to be useful for future long baseline experiments, we need an absolute accuracy of ~1% in the conversion channel.

1300 km

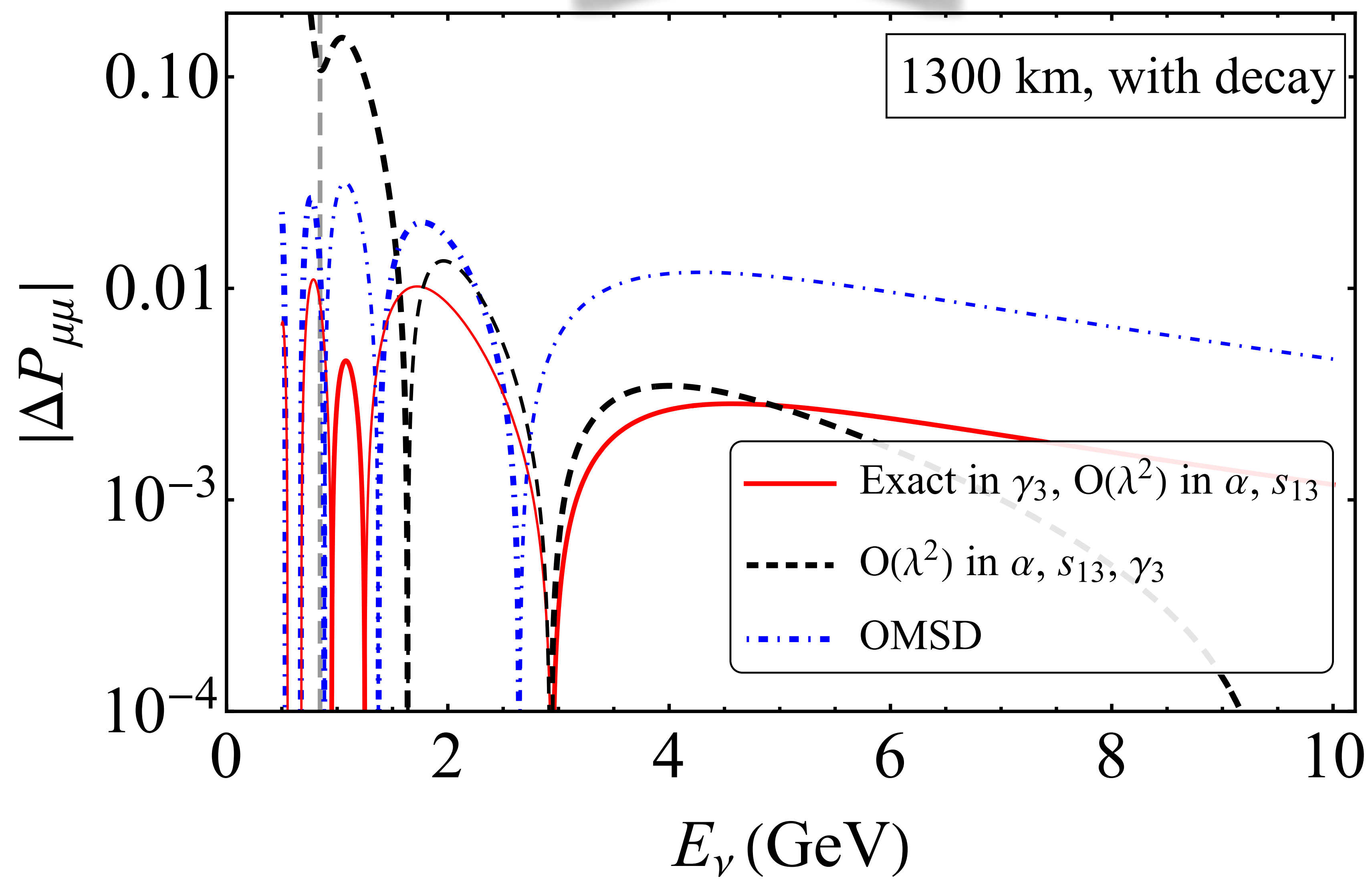
Accuracy Checked!

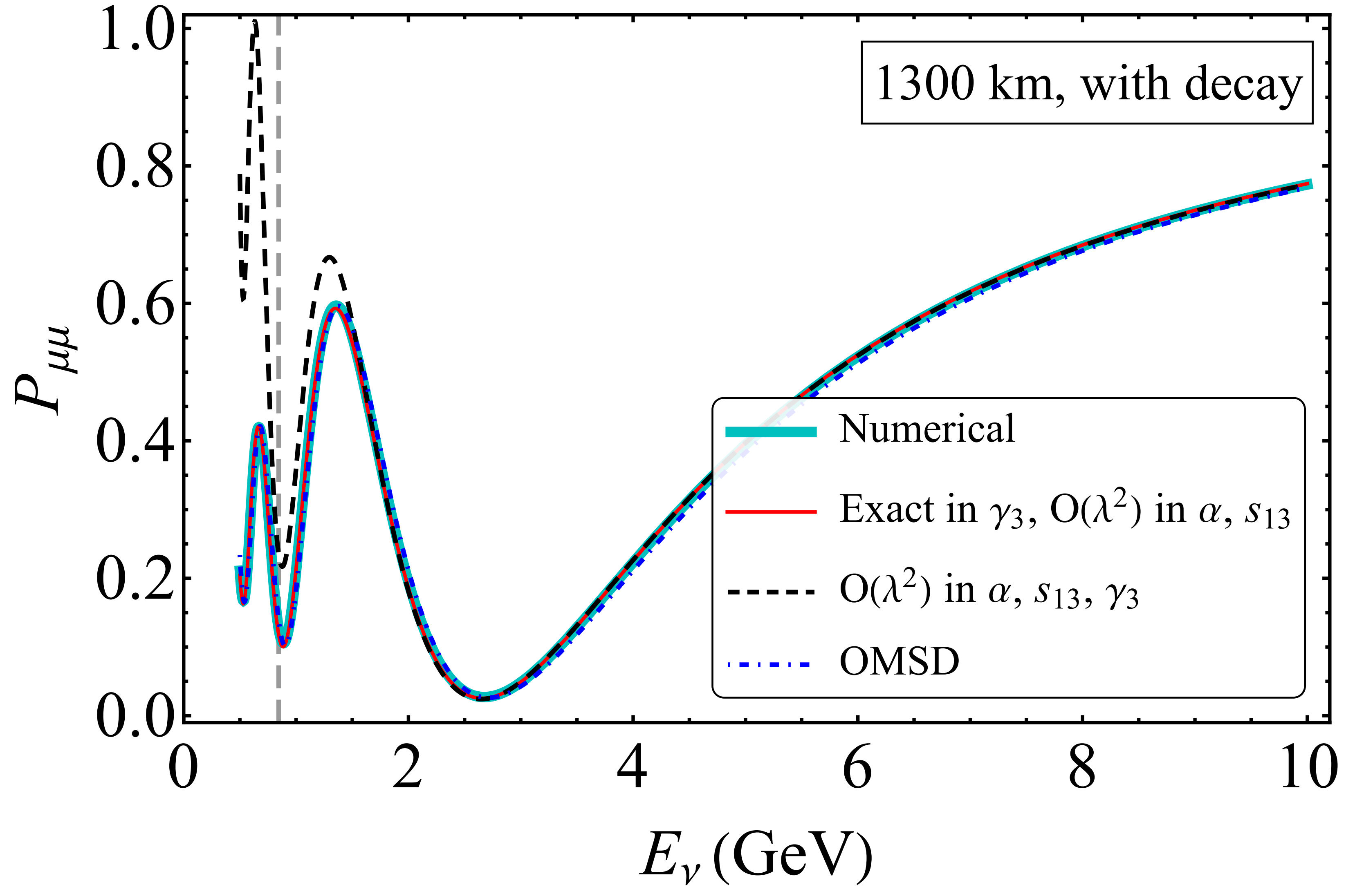
$\nu_\mu \to \nu_e$





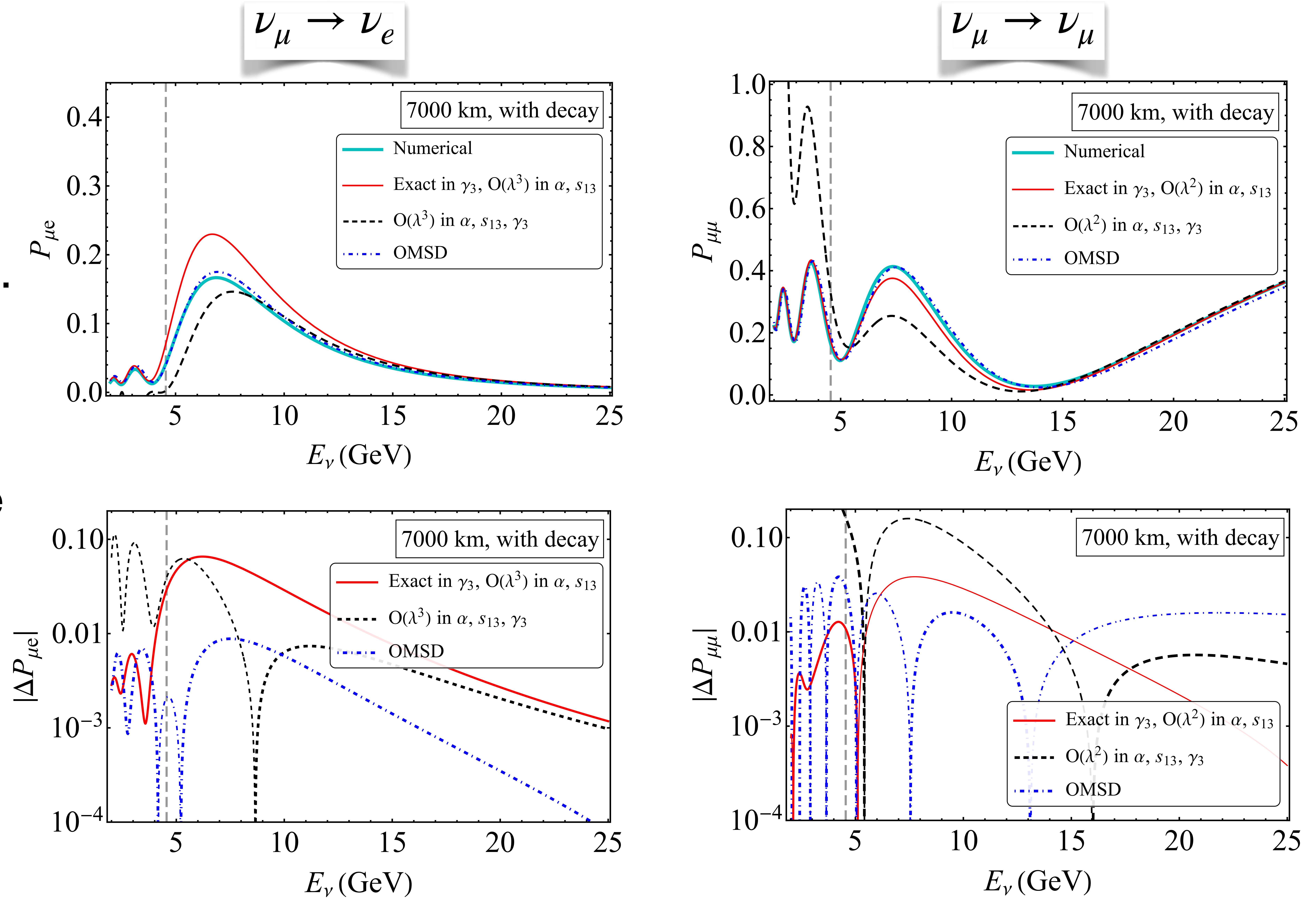
$\nu_\mu \to \nu_\mu$



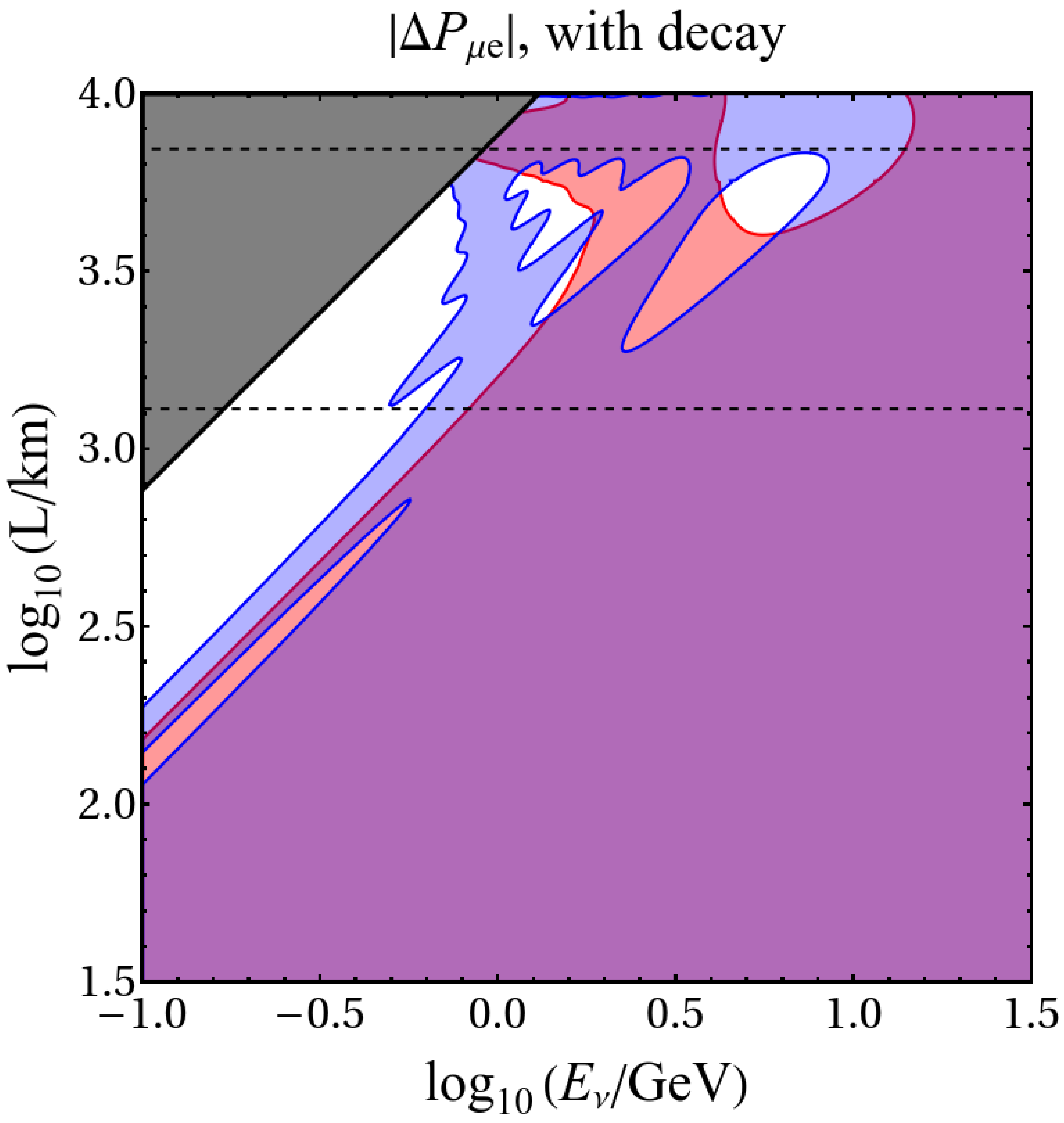


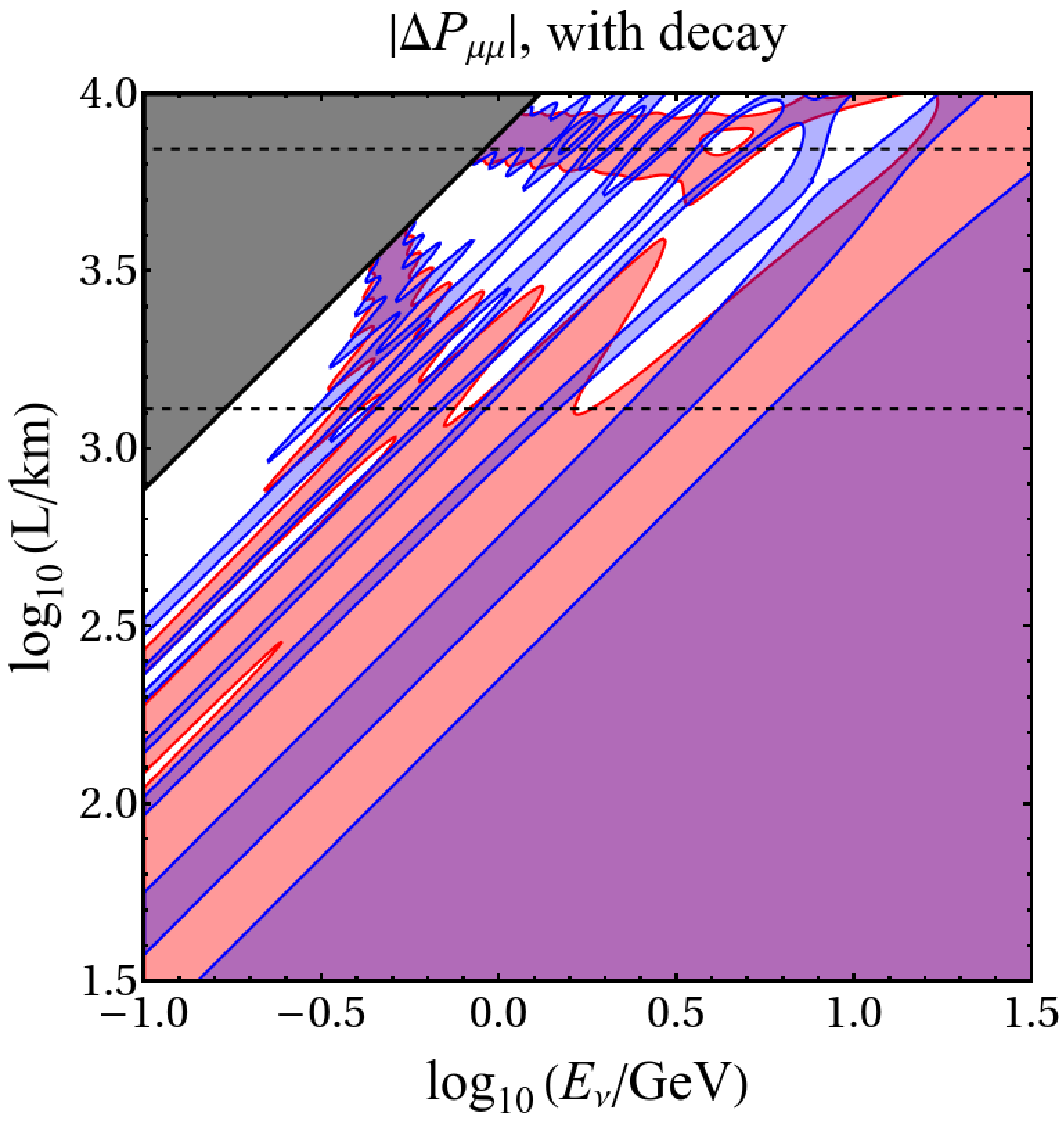
7000 km

- OMSD approx. is very accurate.
- Absolute accuracy ~ 1%
- Depends on the value of δ_{CP} taken.



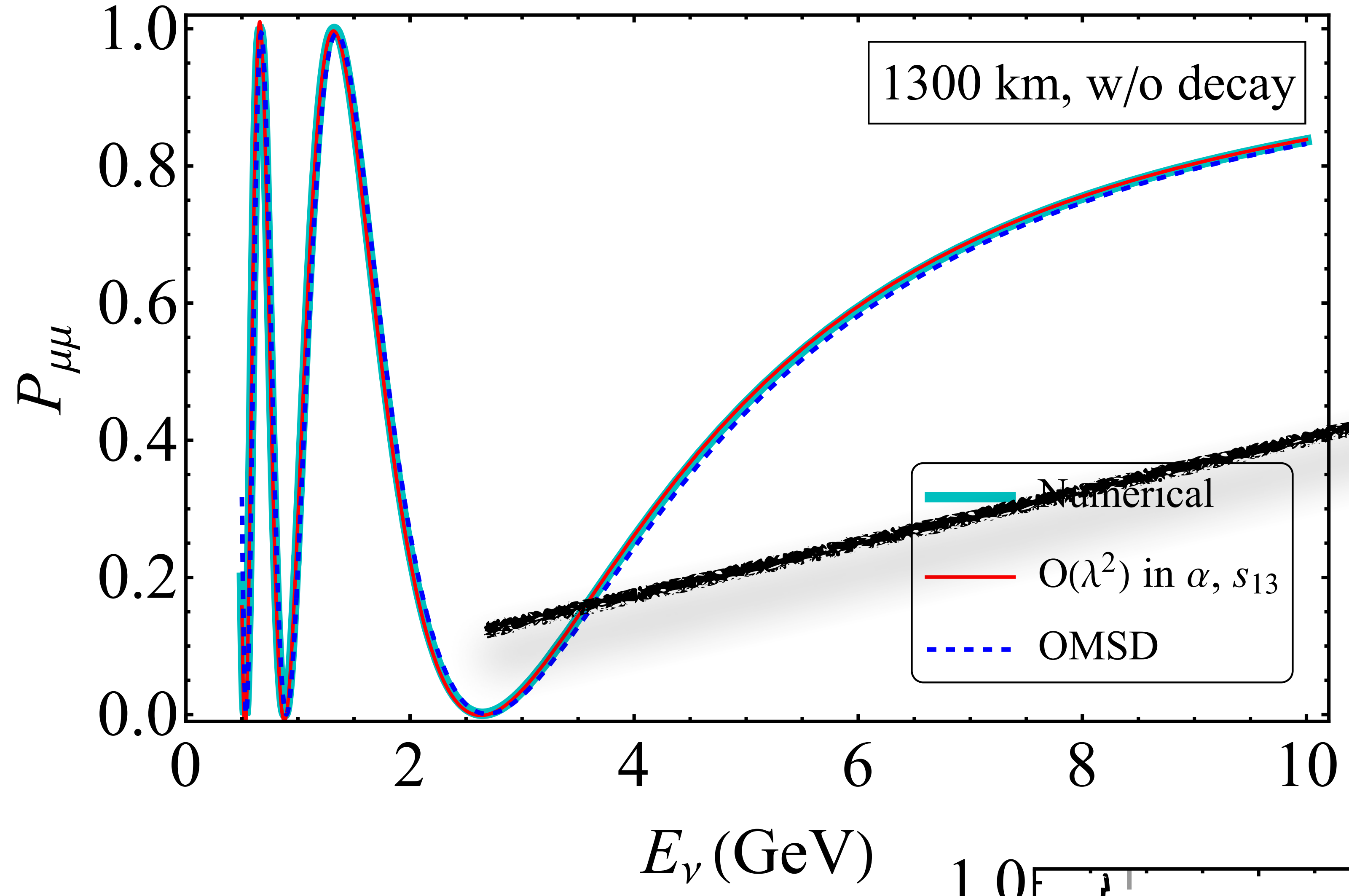
General Baseline

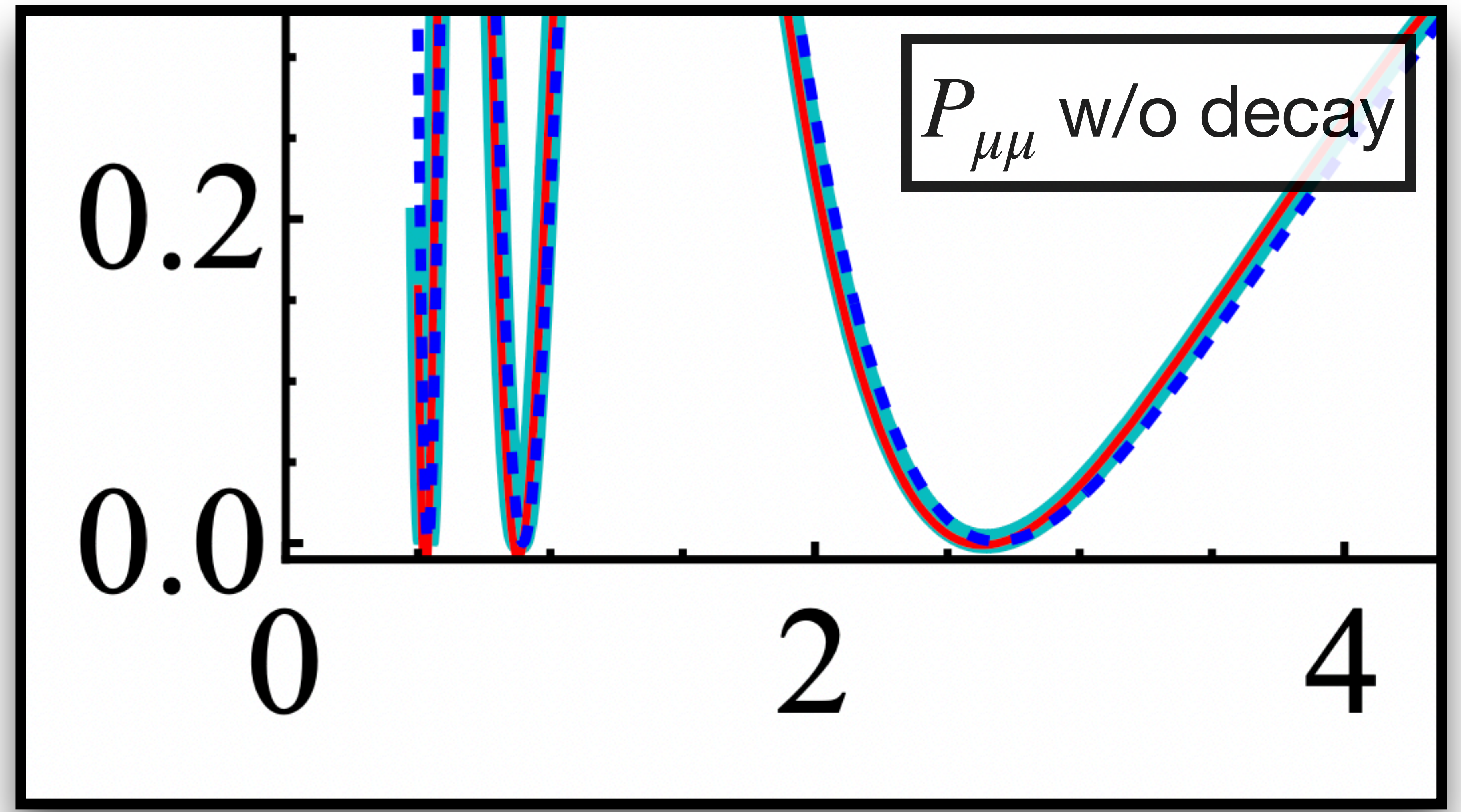


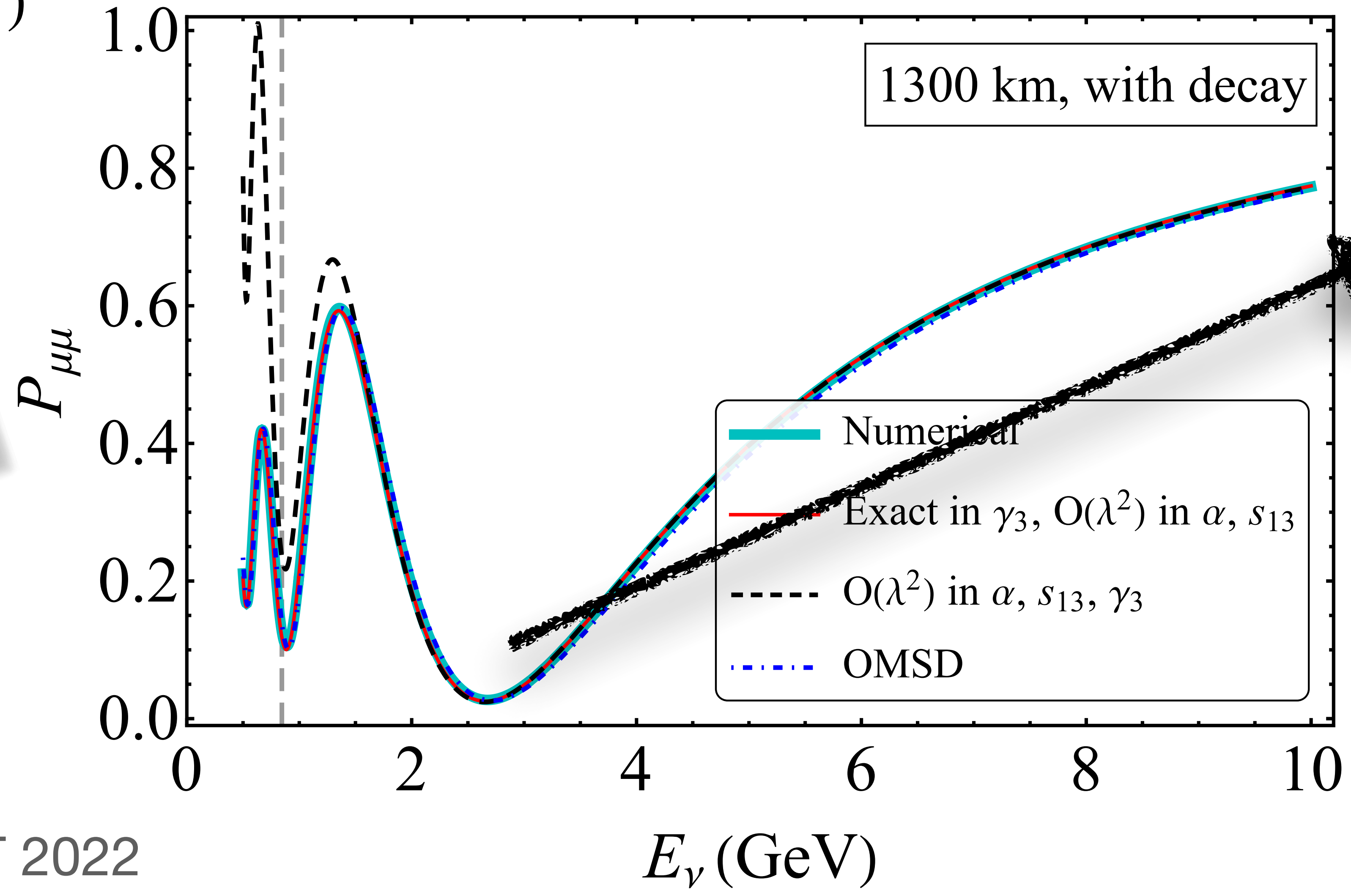


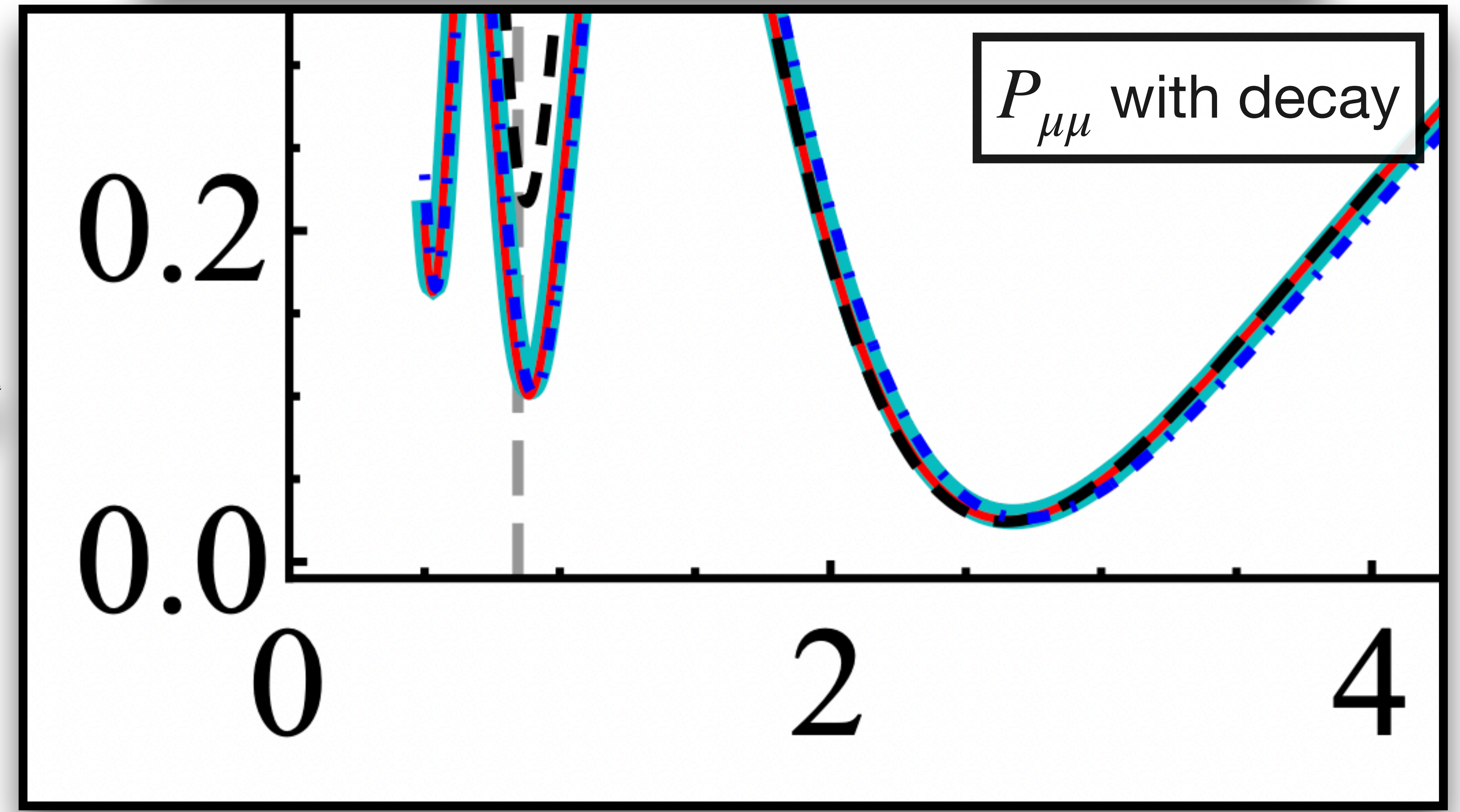
Increase of probability due to decay!

- $P_{\mu\mu}$ at the oscillation dips, 1300 km



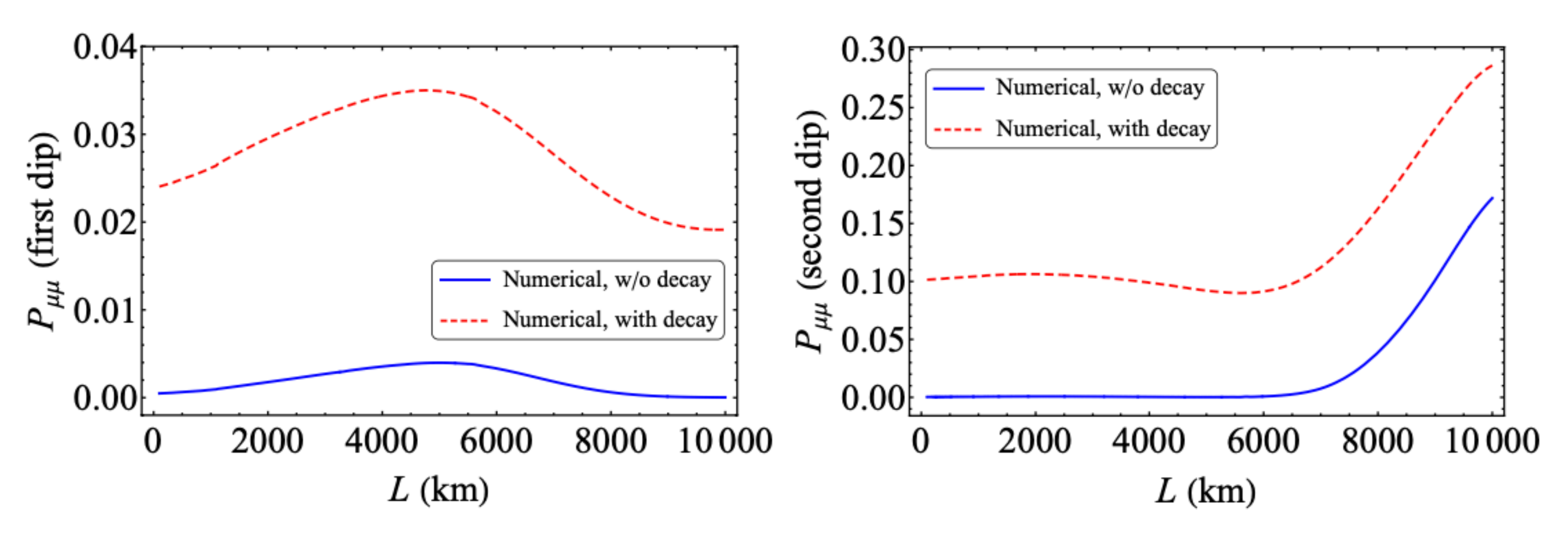






Comparing the probability with and without decay

The first two oscillation dips in $P_{\mu\mu}$



$$P_{\mu\mu}^{\text{leading}}(\text{dip}) = 1 - \sin^2 2\theta_{23} - s_{23}^4 \left(1 - e^{-4\gamma_3 \Delta}\right) + 2 s_{23}^2 c_{23}^2 \left(1 - e^{-2\gamma_3 \Delta}\right)$$

$$P_{\mu\mu}(\text{first dip}) \simeq P_{\mu\mu}^{\text{leading}}(\Delta \simeq \pi/2) = \frac{1}{4}\left(1 - e^{-\pi\gamma_3}\right)^2 \geq 0$$

$$P_{\mu\mu}(\text{second dip}) \simeq P_{\mu\mu}^{\text{leading}}(\Delta \simeq 3\pi/2) \simeq \frac{1}{4}\left(1 - e^{-3\pi\gamma_3}\right)^2 \geq 0$$

- **Increase in probability** at first and second osc. dips due to ν_3 decay.
- For $\gamma_3 = 0.1$, increase of ~0.02 at first and ~0.1 at second oscillation dip.
- **Explained by our analytic expressions (like a damped oscillator**).
- The second osc. dip at: $E_\nu \simeq 0.69\,(L/1000\text{ km})$ GeV. Hence possible to observe at DUNE.

Take Home Message

- If neutrinos decay, **mismatch** between mass and decay eigenstates is **inevitable**.
- We have presented the modifications to the neutrino probabilities due to possible **invisible decay in matter**, in a **compact analytic form**.
- Analytic expressions can explain many features of the probabilities: for example, $P_{\mu\mu}$ at oscillation dips increases due to ν_3 decay.

Thank you for your attention

1. D. S. Chattopadhyay, K. Chakraborty, A. Dighe, S. Goswami and S. M. Lakshmi, *"Neutrino Propagation When Mass Eigenstates and Decay Eigenstates Mismatch"*, **Phys. Rev. Lett. 129, no.1, 011802 (2022)** (arXiv:2111.13128 [hep-ph])

2. Dibya S. Chattopadhyay, Kaustav Chakraborty, Amol Dighe, Srubabati Goswami, *"Analytic treatment of 3-flavor neutrino oscillation and decay in matter"*, **arXiv:2204.05803 [hep-ph]**

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