Oscillation and decay of neutrinos in matter: an analytic treatment

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4 Aug 2022
NuFACT 2022, Utah, USA

and arXiv:2204.05803 [hep-ph]

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Objective

If neutrinos decay, the effective non-Hermitian Hamiltonian needs to be treated carefully due to subtle issues regarding its mass and decay components.

We derive compact analytic expressions for 2-flavor 3-flavor neutrino probabilities with:

- Invisible decay + Oscillation + Explicit matter effects included.

Useful for:

1. Long-baseline neutrino experiments
2. Atmospheric neutrino experiments
3. Reactor anti-neutrino experiments
The problem

• The inclusion of decay makes the effective Hamiltonian non-Hermitian

\[ \mathcal{H} = H - i\Gamma/2 \]

\[ \Gamma_{ij} = 2\pi \sum_k \langle \nu_i | \mathcal{H}' | \phi_k \rangle \langle \phi_k | \mathcal{H}' | \nu_j \rangle \delta(E_k - E_\nu) \]

• The decay and the mass eigenstates need not be the same ⇒ \textit{Mismatch}

\[ [H, \Gamma] \neq 0 \]

• Even if there’s no mismatch in vacuum, due to matter effects, the components will invariably become non-commuting.
Inevitability of the off-diagonal elements

- In the 2-flavor approximation:

\[
\mathcal{H}_m = \begin{pmatrix}
    a_1 - i b_1 & -\frac{1}{2} i \gamma e^{i\chi} \\
    -\frac{1}{2} i \gamma e^{-i\chi} & a_2 - i b_2
\end{pmatrix}
\]

- Even if only \(\nu_2\) in vacuum decays, with \(\alpha_2 = m_2/\tau_2\), in matter, we get:

\[
a_{1,2} = \frac{\tilde{m}^2_{1,2}}{2E} \quad , \quad b_{1,2} = \frac{\alpha_2}{4E} [1 \mp \cos[2(\theta - \theta_m)]] ,
\]

\[
\chi = 0 \quad , \quad \gamma = \frac{\alpha_2}{2E} \sin[2(\theta - \theta_m)] .
\]

- The off-diagonal term \(\gamma\) is generated, even though it was absent in vacuum.

- Inevitable “mismatch” in matter.

- We develop techniques using Zassenhaus (inverse BCH) expansion and Cayley-Hamilton theorem.
2 flavor expressions

- We use the inverse BCH (Zassenhaus) expansion to calculate the probabilities.
- The survival probability of a neutrino flavor is

\[ P_{\alpha\alpha} = \frac{e^{-(b_1+b_2)t}}{2} \left[(1 + |A|^2)\cosh(\Delta_b t) + (1 - |A|^2)\cos(\Delta_\alpha t) - 2\text{Re}(A)\sinh(\Delta_b t) + 2\text{Im}(A)\sin(\Delta_\alpha t)\right]. \]

- The conversion probability is given by

\[ P_{\beta\alpha} = \frac{e^{-(b_1+b_2)t}}{2} |B(\chi)|^2 \left[\cosh(\Delta_b t) - \cos(\Delta_\alpha t)\right]. \]

- Within the 2 flavor approximation, it is possible to get exact results as well.
2 flavor plots

\[ \Delta P_{\mu\mu} \equiv P_{\mu\mu}^{\text{(analytical)}} - P_{\mu\mu}^{\text{(exact)}} \]

- \( L = 295 \text{ km, } E \sim 1 \text{ GeV, } \Delta_a = 2.56 \times 10^{-3} \text{ eV}^2/(2E), \theta_m = 45^\circ, (b_1, b_2, \gamma) = (3,6,8) \times 10^{-5} \text{ eV}^2/(2E), \chi = \pi/4 \).

- Now, let's move on to the more realistic 3-flavor scenario.
Case I: Decay of $\nu_3$ only

- Strong constraints from solar neutrino data on $\nu_1$ and $\nu_2$ decay.

- Therefore, the special case where only $\nu_3$ mass eigenstate in vacuum decays:

$$H_f^{(\nu_3)} = \frac{1}{2E_\nu} U \left[ \Delta m_{31}^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} - i \Delta m_{31}^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \gamma_3 \end{pmatrix} \right] U^\dagger + \begin{pmatrix} V_{cc} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Here, $\gamma_i$ is defined such that $\gamma_i \Delta m_{31}^2 = m_i/\tau_i$.

- Current long-baseline constraints$^\dagger$: $\tau_3/m_3 > 1.5 \times 10^{-12}$ s/eV (3$\sigma$)

$^\dagger$arXiv:1805.01848
Case II: The general decay matrix $\Gamma$

- The solar neutrino constraint on decay is for neutrinos propagating in vacuum.
- For matter induced decay, the solar neutrino constraint may be relaxed.
- For the general decay matrix $\Gamma$ we have:

$$\mathcal{H}_f^{(\Gamma)} = U \left[ \frac{\Delta m_{31}^2}{2E_\nu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{i}{2} \Gamma \right] U^\dagger + \begin{pmatrix} V_{cc} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\Gamma = \frac{\Delta m_{31}^2}{E_\nu} \begin{pmatrix} \gamma_1 & \frac{1}{2} \gamma_{12} e^{i\chi_{12}} & \frac{1}{2} \gamma_{13} e^{i\chi_{13}} \\ \frac{1}{2} \gamma_{12} e^{-i\chi_{12}} & \gamma_2 & \frac{1}{2} \gamma_{23} e^{i\chi_{23}} \\ \frac{1}{2} \gamma_{13} e^{-i\chi_{13}} & \frac{1}{2} \gamma_{23} e^{-i\chi_{23}} & \gamma_3 \end{pmatrix}. $$
Formalism and scales

\[ \alpha \approx 0.03 \sim O(\lambda^2) , \quad s_{13} \equiv \sin \theta_{13} \approx 0.14 \sim O(\lambda) . \]

- Decay has not been observed yet over the timescale of oscillations.
- Decay must be subleading to oscillation, i.e. decay length must be larger.
- Therefore, \( \gamma_3 < O(1) \) and \( \gamma_1, \gamma_2 < O(\alpha) \).

\[
\begin{align*}
\gamma_3 & \sim O(\lambda) , \\
\gamma_1, \gamma_2 & \sim O(\lambda^3) .
\end{align*}
\]

- Decay matrix should be positive definite.

\[
\begin{align*}
\gamma_{12} & \sim O(\lambda^3) , \\
\gamma_{13}, \gamma_{23} & \sim O(\lambda^2) .
\end{align*}
\]
Probabilities expanded in $s_{13}$, $\alpha$ and $\gamma_3$

\[
P^{(0)}_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \Delta - \frac{2}{A-1} s^2_{13} \sin^2 2\theta_{23}
\]
\[
\times \left( \sin \Delta \cos A\Delta \frac{\sin[(A-1)\Delta]}{A-1} - \frac{A}{2} \Delta \sin 2\Delta \right)
\]
\[
- 4s^2_{13}s^2_{23} \sin^2 [(A-1)\Delta] \right.
\]
\[
+ \alpha c^2_{12} \sin^2 2\theta_{23} \Delta \sin 2\Delta + O(\lambda^3),
\]

\[
P^{(\gamma_3)}_{\mu\mu} = -\gamma_3 \Delta \left( \sin^2 2\theta_{23} \cos 2\Delta + 4s^4_{23} \right)
\]
\[
+ \gamma_3^2 \Delta^2 \left( \sin^2 2\theta_{23} \cos 2\Delta + 8s^4_{23} \right) + O(\lambda^3),
\]

\[
P^{(\Gamma)}_{\mu\mu} = \sin 2\theta_{23} \left( \gamma_{13}s_{12} \cos \chi_{13} - \gamma_{23}c_{12} \cos \chi_{23} \right)
\]
\[
\times \sin 2\Delta + O(\lambda^3) .
\]

with $P_{\mu e} = P_{e\mu}(\delta_{\text{CP}} \to -\delta_{\text{CP}}, \chi_{ij} \to -\chi_{ij})$.

\[
A = \frac{2E_cV_{cc}}{\Delta m^2_{31}}, \quad \Delta = \frac{\Delta m^2_{31}L}{4E_c}
\]

\[
P_{\alpha\beta} = P^{(0)}_{\alpha\beta} + P^{(\gamma_3)}_{\alpha\beta} + P^{(\Gamma)}_{\alpha\beta}
\]

\[
P^{(0)}_{e\mu} = 4s^2_{13}s^2_{23} \frac{\sin^2 [(A-1)\Delta]}{(A-1)^2}
\]
\[
+ 2\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos (\Delta - \delta_{\text{CP}})
\]
\[
\times \frac{\sin[(A-1)\Delta] \sin A\Delta}{A-1} + O(\lambda^4),
\]

\[
P^{(\gamma_3)}_{e\mu} = -8\gamma_3 s^2_{13}s^2_{23} \Delta \frac{\sin^2 [(A-1)\Delta]}{(A-1)^2} + O(\lambda^4),
\]

\[
P^{(\Gamma)}_{e\mu} = -4s_{13}s^2_{23} \gamma_{23} s_{12} \sin [\delta_{\text{CP}} + \chi_{23}]
\]
\[
+ \gamma_{13} c_{12} \sin [\delta_{\text{CP}} + \chi_{13}] \frac{\sin^2 [(A-1)\Delta]}{(A-1)^2}
\]
\[
+ O(\lambda^4),
\]
Key observations

• The leading effect of $\nu_3$ decay at the muon neutrino survival channel.

• No matter effects in the muon neutrino survival channel.

• In the conversion channel, the effect of off-diagonal decay terms are as important as effects of $\gamma_3$.

• Matter dependence in the conversion channel decay terms.
Probabilities expanded in $s_{13}$ and $\alpha$, exact in $\gamma_3$

\[
P_{\mu\mu} = c_{23}^2 + s_{23}^2 e^{-2i(1-i\gamma_3)\Delta} - 2i\alpha c_{12}^2 c_{23}^2 \Delta + s_{13}^2 s_{23}^2 \left( e^{-2iA\Delta} \frac{(1-i\gamma_3)^2}{[A-(1-i\gamma_3)]^2} \right)
+ e^{-2i(1-i\gamma_3)\Delta} \left[ 2iA\Delta [A-(1-i\gamma_3)] - (1-i\gamma_3) \right] \frac{1-i\gamma_3}{[A-(1-i\gamma_3)]^2} \right)^2 + O(\lambda^3).
\]

\[
P_{\mu\mu}^{\text{leading}} = c_{23}^4 + s_{23}^4 e^{-4\gamma_3\Delta} + 2s_{23}^2 c_{23}^2 \cos(2\Delta)e^{-2\gamma_3\Delta}
= 1 - \sin^2 2\theta_{23} \sin^2 \Delta - s_{23}^4 \left( 1 - e^{-4\gamma_3\Delta} \right) - 2s_{23}^2 c_{23}^2 \cos(2\Delta) \left( 1 - e^{-2\gamma_3\Delta} \right).
\]

- Exact dependence on $\gamma_3$.
- The region of validity increases to $\alpha \Delta \lesssim 1$ from $\gamma_3 \Delta \lesssim 1$.
- Valid at lower energies.

\[
P_{e\mu} = s_{13}^2 s_{23}^2 \left( 1 + e^{-4\gamma_3\Delta} - 2e^{-2\gamma_3\Delta} \cos[2(A-1)\Delta] \right) \frac{\gamma_3^2 + 1}{(A-1)^2 + \gamma_3^2}
+ \alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin A\Delta}{A}
\times \left[ \left( \sin [(A-2)\Delta + \delta_{CP}] e^{-2\gamma_3\Delta} + \sin [A\Delta - \delta_{CP}] \right) \frac{(A-1) - \gamma_3^2}{(A-1)^2 + \gamma_3^2}
+ \gamma_3 \left( \cos [A\Delta - \delta_{CP}] - \cos [(A-2)\Delta + \delta_{CP}] e^{-2\gamma_3\Delta} \right) \frac{A}{(A-1)^2 + \gamma_3^2} \right] + O(\lambda^4).
\]
Analytic vs Numerical Comparison

• How accurate are our expressions?

• Let us define: \[ \Delta P_{\alpha\beta} = P_{\alpha\beta}\text{(analytic)} - P_{\alpha\beta}\text{(numerical)} \].

• We plot for the values:

\[ \Delta m_{21}^2 = 7.37 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{31}^2 = 2.56 \times 10^{-3} \text{ eV}^2, \]

\[ \theta_{12} = 33^\circ, \quad \theta_{23} \simeq 45^\circ, \quad \theta_{13} \simeq 8.5^\circ, \quad \delta_{\text{CP}} = 0^\circ, \quad \gamma_3 = 0.1 \]

• For the expressions to be useful for future long baseline experiments, we need an absolute accuracy of \(~1\%\) in the conversion channel.
Accuracy Checked!

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7000 km

- OMSD approx. is very accurate.
- Absolute accuracy ~ 1%
- Depends on the value of $\delta_{CP}$ taken.

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General Baseline

$|\Delta P_{\mu e}|$, with decay

$|\Delta P_{\mu\mu}|$, with decay

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Increase of probability due to decay!

- $P_{\mu\mu}$ at the oscillation dips, 1300 km

Comparing the probability with and without decay
The first two oscillation dips in $P_{\mu\mu}$

- Increase in probability at first and second osc. dips due to $\nu_3$ decay.
- For $\gamma_3 = 0.1$, increase of ~0.02 at first and ~0.1 at second oscillation dip.
- Explained by our analytic expressions (like a damped oscillator).
- The second osc. dip at: $E_\nu \simeq 0.69 \,(L/1000 \text{ km})$ GeV. Hence possible to observe at DUNE.
If neutrinos decay, mismatch between mass and decay eigenstates is inevitable.

We have presented the modifications to the neutrino probabilities due to possible invisible decay in matter, in a compact analytic form.

Analytic expressions can explain many features of the probabilities: for example, $P_{\mu\mu}$ at oscillation dips increases due to $\nu_3$ decay.
Thank you for your attention


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