Majorana Phase in Neutrino Oscillation

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Plan of this talk

1. Motivation
2. Majorana phase in neutrino oscillations
3. CP and CPT properties of oscillation probabilities

Introducing Majorana Phase ϕ

- We restrict ourselves to two flavor oscillations. The flavor states ν_e and ν_μ mix to form the mass eigenstates ν_1 and ν_2 with masses m_1 and m_2, respectively.
- In general, flavour states ν_α are related to mass eigenstates ν_i as

$$\nu_\alpha = U\ \nu_i = O\ U_{ph}\ \nu_i ,$$

where $\nu_\alpha = (\nu_e \quad \nu_\mu)^T$ and $\nu_i = (\nu_1 \quad \nu_2)^T$ and

$$O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad U_{ph} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} . \tag{1}$$

- The phase ϕ is unphysical if neutrinos are Dirac particles but is physical if they are Majorana.
- However, both vacuum and matter modified oscillation probabilities are independent of ϕ whether neutrinos are Dirac or Majorana.

Appearance of ϕ in oscillation probabilities

- Benatti and Floreanini (PRD **64** (2001) 085015) considered a novel form of neutrino decoherence with an off-diagonal term in the decoherence matrix.
- For such decoherence, the oscillation probabilities depend on ϕ.

We ask "what are the other possibilities under which the Majorana phase ϕ appears in neutrino oscillation probabilities?"

The Hamiltonian in mass eigenbasis

$$\mathcal{H} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} = \frac{(a_1 + a_2)}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -(a_2 - a_1) & 0 \\ 0 & a_2 - a_1 \end{pmatrix},$$

where $a_1 = m_1^2/2E$ and $a_2 = m_2^2/2E$, $m_i \rightarrow$ mass eigenvalues, $E \rightarrow$ energy.

Evolution equations in mass eigenbasis

$$i\frac{d}{dt}\nu_i(t) = \left[\frac{(a_1 + a_2)}{2}\sigma_0 - \frac{a_2 - a_1}{2}\sigma_z\right]\nu_i(t), \qquad (2)$$

where, $\sigma_0 \equiv \mathbf{I}_{2\times2}$ and σ_z is the diagonal Pauli matrix.

In flavor basis $(\nu_i(0) = U_{mix}^\dagger \nu_f(0) = U_{ph}^\dagger O^T \nu_f(0))$

$$i\frac{d}{dt}\nu_\alpha(t) = \left[\frac{(a_1 + a_2)}{2}\sigma_0 - \frac{(a_2 - a_1)}{2} O U_{ph} \sigma_z U_{ph}^\dagger O^T\right]\nu_\alpha(t). \qquad (3)$$

Since

$$O U_{ph} \sigma_z U_{ph}^\dagger O^T = O \sigma_z O^T \qquad ([U_{ph}, \sigma_z] = 0)$$

$\Rightarrow$ U_{ph} matrix containing ϕ is gone!

If, $\mathcal{H} = ((-\frac{(a_2-a_1)}{2})\sigma_z + b\sigma_x + c\sigma_y)$

$$\boxed{OU_{ph}[-\frac{(a_2-a_1)}{2}\sigma_z + b\sigma_x + c\sigma_y]U_{Ph}^{\dagger}O^T \neq O\sigma_z O^T},$$

where σ_x and σ_y are the off-diagonal Pauli matrices. It is trivial to see

$$[U_{ph}, \sigma_x] \neq 0 \;\; \text{and} \;\; [U_{ph}, \sigma_y] \neq 0$$

- We consider the case of the most general Hamiltonian in neutrino mass basis including the decay terms
$$\mathcal{H} = M - i\Gamma/2,$$
where M and Γ are hemitian matrices.
- Presence of Γ makes $\mathcal{H}$ non-hermitian.
- Since $\mathcal{H}$ is in mass basis, M is diagonal, but Γ need not be in general.
- We assume that the decay eigenstates are not the same as mass eigenstates.
- Hence Γ is non-diagonal and does not commute with M.

Neutrino oscillations with most general decay Hamiltonian

- We consider the following form of the Hamiltonian

$$\mathcal{H} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} - i \begin{pmatrix} b_1 & \frac{1}{2}\eta e^{i\xi} \\ \frac{1}{2}\eta e^{-i\xi} & b_2 \end{pmatrix}.$$

- Γ should be positive semi-definite ($b_i > 0$ and $\eta \leq 4b_1^2 b_2^2$).
- When Γ is non-diagonal ($\eta \neq 0$), *i.e.*, the mass eigenstates are **not** decay eigenstates, the evolution of mass eigenstates

$$i\frac{d}{dt}\nu_i(t) = \left[\frac{(a_1 + a_2)}{2}\sigma_0 - \frac{(a_2 - a_1)}{2}\sigma_z - \frac{i}{2}\left((b_1 + b_2)\sigma_0 + \vec{\sigma}.\vec{\Gamma}\right)\right]\, \nu_i(t),$$

where $\vec{\Gamma} = [\eta\cos\xi, -\eta\sin\xi, -(b_2 - b_1)]$.

- The evolution equation in terms of flavor states is

$$i\frac{d}{dt}\nu_\alpha(t) = \left[\frac{(a_1+a_2)}{2}\sigma_0 - \frac{(a_2-a_1)}{2}O\sigma_z O^T - \frac{i}{2}(b_1+b_2)\sigma_0 \right.$$
$$\left. -\frac{i}{2}OU_{ph}(\vec{\sigma}.\vec{\Gamma})U_{ph}^\dagger O^T\right]\nu_\alpha(t).$$

- The matrix $\vec{\sigma}.\vec{\Gamma}$ does not commute with U_{ph} (since σ_x and σ_y do not commute with U_{ph}), the phase ϕ remains in the evolution equation.
- And hence, ϕ also appears in oscillation probabilities.

- Evolution operator $\mathcal{U} = e^{-i\mathcal{H}t}$ can be expanded in the basis spanned by σ_0 and Pauli matrices σ_i.
- This expansion is parameterized by a complex 4-vector $n_\mu \equiv (n_0, \vec{n})$, with $n_\mu = Tr[(-i\mathcal{H}t).\sigma_\mu]/2$, where

$$n_0 = -\frac{i}{2}(a_1 + a_2)t - \frac{1}{2}(b_1 + b_2)t, \quad n_x = -\frac{1}{2}t\eta\cos\xi,$$
$$n_y = \frac{1}{2}t\eta\sin\xi, \qquad n_z = \frac{i}{2}(a_2 - a_1)t + \frac{1}{2}(b_2 - b_1)t. \tag{4}$$

- The evolution matrix $\mathcal{U}$ is

$$\mathcal{U} = e^{n_0}\left[\cosh n\ \sigma_0 + \frac{\vec{n}.\vec{\sigma}}{n}\sinh n\right], \tag{5}$$

where

$$n = \sqrt{n_x^2 + n_y^2 + n_z^2} = \frac{t}{2}\sqrt{\eta^2 - (a_2 - a_1 - i(b_2 - b_1))^2}. \tag{6}$$

evolution matrix in flavor basis $\mathcal{U}_f = U_{mix}\mathcal{U}U_{mix}^{-1}$.

Neutrino oscillation probabilities

- Neglecting terms of $\mathcal{O}(\eta^2)$ and higher order, we get the survival probabilities

$$P_{ee} = e^{-2bt}\left(P_{ee}^{vac} - \eta\cos(\xi-\phi)\mathcal{A}\right)$$
$$P_{\mu\mu} = e^{-2bt}\left(P_{\mu\mu}^{vac} + \eta\cos(\xi-\phi)\mathcal{A}\right)$$

and the oscillation probabilities

$$P_{e\mu} = e^{-2bt}\left(P_{e\mu}^{vac} + 2\eta\sin(\xi-\phi)\mathcal{B}\right)$$
$$P_{\mu e} = e^{-2bt}\left(P_{\mu e}^{vac} - 2\eta\sin(\xi-\phi)\mathcal{B}\right)$$

where

$$\mathcal{A} = \frac{\sin(2\theta)\sin[(a_2-a_1)t]}{(a_2-a_1)}$$
$$\mathcal{B} = \frac{\sin(2\theta)\sin^2\left[\frac{1}{2}(a_2-a_1)t\right]}{(a_2-a_1)}. \qquad (7)$$

$$P_{e\mu}^{vac} = \sin^2 2\theta\sin^2\left(\frac{(a_2-a_1)t}{2}\right), \qquad a_2-a_1 = \Delta m^2/2E. \qquad (8)$$

The relations between different vacuum oscillation probabilities, $P_{ee}^{vac} = 1 - P_{e\mu}^{vac} = P_{\mu\mu}^{vac}$ and $P_{\mu e}^{vac} = P_{e\mu}^{vac}$.

- Majorana phase ϕ appears in the probability expressions.
- This appearance is proportional to $\Gamma_{12} \propto \eta$.
- Unlike in vacuum oscillations $P_{ee} \neq P_{\mu\mu}$ and $P_{e\mu} \neq P_{\mu e}$.
- $P_{e\mu}$ and $P_{\mu e}$ are sensitive to the mass ordering.

Antineutrino oscillation probabilities

- For antineutrinos, we have

$$\bar{M} = M \quad \text{and} \quad \bar{\Gamma} = \Gamma^*. \tag{9}$$

- Hence, antineutrino probability expressions can be obtained by making the substitutions $\phi \to -\phi$ and $\xi \to -\xi$.

$$P_{\bar{e}\bar{e}} = e^{-2bt}\left(P^{vac}_{\bar{e}\bar{e}} - \eta\cos(\xi-\phi)\mathcal{A}\right) \quad \text{and} \quad P_{\bar{\mu}\bar{\mu}} = e^{-2bt}\left(P^{vac}_{\bar{\mu}\bar{\mu}} + \eta\cos(\xi-\phi)\mathcal{A}\right)$$

$$P_{\bar{e}\bar{\mu}} = e^{-2bt}\left(P^{vac}_{\bar{e}\bar{\mu}} - 2\eta\sin(\xi-\phi)\mathcal{B}\right) \quad \text{and} \quad P_{\bar{\mu}\bar{e}} = e^{-2bt}\left(P^{vac}_{\bar{\mu}\bar{e}} + 2\eta\sin(\xi-\phi)\mathcal{B}\right).$$

- We find $P_{\bar{e}\bar{e}} = P_{ee}$, $P_{\bar{\mu}\bar{\mu}} = P_{\mu\mu}$ and $P_{\bar{\mu}\bar{e}} = P_{e\mu}$, *i.e.*, *CPT* is conserved.
- However, there is *CP*-violation ($P_{\bar{e}\bar{\mu}} \neq P_{e\mu}$) and *T*-violation ($P_{\mu e} \neq P_{e\mu}$).

CP-violation with Majorana phase

- The *CP* violating term in the oscillation probabilities is proportional to $\eta \sin(\xi - \phi)$.
- There are three possibilities of *CP*-violation
  1. *CP*-violation in mass - if $\eta \neq 0$ and $\xi = 0$, but $\phi \neq 0$
  2. *CP*-violation in decay - if $\phi = 0$, but $\eta \neq 0$ and $\xi \neq 0$
  3. *CP*-violation in mass and decay - if $\eta \neq 0$, $\xi \neq 0$ and $\phi \neq 0$
- Non-zero value of η is a necessary condition for *CP*-violation but is not sufficient.
- For the two special cases,(i) $\phi = 0 = \xi$ and (ii) $\phi = \xi$, there is no *CP*-violation even when $\eta \neq 0$.
- However, the presence of η is discernible in the survival probabilities.

Off-diagonal decay matrix Vs. off-diagonal decoherence

- Scenario with off-diagonal decoherence also has ϕ dependent oscillation probabilities (Benatti et al., PRD **64** (2001) 085015)
- This scenario violates both CP and CPT

$$P_{\mu\mu} - P_{\bar{\mu}\bar{\mu}} = \sin^2 2\theta \frac{\alpha \sin(2\phi) \sinh(\sqrt{\alpha^2 - (\Delta m^2)^2}\, t)}{\sqrt{\alpha^2 - (\Delta m^2)^2}} e^{-\gamma t}.$$

(Capolupo et al., PLB **792** (2019), 298-303)

- In our scenario also oscillation probabilities depend on ϕ but only CP and T are violated but CPT is conserved.

These different CPT properties distinguish these two scenarios of ϕ dependent neutrino oscillation probabilities.

Summary & Conclusions

- We look for scenarios in which the Majorana phase of two flavor oscillations can appear in neutrino oscillation probabilities.
- It was demonstrated earlier that off-diagonal terms in the decoherence matrix is one such scenario, which violates CPT.
- We found another scenario which involves neutrino decay, where the decay eigenstates are not the same as the mass eigenstates.
- In this scenario, CP is violated in general but CPT is conserved.
- The CP-violating terms in this scenario are sensitive to the neutrino mass ordering
- The two scenarios can be contrasted by their CPT properties.

THANK
YOU

BACKUP SLIDES

Coherent forward scattering can alter oscillation patterns:

$$\nu_{e,\mu,\tau} + e^-, p, n \to \nu_{e,\mu,\tau} + e^-, p, n \text{ (neutral current)}$$

$$\nu_e + e^- \to \nu_e + e^- \text{ (charged current)}$$

$$H_f = U_{mix} \begin{pmatrix} -\Delta m^2/4E & 0 \\ 0 & \Delta m^2/4E \end{pmatrix} U_{mix}^\dagger + V \begin{pmatrix} 1+\epsilon_{ee} & \epsilon_{e\mu} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} \end{pmatrix}$$

matter density potential $V = \sqrt{2} G_F N_e$

$$i \frac{d}{dt} \nu_\alpha(\mathbf{t}) = \frac{1}{4E} \left(-\Delta m^2 \; \mathbf{O} \sigma_z \mathbf{O}^T + 4EV \vec{\sigma}.\vec{\epsilon} \right) \nu_\alpha(\mathbf{t}),$$

$\vec{\epsilon} = (\Re(\epsilon_{e\mu}), -\Im(\epsilon_{e\mu}), (1+\epsilon_{ee}-\epsilon_{\mu\mu})/2)$; $\epsilon_{\alpha\beta} \to$ non-standard interaction (NSI) strength

$\Rightarrow$ Majorana phase ϕ does not appear in matter modified oscillations.