Earth's Matter Effect in Neutrino Oscillation

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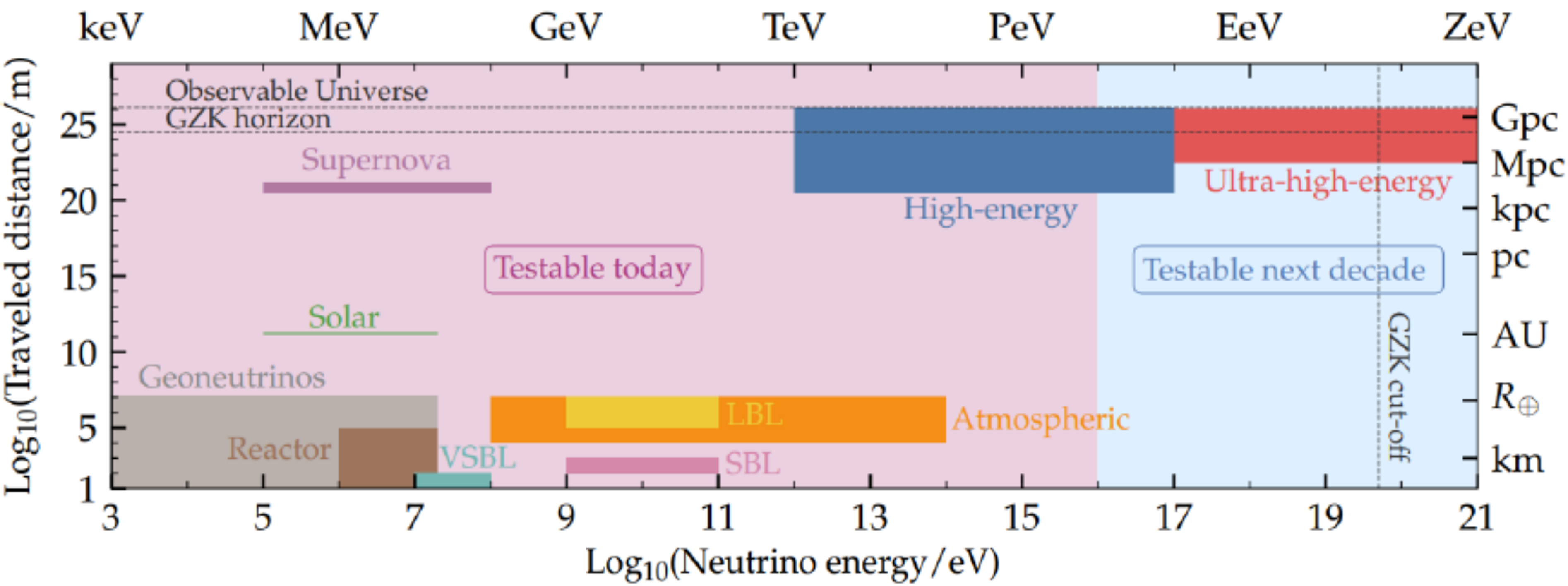
Panorama of Neutrinos: Across 18 orders in E and 25 orders in L



Remarkable progress over the last two decades

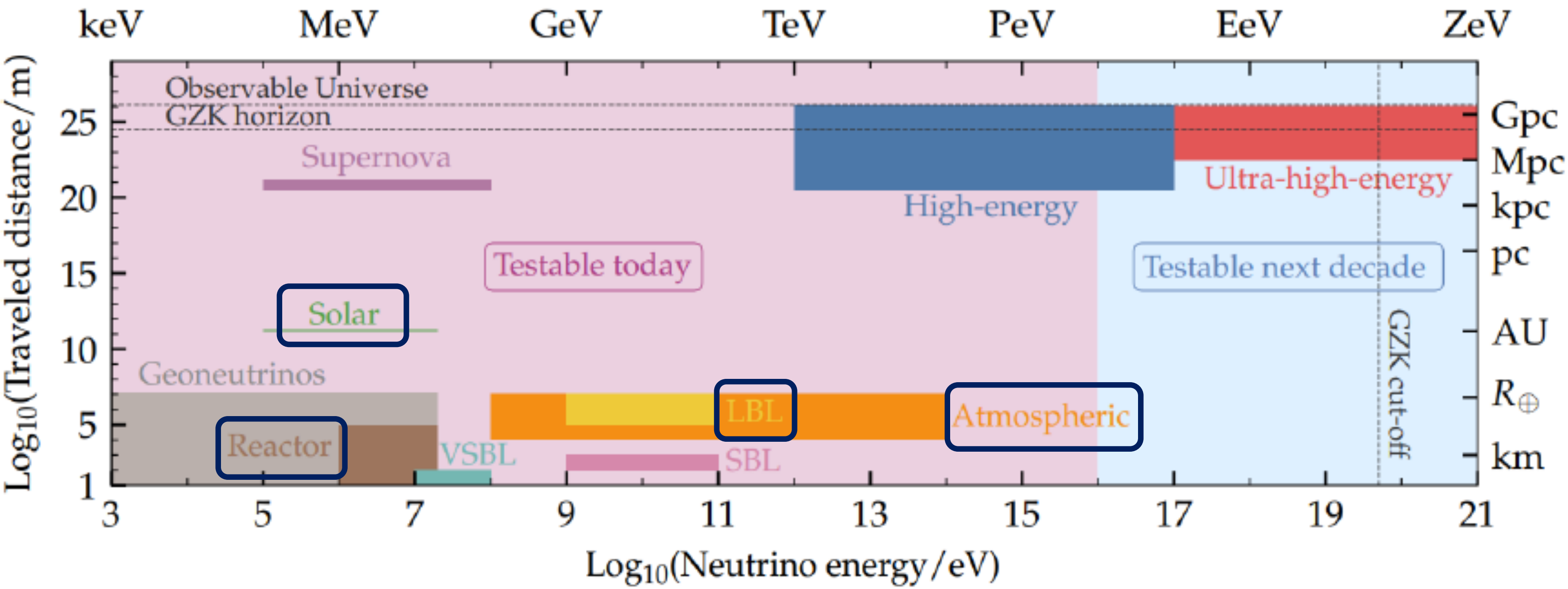
Neutrinos detected from various sources having different energy and distance scales

Detection of cosmic neutrinos opened a new window onto the Universe

Era of Neutrino Astronomy began

2002 Nobel Prize to Raymod Davis Jr. (Sun) and Masatoshi Koshiba (Supernova)

Neutrino Oscillation – A Signature for BSM Physics



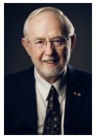
Neutrinos change their flavor as they move in space and time → Neutrinos Oscillate

Solar, Atmospheric, Reactor, and Accelerator (LBL) experiments firmly established Neutrino Flavor Oscillation → implies Neutrinos are Massive and Mix with each other

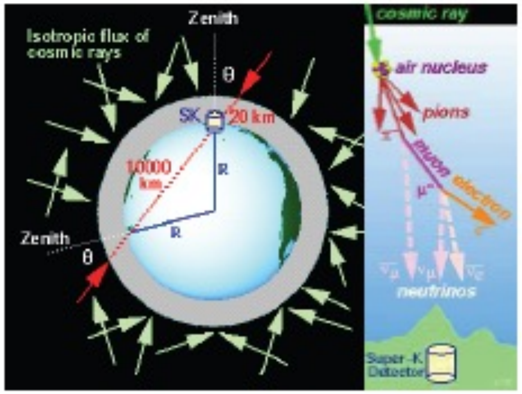
Neutrinos are Massless in the basic Standard Model (SM) of particle physics

Physics beyond the Standard Model (BSM) necessary to explain non-zero ν mass & mixing

2015 Nobel Prize to Takaaki Kajita (Super-K) & Arthur B. McDonald (SNO)



Golden Age of Neutrino Physics (1998 – 2022 & Beyond)

sun	reactors	atmosphere	accelerators
			
Homestake, SAGE, GALLEX SuperK, SNO, Borexino	KamLAND, CHOOZ Double Chooz, Daya Bay, RENO	SuperKamiokande IceCube, DeepCore	K2K, MINOS, T2K NOvA

Over the last two decades or so, marvelous data from world-class experiments

- Solar neutrinos (ν_e)
- Atmospheric neutrinos $(\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e)$
- Reactor anti-neutrinos $(\bar{\nu}_e)$
- Accelerator neutrinos $(\nu_\mu, \bar{\nu}_\mu)$

→ *Data from various neutrino sources and vastly different energy and distance scales*

↓

Neutrinos change their flavor as they move in space and time

We have just started our journey in the mysterious world of neutrinos

Three-Flavor Neutrino Oscillation Framework: Simple & Robust

It happens because flavor (weak) eigenstates do not coincide with mass eigenstates

$c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

θ_{23} : $P(\nu_\mu \to \nu_\mu)$ by Atoms. ν and ν beam

θ_{13}: $P(\nu_e \to \nu_e)$ by Reactor ν
θ_{13} & δ: $P(\nu_\mu \to \nu_e)$ by ν beam

θ_{12} : $P(\nu_e \to \nu_e)$ by Reactor and solar ν

$L/E = 500$ km/GeV $\quad$ $L/E = 15,000$ km/GeV

$\Delta m^2_{31} \sim 2.5 \times 10^{-3}$ eV^2 $\quad$ $P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta_{ij} * \sin^2\left(1.27\Delta m^2_{ij}\frac{L}{E}\right)$ $\quad$ $\Delta m^2_{21} \sim 7.6 \times 10^{-5}$ eV^2

Three mixing angles: $\theta_{23}, \theta_{13}, \theta_{12}$ and one CP-violating (Dirac) phase δ_{CP}

$$\tan^2\theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2\theta_{23} \equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}; \quad U_{e3} \equiv \sin\theta_{13}e^{-i\delta}$$

3 mixing angles simply related to flavor components of 3 mass eigenstates

Over a distance L, changes in the relative phases of the mass states may induce flavor change

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}[U^*_{\alpha i}U_{\alpha j}U_{\beta i}U^*_{\beta j}]\sin^2\Delta_{ij} - 2\sum_{i>j} \mathrm{Im}[U^*_{\alpha i}U_{\alpha j}U_{\beta i}U^*_{\beta j}]\sin 2\Delta_{ij}$$

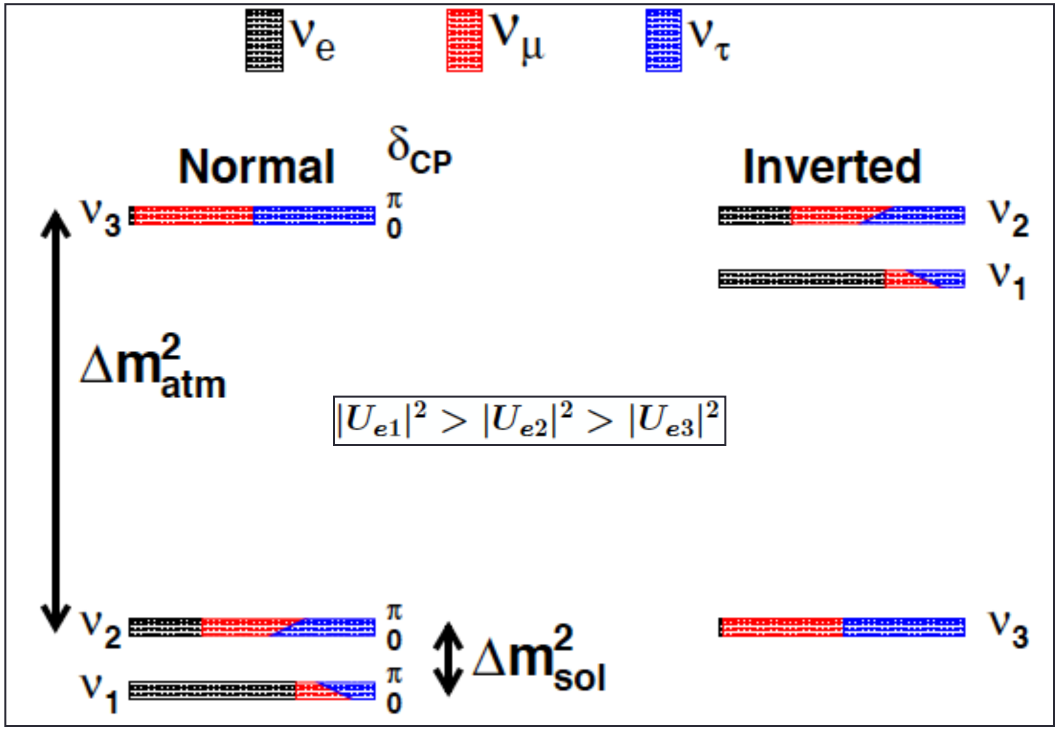
$\Delta_{ij} = \Delta m^2_{ij} L/4E_\nu$

$\Delta m^2_{ij} = m^2_i - m^2_j$

for antineutrinos replace δ_{CP} by $-\delta_{CP}$

Neutrino Mass Ordering: Important Open Question

- The sign of Δm^2_{31} $(m_3^2 - m_1^2)$ is not known



ν_e component of ν_1 > ν_e component of ν_2 > ν_e component of ν_3

Neutrino mass spectrum can be normal or inverted ordered

We only have a lower bound on the mass of the heaviest neutrino

$$\sqrt{2.5 \cdot 10^{-3} \mathrm{eV}^2} \sim 0.05 \text{ eV}$$

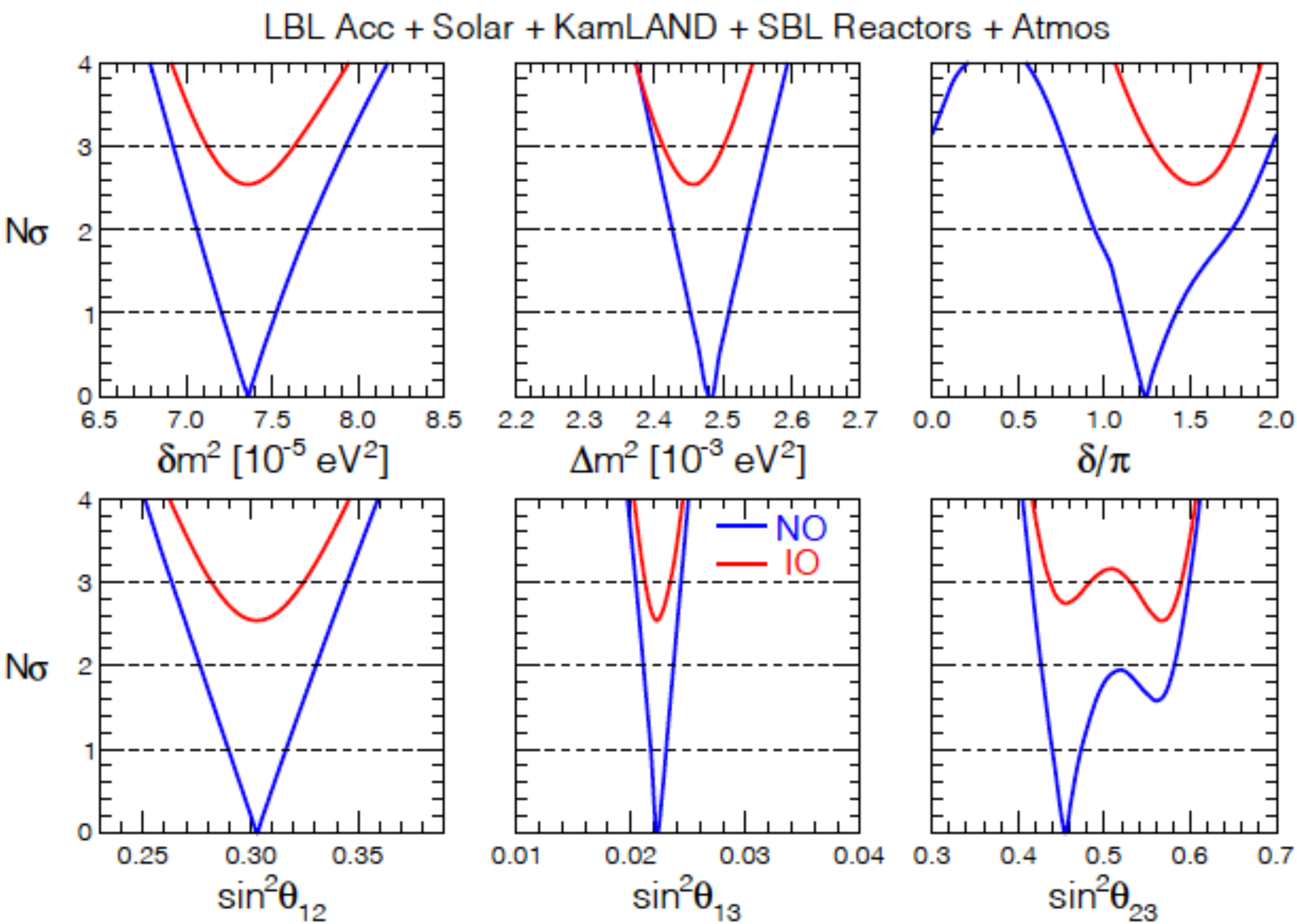
We currently do not know which neutrino is the heaviest

Matter effect inside the Sun played an important role to fix the ordering between m_2 & m_1

Matter effect inside the Earth will play a crucial role to fix the ordering between m_3 & m_1

Mass Ordering Discrimination : A Binary yes-or-no type question

Global Fit of Neutrino Oscillation Parameters Circa 2021



Capozzi, Valentino, Lisi, Marrone, Melchiorri, Palazzo, arXiv:2107.00532v2 [hep-ph]

Present Status of Neutrino Oscillation Parameters Circa 2021

Preference for Normal Mass Ordering (~ 2.5σ), θ_{23} < 45 degree and sin$\boldsymbol{\delta}$ < 0 (both at 90% C.L.)

Parameter	Ordering	Best fit	3σ range	"1σ" (%)
$\delta m^2/10^{-5}$ eV^2	NO, IO	7.36	6.93 – 7.93	2.3
$\sin^2\theta_{12}/10^{-1}$	NO, IO	3.03	2.63 – 3.45	4.5
$\lvert\Delta m^2\rvert/10^{-3}$ eV^2	NO	2.485	2.401 – 2.565	1.1
	IO	2.455	2.376 – 2.541	1.1
$\sin^2\theta_{13}/10^{-2}$	NO	2.23	2.04 – 2.44	3.0
	IO	2.23	2.03 – 2.45	3.1
$\sin^2\theta_{23}/10^{-1}$	NO	4.55	4.16 – 5.99	6.7
	IO	5.69	4.17 – 6.06	5.5
δ/π	NO	1.24	0.77 – 1.97	16
	IO	1.52	1.07 – 1.90	9
$\Delta\chi^2_{\mathrm{IO-NO}}$	IO−NO	+6.5		

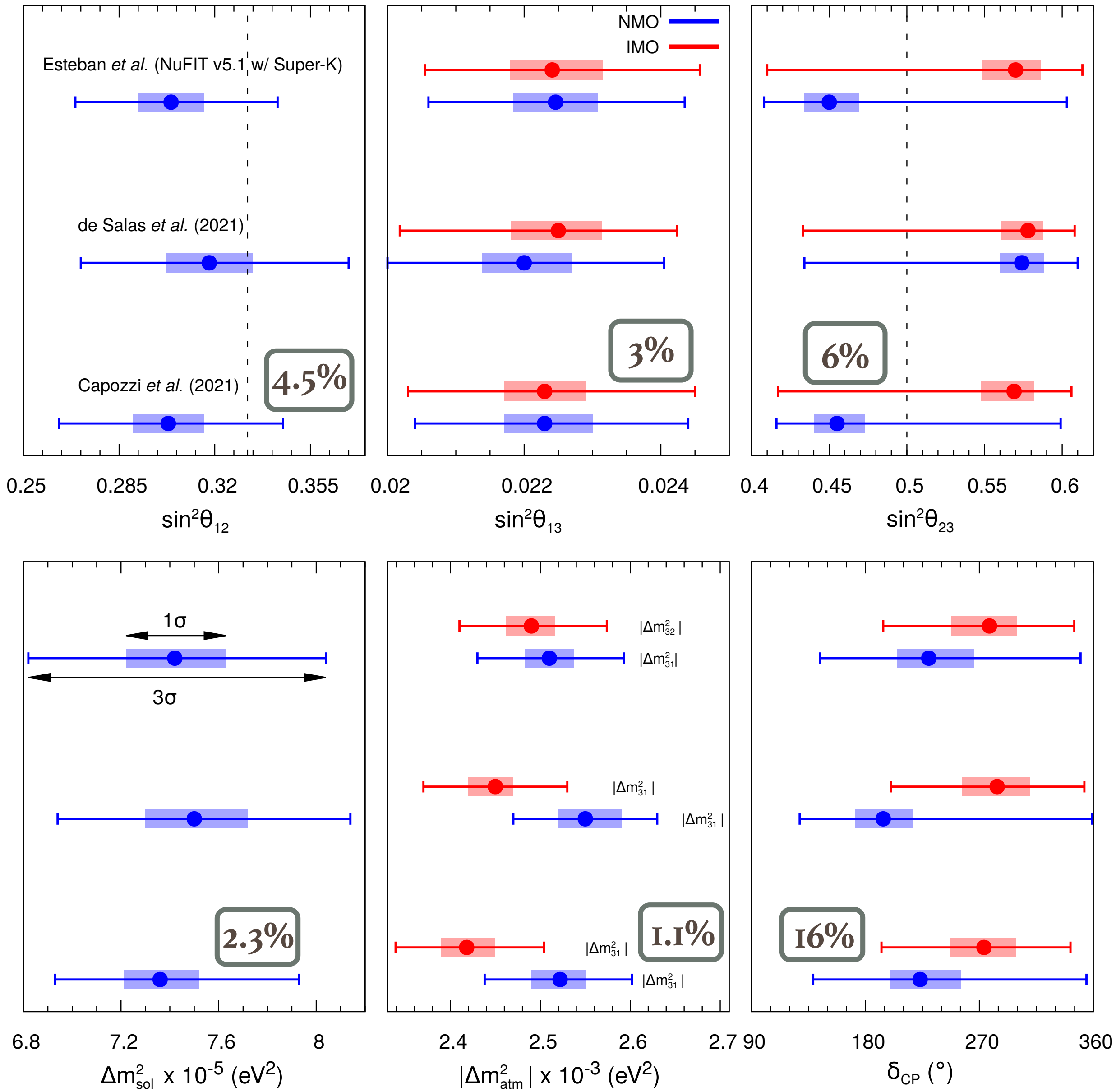
Capozzi, Valentino, Lisi, Marrone, Melchiorri, Palazzo, arXiv:2107.00532v2 [hep-ph]

See also, Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, arXiv:2007.14792v1 [hep-ph], NuFIT v5.1 w/SK

See also, de Salas, Forero, Gariazzo, Martinez-Mirave, Mena, Ternes, Tortolla, Valle, arXiv:2006.11237v2 [hep-ph]

Remarkable Precision on Neutrino Oscillation Parameters

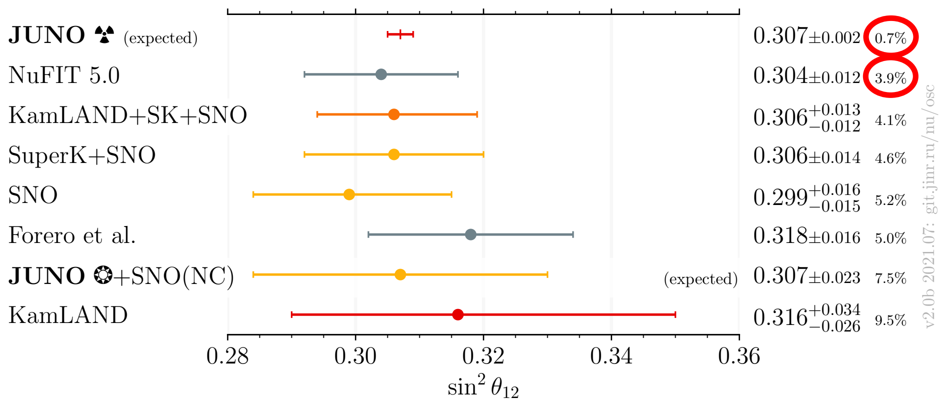
Robust three-flavor neutrino oscillation paradigm

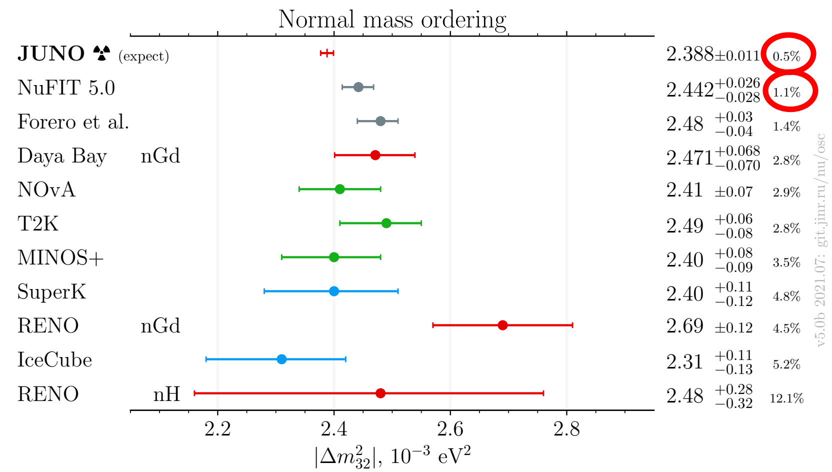


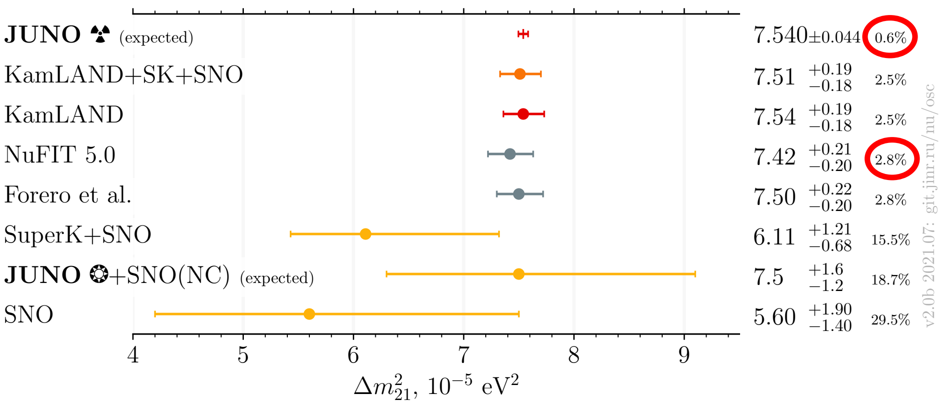
Huge boost for the discovery of NMO, CPV, and θ_{23} Octant

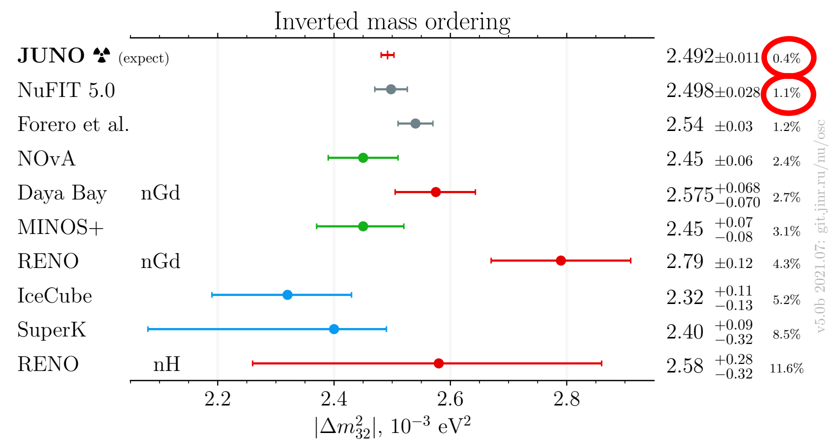
Agarwalla, Kundu, Prakash, Singh, JHEP 03 (2022) 206

Very Bright Future Ahead: Triumph of JUNO







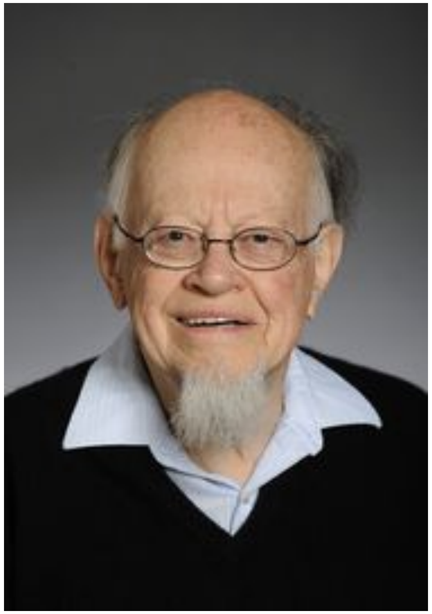


Maxim Gonchar (JUNO Collaboration) EPS-HEP 2021, July 26

JUNO will improve significantly our knowledge on neutrino oscillation parameters. These developments are crucial to probe sub-leading three-flavor effects in next-generation long-baseline experiments for the discovery of NMO, leptonic CPV, and Octant of 2-3 mixing angle

Neutrino Oscillations in Matter: MSW Effect

- The MSW Effect (Wolfenstein, 1978; Mikheyev and Smirnov, 1985)
- Matter can change the pattern of neutrino oscillations significantly
- Resonant enhancement of oscillations and resonant flavor conversion possible
- Responsible for the flavor conversion of solar neutrinos (LMA MSW solution established)


Lincoln Wolfenstein


Stanislav Mikheyev

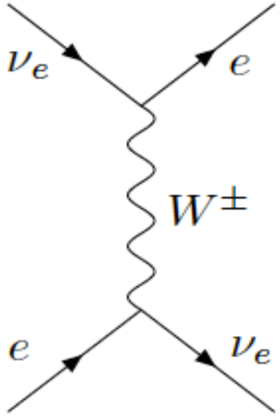

Alexei Smirnov

Neutrino Oscillations in Matter: MSW Effect

Neutrino propagation through matter modify the oscillations significantly

Coherent forward scattering of neutrinos with matter particles

Charged current interaction of ν_e with electrons creates an extra potential for ν_e



MSW matter term: $A = \pm 2\sqrt{2} G_F N_e E$ or $A(\text{eV}^2) = 0.76 \times 10^{-4} \rho \text{ (g/cc)} E(\text{GeV})$

N_e = electron number density , + (-) for neutrinos (anti-neutrinos) , ρ = matter density in Earth

Matter term changes sign when we switch from neutrino mode to antineutrino mode

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq 0$$

→ *even if $\delta_{CP} = 0$, causes fake CP asymmetry*

Matter term modifies oscillation probability differently depending on the sign of Δm^2

$$\Delta m^2 \simeq A \quad \Leftrightarrow \quad E_{\text{res}}^{\text{Earth}} = 6 - 8\,\text{GeV}$$

→ *Resonant conversion – Matter effect*

	ν	$\bar{\nu}$
$\Delta m^2 > 0$	MSW	-
$\Delta m^2 < 0$	-	MSW

→ *Resonance occurs for neutrinos (anti-neutrinos) if Δm^2 is positive (negative)*

Neutrino Oscillation Length Resonance / Parametric Resonance

- Oscillations of atmospheric neutrinos inside the Earth can feel this resonance when neutrino trajectories cross the core of the Earth

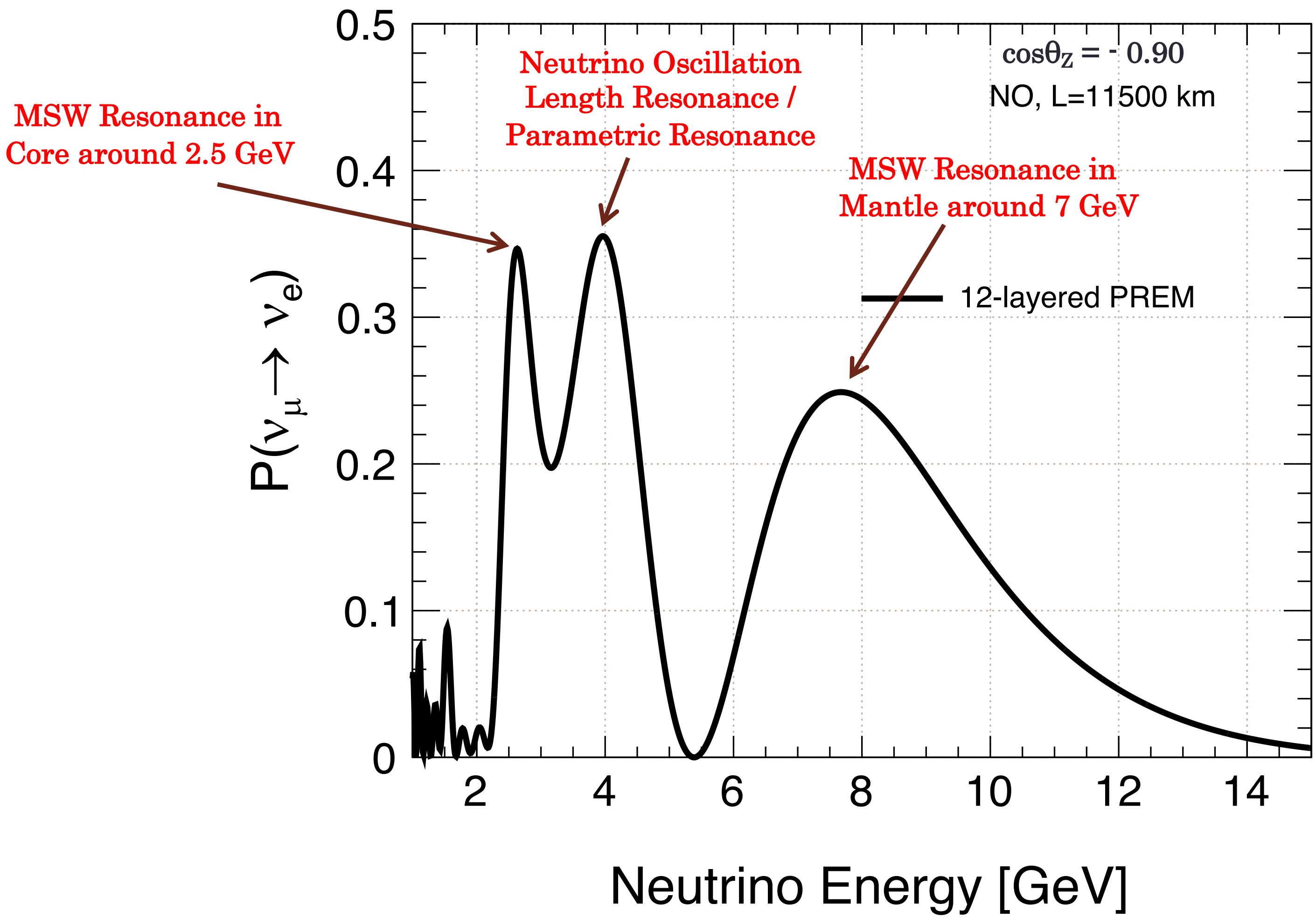
- The probabilities of ν flavor transitions can be strongly enhanced if the oscillation phase undergoes certain modification in matter

- This can happen if the variation of the matter density along the neutrino path is correlated in a certain way with the change of the oscillation phase

- This amplification of the neutrino oscillation probability in matter due to specific phase relationships has an interesting property that it can accumulate if the matter density profile along the neutrino path repeats itself (periodic)

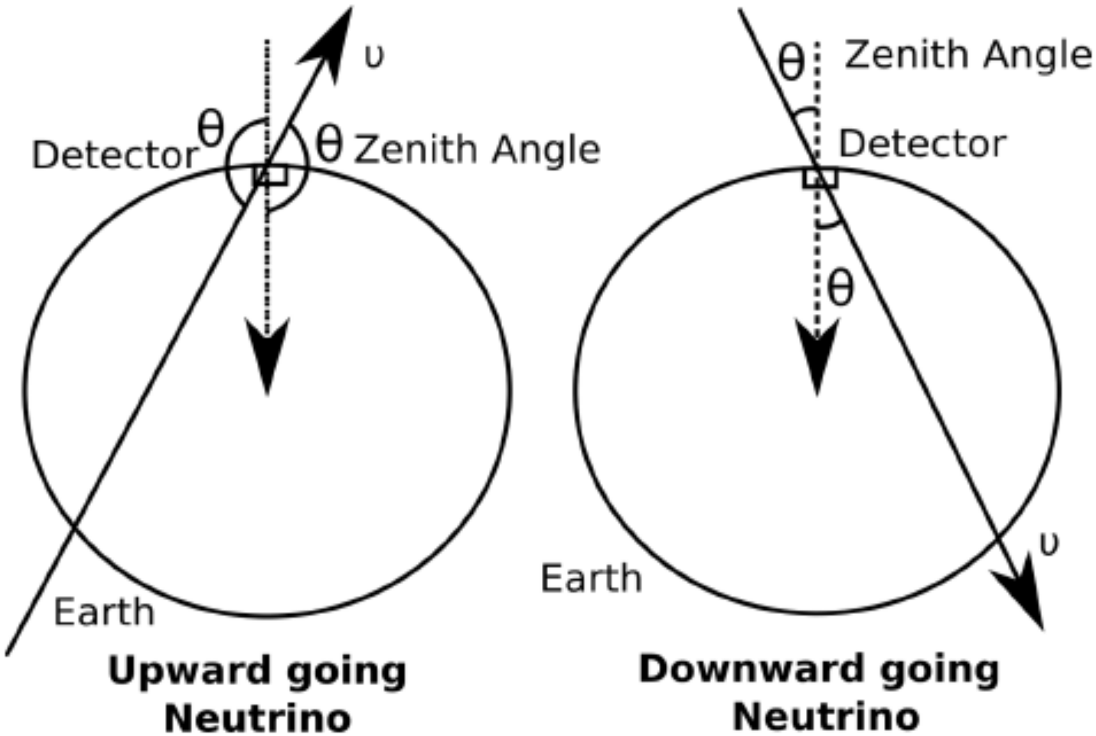
Petcov 1998, Liu and Smirnov 1998, Akhmedov 1998

The Resonances inside the Earth



$$E_{\rm res} \equiv \frac{\Delta m^2_{31}\,\cos 2\theta_{13}}{2\sqrt{2}\,G_F\,N_e} \simeq 7\,{\rm GeV}\left(\frac{4.5\,{\rm g/cm^3}}{\rho}\right)\left(\frac{\Delta m^2_{31}}{2.4\times 10^{-3}\,{\rm eV^2}}\right)\cos 2\theta_{13}$$

Upward and Downward Directions for Atmospheric Neutrinos



Upward-going neutrinos:

$$\pi/2 < \theta < \pi$$

$$-1 < \cos\theta < 0$$

$$L_\nu \approx 2R\cos\theta_\nu$$

Downward-going neutrinos:

$$0 < \theta < \pi/2$$

$$0 < \cos\theta < 1$$

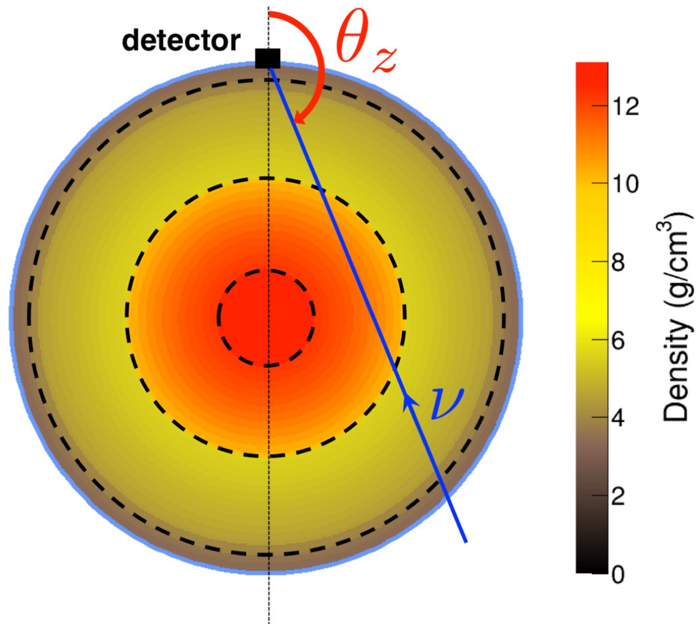
$$L_\nu \approx 0$$

$$L_\nu = \sqrt{(R+h)^2 - (R-d)^2\sin^2\theta_\nu} - (R-d)\cos\theta_\nu$$

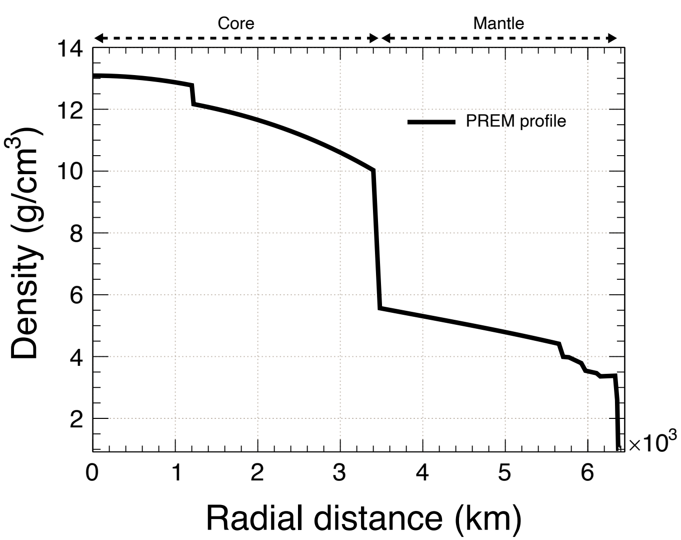
R = radius of Earth (6371 km)
h = average ν production height from surface (~ 10 to 25 km)
d = depth of the detector underground (~ 100 m to 2 km)

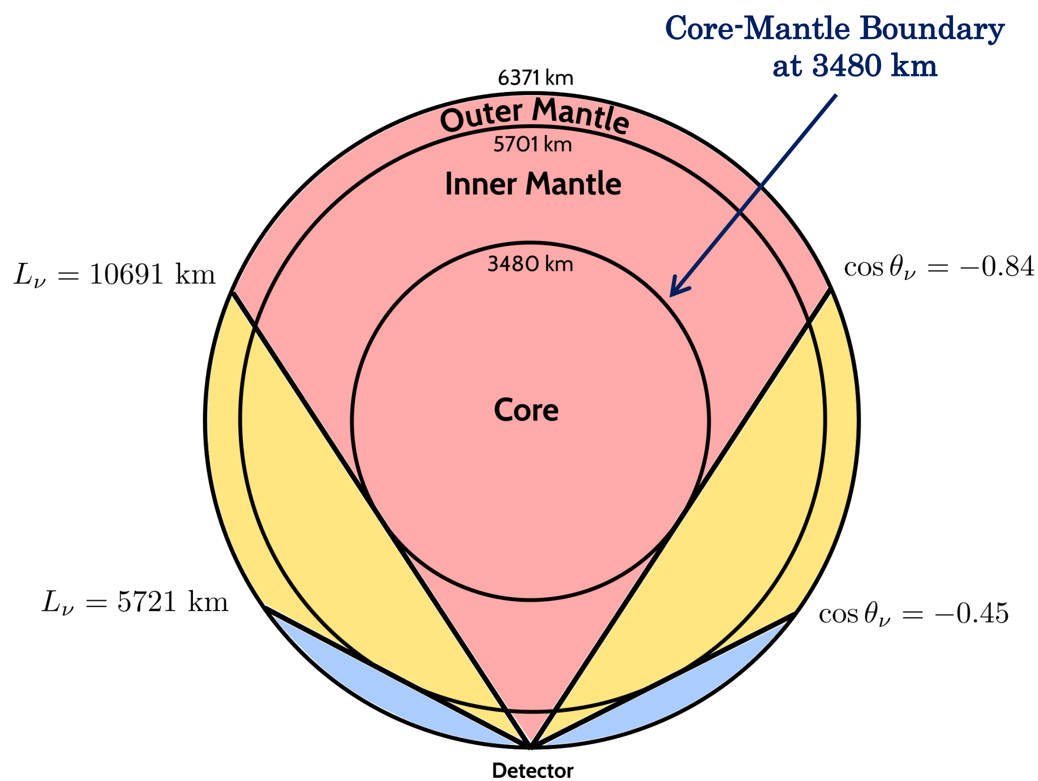
Upward-going neutrinos feel Earth's matter effect during oscillations inside Earth – key for neutrino oscillation tomography of Earth

PREM and Neutrino Trajectories Deep Inside the Earth



The gravitational and seismic measurements are used to infer the radial density distribution inside Earth – known as Preliminary Reference Earth Model (PREM)

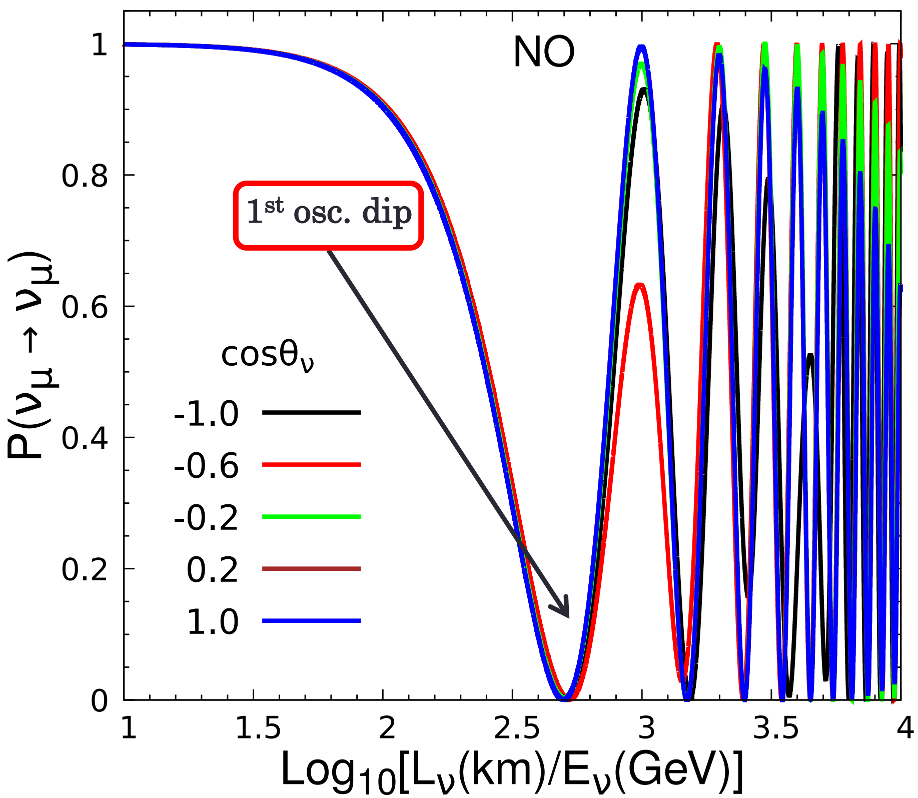


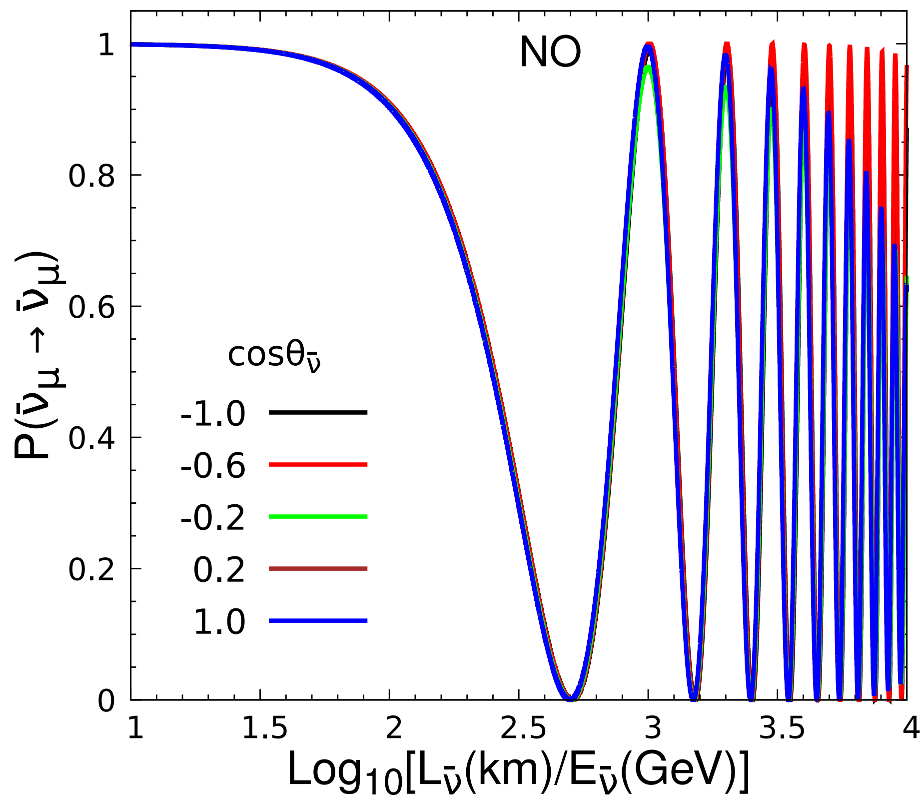


Three-Layered Model of Earth

PREM is not a measured profile!

Oscillation Dip





location of 1st oscillation dip → consider muon survival probability in 2-flavor oscillations

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{23} \cdot \sin^2\left(1.27 \cdot |\Delta m^2_{32}|\,(\mathrm{eV}^2) \cdot \frac{L_\nu\,(\mathrm{km})}{E_\nu\,(\mathrm{GeV})}\right)$$

$\theta = 45^\circ$

$\Delta m^2 = 2.4\text{e-}03\ \mathrm{eV}^2$

$\frac{1.27\Delta m^2 L}{E} = \frac{\pi}{2}$

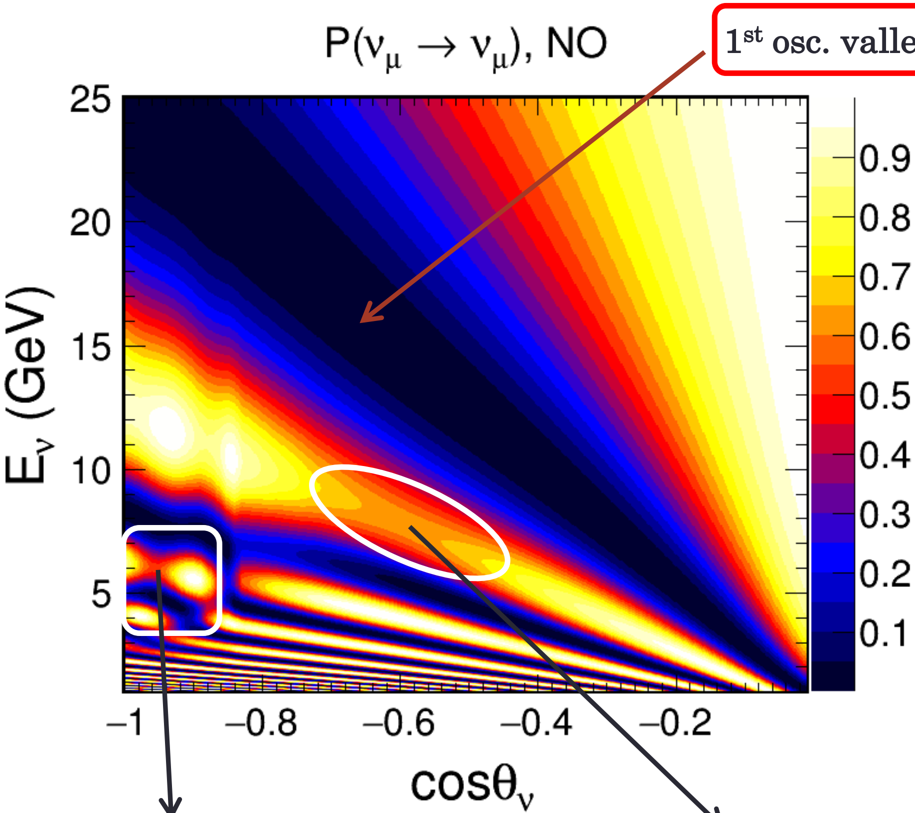
$\frac{L}{E} = \frac{\pi}{2 \times 1.27 \times \Delta m^2} = 515.35$

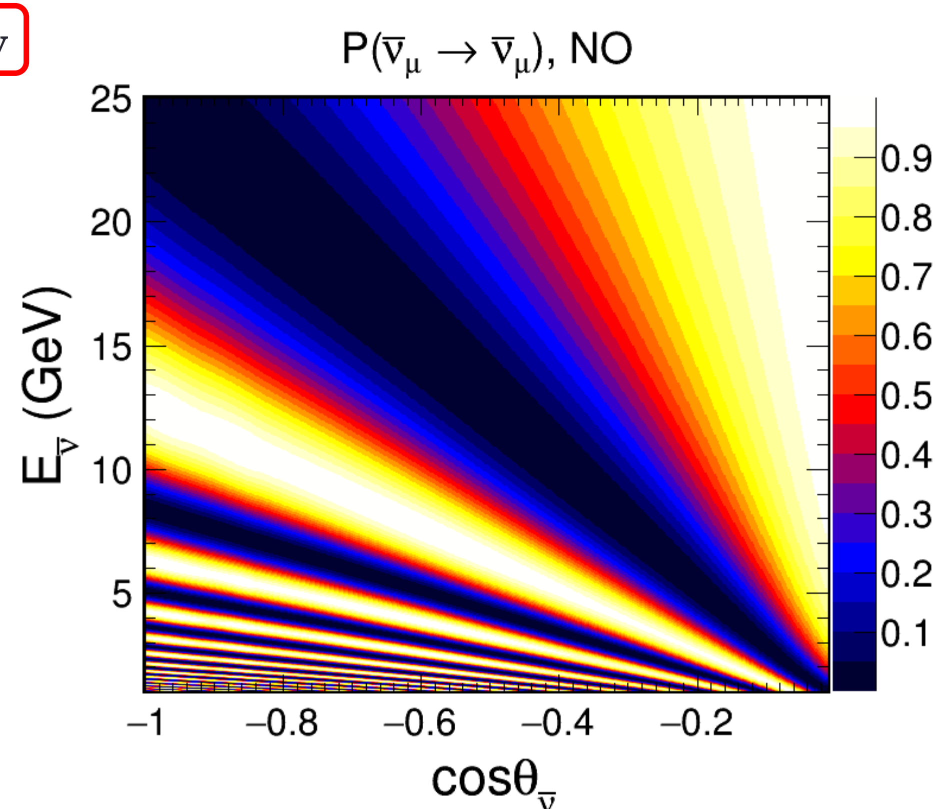
$\log_{10}\left(\frac{L}{E}\right) = 2.71$

Oscillation Valley

Neutrinos (antineutrinos) feel Earth's matter effect for normal (inverted) mass ordering

1[st] osc. valley





NOLR/PR region

$\cos\theta_\nu < -0.8$
$3\ \text{GeV} < E_\nu < 6\ \text{GeV}$
reducing threshold helps

MSW resonance region

$-0.8 < \cos\theta_\nu < -0.5$
$6\ \text{GeV} < E_\nu < 10\ \text{GeV}$

$$V_{CC} = \pm\sqrt{2}G_F N_e \approx \pm 7.6 \times Y_e \times 10^{-14} \left[\frac{\rho}{\text{g/cm}^3}\right]\ \text{eV}$$

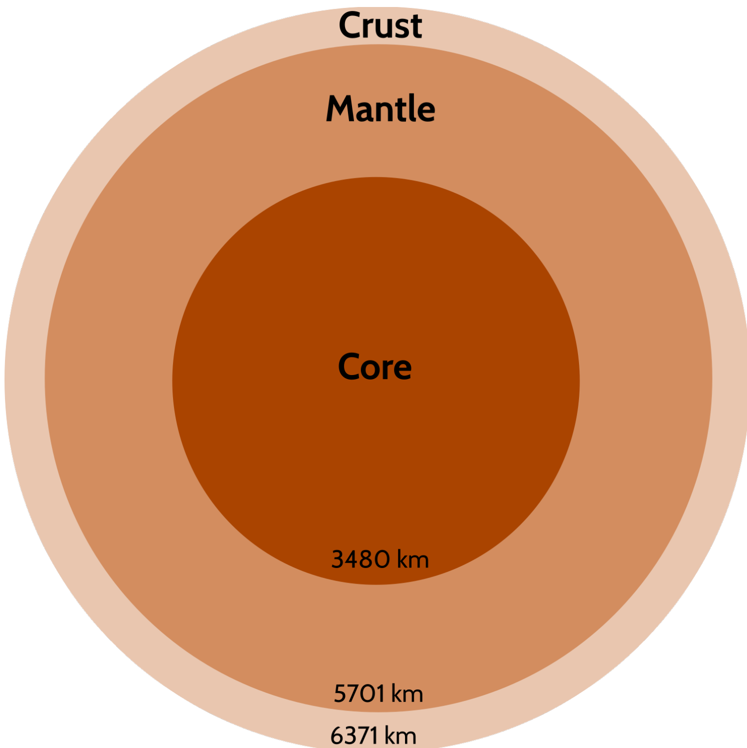
$$Y_e = N_e/(N_p + N_n)$$

ρ denotes the matter density
+1 (-1) for neutrino (antineutrino)

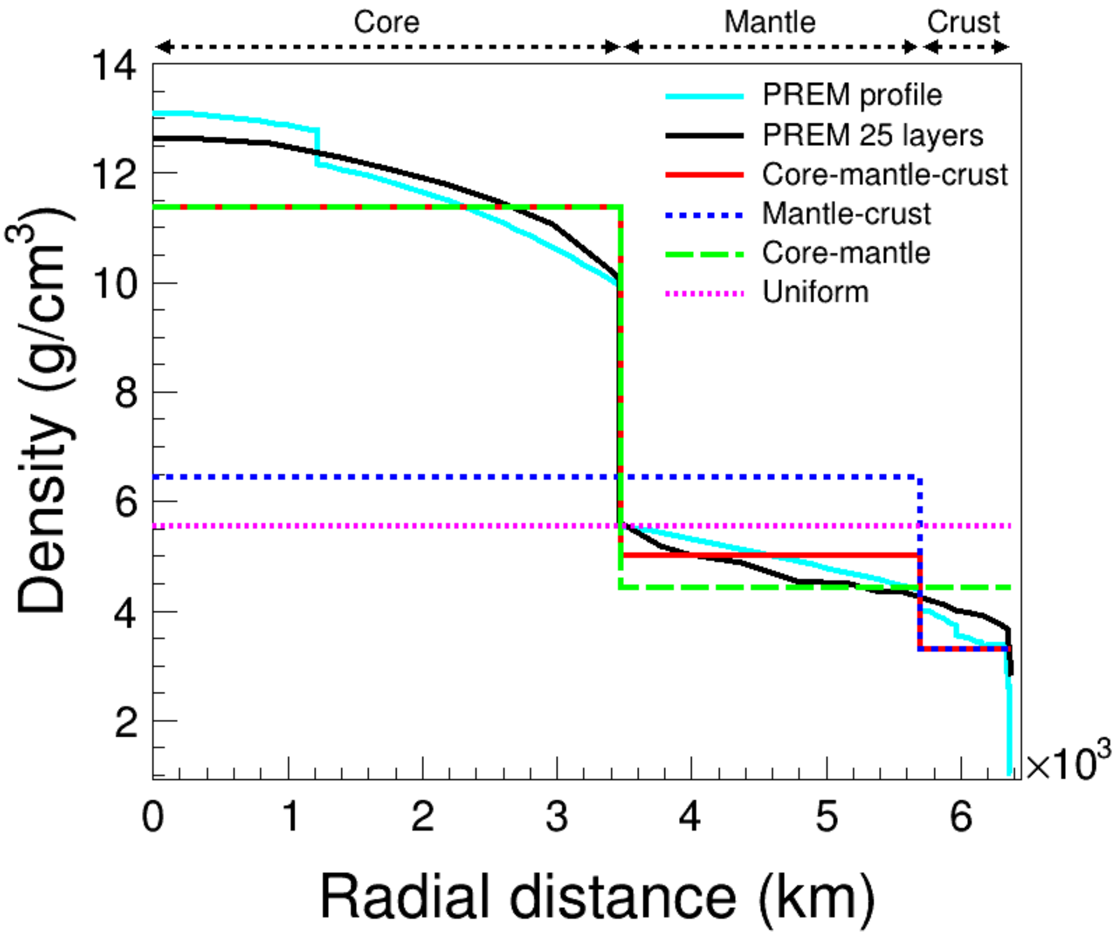
Kumar, Khatun, Agarwalla, Dighe, EPJC 81 (2021) 2, 190

Note: MSW or NOLR/PR resonances have not been observed yet!

Various Radial Density Profiles of Earth



Three-Layered Model of Earth



Anil Kumar and Sanjib Kumar Agarwalla, JHEP 08 (2021) 139

While constructing alternative profiles of Earth, the radius & mass of Earth remain invariant

Atmospheric neutrino experiments can distinguish between these alternative profiles of Earth utilizing neutrino oscillations in the presence of Earth's matter in multi-GeV energy range

Neutrino Oscillation Tomography with Atmospheric Neutrinos

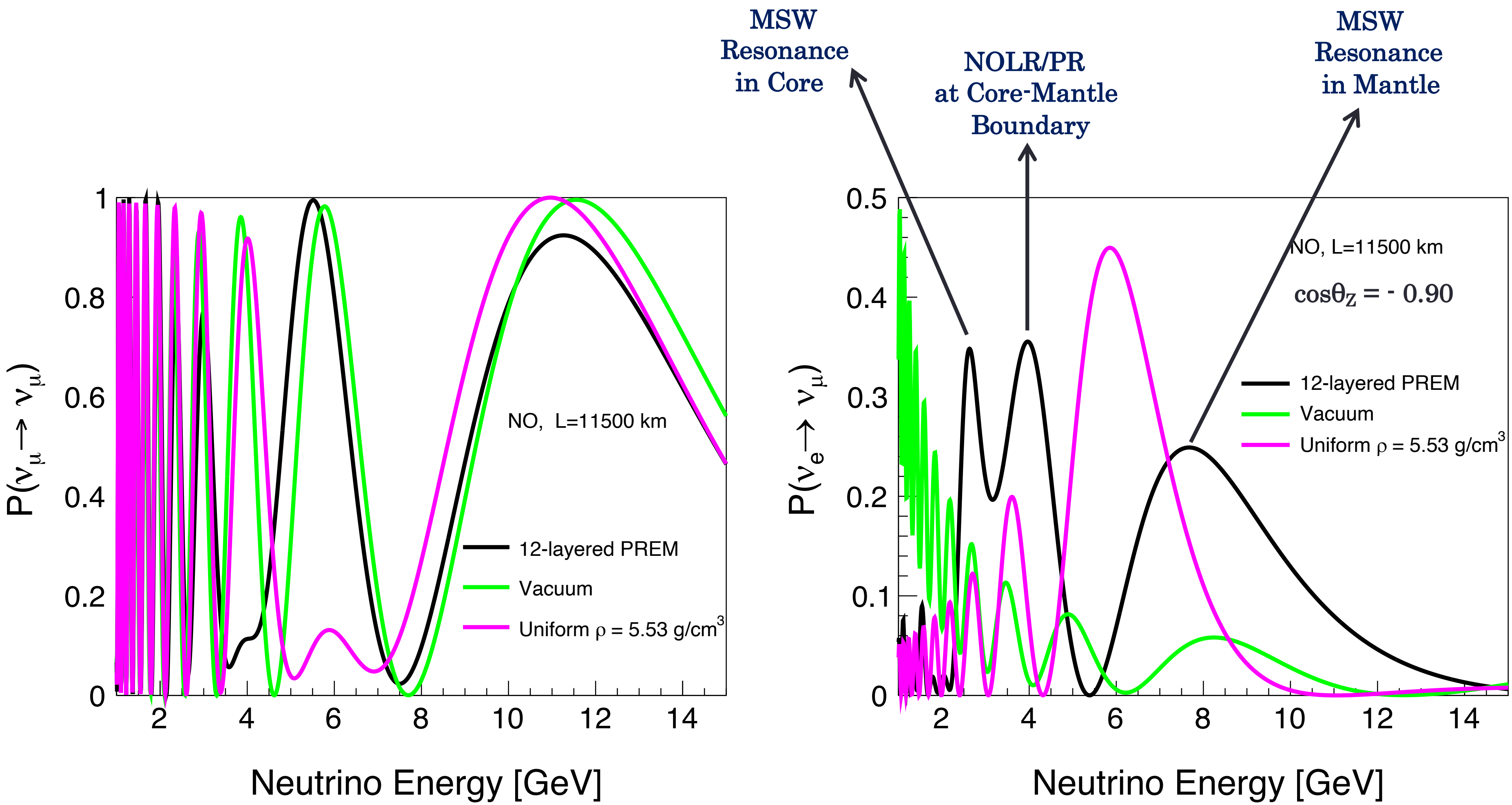
Atmospheric neutrinos have access to sub-GeV (resonance matter effect in core) and multi-GeV (resonance matter effect in mantle) energy ranges with a wide range of baselines passing through Earth's mantle and core

The recent advancement in the precision measurements of neutrino oscillation parameters opens the avenue to perform a rich neutrino oscillation tomography using currently running and upcoming atmospheric neutrino experiments.

One can address the following important issues related to Earth:

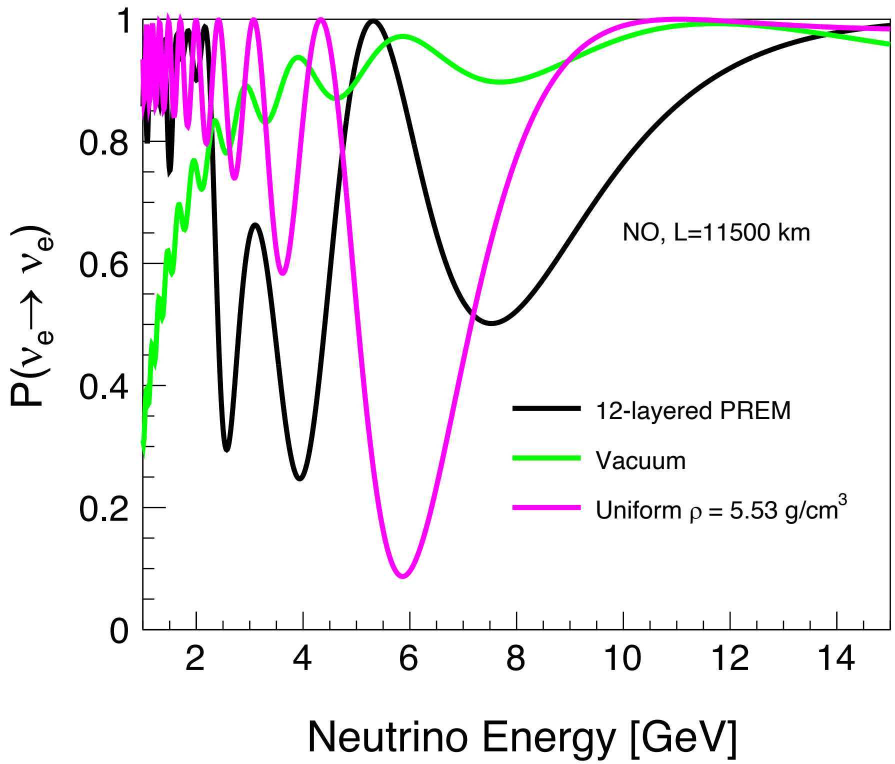
- Observing the presence of Earth matter
- Ruling out the homogeneous matter
- Measurement of mass of Earth
- Validating the presence of core
- Location of core-mantle boundary (CMB)
- Measurement of the density of core and mantle
- Chemical composition of core

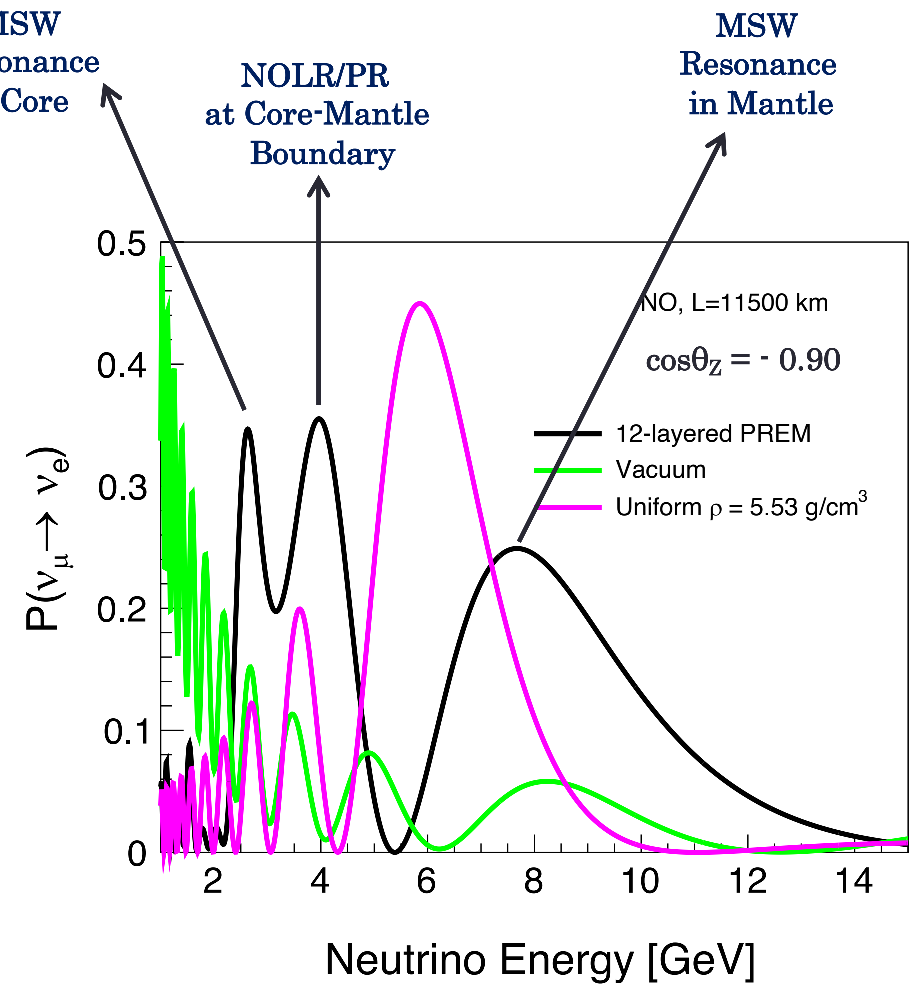
Neutrino Oscillation Probabilities for a Core-Passing Track-Like Events



We get similar oscillation patterns for antineutrinos with inverted mass ordering

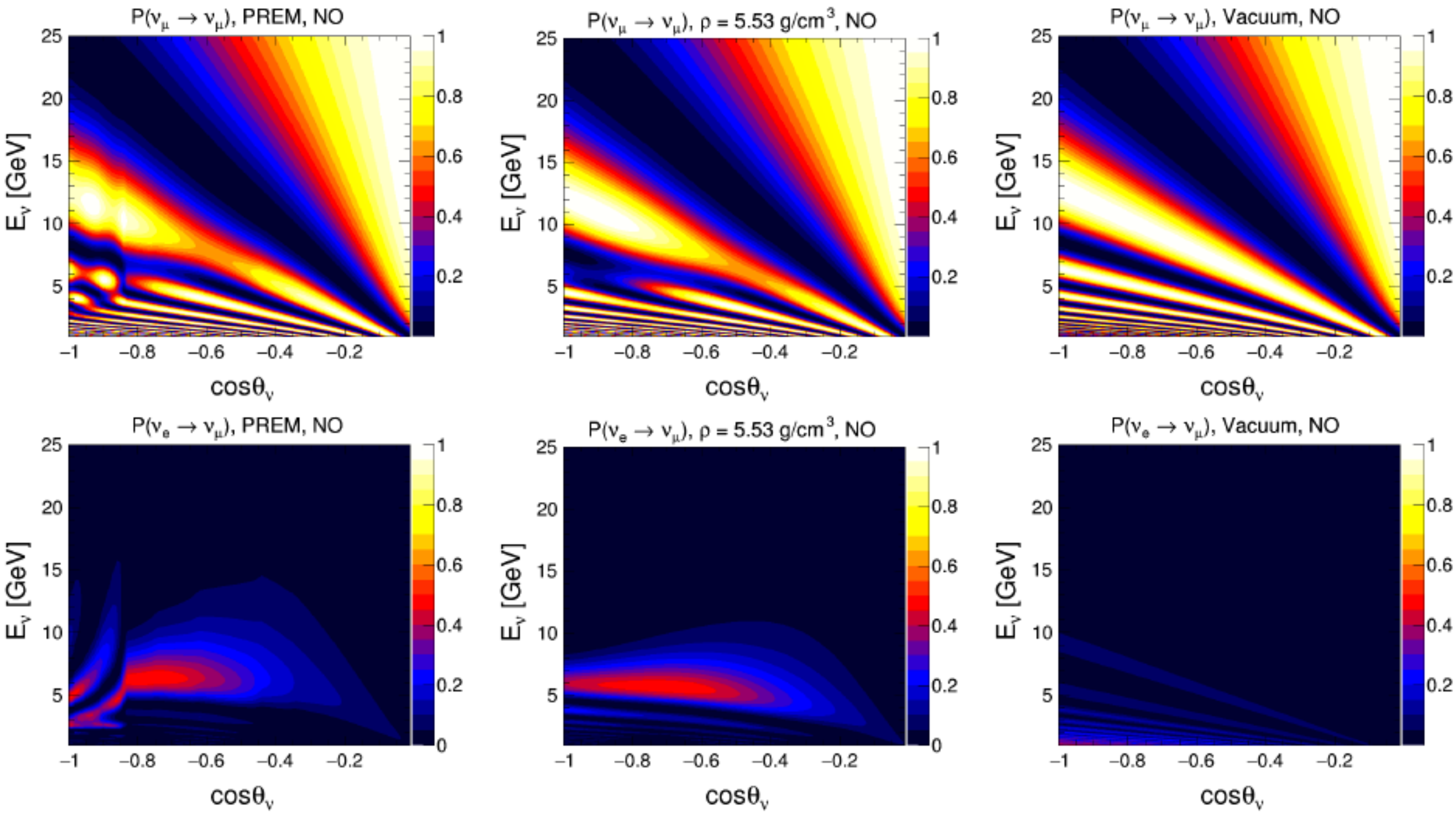
Neutrino Oscillation Probabilities for a Core-Passing Cascade-Like Events





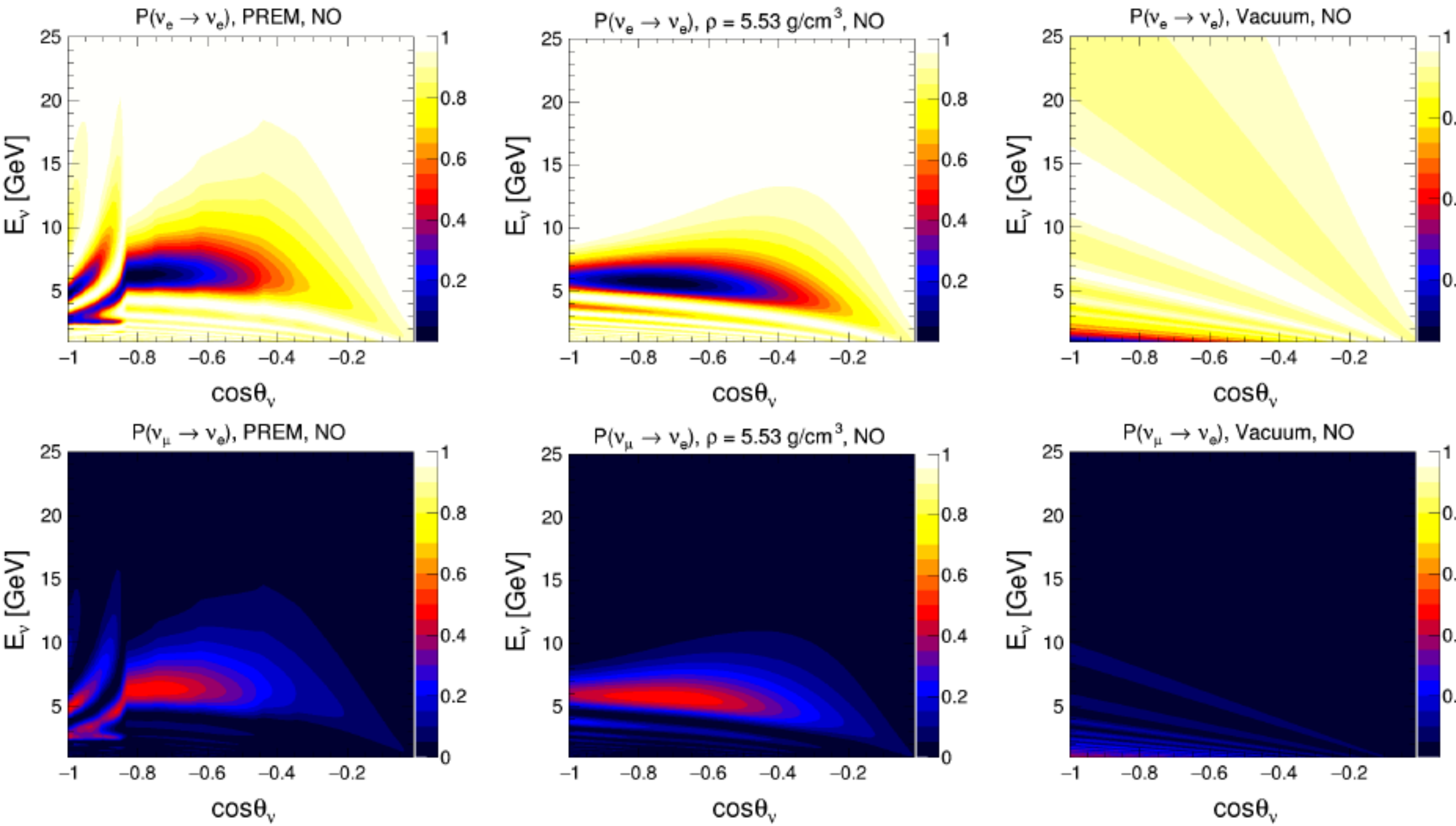
We get similar oscillation patterns for antineutrinos with inverted mass ordering

Neutrino Oscillograms for a Core-Passing Track-Like Events



We get similar oscillograms for antineutrinos with inverted mass ordering

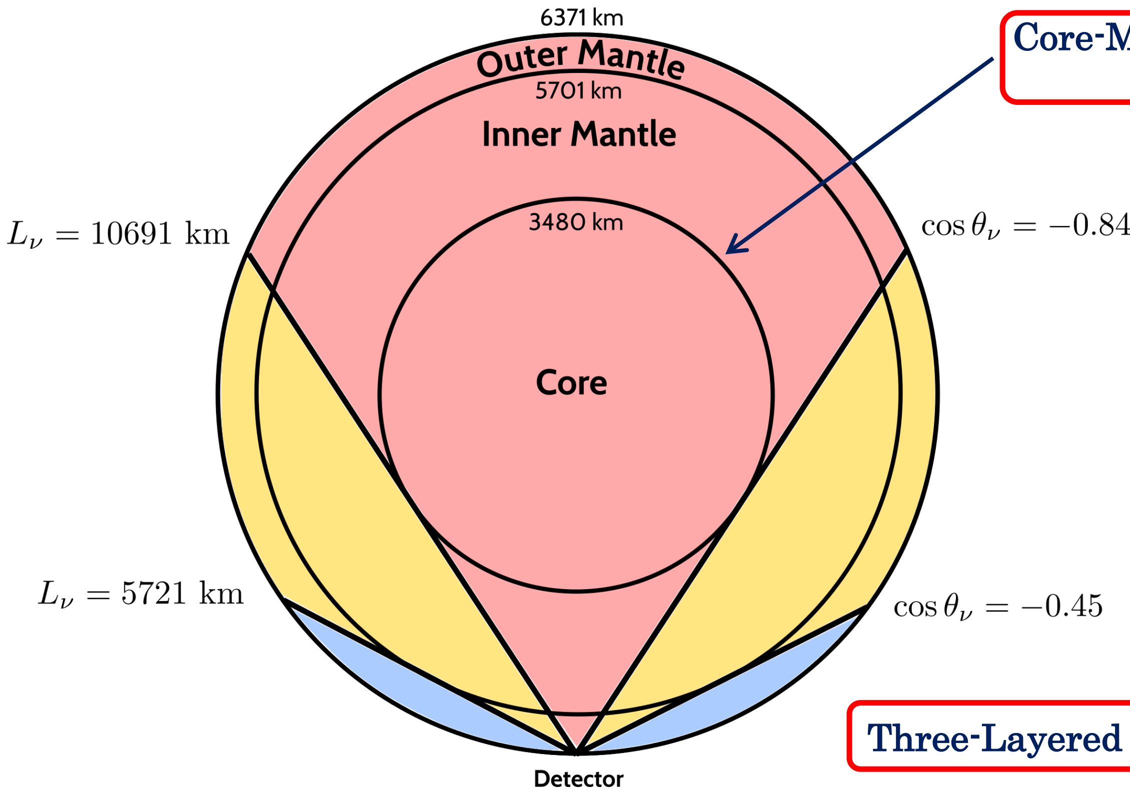
Neutrino Oscillograms for a Core-Passing Cascade-Like Events



We get similar oscillograms for antineutrinos with inverted mass ordering

Locating Core-Mantle Boundary (CMB) using Atmospheric Neutrinos

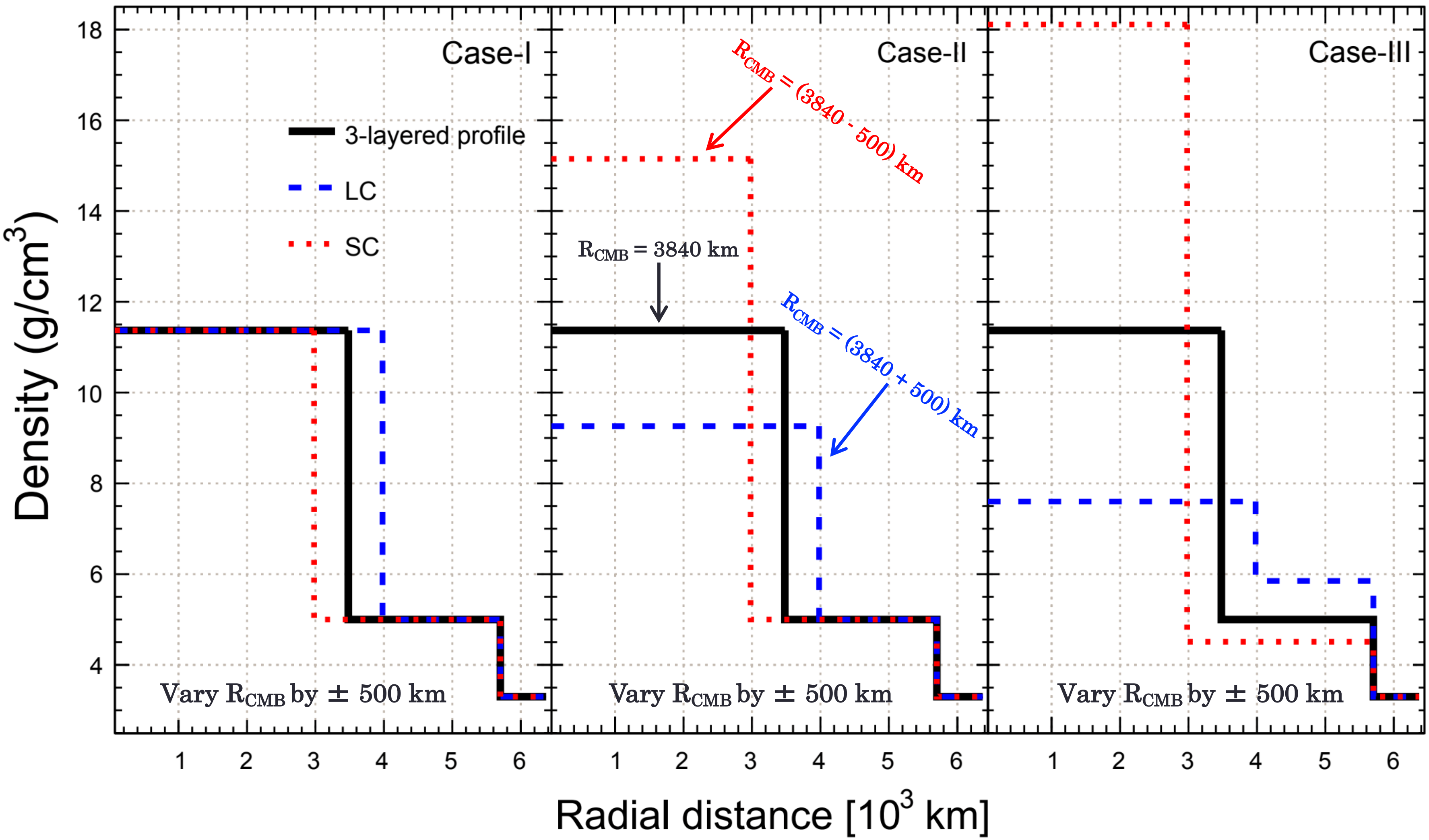
A. K. Upadhyay, A. Kumar, S. K. Agarwalla, and A. Dighe, in preparation



Core-Mantle Boundary at 3480 km

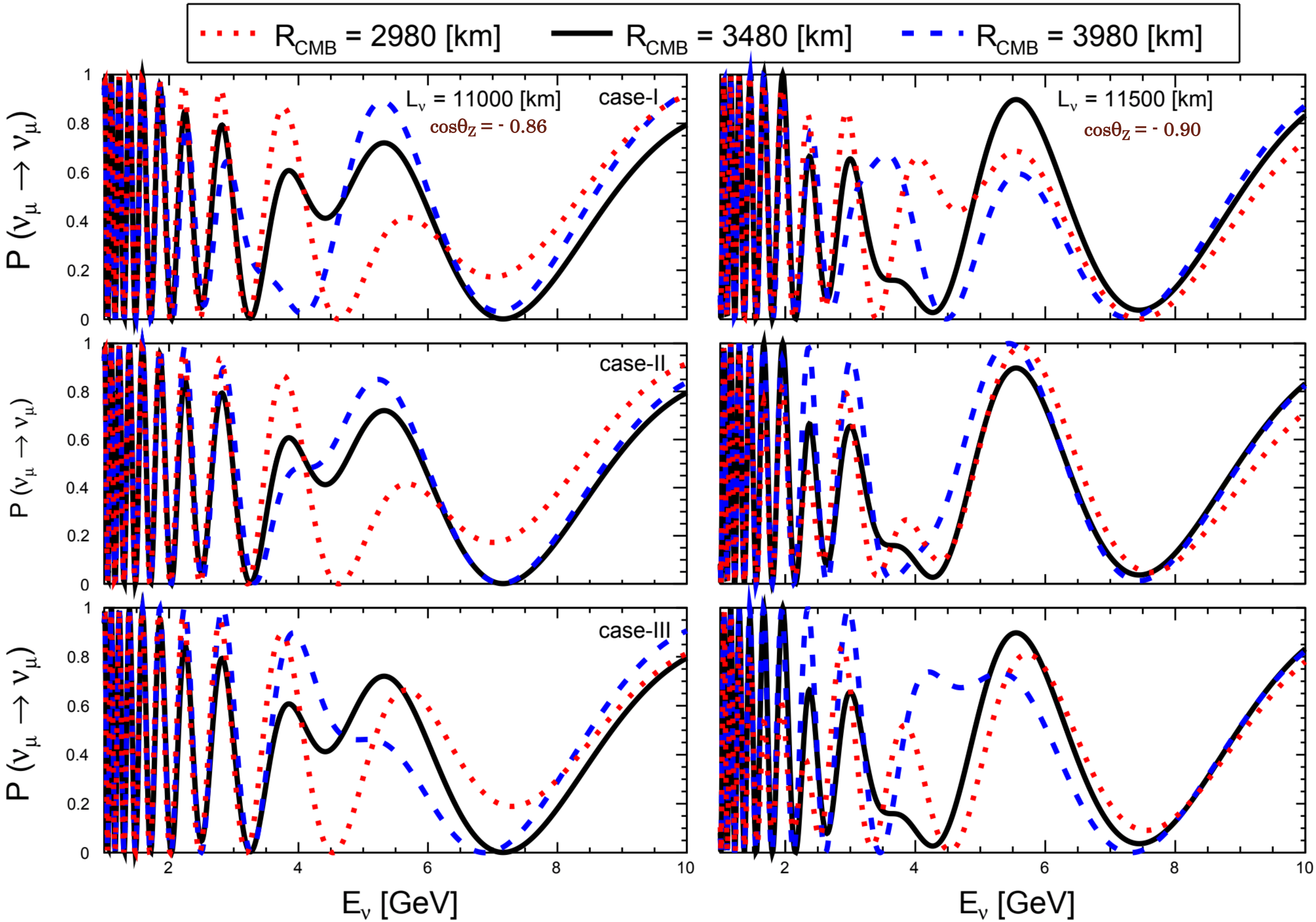
Three-Layered Model of Earth

A Few Toy Models of Earth with Varying CMB

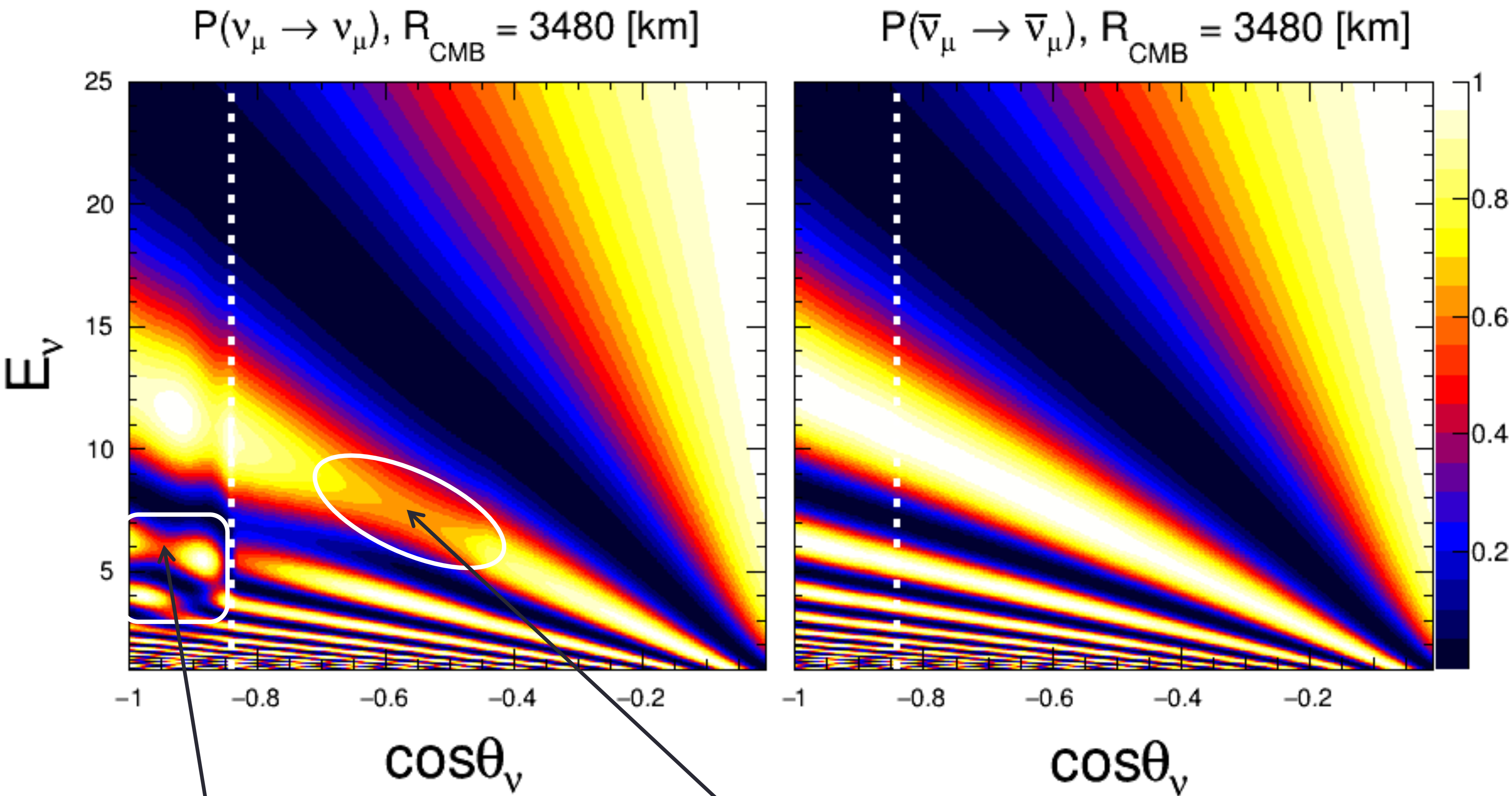


Case-I: Densities of all layers fixed and $M_\oplus$ is not invariant
Case-II: Densities of inner & outer mantle fixed. Core density varies to keep $M_\oplus$ invariant
Case-III: Core & inner mantle densities vary to keep their masses fixed. Outer mantle density fixed & $M_\oplus$ invariant

Muon Neutrino Survival Probabilities with Varying CMB



Muon Neutrino Survival Oscillograms with Standard CMB



Non-Adiabatic Neutrino Oscillation Length Resonance (NOLR) or Parametric Resonance (PR) Region

$\cos\theta_\nu < -0.8$
3 GeV $< E_\nu <$ 6 GeV

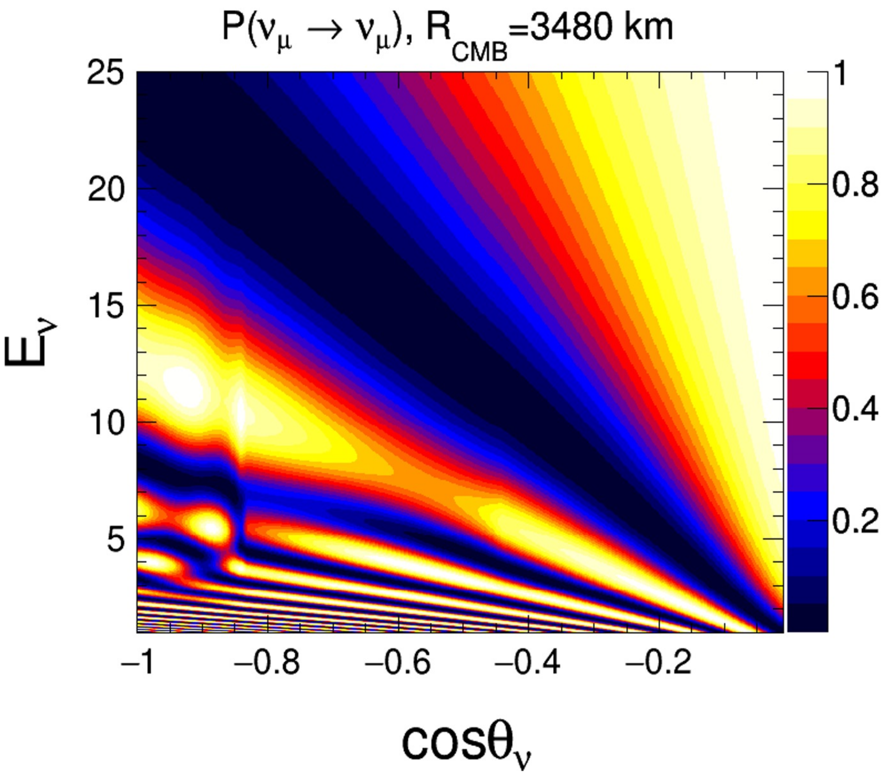
Adiabatic MSW Resonance Region

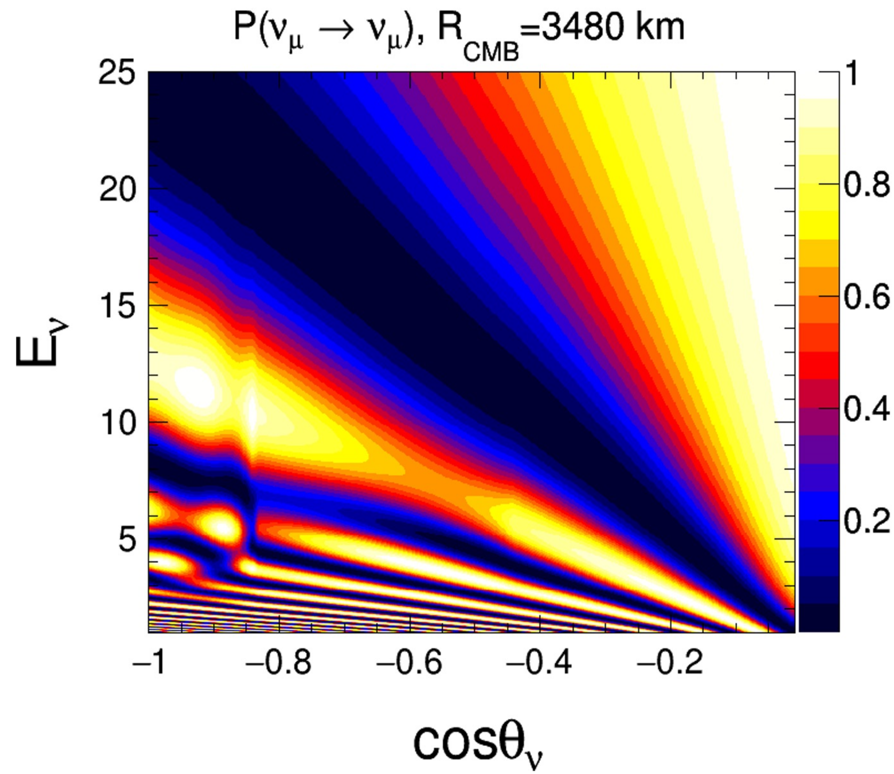
$-0.8 < \cos\theta_\nu < -0.5$
6 GeV $< E_\nu <$ 10 GeV

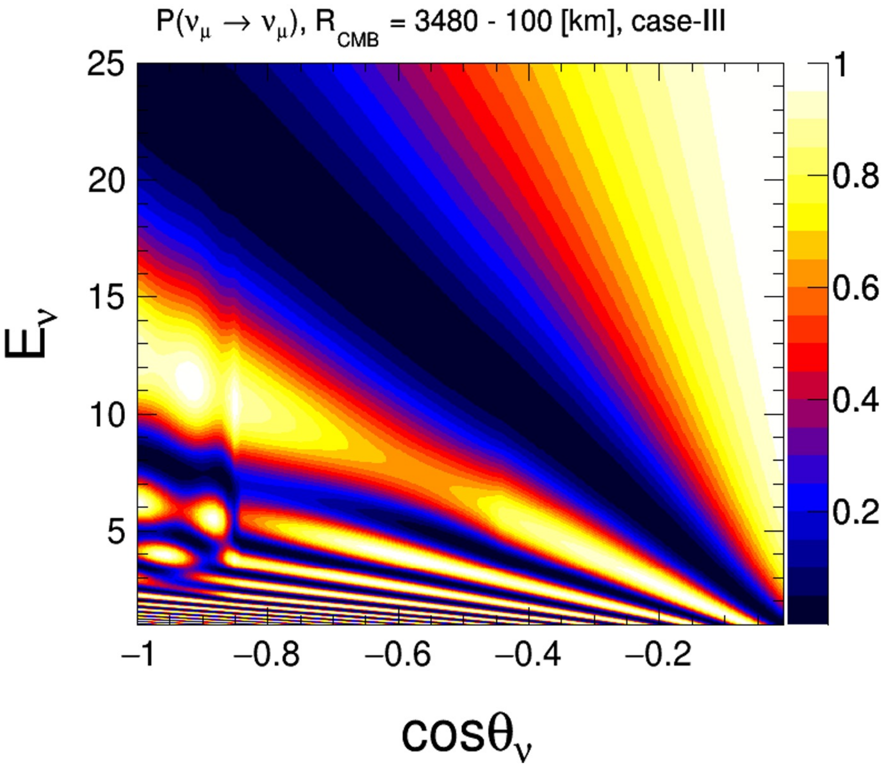
Neutrinos experience matter effect for NO

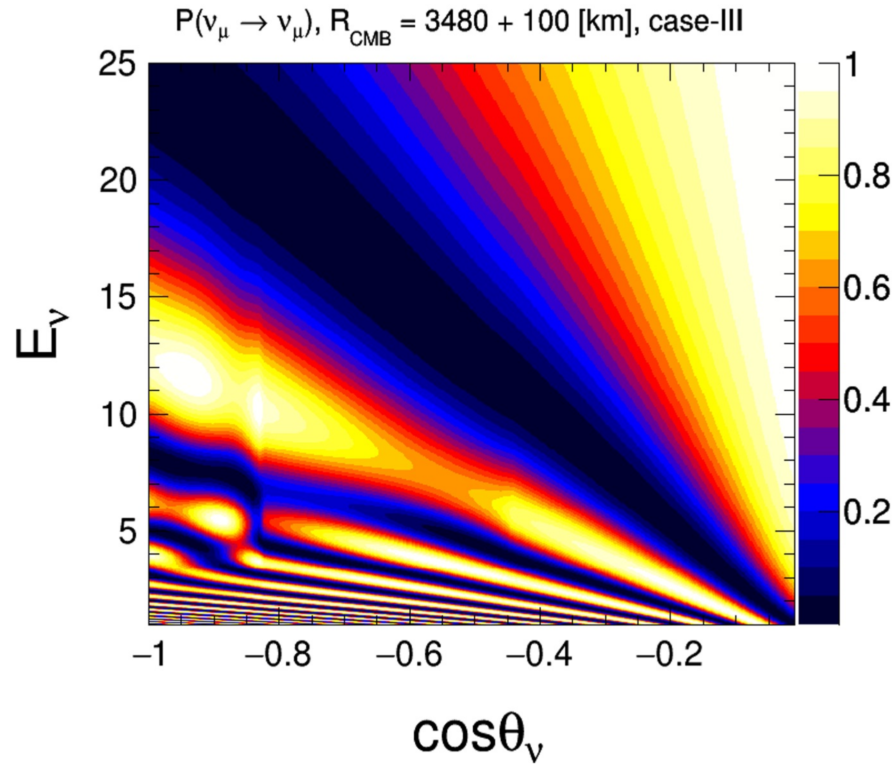
Antineutrinos experience matter effect for IO

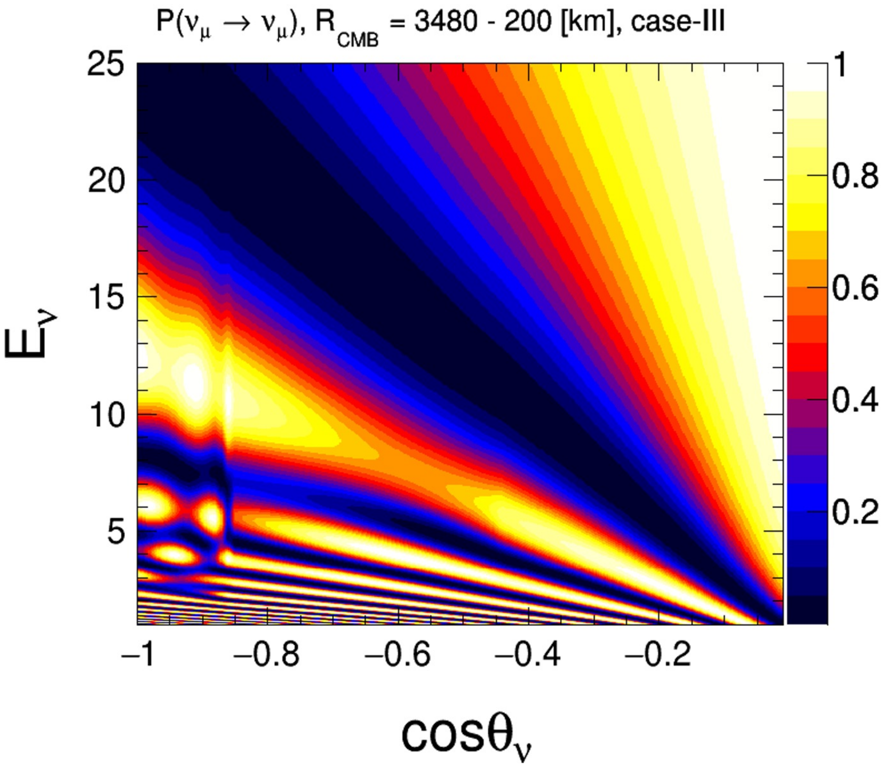
Case - III

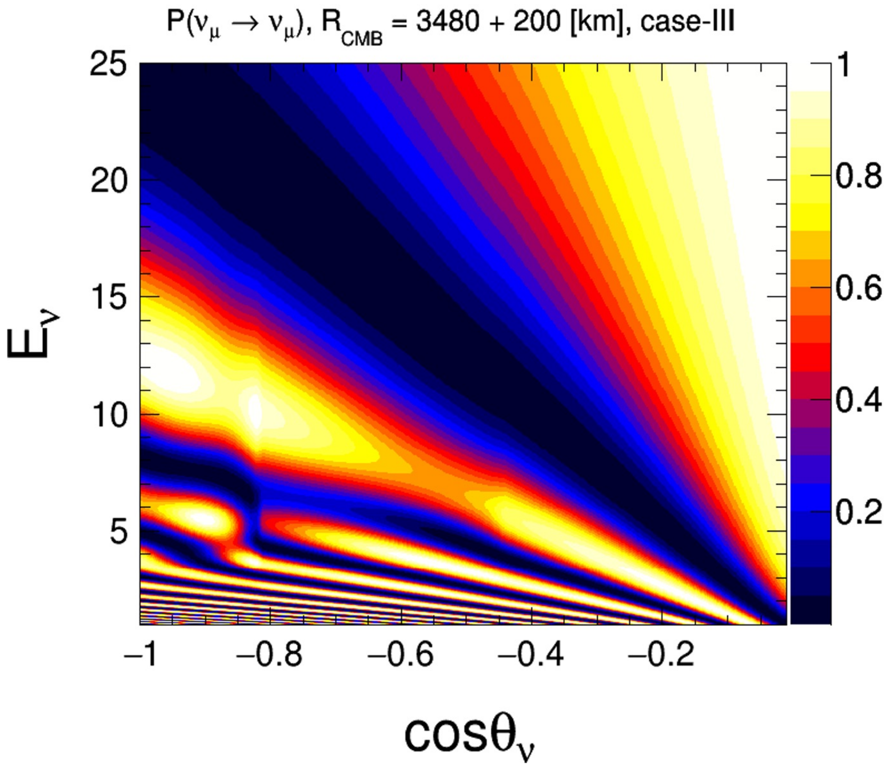

$P(\nu_\mu \rightarrow \nu_\mu)$, R_{CMB}=3480 km
25
20
15
10
5
E_ν
–1
–0.8
–0.6
–0.4
–0.2
$\cos\theta_\nu$
1
0.8
0.6
0.4
0.2

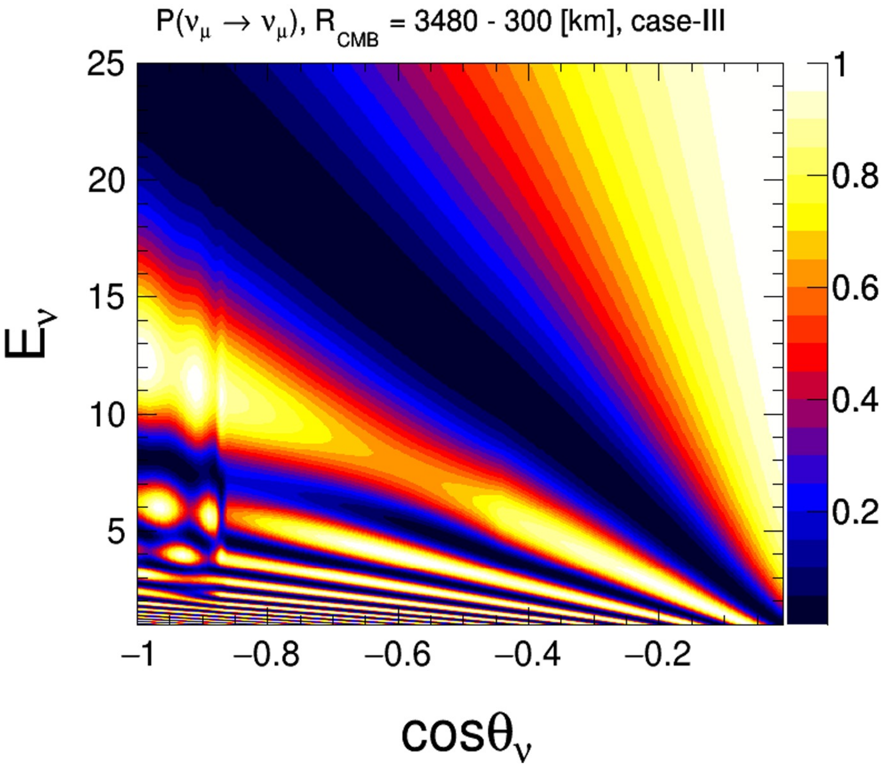

$P(\nu_\mu \rightarrow \nu_\mu)$, R_{CMB}=3480 km
25
20
15
10
5
E_ν
–1
–0.8
–0.6
–0.4
–0.2
$\cos\theta_\nu$
1
0.8
0.6
0.4
0.2


P(νμ → νμ), RCMB = 3480 - 100 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2

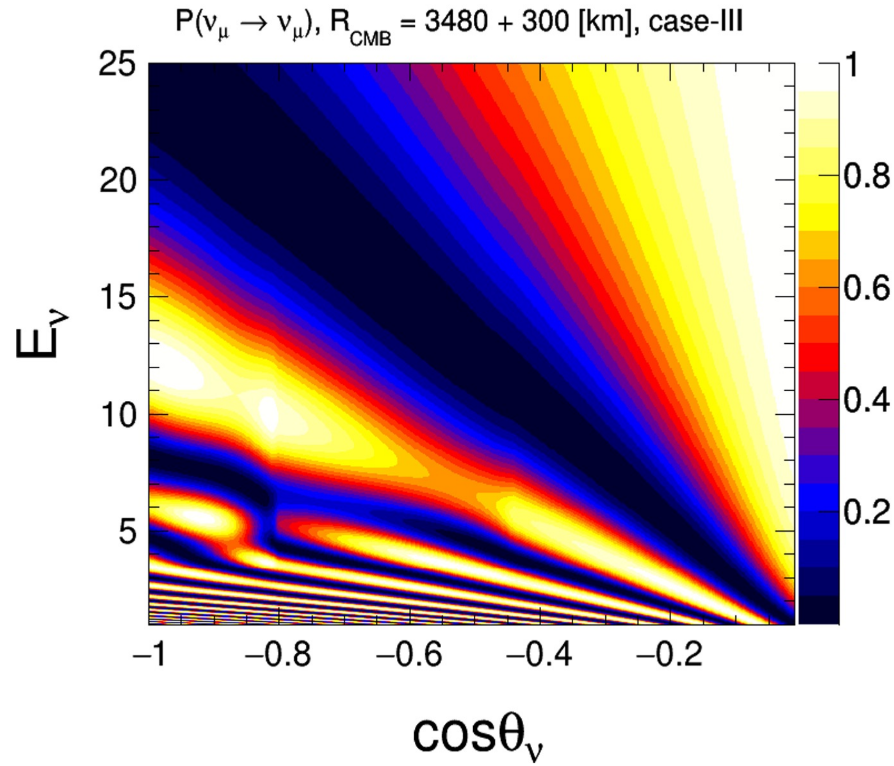

P(νμ → νμ), RCMB = 3480 + 100 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2


P(νμ → νμ), RCMB = 3480 - 200 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2

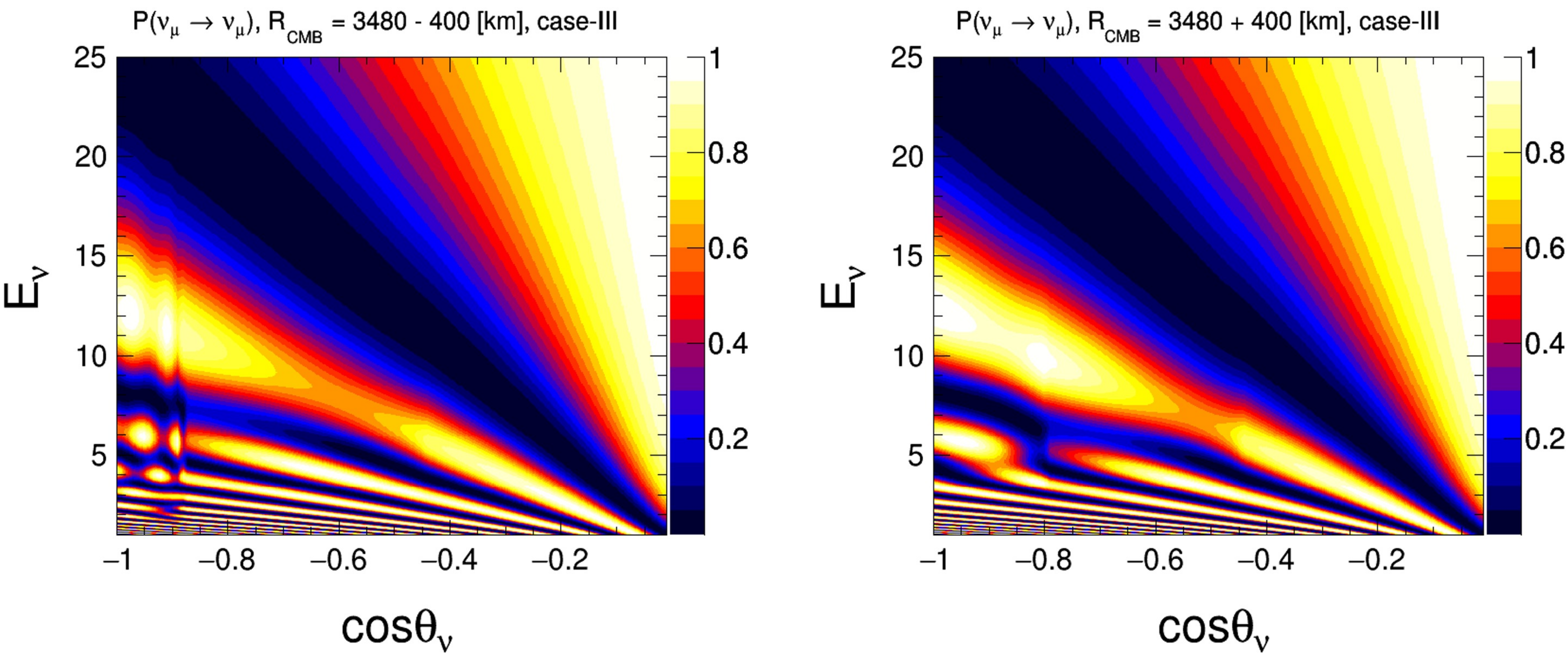

P(νμ → νμ), RCMB = 3480 + 200 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2

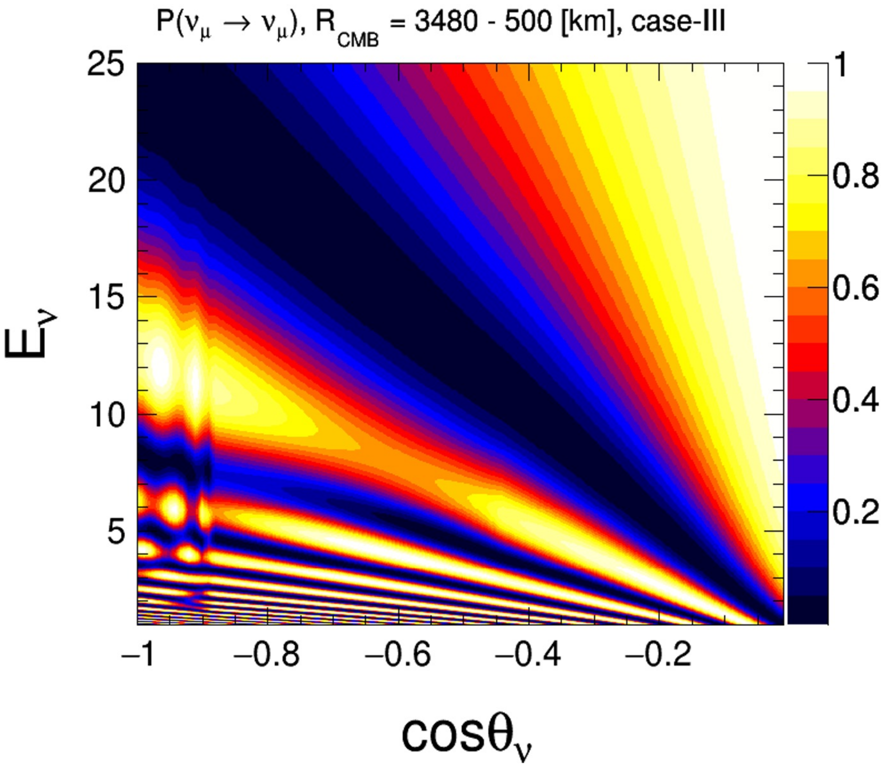


P(νμ → νμ), RCMB = 3480 - 300 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2

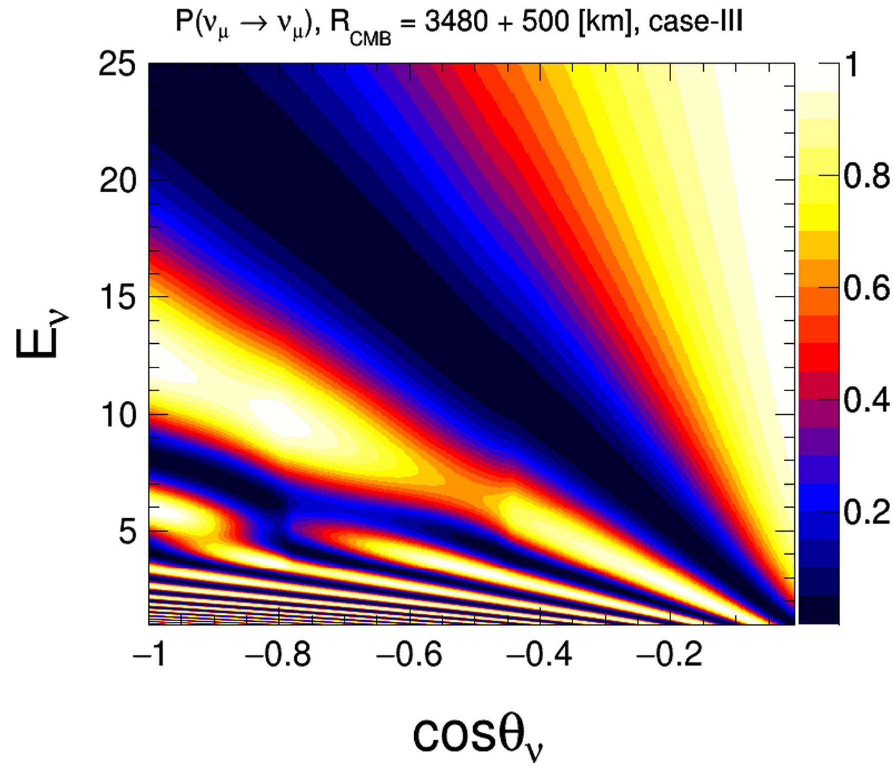


P(νμ → νμ), RCMB = 3480 + 300 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2

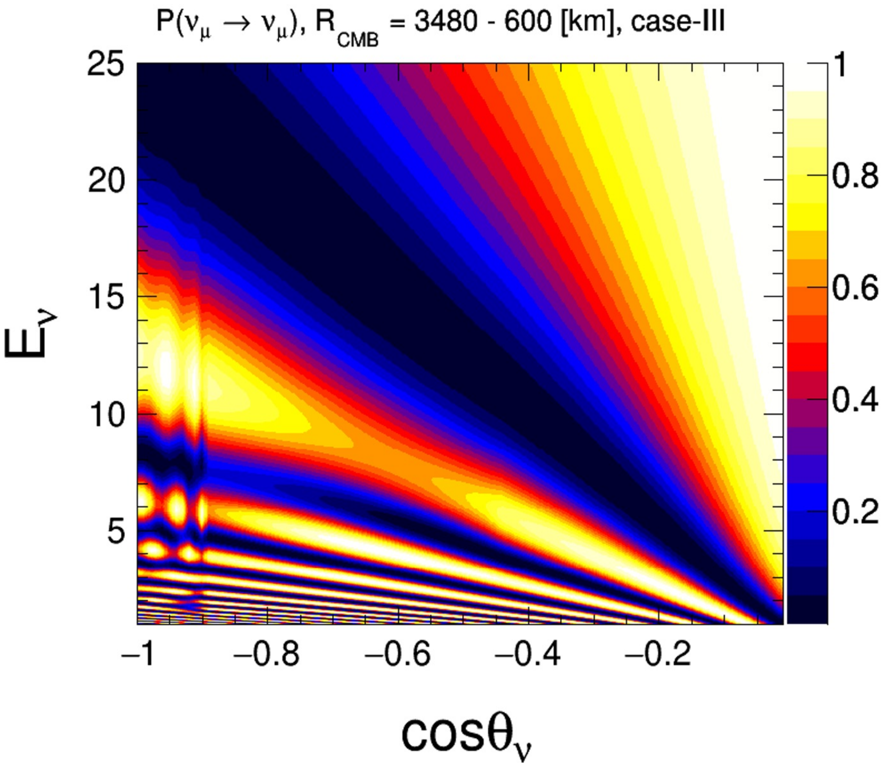

$P(\nu_\mu \to \nu_\mu)$, R_{CMB} = 3480 - 400 [km], case-III
$P(\nu_\mu \to \nu_\mu)$, R_{CMB} = 3480 + 400 [km], case-III
E_ν
$\cos\theta_\nu$
25
20
15
10
5
–1
–0.8
–0.6
–0.4
–0.2
1
0.8
0.6
0.4
0.2



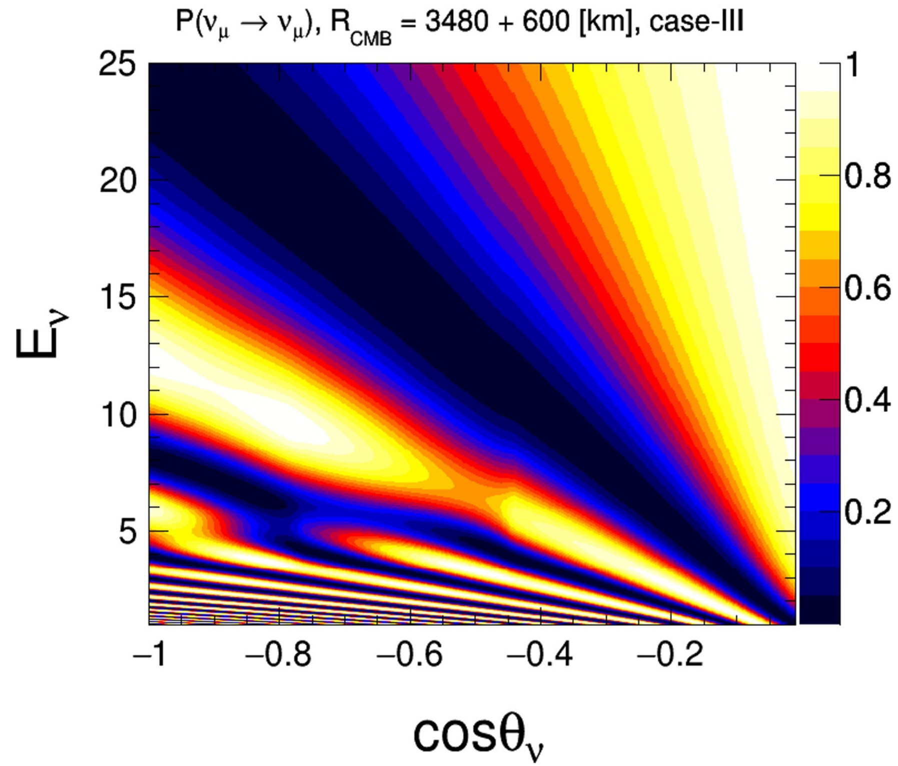
P(ν_μ → ν_μ), R_CMB = 3480 - 500 [km], case-III
25
20
15
10
5
E_ν
–1
–0.8
–0.6
–0.4
–0.2
cosθ_ν
1
0.8
0.6
0.4
0.2



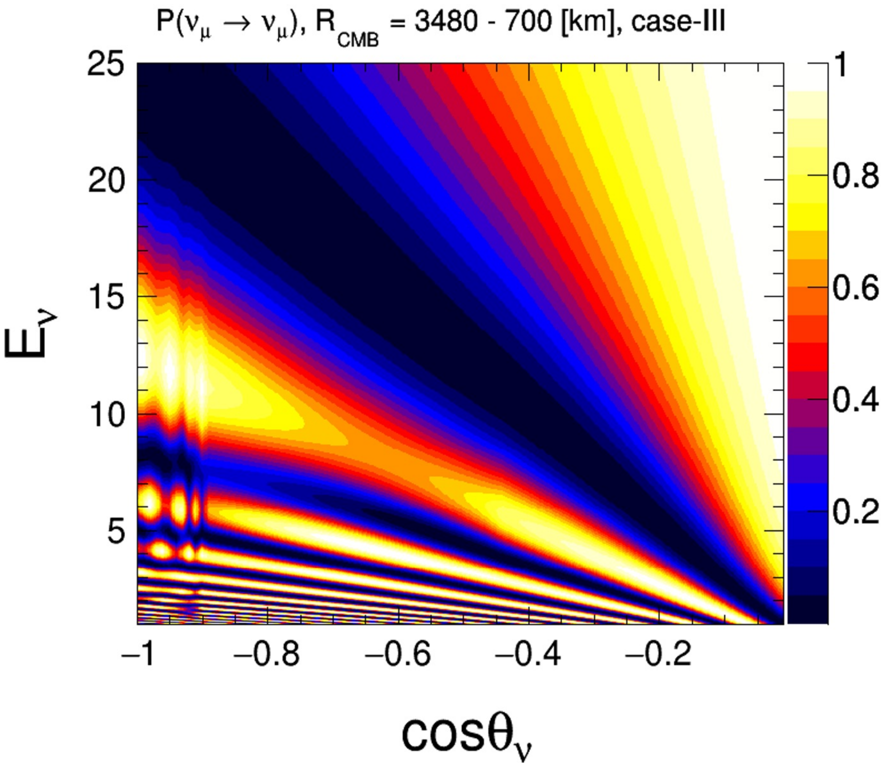
P(ν_μ → ν_μ), R_CMB = 3480 + 500 [km], case-III
25
20
15
10
5
E_ν
–1
–0.8
–0.6
–0.4
–0.2
cosθ_ν
1
0.8
0.6
0.4
0.2

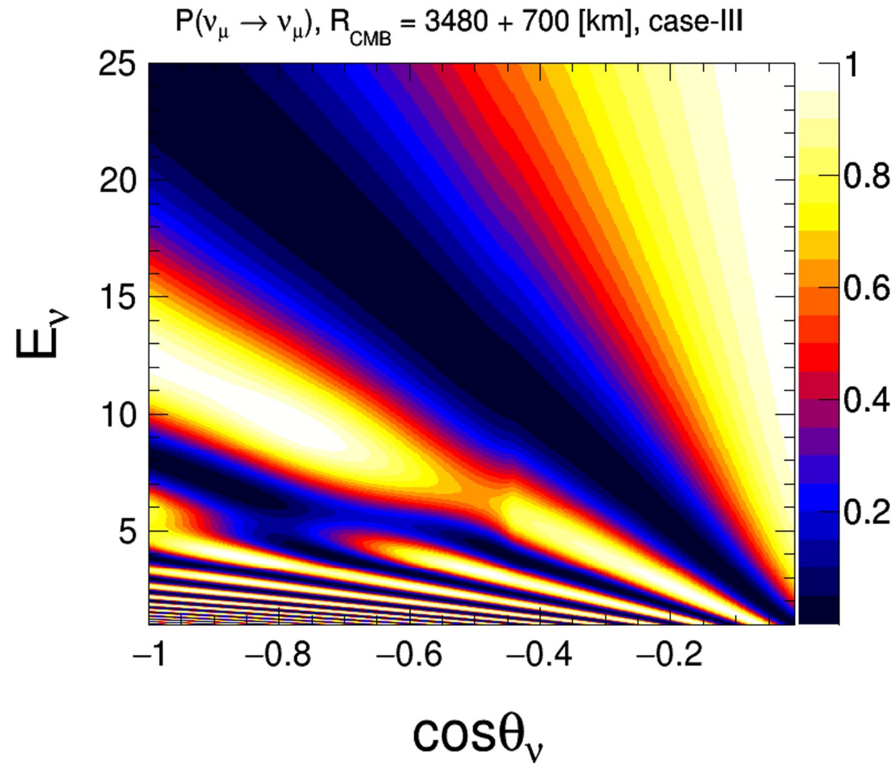


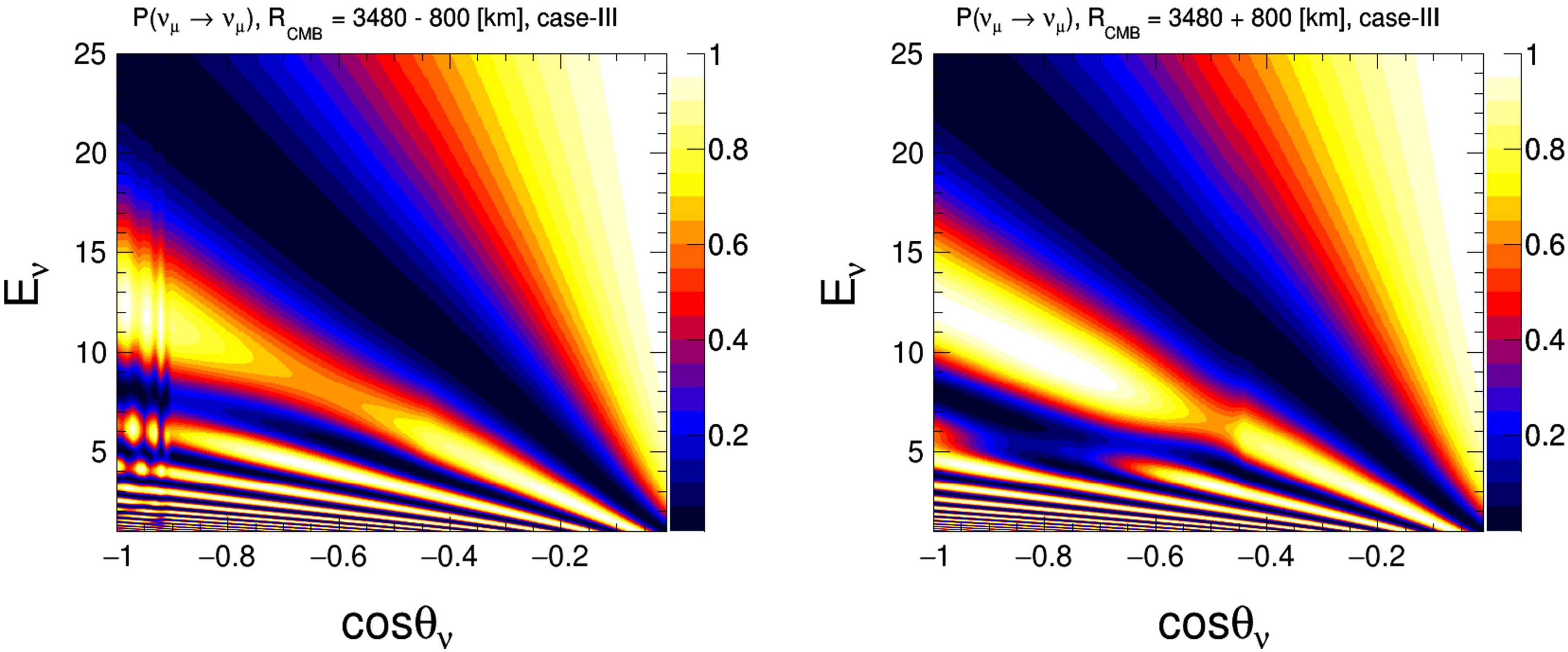
$P(\nu_\mu \to \nu_\mu)$, R_{CMB} = 3480 - 600 [km], case-III
E_ν
$\cos\theta_\nu$

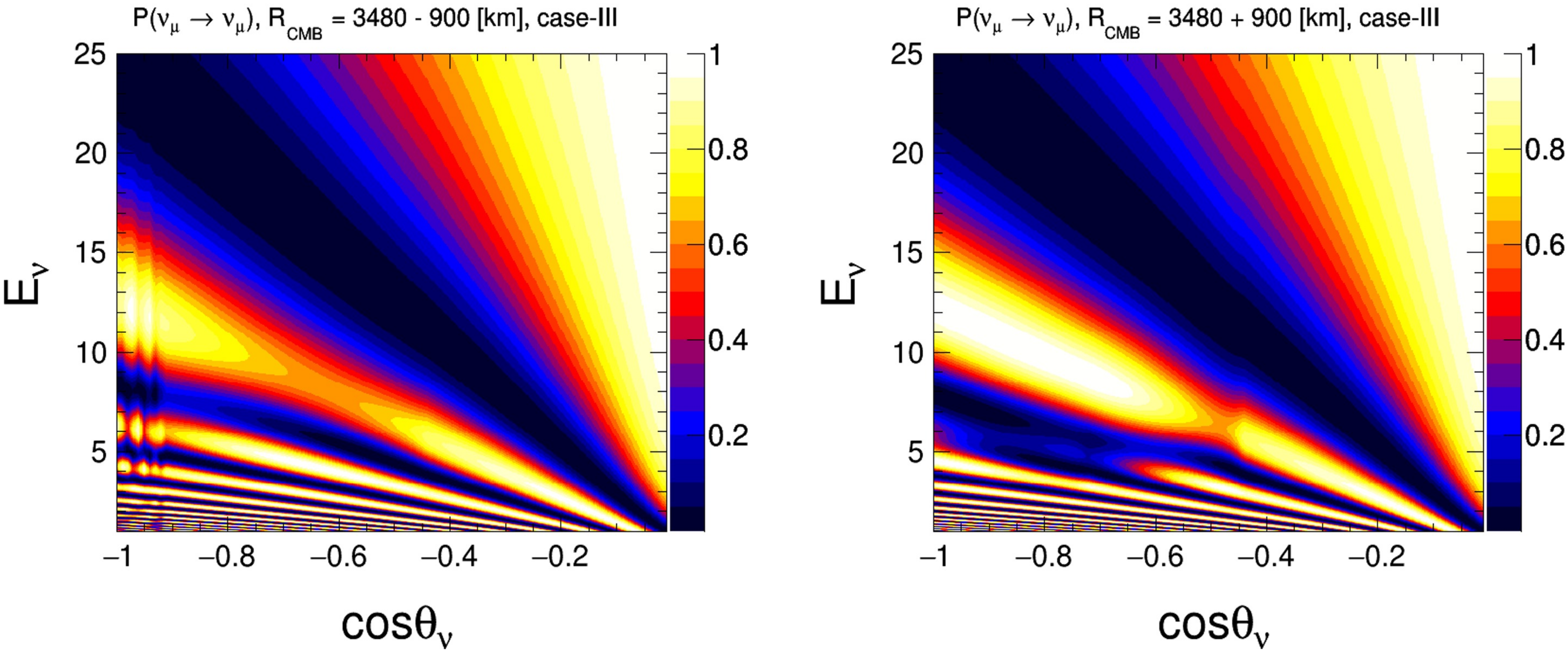


$P(\nu_\mu \to \nu_\mu)$, R_{CMB} = 3480 + 600 [km], case-III
E_ν
$\cos\theta_\nu$

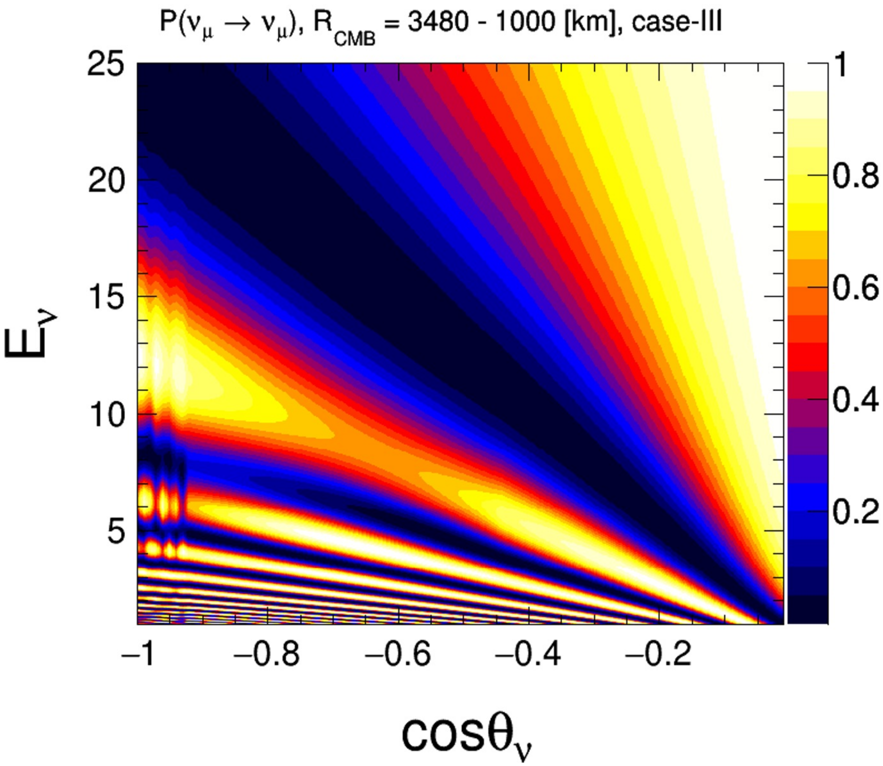

P(νμ → νμ), RCMB = 3480 - 700 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2


P(νμ → νμ), RCMB = 3480 + 700 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2

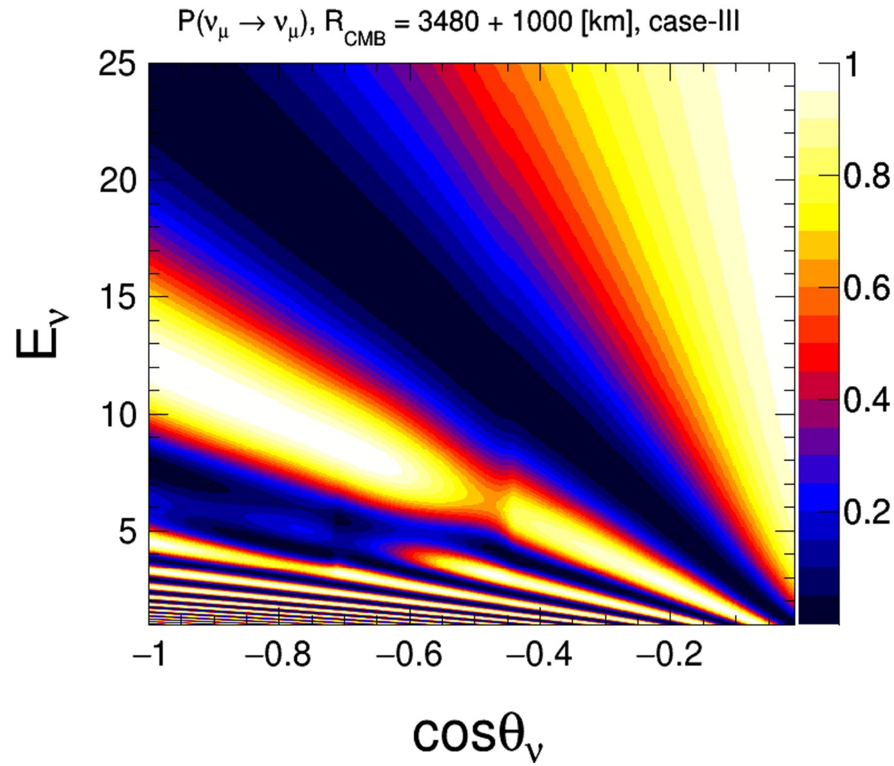

P(νμ → νμ), R_CMB = 3480 - 800 [km], case-III
P(νμ → νμ), R_CMB = 3480 + 800 [km], case-III
25
20
15
10
5
Eν
1
0.8
0.6
0.4
0.2
–1
–0.8
–0.6
–0.4
–0.2
cosθν



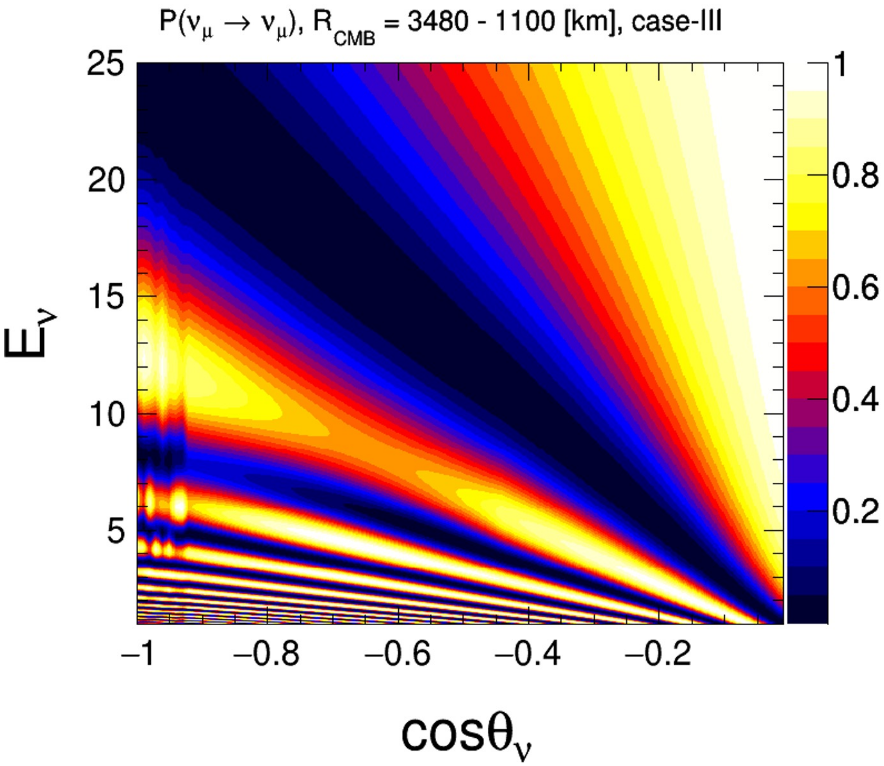
P(ν_μ → ν_μ), R_CMB = 3480 - 900 [km], case-III
25
20
15
10
5
E_ν
1
0.8
0.6
0.4
0.2
−1
−0.8
−0.6
−0.4
−0.2
cosθ_ν
P(ν_μ → ν_μ), R_CMB = 3480 + 900 [km], case-III
25
20
15
10
5
E_ν
1
0.8
0.6
0.4
0.2
−1
−0.8
−0.6
−0.4
−0.2
cosθ_ν

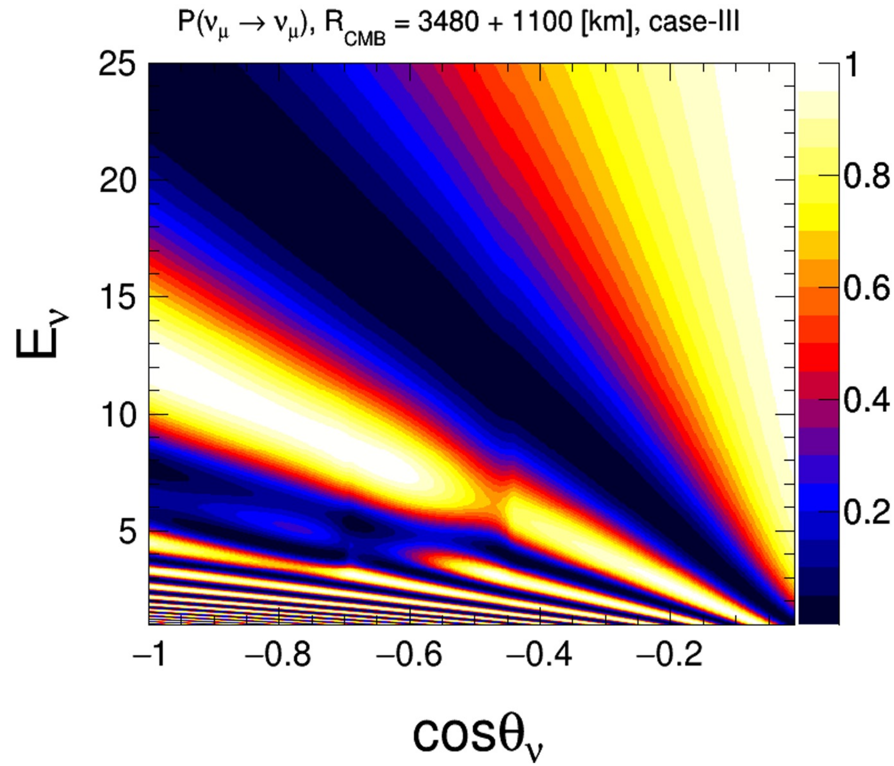


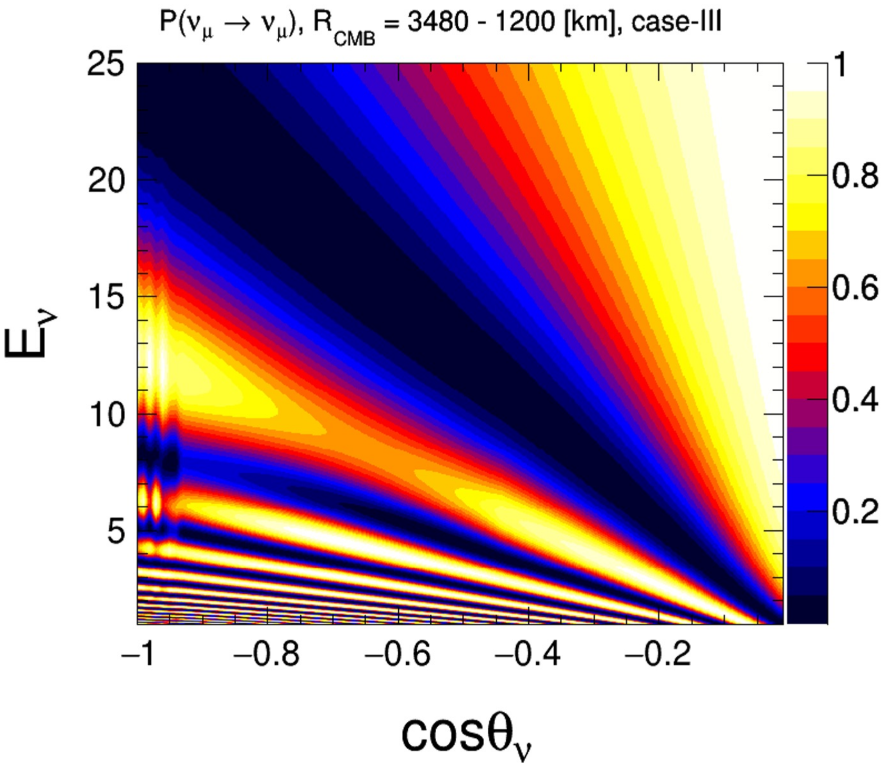
P(ν_μ → ν_μ), R_CMB = 3480 - 1000 [km], case-III
25
20
15
10
5
E_ν
−1
−0.8
−0.6
−0.4
−0.2
cosθ_ν
1
0.8
0.6
0.4
0.2

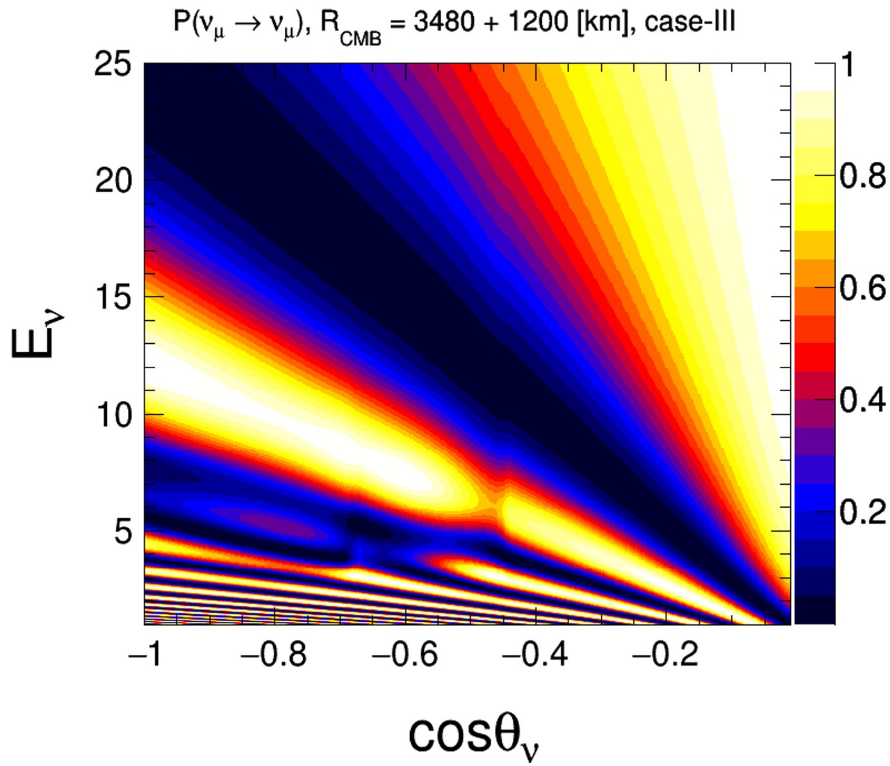


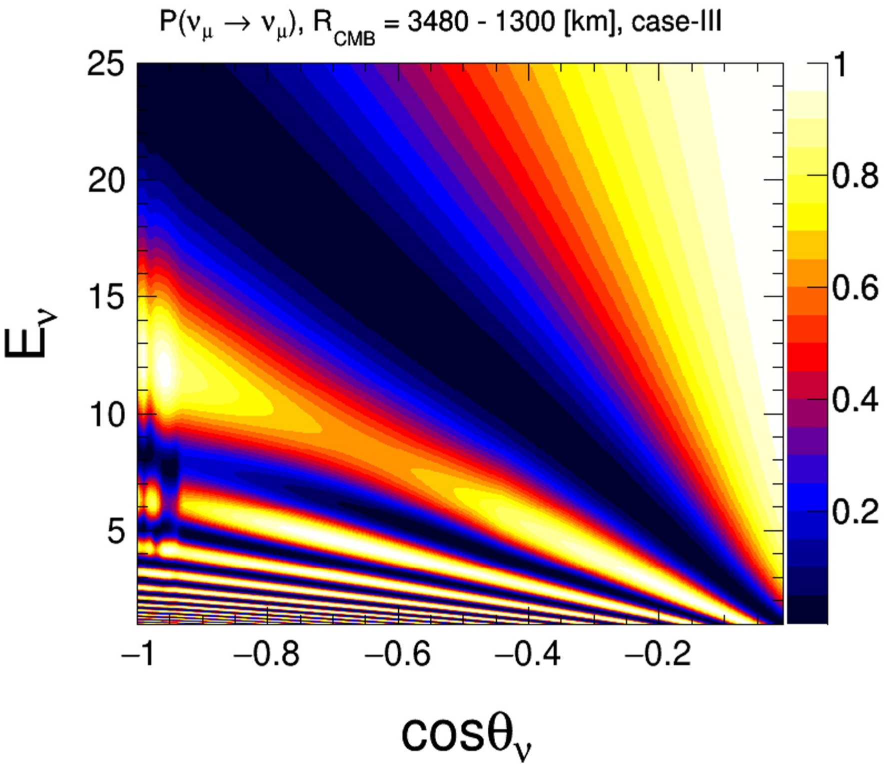
P(ν_μ → ν_μ), R_CMB = 3480 + 1000 [km], case-III
25
20
15
10
5
E_ν
−1
−0.8
−0.6
−0.4
−0.2
cosθ_ν
1
0.8
0.6
0.4
0.2


P(νμ → νμ), RCMB = 3480 - 1100 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2

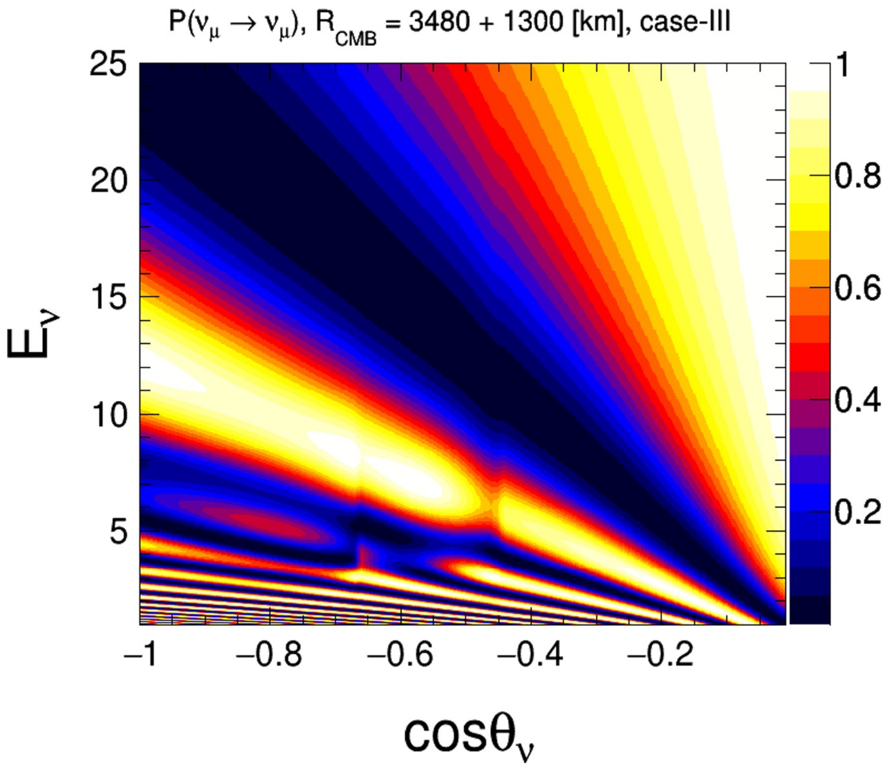

P(νμ → νμ), RCMB = 3480 + 1100 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2


P(νμ → νμ), RCMB = 3480 - 1200 [km], case-III
Eν
cosθν
25
20
15
10
5
−1
−0.8
−0.6
−0.4
−0.2
1
0.8
0.6
0.4
0.2

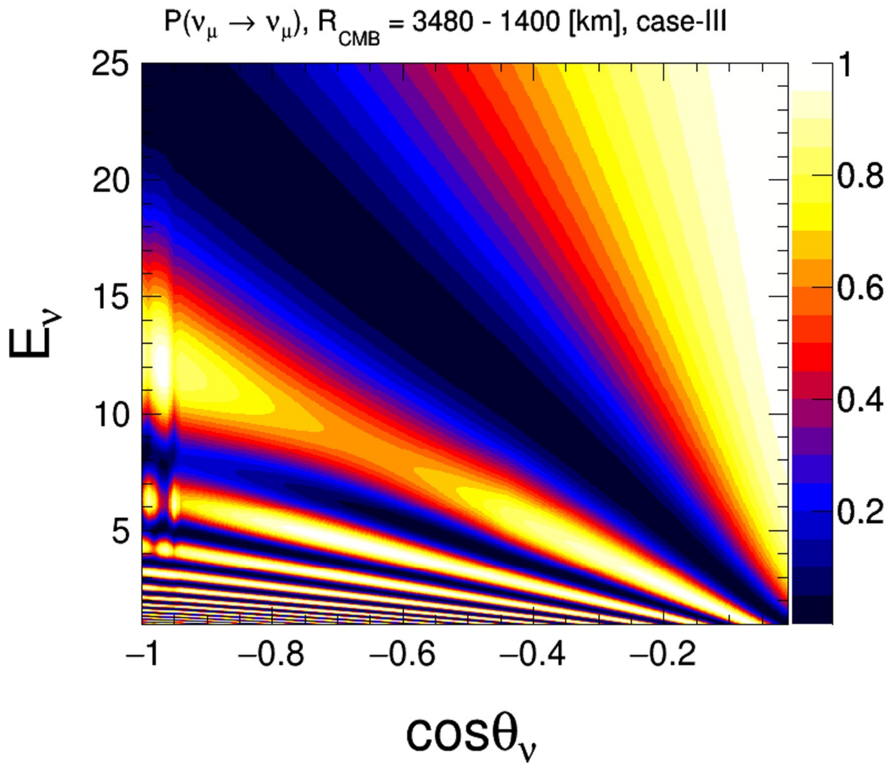

P(νμ → νμ), RCMB = 3480 + 1200 [km], case-III
Eν
cosθν
25
20
15
10
5
−1
−0.8
−0.6
−0.4
−0.2
1
0.8
0.6
0.4
0.2

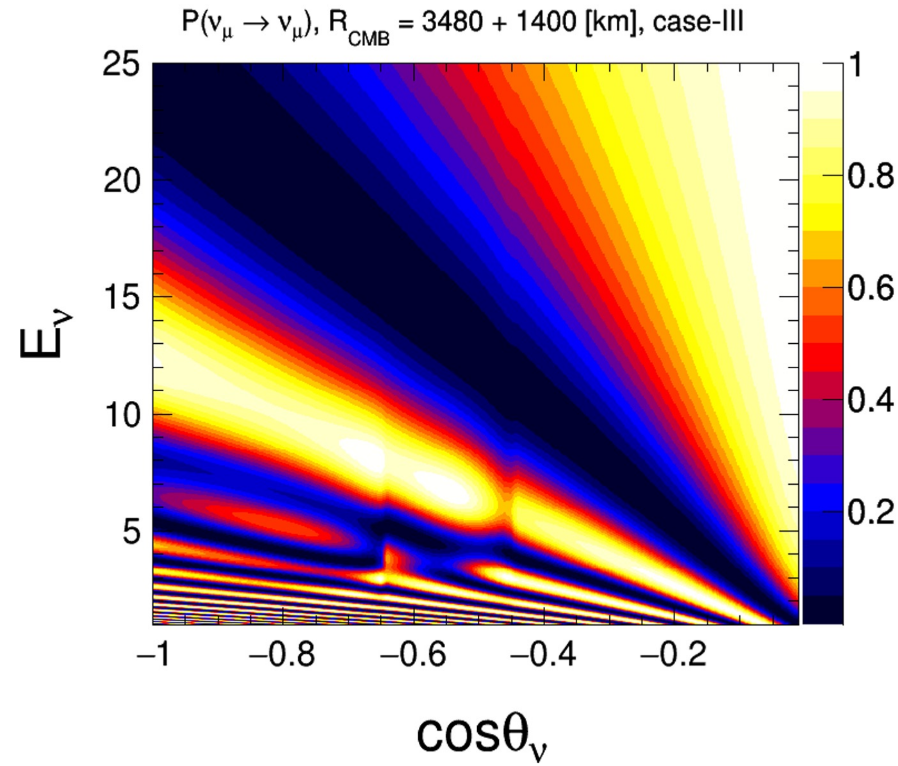


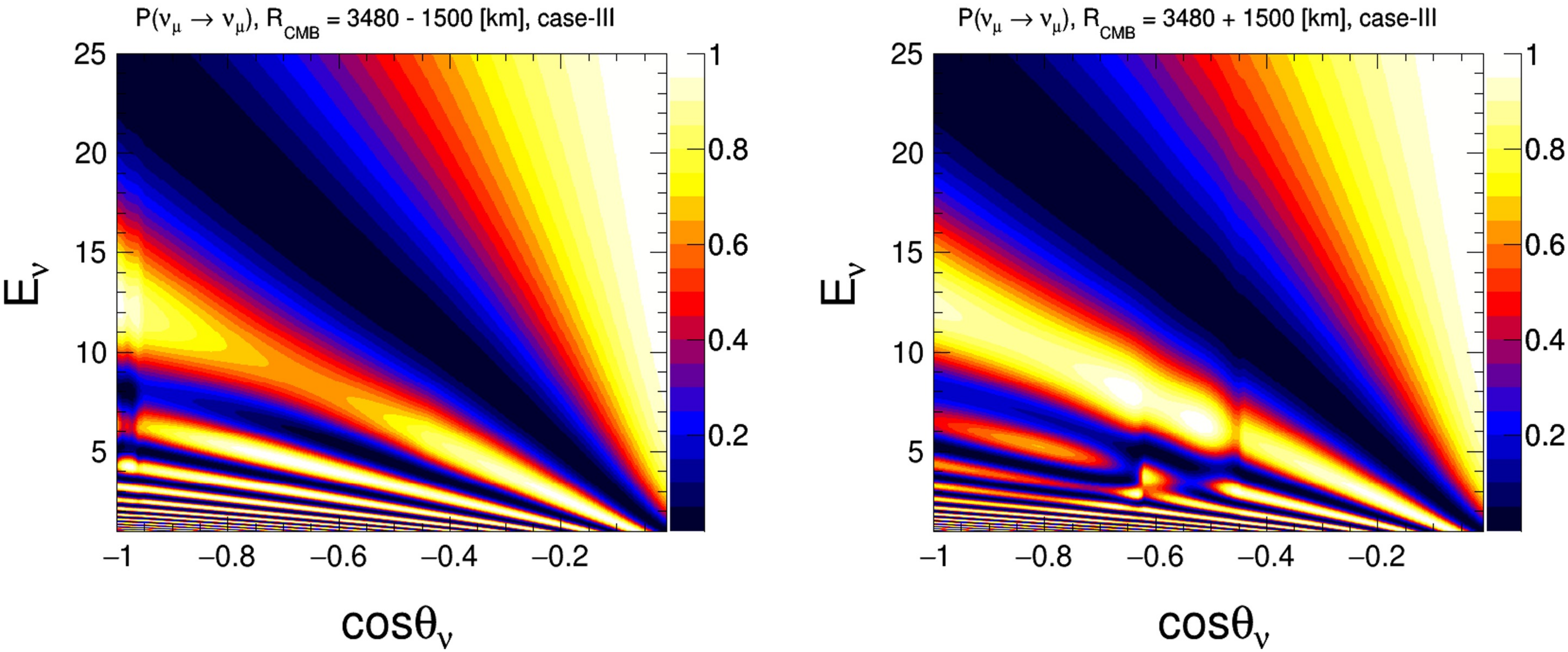
P(νμ → νμ), RCMB = 3480 - 1300 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2



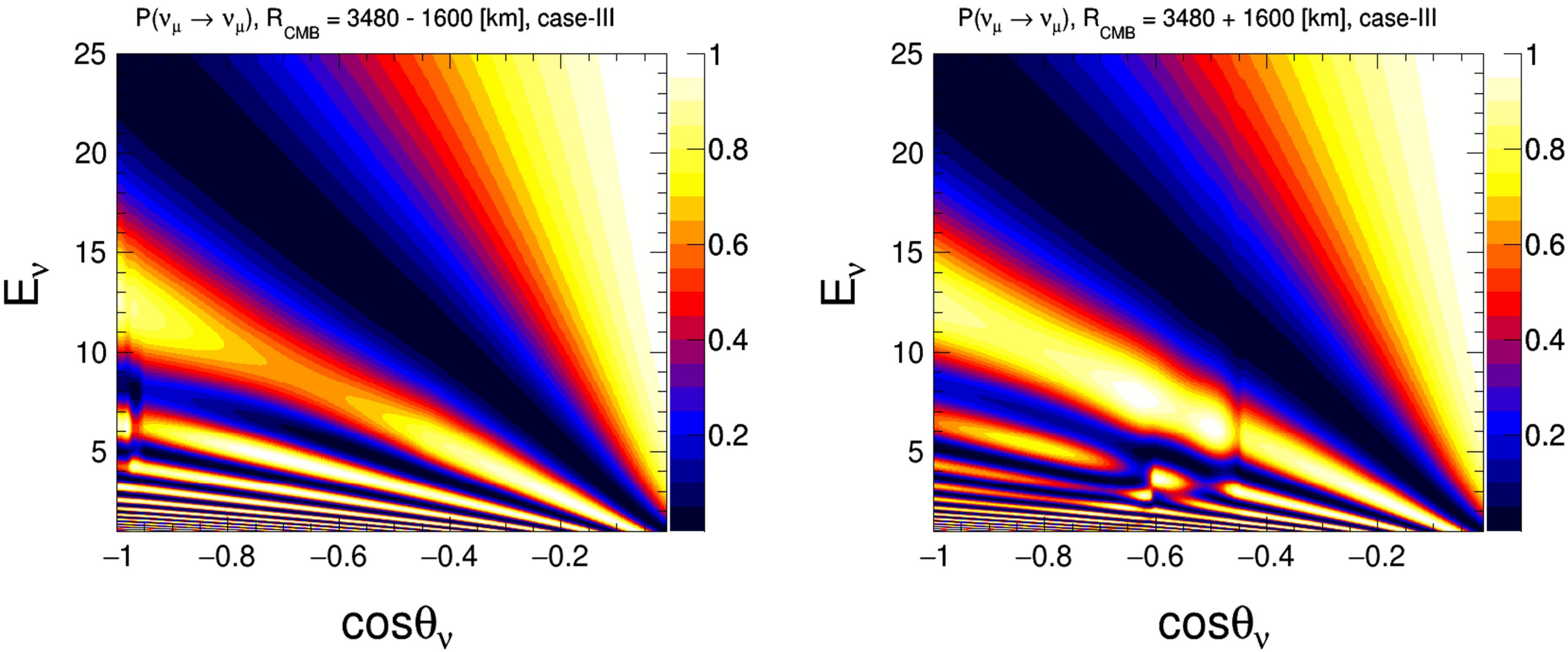
P(νμ → νμ), RCMB = 3480 + 1300 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2

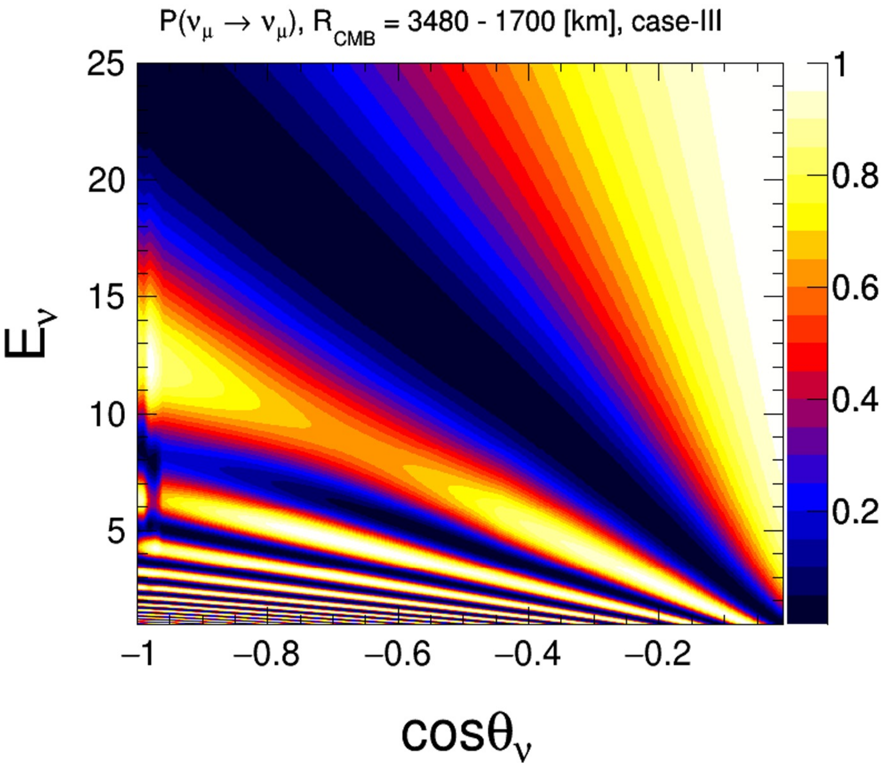

P(ν_μ → ν_μ), R_CMB = 3480 - 1400 [km], case-III
25
20
15
10
5
E_ν
−1
−0.8
−0.6
−0.4
−0.2
cosθ_ν
1
0.8
0.6
0.4
0.2


P(ν_μ → ν_μ), R_CMB = 3480 + 1400 [km], case-III
25
20
15
10
5
E_ν
−1
−0.8
−0.6
−0.4
−0.2
cosθ_ν
1
0.8
0.6
0.4
0.2

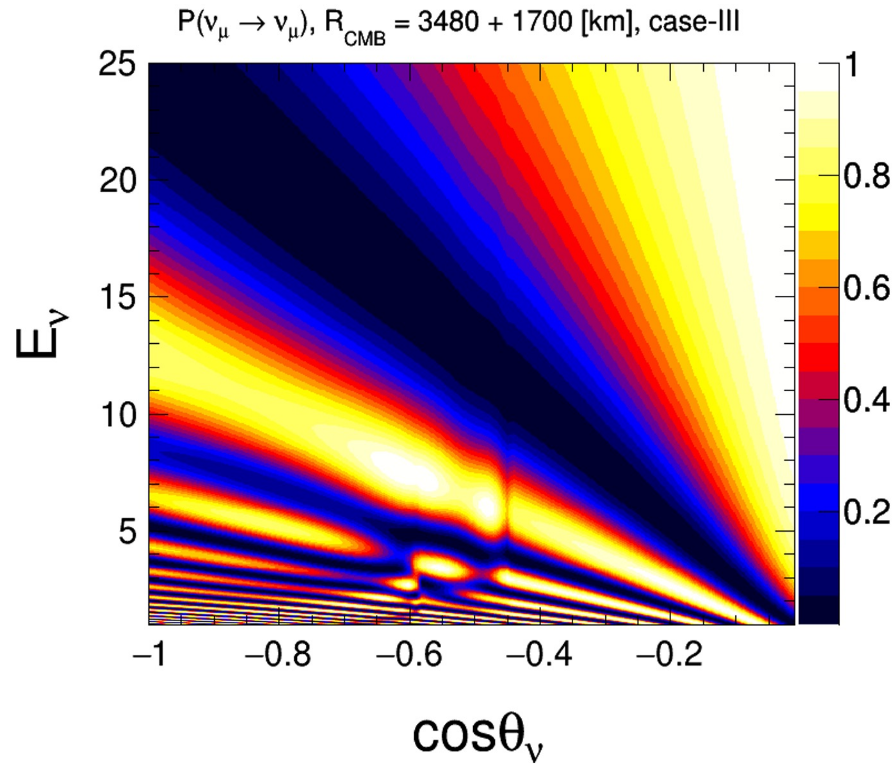


P(νμ → νμ), RCMB = 3480 - 1500 [km], case-III
P(νμ → νμ), RCMB = 3480 + 1500 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2

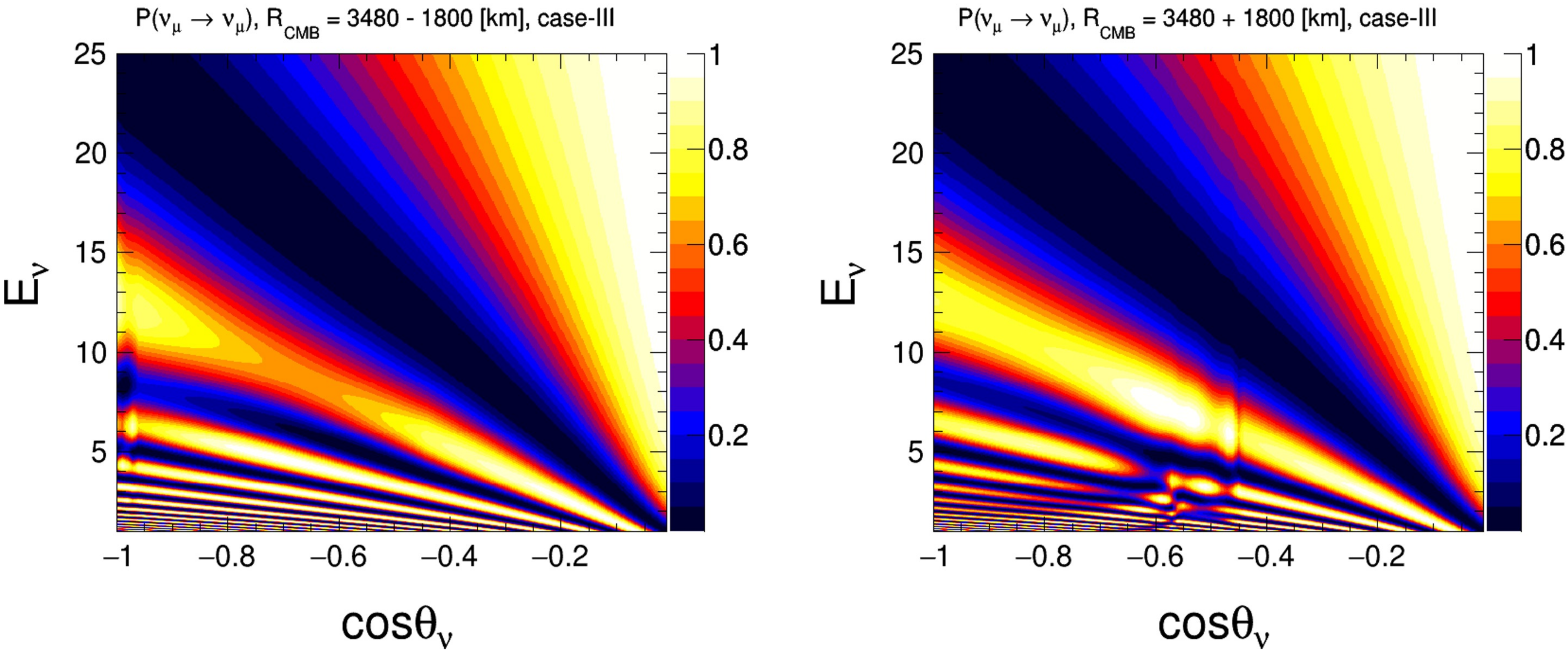

P(νμ → νμ), RCMB = 3480 - 1600 [km], case-III
P(νμ → νμ), RCMB = 3480 + 1600 [km], case-III
Eν
cosθν
25
20
15
10
5
−1
−0.8
−0.6
−0.4
−0.2
1
0.8
0.6
0.4
0.2



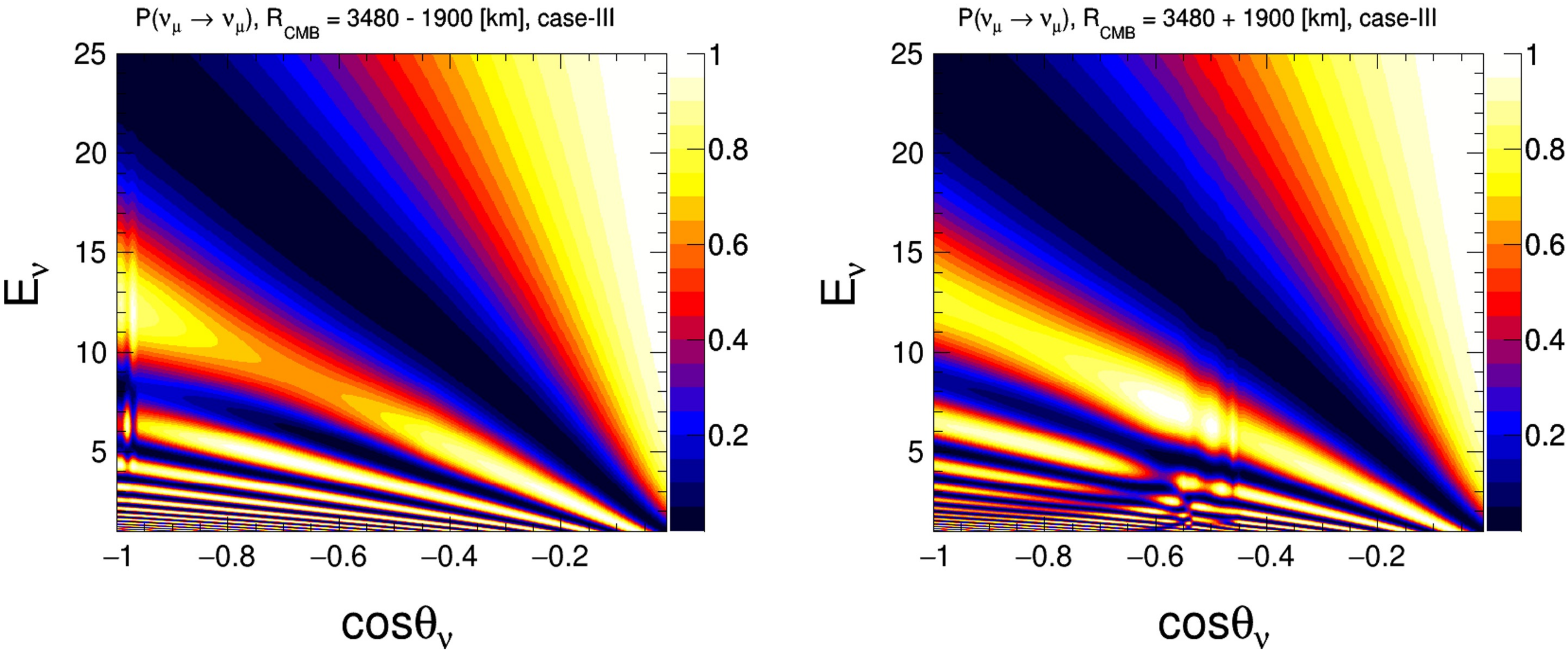
P(νμ → νμ), R_CMB = 3480 - 1700 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2



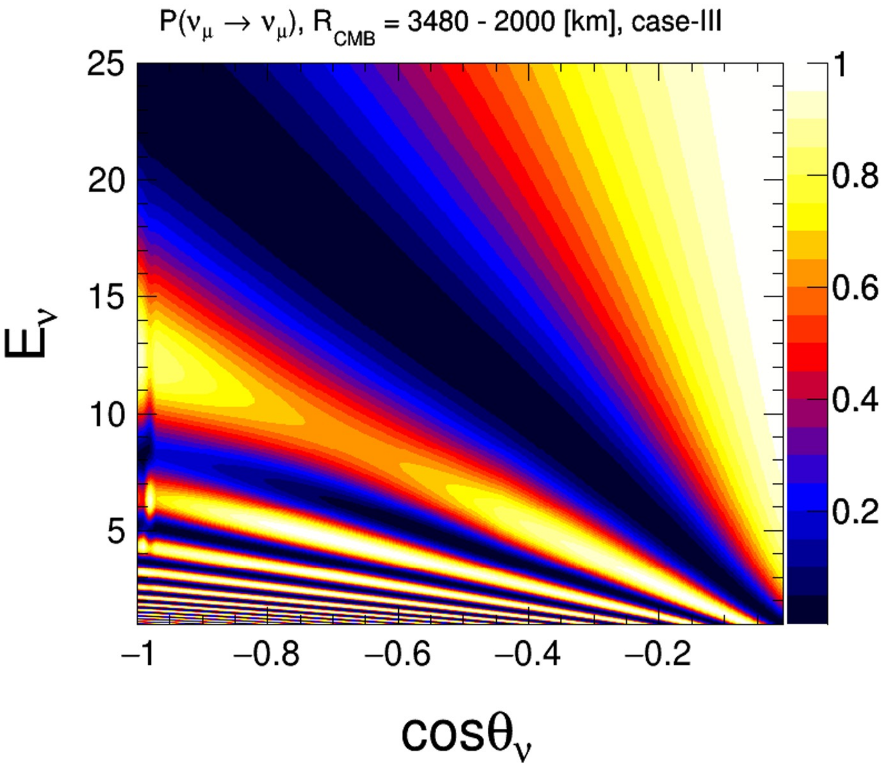
P(νμ → νμ), R_CMB = 3480 + 1700 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2

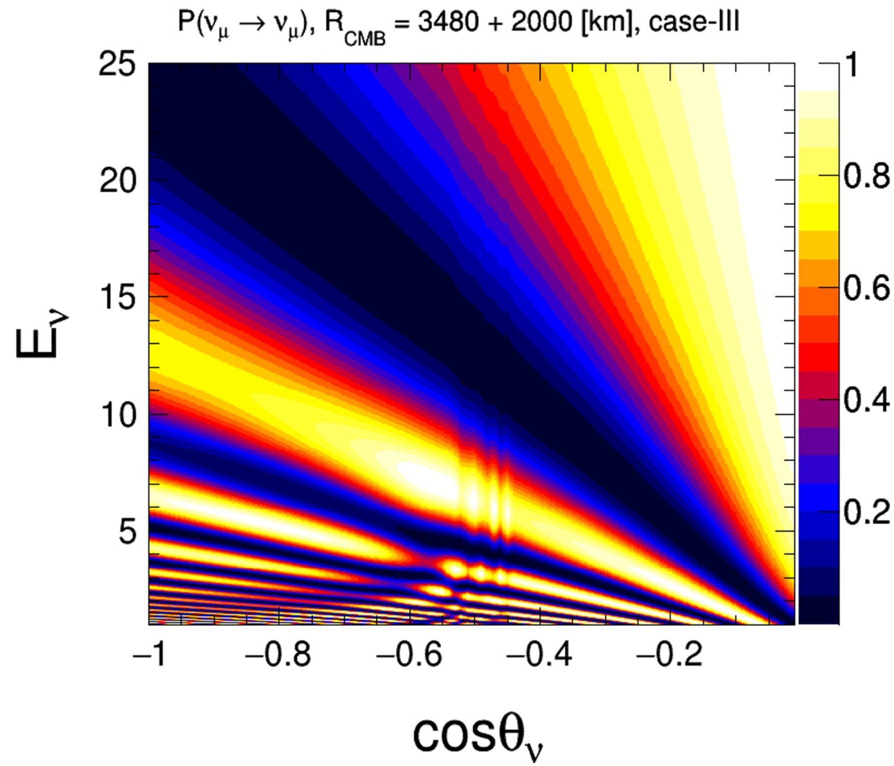


P(νμ → νμ), RCMB = 3480 - 1800 [km], case-III
25
20
15
10
5
Eν
1
0.8
0.6
0.4
0.2
−1
−0.8
−0.6
−0.4
−0.2
cosθν
P(νμ → νμ), RCMB = 3480 + 1800 [km], case-III
25
20
15
10
5
Eν
1
0.8
0.6
0.4
0.2
−1
−0.8
−0.6
−0.4
−0.2
cosθν



P(νμ → νμ), RCMB = 3480 - 1900 [km], case-III
25
20
15
10
5
Eν
1
0.8
0.6
0.4
0.2
−1
−0.8
−0.6
−0.4
−0.2
cosθν
P(νμ → νμ), RCMB = 3480 + 1900 [km], case-III
25
20
15
10
5
Eν
1
0.8
0.6
0.4
0.2
−1
−0.8
−0.6
−0.4
−0.2
cosθν


P(νμ → νμ), RCMB = 3480 - 2000 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2


P(νμ → νμ), RCMB = 3480 + 2000 [km], case-III
25
20
15
10
5
Eν
−1
−0.8
−0.6
−0.4
−0.2
cosθν
1
0.8
0.6
0.4
0.2

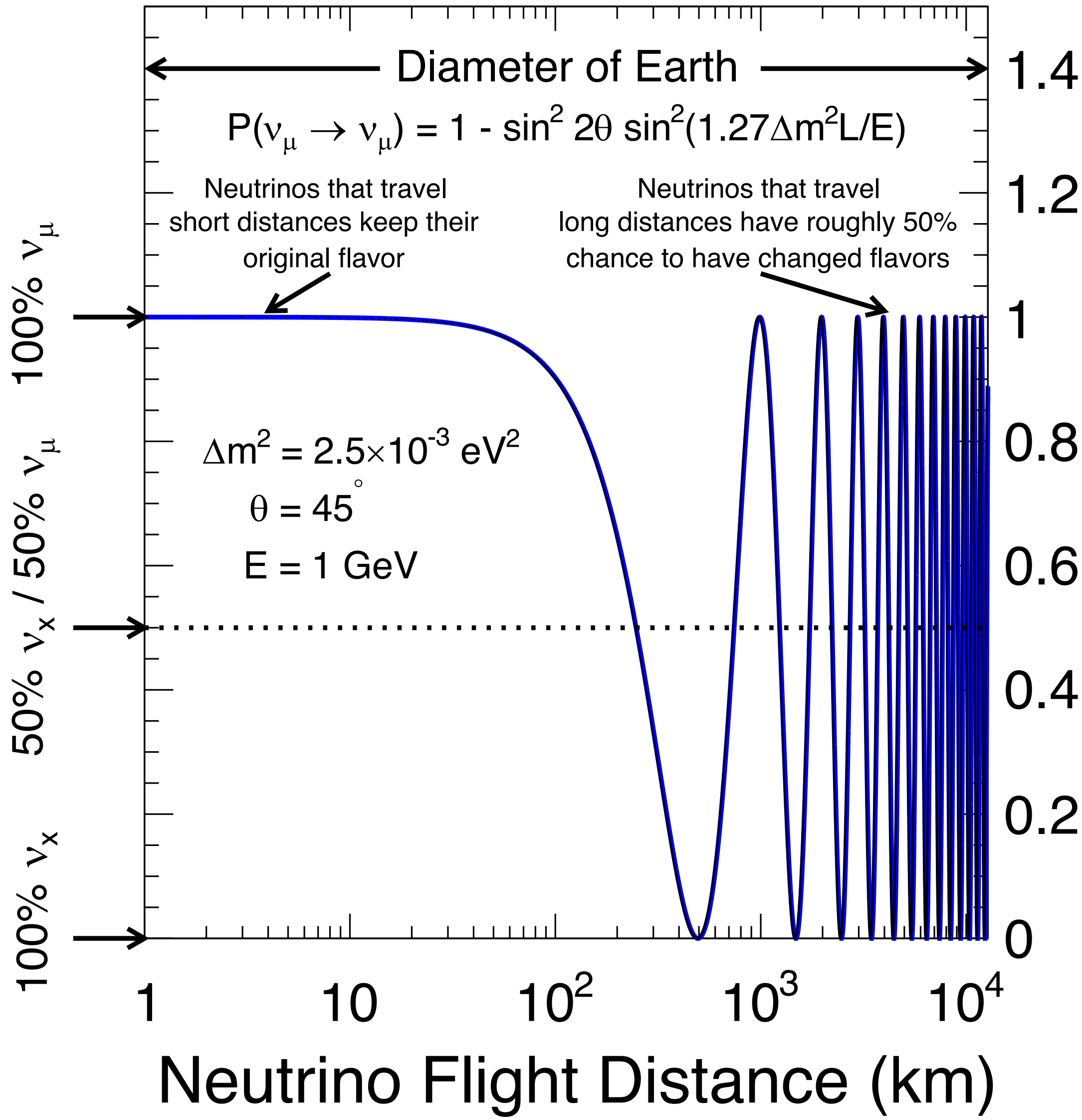
Concluding Remarks

- Earth's matter effect modifies the mass-squared differences and mixing angles when neutrino passes through deep inside Earth which in turn alter the oscillation probabilities significantly

- Earth's matter effect plays an important role to address the pressing fundamental unknowns in three-flavor neutrino oscillation paradigm such as neutrino mass ordering, CP violation, octant of 2-3 mixing angle

- In the precision era, accurate understanding of matter effect opens the avenue to perform neutrino oscillation tomography of Earth complementary to seismic and gravitational measurements

Thank you!



Simple two-flavor neutrino oscillation as seen by Super-K

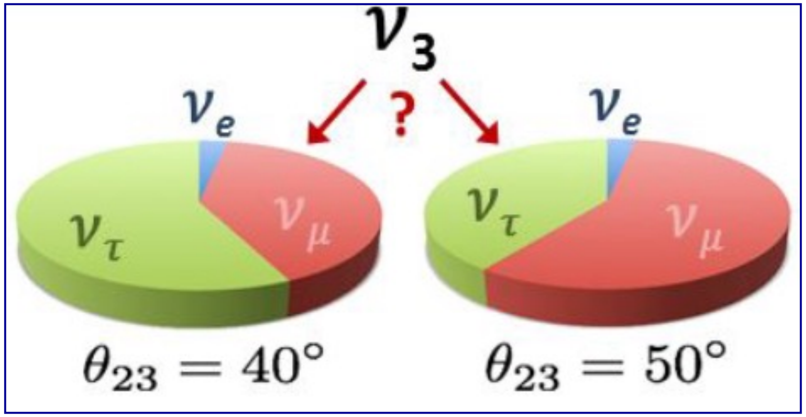
Octant of 2-3 Mixing Angle: Important Open Question

→ In ν_μ survival probability, the dominant term is mainly sensitive to $\sin^2 2\theta_{23}$

→ If $\sin^2 2\theta_{23}$ differs from 1 (recent hints), we get two solutions for θ_{23}

→ One in lower octant (LO: $\theta_{23} < 45$ degree)

→ Other in higher octant (HO: $\theta_{23} > 45$ degree)



Octant ambiguity of θ_{23}

Fogli and Lisi, hep-ph/9604415

ν_μ to ν_e oscillation channel can break this degeneracy
preferred value would depend on the choice of neutrino mass ordering

Leptonic CP-violation: Important Open Question

Is CP violated in the neutrino sector, as in the quark sector?

Mixing can cause CPV in neutrino sector, provided $\delta_{CP} \neq 0^\circ$ *and* 180°

Need to measure the CP-odd asymmetries:

$$\Delta P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta;\, L) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta;\, L) \quad (\alpha \neq \beta)$$

$$\Delta P_{e\mu} = \Delta P_{\mu\tau} = \Delta P_{\tau e} = 4 J_{CP} \times \left[\sin\left(\frac{\Delta m^2_{21}}{2E} L\right) + \sin\left(\frac{\Delta m^2_{32}}{2E} L\right) + \sin\left(\frac{\Delta m^2_{13}}{2E} L\right) \right]$$

Jarlskog CP-odd Invariant → $J_{CP} = \frac{1}{8} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin 2\theta_{12} \sin\delta_{CP}$

Three-flavor effects are key for CPV, need to observe interference

Conditions for observing CPV:
1) *Non-degenerate masses* ✔
2) *Mixing angles* $\neq 0^\circ$ *and* 90° ✔
3) $\delta_{CP} \neq 0^\circ$ *and* 180° *(Hints)*