Evolution of Lepton Number for Neutrinos

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Based on:

A. S. Adam, **NJB**, Y. Kawamura, Y. Matsuo, T. Morozumi, Y. Shimizu, Y. Tokunaga and N. Toyota, PTEP 2021, (2021) [arXiv:2101.07751 [hep-ph]]

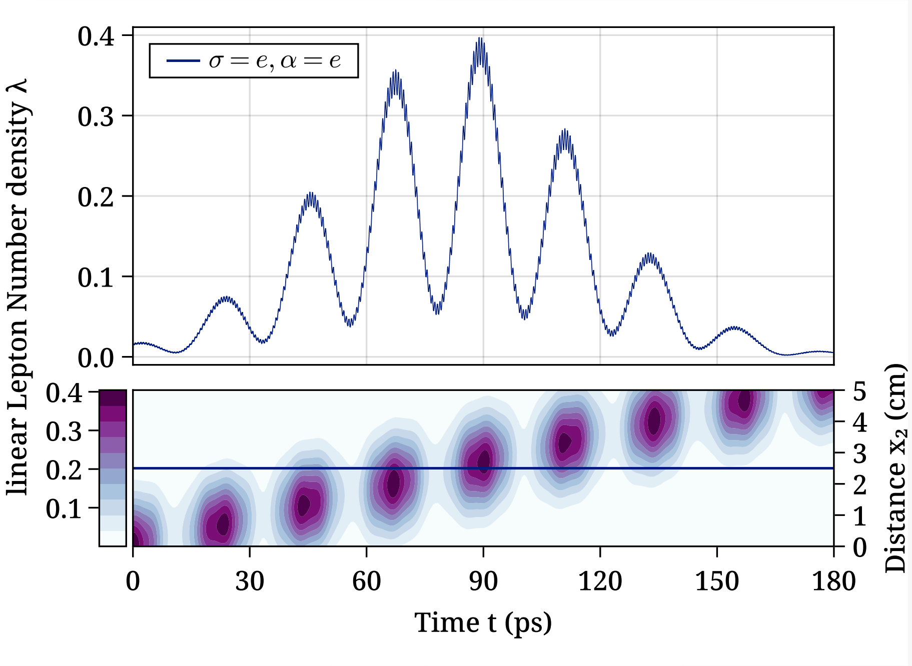
A. S. Adam, **NJB**, Y. Kawamura, Y. Matsuo, T. Morozumi, Y. Shimizu and N. Toyota, [arXiv:2106.02783 [hep-ph]]

Summary of our work

We study **lepton number** in view of **neutrino oscillations and mass**

Point 1: Neutrino oscillations imply lepton family number is a broken symmetry

Point 2: Neutrino mass type modifies lepton family number oscillations



Our result: The expectation value an operator for the lepton family number is modified by neutrino oscillations and mass type. This is true even with decoherence effects.

Motivation to study lepton number

Lepton number in the Standard Model

- We know three neutrino flavors or families exist in the Standard Model (SM),

[LEP (2006)]

$$L_e = n_e - n_{\bar{e}} \qquad L_\mu = n_\mu - n_{\bar{\mu}} \qquad L_\tau = n_\tau - n_{\bar{\tau}}$$

number of leptons minus number of anti-leptons

- Summation over all families results in total lepton number,

$$\sum_\alpha L_\alpha = L_e + L_\mu + L_\tau$$

[Konopinski and Mahmoud (1953)]

- Lepton family numbers are conserved quantum numbers in the SM,

$$\pi^+ \to \pi^0 + e^+ + \nu_e, \qquad \pi^+ \to \mu^+ + \nu_\mu$$

SM Allowed

$$\cancel{\pi^+ \to \mu^+ + \bar{\nu}_e}$$

SM Disallowed

Motivation to study lepton number

Lepton number with massive neutrinos

- We also know neutrinos are massive (proven by oscillation experiments)

$$m_{\nu_2}^2 - m_{\nu_1}^2 \simeq 7.42 \times 10^{-5}\ \mathrm{eV}^2, \qquad |m_{\nu_3}^2 - m_{\nu_2}^2| \simeq 2.517 \times 10^{-3}\ \mathrm{eV}^2$$

[NuFITv5.0 (2020)]

- This means lepton family number and total lepton Number are not necessarily conserved [S. M. Bilenky and C. Giunti (2001)]

$$(A, Z) \to (A, Z+2) + 2e^- + \boxed{0\overline{\nu}_e}$$

Total lepton number violation
(Neutrinoless double beta decay)

$$\pi^+ \to \pi^0 + e^+ + \nu_e \searrow \nu_\mu$$

Lepton family number violation
(Neutrino flavor oscillations)

We are interested in how lepton number is modified by massive neutrinos

Outline

- Lepton Number Setup
- Majorana Calculation, Expectation Value
- Dirac Calculation, Expectation Value
- Lepton Number Density and Wave Packets
- Summary

Framework for the lepton number operator

Heisenberg operator for the lepton family number

- We construct a Heisenberg operator,

$$L_\alpha(t) = \int d^3\mathbf{x} \ : \overline{\nu_\alpha}(t, \mathbf{x}) P_L \gamma^0 \nu_\alpha(t, \mathbf{x}) :$$

- The lepton family numbers are assigned from the *SU(2)*$_L$ doublet of the neutrino and charged lepton partner

$$\alpha = e, \mu, \tau; \quad P_L = (1 - \gamma^5)/2$$

- We consider the two possible neutrino mass types
  - First, Majorana
  - Second, Dirac

Framework for the lepton number operator

Majorana mass case

- the step-function separates our formulation into two regions

$$\mathcal{L}^M = \overline{\nu_{L\alpha}} i\gamma^\mu \partial_\mu \nu_{L\alpha} - \boxed{\theta(t)} \left(\frac{m_{\alpha\beta}}{2} \overline{(\nu_{L\alpha})^C} \nu_{L\beta} + \text{h.c} \right)$$

Region 1 (t<0)	Region 2 (t>0)

Region 1 (t<0)

- Massless neutrinos

$$\mathcal{L}^M = \overline{\nu_{L\alpha}} i\gamma^\mu \partial_\mu \nu_{L\alpha}$$

- The lepton family number is definite
  - i.e. lepton numbers are conserved

$$L^M_\alpha = \int d^3x\, l^M_\alpha(\mathbf{x}) = \int d^3x : \overline{\nu_{\alpha L}}(\mathbf{x}) \gamma^0 \nu_{\alpha L}(\mathbf{x}) :$$

Region 2 (t>0)

- Massive neutrinos

$$\mathcal{L}^M = \overline{\nu_{L\alpha}} i\gamma^\mu \partial_\mu \nu_{L\alpha} - \left(\frac{m_{\alpha\beta}}{2} \overline{(\nu_{L\alpha})^C} \nu_{L\beta} + \text{h.c} \right)$$

- The lepton family number is mixed and time dependent

$$L^M_\alpha(t) = \int d^3x\, l^M_\alpha(t, \mathbf{x}) = \int d^3x : \overline{\nu_{\alpha L}}(t, \mathbf{x}) \gamma^0 \nu_{\alpha L}(t, \mathbf{x}) :$$

- Diagonalize mass matrix with the unitary PMNS matrix

$$\nu_{\alpha L} = U_{\alpha i} \nu_{iL}$$

$$m_i \delta_{ij} = (U^T)_{i\alpha} m_{\alpha\beta} U_{\beta j}$$

Our goal is to connect these two regions

Framework for the lepton number operator

Majorana mass case

- To connect the regions, we enforce continuity of the equations of motion
- We study the time evolution of the lepton number operator in Region 2

$$\lim_{\epsilon \to 0+} \psi_{L\alpha}(-\epsilon, \mathbf{x}) = \lim_{\epsilon \to 0+} U_{\alpha i} P_L \psi_i(+\epsilon, \mathbf{x})$$

$$L_\alpha^M(t) = \int d^3x \, : \overline{\nu_\alpha}(t, \mathbf{x}) \gamma^0 \nu_\alpha(t, \mathbf{x}) :$$

time evolution of the operators

$$a_\alpha(t, \mathbf{p}) = \sum_{\beta=e}^{\tau} \sum_k \left(U_{\alpha k} U_{\beta k}^* \left[\cos E_k(\mathbf{p})t - i v_k \sin E_k(\mathbf{p})t\right] a_\beta(\mathbf{p}) - i U_{\alpha k} U_{\beta k} \sqrt{1 - v_k^2} \sin[E_k(\mathbf{p})t] a_\beta^\dagger(-\mathbf{p}) \right),$$

$$b_\alpha(t, \mathbf{p}) = \sum_{\gamma=e}^{\tau} \sum_j \left(U_{\alpha j}^* U_{\gamma j} \left[\cos E_j(\mathbf{p})t - i v_j \sin E_j(\mathbf{p})t\right] b_\gamma(\mathbf{p}) - i U_{\alpha j}^* U_{\gamma j}^* \sqrt{1 - v_j^2} \sin[E_j(\mathbf{p})t] b_\gamma^\dagger(-\mathbf{p}) \right),$$

$$\sqrt{1 - v_k^2} = m_k / E_k(\mathbf{p})$$
$$v_k = |\mathbf{p}| / E_k(\mathbf{p})$$

We would like to highlight the non-trivial mixing for the time evolution operators

Time evolution of the lepton family numbers

Majorana lepton family number

- We treat the lepton family numbers as Heisenberg operators,

$$L_\alpha^M(t) = \int' \frac{d^3p}{(2\pi)^3|2\mathbf{p}|} \left(a_\alpha^\dagger(t,\mathbf{p})a_\alpha(t,\mathbf{p}) - b_\alpha^\dagger(t,\mathbf{p})b_\alpha(t,\mathbf{p})\right).$$

time dependent operators are from our mixing relations and the continuity condition

- Substitute the time evolution operators to solve for the time evolution of the Majorana operator $L_\alpha^M(t)$

$$a_\alpha(t,\mathbf{p}) = \sum_{\beta=e}^{\tau}\sum_k \left(U_{\alpha k}U^*_{\beta k}\left[\cos E_k(\mathbf{p})t - iv_k \sin E_k(\mathbf{p})t\right]a_\beta(\mathbf{p}) - iU_{\alpha k}U_{\beta k}\sqrt{1-v_k^2}\sin[E_k(\mathbf{p})t]a_\beta^\dagger(-\mathbf{p})\right),$$

$$b_\alpha(t,\mathbf{p}) = \sum_{\gamma=e}^{\tau}\sum_j \left(U^*_{\alpha j}U_{\gamma j}\left[\cos E_j(\mathbf{p})t - iv_j \sin E_j(\mathbf{p})t\right]b_\gamma(\mathbf{p}) - iU^*_{\alpha j}U^*_{\gamma j}\sqrt{1-v_j^2}\sin[E_j(\mathbf{p})t]b_\gamma^\dagger(-\mathbf{p})\right),$$

Time evolution of the lepton family numbers

Majorana expectation value

- For an observable time evolution, we take the expectation value of the Majorana operator

$$\sigma = (e, \mu, \tau)$$

definite initial family

$$|\sigma(\mathbf{q})\rangle = \frac{a_\sigma^\dagger(\mathbf{q})\,|0\rangle}{\sqrt{(2\pi)^3\delta^{(3)}(0)2|\mathbf{q}|}}$$

$$\langle\sigma(\mathbf{q})|L_\alpha^M(t)|\sigma(\mathbf{q})\rangle = \frac{\langle 0|a_\sigma(\mathbf{q})L_\alpha^M(t)a_\sigma^\dagger(\mathbf{q})|0\rangle}{(2\pi)^3\delta^{(3)}(0)2|\mathbf{q}|}$$

Majorana expectation value

$$\begin{aligned}\langle\sigma(\mathbf{q})|L_\alpha^M(t)|\sigma(\mathbf{q})\rangle = \sum_{k,j}\Big[&\mathrm{Re}\left(U_{\alpha k}^*U_{\sigma k}U_{\alpha j}U_{\sigma j}^*\right)\left(\cos E_k(\mathbf{q})t\cos E_j(\mathbf{q})t + v_kv_j\sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\right)\\ &-\mathrm{Im}\left(U_{\alpha k}^*U_{\sigma k}U_{\alpha j}U_{\sigma j}^*\right)\left(v_k\sin E_k(\mathbf{q})t\cos E_j(\mathbf{q})t - v_j\cos E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\right)\\ &-\mathrm{Re}\left(U_{\alpha k}^*U_{\sigma k}^*U_{\alpha j}U_{\sigma j}\right)\sqrt{1-v_k^2}\sqrt{1-v_j^2}\sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\Big],\end{aligned}$$

$$\sqrt{1-v_k^2} = m_k/E_k(\mathbf{p})$$
$$v_k = |\mathbf{p}|/E_k(\mathbf{p})$$

We will discuss more later

Framework for the lepton number operator

Dirac mass case

- Follows a similar derivation, so we list differences from the Majorana case

$$\mathcal{L}^D = \overline{\nu_{L\alpha}} i\gamma^\mu \partial_\mu \nu_{L\alpha} + \overline{\nu_{R\alpha}} i\gamma^\mu \partial_\mu \nu_{R\alpha} - \theta(t)\left(\overline{\nu_{R\alpha}} m_{\alpha\beta} \nu_{L\beta} + \text{h.c.}\right)$$

two types of fields, left-handed and right-handed

This creates 2 lepton number operators and 2 continuity conditions

$$\lim_{\epsilon\to 0+} \nu_{Lk}(+\epsilon, \mathbf{x}) = \lim_{\epsilon\to 0+} \sum_\alpha U^*_{\alpha k} \nu_{L\alpha}(-\epsilon, \mathbf{x}),$$

$$\lim_{\epsilon\to 0+} \nu_{Rk}(+\epsilon, \mathbf{x}) = \lim_{\epsilon\to 0+} \sum_\beta V^*_{\beta k} \nu_{R\beta}(-\epsilon, \mathbf{x}).$$

Dirac mass term is diagonalized by 2 mixing matrices

$$\nu_{L\beta} = U_{\beta j} \nu_{Lj}$$

$$\nu_{R\alpha} = V_{\alpha k} \nu_{Rk}$$

$$(V^\dagger)_{k\alpha} m_{\alpha\beta} U_{\beta j} = m_k \delta_{kj}.$$

Time evolution of the lepton family numbers

Dirac lepton family number

- The Dirac case has two Heisenberg operators for the lepton family numbers based on handedness,

$$L^L_\alpha(t) = \int d^3x\, l^L_\alpha(t,\mathbf{x}) = \int' \frac{d^3p}{(2\pi)^3 2|\mathbf{p}|} \left(a^\dagger_{L\alpha}(\mathbf{p},t) a_{L\alpha}(\mathbf{p},t) - b^\dagger_{L\alpha}(\mathbf{p},t) b_{L\alpha}(\mathbf{p},t) \right)$$

$$L^R_\alpha(t) = \int d^3x\, l^R_\alpha(t,\mathbf{x}) = \int' \frac{d^3p}{(2\pi)^3 2|\mathbf{p}|} \left(a^\dagger_{R\alpha}(\mathbf{p},t) a_{R\alpha}(\mathbf{p},t) - b^\dagger_{R\alpha}(\mathbf{p},t) b_{R\alpha}(\mathbf{p},t) \right)$$

- The addition of the left-handed operator with with right-handed operator is the Dirac operator for lepton family numbers

$$L^D_\alpha(t) = L^L_\alpha(t) + L^R_\alpha(t)$$

Time evolution of the lepton family numbers

Dirac expectation value

- The summation of the left- and right-handed operators forms our Dirac expectation value

$$\langle\sigma_L(\mathbf{q})|L^D_\alpha(t)|\sigma_L(\mathbf{q})\rangle = \frac{\langle 0|a_{L\sigma}(\mathbf{q})L^L_\alpha(t) + L^R_\alpha(t)a^\dagger_{L\sigma}(\mathbf{q})|0\rangle}{(2\pi)^3\delta^{(3)}(0)2|\mathbf{q}|}$$

$$\sigma_L = (e, \mu, \tau)$$

definite initial family

$$|\sigma_L(\mathbf{q})\rangle = \frac{a^\dagger_{L\sigma}(\mathbf{q})\,|0\rangle}{\sqrt{(2\pi)^3\delta^{(3)}(0)2|\mathbf{q}|}}$$

Dirac expectation value

$$\begin{aligned}\langle\sigma_L(\mathbf{q})|L^D_\alpha(t)|\sigma_L(\mathbf{q})\rangle = \sum_{k,j}\Big[&\mathrm{Re}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(\cos E_k(\mathbf{q})t\cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)\\ &- \mathrm{Im}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(v_k \sin E_k(\mathbf{q})t\cos E_j(\mathbf{q})t - v_j \cos E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)\\ &+ \mathrm{Re}\left(V_{\alpha k}U^*_{\sigma k}V^*_{\alpha j}U_{\sigma j}\right)\sqrt{1-v_k^2}\sqrt{1-v_j^2}\sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\Big],\end{aligned}$$

$$\sqrt{1-v_k^2} = m_k/E_k(\mathbf{p})$$
$$v_k = |\mathbf{p}|/E_k(\mathbf{p})$$

Let us compare and discuss with Majorana next,

Comparison of the expectation values

Dirac vs. Majorana expectation values

- We compare the two expectation values

Majorana

$$\langle\sigma(\mathbf{q})|L^M_\alpha(t)|\sigma(\mathbf{q})\rangle = \sum_{k,j}\Big[\mathrm{Re}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(\cos E_k(\mathbf{q})t\cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Im}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(v_k\sin E_k(\mathbf{q})t\cos E_j(\mathbf{q})t - v_j\cos E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Re}\left(U^*_{\alpha k}U^*_{\sigma k}U_{\alpha j}U_{\sigma j}\right)\sqrt{1-v_k^2}\sqrt{1-v_j^2}\sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\Big],$$

Dirac

$$\langle\sigma_L(\mathbf{q})|L^D_\alpha(t)|\sigma_L(\mathbf{q})\rangle = \sum_{k,j}\Big[\mathrm{Re}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(\cos E_k(\mathbf{q})t\cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Im}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(v_k\sin E_k(\mathbf{q})t\cos E_j(\mathbf{q})t - v_j\cos E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\right)$$
$$+ \mathrm{Re}\left(V_{\alpha k}U^*_{\sigma k}V^*_{\alpha j}U_{\sigma j}\right)\sqrt{1-v_k^2}\sqrt{1-v_j^2}\sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\Big],$$

Comparison of the expectation values

Dirac vs. Majorana expectation values

- What is common?

Exactly the same terms between formulations, time dependent sine and cosine lead to oscillations. Can recover the QM probability equation in ultra-relativistic limit

Majorana

$$\langle \sigma(\mathbf{q})|L_\alpha^M(t)|\sigma(\mathbf{q})\rangle = \sum_{k,j}\Big[\mathrm{Re}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(\cos E_k(\mathbf{q})t\cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Im}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(v_k \sin E_k(\mathbf{q})t\cos E_j(\mathbf{q})t - v_j \cos E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Re}\left(U^*_{\alpha k}U^*_{\sigma k}U_{\alpha j}U_{\sigma j}\right)\sqrt{1-v_k^2}\sqrt{1-v_j^2}\sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\Big],$$

Dirac

$$\langle \sigma_L(\mathbf{q})|L_\alpha^D(t)|\sigma_L(\mathbf{q})\rangle = \sum_{k,j}\Big[\mathrm{Re}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(\cos E_k(\mathbf{q})t\cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Im}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(v_k \sin E_k(\mathbf{q})t\cos E_j(\mathbf{q})t - v_j \cos E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$
$$+ \mathrm{Re}\left(V_{\alpha k}U^*_{\sigma k}V^*_{\alpha j}U_{\sigma j}\right)\sqrt{1-v_k^2}\sqrt{1-v_j^2}\sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\Big],$$

Comparison of the expectation values

Dirac vs. Majorana expectation values

- What are the differences?

Majorana

$$\langle\sigma(\mathbf{q})|L_\alpha^M(t)|\sigma(\mathbf{q})\rangle = \sum_{k,j}\Big[\mathrm{Re}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(\cos E_k(\mathbf{q})t\cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Im}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(v_k\sin E_k(\mathbf{q})t\cos E_j(\mathbf{q})t - v_j\cos E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Re}\left(U^*_{\alpha k}U^*_{\sigma k}U_{\alpha j}U_{\sigma j}\right)\sqrt{1-v_k^2}\sqrt{1-v_j^2}\sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\Big],$$

Majorana phase dependence appears

Dirac

$$\langle\sigma_L(\mathbf{q})|L_\alpha^D(t)|\sigma_L(\mathbf{q})\rangle = \sum_{k,j}\Big[\mathrm{Re}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(\cos E_k(\mathbf{q})t\cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Im}\left(U^*_{\alpha k}U_{\sigma k}U_{\alpha j}U^*_{\sigma j}\right)\left(v_k\sin E_k(\mathbf{q})t\cos E_j(\mathbf{q})t - v_j\cos E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\right)$$
$$+ \mathrm{Re}\left(V_{\alpha k}U^*_{\sigma k}V^*_{\alpha j}U_{\sigma j}\right)\sqrt{1-v_k^2}\sqrt{1-v_j^2}\sin E_k(\mathbf{q})t\sin E_j(\mathbf{q})t\Big],$$

Depends on the right-handed mixing matrix

Comparison of the expectation values

Dirac vs. Majorana expectation values

- What are the differences?

Majorana

$$\langle\sigma(\mathbf{q})|L_\alpha^M(t)|\sigma(\mathbf{q})\rangle = \sum_{k,j}\Big[\mathrm{Re}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right)\left(\cos E_k(\mathbf{q})t \cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$

$$- \mathrm{Im}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right)\left(v_k \sin E_k(\mathbf{q})t \cos E_j(\mathbf{q})t - v_j \cos E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$

$$\boxed{-}\,\mathrm{Re}\left(U_{\alpha k}^* U_{\sigma k}^* U_{\alpha j} U_{\sigma j}\right)\sqrt{1-v_k^2}\sqrt{1-v_j^2}\,\sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\Big],$$

Minus sign leads to total lepton number violation

Dirac

$$\langle\sigma_L(\mathbf{q})|L_\alpha^D(t)|\sigma_L(\mathbf{q})\rangle = \sum_{k,j}\Big[\mathrm{Re}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right)\left(\cos E_k(\mathbf{q})t \cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$

$$- \mathrm{Im}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right)\left(v_k \sin E_k(\mathbf{q})t \cos E_j(\mathbf{q})t - v_j \cos E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$

$$\boxed{+}\,\mathrm{Re}\left(V_{\alpha k} U_{\sigma k}^* V_{\alpha j}^* U_{\sigma j}\right)\sqrt{1-v_k^2}\sqrt{1-v_j^2}\,\sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\Big],$$

Total lepton number is conserved because of the plus sign

Comparison of the expectation values

Dirac vs. Majorana expectation values

- What are the differences?

Majorana

$$\langle\sigma(\mathbf{q})|L_\alpha^M(t)|\sigma(\mathbf{q})\rangle = \sum_{k,j}\Big[\mathrm{Re}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right)\left(\cos E_k(\mathbf{q})t\cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Im}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right)\left(v_k \sin E_k(\mathbf{q})t\cos E_j(\mathbf{q})t - v_j \cos E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Re}\left(U_{\alpha k}^* U_{\sigma k}^* U_{\alpha j} U_{\sigma j}\right)\boxed{\sqrt{1-v_k^2}\sqrt{1-v_j^2}}\sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\Big],$$

Dirac

$$\langle\sigma_L(\mathbf{q})|L_\alpha^D(t)|\sigma_L(\mathbf{q})\rangle = \sum_{k,j}\Big[\mathrm{Re}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right)\left(\cos E_k(\mathbf{q})t\cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Im}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right)\left(v_k \sin E_k(\mathbf{q})t\cos E_j(\mathbf{q})t - v_j \cos E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$
$$+ \mathrm{Re}\left(V_{\alpha k} U_{\sigma k}^* V_{\alpha j}^* U_{\sigma j}\right)\boxed{\sqrt{1-v_k^2}\sqrt{1-v_j^2}}\sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\Big],$$

All differences are mass suppressed $\sqrt{1-v_k^2} = m_k/E_k(\mathbf{p})$

Lepton number density and wave packets

Dirac vs Majorana expectation values

- Differences in the formulations are mass suppressed
  - Non-relativistic energies emphasize the differences
- Coherence of the Lepton number evolution becomes important

- Coherence effects in neutrino oscillations have been described using wave packets [Akhmedov and Smirnov (2009), Beuthe (2003), Giunti (2002), Giunti et al. (1991), Kayser (1981), etc.]

- For coherent propagation we consider the evolution of a lepton family number density,

$$L_\alpha^M(t) \xrightarrow{\text{Spacetime Density}} l_\alpha^M(t,\mathbf{x}) =: \overline{\nu_\alpha}(t,\mathbf{x})\gamma^0\nu_\alpha(t,\mathbf{x}) :$$

Outline

- Lepton Number Setup
- Majorana Calculation, Expectation Value
- Dirac Calculation, Expectation Value
- Lepton Number Density and Wave Packets
- Summary

Lepton Number Density and Wave Packets

Expectation value of the density

- We set the initial shape of the density with a 1-D Gaussian state
  - Gaussian chosen so we can calculate the momentum integral later
  - Acts like momentum wave packet centered at mean momentum

$$|\psi_{\sigma}^{(L)}(q^0;\sigma_q)\rangle = \frac{1}{\sqrt{\sigma_q}(2\pi)^{3/4}\delta(0)} \int' \frac{dq}{2\pi\sqrt{2|q|}} e^{-\frac{(q-q^0)^2}{4\sigma_q^2}} a^{\dagger}_{(L)\sigma}(0,q,0)|0\rangle$$

initial family
$\sigma = (e,\mu,\tau)$

width of the Gaussian distribution in the second component of the momentum

mean momentum we take as positive

Majorana expectation value	Left-handed Dirac expectation value
$\langle\psi_{\sigma}(q^0;\sigma_q)\|l_{\alpha}^{M}(t,\mathbf{x})\|\psi_{\sigma}(q^0;\sigma_q)\rangle$	$\langle\psi_{\sigma}^{L}(q^0;\sigma_q)\|l_{\alpha}^{L}(t,\mathbf{x})\|\psi_{\sigma}^{L}(q^0;\sigma_q)\rangle$

Lepton Number Density and Wave Packets

Example, expectation value of the Majorana density operator

- Two points to compare to the plane wave expectation value calculation

New term from the Gaussian initial states

$$\langle\psi_\sigma(q^0;\sigma_q)|l_\alpha^M(t,\mathbf{x})|\psi_\sigma(q^0;\sigma_q)\rangle = \boxed{\frac{1}{\sigma_q(2\pi)^{3/2}\delta(0)^2}\iint' \frac{dq'dq}{(2\pi)^2} e^{-\frac{(q'-q^0)^2+(q-q^0)^2}{4\sigma_q^2}-i(q'-q)\mathbf{e}_2\cdot\mathbf{x}}}$$

$$\times\left[\sum_{i,j} U_{\alpha i}^* U_{\sigma i} U_{\alpha j} U_{\sigma j}^* \left(\cos E_i(q')t + i\frac{|q'|}{E_i(q')}\sin E_i(q')t\right)\left(\cos E_j(q)t - i\frac{|q|}{E_j(q)}\sin E_j(q)t\right)\right.$$

$$\left. - \sum_{i,j} U_{\alpha i}^* U_{\sigma i}^* U_{\alpha j} U_{\sigma j} \frac{m_j}{E_j(q')}\sin E_j(q')t \frac{m_i}{E_i(q)}\sin E_i(q)t\right]$$

Same as the plane wave calculation

Lepton Number Density and Wave Packets

Example, expectation value of the Majorana density operator

- To preform integration over q and q-dash we must assume two things,

Distributions are sharply peaked around the mean momentum

$$\left.\begin{array}{l} \sigma_q \ll \dfrac{E_{i,j}(q^0)}{m_{i,j}}, \\ \dfrac{d^n}{(dt)^n}\sigma_q = 0 \end{array}\right\} \rightarrow E_{i,j}(q^{(\prime)}) \simeq E_{i,j}(q^0) + \frac{q^0}{E_{i,j}(q^0)}(q^{(\prime)} - q^0)$$

Variance or width of the distribution does not evolve

- Lastly, to match our initial condition we consider a 1-D linear density

$$\lambda^M_{\sigma\rightarrow\alpha}(q^0;\sigma_q;t,x_2) \equiv \iint dx_1 dx_3 \langle \psi_\sigma(q^0;\sigma_q)| l^M_\alpha(t,\mathbf{x}) |\psi_\sigma(q^0;\sigma_q)\rangle$$

Lepton Number Density and Wave Packets

$$\sqrt{1-v_k^2} = m_k/E_k(q^0)$$
$$v_k = |q^0|/E_k(q^0)$$

Linear density expectation value for the Majorana density operator

- Important features,

Common terms as plane wave Majorana expectation value are responsible for oscillations

$$\lambda^M_{\sigma\to\alpha}(q^0;\sigma_q;t,L) \simeq \frac{\sigma_q}{(2\pi)^{1/2}} \sum_{i,j} U^*_{\alpha i} U_{\sigma i} U_{\alpha j} U^*_{\sigma j}$$
$$\times \frac{1}{2}\Big[(v_{i0}+v_{j0}+1+v_{i0}v_{j0}) e^{i(E_i(q^0)-E_j(q^0))t} e^{-\sigma_q^2[(L-v_{i0}t)^2+(L-v_{j0}t)^2]}$$
$$-(v_{i0}+v_{j0}-1-v_{i0}v_{j0}) e^{-i(E_i(q^0)-E_j(q^0))t} e^{-\sigma_q^2[(L+v_{i0}t)^2+(L+v_{j0}t)^2]}$$
$$+(v_{i0}-v_{j0}+1-v_{i0}v_{j0}) e^{i(E_i(q^0)+E_j(q^0))t} e^{-\sigma_q^2[(L-v_{i0}t)^2+(L+v_{j0}t)^2]}$$
$$-(v_{i0}-v_{j0}-1+v_{i0}v_{j0}) e^{-i(E_i(q^0)+E_j(q^0))t} e^{-\sigma_q^2[(L+v_{i0}t)^2+(L-v_{j0}t)^2]}\Big]$$
$$-\frac{\sigma_q}{(2\pi)^{1/2}} \sum_{i,j} U^*_{\alpha i} U^*_{\sigma i} U_{\alpha j} U_{\sigma j} \sqrt{1-v_{i0}^2}\sqrt{1-v_{j0}^2}$$
$$\times \frac{1}{2}\Big[e^{i(E_i(q^0)-E_j(q^0))t} e^{-\sigma_q^2[(L-v_{i0})^2+(L-v_{j0}t)^2]} + e^{-i(E_i(q^0)-E_j(q^0))t} e^{-\sigma_q^2[(L+v_{i0})^2+(L+v_{j0}t)^2]}$$
$$-e^{i(E_i(q^0)+E_j(q^0))t} e^{-\sigma_q^2[(L-v_{i0})^2+(L+v_{j0}t)^2]} - e^{-i(E_i(q^0)+E_j(q^0))t} e^{-\sigma_q^2[(L+v_{i0})^2+(L-v_{j0}t)^2]}\Big]$$

Lepton Number Density and Wave Packets

$$\sqrt{1-v_k^2} = m_k/E_k(q^0)$$
$$v_k = |q^0|/E_k(q^0)$$

Linear density expectation value for the Majorana density operator

- Important features,

The real exponentials are non-linear damping terms responsible for decoherence

$$\begin{aligned}
\lambda^M_{\sigma\to\alpha}(q^0;\sigma_q;t,L) \simeq & \frac{\sigma_q}{(2\pi)^{1/2}} \sum_{i,j} U^*_{\alpha i} U_{\sigma i} U_{\alpha j} U^*_{\sigma j} \\
& \times \frac{1}{2}\Big[(v_{i0}+v_{j0}+1+v_{i0}v_{j0}) e^{i(E_i(q^0)-E_j(q^0))t} \boxed{e^{-\sigma_q^2[(L-v_{i0}t)^2+(L-v_{j0}t)^2]}} \\
& - (v_{i0}+v_{j0}-1-v_{i0}v_{j0}) e^{-i(E_i(q^0)-E_j(q^0))t} \boxed{e^{-\sigma_q^2[(L+v_{i0}t)^2+(L+v_{j0}t)^2]}} \\
& + (v_{i0}-v_{j0}+1-v_{i0}v_{j0}) e^{i(E_i(q^0)+E_j(q^0))t} \boxed{e^{-\sigma_q^2[(L-v_{i0}t)^2+(L+v_{j0}t)^2]}} \\
& - (v_{i0}-v_{j0}-1+v_{i0}v_{j0}) e^{-i(E_i(q^0)+E_j(q^0))t} \boxed{e^{-\sigma_q^2[(L+v_{i0}t)^2+(L-v_{j0}t)^2]}}\Big] \\
& - \frac{\sigma_q}{(2\pi)^{1/2}} \sum_{i,j} U^*_{\alpha i} U^*_{\sigma i} U_{\alpha j} U_{\sigma j} \sqrt{1-v_{i0}^2}\sqrt{1-v_{j0}^2} \\
& \times \frac{1}{2}\Big[e^{i(E_i(q^0)-E_j(q^0))t} \boxed{e^{-\sigma_q^2[(L-v_{i0})^2+(L-v_{j0}t)^2]}} + e^{-i(E_i(q^0)-E_j(q^0))t} \boxed{e^{-\sigma_q^2[(L+v_{i0})^2+(L+v_{j0}t)^2]}} \\
& - e^{i(E_i(q^0)+E_j(q^0))t} \boxed{e^{-\sigma_q^2[(L-v_{i0})^2+(L+v_{j0}t)^2]}} - e^{-i(E_i(q^0)+E_j(q^0))t} \boxed{e^{-\sigma_q^2[(L+v_{i0})^2+(L-v_{j0}t)^2]}}\Big]
\end{aligned}$$

Lepton Number Density and Wave Packets

$$\sqrt{1-v_k^2} = m_k/E_k(q^0)$$
$$v_k = |q^0|/E_k(q^0)$$

Linear density expectation value for the Majorana density operator

- Important features,

Same term as QM wave packet calculations

$$\lambda^M_{\sigma\to\alpha}(q^0;\sigma_q;t,L) \simeq \frac{\sigma_q}{(2\pi)^{1/2}} \sum_{i,j} U^*_{\alpha i}U_{\sigma i}U_{\alpha j}U^*_{\sigma j}$$
$$\times \frac{1}{2}\Big[(v_{i0}+v_{j0}+1+v_{i0}v_{j0})e^{i(E_i(q^0)-E_j(q^0))t}e^{-\sigma_q^2[(L-v_{i0}t)^2+(L-v_{j0}t)^2]}$$
$$-(v_{i0}+v_{j0}-1-v_{i0}v_{j0})e^{-i(E_i(q^0)-E_j(q^0))t}e^{-\sigma_q^2[(L+v_{i0}t)^2+(L+v_{j0}t)^2]}$$
$$+(v_{i0}-v_{j0}+1-v_{i0}v_{j0})e^{i(E_i(q^0)+E_j(q^0))t}e^{-\sigma_q^2[(L-v_{i0}t)^2+(L+v_{j0}t)^2]}$$
$$-(v_{i0}-v_{j0}-1+v_{i0}v_{j0})e^{-i(E_i(q^0)+E_j(q^0))t}e^{-\sigma_q^2[(L+v_{i0}t)^2+(L-v_{j0}t)^2]}\Big]$$
$$-\frac{\sigma_q}{(2\pi)^{1/2}} \sum_{i,j} U^*_{\alpha i}U^*_{\sigma i}U_{\alpha j}U_{\sigma j}\sqrt{1-v_{i0}^2}\sqrt{1-v_{j0}^2}$$
$$\times \frac{1}{2}\Big[e^{i(E_i(q^0)-E_j(q^0))t}e^{-\sigma_q^2[(L-v_{i0})^2+(L-v_{j0}t)^2]} + e^{-i(E_i(q^0)-E_j(q^0))t}e^{-\sigma_q^2[(L+v_{i0})^2+(L+v_{j0}t)^2]}$$
$$-e^{i(E_i(q^0)+E_j(q^0))t}e^{-\sigma_q^2[(L-v_{i0})^2+(L+v_{j0}t)^2]} - e^{-i(E_i(q^0)+E_j(q^0))t}e^{-\sigma_q^2[(L+v_{i0})^2+(L-v_{j0}t)^2]}\Big]$$

Majorana phase dependence and the minus sign leads to total lepton number violation

Lepton Number Density and Wave Packets

$$\sqrt{1-v_k^2} = m_k/E_k(q^0)$$
$$v_k = |q^0|/E_k(q^0)$$

Linear density expectation value for the left-handed Dirac density operator

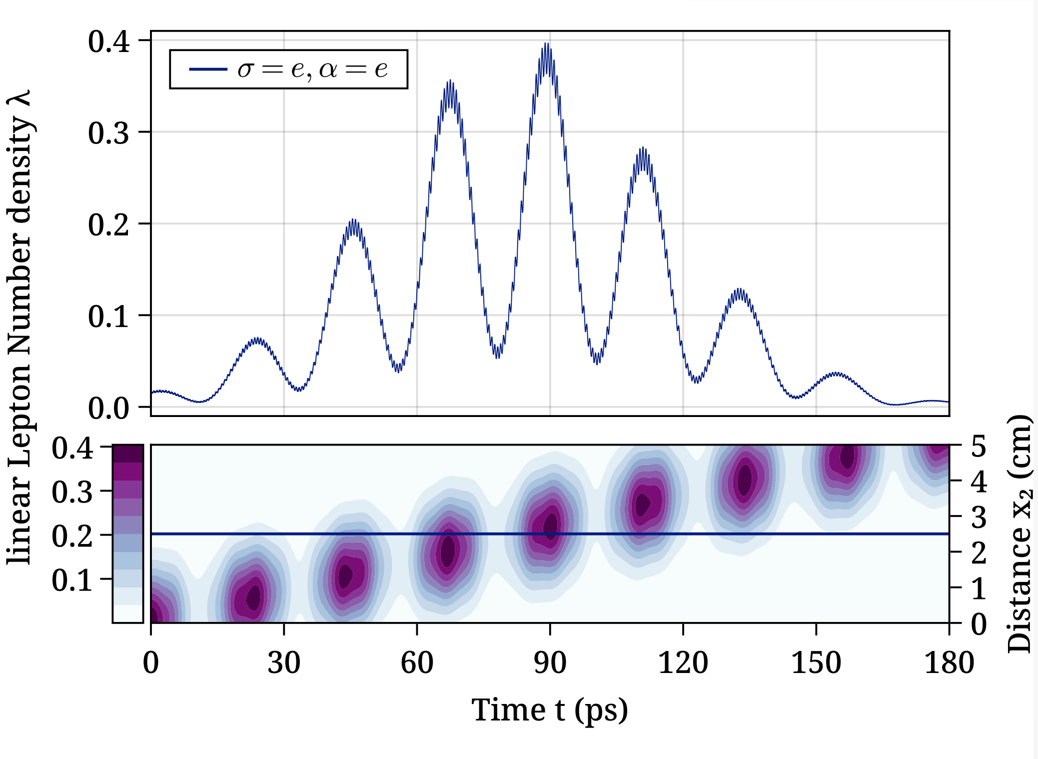
- Similar result for the left-handed Dirac case
- Mass suppressed term only appears with the right-handed contribution

$$\begin{aligned}\lambda^L_{\sigma\to\alpha}(q^0;\sigma_q;t,x_2) \simeq & \frac{\sigma_q}{(2\pi)^{1/2}}\sum_{i,j} U^*_{\alpha i}U_{\sigma i}U_{\alpha j}U^*_{\sigma j} \\ & \times \frac{1}{2}\Big[(v_{i0}+v_{j0}+1+v_{i0}v_{j0})e^{i(E_i(q^0)-E_j(q^0))t}e^{-\sigma_q^2[(x_2-v_{i0}t)^2+(x_2-v_{j0}t)^2]} \\ & \quad -(v_{i0}+v_{j0}-1-v_{i0}v_{j0})e^{-i(E_i(q^0)-E_j(q^0))t}e^{-\sigma_q^2[(x_2+v_{i0}t)^2+(x_2+v_{j0}t)^2]} \\ & \quad +(v_{i0}-v_{j0}+1-v_{i0}v_{j0})e^{i(E_i(q^0)+E_j(q^0))t}e^{-\sigma_q^2[(x_2-v_{i0}t)^2+(x_2+v_{j0}t)^2]} \\ & \quad -(v_{i0}-v_{j0}-1+v_{i0}v_{j0})e^{-i(E_i(q^0)+E_j(q^0))t}e^{-\sigma_q^2[(x_2+v_{i0}t)^2+(x_2-v_{j0}t)^2]}\Big] \,.\end{aligned}$$

Lepton Number Density and Wave Packets

Linear density expectation values

- We consider momentum of 0.2eV and lightest mass of 0.01eV.
- Density propagates in time (contour)



- At distance slice damping occurs
  - Due to the real exponentials

$$e^{-\sigma_q^2[(x_2 \pm v_{i0}t)^2 + (x_2 \pm v_{j0}t)^2]}$$

$$e^{-\sigma_q^2[(x_2 \pm v_{i0}t)^2 + (x_2 \mp v_{j0}t)^2]}$$

- Similar to wave packet damping

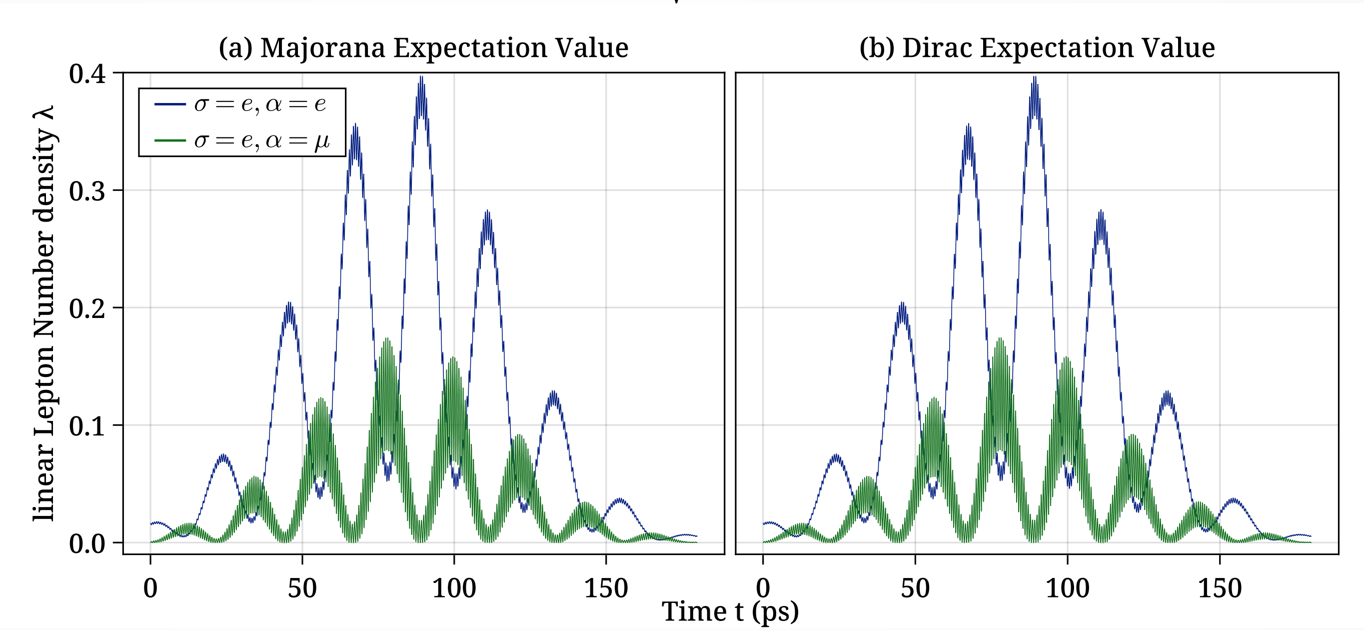
The initial momentum density is a Gaussian distribution with a width of $_q$ = 0.00001 and a mean momentum of q^0=0.2eV. Normal mass hierarchy for neutrinos is considered. Lepton mixing angles, the Dirac CP phase, and the mass squared differences are the reported best fit values from the work of the NuFIT 5.0 (2020) collaboration.

Lepton Number Density and Wave Packets

Dirac vs Majorana linear density expectation values

- At momentum larger than mass no difference between expectation values
  - Due to mass suppression in the third terms $\sqrt{1 - v_k^2} = m_k / E_k(q^0)$

momentum is 0.2eV
lightest mass is 0.01eV



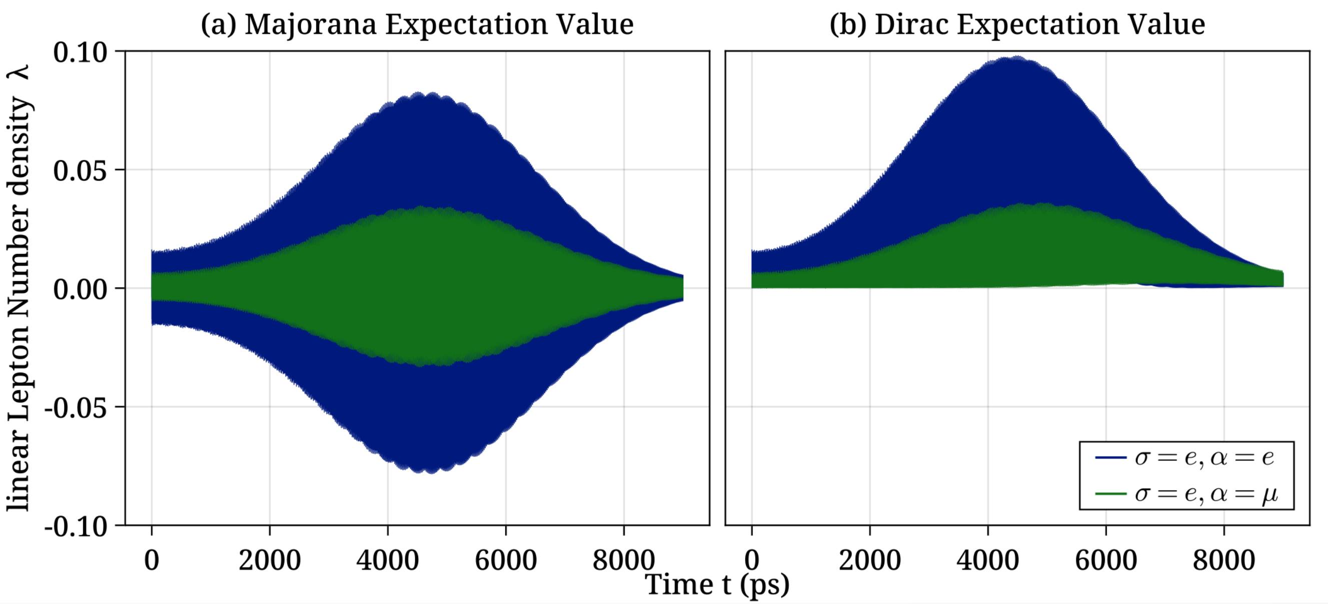
We take an arbitrary distance slice at x_2=2.5cm. The initial momentum density is a Gaussian distribution with a width of $_q$ = 0.00001 and a mean momentum of q^0=0.2eV. Normal mass hierarchy is considered. Oscillation parameters are the best fit values from the NuFIT 5.0 (2020) collaboration.

Lepton Number Density and Wave Packets

Dirac vs Majorana linear density expectation values

- At lower momentum, differences become clear; however, the expectation value is suppressed by damping
  - Peak value is small due to wave packet-like decoherence from damping

momentum is 0.0002eV
lightest mass is 0.01eV



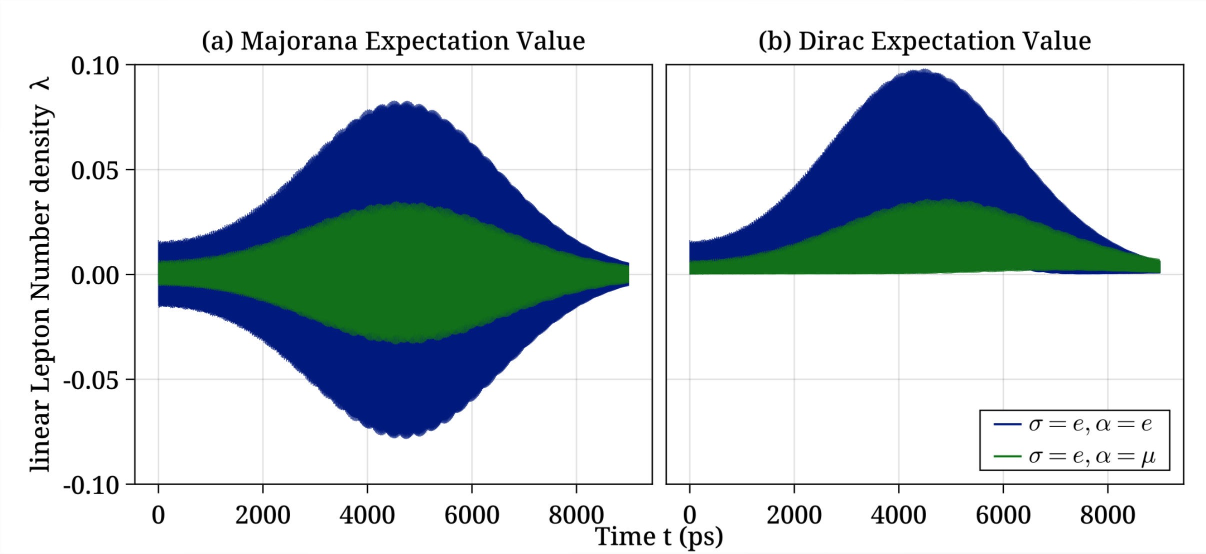
Same as previous slide except momentum is now q^0 = 0.0002eV.

Summary and Conclusions

We study **lepton number** in view of **neutrino oscillations and mass**

Point 1: Neutrino oscillations imply lepton family number is a broken symmetry

Point 2: Neutrino mass type modifies lepton family number oscillations



Our result: The expectation value an operator for the lepton family number is modified by neutrino oscillations and mass type.
This result is true even when we consider decoherence effects.

Thank you to WG5 for your attention!

Backup

Framework for the lepton number operator

Majorana mass case

- To connect the regions, we enforce continuity of the equations of motion
  - The continuity condition connects the fields as we approach zero

$$\lim_{\epsilon \to 0+} \psi_{L\alpha}(-\epsilon, \mathbf{x}) = \lim_{\epsilon \to 0+} U_{\alpha i} P_L \psi_i(+\epsilon, \mathbf{x})$$

Region 1 (t<0)

Framework for the lepton number operator

Majorana mass case

- To connect the regions, we enforce continuity of the equations of motion
  - The continuity condition connects the fields as we approach zero

$$\lim_{\epsilon\to 0+} \psi_{L\alpha}(-\epsilon,\mathbf{x}) = \lim_{\epsilon\to 0+} U_{\alpha i} P_L \psi_i(+\epsilon,\mathbf{x})$$

Region 1 (t<0)

- We approach the zero time from below
- Fourier expand the massless fields as Weyl fermions

$$\psi_{L\alpha}(-\epsilon,\mathbf{x}) = \int' \frac{d^3p}{(2\pi)^3 2|\mathbf{p}|} \left(a_\alpha(\mathbf{p}) u_L(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{x}} + b^\dagger_\alpha(\mathbf{p}) v_L(\mathbf{p}) e^{-i\mathbf{p}\cdot\mathbf{x}}\right)$$

- The operators obey the usual anti-commutation relations

$$\left.\begin{array}{c}\{a_\alpha(\mathbf{p}), a^\dagger_\beta(\mathbf{q})\} \\ \{b_\alpha(\mathbf{p}), b^\dagger_\alpha(\mathbf{q})\}\end{array}\right\} = 2\mathbf{p}(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{q})\delta_{\alpha\beta}$$

Framework for the lepton number operator

Majorana mass case

- To connect the regions, we enforce continuity of the equations of motion
  - The continuity condition connects the fields as we approach zero

$$\lim_{\epsilon\to 0+} \psi_{L\alpha}(-\epsilon,\mathbf{x}) = \lim_{\epsilon\to 0+} U_{\alpha i} P_L \psi_i(+\epsilon,\mathbf{x})$$

Region 2 (t>0)

- Approach zero from above
- The neutrinos are Majorana fermions that are Fourier expanded,

$$U_{\alpha k}\psi_{Lk}(+\epsilon,\mathbf{x}) = U_{\alpha k} P_L \int' \frac{d^3\mathbf{p}}{2E_k(\mathbf{p})(2\pi)^3} \sum_{\lambda=\pm} \left(a_{Mk}(\mathbf{p},\lambda) u_k(\mathbf{p},\lambda) e^{i\mathbf{p}\cdot\mathbf{x}} + a^\dagger_{Mk}(\mathbf{p},\lambda) v_k(\mathbf{p},\lambda) e^{-i\mathbf{p}\cdot\mathbf{x}} \right)$$

energy of each mass state

$$E_k(\mathbf{p}) = \sqrt{|\mathbf{p}|^2 + m_k^2}$$

spinor helicity

Framework for the lepton number operator

Majorana mass case

- To connect the regions, we enforce continuity of the equations of motion
  - The continuity condition connects the fields as we approach zero

$$\lim_{\epsilon \to 0+} \psi_{L\alpha}(-\epsilon, \mathbf{x}) = \lim_{\epsilon \to 0+} \boxed{U_{\alpha i} P_L \psi_i(+\epsilon, \mathbf{x})}$$

Region 2 (t>0)

- Approach zero from above
- The neutrinos are Majorana fermions that are Fourier expanded,

$$\boxed{U_{\alpha k}\psi_{Lk}(+\epsilon, \mathbf{x})} = U_{\alpha k} P_L \int' \frac{d^3\mathbf{p}}{\boxed{2E_k(\mathbf{p})}(2\pi)^3} \boxed{\sum_{\lambda=\pm}} \left(a_{Mk}(\mathbf{p}, \lambda) u_k(\mathbf{p}, \lambda) e^{i\mathbf{p}\cdot\mathbf{x}} + a^\dagger_{Mk}(\mathbf{p}, \lambda) v_k(\mathbf{p}, \lambda) e^{-i\mathbf{p}\cdot\mathbf{x}} \right)$$

energy of each mass state

$$E_k(\mathbf{p}) = \sqrt{|\mathbf{p}|^2 + m_k^2}$$

spinor helicity

- The Majorana operators are distinct from the operators in Region 1,

$$\{a_{Mk}(\mathbf{p}, \lambda), a^\dagger_{Mj}(\mathbf{q}, \lambda')\} = 2E(\mathbf{p})(2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{q}) \delta_{kj} \delta_{\lambda\lambda'}$$

Lepton Number

Majorana case, resulting Lepton Number

$$
\begin{aligned}
L_\alpha(t) = \int'_{\mathbf{p}\in A} \Bigg[& -\frac{m_i m_j \sin(E_i(\mathbf{p})t)\sin(E_j(\mathbf{p})t)}{E_i(\mathbf{p})E_j(\mathbf{p})} \left\{ V^*_{\alpha i}V^*_{\beta i}V_{\alpha j}V_{\gamma j}\left(a^\dagger_\gamma(\mathbf{p})a_\beta(\mathbf{p}) + a^\dagger_\gamma(-\mathbf{p})a_\beta(-\mathbf{p})\right) - V_{\alpha i}V_{\beta i}V^*_{\alpha j}V^*_{\gamma j}\left(b^\dagger_\gamma(\mathbf{p})b_\beta(\mathbf{p}) + b^\dagger_\gamma(-\mathbf{p})b_\beta(-\mathbf{p})\right)\right\} \\
& + \Bigg(\cos(E_i(\mathbf{p})t)\cos(E_j(\mathbf{p})t) + \frac{|\mathbf{p}|^2}{E_i(\mathbf{p})E_j(\mathbf{p})}\sin(E_i(\mathbf{p})t)\sin(E_j(\mathbf{p})t) \\
& \qquad + \frac{i|\mathbf{p}|}{E_i(\mathbf{p})}\sin(E_i(\mathbf{p})t)\cos(E_j(\mathbf{p})t) - \frac{i|\mathbf{p}|}{E_j(\mathbf{p})}\cos(E_i(\mathbf{p})t)\sin(E_j(\mathbf{p})t)\Bigg) \\
& \quad \times \left\{ V^*_{\alpha i}V_{\beta i}V_{\alpha j}V^*_{\gamma j}\left(a^\dagger_\beta(\mathbf{p})a_\gamma(\mathbf{p}) + a^\dagger_\beta(-\mathbf{p})a_\gamma(-\mathbf{p})\right) - V_{\alpha i}V^*_{\beta i}V^*_{\alpha j}V_{\gamma j}\left(b^\dagger_\beta(\mathbf{p})b_\gamma(\mathbf{p}) + b^\dagger_\beta(-\mathbf{p})b_\gamma(-\mathbf{p})\right)\right\} \\
& - \left(\frac{m_j \sin(E_j(\mathbf{p})t)}{E_j(\mathbf{p})}\left(\cos(E_i(\mathbf{p})t) + \frac{i|\mathbf{p}|}{E_i(\mathbf{p})}\sin(E_i(\mathbf{p})t)\right)\right) \\
& \quad \times \left\{ V^*_{\alpha i}V_{\beta i}V_{\alpha j}V_{\gamma j}\left(a^\dagger_\beta(\mathbf{p})a^\dagger_\gamma(-\mathbf{p}) - a^\dagger_\beta(-\mathbf{p})a^\dagger_\gamma(\mathbf{p})\right) - V_{\alpha i}V^*_{\beta i}V^*_{\alpha j}V^*_{\gamma j}\left(b^\dagger_\beta(\mathbf{p})b^\dagger_\gamma(-\mathbf{p}) - b^\dagger_\beta(-\mathbf{p})b^\dagger_\gamma(\mathbf{p})\right)\right\} \\
& + \left(\frac{m_i \sin(E_i(\mathbf{p})t)}{E_i(\mathbf{p})}\left(\cos(E_j(\mathbf{p})t) - \frac{i|\mathbf{p}|}{E_j(\mathbf{p})}\sin(E_j(\mathbf{p})t)\right)\right) \\
& \quad \times \left\{ V^*_{\alpha i}V^*_{\beta i}V_{\alpha j}V^*_{\gamma j}(a_\beta(-\mathbf{p})a_\gamma(\mathbf{p}) - a_\beta(\mathbf{p})a_\gamma(-\mathbf{p})) - V_{\alpha i}V_{\beta i}V^*_{\alpha j}V_{\gamma j}\left(b_\beta(-\mathbf{p})b_\gamma(\mathbf{p}) - b_\beta(\mathbf{p})b_\gamma(-\mathbf{p})\right)\right\}\Bigg].
\end{aligned}
$$

Framework for the lepton number operator

Dirac mass case

- After connecting the regions with the continuity condition we can write the time evolution form of the operators

time evolution of the operators

$$a_{L\alpha}(\pm\mathbf{p},t) = \sum_{\beta=e}^{\tau}\sum_{k}\left[U_{\alpha k}U^{*}_{\beta k}\left(\cos E_k(\mathbf{p})t - iv_k \sin E_k(\mathbf{p})t\right)a_{L\beta}(\pm\mathbf{p}) \mp U_{\alpha k}V^{*}_{\beta k}\sqrt{1-v_k^2}\sin E_k(\mathbf{p})t\, b^{\dagger}_{R\beta}(\mp\mathbf{p})\right]$$

$$b^{\dagger}_{L\alpha}(\pm\mathbf{p},t) = \sum_{\gamma=e}^{\tau}\sum_{k}\left[U_{\alpha k}U^{*}_{\gamma k}\left(\cos E_k(\mathbf{p})t + iv_k \sin E_k(\mathbf{p})t\right)b^{\dagger}_{L\gamma}(\pm\mathbf{p}) \mp U_{\alpha k}V^{*}_{\beta k}\sqrt{1-v_k^2}\sin E_k(\mathbf{p})t\, a_{R\gamma}(\mp\mathbf{p})\right]$$

- Right-handed operators are found by replacement of the mixing matrices and handedness of the operators

The non-trivial mixing will lead to phenomena similar to **neutrino flavor oscillations**

Time evolution of the lepton family numbers

Left-handed lepton family number

- We substitute the time dependent operators to find how the left-handed lepton family number evolves in time

$$L^L_\alpha(t) = \int d^3x\, l^L_\alpha(t,\mathbf{x}) = \int' \frac{d^3p}{(2\pi)^3 2|\mathbf{p}|} \left(a^\dagger_{L\alpha}(\mathbf{p},t) a_{L\alpha}(\mathbf{p},t) - b^\dagger_{L\alpha}(\mathbf{p},t) b_{L\alpha}(\mathbf{p},t) \right)$$

time dependent operators are from our mixing relations and the continuity condition

time evolution of the operators

$$a_{L\alpha}(\pm\mathbf{p},t) = \sum_{\beta=e}^{\tau} \sum_k \left[U_{\alpha k} U^*_{\beta k} \left(\cos E_k(\mathbf{p})t - i v_k \sin E_k(\mathbf{p})t \right) a_{L\beta}(\pm\mathbf{p}) \mp U_{\alpha k} V^*_{\beta k} \sqrt{1 - v_k^2}\, \sin E_k(\mathbf{p})t\, b^\dagger_{R\beta}(\mp\mathbf{p}) \right]$$

$$b^\dagger_{L\alpha}(\pm\mathbf{p},t) = \sum_{\gamma=e}^{\tau} \sum_k \left[U_{\alpha k} U^*_{\gamma k} \left(\cos E_k(\mathbf{p})t + i v_k \sin E_k(\mathbf{p})t \right) b^\dagger_{L\gamma}(\pm\mathbf{p}) \mp U_{\alpha k} V^*_{\beta k} \sqrt{1 - v_k^2}\, \sin E_k(\mathbf{p})t\, a_{R\gamma}(\mp\mathbf{p}) \right]$$

Time evolution of the lepton family numbers

Majorana expectation value

- Three important facts

1st cosine and sine terms are responsible for time dependent oscillations of the expectation value

$$\langle \sigma(\mathbf{q})|L_\alpha^M(t)|\sigma(\mathbf{q})\rangle = \sum_{k,j} \Big[\mathrm{Re}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right) \left(\cos E_k(\mathbf{q})t \cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Im}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right) \left(v_k \sin E_k(\mathbf{q})t \cos E_j(\mathbf{q})t - v_j \cos E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right)$$
$$- \mathrm{Re}\left(U_{\alpha k}^* U_{\sigma k}^* U_{\alpha j} U_{\sigma j}\right) \sqrt{1-v_k^2}\sqrt{1-v_j^2} \sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t \Big],$$

2nd PMNS matrix combination is dependent on the Majorana phases, which are observable CP phases and could be determined by some experiments

3rd Quantum mechanics equation for neutrino flavor oscillation is recovered in the ultra-relativistic limit

$$\lim_{|\mathbf{q}|^2 \gg m_k m_j} \langle \sigma(\mathbf{q})|L_\alpha^M(t)|\sigma(\mathbf{q})\rangle \approx \sum_{k,j} \left(\mathrm{Re}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right) \cos \frac{\Delta m_{kj}^2 t}{2|\mathbf{q}|} - \mathrm{Im}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right) \sin \frac{\Delta m_{kj}^2 t}{2|\mathbf{q}|} \right)$$
$$= \lim_{\vec{p} \gg m_{i,j}} P_{\alpha \to \beta}(t)$$

Time evolution of the lepton family numbers

Dirac expectation value

- Interesting parts

1st cosine and sine terms are responsible for time dependent oscillations of the expectation value

$$
\begin{aligned}
\langle \sigma_L(\mathbf{q}) | L_\alpha^D(t) | \sigma_L(\mathbf{q}) \rangle = \sum_{k,j} \Big[& \mathrm{Re}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right) \left(\cos E_k(\mathbf{q})t \cos E_j(\mathbf{q})t + v_k v_j \sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right) \\
& - \mathrm{Im}\left(U_{\alpha k}^* U_{\sigma k} U_{\alpha j} U_{\sigma j}^*\right) \left(v_k \sin E_k(\mathbf{q})t \cos E_j(\mathbf{q})t - v_j \cos E_k(\mathbf{q})t \sin E_j(\mathbf{q})t\right) \\
& + \mathrm{Re}\left(V_{\alpha k} U_{\sigma k}^* V_{\alpha j}^* U_{\sigma j}\right) \sqrt{1 - v_k^2} \sqrt{1 - v_j^2} \sin E_k(\mathbf{q})t \sin E_j(\mathbf{q})t \Big],
\end{aligned}
$$

2nd There are mixing matrices related to the right-handed neutrinos, which are suppressed by the masses of the neutrinos

$$\sqrt{1 - v_k^2} = m_k / E_k(\mathbf{p})$$

3rd Quantum mechanics equation for neutrino flavor oscillation is recovered in the ultra-relativistic limit

Lepton Number Density and Wave Packets

Dirac vs Majorana linear density expectation values

- Total Lepton Number density
- Dirac expectation is conserved with a forward-backward evolution

$$\sum_\alpha \lambda^D_{\sigma\to\alpha}(t,x_2) \simeq \frac{\sigma_q}{\sqrt{2\pi}} \sum_i |V_{\alpha i}|^2 \left[(1+v_{i0})e^{-2\sigma_q^2(x_2-v_{i0}t)^2} + (1-v_{i0})e^{-2\sigma_q^2(x_2+v_{i0}t)^2}\right],$$

$$\sum_\alpha \lambda^M_{\sigma\to\alpha}(t,x_2) \simeq \frac{\sigma_q}{\sqrt{2\pi}} \sum_i |V_{\sigma i}|^2 \left[v_{i0}(1+v_{i0})e^{-2\sigma_q^2(x_2-v_{i0}t)^2} - v_{i0}(1-v_{i0})e^{-2\sigma_q^2(x_2+v_{i0}t)^2} \right.$$

$$\left. + 2(1-v_{i0}^2)e^{-2\sigma_q^2(v_{i0}^2t^2+x_2^2)}\cos 2E_i(q^0)t\right].$$

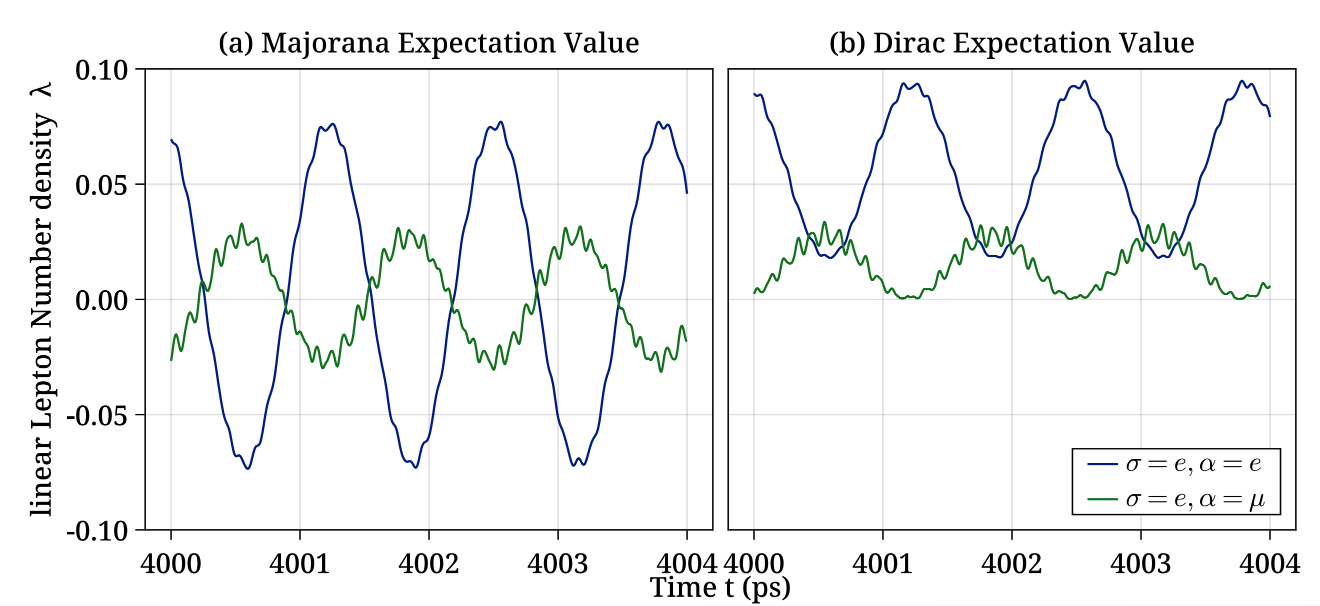
- Majorana expectation value is violated with a time dependent oscillation

Lepton Number Density and Wave Packets

Dirac vs Majorana linear density expectation values

- At lower momentum, differences become clear; however, the expectation value is suppressed by damping
  - Peak value is small due to wave packet-like decoherence from damping

momentum is 0.0002eV
lightest mass is 0.01eV

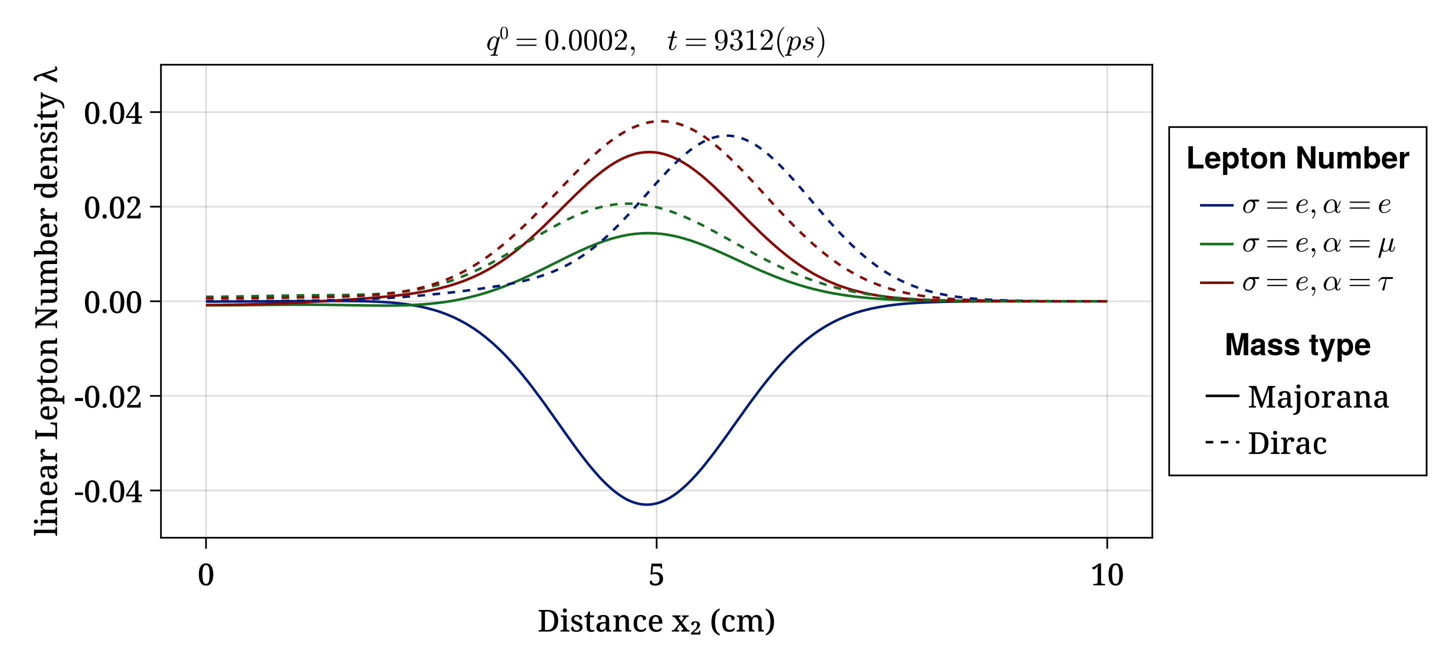


Same as previous slide except momentum is now q^0 = 0.0002eV.

Lepton Number Density and Wave Packets

Dirac vs Majorana linear density expectation values

- The same distinguishing features occur if we take a time slice
  - Peak value is still suppressed by the damping terms



Normal mass hierarchy is considered, and we choose the Majorana phases to be arbitrary values of $\alpha_{21} = \pi$ and $\alpha_{31} = 0.5\pi$.
Oscillation parameters are the reported best fit values from the work of the NuFIT 5.0 (2020) collaboration.