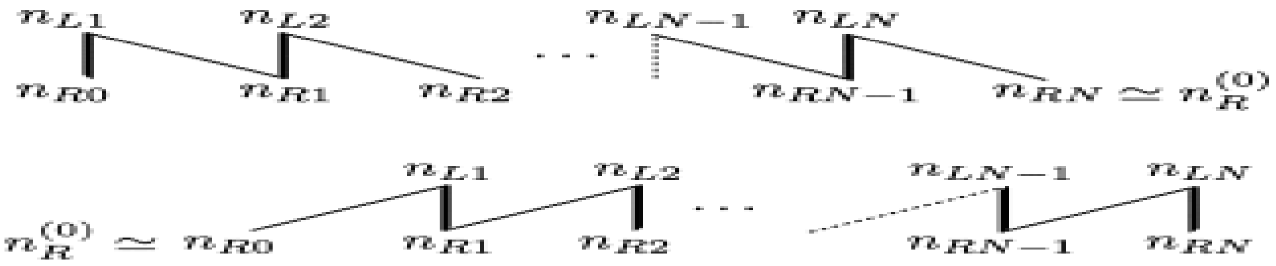
Clockwork Fermions Contribution to neutrino mass generation and Charged Lepton Flavour Violation $l_i \rightarrow l_j + \gamma$



Gayatri Ghosh

Department of Physics, PDDUAM, Assam University, Silchar, India

Nufact 2022, 31st July-06 August, Salt Lake City US

Neutrinos in Clockwork

- We extend the Standard Model with n left-handed and n + 1 right-handed chiral fermions, singlets under the Standard Model gauge group, which we denote as $\psi_{Li}(i = 0, ..., n - 1)$ and $\psi_{Ri}(i = 0, ..., n)$ respectively.
- The Lagrangian of the model reads:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{Clockwork} + \mathcal{L}_{int} \, ,$$

- L_{SM} is the Standard Model Lagrangian, $L_{Clockwork}$ is the part of the Lagrangian involving only the new fermion singlets, and L_{int} is the interaction term of the new fields with the Standard Model fields

$$\mathcal{L}_{int} = -Y \widetilde{H} \overline{L}_L \psi_{Rn} \, ,$$

← The Standard Model only couples to the last site of the fermionic clockwork, therefore,

- In full generality, the clockwork Lagrangian can be cast as:

$$\mathcal{L}_{\text{Clockwork}} = \mathcal{L}_{\text{kin}} - \sum_{i=0}^{n-1}\left(m_i \overline{\psi}_{Li}\psi_{Ri} - m_i' \overline{\psi}_{Li}\psi_{Ri+1} + \text{h.c.}\right) - \sum_{i=0}^{n-1}\frac{1}{2}M_{Li}\overline{\psi_{Li}^{c}}\psi_{Li} - \sum_{i=0}^{n}\frac{1}{2}M_{Ri}\overline{\psi_{Ri}^{c}}\psi_{Ri}\,,$$

- where L_{kin} denotes the kinetic term for all fermions, and m, m_0 and $M_{L,R}$ are mass parameters. Denoting Ψ = (ψ_{L0}, ψ_{L1}, …ψ_{Ln-1}, $\psi^c{}_{R0}$, $\psi^c{}_{R1}$, …, $\psi^c{}_{Rn}$)
- The clockwork Lagrangian can be written in the compact form: → $\mathcal{L}_{\text{Clockwork}} = \mathcal{L}_{\text{Kin}} - \frac{1}{2}(\overline{\Psi^c}\mathcal{M}\Psi + \text{h.c.})$

- We assume for simplicity universal Dirac masses, Majorana masses and nearest neighbor interactions, namely $m_i = m$, $m'_i = mq$, $M_{Ri} = M_{Li} = m\tilde{q}$ for all i.

Under this assumption, the mass matrix reads:

$$\mathcal{M} = m \begin{pmatrix} \tilde{q} & 0 & \cdots & 0 & 1 & -q & \cdots & 0 \\ 0 & \tilde{q} & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \tilde{q} & 0 & 0 & 0 & -q \\ 1 & 0 & \cdots & 0 & \tilde{q} & 0 & \cdots & 0 \\ -q & 1 & \cdots & 0 & 0 & \tilde{q} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -q & 0 & 0 & 0 & \tilde{q} \end{pmatrix},$$

- which has eigen values M_k given by:

$$M_0 = m\widetilde{q} \, ,$$
$$M_k = m\widetilde{q} - m\sqrt{\lambda_k} \, , \qquad k = 1, \ldots, n \, ,$$
$$M_{n+k} = m\widetilde{q} + m\sqrt{\lambda_k} \, , \qquad k = 1, \ldots, n \, ,$$

with λ_k defined as

$$\lambda_k \equiv q^2 + 1 - 2q \cos \frac{k\pi}{n+1} \, .$$

- The mass eigenstates, are related to the interaction eigenstates Ψj by the unitary transformation U, namely

$$\Psi_j = \sum_j \mathcal{U}_{jk} \chi_k .$$

The matrix U can be explicitly calculated, the result being

$$\mathcal{U} = \begin{pmatrix} \vec{0} & \frac{1}{\sqrt{2}} U_L & -\frac{1}{\sqrt{2}} U_L \\ \vec{u}_R & \frac{1}{\sqrt{2}} U_R & \frac{1}{\sqrt{2}} U_R \end{pmatrix} .$$

where $\vec{0}$ and $\vec{u}_R$ are n-dimensional vectors, with entries:

$$\vec{0}_j = 0 \,, \quad j = 1, ..., n \,,$$
$$(u_R)_j = \frac{1}{q^j}\sqrt{\frac{q^2-1}{q^2-q^{-2n}}} \,, \quad j = 1, ..., n \,,$$

while U_L and U_R are, respectively, $n \times n$ and $(n+1) \times n$ matrices with elements

$$(U_L)_{jk} = \sqrt{\frac{2}{n+1}} \sin\frac{jk\pi}{n+1} \,, \qquad j, k = 1, ..., n \,,$$
$$(U_R)_{jk} = \sqrt{\frac{2}{(n+1)\lambda_k}} \left[q \sin\frac{jk\pi}{n+1} - \sin\frac{(j+1)k\pi}{n+1} \right] , \quad j = 0, .., n, \quad k = 1, ..., n \,,$$

- ***The interaction Lagrangian of the clockwork fields to the Standard Model fields, can now be recast in terms of mass eigenstates:***

$$\mathcal{L}_{\rm int} = -Y\,\overline{L}_L \widetilde{H} \mathcal{U}_{nk} \chi_k \equiv -\sum_{k=0}^{2n} Y_k\,\overline{L}_L \widetilde{H} \chi_k \ ,$$

where

$$Y_0 \equiv Y(u_R)_n = \frac{Y}{q^n}\sqrt{\frac{q^2-1}{q^2-q^{-2n}}}\ ,$$

$$Y_k = Y_{k+n} \equiv \frac{1}{\sqrt{2}} Y(U_R)_{nk} = Y\sqrt{\frac{1}{(n+1)\lambda_k}}\left[q \sin\frac{nk\pi}{n+1}\right]\ , \qquad k = 1, ..., n\ .$$

- The components (uR)n and (UR)np, which describe the fraction of the nth “gear” in the zero mode, will play a major role in the phenomenology, as they parametrize the portal strength between the Standard Model sector and the clockwork sector. After electroweak symmetry breaking new mass terms arise which mix the Standard Model neutrino with the clockwork fermions. The mass matrix of the 2n + 2 electrically neutral fermion fields of the model reads:

$$m_\nu = \begin{array}{c} \\ \nu_L \\ \chi_0 \\ \chi_1 \\ \chi_2 \\ \vdots \\ \chi_{2n} \end{array} \begin{array}{c} \begin{array}{cccccc} \nu_L & \chi_0 & \chi_1 & \chi_2 & \cdots & \chi_{2n} \end{array} \\ \begin{pmatrix} 0 & vY_0 & vY_1 & vY_2 & \cdots & vY_{2n} \\ vY_0 & M_0 & 0 & 0 & \cdots & 0 \\ vY_1 & 0 & M_1 & 0 & & \\ vY_2 & 0 & 0 & M_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ vY_{2n} & 0 & 0 & 0 & \cdots & M_{2n} \end{pmatrix} \end{array},$$

Lepton Flavor Violation

- The clockwork mechanism suppresses the Yukawa couplings for the zero mode, hence explaining the smallness of neutrino masses.
- However the Yukawa couplings for the higher modes are in general unsuppressed and can lead to observable effects at low energies.
- In particular, the lepton flavor violation generically present in the Yukawa couplings of the higher modes contributes, through quantum effects induced by clockwork fermions, to generate rare leptonic decays (such as $l_i \to l_j + \gamma$) or μ-e conversion in nuclei, with rates that could be at the reach of current or future experiments if the gear masses are sufficiently low.

- We calculate the rate for $l_i \rightarrow l_j + \gamma$ following. For N clockwork generations, we obtain:

$$B(\mu \to e\gamma) \simeq \frac{3\alpha_{\rm em} v^4}{8\pi} \left| \sum_{\alpha=1}^{N} \sum_{k=1}^{n_\alpha} \frac{Y_k^{e\alpha} Y_k^{\mu\alpha}}{M_k^{\alpha 2}} F(x_k^\alpha) \right|^2 ,$$

where α_{em} is the fine structure constant, n_α is the number of gears in the α-th generation,
M_α^k is the mass of the k-th mode in the α-th generation ($k = 1, \ldots, n_\alpha$), and $x_\alpha^k \equiv \{M_\alpha^k\}^2 / M_W$. The loop function F(x) is defined as

$$F(x) \equiv \frac{1}{6(1-x)^4}(10 - 43x + 78x^2 - 49x^3 + 4x^4 - 18x^3 \log x)$$

and has limits F (0) = 5/3 and F (∞) = 2/3.

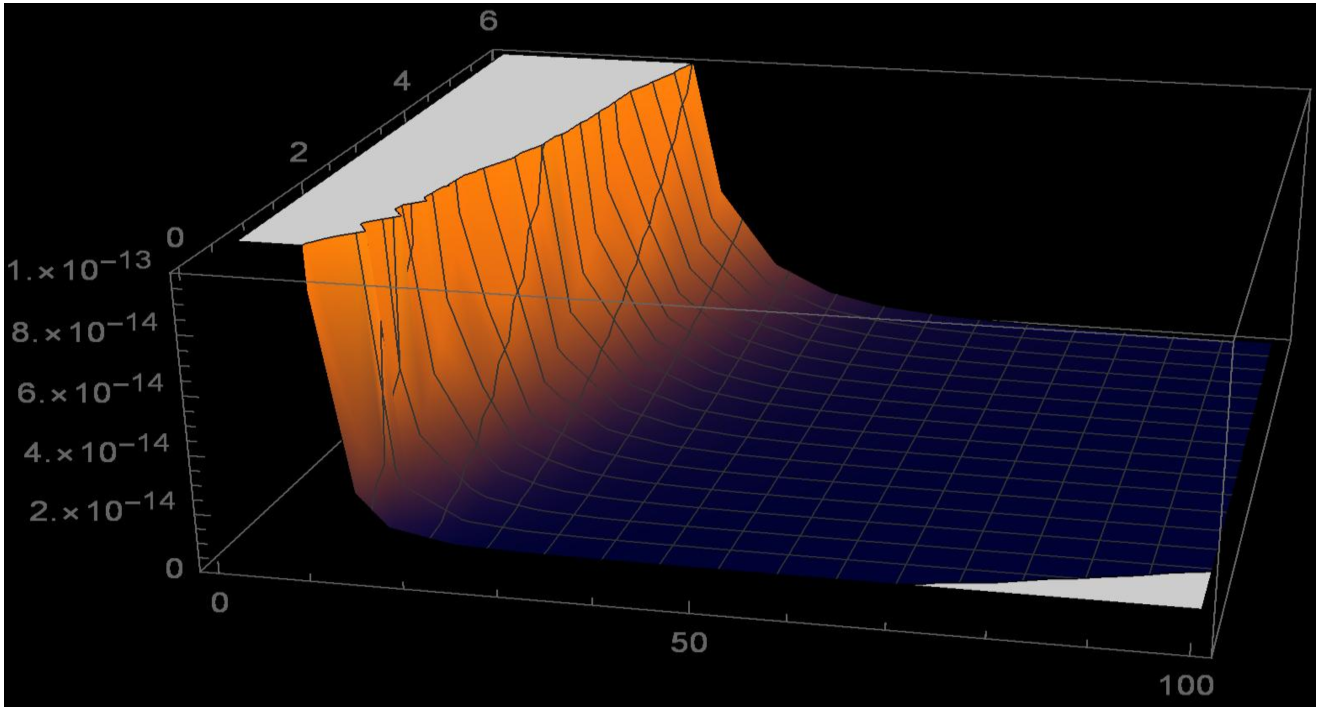


Fig. 1 Predicted value of Br(μ → eγ) for points of the parameter space reproducing the observed neutrino oscillation parameters, as a function of the mass of the first clockwork gear.

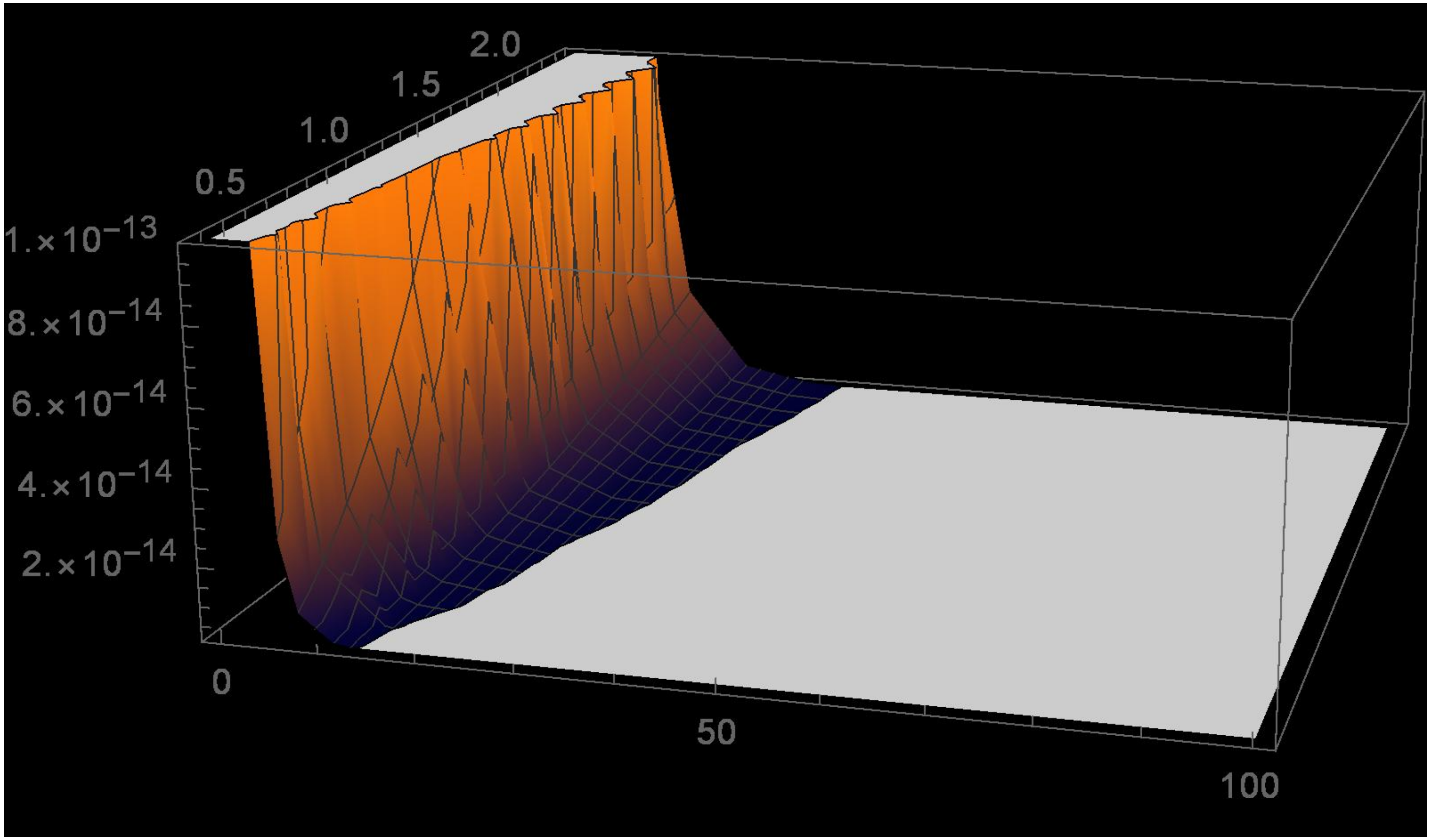


Fig. 2 Predicted value of Br($\tau \to e\gamma$) for points of the parameter space reproducing the observed neutrino oscillation parameters, as a function of the mass of the first clockwork gear.

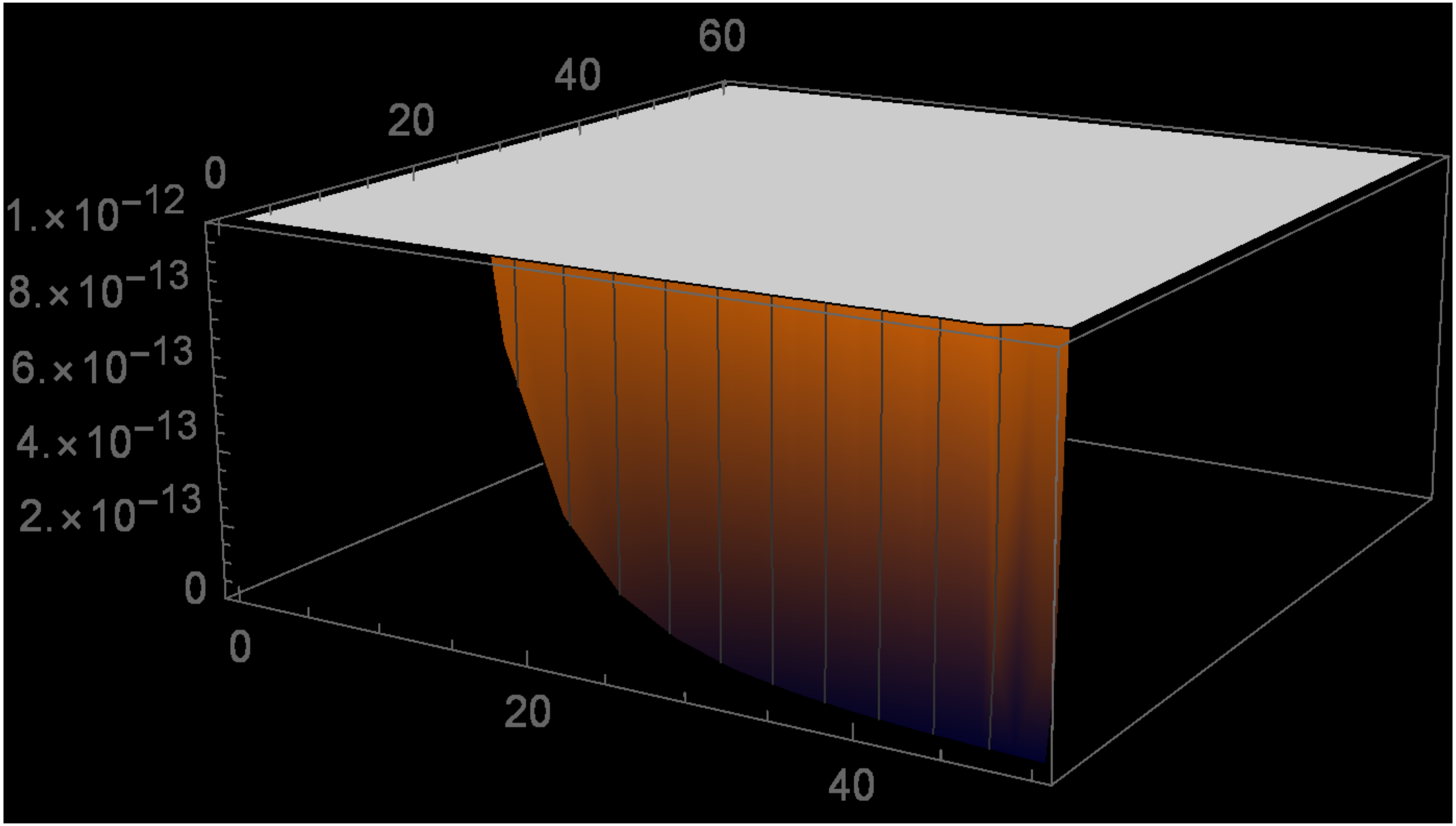


Fig. 2 Predicted value of Br($\tau \rightarrow \mu\gamma$) for points of the parameter space reproducing the observed neutrino oscillation parameters, as a function of the mass of the first clockwork gear.

- The current upper bound $Br(\mu \rightarrow e\gamma) \leq 4.2 \times 10^{-13}$ from the MEG experiment poses stringent constraints on the mass scale of the clockwork. In Fig.1 we show the branching ratio expected for points reproducing the measured neutrino parameters, assuming two clockwork generations, as obtained in the scan presented in section , as a function of the mass of the first clockwork gear. It follows from the figure that the clockwork gears must be larger than $\sim$ 15 TeV in order to evade the experimental constraints, unless very fine cancellations among all contributions to this process exist. For a larger number of clockwork generations we expect even stronger lower limits on the lightest gear mass, due to the larger number of particles in the loop.

Thank You