Status of Lattice QCD Determination of Nucleon Form Factors and their Relevance for the Few-GeV Neutrino Program

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Joint Neutrino Theory/Expt Meeting

Outline

- Argument for LQCD
- LQCD Intro
- $F_A(Q^2)$ from LQCD
  - Axial radius
  - Electro Pion Production
- Free Nucleon XSec
- T2K/DUNE Implications

LQCD in ν Oscillation

- Next gen ν oscillation experiments have strict precision requirements. Precise predictions of xsecs for variety of interaction topologies needed. Nucleon axial form factor is first target for greatest impact.

- $F_A(Q^2)$ not constrained precisely by experiment. Experimental constraints from
  - D_2 scattering (low statistics),
  - electro pion production (hard pion model dependence),
  - large nuclear targets (nuclear model dependence).

  Ideally want elementary target expt data, but explore all methods.

- LQCD computation offers alternative constraint, complementary to expt. LQCD is well established in flavor physics. Access $F_A(Q^2)$ directly, without nuclear corrections, from systematically improvable procedure.

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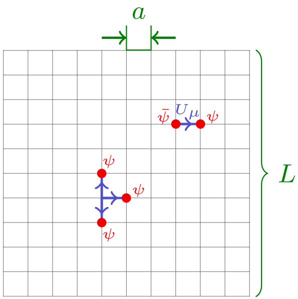
What is the current status of LQCD constraints on $F_A(Q^2)$?

Lattice QCD Formalism

Numerical eval of path integral
Quark, gluon DOFs —

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi\, \mathcal{D}\overline{\psi}\, \mathcal{D}U\, \exp(-S)\, \mathcal{O}_\psi\,[U]$$



Inputs —

Computational: $am_{(u,d),\mathrm{bare}}$
$am_{s,\mathrm{bare}}$
$\beta = 6/g^2_{\mathrm{bare}}$

Scale setting: e.g. $\frac{M_\pi}{M_\Omega}$, $\frac{M_K}{M_\Omega}$, M_Ω
1-to-1 w/ computational input

Results — first principles predictions from QCD Lagrangian, gluons to all orders

"Complete" error budget $\implies$ extrap in a, L, M_π guided by EFT, FVχPT

- $a \to 0$ (continuum limit)
- $L \to \infty$ (infinite volume limit)
- $M_\pi \to M_\pi^{\mathrm{phys}}$ (chiral limit)

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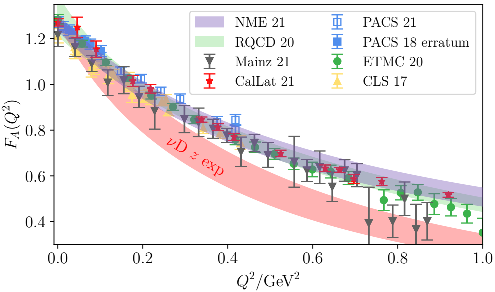
Neutrino Oscillations, Nucleon Form Factors, Lattice QCD

Abstract

Neutrino-nucleon interactions provide the dominant contribution to neutrino-nucleus cross sections, which are critical inputs to billion-dollar experimental efforts aimed at measuring neutrino oscillation parameters. The neutrino-nucleon interactions are difficult to measure experimentally and current parameterizations rely on low-statistics measurements from a handful of historic measurements to inform a number of nucleon form factors and other key quantities. Lattice QCD can be used to determine these interactions directly from the Standard Model with fully quantified theoretical uncertainties. Recent lattice QCD results of g_A are in excellent agreement with experimental data, offering hope that soon, results for the (quasi-)elastic nucleon form factors will be available. We review the status of the field and lattice QCD results for the nucleon axial form factor, $F_A(Q^2)$, a major source of uncertainty in neutrino-nucleon interaction parameterizations for $E_\nu \lesssim 1$ GeV. Results from different lattice calculations are in good agreement with each

Nucleon Axial Form Factor



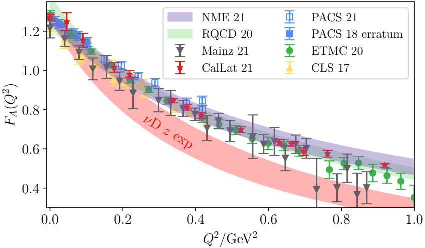
LQCD results maturing:

- Many results: independent data & different methods
- Full error budget (bands) vs. single ensemble (scatter points)
- Agreement w/ single ensemble $\implies$ uncontrolled systematics are small
- Extrapolated results (bands) satisfy GGT/PCAC checks
  Lots of recent effort to understand

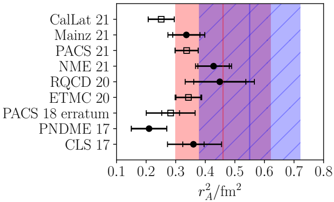
Indication of slow Q^2 falloff

Situation unlikely to change drastically

Axial Radius



Filled circle: full error budget
Open square: incomplete
elec π prod: [Phys.Rev.D 84 (2011)]
D_2 scatter: [Phys.Rev.D 93 (2016)]

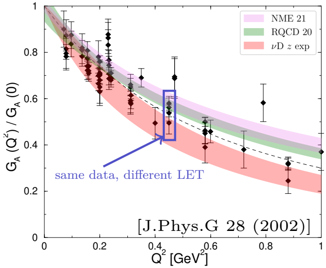


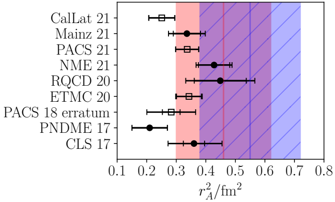
- Dipole shape fixed by $r_A^2 = -\frac{6}{g_A}\frac{dF_A}{dQ^2} \equiv \frac{12}{m^2_{A,\text{dipole}}}$
- LQCD, D_2 agree on r_A^2 (slope) at $Q^2 = 0$
- Dipole tracks with D_2 over Q^2, tension w/ LQCD at large Q^2
- If LQCD prefers dipole, must also track with D_2

LQCD (c.f. NME, RQCD) disfavors dipole ansatz

→ Important when connecting to Electro Pion Production

Electro Pion Production





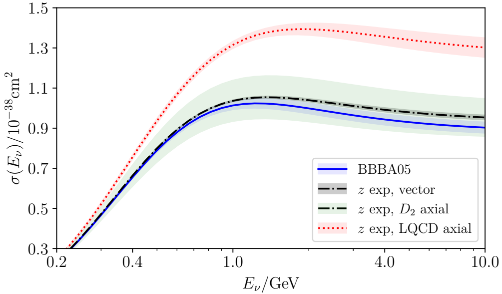
LQCD appropriately consistent with EPP data:

- EPP strictly valid in $M_\pi \to 0$, $k \sim q \to 0$ limits
- LET includes BχPT corrections to $O(M_\pi^2, k^2)$
  $\implies$ implicit dipole assumption
- BχPT corrections assume no Δ resonance?
- Bernard *et al.* EPP average $M_{A,\text{dipole}} = 1.069(16)$
  does not include uncertainty for LET variation (> stat error)

Large Q^2 could suffer from model dependence, k^2 truncation

At small Q^2, LQCD & EPP agree. No sign of tension at large Q^2

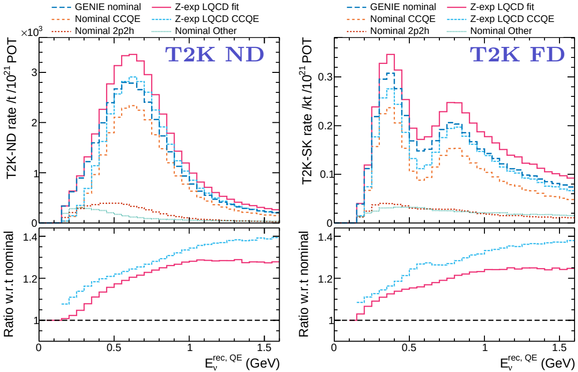
Free Nucleon Cross Section



Slow falloff integrated over Q^2 magnifies tensions:

- Uncertainty reduced $\times 2$ (red vs. green)
  $\Longrightarrow$ better precision than D_2 scattering data
- LQCD prefers 30-40% enchancement of ν_μ QE cross section
- With improved precision, sensitive to vector FF tension (black vs blue)
  [Phys.Rev.D 102 (2020)] vs [Nucl.Phys.B Proc.Suppl. 159 (2006)]

T2K/DUNE Implications



- Dashed dark blue (GENIE nominal) vs solid magenta (z exp LQCD fit)
- QE xsec enhancements produce 10-20% ν_μ event rate enhancement
- E_ν-dependent modifications to event rate
- xsec changes at ND $\neq$ *effective* xsec changes at FD: insufficient CCQE model freedom $\rightarrow$ bias in FD prediction

Risk of bias in oscilliation predictions

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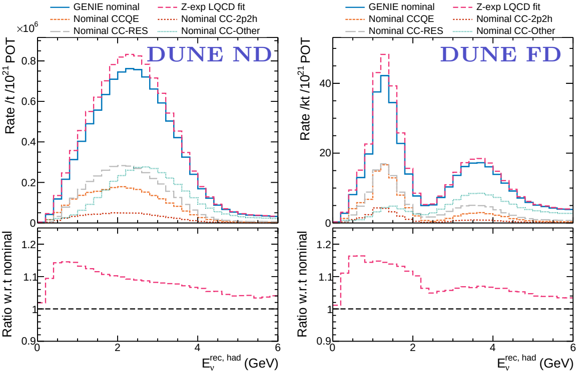

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Risk of bias in oscilliation predictions

Outlook

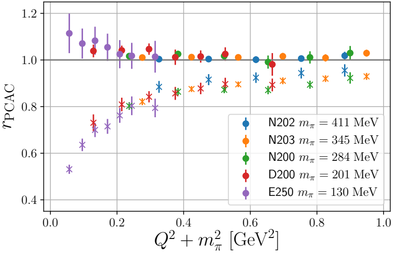
- LQCD prefers F_A with slow Q^2 falloff relative to D_2 scattering
  $\rightarrow$ consistency btw collabs w/ independent data, different methods
- LQCD disfavors dipole ansatz
- LQCD consistency with EPP fine
- Slower Q^2 falloff translates to 30-40% enhancement of QE xsec
- Implies significant changes to xsec model
  $\implies$ care needed to avoid bias

Thanks for your attention!

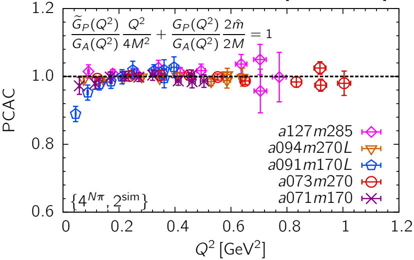
Backup

Generalized Goldberger-Triemann

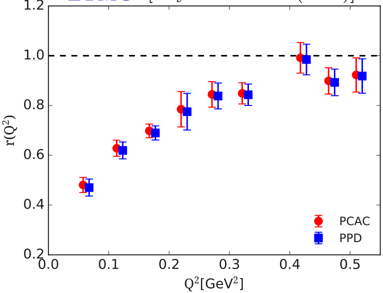
RQCD [JHEP 05 (2020)]



NME [2103.05599]

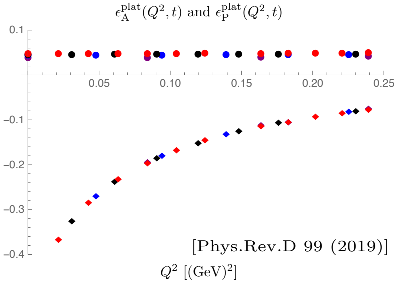


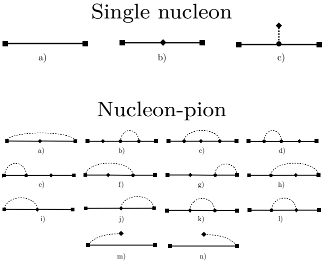
ETMC [Phys.Rev.D 103 (2021)]



- Relation btw F_A, F_P, $\tilde{F}_P$ via PCAC
- $N\pi$ excited states important
  [Phys.Rev.Lett. 124 (2020)]
- Contamination in F_A and $\tilde{F}_P$, F_P
  very different [Phys.Rev.D 99 (2019)]
  $\Longrightarrow$ nontrivial consistency check
- Two approaches:
  χPT-inspired fit function (RQCD)
  Spectrum from A_4 current (NME)

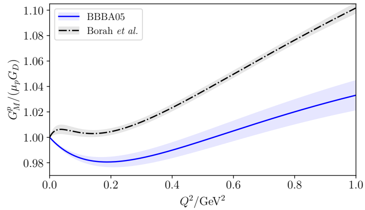
χPT Expectation





- (induced) pseudoscalar ∼ tree level (strong Q^2 dep.)
- axial ∼ loop level (mild Q^2 dep.)

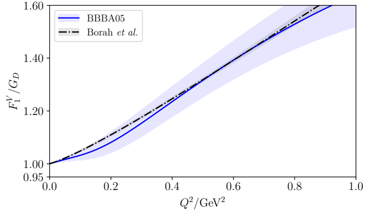
Vector Form Factors



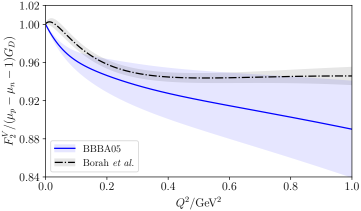
Proton Magnetic

- Tension in $G_{M,p}$ btw BBBA05 [Nucl.Phys.B Proc.Suppl. 159 (2006)] vs Borah *et al.* [Phys.Rev.D 102 (2020)]
- Borah *et al.* uses z expansion, modern data
- Mainz data included, only low Q^2

Isovector F_1



Isovector F_2



Dipole Axial Form Factor

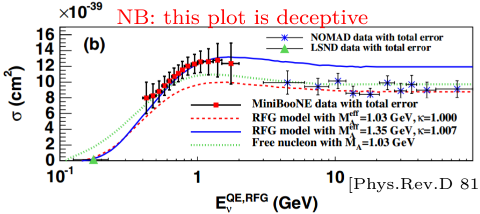
Most widely used: dipole ansatz [Phys.Rept.3 (1972)]

$$F_A^{\mathrm{dipole}}(Q^2) = g_A \left(1 + Q^2/M_A^2\right)^{-2}$$

Large variation in M_A ("axial mass problem"):

- $M_A = 1.026 \pm 0.021$ [J.Phys.G 28 (2002)]
- $M_A^{\mathrm{eff}} = 1.35 \pm 0.17$ [Phys.Rev.D 81 (2010)]

NB: this plot is deceptive



[Phys.Rev.D 81 (2010)]

- M_A^{eff}: nuclear modeling & nucleon FF entangled
- Expts: different selection criteria, sensitivity, MC model, ...

Goal: isolate nucleon FF, then address modeling

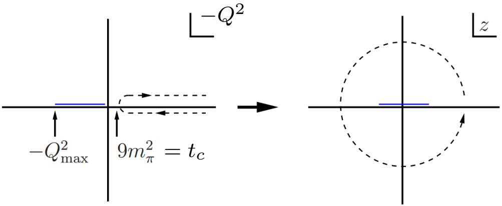
z Expansion

Want model independence, Q^2 expansion only good for $Q^2 \ll 1$

Conformal mapping: [Phys.Rev.D 84 (2011)]

$$z(-Q^2; t_0, t_c) = \frac{\sqrt{t_c + Q^2} - \sqrt{t_c - t_0}}{\sqrt{t_c + Q^2} + \sqrt{t_c - t_0}} \qquad F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad t_c = 9m_\pi^2$$

$-Q^2 \leq 0$ kinematically allowed $\quad \rightarrow \quad |z| < 1$



- Long history w/ flavor physics & CKM determination ($\leq$ 1971)
- Model independent: motivated by analyticity of QCD
- $|z|^k$, $|a_k| \rightarrow 0$ as $k \rightarrow \infty$
- Truncate at finite k_{max}, use sum rules to regulate large-Q^2

Axial Charge: $g_A(Q^2 = 0)$

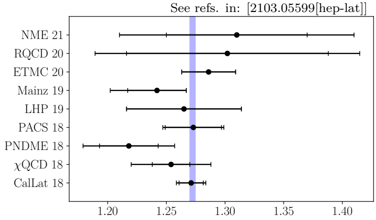
g_A is benchmark for
nucleon matrix elements in LQCD

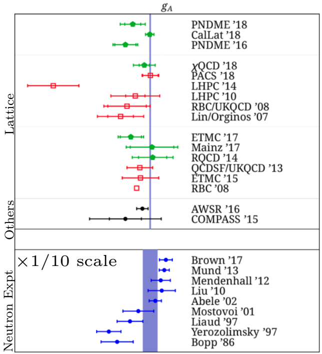
Status circa 2018 summarized by
USQCD white paper
[Eur.Phys.J.A 55 (2019)]

See also: FLAG review
[Eur.Phys.J.C 80 (2020)]

Historically g_A low compared to expt
excited states (+other...)

Lots of activity since 2018,
consistent agreement with PDG
full error budgets available





[Eur.Phys.J.A 55 (2019)]